Technological Rivalry and Optimal Dynamic Policy in an Open Economy

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Abstract

In the context of technological competition and international trade, a country may attempt to influence a rival's innovation efforts and use trade and innovation policies to gain at another's expense. In a multi-country, multi-sector, dynamic model with endogenous technology accumulation through R&D innovation, we show that there is an additional incentive (beyond conventional terms of trade considerations) for Home to shift its demand for particular foreign goods and in turn affect foreign's innovation efforts. We derive explicit expressions for optimal policies under an efficient baseline case, and general results for a wide range of specifications. In a dynamic setting, Ramsey optimal policies do not distort domestic R&D efforts if a country can commit to a schedule of trade policies, but time consistent policies employ both innovation and trade policies to implement the optimal foreign allocation, viewed from the Home country's perspective.

Keywords: Endogenous Technology, Innovation, Trade, Optimal Policies

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1 Introduction

Trade disputes are often not about trade, but about technological rivalry. When Japan started to build up its semi-conductor industry in the 1970s from a barren land, eventually grabbing 80% of the market share up from nil, it caused alarm in the former leader of the industry–the U.S. The U.S. then started to impose a set of protectionist measures, along with massive subsidies in the domestic semiconductor sectors. The outcome was a renewal of its competitiveness. Another example is the more recent U.S. China competition. When China started to produce and export solar panels en masse, some U.S. policymakers believed that rather than just importing solar panels from China, it should strengthen its own solar panel industry and subsidize its innovation. In both of these examples, countries did not passively accept changes to foreign competition but deployed active policies as a response.

Throughout history, we have seen how one country's gain in market share in leading-edge sectors has spurred national competition, protectionism, sometimes under the guise of trade wars. That was the case when Germany caught up with Great Britain in the 19th century, and when European countries such as France and Germany were catching up with U.S. innovation in the 1960s. The same is happening as Chinese technology is rapidly catching up, in some cases outstripping, U.S. technology. One of the core disputes in the 2018 U.S-China trade war was over technology, an event that led to the U.S. banning Huawei and other Chinese companies from accessing critical supplies in the U.S.. Increasingly, as the U.S. restricts more and more Chinese companies, domestic demand and procurement for technology and equipment have soared inside China, accelerating their own development.

The spirit of contest is captivatingly explored in Samuelson (2004), which argues that developing countries' technological advancement can sometimes harm the interests of advanced economies, by becoming more productive in sectors in which rich countries have a comparative advantage (or conversely, benefit them if rapid productivity growth happens in developing countries' comparative advantage sectors). This simple but powerful argument, however, does not consider the option that a country can influence innovation efforts of its chief rival. To re-evaluate Samuelson's thesis when technology endogenously evolves, and countries have a set of instruments to influence trade and technology dynamics, we theoretically characterize optimal taxation in a workhorse Ricardian model with endogenous technology. We propose a new mechanism whereby countries employ trade and industrial policies to manipulate innovation incentives of *other* countries. This motivation goes beyond what conventionally animates trade and innovation policies. Classic industrial policies justify subsidizing or protecting some sectors at the expense of other sectors on the basis of externalities and spillovers. Classic trade policies stipulate manipulating a country's terms of trade (raising its price of production relative to that of consumption) so as to benefit domestic consumers at the expense of foreign ones. Specifically, by imposing a country-wide tariff on the foreign country, a large economy can reduce the demand for foreign goods and in turn suppress wages abroad—making foreign imports cheaper. But in these models, technology is invariably taken as given. In light of historical and recent events of international technology competition and trade disputes, understanding the sources of technological differences across countries and relevant policies that can impact them may be important.

The framework we propose to study optimal innovation and trade policy is a dynamic multi-country, multi-sector model with comparative advantage based trade and endogenous technology accumulation through R&D and innovation. Specifically, our model examines optimal taxation in the workhorse model of Eaton and Kortum (2001), which is expanded along several important dimensions. The economy's government is benevolent and can choose a set of heterogeneous domestic taxes on R&D, as well as differential trade policies across sectors and trading partners. When choosing these policies, the government internalizes its choices on trade and technology development in its own country as well as in others. Other economies' government is taken to be passive.

To isolate the new mechanism underpinning optimal policy, we first consider a baseline economy that is efficient. Productivity differences across sectors shape comparative advantage, which determines trade across countries. There are constant returns to scale in production and innovation, and a free flow of labor between research and production. The model features Bertrand competition between producers for each goods, where each producer competes with all other producers in the world. Although the realized markup of each firm follows a distribution, the aggregate endogenous distribution of markups is

Pareto, and the expected profit of a firm is a constant share of sales in the industry. Thus, there are constant and identical aggregate markups in each sector. This set of assumptions is standard in the trade literature, and is convenient as it permits an explicit characterization of optimal policy under certain conditions. In addition, there are no taxes, distortions, or externalities in the baseline case.

In a closed economy, the planner would choose the same allocation as the market. In addition, openness itself does not affect the level of private innovation: the increased foreign competition effect that stimulates innovation is exactly offset by the larger foreign market effect that tends to reduce innovation effort. But in the open economy environment, there is room for optimal policy: Home would want to impose a higher tariff in sectors that see larger net exports, and increase that tariff when net exports increase in that sector (for example, due to openness or a rise in global demand). By reducing the demand for foreign goods in those sectors, these tariffs can affect foreigners' research efforts in the same sectors. Importantly, there is scope for policy even when the private equilibrium is *efficient*.

To demonstrate the crucial role that endogenous technology plays, we show in our baseline model that if technology is fixed and if the Home country can freely choose trade policies without facing retaliation, Home would exercise monopoly power to affect relative prices to its own benefit. Specifically, it would opt for a higher export tax in sectors with greater comparative advantage (or a higher subsidy in the comparative dis-advantage sectors), so as to restrict that sector's exports and enjoy a terms of trade improvement. The prescription of 'heterogeneous' trade policy across sectors makes it different from classic optimal trade policy which deploys a uniform import tariff to manipulate a country's terms of trade through its general equilibrium effect on foreign wages. Recent works such as Itoh, Itoh and Kiyono (1987) and the recent work of Costinot, Donaldson, Vogel, and Werning (2015) make the important point that when there are multiple sectors, there is further scope to manipulate the terms of trade by raising specifically the prices of its comparative advantage goods relative to other good.[1] Import tariffs are avoided as they affect foreign prices only through a reduction in wages, which a uniform export tax can achieve—thus obviating the need to further distort Home consumption prices. The Home government would

[1] Costinot et al. (2015) shows that export tax relates to foreign demand elasticity.

use solely heterogeneous export taxes/subsidies to improve welfare.[2]

But when technology is endogenous, and a full set of import tariffs is available, heterogeneous import tariffs become useful in reducing demand for corresponding foreign goods and hence curb its incentives to innovate in these sectors. This motivation is different from those explored in the past—-tariffs to affect foreign wages (classic trade policy) or relative prices (Costinot et al. (2015)). The latter two still constitute a part of optimal policies, but can be implemented with heterogeneous import tariffs in conjunction with sector level export taxes.

A recent example is in order: in the context of US and China competition, our new mechanism rationalizes why the U.S. would want to discourage innovation in China's semiconductor industry if the US and China are both net exporters of semiconductor to the rest of the world. If, say, there is a rise in the global demand for semiconductors, and consequentially a rise in the net export of semiconductors from both countries, then, assuming the other is passive, both would like to increase import tariffs on each other in the sector so as to discourage further innovation from its competitor. China would also impose a higher tariff on the rest of the world's textiles (the 'other' sector) so as to induce US to shift innovation efforts into textiles and away from semiconductors—and conversely, the U.S. would do the same to induce China to shift labor into textiles away from semiconductors.

The preceding discussions revolve around the case when sectors are not inherently different. But when they are subject to varying increasing returns or scale, or have spillovers or other distortions, there is a gap between the private and social return for R&D. Under exogenous technology, industrial policies can correct for domestic wedges/inefficiency, and heterogeneous export taxes is used to manipulate terms of trade. This is shown in Bartelme, Costinot, Donaldson, and Rodriguez-Clare (2019) and Lashkaripour and Lugovskyy (2016), which feature increasing returns to scale to production in an open economy. In both papers, the model is static, with no investment decisions that can influence technol-

[2] Trade policy under fixed technology in a partial equilibrium setting is explored in Gros (1987), and Broda, Limao, and Weinstein (2008), which show that industry tariff is related to the foreign export supply elasticity. Demidova and Rodriguez-Clare (2009) characterize optimal tariffs in a single industry, Melitz-Pareto setting. Trade policy analyzed in quantitative or new trade theories include Costinot and Rodriguez-Clare (2014); Caliendo, Feenstra, Romalis, and Taylor (2015); Demidova (2017); Beshkar and Lashkaripour (2019), Costinot, Rodríguez-Clare, and Werning (2020) characterizes optimal firm-level trade policy in a single-sector two-country Melitz model. Ossa (2014).

ogy.[3] Our generalized framework can also encapsulate the set of features such as imperfect competition, knowledge spillover, congestion externalities or creative destruction, and nest our baseline model as a special case. Under the general setup, optimal policy depends on the specification of these features, as well as on the gap between the private and social choice of R&D generated. The main mechanism of affecting foreign innovation incentives still stands as a key motive for intervention.

Our technical contribution is two-fold. First, we theoretically characterize optimal policy and derive general results in a framework with elemental features, while deriving explicit formulas under certain special cases. This is different from the numerical approaches to computing optimal policy under a set of calibrated parameters that captures a particular environment at a moment in time. Second, we solve a dynamic model which is crucially different from the static model explored in the past literature. We show that Ramsey optimal policies do not distort domestic R&D efforts if Home can commit to a schedule of trade policies. That is, being able to commit to today's and tomorrow's trade policies (a path of import tariffs and export taxes) would be sufficient to implement the optimal foreign allocation without needing to distort its own investment. But time consistent policies employ both innovation and trade policies for the reason that Home would like to see its innovation and trade policies deviate from the optimal Ramsey ones in subsequent periods.

Lastly, this paper sidesteps from important issues such as the gains to trade, on which there is already a large and expansive literature. Papers abound on the topic of international technology diffusion in the global economy, but few consider optimal policy in these settings.[4] The paper is related to, but has little overlap with growth theories that have emphasized the importance of R&D on long run growth. Optimal policies in these contexts depend on assumptions of each theory—featuring either imperfect competition pricing, knowledge spillovers, congestion externalities, or creative destruction. Akcigit, Ates, and Impullitti (2019) explore dynamic policies with these features in an open economy, but

[3] Bartelme et al. (2019) characterize optimal policy for a small open economy in a multi-sector Ricardian model with Marshallian externalities. Lashkaripour and Lugovskyy (2016) study optimal industry and trade policy with scale economies.

[4] Innovation and international technology diffusion in the global economy include works such as Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018), Bloom, Draca, and Van Reenen (2016), Cai, Li, and Santacreu (2022), Eaton and Kortum (1999), Grossman and Helpman (1990), Hsieh, Klenow, and Nath (2019), Perla, Tonetti, and Waugh (2021), and Somale (2021).

the framework is a one sector model and does not have comparative advantage aspects to trade (and hence heterogeneous tariffs) that are essential to our framework. Liu and Ma (2021) examines optimal policy when there are international spillovers and externalities. In both of these papers, it is the presence of externalities, spillovers or distortions that justify interventionist policies, whereas optimal policies in our setting arises from comparative advantage and exist even in a frictionless setting.

2 Theoretical Framework

2.1 Model

The model extends the endogenous technology model in Eaton and Kortum (2001), henceforward EK2001, to one that features multiple sectors and countries, and derive optimal R&D and trade policies therein. We then extend the model to allow for various externalities and international spillover.

The world has many countries and sectors. Within a sector, there is a continuum of varieties of consumption goods. Preferences for consumers in each country is Cobb-Douglas both across sectors and across varieties. All consumers' discount factor is β. Country $n \in N$ has a measure L_n of labor, which can freely flow into the production sector as a worker or the research sector as an innovator.

Consumer preference in each country n is $\sum_{t=0}^{\infty} \beta^t \frac{C_{nt}^{1-\sigma}}{1-\sigma}$, where final goods is a Cobb-Douglas function across the consumption of different sector j goods $C_n = \Pi_{j \in N_s} \left(C_n^j \right)^{\beta_j}$, where β_j is constant and reflects the share of sector j. Within each sector, consumption is also aggregated with a Cobb-Douglas function across individual varieties $C_n^j = \exp \int_0^1 \ln c_n^j(\omega) d\omega$. All goods are tradable with an iceberg trade cost d_{nm} between country n and m.

Innovation incentive and research decision. We start by explaining innovation efforts within each sector, as in the one-sector economy model of Eaton and Kortum (2001). All countries n are capable of producing any variety ω of good with technology $z_n(\omega)$ (where industry j is suppressed for notational convenience), the distribution of which is endogenous and depends on the number of researchers and research productivity.

Researchers draw ideas about how to produce goods. At a Poisson rate α_n, a researcher in country n draws an idea, which consists of the realization of two random variables. One is the good ω to which the idea applies, drawn from the uniform distribution over $[0, 1]$. The other is the efficiency $q(\omega)$, drawn from a Pareto distribution with a parameter θ.

Let the measure of researchers in country n at t be L_{nrt}, and the cumulative stock of ideas be T_{nt}. Under a unit interval of varieties, the number of ideas for producing a specific good is Poisson distributed with parameter T_{nt}. Ideas retire with probability δ and hence the evolution of the stock of ideas T_{nt} is:

$$T_{nt} = (1 - \delta)T_{n,t-1} + \alpha_{nt}L_{nrt}. \tag{1}$$

Kortum (1997) proves that when the quality of each idea is Pareto distributed, the distribution of technology efficiency frontier is a Frechet distribution with parameter T_{nt} and θ.

Firms engage in Bertrand competition: the lowest-cost producer of each good in each market claims the entire market for that good, charging a markup just enough to keep the second-lowest-cost producer out of the market. In equilibrium, the distribution of the markup is Pareto with the parameter θ. Since all firms selling in the market charge a markup drawn from the same distribution, total profits $prof_{nt}$ at period t earned by firms in the market are a constant share of total sales. Let x_{mt} denote market m's total spending at t (also expenditure per variety in country m given Cobb-Douglas preferences). Thus profits earned by either domestic or foreign firms who sell in that market is $x_{mt}/(1 + \theta)$. The probability that a researcher in n draws a q that is a the highest in market m at t is π_{mnt}/T_{nt} (proof in Appendix): a firm innovates and surpasses the current set of ideas with probability $1/T_{nt}$ at time t, but then needs to be the cheapest source of a particular good in in country mt, with probability π_{mnt}. In that case, total global profits earned by n at time t is:

$$prof_{nt} = \sum_m \frac{\pi_{mnt}}{T_{nt}} \frac{x_m}{1+\theta} = \frac{1}{1+\theta} \frac{x_{nt}}{T_{nt}} = \frac{1}{\theta} \frac{w_{nt} L_{npt}}{T_{nt}}.$$

The second equality uses the balanced trade condition, $\sum_m \pi_{mnt} x_{mt} = x_{nt}$, while the third is obtained using the fact that on expectation a constant fraction of sales goes to profit while the remaining goes to labor income paid to production workers.

We can write the expected discounted value of an idea as

$$v_{nt} = \sum_{s=t}^{\infty} [\beta(1-\delta)]^{s-t} \frac{u'_{ns}}{u'_{nt}} \frac{P_{nt}}{P_{ns}} prof_{ns}. \tag{2}$$

where u_{ns} is country n's marginal utility of consumption at period s and P_{ns} is the consumer price.

A researcher is motivated by the possibility of coming up with an idea with value. Free mobility across sectors ensures that the present value of the expected profits of being a researcher is equal to the wage of being a worker in the production sector w, i.e. $\alpha_{nt} v_{nt} = w_{nt}$. This determines the level of R&D conducted. Workers engaged in research do not know how good their ideas will be ex-ante. Since each idea is worth v_{nt} in expectations, the total value of research output at time t is $\alpha_{nt} L_{nrt} v_{nt}$. The average value of a researcher is $\alpha_{nt} v_{nt}$. Total number of research workers is $L_{nrt} = r_{nt} L_{nt}$, where r_{nt} is the equilibrium share of research workers—or, research intensity. Thus,

$$\alpha_{nt} v_{nt} = w_{nt} \quad r_{nt} \in [0,1] \tag{3}$$

$$\alpha_{nt} v_{nt} < w_{nt} \quad r_{nt} = 0 \tag{4}$$

$$\alpha_{nt} v_{nt} > w_{nt} \quad r_{nt} = 1. \tag{5}$$

The poisson rate α_{nt} reflects how effective the researchers are in country n's innovation process—-or, innovation efficiency. Innovation can exhibit CRS, that is, $\alpha_{nt} = \alpha_n$, or have domestic externality, where $\alpha_{nt} = \alpha_n (L_{nrt})^{\epsilon-1} (T_{n,t-1})^{\eta}$, including potential DRS ($\epsilon < 1$), and intertemporal diffusion $\eta \neq 0$; or foreign externality/diffusion, where $\alpha_{nt} = \alpha_n (L_{nrt})^{\epsilon-1} (T_{j,t-1})^{\eta}$. The main mechanism, however, is independent of these assumptions.

Multi-country, multi-sector trade and endogenous technology.

The equilibrium is an allocation $\{L^j_{nr}, L^j_{np}, C^j_n\}$, $\{T^j_n,\}$with prices $\{P^j_n,\}$ and wages $\{w_n\}$ such that consumer maximize expected discounted utility and firms maximize expected discounted profits. Summarized below, there is also the free entry for researchers, evolution of technology through innovation, and the clearing of the goods and labor market, along with balanced budgets for the government:

1. Free entry conditions for researchers

$$w_n = \alpha^j_n(L^j_{nr}, T^j_{n,-1}) \left(\frac{1}{\theta} \frac{w_n L^j_{np}}{T^j_n} + E\tilde{\beta}_n v^j_{n,+1} \right), \quad (\forall n \in N, \forall j \in N_s)$$

where $\tilde{\beta_{nt}} = \beta(1-\delta)\frac{u'_{n,t+1}P_{nt}}{u'_{nt}P_{n,t+1}}$

2. Evolution of technology

$$T^j_n = \alpha^j_n(L^j_{nr}, T^j_{n,-1})L^j_{nr} + (1-\delta)T^j_{n,-1}, \quad (\forall n \in N, \forall j \in N_s)$$

3. Goods market clearing conditions

$$x_n = \frac{1+\theta}{\theta} w_n \sum_j L^j_{np} = \sum_j \beta_j \left[\sum_m \pi^j_{mn} x_m \right]$$

4. The labor market clearing conditions

$$\sum_j \left(L^j_{nr} + L^j_{np} \right) = L_n.$$

5. Normalizing Home country's (country 1) price index to 1:

$$P_1 = \Pi_j \left[T^j_1 w_1^{-\theta} + \sum_{n \neq 1} T^j_n \left(w_n d_{1n} \right)^{-\theta} \right]^{-\frac{\beta_j}{\theta}} = 1$$

The private equilibrium derived from the above conditions satisfy:

Proposition 1. *At the steady state of the multiple-sector open economy, the market private research intensity r^j_n is the same as in the closed economy for all sectors j in country n. Openness reallocates* *more labor into the comparative-advantage sectors and increases the endogenous level of technology in these sectors.*

The optimal research intensity r^j_{nt} in each sector j does not depend on a country's size, research productivity, or trade openness. The intuition is that accessing foreign markets increases the potential profits, but competition from foreign inventions decreases them. These two effects exactly cancel out, and the level of openness does not affect research intensity. Openness increases profits—thus, given the same level of research intensity r^j_n, more labor reallocated to the comparative advantage sector increases the total amount of researchers in that sector and hence its technology T^j_n.

2.2 Domestic optimal trade and innovation policies

The Home government (country 1) chooses the optimal unilateral trade policies and domestic $R\&D$ policies by maximizing the aggregate of individuals' instantaneous utilities discounted by β. Foreigners are taken to be passive. Trade policy instruments are restricted to the country-industry level, comprising country-sector-specific import tariffs t^j_n and export taxes τ^j_{xn} directed at country $n \neq 1$. The government rebates the tax income to households in a lump-sum fashion. Domestic $R\&D$ policies are sector-specific innovation tax/subsidy. We first derive optimal domestic innovation captured by L^j_{1r}, before showing how to implement it with taxes. Thus, government policy constitutes $\{t^j_{n\neq 1}, \tau^j_{xn\neq 1}, L^j_{1r}\}$.

The Home government determines researchers and workers in each sector j, taking into account foreign private innovation decision and equilibrium production and trade. Specifically, the government chooses L^j_{1r}, L^j_{1p} with $j \in \{1, 2, .., N_s\}$, country-sector-specific import tariff t^j_n and export taxes τ^j_{xn} toward country $n \neq 1$ to solve the following problem:

$$V\left(\left\{T^j_{n,-1}\right\}\right) = \max_{\left\{L^j_{1r}, \tau^j_{xn}, t^j_n\right\}} \frac{x_1^{1-\sigma}}{1-\sigma} + \beta E\left[V\left(\left\{T^j_n\right\}\right)\right]$$

Subject to

$$\frac{w_n}{\alpha^j_n(L^j_{nr}, T^j_{n,-1})} = \frac{1}{\theta}\frac{w_n L^j_{np}}{T^j_n} + E\tilde{\beta}_n v'^j_n, \qquad (\gamma^j_{rn}, \quad N_s \times (N-1)) \tag{6}$$

$$T_n^j = \alpha_n^j(L_{nr}^j, T_{n,-1}^j)L_{nr}^j + (1-\delta)T_{n,-1}^j, \quad (\gamma_{Tn}^j, \quad N_s \times N) \tag{7}$$

$$\frac{1+\theta}{\theta} w_1 L_{1p}^j = \beta_j \left[\pi_{11}^j x_1 + \sum_{m\neq 1} \frac{1}{1+\tau_{xm}^j}\pi_{m1}^j x_m\right], \quad (\gamma_{L1}^j, \quad N_s) \tag{8}$$

$$\frac{1+\theta}{\theta} w_n L_{np}^j = \beta_j \left[\frac{1}{1+t_n^j}\pi_{1n}^j x_1 + \sum_{m\neq 1} \pi_{m,n}^j x_m\right] \quad (\gamma_{Ln}^j \quad N_s \times (N-1)) \tag{9}$$

$$\sum_j \left(L_{nr}^j + L_{np}^j\right) = L_n, \quad (\mu_n, \quad N)$$

$$x_1 = \frac{1+\theta}{\theta} w_1 \sum_j L_{1p}^j + \sum_{m\neq 1}^{N}\sum_{j=1}^{N_s} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j x_m + \sum_{m\neq 1}^{N}\sum_{j=1}^{N_s} \beta_j \frac{t_m^j}{1+t_m^j}\pi_{1m}^j x_1, \quad (\gamma_4)$$

$$P_1 = \Pi_j \left[T_1^j w_1^{-\theta} + \sum_{n\neq 1} T_n^j (w_n(1+t_n^j)d_{1n})^{-\theta}\right]^{-\frac{\beta_j}{\theta}} = 1 \quad (\gamma_P)$$

where

$$x_m = \frac{1+\theta}{\theta} w_m \sum_j L_{mp}^j$$

$$P_m = \Pi_j \left[T_1^j (w_1(1+\tau_{xm}^j)d_{m1})^{-\theta} + \sum_{n\neq 1} T_n^j (w_n d_{mn})^{-\theta}\right]^{-\frac{\beta_j}{\theta}}$$

$$\pi_{11}^j = \frac{T_1^j (w_1)^{-\theta}}{T_1^j (w_1)^{-\theta} + \sum_{n\neq 1} T_n^j (w_n(1+t_n^j)d_{1n})^{-\theta}}$$

$$\pi_{m1}^j = \frac{T_1^j (w_1(1+\tau_{xm}^j)d_{m1})^{-\theta}}{T_1^j (w_1(1+\tau_{xm}^j)d_{m1})^{-\theta} + \sum_{n\neq 1} T_n^j (w_n d_{mn})^{-\theta}}$$

$$\pi_{mn}^j = \frac{T_n^j (w_n d_{mn})^{-\theta}}{T_1^j (w_1(1+\tau_{xm}^j)d_{m1})^{-\theta} + T_n^j (w_n d_{mn})^{-\theta} + \sum_{i\neq \{m,n\}} T_i^j (w_i d_{mi})^{-\theta}}$$

$$\pi_{1m}^j = \frac{T_m^j (w_m(1+t_m^j)d_{1m})^{-\theta}}{T_1^j (w_1)^{-\theta} + T_m^j (w_m(1+t_m^j)d_{1m})^{-\theta} + \sum_{n\neq \{1,m\}} T_n^j (w_n(1+t_n^j)d_{1n})^{-\theta}}$$

Note that $\sum_n \pi_{m,n}^j = 1$ for any m, and without loss of generality, we assume that countries other than 1 (Home) do not impose tariffs on each other as reflected in Eq. 9.

2.3 Theoretical Comparative Static: Baseline model

We first consider a baseline model where the private equilibrium is efficient: innovation exhibits CRS, i.e., $\alpha_{nt} = \alpha_n$, and the result is an efficient allocation of researcher and workers. Under Bertand competition, endogenous markup of each firms follows a distribution but is invariant with time and destination to which the firm sells; there are constant and identical aggregate markups in each sector. Note that in the baseline model there are no other distortions, externalities or international spillovers.

In what follows, we compare long-run optimal policies when technology T_n for $n \neq 1$ is fixed and when it is endogenous. The following two propositions describe these optimal policies:

Proposition 2. *If tax instruments include sector-specific import tariff and export tax:*

1. *Under exogenous technology for each sector in each country, the optimal trade policies should have a uniform import tariff, along with an export tax that rises with a sector's degree of comparative advantage (or a subsidy that rises with a sector's degree of comparative disadvantage).*
2. *Openness affects the optimal trade policies: with lower trade cost, Home charges a higher tax on the comparative advantage sectors or a higher subsidy on the comparative disadvantage sector, i.e., Home allows for a greater differentiation of taxes across sectors.*

Proof: in Appendix. When T is exogenous, we have

$$t_n^j = \bar{t}, \qquad (1+\tau_{xn}^j)(1+\bar{t}) = \frac{1+\theta(1-\pi_{n1}^j)}{\theta(1-\pi_{n1}^j)},$$

where Home is denoted as country 1. Tariffs are uniform across sectors, and export taxes exploit the country's monopoly power. The levels of import tariffs are not pinned down, and thus, letting $t = 0$ for all sectors, we can see that export taxes τ_{xn}^j are rising with π_{n1}^j, the share of good j that country n imports from Home. That is, the higher the net exports (or comparative advantage) of a sector, the higher the sector-specific export tax. This schedule of trade policies is the same as Costinot et al. (2015): the government can manipulate relative prices in its favor by restricting the supply of its export goods.

Proposition 3. *When technology is endogenous and the private equilibrium efficient, and when tax instruments include country-sector-specific import tariffs, export taxes, and domestic innovation taxes, then in the steady state:*

1. *optimal policies do not distort domestic innovation, but consist of heterogenous import tariffs and export taxes across countries and sectors*
2. *tariffs are higher for sectors with relative higher net exports*
3. *a rise in openness or technology affect optimal policies: sectors that have relatively higher net exports raise tariffs by more*

When T is endogenous, Appendix D proves that

$$1+\tau_{xn}^{j}=\frac{1+\theta(1-\pi_{n1}^{j})}{\theta\sum_{m\neq1}(1+t_{m}^{j})\pi_{nm}^{j}},$$

and

$$t_{n}^{j}=-\gamma_{Ln}^{j}\frac{1}{u_{c}}=-\gamma_{rn}^{j}\frac{1}{(1+\theta)T_{n}^{j}u_{1}}+Const_{n}.$$

where γ_{rn}^{j} is the multiplier on the researcher free entry condition (6), γ_{Ln}^{j} multiplier on the private optimal labor choice (9), for $n>1$ and $j\in[1,N_s]$, and $Const_n$ is the same across all sectors in country n.

Tariffs are now country-sector specific. The second equation shows that they are related to the multiplier γ_{rn}^{j}, which reflects how Foreign's innovation affects Home's welfare, as well as to the multiplier γ_{Ln}^{j}, which affects the excess demand of a foreign good. Home would like to use tariffs to affect foreign's choice of labor and production across sectors. The first equation shows that the government will still want to use export taxes to exploit the country's monopoly power at the steady state, as in the case with exogenous technology—but in conjunction to heterogenous import tariffs across sectors aimed at influencing Foreign's innovation. Moreover, in the multi-country case, these taxes depend on how much other countries export to country n (π_{mn}), capturing the extent of monopoly power Home has over country n in sector j.

As we have seen—in a closed economy with multiple sectors and endogenous technology, the Home government will choose the same R&D intensity as in the private equilibrium, as it is efficient. In the open economy, however, the domestic government would like to use tariffs to influence Foreign's innovation. The sign of γ^j_{rn} determines whether Home will impose a tariff on country n in sector j:

Proposition 4. *1. For two countries*

$$t^j = \frac{1}{w_2(1-\bar{r}_2)(1+\theta)}\left(\frac{\beta_j(\frac{1}{1+\tau^j_x}\pi^j_{21}x_2 - \pi^j_{12}x_1)}{L^j_2} - \frac{\beta_0(\frac{1}{1+\tau^0_x}\pi^0_{21}x_2 - \pi^0_{12}x_1)}{L^0_2}\right),$$

where $\bar{r}_2$ is the steady state share of researchers in country 2. Home (country 1) imposes a higher import tariff on its own sectors with higher net exports (relative to sector 0)

2. For multiple countries, the special case that only Home and one foreign country (country 2) produce sector j goods, e.g. rest of the world $\alpha^j = 0$, yields:

$$t^j_2 - t^0_2 = \frac{1}{w_2(1-\bar{r}_2)(1+\theta)}\left(\frac{\beta_j(\sum_m \frac{1}{1+\tau^j_{xm}}\pi^j_{m1}x_m - \pi^j_{12}x_1)}{L^j_2} - \frac{\beta_0(\sum_m \frac{1}{1+\tau^0_{xm}}\pi^0_{m1}x_m - \sum_n \pi^0_{1n}x_1)}{L^0_2}\right)$$

Home imposes a higher import tariff on country 2's sectors with higher net exports (relative to sector 0).

Proof in Appendix E. From the expressions above, the tariff Home imposes on sector j is related to j's net exports, which is by definition $\pi^j_{21}x_2 - \pi^j_{12}x_1$ in the two-country case, and analogously in the multi-country case.

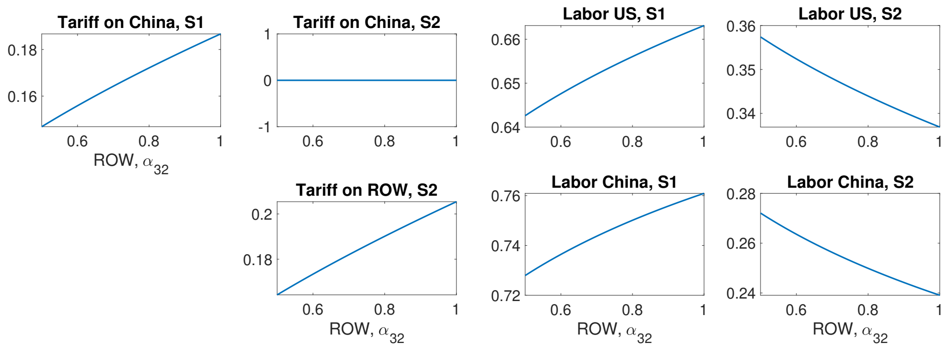
To give some intuition behind these results, we consider a numerical example of three countries in the steady state: US, China, and ROW (rest of the world). The first example shows what happens when there is a rise in global demand of sector 1 goods, which is the U.S.' comparative advantage sector, and China's comparative disadvantage sector, determined by the fact that innovation efficiency is US $(1, 0.9)$, CN $(0.9, 1)$, ROW $(0, 0.5 \sim 1)$. However, both countries are net exporters of sector 1 to the rest of the world as ROW cannot produce sector 1 goods. Figure 1 shows US tariffs on Chinese goods: tariffs imposed on sector 1 goods (relative to tariffs on sector 2 goods) rises with ROW demand. The U.S. also imposes a relatively higher tariff on ROW in sector 2 so as to induce Chinese labor to flow into sector 2. But over all, this force is not strong enough to induce Chinese labor in sector 2 to rise, but acts to dampen the flow of labor into sector 1. The same optimal policy applies symmetrically to China levying a higher tariff on the US in sector 1. These motives are driven by the fact that there are more profits to be gained as world demand increases; and the larger is the rise in global demand—the larger are the tariffs levied for the purpose of endogenously improving one's own technology in sector 1.

Next consider the case where China's innovation efficiency α_1^1 rises, and takes on the value between 0.9 and 1. Figure 3 shows that China imposes a tariff on US's sector 1, and would levy a relatively higher tariff on the U.S. in this sector as its own innovation efficiency rises, so as to discourage innovation in the U.S.. It would levy a higher tariff on ROW's sector 2 so as to induce U.S. labor to flow towards sector 2. Overall, this results in U.S. labor (and hence innovation) falling in sector 1.

We have shown examples where a rise in net exports induces an import tariff increase. But there are other more subtle points to be made when T is endogenous and optimal policies across sectors and countries are jointly determined. First, the larger the comparative advantage, the larger the motive to impose tariffs on one another. For instance, as the overall trade costs fall, there is more trade between all countries, and hence there is a larger incentive to undertake policies. Also, when there are only two countries, if China becomes more and more like the U.S., there is actually less room to impose tariffs, as they trade less (of courses, the presence of other countries makes it different, as has been illustrated in the previous example.) Second, for example, if sector 1 is China/U.S.'s comparative advantage sector to the world, it will sell more of this good to the rest of the world, including to US/China. When itself becomes more productive or trade cost decrease, it will raise tariffs on the other country's sector 1 to discourage innovation there, and also raise tariffs on ROW in sector 2 to induce the other's labor to flow to sector 2. But if the main competitier becomes more productive, it's action depend on if the sector's net export increase or not. If trade cost reduction or U.S innovation in sector 1 improves due to surging demand, China will levy lower tariffs on the U.S as it is efficient to import more good 1 from the U.S.

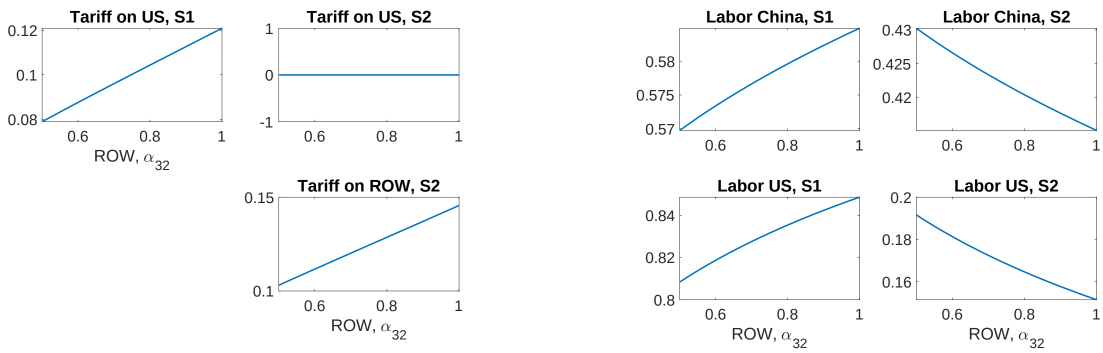
Optimal Innovation Policies without Tariff. It is possible that a full set of country-sector specific tariffs are not available to countries, for instance, due to WTO rules. In this

Figure 1: Optimal Policies for U.S. when Global Demand for S1 Rises



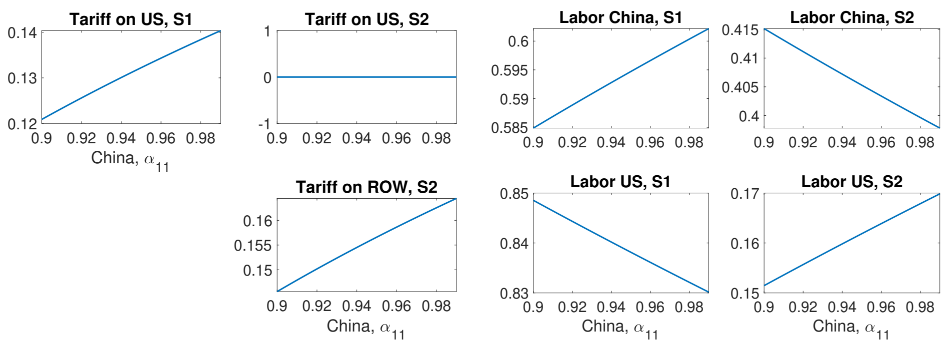
(a) Tariff (b) Labor Allocation

Figure 2: Optimal Policies for China when Global Demand for S1 Rises



(a) Tariff (b) Labor Allocation

Figure 3: Optimal Policies for China when S1 Efficiency Rises



(a) Tariff (b) Labor Allocation

case, Home would resort to subsidizing/taxing some its own industries' innovation efforts.

In the two country case, Home would implement heterogenous export tax, and

$$[1-\beta(1-\delta)]\frac{w_1}{\alpha_1^j} = \frac{1}{\theta}\frac{w_1(1-r_1^j)L_1^j}{T_1^j} + \frac{1}{1+\theta}\frac{\beta_j}{T_1^j}\frac{\pi_{11}^j x_1}{\gamma_4-\gamma_{L1}}\left[\left(\gamma_{L1}-\gamma_{L2}^j\right)\theta\pi_{12}^j + \sum_j(\gamma_{L1}-\gamma_{L2}^j)\beta_j\pi_{12}^j\right]$$

The last two terms (in blue) represent wedges on Home's innovation. The wedges across sectors average to zero. The sign of $\left(\gamma_{L1}-\gamma_{L2}^j\right)\theta\pi_{12}^j$ determines which sector's R&D should be subsidized. It says that the government should subsidize innovation efforts in sectors that have higher net exports. See Appendix F for proofs. This is inferior to country-specific import tariffs because it distorts Home's own innovation efforts, and cannot target specific countries' innovation efforts in particular sectors. Affecting its own innovation would also impact Foreign's innovation efforts (through γ_{L1}), but it is not precisely targeted. In the case of multiple countries:

$$[1-\beta(1-\delta)]\frac{w_1}{\alpha_1^j} = \frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j}$$
$$+\frac{1}{1+\theta}\frac{1}{T_1^j}\frac{\beta_j\pi_{11}^j x_1}{\gamma_4-\gamma_{L1}}\left\{\theta\gamma_{L1}(1-\pi_{11}^j)-\theta\sum_{n\neq 1}^N\gamma_{Ln}^j\pi_{1n}^j+\gamma_{L1}-\sum_{n=1}^N\sum_j\gamma_{Ln}^j\beta_j\pi_{1n}^j\right\}$$

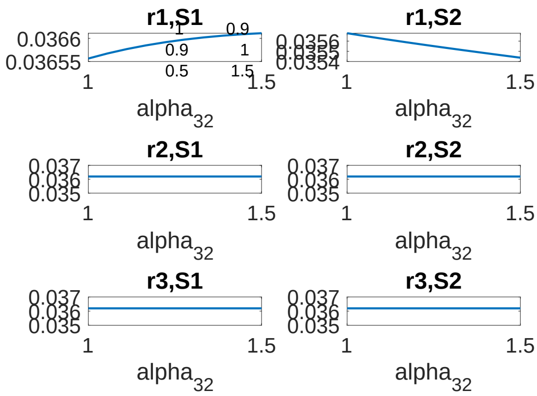
The wedge on innovation cross sectors average to zero. The sign of $\theta\gamma_{L1}(1-\pi_{11}^j) - \theta\sum_{n\neq1}^N\gamma_{Ln}^j\pi_{1n}^j = \theta\sum_{n\neq1}^N(\gamma_{L1}-\gamma_{Ln}^j)\pi_{1n}^j$ determines which sector's R&D should be subsidized. Again, these policies distort one's own innovation in order to affect foreign demand and innovation, but it is inferior to first-best policies with tariffs as it cannot target countries differentially. Nevertheless, it is better than no policies at all.

Consider the numerical illustration of the case when country sector specific tariff are not available. Figure 4 shows that the U.S. would subsidize its own sector 1.

Time-Consistent Optimal Policies: During Transition

During transition, the time consistent optimal policies are that Home government uses country-sector specific export tax, country-sector specific tariff and domestic innovation

Figure 4: Optimal Policies for the US when tariff are not available



policies.

Trade policies:

$$1+\tau_{xn}^{j}=\frac{\left(1+\theta(1-\pi_{n1}^{j})\right)}{\theta\sum_{m\neq 1}(1+t_{m}^{j})\pi_{nm}^{j}+\left(\sum_{k}\gamma_{rn}^{k}(\sigma-1)E\tilde{\beta}_{n}v_{n}^{k}\right)/x_{n}u_{c}},$$

$$1+t_{n}^{j}=\frac{u_{c}-\gamma_{Ln}^{j}}{u_{c}},$$

Innovation policies

$$\frac{w_1}{\alpha_1^j}=\frac{1}{\theta}\frac{w_1 L_{1p}^{j}}{T_1^j}+\tilde{\beta}\frac{Ev_1^j}{u_c}$$
$$+\beta(1-\delta)\frac{\theta}{1+\theta}\frac{1}{u_c}\sum_{n\neq 1}^{N}\sum_{k}^{N_s}\gamma_{r,n}^{k}\frac{\partial Ev_n^k}{\partial T_1^j}x_n^{\sigma}P_n^{1-\sigma}$$

where $\frac{\partial E v_n^k}{\partial T_1^j} = \frac{\partial E\left\{x_n'^{-\sigma} P_n'^{\sigma-1}\frac{w_n'}{\alpha_n^j}\right\}}{\partial T_1^j}$.

The incentives of policies are the same as in the SS, while the additional wedges on innovation reflect Home government could use domestic innovation to change T_1 hence the expected return for foreign innovation.

3 Dynamic Optimal Policies: General Case

3.1 Ramsey optimal policies

First, we consider Ramsey optimal policies for general cases, i.e., when Home government can commit to the path of policies. The government decides the entire path of policies which will be honored in the future, as in particular, future policies would affect the foreign individual expected value of innovation

$$\frac{w_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \beta(1-\delta)E\left\{\left(x_n'\right)^{-\sigma}\left(P_n'\right)^{\sigma-1}\frac{w_n'}{\alpha_n^j(L_{nr}^{j'}, T_n^j)}\right\} x_n^{\sigma} P_n^{1-\sigma}$$

Define

$$\bar{v}_n = w_n x_n^{-\sigma} P_n^{\sigma-1}$$

The constraints become

$$\frac{\bar{v}_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)}\left[1 - \frac{1}{\theta}\frac{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)L_{np}^j}{T_n^j}\right] = \beta(1-\delta)E\left\{\frac{\bar{v}_n'}{\alpha_n^j(L_{nr}^{j'}, T_n^j)}\right\}$$

Home government commits to $\bar{v}_n$, L_{nr}^j from the last period (which gave the expected $E\left\{\frac{\bar{v}_n'}{\alpha_n^j(L_{nr}^{j'}, T_n^j)}\right\}$ in last period). Hence Home government solves the following problem,

$$V\left(\left\{T_{n,-1}^j\right\}, \bar{v}_n, L_{nr}^j\right) = \max_{L_{1r}^j, \tau_{xn}^j, t_n^j} \frac{x_1^{1-\sigma}}{1-\sigma} + \beta E\left[V\left(\left\{T_n^j\right\}, \bar{v}_n', L_{nr}^{j'}\right)\right]$$

Subject to competitive equilibrium with taxes.

Government Euler on home research

$$\frac{w_1}{\alpha_1^j(L_{1r}^j, T_{1,-1}^j) + \frac{\partial \alpha_1^j}{\partial L_{1r}^j} L_{1r}^j} = \frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j} + \beta E\left[\frac{1}{u_c}\left(\frac{u_c' w_1'}{\alpha_1^j(L_{1r}^{j\prime}, T_1^j) + \frac{\partial \alpha_1^{j\prime}}{\partial L_{1r}^{j\prime}} L_{1r}^{j\prime}}\right)\left(1 - \delta + \frac{\partial \alpha_1^{j\prime}}{\partial T_{n,}^j}\right)\right]$$

With CRS, this Euler is the same as private decision

$$\frac{w_1}{\alpha_1^j} = \frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j} + \beta\left(1 - \delta\right) E\left[\frac{1}{u_c}\frac{u_c' w_1'}{\alpha_1^j}\right]$$

and Ramsey optimal policies do not distort domestic innovation, but commit to a path of heterogenous export tax AND tariff across sectors.

With externalities and intertemporal diffusion,

$$\text{wedge} = \frac{(\varepsilon^j - 1)}{\theta}\frac{w_1 L_{1p}^j}{T_1^j} + \beta\eta^j \frac{w_1 r_1^j L_1^j}{T_1^j}$$

Innovation policies correct domestic externalites and diffusion.

3.2 Time-consistent optimal policies

Second, we consider time-consistent optimal policies with externalities and intertemporal diffusion

$$1 + \tau_{xn}^j = \frac{\left(1 + \theta(1 - \pi_{n1}^j)\right)}{\theta \sum_{m \neq 1}(1 + t_m^j)\pi_{nm}^j + \left(\sum_k \gamma_{rn}^k(\sigma - 1)E\tilde{\beta}_n v_n^k\right) / x_n u_c},$$

$$1 + t_n^j = \frac{u_c - \gamma_{Ln}^j}{u_c}$$

Innovation: innovation wedges depend on ϵ, η and $\sum_m \gamma_{r2}^m \frac{\partial G_2^m}{\partial T_1^j}(T_2^m)^{-\eta}$,

$$\text{wedge} = \frac{(\varepsilon^j - 1)}{\theta}\frac{w_1 L^j_{1p}}{T^j_1} + \beta\eta^j\frac{w_1 r^j_1 L^j_1}{T^j_1} + \beta(1-\delta)\frac{\theta}{1+\theta}\frac{1}{u_c}\sum_{n\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{r,n}\frac{\partial E v^k_n}{\partial T^j_1}x^\sigma_n P^{1-\sigma}_n$$

When $\varepsilon^j = 1$ and $\eta^j = 0$, even Home industries have CRS, Home may still implement innovation policies considering their impact on foreign's expected return.

Innovation policies correct externalities ($\frac{\partial \alpha^j_1}{\partial L^j_{1r}} L^j_{1r}$ and $\frac{\partial \alpha^{j'}_1}{\partial T^j_{n,}}$), these are traditional incentive of industrial policies. However, the third term is the extra wedge Home considers own innovation and technology would affect foreign's incentive to do innovation hence their T_n. (Again $\gamma^k_{r,n}$ is the multiplier on Foreign innovation decision constraint Home faces)

Markov optimal policies: use both domestic innovation policies and heterogenous export tax AND tariff across sectors every period. With externalites, both Ramsey and Markov use domestic innovation policies and heterogenous export and import tariff across sectors.

4 Quantitative Gains from Optimal Policies

In this quantitative analysis, the world consists of three countries, China, U.S., and the rest of the world (ROW). We take China as the home country and evaluate the quantitative gains associated with implementing unilateral optimal policies. To evaluate these gains, we extend the 'exact hat method' to compute the counterfactual equilibrium under optimal policies.

4.1 Data and Measurement

We need sectoral level data on gross production and bilateral trade for each country. Bilateral trade flows are from the United Nations' Statistical Division Commodity Trade (COMTRADE) database, and the annual gross production is from the OECD Structural Analysis Database (STAN) and National Accounts and Industrial Statistics Database (UNIDO) compiled by United Nations. The gross production data are available at the 2-digit level ISIC industries. The 6-digit H.S. trade data is then mapped onto two-digit ISIC industries, resulting in18 two-digit manufacturing sectors in 2018.

For a sector in ROW, we sum up sectoral production across the countries in ROW while ignoring bilateral trade between these countries. An issue that arises is that ROW could become too large and productive in this model with endogenous technology. Thus, we consider an alternative exercise where we exclude major economies and China's trading partners from the ROW and check results for robustness (results to be added).

4.2 Gains from Optimal Policies

We use the 'exact hat method' to compute the counterfactual equilibrium under optimal policies. This method allows us to calculate welfare gains using bilateral trade and sector-level production data without needing to back out fundamental research efficiency $\{\alpha_n^j\}$ and trade costs $\{d_{nm}^j\}$, for both the long-run and the short-run transition paths. We adapt the standard exact hat method but incorporate endogenous technology adoption and optimal policies. In particular, the counterfactuals include the calculation of the multipliers and optimal policies.

Let variables without 'prime' denote the observed variables, which includes the trade matrix $\{\pi_{ni}^j\}$ and sectoral production $\{\frac{1+\theta}{\theta} w_n L_{pn}^j\}$. Variables denoted with 'prime' represent counterfactuals after implementing the optimal policies, and variables with 'hats' denote the ratios of prime variables to the observed ones. The following equations characterize the counterfactual equilibrium.

$$\hat{P}_n = \Pi_j \left[\pi^j_{n1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xn}) \right)^{-\theta} + \sum_{i=2}^N \pi^j_{ni} \hat{T}^j_i \hat{w}_i^{-\theta} \right]^{-\beta^j_n/\theta}$$

$$\hat{P}_1 = \Pi_j \left[\pi^j_{11} \hat{T}^j_1 \hat{w}_1^{-\theta} + \sum_{i=2}^N \pi^j_{1i} \hat{T}^j_i (\hat{w}_i (1 + t'^j_{xi}))^{-\theta} \right]^{-\beta^j_1/\theta}$$

$$\pi'^j_{11} = \frac{\pi^j_{11} \hat{T}^j_1 \hat{w}_1^{-\theta}}{\pi^j_{11} \hat{T}^j_1 (\hat{w}_1)^{-\theta} + \sum_{n=2}^N \pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^{j\prime}_n) \right)^{-\theta}}$$

$$\pi'^j_{m1} = \frac{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta}}{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta} + \sum_{i=2}^N \pi^j_{mi} \hat{T}^j_i (\hat{w}_i)^{-\theta}}$$

$$\pi'^j_{mn} = \frac{\pi^j_{mn} \hat{T}^j_n (\hat{w}_n)^{-\theta}}{\pi^j_{m1} \hat{T}^j_1 \left(\hat{w}_1 (1 + \tau'^j_{xm}) \right)^{-\theta} + \pi^j_{mn} \hat{T}^j_n (\hat{w}_n)^{-\theta} + \sum_{i=\{m,n\}}^N \pi^j_{mi} \hat{T}^j_i (\hat{w}_i)^{-\theta}}$$

$$\pi'^j_{1n} = \frac{\pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^j_n) \right)^{-\theta}}{\pi^j_{11} \hat{T}^j_1 (\hat{w}_1)^{-\theta} + \pi^j_{1n} \hat{T}^j_n \left(\hat{w}_n (1 + t^{j\prime}_n) \right)^{-\theta} + \sum_{m \neq \{1,n\}}^N \pi^j_{1m} \hat{T}^j_m \left(\hat{w}_m (1 + t^{j\prime}_m) \right)^{-\theta}}$$

$$x'_1 = \frac{1+\theta}{\theta} \hat{w}_1 w_1 L_{1p} + \sum_{n=2}^N \sum_j \frac{\tau'^j_{xn}}{1 + \tau'^j_{xn}} \pi'^j_{n1} \beta^j_n x'_n + \sum_{i=2}^N \sum_j \frac{t'^j_i}{1 + t'^j_n} \pi'^j_{1i} \beta^j_1 x'_1$$

$$x'_n = \frac{1+\theta}{\theta} \hat{w}_n w_n L_{np}$$

$$\frac{1+\theta}{\theta} \hat{w}_1 \hat{L}^j_1 w_1 L^j_{1p} = \pi'^j_{11} \beta^j_1 x'_1 + \sum_{m \neq 1} \frac{1}{1 + \tau'^j_{xm}} \pi'^j_{m1} \beta^j_m x'_m$$

$$\frac{1+\theta}{\theta} \hat{w}_n \hat{L}^j_{np} w_n L^j_{np} = \frac{1}{1 + t'^j_n} \pi'^j_{1n} \beta^j_1 x'_1 + \sum_{m \neq 1} \pi'^j_{m,n} \beta^j_m x'_m$$

$$\sum_j \left(\hat{L}^j_{np} w_n L^j_{np} \right) = w_n L_{np}$$

The first order conditions become

$$(1 - \beta(1-\delta)) \frac{\mu_n}{w'_n} \frac{r \hat{w}_n \hat{L}^j_n w_n L^j_n}{\delta} = \gamma_P \frac{\beta^j_1}{\theta} \pi'^j_{1n} - \sum_{i=1}^N \gamma^j_{Li} \sum_{m \neq 1} \pi'^j_{m,i} \pi'^j_{mn} \beta^j_m x'_m + \gamma^j_{Ln} \sum_{m \neq 1} \pi'^j_{m,n} \beta^j_m x'_m$$

$$-u_c \sum_{m\neq 1}^{N} \beta_m^j \frac{\tau_{xm}'^j}{1+\tau_{xm}'^j} \pi_{m,1}'^j \pi_{mn}'^j x_m' - \frac{\gamma_{rn}^j}{\left(T_n'^j\right)} \frac{1}{\theta} \hat{w}_n \hat{L}_{np}^j w_n L_{np}^j, \qquad (T_n^j, \quad N_s(N-1))$$

$$\frac{\gamma_{rn}^j}{T_n'^j} \frac{1}{\theta} - \gamma_{Ln}^j \frac{1+\theta}{\theta} - \frac{\mu_n}{w_n'} + \sum_{k=1}^{N_s} \gamma_{L1}^k \beta_n^k \frac{1}{1+\tau_{xn}'^k} \pi_{n1}'^k \frac{1+\theta}{\theta}$$

$$+ \sum_{i\neq 1}^{N} \sum_{k}^{N_s} \gamma_{Li}^k \beta_n^k \pi_{n,i}'^j \frac{1+\theta}{\theta} + \gamma_4 \sum_{k}^{N_s} \beta_n^k \frac{\tau_{xn}'^k}{1+\tau_{xn}'^k} \pi_{n1}'^k \frac{1+\theta}{\theta} = 0 \qquad (L_{np}^j, \quad N_s(N-1))$$

$$-u_c x_1' \sum_{j}^{N_s} \beta_1^j \pi_{1n}'^j + \gamma_{L1} \left[\sum_{m\neq 1}^{N} \sum_{j}^{N_s} \beta_m^j \frac{1}{1+\tau_{xm}'^j} \theta \pi_{m1}'^j \pi_{mn}'^j x_m' + \sum_{j}^{N_s} \beta_n^j \frac{1}{1+\tau_{xn}'^j} \pi_{n1}'^j x_n' \right]$$

$$+ \sum_{i\neq\{1\}}^{N} \sum_{j}^{N_s} \gamma_{Li}^j \left(\sum_{m\neq 1} \theta \pi_{mn}'^j \pi_{m,n}'^j \beta_m^j x_m' + \pi_{n,i}'^j \beta_n^j x_n' \right) - \sum_{j}^{N_s} \gamma_{Ln}^j \left(\theta \sum_{m\neq 1} \pi_{m,n}'^j \beta_m^j x_m' + \frac{1+\theta}{\theta} \hat{w}_n \hat{L}_{np}^j w_n L_{np}^j \right)$$

$$+\gamma_4 \sum_{j=1}^{N_s} \left(\sum_{m\neq 1}^{N} \frac{\tau_{xm}'^j}{1+\tau_{xm}'^j} \theta \pi_{m1}'^j \pi_{mn}'^j \beta_m^j x_m' + \frac{\tau_{xn}'^j}{1+\tau_{xn}'^j} \pi_{n1}'^j \beta_n^j x_n' \right) = 0, \qquad (w_n, N-1)$$

and the optimal trade policies:

$$1 + t_n'^j = \frac{u_c' - \gamma_{Ln}^{j\prime}}{u_c'}, \qquad 1 + \tau_{xn}'^j = \frac{\left(1 + \theta(1 - \pi_{n1}'^j)\right)}{\theta \sum_{m\neq 1} (1 + t_m'^j) \pi_{nm}'^j}.$$

In the data, China runs a large surplus, but our exercise features a long-run counterfactual equilibrium with balanced trade. This implies that the welfare gains computed include both the gains from eliminating trade imbalances and those from implementing optimal policies. To separate them, we first run a counterfactual to eliminate the imbalances. We then take the new equilibrium to be our private equilibrium observations which are used to calculate the optimal policies and welfare changes. The results show additional changes brought about by optimal policies.[5]

Table 1 shows the optimal policies and the induced labor movement due to heterogenous

[5]When exogenous imbalances are eliminated, Lerner symmetry holds. Hence, a uniform increase in export tax is equivalent to a uniform increase in tariff. We can also solve the optimal policies with trade imbalances. In this case, Lerner symmetry no longer holds, and there is an additional valuation effect for the fixed amount of deficit. Home would like to change its optimal policies to increase its domestic prices and inflate away the deficit. To avoid this complication and focus on our main mechanism, we only consider the balanced trade in the long run.

Table 1: Optimal Policies

	Export Tax		Import Tariff		L change due to tariff hetero.		
	US	ROW	US	ROW	CN	US	ROW
Textiles	0.91	-3.58	22.08	35.10	2.68	4.06	-2.95
Furniture	0.34	-4.10	24.38	33.58	3.68	1.32	-1.99
Electrical machinery	6.29	-3.82	11.02	46.08	1.23	9.12	-7.73
Office, accounting and computing machinery	-4.40	0.91	3545.11	18.04	-9.19	-93.72	6.44
Machinery and equipment	2.66	-3.48	18.50	33.43	1.37	3.62	-1.79
Rubber and plastics	0.26	-3.86	24.51	31.60	1.54	0.77	-0.80
Fabricated metal	-0.38	-3.93	25.67	31.34	1.36	0.26	-0.64
Non-metallic mineral	-0.18	-3.92	25.35	31.28	0.39	0.43	-0.57
Printing	-0.47	-3.98	25.80	30.67	0.29	0.14	-0.22
Other transport equipment	2.86	-3.96	18.59	34.89	2.45	1.62	-1.84
Wood	-0.34	-3.70	25.21	30.56	0.54	0.28	-0.35
Tobacco	-0.69	-3.98	25.75	30.40	-0.07	0.04	-0.10
Food Beverage	-0.34	-3.84	25.00	30.37	0.12	0.23	-0.15
Fuel	0.15	-3.74	24.24	30.82	0.25	0.40	-0.32
Vehicles	0.71	-4.03	21.26	31.27	0.13	1.23	-0.44
Basic metals	0.52	-3.18	22.23	30.60	0.17	1.09	-0.49
Paper	-0.01	-3.47	24.77	30.37	0.36	0.34	-0.31
Chemicals	1.35	-3.14	21.08	31.88	0.47	1.57	-1.00

Table 2: Optimal Policies if technology would not change

	Export Tax		Import Tariff		L change		
	US	ROW	US	ROW	CN	US	ROW
Textiles	2.47	2.44	25.00	25.00	-12.00	4.40	11.19
Furniture	1.50	1.29	25.00	25.00	-19.13	4.06	7.66
Electrical machinery	2.25	2.30	25.00	25.00	-4.79	-10.69	9.71
Office, accounting and computing machinery	2.15	2.36	25.00	25.00	1.20	17.68	-9.93
Machinery and equipment	1.25	1.15	25.00	25.00	-3.00	-4.48	2.18
Rubber and plastics	0.88	0.73	25.00	25.00	-5.04	0.13	2.05
Fabricated metal	0.58	0.66	25.00	25.00	-4.40	0.68	1.98
Non-metallic mineral	0.71	0.66	25.00	25.00	-0.25	0.25	1.63
Printing	0.34	0.24	25.00	25.00	-0.21	0.26	0.62
Other transport equipment	0.18	0.82	25.00	25.00	2.03	-9.51	4.13
Wood	0.39	0.43	25.00	25.00	1.28	-0.58	-0.18
Tobacco	0.01	0.11	25.00	25.00	1.61	-0.21	0.20
Food	0.09	0.16	25.00	25.00	2.37	-0.88	-0.30
Fuel	0.02	0.27	25.00	25.00	2.50	-1.51	-0.23
Vehicles	0.26	0.23	25.00	25.00	2.71	-4.84	0.25
Basic metals	0.19	0.70	25.00	25.00	2.66	-2.37	-1.71
Paper	0.21	0.39	25.00	25.00	3.58	-1.17	-1.09
Chemicals	0.32	0.84	25.00	25.00	3.62	-3.17	-1.64

tariffs across sectors. We take China to be the home country and consider its optimal unilateral policies. Table 1 ranks sectors by China's net export as a share of world production in that sector. For example, relative to other sectors, China's textile has the highest share of net export, and China is a net importer of Chemicals.

Three points are brought to bear from our results. First, export tax and import tariffs are heterogeneous across sectors and countries and exhibit a wide range of values. China's optimal import tariffs for the U.S. range from 11% to 35 times. Tariffs also vary across countries. For electrical machinery, the optimal tariff imposed on the U.S. is11% where as it is 46% for ROW. Similarly, China imposes a tax of 6.29% for exporting electrical machinery to the U.S. but subsidizes the ROW with an export tax of -3.82%. Of course, all optimal taxes and tariffs are relative, as Lerner symmetry holds.

Second, as is obvious from the table, there is no discernible pattern between optimal policy and an ordering of comparative advantage. The reason is that all country-sector specific tax and tariffs are jointly determined, and hence no bilateral relationship between tariff/export tax and some revealed country pair-wise comparative advantage exists. However, sectors that display a large tariff gap between the U.S. and ROW are those where China features a relatively higher net exports. These results arise from endogenous technology adoption.

Third, heterogeneity in both export taxes and import tariffs helps bridge the gap between the existing theories and the data. As pointed out by Caliendo and Parro (2021), the literature remains highly disconnected from the data: " theoretical result on uniform import tariffs and heterogenous export tax across goods" is in stark contrast to “ the observed wide range of tariff changes across products during the recent trade war...." If the conventional wisdom guiding trade policy changes relates to manipulating the terms of trade, then it fails to explain the recent trade war between the U.S. and China, for instance. To explain these observed trade policies, the recent literature pursues mechanisms other than terms of trade manipulation, such as externalities or political considerations. In contrast, our model shows that optimal tariffs should differ across products under endogenous technology and technology rivalry, even absent externalities and political considerations.

For comparison, we run the counterfactual equilibrium under exogenous technology in

Table 2. As consistent with theory, optimal export taxes tend to be higher in sectors in which China exports relatively more (on net). For example, textiles, furniture, office, and machinery and equipment face about 2% export taxes, while food, paper, and chemicals have export taxes less than 0.5%. Since Home has a comparative advantage and high monopoly power in these high-tax sectors, it can charge a higher markup upon exporting, whilst technology remains immune to policies. Note that there is also no strict relationship between the ordering of sector based on comparative advantage and attendant export taxes as there are trade costs that can differ across sectors and multiple countries.

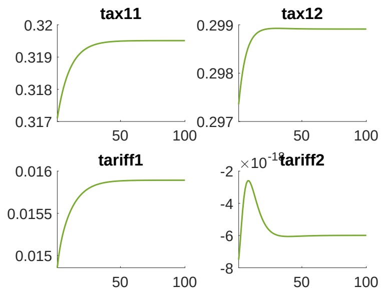
Table 3 reports the welfare gains from the optimal policies and two alternative, simple policies. As shown in Table 1, the optimal policies of export taxes and import tariffs might be complex to implement. We, therefore, consider two simple alternative policies: one with heterogeneous sectorial tariffs as in the optimal case, the other one with a uniform tariff of 25%. In both cases, we set zero export taxes. Under the optimal unilateral policies, China gains by 0.69%, while ROW loses about 0.9%. With the only heterogenous tariff, China's gain can go as high as 0.67%. On the other hand, the only uniform tariff generates about 0.6% of welfare gain.

Note that the magnitude of our welfare gains is higher than the standard estimations. It is common to have small welfare gains in the literature. Higher welfare gains may arise due to input-output structure or externality. For example, Bartelme et al. (2019) use a model with externality and find an average gain from optimal industry policies of 0.98%. Caliendo and Parro (2015) evaluate the welfare gains for NAFTA with input-output structure. They find that the gain for the U.S. is 0.08% and 1.3% for Mexico. In contrast, our model emphasizes the role of endogenous technology adoption in trade policy. The welfare gain is about 0.7% for China.

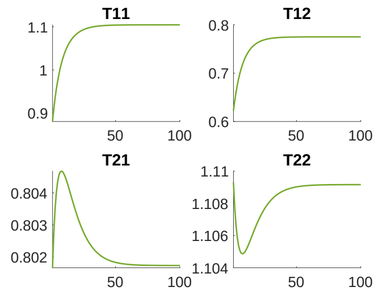
Table 3: Welfare Gain Implications

	Optimal policies	Only heterogenous sectorial tariff	Only uniform tariff
CN	0.69	0.67	0.60
US	0.36	0.27	−0.48
ROW	−0.91	−0.93	−0.59

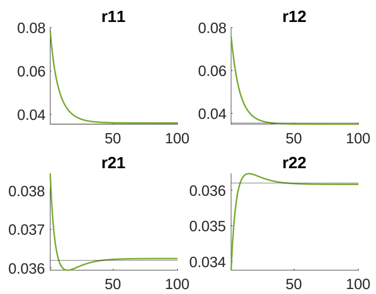
Figure 5: Dynamic Optimal Policies



(a) Tariff



(b) Labor Allocation



(c) Research Intensity

5 Conclusion

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Online Appendix to "Technological Rivalry and Optimal Policy in an Open Economy"

by Yan Bai, Keyu Jin, and Dan Lu

This appendix is organized as follows.

A. Government problem: General cases

B. Optimal conditions: General cases

C. Efficiency of the baseline (Optimal Cooperative Policies)

D. Optimal unilateral policies: Baseline

E. Proof for Proposition 4

F. Optimal innovation policy without tariff: Baseline

??. Numerical example with three countries

??. Proofs for the dynamic optimal policies: Ramsey

A Government Problem: Optimal Time Consistent Policies

$$V\left(\left\{T_{n,-1}^{j}\right\}\right)=\max_{\left\{L_{nr}^{j},L_{np}^{j}T_{n}^{j},w_{n},x_{n},\tau_{xn}^{j},t_{n}^{j}\right\}}\frac{x_{1}^{1-\sigma}}{1-\sigma}+\beta E\left[V\left(\left\{T_{n}^{j}\right\}\right)\right]$$

Subject to

$$\frac{w_n}{\alpha_n^j(L_{nr}^j,T_{n,-1}^j)}=\frac{1}{\theta}\frac{w_nL_{np}^j}{T_n^j}+E\tilde{\beta}_nv_n^{\prime j},\qquad(\gamma_{rn}^j,\quad N_s\times(N-1)),\qquad(\gamma_{rn}^j,\quad N_s\times(N-1))$$

$$T_n^j=\alpha_n^j(L_{nr}^j,T_{n,-1}^j)L_{nr}^j+(1-\delta)T_{n,-1}^j,\quad(\gamma_{Tn}^j,\quad N_s\times N)$$

$$P_1=\Pi_j\left[T_1^jw_1^{-\theta}+\sum_{n\neq1}T_n^j(w_n(1+t_n^j)d)^{-\theta}\right]^{-\frac{\beta_j}{\theta}}=1\quad(\gamma_P)$$

$$\frac{1+\theta}{\theta} w_1 L_{1p}^j = \beta_j \left[\pi_{11}^j x_1 + \sum_{m \neq 1} \frac{1}{1+\tau_{xm}^j} \pi_{m1}^j x_m \right], \quad (\gamma_{L1}^j, \quad N_s) \tag{A.1}$$

$$\frac{1+\theta}{\theta} w_n L_{np}^j = \beta_j \left[\frac{1}{1+t_n^j} \pi_{1n}^j x_1 + \sum_{m \neq 1} \pi_{m,n}^j x_m \right] \quad (\gamma_{Ln}^j \quad N_s \times (N-1)) \tag{A.2}$$

$$\sum_j \left(L_{nr}^j + L_{np}^j \right) = L_n, \quad (\mu_n, \quad N)$$

$$x_1 = \frac{1+\theta}{\theta} w_1 \sum_j L_{1p}^j + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau_{xm}^j}{1+\tau_{xm}^j} \pi_{m1}^j x_m + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t_m^j}{1+t_m^j} \pi_{1m}^j x_1, \quad (\gamma_4)$$

where

$$x_m = \frac{1+\theta}{\theta} w_m \sum_j L_{mp}^j$$

$$P_m = \Pi_j \left[T_1^j (w_1 (1+\tau_{xm}^j) d)^{-\theta} + \sum_{n \neq 1} T_n^j (w_n d)^{-\theta} \right]^{-\frac{\beta_j}{\theta}}$$

$$\pi_{11}^j = \frac{T_1^j (w_1)^{-\theta}}{T_1^j (w_1)^{-\theta} + \sum_{n \neq 1} T_n^j (w_n (1+t_n^j) d)^{-\theta}}$$

$$\pi_{m1}^j = \frac{T_1^j (w_1 (1+\tau_{xm}^j) d)^{-\theta}}{T_1^j (w_1 (1+\tau_{xm}^j) d)^{-\theta} + \sum_{n \neq 1} T_n^j (w_n d)^{-\theta}}$$

$$\pi_{mn}^j = \frac{T_n^j (w_n d)^{-\theta}}{T_1^j (w_1 (1+\tau_{xm}^j) d)^{-\theta} + T_n^j (w_n d)^{-\theta} + \sum_{i \neq \{m,n\}} T_n^j (w_n d)^{-\theta}}$$

$$\pi_{1m}^j = \frac{T_m^j (w_m (1+t_m^j) d)^{-\theta}}{T_1^j (w_1)^{-\theta} + T_m^j (w_m (1+t_m^j) d)^{-\theta} + \sum_{n \neq \{1,m\}} T_n^j (w_n (1+t_n^j) d)^{-\theta}}$$

Note that $\sum_n \pi_{m,n}^j = 1$ for any m.

Note that sum up (A.1) we get the balanced trade condition for country 1

$$\sum_{m=1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{1}{1+t_m^j} \pi_{1m}^j x_1 = \sum_{m=1}^{N} \sum_{j=1}^{N_s} \beta_j \pi_{m1}^j x_m$$

and sum up (A.2) we get the balanced trade condition for country $n \neq 1$,

$$\sum_{m=1}^{N}\sum_{j=1}^{N_s} \beta_j \pi_{n,m}^{j} x_n = \sum_{m\neq 1}^{N}\sum_{j=1}^{N_s} \beta_j \pi_{m,n}^{j} x_m + \sum_{j=1}^{N_s} \beta_j \frac{1}{1+t_n^j} \pi_{1,n}^{j} x_1.$$

As one of them is redundant, we can drop one of the (A.2), so we end up with number $(N-1)N_s - 1$ for γ_{Ln}^j with $n \neq 1$.

B Optimal Conditions

Equilibrium under optimal policies have unknowns:

$$\left\{ \underbrace{L_{nr}^j}_{N_s\times N}, \underbrace{L_{np}^j}_{N_s\times N}, \underbrace{T_n^j}_{N_s\times N}, \underbrace{\tau_{xn}^j}_{N_s\times(N-1)}, \underbrace{t_n^j}_{N_s\times(N-1)}; \underbrace{\gamma_{Ln}^j}_{N_s\times(N-1)-1}, \underbrace{\gamma_{rn}^j}_{N_s\times(N-1)}, \underbrace{\gamma_{Tn}^j}_{N_s\times(N-1)}, \underbrace{\mu_n}_{N-1}, \underbrace{w_n}_{N}, \gamma_{L1}, \gamma_4 \right\}$$

The optimal conditions are:

For L_{nr}^j other than Home:

$$\frac{w_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)} = \frac{1}{\theta}\frac{w_n L_{np}^j}{T_n^j} + \beta(1-\delta) G_n^j\left(\left\{T_n^j\right\}\right) x_n^{\sigma} P_n^{1-\sigma}, \qquad (\gamma_{rn}^j, \quad N_s \times (N-1))$$

For L_{1r}^j, we combine the FOC for L_{1r}^j and T_1^j, and we have the optimal condition for Home innovation across sectors:

$$\frac{w_1}{\alpha_1^j(L_{1r}^j, T_{1,-1}^j) + \frac{\partial \alpha_1^j}{\partial L_{1r}^j} L_{1r}^j} = \frac{1}{\theta}\frac{w_1 L_{1p}^j}{T_1^j} + \beta E\left[\frac{1}{u_c}\left(\frac{u_c' w_1'}{\alpha_1^j(L_{1r}^{j\prime}, T_1^j) + \frac{\partial \alpha_1^{j\prime}}{\partial L_{1r}^{j\prime}} L_{1r}^{j\prime}} + \frac{\theta}{1+\theta}\frac{\xi_{1p}^{j\prime} - \xi_{1r}^{j\prime}}{\alpha_n^j(L_{nr}^{j\prime}, T_n^j) + \frac{\partial \alpha_n^{j\prime}}{\partial L_{nr}^{j\prime}} L_{nr}^{j\prime}}\right)\left(1\right.\right.$$

$$+ \frac{\theta}{1+\theta}\frac{1}{u_c}\sum_{n\neq 1}^{N}\sum_{k}^{N_s} \gamma_{r,n}^k \beta(1-\delta)\frac{\partial G_n^k}{\partial T_1^j} x_n^{\sigma} P_n^{1-\sigma} - \frac{1}{\gamma_4 - \gamma_{L1}}\frac{\theta}{1+\theta}\frac{\xi_{1p}^j - \xi_{1r}^j}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j) + \frac{\partial \alpha_n^j}{\partial L_{nr}^j}}$$

For L^j_{np}:

$$\frac{1+\theta}{\theta} w_1 L^j_{1p} = \beta_j \left[\pi^j_{11} x_1 + \sum_{m \neq 1} \frac{1}{1+\tau^j_{xm}} \pi^j_{m1} x_m \right], \quad (\gamma^j_{L1}, \quad N_s)$$

$$\frac{1+\theta}{\theta} w_n L^j_{np} = \beta_j \left[\frac{1}{1+t^j_n} \pi^j_{1n} x_1 + \sum_{m \neq 1} \pi^j_{m,n} x_m \right] \quad (\gamma^j_{Ln} \quad N_s \times (N-1))$$

For T^j_n:

$$T^j_n = \alpha^j_n (L^j_{nr}, T^j_{n,-1}) L^j_{nr} + (1-\delta) T^j_{n,-1}, \quad (\gamma^j_{Tn}, \quad N_s \times N)$$

From the FOC of τ^j_{xn}

$$1+\tau^j_{xn} = \frac{(\gamma_4 - \gamma_{L1}) \left[1 + \theta (1 - \pi^j_{n1}) \right]}{\gamma_4 \theta (1 - \pi^j_{n1}) - \theta \left(\sum_{i \neq 1}^N \gamma^j_{Li} \pi^j_{n,i} \right) + \left(\sum_k \gamma^k_{rn} \beta (1-\delta)(\sigma - 1) M^k_n \right) x_n^{-1}}, \qquad (N_s(N-1))$$

From the FOC of t^j_n

$$1 + t^j_n = \frac{\gamma_4 - \gamma^j_{Ln}}{\gamma_4 - \gamma_{L1}}, \qquad (N_s(N-1))$$

The FOC of T^j_n other than Home:

$$\begin{aligned} \gamma^j_{Tn} =& \beta E \left[\gamma^{j\prime}_{Tn} \left(1 - \delta + \frac{\partial \alpha^{j\prime}_n}{\partial T^j_n} L^j_{nr} + \gamma^j_{rn} \frac{w^\prime_n}{\left(\alpha^{j\prime}_n \right)^2} \frac{\partial \alpha^{j\prime}_n}{\partial T^j_n} \right) \right] + \frac{1}{T^j_n} \{ \gamma_P \frac{\beta_j}{\theta} \pi^j_{1n} \\ & - \sum_{i=1}^N \gamma^j_{Li} \beta_j \sum_{m \neq 1} \pi^j_{m,i} \pi^j_{mn} x_m + \gamma^j_{Ln} \beta_j \sum_{m \neq 1} \pi^j_{m,n} x_m - u_c \sum_{m \neq 1}^N \beta_j \frac{\tau^j_{xm}}{1 + \tau^j_{xm}} \pi^j_{m,1} \pi^j_{mn} x_m \\ & - \gamma^j_{rn} \frac{1}{\theta} \frac{w_n L^j_{np}}{\left(T^j_n \right)} - \sum_{i \neq 1}^N \left(\sum_k^{N_s} \gamma^k_{ri} \beta (1-\delta)(1-\sigma) M^k_n \right) \frac{\beta_j}{\theta} \pi^j_{in} \} \\ & + \sum_{i \neq 1}^N \sum_k^{N_s} \gamma^k_{ri} \beta (1-\delta) \frac{\partial G^k_n}{\partial T^j_n} x^\sigma_n P^{1-\sigma}_n = 0, \qquad (T^j_n, \quad N_s(N-1)) \end{aligned}$$

The FOC of L^j_{np} other than Home:

$$\gamma^j_{rn}\frac{1}{\theta}\frac{w_n}{T^j_n}-\gamma^j_{Ln}\frac{1+\theta}{\theta}w_2-\mu_n+\sum_{k=1}^{N_s}\gamma^k_{L1}\beta_k\frac{1}{1+\tau^k_{xn}}\pi^k_{n1}\frac{1+\theta}{\theta}w_n$$

$$+\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi_{n,i}\frac{1+\theta}{\theta}w_n+\gamma_4\sum_{k}^{N_s}\beta_k\frac{\tau^k_{xn}}{1+\tau^k_{xn}}\pi^k_{n1}\frac{1+\theta}{\theta}w_n+\sum_{k}^{N_s}\gamma^k_{rn}\beta(1-\delta)\sigma M^k_n\frac{1+\theta}{\theta}\frac{w_n}{x_n}+\xi^j_{np}=0 \qquad (L$$

The FOC of L^j_{nr} other than Home:

$$\mu_n=\gamma^j_{Tn}\left[\alpha^j_n(L^j_{nr},T^j_{n,-1})+\frac{\partial\alpha^j_n}{\partial L^j_{nr}}L^j_{nr}\right]+\gamma^j_{rn}\frac{w_n}{\left[\alpha^j_n(L^j_{nr},T^j_{n,-1})\right]^2}\frac{\partial\alpha^j_n}{\partial L^j_{nr}}+\xi^j_{nr}, \qquad (L^j_{nr},\quad N_s(N-1))$$

The FOC of w_n ($n>2$ as one of the FOC for w_n is redundant)

$$-u_c x_1\sum_{j}^{N_s}\beta_j\pi^j_{1n}+\gamma_{L1}\left[\sum_{m\neq 1}^{N}\sum_{j}^{N_s}\beta_j\frac{1}{1+\tau^j_{xm}}\theta\pi^j_{m1}\pi^j_{mn}x_m+\sum_{j}^{N_s}\beta_j\frac{1}{1+\tau^j_{xn}}\pi^j_{n1}x_n\right]$$

$$+\sum_{i\neq\{1\}}^{N}\sum_{j}^{N_s}\gamma^j_{Li}\beta_j\left(\sum_{m\neq 1}\theta\pi^j_{mn}\pi^j_{m,n}x_m+\pi^j_{n,i}x_n\right)-\sum_{j}^{N_s}\gamma^j_{Ln}\left(\theta\beta_j\sum_{m\neq 1}\pi^j_{m,n}x_m+\frac{1+\theta}{\theta}w_nL^j_{np}\right)$$

$$+\gamma_4\sum_{j=1}^{N_s}\beta_j\left(\sum_{m\neq 1}^{N}\frac{\tau^j_{xm}}{1+\tau^j_{xm}}\theta\pi^j_{m1}\pi^j_{mn}x_m+\frac{\tau^j_{xn}}{1+\tau^j_{xn}}\pi^j_{n1}x_n\right)$$

$$+\sum_{j}^{N_s}\gamma^j_{rn}\left[\beta(1-\delta)(\sigma-1)M^j_n\right]+\sum_{m\neq 1}^{N}\sum_{j}^{N_s}\gamma^j_{rm}\left[\beta(1-\delta)(1-\sigma)M^j_m\left(\sum_{k}^{N_s}\beta_k\pi^k_{mn}\right)\right]=0, \qquad (w_n,N-1)$$

$$P_1=\Pi_j\left[T^j_1(w_1d)^{-\theta}+\sum_{n\neq 1}T^j_n(w_n(1+t^j_n)d)^{-\theta}\right]^{-\frac{\beta_j}{\theta}}=1 \quad (\gamma_P)$$

$$\sum_j\left(L^j_{nr}+L^j_{np}\right)=L_n, \quad (N-1)$$

The FOC of L_{1r} and x_1 implies $\gamma_{L1} = const$, and

$$\gamma_4 = u_c + \gamma_{L1}.$$

In addition,

$$G_n^j\left(\left\{T_n^j\right\}\right) = E\left[(x_n')^{-\sigma}(P_n')^{\sigma-1}\frac{w_n}{\alpha_n^j(L_{nr}^j, T_{n,-1}^j)}\right]$$

$$M_n^j = G_n^j x_n^\sigma P_n^{1-\sigma}$$

$$x_1 = \frac{1+\theta}{\theta}w_1\sum_j L_{1p}^j + \sum_{m\neq 1}^{N}\sum_{j=1}^{N_s}\beta_j\frac{\tau_{xm}^j}{1+\tau_{xm}^j}\pi_{m1}^j x_m + \sum_{m\neq 1}^{N}\sum_{j=1}^{N_s}\beta_j\frac{t_m^j}{1+t_m^j}\pi_{1m}^j x_1, \quad (\gamma_4)$$

$$x_m = \frac{1+\theta}{\theta}w_m\sum_j L_{mp}^j$$

$$P_m = \Pi_j\left[T_1^j(w_1(1+\tau_{xm}^j)d)^{-\theta} + \sum_{n\neq 1}T_n^j(w_n d)^{-\theta}\right]^{-\frac{\beta_j}{\theta}}$$

C Optimal Cooperative Policies/Efficiency in the Baseline

This appendix shows that in the baseline, the polices maximize global welfare are zero export taxes and tariff, and no distortions on domestic innovation.

$$V\left(\left\{T_{n,-1}^j\right\}\right) = \max_{\left\{L_{nr}^j, \tau_{xnm}^j, t_{nm}^j\right\}}\sum\frac{\frac{x_m}{P_m}^{1-\sigma}}{1-\sigma} + \beta E\left[V\left(\left\{T_n^j\right\}\right)\right]$$

Subject to

$$T_n^j = \alpha_n^j L_{nr}^j + (1-\delta)T_{n,-1}^j, \quad (\gamma_{Tn}^j, \quad N_s\times N)$$

$$\frac{1+\theta}{\theta} w_n L^j_{np} = \beta_j \sum_m \frac{1}{(1+t^j_{mn})(1+\tau^j_{xmn})} \pi^j_{mn} x_m, \quad (\gamma^j_{Ln} \quad N_s \times N) \tag{A.3}$$

$$\sum_j \left(L^j_{nr} + L^j_{np} \right) = L_n, \quad (\mu_n, \quad N)$$

$$x_n = \frac{1+\theta}{\theta} w_n \sum_j L^j_{np} + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{\tau^j_{xmn}}{1+\tau^j_{xmn}} \pi^j_{mn} x_m + \sum_{m \neq 1}^{N} \sum_{j=1}^{N_s} \beta_j \frac{t^j_{nm}}{1+t^j_{nm}} \pi^j_{nm} x_n, \quad (\gamma_{4n})$$

$$P_m = \Pi_j \left[\sum_n T^j_n ((1+\tau^j_{xmn})(1+t^j_{xmn}) w_n d)^{-\theta} \right]^{-\frac{\beta_j}{\theta}}$$

The optimal policies are, $t^j_{nm} = 0, \tau^j_{xnm} = 0$, and no wedges on innovation as it can be derived that:

$$\frac{w_n}{\alpha^j_n} = \frac{1}{\theta} \frac{w_n L^j_{np}}{T^j_n} + \beta(1-\delta) G^j_n \left(\left\{ T^j_n \right\} \right) x^\sigma_n P^{1-\sigma}_n, \qquad (\gamma^j_{rn}, \quad N_s \times N)$$

D Optimal Policies at the SS: Baseline

$$\frac{w_1}{\alpha^j_1(L^j_{1r}, T^j_{1,-1}) + \frac{\partial \alpha^j_1}{\partial L^j_{1r}} L^j_{1r}} = \frac{1}{\theta} \frac{w_1 L^j_{1p}}{T^j_1} + \beta E \left[\frac{1}{u_c} \left(\frac{u'_c w'_1}{\alpha^j_1(L^{j\prime}_{1r}, T^j_1) + \frac{\partial \alpha^{j\prime}_1}{\partial L^{j\prime}_{1r}} L^{j\prime}_{1r}} \right) \left(1 - \delta + \frac{\partial \alpha^{j\prime}_1}{\partial T^j_{n,}} \right) \right]$$
$$+ \frac{\theta}{1+\theta} \frac{1}{u_c} \sum_{n \neq 1}^{N} \sum_{k}^{N_s} \gamma^k_{r,n} \beta(1-\delta) \frac{\partial G^k_n}{\partial T^j_1} x^\sigma_n P^{1-\sigma}_n, \qquad (N_s)$$

From the FOC of τ^j_{xn}

$$1+\tau^j_{xn} = \frac{(\gamma_4 - \gamma_{L1}) \left[1 + \theta(1-\pi^j_{n1}) \right]}{\gamma_4 \theta (1-\pi^j_{n1}) - \theta \left(\sum^N_{i \neq 1} \gamma^j_{Li} \pi^j_{n,i} \right) + \left(\sum_k \gamma^k_{rn} \beta (1-\delta)(\sigma-1) M^k_n \right) x^{-1}_n}, \qquad (N_s(N-1))$$

From the FOC of t_n^j

$$1+t_n^j = \frac{\gamma_4 - \gamma_{Ln}^j}{\gamma_4 - \gamma_{L1}}, \qquad (N_s(N-1))$$

In the Baseline, there are no externality and spillover, so $\frac{\partial \alpha_1^j}{\partial L_{1r}^j} = 0$ and $\frac{\partial \alpha_1^{j'}}{\partial T_{n,}^j} = 0$. At the SS, it can be proved $\sum \gamma_{r2}^j / \alpha_2^j = 0$ and given α_s are constant and do not depend on endogenous variables, we have $\sum_{n\neq 1}^N \sum_k^{N_s} \gamma_{r,n}^k \beta(1-\delta) \frac{\partial G_n^k}{\partial T_1^j} x_n^\sigma P_n^{1-\sigma} = 0$ and $\left(\sum_k \gamma_{rn}^k \beta(1-\delta)(\sigma-1) M_n^k\right) x_n^{-1} = 0$.

The optimal policies simplify to:

$$1+\tau_{xn}^j = \frac{(\gamma_4 - \gamma_{L1})\left[1+\theta(1-\pi_{n1}^j)\right]}{\gamma_4 \theta(1-\pi_{n1}^j) - \theta\left(\sum_{i\neq 1}^N \gamma_{Li}^j \pi_{n,i}^j\right)}, \qquad (N_s(N-1))$$

From the FOC of t_n^j

$$1+t_n^j = \frac{\gamma_4 - \gamma_{Ln}^j}{\gamma_4 - \gamma_{L1}}, \qquad (N_s(N-1))$$

The innovation:

$$\frac{w_n}{\alpha_n^j} = \frac{1}{\theta} \frac{w_n L_{np}^j}{T_n^j} + \beta(1-\delta) G_n^j\left(\left\{T_n^j\right\}\right) x_n^\sigma P_n^{1-\sigma}, \qquad (\gamma_{rn}^j, \quad N_s \times N)$$

So there are no distortions on Home innovation, Home government uses heterogenous tariff and tax across sectors.

E Proposition 4 Proof

Sum up the FOCs for T_n^j and τ_{xm}^j, we get

$$\sum_{n\neq 1} \theta(1-\beta(1-\delta))\gamma_{Tn}^j T_n^j = -\sum_{n\neq 1} \frac{\gamma_{rn}^j}{T_n^j} w_n L_{np}^j + u_c \left[\sum_{n\neq 1} \beta_j \pi_{1n}^j x_1 - \sum_{m\neq 1} \beta_j \frac{1}{(1+\tau_{xm}^j)} \pi_{m1}^j x_m\right]$$

At the steady state

$$\sum_{n\neq 1}\theta(1-\beta(1-\delta))\mu_n\frac{L^j_{nr}}{\delta}=-\sum_{n\neq 1}\frac{\gamma^j_{rn}}{T^j_n}w_nL^j_{np}+u_c\left[\sum_{n\neq 1}\beta_j\pi^j_{1n}x_1-\sum_{m\neq 1}\beta_j\frac{1}{(1+\tau^j_{xm})}\pi^j_{m1}x_m\right]$$

$$t^j_n=-\frac{\gamma^j_{Ln}}{u_c}$$

$$\gamma^j_{Tn}\alpha^j_n-\mu_n=0$$

Recall L_{np}

$$-\gamma^j_{rn}\frac{1}{\theta}\frac{w_n}{T^j_n}=-\gamma^j_{Ln}\frac{1+\theta}{\theta}w_n+\mu_n-\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi_{n,i}\frac{1+\theta}{\theta}w_n-u_c\sum_{k}^{N_s}\beta_k\frac{\tau^k_{xn}}{1+\tau^k_{xn}}\pi^k_{n1}\frac{1+\theta}{\theta}w_n$$

$$-\gamma^j_{rn}\frac{1}{\theta}\frac{w_n}{T^j_n}=u_ct^j_n\frac{1+\theta}{\theta}w_n+\mu_n-\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi_{n,i}\frac{1+\theta}{\theta}w_n-u_c\sum_{k}^{N_s}\beta_k\frac{\tau^k_{xn}}{1+\tau^k_{xn}}\pi^k_{n1}\frac{1+\theta}{\theta}w_n$$

$$-\sum_{n\neq 1}\gamma^j_{rn}\frac{1}{\theta}\frac{w_n}{T^j_n}L^j_{np}=u_c\sum_{n\neq 1}t^j_n\frac{1+\theta}{\theta}w_nL^j_{np}+\sum_{n\neq 1}\mu_nL^j_{np}-\sum_{n\neq 1}\sum_{i\neq 1}^{N}\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi_{n,i}\frac{1+\theta}{\theta}w_nL^j_{np}-u_c\sum_{n\neq 1}\sum_{k}^{N_s}\beta_k\frac{\tau}{1+}$$

Go back and

$$\sum_{n\neq 1}\frac{\theta(1-\beta(1-\delta))}{\delta}\mu_nL^j_{nr}-\sum_{n\neq 1}\mu_nL^j_{np}+\sum_{n\neq 1}\sum_{i\neq 1}^{N}\left(\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi^k_{n,i}\right)\frac{1+\theta}{\theta}w_nL^j_{np}+u_c\sum_{n\neq 1}\left(\sum_{k}^{N_s}\beta_k\frac{\tau^k_{xn}}{1+\tau^k_{xn}}\pi^k_{n1}\right)$$

$$=u_c\sum_{n\neq 1}t^j_n\frac{1+\theta}{\theta}w_nL^j_{np}+u_c\left[\sum_{n\neq 1}\beta_j\pi^j_{1n}x_1-\sum_{m\neq 1}\beta_j\frac{1}{(1+\tau^j_{xm})}\pi^j_{m1}x_m\right]$$

It can be simplified to

$$\sum_{n\neq 1}\sum_{i\neq 1}^{N}\left(\sum_{k}^{N_s}\gamma^k_{Li}\beta_k\pi^k_{n,i}\right)\frac{1+\theta}{\theta}w_nL^j_{np}+u_c\sum_{n\neq 1}\left(\sum_{k}^{N_s}\beta_k\frac{\tau^k_{xn}}{1+\tau^k_{xn}}\pi^k_{n1}\right)\frac{1+\theta}{\theta}w_nL^j_{np}$$

$$=u_c\sum_{n\neq 1}t^j_n\frac{1+\theta}{\theta}w_nL^j_{np}+u_c\underbrace{\left[\sum_{n\neq 1}\beta_j\pi^j_{1n}x_1-\sum_{m\neq 1}\beta_j\frac{1}{(1+\tau^j_{xm})}\pi^j_{m1}x_m\right]}_{\text{total net export of ROW on sector } j}$$

1. In the special case where there are symmetry among foreign countries, Home impose

tariff on its own net export sectors.

2. In the special cases of two countries or multiple countries but only Home and one foreign country (country 2) produce sector j goods, we don't need to consider the supply of other countries that been affected by the tariff. Then

$$t_2^j - t_2^0 = \frac{1}{w_2(1-\bar{r}_2)(1+\theta)} \left(\frac{\beta_j(\sum_m \frac{1}{1+\tau_{xm}^j}\pi_{m1}^j x_m - \pi_{12}^j x_1)}{L_2^j} - \frac{\beta_0(\sum_m \frac{1}{1+\tau_{xm}^0}\pi_{m1}^0 x_m - \sum_n \pi_{1n}^0 x_1)}{L_2^0} \right)$$

F Optimal Innovation Policies without tariff at the SS: Baseline

1. Steady state government problem: we can prove $\sum \gamma_{r2}^j / \alpha_2^j = 0$ (endT-Msector-2Country.tex, L893).

So without tariff, the SS optimal innovation satisfies:

$$[1-\beta(1-\delta)]\frac{w_1}{\alpha_1^j} = \frac{1}{\theta}\frac{w_1(1-r_1^j)L_1^j}{T_1^j} + \frac{1}{1+\theta}\frac{\beta_j}{T_1^j}\frac{\pi_{11}^j x_1}{\gamma_4 - \gamma_{L1}}\left[\left(\gamma_{L1}-\gamma_{L2}^j\right)\theta\pi_{12}^j + \sum_j(\gamma_{L1}-\gamma_{L2}^j)\beta_j\pi_{12}^j\right] \tag{A.4}$$

and the optimal export tax are:

$$1+\tau_x^j = \frac{(\gamma_4-\gamma_{L1})\left(1+\theta\pi_{22}^j\right)}{\left(\gamma_4-\gamma_5^j\right)\theta\pi_{22}^j}$$

2. Sum over j of the FOC for L_{1r}, (A.4), we get:

$$(1-\beta(1-\delta))\sum_j L_{1r}^j = \frac{\delta}{\theta}\sum_j L_{1p}^j$$

which hold in market equilibrium, i.e., government distort own innovation across sectors but not overall. The wedges across sectors average to zero. The sign of $\left(\gamma_{L1}-\gamma_{L2}^j\right)\theta\pi_{12}^j$ determines which sector's R&D should be subsidized.

3.

G Optimal Ramsey Policies