LEASING AS A MITIGATION CHANNEL OF CAPITAL MISALLOCATION

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Abstract

Leased capital accounts for about 20% of the total productive physical assets used by US publicly listed firms, and this proportion is even higher among small and financially constrained firms - over 40%. In this paper, we argue that leasing is an important alternative way of capital reallocation, complementary to directly purchasing capital from the reallocation market, and it significantly mitigates credit-constraint-induced capital misallocation. However, in the existing literature, leased capital is "unmeasured" capital in quantifying capital reallocation efficiency. Empirically, we show that neglecting leased capital and overlooking its mitigation effect leads to significant overestimations of capital misallocation (Hsieh and Klenow, 2009) and the cyclicality of capital reallocation (Eisfeldt and Rampini, 2006). Theoretically, we develop a general equilibrium model with heterogeneous firms, collateral constraint, and an explicit buy versus lease decision to demonstrate leasing's novel role in mitigating capital misallocation.

JEL Codes: D2, E22, E32, E44

Keywords: Leased capital, Collateral constraint, Capital misallocation, Capital reallocation

This Draft: Dec 2021

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I Introduction

Leasing is extensively used in capital markets and production. We document that leased capital accounts for about 20% of the total productive physical assets used by US publicly listed firms, and this proportion is even higher among small and financially constrained firms - over 40%. In this paper, we argue that leasing is an important alternative way of capital reallocation, complementary to directly purchasing capital from the reallocation market, and it mitigates the credit-constraint-induced capital misallocation.[1] When firms are financially constrained, the possibility for them to rent capital offers an alternative channel to improve capital reallocation efficiency and mitigates capital misallocation.

The significant magnitude of leased capital and the above intuition on the role of leasing in mitigating capital misallocation suggest that it is important for us to explicitly account for leased capital in measuring capital productivity, misallocation, and capital reallocation. However, before the recent lease accounting rule changes in ASC 842, operating lease was treated as an off-balance-sheet item, leaving leased capital as an important source of "unmeasured" capital.[2] This "unmeasured" leased capital leads to significant mis-measurements in measuring capital misallocation and the cyclical pattern of capital reallocation.[3] First, and unconditionally, neglecting leased capital produces an overestimation of capital misallocation, measured as dispersion in the marginal product of capital (MPK) (Hsieh and Klenow, 2009). The current literature (for instance, Chen and Song (2013)) does not consider the fact that firms use not only purchased capital but also leased capital to produce. Ignoring leased capital in the denominator leads to an overestimation of capital productivity, and such overestimation is disproportionately larger for small and financially constrained firms that rent more capital. And it in turn tends to exaggerate the capital misallocation measured as

[1] We use "lease" and "rent", "purchase" and "own" interchangeably in this paper.

[2] In February 2016, the Financial Accounting Standards Board (FASB) issued updated accounting standards for leases (ASU 2016-02, Topic 842). Effective from 2019, firms are required to recognize lease assets and lease liabilities from off-balance-sheet activities on their balance sheets, thereby increasing the transparency and comparability among organizations. The exact adoption rule differs across public and private firms. After adopting the new accounting rule, firms now report "Lease right-of-use asset" on the asset side, and both short-term and long-term lease liabilities on the liability side. These items were absent before the adoption of the new operating lease accounting rule. Additionally, firms are required to report the estimates of their operating leases, including the value, average regaining life, and discount rate, and disclose the possibility of renewing or extending existing leases. Similarly, the International Accounting Standards Board (IASB) also released IFRS 16 on new lease standards, requiring nearly all leases to be reported on lessees' balance sheets as assets and liabilities in 2016, effective for annual periods beginning on or after January 1, 2019.

[3] The focus of this paper is the off-balance-sheet item operating lease, whose ownership belongs to the lessor. There is another type of lease – capital lease, in which the lessee acquires ownership of the asset at the end of the lease's term. Capital lease was already on balance sheet before the lease accounting rule change, and is much lower in magnitude than operating lease in the US.

capital productivity dispersion. Focusing on US publicly listed firms, we find that Hsieh and Klenow (2009) type of capital misallocation can drop by nearly 50% when leased capital is correctly accounted for. This reduction is more salient for small and financially constrained firms.

Second, and conditionally, explicitly accounting for leased capital as an alternative capital reallocation is also important for us to correct for an overestimation of the cyclical pattern of capital reallocation. Eisfeldt and Rampini (2006) consider sales of property, plant and equipment (PPE) and acquisitions as capital reallocation, and document important evidence that capital reallocation is procyclical while the benefit of capital reallocation is countercyclical. Guided by our theoretical framework, we suggest a broader definition of capital reallocation that includes not only purchased capital on the reallocation market but also leased capital, which we identify as the lease-adjusted capital reallocation. Since the leased capital to physical capital ratio is highly countercyclical in the data, we find that the lease-adjusted capital reallocation is less procyclical, while the benefit of capital reallocation, as measured by the MPK dispersion, also becomes less countercyclical for the entire sample of US publicly listed firms. Moreover, if we look further at small firms and financially constrained firms, we find that their capital reallocation becomes acyclical, due to the fact that these firms rent more capital, in particular, in recessions. This evidence again strongly confirms our intuition that leasing provides an important mitigation effect to capital misallocation.

In order to formalize our intuition and provide precise empirical guidance, we develop a two-period general equilibrium model with heterogeneous firms, collateral constraints, and an explicit buy versus lease decision. Within this model framework, we demonstrate our novel economic mechanism: the possibility for firms to rent capital when they are financially constrained mitigates capital misallocation. Moreover, our model setup provides an explicit separation between intertemporal capital investment and capital purchase on the reallocation market, so that we can directly compare capital reallocation measures with and without adjusting for leased capital. In our model, we explicitly consider firms' buy versus lease decisions and formalize the trade-off between these two options. As in Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2010, 2013), we model leasing as highly collateralized albeit costly financing. On the one hand, leasing has a higher debt capacity and is highly collateralized because of its repossession advantage, which helps firms relax their financial constraints; on the other hand, leasing is costly because the lessor incurs a monitoring cost to avoid agency problems associated with the separation of capital ownership and control. Financially constrained firms, whose shadow cost of obtaining secured loans has increased, value the additional debt capacity more and find it optimal to lease capital despite of its expensive rents. Whether a firm chooses to buy or lease capital depends on the firm's initial

wealth and its idiosyncratic productivity. Consequently, the heterogeneity in wealth and financing needs translates into differences in the leased capital ratio in equilibrium: firms with a high need for capital but low initial wealth lease more of their capital to support their production.

In this theoretical setup, we show that in the cross-section, firms with higher financing needs (i.e., higher idiosyncratic productivity given the same wealth) are more likely to become constrained. Additionally, these firms use more leased capital. Their MPK is thus more overestimated compared to the traditional marginal product measure, which is unadjusted for leased capital. Ignoring leased capital also greatly overestimates the dispersion in MPK and exaggerates its implied capital allocation inefficiency due to credit constraints. Meanwhile, our model sheds light on the cyclical pattern of capital reallocation in the time series. When firms' initial wealth decreases, the leased capital ratio becomes higher, suggesting that leased capital ratio is countercyclical in equilibrium. The countercyclicality of leased capital also has implications for capital misallocation: adjusting for leased capital, the amount of capital reallocation is less procyclical, while the dispersion in MPK, interpreted as the benefit of capital reallocation, becomes less countercyclical.

Next, we conduct a counterfactual analysis to illustrate how the dispersion in MPK will deteriorate when the rental market is artificially closed. Without the rental market, our model implies a larger amount of capital misallocation and a correspondingly lower aggregate efficiency. These results point out the potential roles that leasing plays in mitigating capital misallocation induced from credit constraints and hence in improving aggregate productivity (i.e., lease contract provides an alternative capital reallocation channel from the perspective of financially constrained capital borrowers).

We then extend our model to consider monopolistic competition and the fixed cost of leasing. We find that none of the model implications on capital misallocation and reallocation are affected. Moreover, under different setups, we show why the existing measures of MPK and MPK dispersion are biased. We demonstrate that our adjustment to the denominator of MPK is robust (up to a constant) among all specifications.

Finally, we provide empirical evidence supporting our theory. We first document that the leased capital ratio is countercyclical. Further, the MPK dispersion adjusted for leased capital is less countercyclical than the unadjusted dispersion measures. The less countercyclical feature of the adjusted MPK dispersion is more pronounced among small and financially constrained firms. Our empirical findings also provide an important caveat to previous evidence on procyclical capital reallocation, as in Eisfeldt and Rampini (2006). If we recognize leasing as an important way of capital reallocation, we find that the adjusted capital reallocation

becomes less procyclical for the whole sample. The adjusted capital reallocation even becomes acyclical and mildly countercyclical for small and financially constrained firms, since they rent much capital to mitigate financial constraints, particularly in recessions.

Related literature Our study builds on the theories of corporate leasing decisions. Miller and Upton (1976), Myers, Dill and Bautista (1976), Smith Jr and Wakeman (1985), Lewis and Schallheim (1992), and Graham (2000) all show that taxes create incentives to lease. However, our model focuses on other dimensions – i.e., financial frictions and agency costs associated with the separation of capital ownership and control. The papers most related to ours are Eisfeldt and Rampini (2009), Rampini and Viswanathan (2010, 2013), Zhang (2012), Gal and Pinter (2017) and Li and Tsou (2019).[4] We draw elements from these papers to construct both collateral constraints and a firm's buy versus lease decision, and the differences lie in two dimensions. First, Eisfeldt and Rampini (2009) is a static model, and Rampini and Viswanathan (2010, 2013) and Zhang (2012) are dynamic models with a partial equilibrium framework. Gal and Pinter (2017) adopt a general equilibrium framework to study the aggregate properties, while Li and Tsou (2019) study a general equilibrium model with heterogeneous firms but without misallocation features. For our study, we set up a two-period general equilibrium model with heterogeneous firms, and this model can generate capital misallocation. Second, we focus on the asset side of leased capital, and analyze the economic consequences by aggregating micro-level firm behavior. Kermani and Ma (2020) study the heterogeneity in corporate debt contracts: asset-based debt focuses on the liquidation value of discrete assets as collateral, whereas cash flow-based debt relies on the borrower's going-concern value of the business and reflects more intensive performance monitoring. Operating leases are akin to the asset-based debt when lessors eventually repossess the leased assets at the end of leasing contracts.[5] Using the anti-recharacterization laws as an exogenous shock, Chu (2020) provides supporting empirical evidence with respect to the dynamic buy versus lease trade-off argued in the papers we described previously. Binfare et al. (2020) examine firms' choice of discount rates in valuing their leased assets.[6]

[4] Eisfeldt and Rampini (2009) provide a comprehensive review of this literature.

[5] Lian and Ma (2021) document that 80% of corporate borrowing for US nonfinancial firms is cash flow-based debt.

[6] The finance literature that connects firms' capital structure to asset collateralizability is also closely related to our paper. See Albuquerque and Hopenhayn (2004); Schmid (2008); Eisfeldt and Rampini (2007); Rampini and Viswanathan (2010, 2013); Nikolov, Schmid and Steri (2021); and Ai et al. (2020). Unlike most of these studies emphasizing the financing role of collateral, we investigate its implications on the asset side - in particular, the marginal product of capital. Moreover, most of these studies, with the exceptions of Rampini and Viswanathan (2010, 2013), do not consider the possibility for firms to rent capital, while we explicitly model firms' buy versus lease decision and focus on the macroeconomic implications.

Our study belongs to the macroeconomics literature with financial frictions. Brunnermeier, Eisenbach and Sannikov (2012) provide an excellent survey. Specifically, the papers that are most related to our study include Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010), Kiyotaki and Moore (2012), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2013), and Elenev, Landvoigt and Van Nieuwerburgh (2018). They all emphasize the importance of borrowing constraints and limited contract enforceability. Gomes, Yamarthy and Yaron (2015) develop a production-based asset pricing model to discuss the impact of financial frictions on risk premia. We differ from these studies by introducing leasing as a strongly collateralized, albeit costly, financing and explore the implications with respect to increasing the aggregate efficiency in the real economy.

Our paper relates to the literature that links aggregate total factor productivity (TFP) to capital misallocation caused by financial frictions at the firm level (for example, Buera, Kaboski and Shin (2011), Moll (2014), Buera and Moll (2015), and Ai et al. (2019)). Gilchrist, Sim and Zakrajšek (2013) focus on the cost of debt and study firm-specific borrowing costs. Using plant-level data, Midrigan and Xu (2014) quantify the relationship between financial constraints, capital misallocation, and aggregate TFP in South Korea. Gopinath et al. (2017) explain the decline of TFP and the increase of capital misallocation in South Europe, alongside declining real interest rates. Kehrig and Vincent (2017) study the interaction of financial frictions and adjustment costs in explaining recent dynamics of misallocation within firms. Also, Cavalcanti et al. (2021) utilize rich administrative firm-level data and explore the role of dispersion in financing costs on aggregate development and firm dynamics in Brazil. None of these papers, however, focus on the effect of leasing on financial-friction-induced capital misallocation as we do.

More broadly, our paper is connected to the general capital misallocation literature, seminal examples of which include Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) (see Restuccia and Rogerson (2017) and Hopenhayn (2014) for extensive reviews). Apart from the role of financial frictions in capital misallocation mentioned earlier, additional potential sources include adjustment costs (Asker, Collard-Wexler and De Loecker, 2014), information frictions (David, Hopenhayn and Venkateswaran, 2016), markups (Peters, 2020; Edmond, Midrigan and Xu, 2018; Haltiwanger, Kulick and Syverson, 2018), as well as firm-level risk premia (David, Schmid and Zeke, 2020). David and Venkateswaran (2019) develop a methodology to disentangle various sources of capital misallocation. Whited and Zhao (2021) study financial misallocation and find large real losses in China. Dou et al. (2021) discover that misallocation measures provide a more informative stochastic discount factor for capital market valuations. Meanwhile, Lanteri and Rampini (2021) study the effect of externalities on capital misallocation and reallocation. Empirically, Hsieh and Klenow (2009), David, Hopen-

hayn and Venkateswaran (2016), David and Venkateswaran (2019) and David, Schmid and Zeke (2020) find substantial capital misallocation in the US.[7] The extant literature neglects leased capital in quantifying capital misallocation, whereas our paper appropriately accounts for the "unmeasured" leased capital to re-estimate the capital misallocation for US public firms.

Our paper also contributes to the literature that emphasizes the importance of the cyclical properties of capital reallocation and capital misallocation. Eisfeldt and Rampini (2006), Kehrig (2015) and Ai, Li and Yang (2020) empirically document that the amount of capital reallocation is procyclical while the benefit of capital reallocation is countercyclical. Eisfeldt and Rampini (2006) rationalize these puzzling facts using a model in which the cost of capital reallocation is correlated with TFP shocks. Ai, Li and Yang (2020) study the link between financial intermediation and capital reallocation. Lanteri (2018) analyzes a model with endogenous partial irreversibility and used investment goods. Dong, Wang and Wen (2020) develop a search-based neoclassical model with capacity utilization to explain these facts.[8] Also, Eisfeldt and Rampini (2008), Kurlat (2013), Fuchs, Green and Papanikolaou (2016) and Li and Whited (2015) all analyze models of capital reallocation with adverse selection. Our paper emphasizes the role of leasing as an alternative capital reallocation channel that allows us to correct for an overestimation of the cyclical pattern of capital reallocation, partially reconciling the Eisfeldt and Rampini (2006) puzzle.

The rest of our paper is organized as follows. We summarize several empirical facts on the importance of leased capital and mismeasurement of MPK dispersion in Section II. We describe an equilibrium model with heterogeneous firms in which firms are subject to collateral constraints and have the option to lease capital in Section III and analyze our model implications in Section IV. We then use Section V to emphasize our model guidance on the empirical adjustment. In Section VI, we provide additional supporting evidence for our model. We conclude this paper in Section VII. Details on model solutions, proofs for propositions, alternative setups, and data construction are delegated to Appendices A to D.

[7] Bils, Klenow and Ruane (2021) propose a method for estimating the role of additive measurement errors. David and Venkateswaran (2019) apply this method to US public firms and find only about 10% of the observed MPK dispersion can be accounted for by such additive measurement errors.

[8] Cui (2017) studies the effects of financing constraints and partial irreversibility on the cyclicality of capital liquidation. See Eisfeldt and Shi (2018) for a survey of the literature on capital reallocation.

II Empirical facts

In this section, we provide aggregate and cross-sectional evidence that highlights the role of leasing as a source of external finance and as an important source of "unmeasured" capital in measuring the dispersion of the marginal product of capital (MPK).

II.A Importance of leased capital

We follow Rampini and Viswanathan (2013) and Lim, Mann and Mihov (2017) to estimate the amount of leased capital. We denote this direct capitalized item as leased capital (multiplier). Our second measure, which we denote as leased capital (commitment), follows Li, Whited and Wu (2016) and is equal to the present value of current and future lease commitments. We use Property, Plant and Equipment - Total (Net), i.e., PPENT, to measure purchased (owned) tangible capital and further define leased capital ratio as leased capital divided by the sum of leased and owned capital. Similarly, we define rental share as the ratio between rental expense over the sum of capital expenditure plus rental expense. The leased capital ratio and the rental share measure the proportion of total capital input in a firm's production obtained from leasing activity. We use total book assets (AT) to determine size groups. We measure the firm-level constraint by the Whited-Wu index (Whited and Wu (2006), Hansen et al. (2007), WW index hereafter).[9] Table I reports the summary statistics of leased capital ratio and leverage for the aggregate and for the cross-sectional firms in Compustat.

[Place Table I about here]

At the aggregate level, leased capital accounts for a substantial portion of overall productive assets - over 20%. Using lease commitment and rental share gives slightly lower proportions of 13% and 18%, respectively. The magnitude is consistent with Eisfeldt and Rampini (2009) and Rauh and Sufi (2012).[10] These observations serve as important evidence to illustrate how leased capital might be utilized in production. For leverage, considering leased capital will increase its overall level by 50% in our sample.

In the cross-section, the average leased capital ratio of small firms (0.48) is significantly higher than that of large firms (0.22); that is to say, small firms lease more. Meanwhile, the average debt leverage of small firms (0.10) is lower than that of large firms (0.20). Defined as the sum of debt and rental leverage, the lease-adjusted leverage ratios across two different

[9] The results are very similar when we use other financial constraint measures, such as SA index.

[10] Eisfeldt and Rampini (2009) emphasize that using lease commitments usually leads to a lower bound of leased capital estimation due to many missing observations.

groups are comparable to each other. A similar pattern holds for financial-constraint-sorted groups. These imply that leasing is an important source of external finance for small and financially constrained firms, and complements financial debt.

Our results in Table I are in line with Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013), who further show that ignoring leased capital brings significant bias in terms of a firm's investment, capital structure, and risk management, and this bias is asymmetric in that it is particularly severe for small and financially constrained firms. In what follows, we will present evidence to show that, due to the leasing accounting procedure, leased capital is an important type of "unmeasured" capital that leads to a significant mismeasurement in Hsieh and Klenow (2009) type of capital misallocation.

II.B Unadjusted and lease-adjusted MPK dispersion

Efficient capital allocation requires the marginal product of capital (MPK) to equalize across firms. MPK dispersion can therefore be interpreted as capital misallocation. Based on this insight, studies such as Hsieh and Klenow (2009) develop a measure of capital misallocation using dispersion of $\log(MPK)$ (or mpk). Using (average) capital productivity to measure mpk, these studies observe substantial capital misallocation in the US economy, and hence document large TFP losses. However, most of these mpk measures utilize owned tangible capital in the denominator, but ignore the contribution of leased capital.

Our stylized facts suggest that leasing activities account for a non-negligible proportion of total capital input in a firm's production, and hence we should adjust the "traditional" mpk measure by explicitly considering leased capital. To empirically adjust the MPK measure, we start by assuming that each firm faces a Cobb-Douglas production technology:

$$y_{it} = A_t z_{it} \left(K_{it}^o + K_{it}^l\right)^{\alpha} L_{it}^{1-\alpha} \tag{1}$$

where A_t is aggregate productivity, z_{it} is idiosyncratic productivity of firm i, K_{it}^o is owned capital, K_{it}^l is leased capital, L_{it} is the labor employed, and α is the elasticity of output with respect to total capital. Owned capital and leased capital are assumed to be perfect substitutes in production.[11]

Denoting the price of output i by p_{it}, firms take the price as given and choose their capital

[11]The assumption that owned capital and leased capital are perfect substitutes is consistent with previous studies (i.e., Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013)). This assumption can be verified in the data. It is also how Compustat records the data after the accounting rule change.

and labor to maximize profits:

$$\max_{K_{it}^o, K_{it}^l, L_{it}} \Pi_{it} = p_{it}y_{it} - MPK_{it}\left(K_{it}^o + K_{it}^l\right) - W_t L_{it} \tag{2}$$

where W_t is wage, and MPK_{it} denotes the marginal product of capital of firm i. For now we write the MPKs into reduced forms and endogenize them later in the model.

The first-order condition with respect to total capital is given by:

$$MPK_{it} = \alpha\left(\frac{p_{it}y_{it}}{K_{it}^o + K_{it}^l}\right) \tag{3}$$

With the Cobb-Douglas production function, the true marginal product of capital is proportional to the average product of capital, and equals the capital share multiplied by value-added $p_{it}y_{it}$ over the sum of owned capital and leased capital $K_{it}^o + K_{it}^l$. We denote it as the lease-adjusted MPK, MPK_{it}^{adj}.

However, prior literature, such as Hsieh and Klenow (2009) and Chen and Song (2013), doesn't correctly adjust MPK for leased capital.[12] Ignoring leased capital would overestimate the MPK. As noted before, in the cross-section, firms use leased capital differently: small and financially constrained firms lease more (and they tend to have high MPK). Intuitively, bringing back leased capital effectively narrows the divergence of MPKs, resulting in a lower adjusted MPK dispersion among all firms.

[Place Table II about here]

Table II confirms our intuitions. In Row 1, we present the *mpk* dispersion under the existing measure, following Chen and Song (2013) and the large literature. We denote it as the unadjusted measure. The unadjusted *mpk* dispersion is 0.48, consistent with that in David, Hopenhayn and Venkateswaran (2016) and David and Venkateswaran (2019).[13] We adjust for leased capital and report the adjusted *mpk* dispersion in Row 2. The adjusted *mpk* dispersion is 0.26. The difference between these two measures is 0.22, implying a 46% reduction. Motivated by the insight in Midrigan and Xu (2014) and David, Schmid and Zeke (2020), this significant drop in *mpk* dispersion would translate into a 10% overestimation of TFP losses in the US economy, suggesting that the US economy is more efficient in capital allocation than previously expected. Moreover, this reduction is clearly more prominent in

[12] We discuss in detail why these existing measures are biased in Section V.

[13] To isolate the firm-specific variation in our data series, we extract a time-by-industry fixed-effect from each and use the residuals. Industries are classified at the SIC 4-digit level. This is equivalent to deviating each firm from the unweighted average within its industry in each period.

small and financially constrained firms, consistent with the fact that small and financially constrained firms rely heavily on leased capital.[14]

III A general equilibrium model with heterogeneous firms

In this section, we present a two-period general equilibrium model with collateral constraints, capital reallocation, and leased capital to highlight our mechanism. The economy is populated by a representative household and heterogeneous firms.

III.A The household

We assume that a representative household is endowed with an initial wealth ϵ_0. The household maximizes log utility subject to standard intertemporal budget constraints:

$$\max_{C_0,C_1,B_0,w_0^i,K^l} E\left[\sum_{t=0}^{1}\beta^t u\left(C_t\right)\right] \tag{4}$$

$$s.t.: C_0 + B_0 + \int w_0^i V_0^i di + K_1^l = \epsilon_0 \tag{5}$$

$$\tau_l K_1^l + (1-\delta-h)K_1^l + R_f B_0 + \int w_0^i (V_1^i + D_1^i) di + W = C_1 \tag{6}$$

where β is the discount factor, and R_f is the gross risk-free interest rate. Eq. (5) is the budget constraint for the household in period 0. At period 0, the household purchases w_0^i shares of stock from firm i and preserves B_0 amount of cash for purchasing risk-free bonds. The household also serves as the lessor: he can transform net worth into K_1^l amount of leased capital and rent to firms. Eq. (6) is the budget constraint for the household at period 1. At period 1, the household rents out the leased capital to firms and gets the leasing payment $\tau_l K_1^l$. Additionally, he receives the debt repayment $R_f B_0$ and the labor income W, collects the dividend and return from holding stock shares $\int w_0^i (V_1^i + D_1^i) di$, as well as gets the resale value of leased capital $(1-\delta-h)K_1^l$.[15] δ is the rate of capital depreciation, and h is the monitoring cost of leased capital due to the separation of ownership and control, as in Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013). h captures the

[14] In untabulated results, we consider the existing measure of MPK using sales over owned capital. The monotonic relations between size groups and financially constrained groups remain the same.

[15] Since labor supply is inelastic and normalized to one, the total amount of labor income is W.

disadvantages of leased capital related to its faster depreciation rate in production and more costly maintenance.[16] The household uses all these resources to consume at period 1.

We denote M_1 as the stochastic discount factor defined by the household. Under the assumption of log utility, we have $M_1 = \beta \frac{C_0}{C_1}$ and $E[M_1 R_f] = 1$. The first-order condition of K_1^l indicates that $\tau_l + 1 - \delta - h = R_f$.

III.B Nonfinancial firms

There are two types of nonfinancial firms in our model: final goods producers and intermediate goods producers.[17]

1 Final goods producers

Final goods are produced by a representative firm using a continuum of intermediate inputs indexed by $i \in [0, 1]$. Because a final goods producer does not make intertemporal decisions in our model, we suppress time t in this section to save notation. We normalize the price of final goods to one and write the profit maximization problem of a final goods producer as:

$$\max_{\{y_i\}} \left\{ Y - \int_{[0,1]} p_i y_i di \right\}_{Y = \left[\int_{[0,1]} y_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}}$$

where p_i and y_i are the price and quantity of input i, respectively, and Y stands for the total output of final goods. The parameter η is the elasticity of substitution across input varieties. The optimality condition implies that the demand of the final goods producer is $y_i = {p_i}^{-\eta} Y$.

2 Intermediate goods producers

There is a unit measure of competitive intermediate goods producers, $i \in [0, 1]$, each of which produces a different variety of goods.[18] We assume firm i is endowed with initial wealth N_0^i.

In order to motivate an interesting reallocation market, we explicitly split the end of period 0 into two subperiods: the afternoon and the evening. In the afternoon of period

[16] Note that we assume firms will return the leased capital to the household after production and the world ends at period 1.

[17] We feature perfect competition among intermediate goods producers in our baseline model. We can easily extend to the monopolistic competition case and a more general form in which leasing involves additional costs compared to using owned capital. We discuss these extensions in Section V.

[18] We use "firm" and "intermediate goods producer" interchangeably in the remainder of the paper.

0, before knowing the idiosyncratic productivity z_i and the aggregate productivity A_1, firm i determines its owned capital stock K_1^o. In the evening, firm i observes its idiosyncratic productivity z_i in advance but does not observe the aggregate productivity. This assumption of "observing idiosyncratic shock ahead of time" is standard in the investment literature, as in Moll (2014) and Midrigan and Xu (2014). It is also consistent with the view that managers enjoy information advantage because of their potential insider information. Upon observing the idiosyncratic productivity shock, the market for capital reallocation opens.

Firm i's budget constraint in the evening can then be summarized as:

$$D_0^i + K_1^o + qRA_1^i = N_0^i + B_0^i, \tag{7}$$

where q is the market clearing price in the reallocation market.[19] Firm i determines its reallocation amount RA_1^i (i.e., purchasing or selling owned capital) on the reallocation market, through borrowing B_0^i from the household. The amount of borrowing is subject to a collateral constraint:

$$B_0^i \leq \theta(K_1^o + RA_1^i) \tag{8}$$

in which θ is the collateralizability characterizing the collateral constraint. It means that a maximum of θ fraction of the owned capital can be retrieved upon default.

Apart from purchasing or selling owned capital on the reallocation market, firm i can also lease capital (but pays the leasing fee at period 1). D_0^i is the dividend at period 0. Without loss of generality, we assume firms only pay dividends at period 1, i.e., $D_0^i = 0$.

In the morning of period 1, production occurs. Each firm faces a Cobb-Douglas production technology:

$$y_i = A_1 z_1^i \left(K_1^o + RA_1^i + (K_1^l)^i\right)^\alpha L_i^{1-\alpha} \tag{9}$$

Here $\alpha < 1$ is the capital share in production, L_i is labor, and y_i is the output of firm i. K_1^o is the predetermined capital stock before any information of aggregate shock or idiosyncratic shock is revealed. Additionally, firm i relies on RA_1^i and $(K_1^l)^i$ for its production. These components represent two sources of capital reallocation after observing the idiosyncratic productivity: one is the traditional purchasing or selling owned capital on the reallocation market, and the other is leasing. The second source is our focus and differs from the prior literature: we consider leasing to be an alternative capital reallocation channel.

[19] In Appendix A, we show that q is equal to 1 in equilibrium.

We can write firm i's dividend in period 1, D_1^i, as:

$$D_1^i = p_i y_i - \tau_l (K_1^l)^i - R_f B_0^i + (1-\delta)(K_1^o + RA_1^i) - WL_i \tag{10}$$

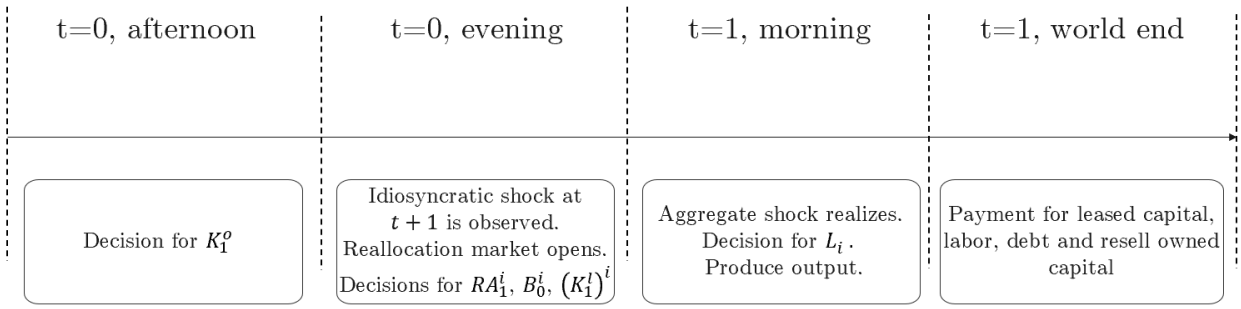
where p_i is taken as given since intermediate goods producers compete perfectly.

After production, both types of capital suffer depreciation. The firm has to pay back bond, interest, wage, and leasing fees. It resells owned capital and gives back the depreciated leased capital to the lessor.

The timing of events is illustrated in Figure I.

Figure I

Timing of events



As the world finishes at the end of period 1, $V_1^i = 0$. For each firm i, its objective is to maximize:

$$\max_{K_1^o, RA_1^i, B_0^i, (K_1^l)^i, L_i, D_1^i} V_0^i = E\left[M_1 D_1^i\right] \tag{11}$$

by choosing the initial owned capital stock K_1^o, a state-contingent plan for capital reallocation RA_1^i, borrowing from the household B_0^i, leased capital $(K_1^l)^i$, labor L_i, and its dividend D_1^i, subject to the budget constraint (7), the collateral constraint (8), and the law of motion for dividend in period 1 (10).

The firm problem is where our model departs from the frictionless neoclassical setup. The key constraint for borrowing is $B_0^i \leq \theta(K_1^o + RA_1^i)$. Without this collateral constraint, our model reduces to the frictionless neoclassical model.

III.C Market clearing conditions

To complete the specification of the model, we list the market clearing conditions as follows:

$$C_0 + \int w_0^i V_0^i di + \int D_0^i di + \int K_1^o di + K_1^l = \epsilon_0 + \int N_{i,0} di; \tag{12}$$

$$Y + \int (1-\delta)(K_1^o + RA_1^i) di + (1-\delta-h)K_1^l = C_1; \tag{13}$$

$$B_0 = \int (B_0)^i di; \tag{14}$$

$$w_0^i = 1, for\ \ all\ \ i \tag{15}$$

$$K_1^l = \int (K_1^l)^i di; \tag{16}$$

$$\int RA_1^i di = 0 \tag{17}$$

$$\int L_i di = 1; \tag{18}$$

The first two equations are the market clearing conditions for final output at period 0 and period 1, respectively. Eqs. (14) and (15) correspond to the bond and stock market clearing conditions, respectively. Eq. (16) is the leased capital market clearing condition, while Eq. (17) refers to the market clearing condition for capital reallocation. The last equation represents the labor market clearing condition, where we have normalized total labor supply to one.

IV Model implications

For simplicity, we assume there is no aggregate uncertainty, i.e., $A_1 = 1$. We also assume that there are only two possible realizations of idiosyncratic productivity shocks, z_L and z_H, with $Prob(z = z_H) = 1 - Prob(z = z_L) = \pi$. Firms are endowed with the same initial net worth N_0. These assumptions enable us to derive analytical results.

To facilitate discussion, we plot the equilibrium results of our two-period model to illustrate the mechanism through which a firm's initial wealth affects macro quantities. Table III lists the set of plausible parameters used in the numerical example.

[Place Table III about here]

IV.A Aggregation of the product market

We denote K_H as the total amount of capital used by a high productivity firm at time 1, K_L as the total capital used by a low productivity firm at time 1, and $K = \pi K_H + (1-\pi) K_L$ as the total capital in the economy. Note that K_H, K_L and K are the total capital amount after reallocation has taken place, i.e., $K_L = K_1^o + RA_1^L + (K_1^l)^L$ and $K_H = K_1^o + RA_1^H + (K_1^l)^H$. We define a capital ratio between high productivity and low productivity firm as $\phi = \frac{K_H}{K_L}$.

We can write aggregate output as a function of ϕ, which is specified in the following proposition.

Proposition 1. *The total output of the economy at period 1 is $Y = f(\phi)K^\alpha$, where the function $f : \left[1, \hat{\phi}\right] \to [0,1]$ is defined as:*

$$f(\phi) = \left\{ (1-\pi) z_L^{\frac{(\eta-1)}{\alpha\eta-\alpha+1}} \left(\frac{1}{1-\pi+\pi\phi} \right)^{\frac{\alpha(\eta-1)}{\alpha\eta-\alpha+1}} + \pi z_H^{\frac{(\eta-1)}{\alpha\eta-\alpha+1}} \left(\frac{\phi}{1-\pi+\pi\phi} \right)^{\frac{\alpha(\eta-1)}{\alpha\eta-\alpha+1}} \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}}$$

and $\hat{\phi} = \left(\frac{z_H}{z_L}\right)^{\eta-1}$.

We assume the following normalization: $(1-\pi) z_L^{\eta-1} + \pi z_H^{\eta-1} = 1$.

The (true) marginal product of low and high productivity firms, MPK_L and MPK_H,[20] *can be written as:*

$$MPK_L = \alpha f(\phi) K^{\alpha-1} \left[(1-\pi) + \pi\phi\right] \left\{ (1-\pi) + \pi \hat{\phi}^{\frac{1}{\alpha\eta-\alpha+1}} \phi^{\frac{\alpha(\eta-1)}{\alpha\eta-\alpha+1}} \right\}^{-1}$$

$$MPK_H = \hat{\phi}^{\frac{1}{1+\alpha\eta-\alpha}} \phi^{\frac{-1}{1+\alpha\eta-\alpha}} MPK_L$$

Proof. See Appendix B.1.

It is not hard to show that the efficient level of ϕ, which implies an equalization of MPK across all firms, is $\hat{\phi} = \left(\frac{z_H}{z_L}\right)^{\eta-1}$ and $f(\hat{\phi}) = 1$. The function $f(\phi)$ is a measure of the efficiency in the entire economy and is increasing in ϕ.

IV.B Collateral constraint and buy versus lease decision

We use this section to discuss whether a firm becomes constrained and whether it leases capital.

[20] These MPK expressions correspond to the true adjusted $MPK^{adj.}$s.

1 Collateral constraint

Given that firms are endowed with the same N_0 but have two types of productivities, they naturally become constrained differently. The following proposition characterizes the nature of the binding constraints.

Proposition 2. *There exist cutoff values $\widehat{N}$ and $\overline{N}$, such that*

- *If $N_0 \geq \widehat{N}$, then the first best allocation is achieved.*
- *If $\overline{N} \leq N_0 < \widehat{N}$, then the collateral constraints for high productivity firms bind.*
- *If $0 < N_0 < \overline{N}$, then the collateral constraints for both types of firms bind.*

Proof. See Appendix B.2.

The above proposition implies that given the initial wealth N_0, whether a firm is constrained is completely determined. When initial wealth N_0 is higher than $\widehat{N}$, the wealth level is high enough so the collateral constraints never bind. As the wealth level decreases, when $\overline{N} \leq N_0 < \widehat{N}$, the collateral constraint binds only if the firm receives a high productivity shock because it has higher financing needs and requires more capital to arrive at the first best allocation. In the region where $0 < N_0 < \overline{N}$, firms are endowed with little wealth and the collateral constraints bind for both realizations of idiosyncratic productivity shocks.

Proposition 2 has several important implications. First, in the cross-section, the collateral constraint is more likely to bind for firms with high idiosyncratic productivity. Second, in the time series, the collateral constraint is more likely to bind when firms' initial wealth is low. This is the amplification mechanism in our model. Adverse shocks to firms' initial wealth are amplified because they tighten the collateral constraints.

2 Buy versus lease decision

Leasing has its benefits and costs. In the following, we present the user costs to analyze firms' decision on whether to lease or to buy owned capital on the capital reallocation market. We first set up the Lagrangian of a typical firm, before capital reallocation occurs. We outline these details in Appendix A.

If the firm turns out to be a high productivity firm, we denote the multipliers on Eqs. (7), (10), (8) and the non-negativity of $K_1^o + RA_1^H$, $(K_1^l)^H$ and D_1^H by $\pi\eta_{H0}$, $\pi\eta_{H1}$, $\pi\xi_{H0}\eta_{H0}$, $\pi\bar{\nu}_{H0}\eta_{H0}$, $\pi\underline{\nu}_{H0}\eta_{H0}$, and πd_{H1}, respectively. If the firm turns out to be a low productivity firm, we denote the multipliers on Eqs. (7), (10), (8) and the non-negativity of $K_1^o + RA_1^L$,

$(K_1^l)^L$ and D_1^L by $(1-\pi)\eta_{L0}$, $\pi\eta_{L1}$, $(1-\pi)\xi_{L0}\eta_{L0}$, $(1-\pi)\bar{\nu}_{L0}\eta_{L0}$, $(1-\pi)\underline{\nu}_{L0}\eta_{L0}$, and $(1-\pi)d_{L1}$, respectively.

With a slight abuse of notation, we use i to nest both firm types in period 1. For type i, the user cost of leased capital is:

$$\tilde{\tau}_{l,i} = M_1 \frac{\tau_l}{\eta_{i0}} = \frac{M_1}{\eta_{i0}} \tau_l = \tilde{M}_i (R_f - 1 + \delta + h) \tag{19}$$

that is, the leasing fee in terms of the marginal value of net worth for firm i, $\frac{\tau_l}{\eta_{i0}}$, discounted by the SDF M_1. η_{i0} is the marginal value of net worth for firm i at time 0.

We define firm i's user cost of buying owned capital on the capital reallocation market as:

$$\tilde{\tau}_{o,i} = 1 - \tilde{M}_i (1 - \delta) - \theta \xi_{i0} \tag{20}$$

The interpretation is that the user cost of buying owned capital on the reallocation market is equal to the current price, 1, minus the discounted resale value, and also subtract the marginal value of relaxing the collateral constraint for owning this capital.

To discuss the trade-off through comparing the user costs of buying owned versus leasing capital, we start by defining a shadow interest rate R_I for the borrowing and lending among firms:

$$R_{I,i} = \frac{1}{\tilde{M}_i}$$

and hence a wedge $\Delta_i = R_{I,i} - R_f = R_f (\eta_{i0} - 1) = R_f \frac{\xi_{i0}}{1-\xi_{i0}} \equiv \Delta(\xi_{i0})$, which is an increasing function of ξ_{i0}. When the collateral constraint is binding, this wedge becomes strictly positive. Specifically, it reflects a premium that firms must pay for the loans among themselves, when cheaper household loans become inaccessible due to a binding collateral constraint.

Using this wedge and the net interest rate $r_f = R_f - 1$, we can re-write the user costs as:

$$\tilde{\tau}_{l,i} = \tilde{M}_i \tau_l = \frac{r_f + \delta + h}{R_f + \Delta_i} \tag{21}$$

and

$$\tilde{\tau}_{o,i} = \frac{r_f + \delta + \Delta_i}{R_f + \Delta_i} - \theta \xi_{i0} \tag{22}$$

The difference between two user costs (lease - own) is hence:

$$\tilde{\tau}_{l,i} - \tilde{\tau}_{o,i} = \frac{h}{R_f + \Delta_i} - \frac{\Delta_i}{R_f + \Delta_i} + \theta\xi_{i0} = \frac{\eta_{i1}}{\eta_{i0}}h + \xi_{i0}(\theta - 1) \tag{23}$$

The benefit of leasing is the premium saved on internal funds due to constraints, while the cost of leasing includes the additional monitoring cost and the cost of giving up the marginal value of relaxing the collateral constraint when buying this capital. In the environment of collateral constraint, $\theta < 1$, and multipliers are non-negative. When firms become sufficiently constrained (ξ_{i0} sufficiently large), the benefit of leasing dominates its cost, and firms start to lease. The benefit of leasing has been emphasized in the literature (e.g., Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013)).

Firms that are differently constrained will naturally make different leasing decisions. The following proposition characterizes the properties of the leasing decisions.

Proposition 3. *There exist cutoff values $\widehat{N}_L$ and $\overline{N}_L$, such that*

- *If $N_0 \geq \widehat{N}_L$, then no firms lease capital.*
- *If $\overline{N}_L \leq N_0 < \widehat{N}_L$, then only high productivity firms lease capital.*
- *If $0 < N_0 < \overline{N}_L$, then both types of firms lease capital.*
- *Under reasonable parameter value for agency cost, h, $\overline{N}_L < \overline{N} < \widehat{N}_L < \widehat{N}$.*

Proof. See Appendix B.3.

Proposition 3 implies that given the initial wealth N_0, firms' buy versus lease decisions are completely determined. When initial wealth N_0 is higher than $\widehat{N}_L$, the wealth level is high enough and no firms will use the costly leased capital. As the wealth level decreases, when $\overline{N}_L \leq N_0 < \widehat{N}_L$, only the firms that receive a high idiosyncratic productivity shock lease. In the region in which $0 < N_0 < \overline{N}_L$, both types of firms lease. Compared with Proposition 2, we have $\overline{N}_L < \overline{N} < \widehat{N}_L < \widehat{N}$ under reasonable parameters. That is, firms use leased capital only if they become sufficiently constrained. High productivity firms lease before low productivity firms become constrained. Meanwhile, leased capital is more likely to be used when firms' initial wealth is low, indicating that leasing is countercyclical.

3 Graphic illustration

In Figure II, we plot the collateral constraint multipliers on the left and leased capital ratio on the right. We denote the thresholds $\widehat{N}$, $\overline{N}$, $\widehat{N}_L$, and $\overline{N}_L$ as in our propositions. Indeed, we

find that increases in firms' initial net worth above $\widehat{N}$ do not affect the collateral multipliers, because productivity is constant and capital allocation stays at its first-best level. As N_0 decreases toward $\overline{N}$, only the collateral constraint for high productivity firms binds. When high productivity firms become sufficiently constrained ($N_0 < \widehat{N}_L$), they begin to lease. As N_0 drops below $\overline{N}$, the collateral constraint for both firms binds. Based on our parameter choice, $\overline{N} < \widehat{N}_L$. Thus, high productivity firms lease capital before low productivity firms become financially constrained. Similarly, when low productivity firms become constrained above a certain level ($N_0 < \overline{N}_L$), they start to lease capital.

[Place Figure II about here]

The fact that leased capital ratio increases when N_0 decreases sheds light on its cyclical pattern. Since N_0 is positively related to the aggregate shock, the leased capital ratio is countercyclical.

IV.C Misallocation

We measure misallocation using the cross-sectional dispersion of $\log(MPK)$. In this part, we first discuss two types of MPK: the adjusted MPK and the unadjusted MPK. We then analyze two corresponding types of misallocation: the adjusted as well as the unadjusted misallocation.

1 MPK

In the data, the first-order derivative (MPK) is not directly available. Thanks to the above production function assumption, we have the adjusted **true** MPK as:

$$MPK^{adj.} = \alpha \frac{p_i y_i}{K_1^o + RA_1^i + (K_1^l)^i} = \alpha \frac{Value\text{-}Added}{Total\ \ Capital}$$

The unadjusted MPK is:

$$MPK^{unadj.} = \alpha \frac{p_i y_i}{K_1^o + RA_1^i} = \alpha \frac{Value\text{-}Added}{Owned\ \ Capital}$$

These two forms of MPK are only different when leased capital is used in production at the corresponding productivity. Using $MPK^{unadj.}$ leads to an overestimation of the true MPK level in this case. The overestimation is more severe for firms with substantial amounts of leased capital. We note that the $MPK^{adj.}$s are the MPKs mentioned in Proposition 1.

Propositions 2 and 3 together show that each firm's adjusted and unadjusted MPKs are completely determined. Given the same wealth, firms with high idiosyncratic productivities are more likely to lease capital. Their adjusted MPKs are thus more likely to be overstated.

Empirically, small and constrained firms lease more of their capital and tend to have high $MPK^{unadj.}$ when such capital is ignored. Hence, adjusting for leased capital reduces high $MPK^{unadj.}$ more (than it reduces low $MPK^{unadj.}$). Consequently, the difference of measured MPK is lower and hence lower misallocation is expected. We discuss misallocation in detail in the following.

2 Adjusted misallocation

Because we assume two types of idiosyncratic productivity, the cross-sectional variance of $\log(MPK^{adj.})$ can be computed as:

$$\begin{aligned} Var[\log\left(MPK^{adj.}\right)] &= E\left[\left\{\log\left(MPK^{adj.}\right) - E\log(MPK^{adj.})\right\}^2\right] \\ &= \pi(1-\pi)\left[\log\left(MPK_H^{adj.}\right) - \log(MPK_L^{adj.})\right]^2 \end{aligned} \tag{24}$$

This equation suggests that given the probability π, the dispersion of $\log(MPK^{adj.})$ is measured by the distance between $\log(MPK_H^{adj.})$ and $\log(MPK_L^{adj.})$, or the ratio of $\frac{MPK_H^{adj.}}{MPK_L^{adj.}}$.

From Proposition 1, we know $\frac{MPK_H^{adj.}}{MPK_L^{adj.}} = \hat{\phi}^{\frac{1}{1+\alpha\eta-\alpha}}\phi^{\frac{-1}{1+\alpha\eta-\alpha}}$, which is decreasing in ϕ.

When all firms are unconstrained, they optimally choose their capital and hence $\phi = \widehat{\phi}$. There is no capital misallocation. When some firms become constrained (but not to the point where all firms lease), firms cannot choose the optimal allocation, $\phi < \widehat{\phi}$, and misallocation occurs.

Specifically, in the region in which high productivity firms become constrained but low productivity firms are unconstrained, high productivity firms cannot choose the optimal allocation, in this case $\phi < \widehat{\phi}$, and misallocation is non-zero. When high productivity firms become more constrained while low productivity firms are still unconstrained, ϕ goes down and misallocation rises. If high productivity firms begin to lease capital while low productivity firms can still achieve the optimal allocation, we see a constant wedge h between two MPKs by comparing Eqs. (A5) and (A6). ϕ achieves a relatively low position, and hence the misallocation still exists. When high productivity firms lease and low productivity firms become constrained, the MPK difference h is offset by the higher user cost of owned capital

for low productivity firms. As a result, ϕ goes up and misallocation decreases.[21]

3 Unadjusted misallocation

The cross-sectional variance of $\log(MPK^{unadj.})$ is calculated as:

$$\begin{aligned} Var[\log\left(MPK^{unadj.}\right)] &= E\left[\left\{\log\left(MPK^{unadj.}\right) - E\log(MPK^{unadj.})\right\}^2\right] \\ &= \pi(1-\pi)\left[\log\left(MPK_H^{unadj.}\right) - \log(MPK_L^{unadj.})\right]^2 \end{aligned} \tag{25}$$

This expression indicates that given the probability π, the dispersion of $\log(MPK^{unadj.})$ is measured by the distance between $\log(MPK_H^{unadj.})$ and $\log(MPK_L^{unadj.})$, or the ratio of $\frac{MPK_H^{unadj.}}{MPK_L^{unadj.}}$.

We combine the unadjusted MPK dispersion with the adjusted one. Denote $s_i = \frac{(K_1^l)^i}{K_i}$, where $K_i = K_1^o + RA_1^i + (K_1^l)^i$ and $i = H, L$, we have:

$$\frac{MPK_H^{unadj.}}{MPK_L^{unadj.}} = \frac{MPK_H^{adj.}}{MPK_L^{adj.}} \times \frac{1-s_L}{1-s_H} = \hat{\phi}^{\frac{1}{1+\alpha\eta-\alpha}} \phi^{\frac{-1}{1+\alpha\eta-\alpha}} \frac{1-s_L}{1-s_H}$$

In cases in which no firms lease capital, the unadjusted MPK dispersion is the same as the adjusted MPK dispersion: there is no capital misallocation when all firms are unconstrained since they can optimally choose their capital; misallocation occurs when some firms become constrained.

From Proposition 3, we know firms with the same initial wealth but high productivity are more likely to lease capital. When high productivity firms use leased capital (but low productivity firms don't yet lease), the unadjusted MPK dispersion overstates the adjusted MPK dispersion because of the additional term of $(1 - s_H)$ in the denominator.

When both types of firms use leased capital, the unadjusted misallocation achieves the highest level, unlike the adjusted misallocation being zero. This is because when all firms lease, they have the same adjusted MPK, implying that ϕ is constant. The term $\frac{1-s_L}{1-s_H}$ is also constant because firms will buy the same amount of owned capital (up to the constraint).

[21] Eventually, when both types begin to lease, $\widehat{\phi} = \phi$ and no misallocation occurs in the economy since all firms have the adjusted MPK equal to $r_f + \delta + h$, as indicated by Eqs. (A6) and (A7) with $\underline{\nu}_{H0}$, $\underline{\nu}_{L0}$ both equal to 0. Under the setup with fixed cost in Section V, we show when both firms lease, misallocation still exists. Nevertheless, all our results indicate that misallocation doesn't necessarily go up when initial net worth drops.

4 Graphic illustration

We denote the thresholds $\widehat{N}$, $\overline{N}$, $\widehat{N}_L$, and $\overline{N}_L$ as in our propositions. Then we plot MPKs under our parameter choices.

[Place Figure III about here]

In the left panel of Figure III, we plot the adjusted MPKs for both firm types. For initial wealth above $\widehat{N}$, there is no capital misallocation because both firms are unconstrained. When N_0 decreases to the level lower than $\widehat{N}$, high productivity firms become constrained and their MPK increases because of an additional element on the collateral multiplier. This is the capital misallocation induced by financial frictions, as argued in a large body of literature. As long as $N_0 > \widehat{N}_L$, no firms lease capital and the increase of misallocation is associated with the drop in N_0. Meanwhile, the adjusted MPK is the same as the unadjusted MPK since no leased capital is utilized in the economy yet. As soon as $N_0 < \widehat{N}_L$, high productivity firms begin to lease in order to relax the collateral constraints. That is, leased capital adds an upper bar to the adjusted MPK for high productivity firms (but the adjusted MPK does change since R_f is endogenous and changes with initial wealth). Within this region, high productivity firms have an adjusted MPK of $r_f + \delta + h$. The adjusted MPK for low productivity firms is $r_f + \delta$. We can easily see there is a constant wedge h. When N_0 further drops below $\overline{N}$, the divergence between two adjusted MPKs is lower since low productivity firms become more constrained and have a higher MPK. Eventually, both firms lease capital ($N_0 < \overline{N}_L$) and the MPK divergence disappears in the economy. In summary, the adjusted MPK dispersion does not necessarily increase when net worth decreases - suggesting that the adjusted MPK dispersion is less countercyclical, or even acyclical.

We plot the unadjusted MPKs on the right panel in Figure III. We notice that only when initial wealth is below $\widehat{N}_L$ will the unadjusted MPKs start to deviate with the adjusted ones. Indeed, it shows that dispersion of the unadjusted MPK overstates the adjusted MPK dispersion when leased capital is in use ($(K_1^l)^i > 0$).

Further decreases in initial wealth bring greater dispersion between the unadjusted MPKs. This implies that Hsieh and Klenow (2009) type of misallocation rises as firms' net worth declines. Our model features countercyclical unadjusted MPK dispersion, consistent with the evidence in Eisfeldt and Rampini (2006) and Ai, Li and Yang (2020).

IV.D Reallocation

When the initial wealth is high, both types of firms are unconstrained. They optimally and freely reallocate capital across themselves. When some firms become constrained, they cannot achieve the first best outcome by purchasing or selling owned capital on the reallocation market.

We define the total amount of capital reallocation as the sum of all capital sales plus all capital purchases. We use the sum of absolute value change of leased capital (compared to period 0) to obtain the lease-adjusted capital reallocation.

In Figure IV, we plot the capital reallocation of high and low productivity firms as functions of firms' initial wealth. We denote the thresholds $\widehat{N}$, $\overline{N}$, $\widehat{N}_L$, and $\overline{N}_L$ as in our propositions.

[Place Figure IV about here]

We can see that capital reallocates from low productivity firms to high productivity firms when wealth is high (i.e., when $N_0 > \overline{N}$). As initial wealth decreases, the amount of capital reallocation drops. A further decrease in initial wealth below $\overline{N}$ is associated with a zero capital reallocation amount. That is, capital reallocation is procyclical. Our model is consistent with the fact documented in Eisfeldt and Rampini (2006). We see that the lease-adjusted reallocation is less procyclical, as shown in the right panel.

IV.E The mitigation effect of leasing

When there is no rental market, both firms are constrained eventually. According to the budget constraint (7) and collateral constraint (8), they can only have capital up to the level of $\frac{1}{1-\theta}N_0$. In this case, ϕ is as low as 1 and misallocation is large. However, in the economy with leasing, both firms can turn to leased capital and achieve an optimal ϕ with no misallocation. This is the mitigation effect of leasing on capital misallocation.

We discuss the variables of interest below. We note that in this economy, the adjusted and unadjusted MPKs are the same since no leased capital is allowed.

Counterfactual analysis: MPK dispersion In the bottom panel of Figure V(a), when N_0 decreases to a level lower than $\widehat{N}$, the MPKs for two types of firms diverge since only the high productivity firms become constrained. With a further drop in initial wealth N_0, the constraints for low productivity firms will also bind. The MPK dispersion still exists and

further increases since firms are constrained differently. This is the mechanism that generates countercyclical dispersion of MPKs in most macroeconomic models without a rental market.

[Place Figure V about here]

It's noteworthy that the point in which both firms become constrained (i.e., $\tilde{N}$) in this economy is different from that threshold in the model with a rental market ($\overline{N}$). Because leasing endogenously improves the capital allocation and lifts R_f, shutting it down will lead to a low interest rate; hence, a higher initial net worth is needed for low productivity firms to be unconstrained.

To gain intuitions for higher misallocation without a rental market, we look at the region between $\tilde{N}$ and $\widehat{N}_L$. When the rental market shuts down, naturally the adjusted MPK for high productivity firm is higher than its counterpart in the benchmark model since there is no additional leased capital to utilize and the production function is of decreasing return to scale. The adjusted MPK for a low productivity firm without the rental market is lower than its counterpart because of a lower interest rate. A higher-than-before adjusted MPK for high productivity firms, along with a lower-than-before adjusted MPK for low productivity firms, implies higher misallocation when the rental market is closed.

Counterfactual analysis: aggregate outcome We demonstrate the impact of leasing on aggregate efficiency in Figure V(b). We can clearly see the difference between economies with and without the rental market: when initial wealth is relatively low, the aggregate efficiency measure is higher in the benchmark economy than that in the economy without the rental market. It seems puzzling that leasing will give us the first-best outcome. We argue that leasing generates a constrained efficient outcome since firms are already endowed with low and insufficient initial wealth. Nevertheless, Figure V(b) strongly suggests that the impact of leasing on the whole economy is more pronounced in the crisis region.

IV.F Testable implications

From the equilibrium quantities discussed above, we see that our model generates rich testable implications for the cyclical pattern of leased capital ratio, capital reallocation, unadjusted MPK dispersion, and adjusted MPK dispersion (Capital misallocation and capital reallocation are naturally linked since capital misallocation reflects the benefits of capital reallocation):

1. The leased capital ratio (rental share) is countercyclical, as shown in Figure II(b).

2. Adjusting for lease, capital reallocation becomes less procyclical, as suggested by Figure IV(b).

3. Adjusting for lease, the benefit of capital reallocation (as measured by MPK dispersion) becomes less countercyclical, corresponding to Figure III(a).

In Section VI, we provide the empirical evidence to support these testable implications.

V Model guidance on empirical adjustment and extensions

Our model provides precise guidance on the empirical adjustment to MPK, by correcting the amount of utilized capital in the denominator. For the numerator, however, our data sample Compustat does not include a direct measure of value-added in all years, nor does Compustat have information on firm-specific wage compensation. Nevertheless, Compustat contains information on operating income (ex rental expense), corresponding to $py - WL$. In our baseline model, $py - WL$ is equal to αpy. That is to say, operating income (ex rental expense) in Compustat corresponds to αpy. Hence, we can compute the adjusted mpk as the log difference between operating income (ex rental expense) (OIBDP+XRENT) and total utilized capital.[22]

Our analysis shows that there is another route of adjusting towards the true MPK - through adjusting the numerator while keeping the denominator at owned capital. The baseline model suggests that we may directly subtract the rental fees to adjust the numerator. This implies that the measure of MPK in Chen and Song (2013), which uses OIBDP in the numerator and PPENT in the denominator, is correct, as shown below:

$$MPK^{C.\&S.} = \frac{OIBDP}{Owned\ Capital} = \frac{(OIBDP + XRENT) - XRENT}{Owned\ Capital} = \frac{\alpha p_i y_i - \tau_l (K_1^l)^i}{K_1^o + RA_1^i}$$

where $\alpha p_i y_i$ can be replaced by $\tau_l \left(K_1^o + RA_1^i + (K_1^l)^i\right)$ when a firm starts to lease, indicating $MPK^{C.\&S.}$ is equal to the per unit rental fee τ_l. This is the true MPK when a firm leases in the baseline model.

The above numerator adjustment suggests the mpk dispersion using Chen and Song (2013) should yield the same estimates with the adjusted measure using operating income (ex rental

[22] This operating income (ex rental expense), OIBDP+XRENT, corresponds to the accounting variable EBITDAR - earnings before interest rate, depreciation, amortization and rental expense. Indeed, as emphasized in Rauh and Sufi (2012), incorporating operating leases as a form of capital requires adding back the rental expense to operating cash flows.

expense) and total utilized capital. Obviously, the equivalence between these two types of adjustment is inconsistent with the empirical evidence presented in Section II.B.

In fact, this numerator adjustment type is subject to model specification errors and can be easily contaminated by different model extensions. We consider two model extensions - one under the monopolistic competition setup, while the other considers the fixed cost of renting capital. The detailed setup, optimality conditions, and MPK can be found in Appendix C.

These two simple extensions, along with our benchmark model, suggest various adjustment to the numerator. This confirms that model specification errors are indeed severe for the numerator adjustment. On the other hand, all model variations imply that the adjustment to the denominator is robust and subject to minimum (and reasonable) assumptions (e.g., the assumption that firms have the same market power within the same industry). There are no changes of model implications for misallocation, as misallocation is measured by within-industry dispersion of $\log(MPK)$. The implications on reallocation and the mitigation effect of leasing are also preserved, since considering these additional features only creates gaps between total output and total cost, or, between marginal benefit and rental rate. We therefore conclude that our adjustment to the denominator is robust and should work best.

VI Empirical evidence

In this section, we present empirical evidence that supports our model predictions in Section IV.F. Our results provide additional caveat to prior literature on capital misallocation and capital reallocation (Eisfeldt and Rampini, 2006; Ai, Li and Yang, 2020).

VI.A Countercyclical leased capital ratio

We focus on the cash-flow-based leased capital ratio (i.e., rental share) by computing the percentage of aggregate rental fees in total expenditure (sum of capital expenditure and rental fees) each year.[23] We plot the time series of the leased capital ratio in the top panel of Figure VI. In the bottom panel, we show the H-P filtered cyclical components of the leased capital ratio and output. The output data is published by the Federal Reserve Bank of St. Louis. The shaded areas in both panels indicate NBER-classified recessions. Clearly,

[23]Using the capital-stock-based leased capital ratio also produces a negative, yet less significant, correlation with output. This is because the flow-based measure is naturally more sensitive to macroeconomic fluctuations, while the stock-based measure is less sensitive due to its time-to-build features.

whenever there is a recession, the leased capital ratio rises. The leased capital ratio exhibits strong countercyclicality, with a correlation of -0.50 (t-stat$= -3.25$) with output. A similar conclusion has been documented in Gal and Pinter (2017) and Zhang (2012).

VI.B Adjusting for leased capital in MPK dispersion and capital reallocation

It is well documented in Eisfeldt and Rampini (2006) and Ai, Li and Yang (2020) that the amount of capital reallocation is procyclical, and that the cross-sectional dispersion of MPK is countercyclical. The countercylicality of leased capital ratio, however, suggests that adjusting for leased capital leads to significant implications in measured MPK dispersion and capital reallocation. We test these two implications in this section.

1 Cyclical pattern of capital reallocation

Eisfeldt and Rampini (2006) measure capital reallocation using the sum of sales of PPE (SPPE) and acquisitions (AQC):

$$RA^{unadj.} = AQC + SPPE$$

As we argued before, in each period, firms could also turn to leased capital as a reallocation channel. That is, the lease-adjusted capital reallocation amount is the sum of sales of PPE (SPPE), acquisitions (AQC), and the leased capital investment:

$$RA^{adj.} = AQC + SPPE + |K^l - (1-\delta)\, K^l_{-1}|$$

where K^l is the amount of leased capital at current year, K^l_{-1} is the amount of leased capital of the previous year, and δ is the depreciation rate. The term '$K^l - (1-\delta)\, K^l_{-1}$' captures a "synthetic" investment on leased capital. The fact that leasing behavior is countercyclical is manifested in the additional term, which makes the adjusted reallocation less procyclical.

[Place Table IV about here]

As can be seen from the first two rows in Table IV, when leased capital is factored in, the roughly 30% reduction in the correlation of capital reallocation with output is substantial. We further sort firms into size groups and financially constrained groups. Consistent with the previous literature, the unadjusted capital reallocation amount exhibits strong and consistent

procyclicality with output across all subgroups. Surprisingly, for the lease-adjusted capital reallocation, the correlation with output is much lower - it is even negative in small and financially constrained groups. The results suggest that the lease-adjusted capital reallocation measures are indeed less procyclical.

To alleviate the effects of variations in capital prices, we study the capital turnover rates (defined as reallocation normalized by the lagged total assets) in Rows 3 and 4 of Table IV. We find that a similar pattern holds: the turnover rate of capital reallocation becomes less procyclical, and the effect is more pronounced within small and financially constrained firms. The empirical evidence of capital reallocation is consistent with our model predictions.

2 Cyclical pattern of *mpk* dispersion

Under the assumption that owned capital and leased capital are perfect substitutes, we know the lease-adjusted MPK is:

$$MPK^{adj.} = MPK^{unadj.} \times (1 - s)$$

Therefore, the ratio between lease-adjusted MPK can be written as:

$$\frac{MPK_H^{adj.}}{MPK_L^{adj.}} = \frac{MPK_H^{unadj.}}{MPK_L^{unadj.}} \times \frac{1 - s_H}{1 - s_L}$$

This equation indicates that the lease-adjusted MPK dispersion is a joint product of the unadjusted MPK dispersion and leased capital ratio dispersion. In the area where low productivity firms don't lease, we can clearly see the countercyclicality of unadjusted MPK dispersion is weakened by the procyclical term $(1 - s_H)$, since leasing behavior is countercyclical.

[Place Table V about here]

In Table V, we report the correlation of MPK dispersion with output. At the aggregate level, the lease-adjusted MPK dispersion is acyclical: though the correlation with output is negative, it is insignificant. The weakening effect on countercyclical MPK dispersion is more salient among small and financially constrained firms, within which the correlation coefficients are closer to 0.

These empirical results and implications are consistent with our model predictions that leasing has a mitigation effect on capital misallocation.

VII Conclusion

As an important proportion of productive assets, leased capital has been largely ignored in the macro-finance literature, due to the fact that it does not show up on firms' balance sheets under previous lease accounting standards. In this study, we empirically document that leased capital accounts for around 20% of the total productive physical assets among US publicly listed firms, and this proportion is more than 40% for small and financially constrained firms. We also find that considering leased capital will substantially reduce the observed dispersion of the marginal product of capital in the US, as measured by Hsieh and Klenow (2009), David, Hopenhayn and Venkateswaran (2016) and David and Venkateswaran (2019), among others. Through our general equilibrium model with heterogeneous firms and buy versus lease decision, we demonstrate that explicitly accounting for leased capital generates a significant mitigation effect on capital misallocation, and results in new interesting features to the cyclical patterns of capital reallocation as compared to the measure in Eisfeldt and Rampini (2006). The empirical evidence supports our key model implication: as an additional reallocation channel, leasing mitigates capital misallocation induced by credit constraints.

Table I

SUMMARY STATISTICS

	Aggregate	Size			WW index		
Variables	**Mean**	**S**	**M**	**L**	**C**	**MC**	**UC**
Lease Capital Ratio (multiplier)	0.24	0.48	0.40	0.22	0.49	0.40	0.22
Lease Capital Ratio (commitment)	0.13	0.32	0.25	0.12	0.33	0.26	0.12
Rental Share	0.18	0.31	0.28	0.17	0.32	0.27	0.17
Debt Leverage	0.20	0.10	0.17	0.20	0.13	0.18	0.20
Lease Adjusted Leverage	0.30	0.29	0.33	0.29	0.33	0.34	0.29

This table presents summary statistics for variables of interest in our sample. Leased capital ratio is the ratio of leased capital over the sum of leased capital and owned capital (PPENT). Leased capital (multiplier) is defined as 8 times the rental expense (XRENT). Leased capital (commitment) is calculated as the sum of current rental expense and the present value of future lease commitments. Rental share is defined as the ratio between rental expense over the sum of capital expenditure (CAPX) plus rental expense. Debt leverage is the ratio of the long-term debt (DLTT) over the sum of leased capital and total assets (AT). Leased adjusted leverage is the sum of debt leverage and rental leverage, the latter of which is defined as the ratio of leased capital (multiplier) over the sum of leased capital and total assets (AT). On the right panel, we split the whole sample into subgroups according to their size, and by financial constraint level each year. Size is defined by total assets, while the financial constraint level is classified by WW index, according to Whited and Wu (2006). We use "S", "M", and "L" to denote small, medium, and large firm groups, respectively. We use "UC", "MC", and "C" to denote unconstrained, mildly constrained, and constrained firm groups, respectively. We report time series averages of the cross section averages in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from the analysis. Firms that are not incorporated in the US and/or do not report in US dollars are also eliminated.

Table II

MPK DISPERSION (UNADJUSTED V.S. ADJUSTED)

	Aggregate	Size			WW index		
Variables	**Mean**	**S**	**M**	**L**	**C**	**MC**	**UC**
mpk dispersion- unadjusted	0.48	0.68	0.45	0.31	0.68	0.43	0.29
mpk dispersion- adjusted	0.26	0.34	0.25	0.18	0.33	0.24	0.17
Level Diff.	-0.22	-0.34	-0.20	-0.13	-0.35	-0.19	-0.12
Percentage Diff.	-46%	-50%	-44%	-42%	-51%	-44%	-41%

This table presents the time series average of *mpk* ($\log(MPK)$) dispersion in our sample. Dispersion is defined as the cross-sectional variance. We subtract each *mpk* from its industry and year mean and work on the residuals. The unadjusted *mpk* is defined as the log difference between operating income (OIBDP) and owned capital (PPENT), while the adjusted *mpk* is defined as the log difference between adjusted operating income (OIBDP+XRENT) and the sum of owned capital and leased capital. On the right panel, we split the whole sample into subgroups according to their size and financial constrained level each year. Size is defined by total assets, while the financial constraint level is classified by WW index, according to Whited and Wu (2006). We use "S", "M", and "L" to denote small, medium, and large firm groups, respectively. We use "UC", "MC", and "C" to denote unconstrained, mildly constrained, and constrained firm groups, respectively. We report time series averages in the table. The sample is from 1977 to 2017 and excludes financial, utility, public administrative, and lessor industries from the analysis. Firms that are not incorporated in the US and/or do not report in US dollars are also eliminated.

Table III

BENCHMARK PARAMETERS

β	α	θ	η	z_L	z_H	π	δ	h	ϵ_0
0.90	0.30	0.40	3	0.50	1.32	0.50	0.10	0.10	3.00

This table lists the set of plausible parameters used in the numerical example. β is the discount factor, α is the capital share in production, and θ is collateralizability in the collateral constraint. The parameter η is the elasticity of substitution across input varieties. z_L and z_H are two possible realizations of idiosyncratic productivities, with $Prob(z = z_H) = 1 - Prob(z = z_L) = \pi$. δ is the depreciation rate, and h is the monitoring cost of leased capital due to the separation of ownership and control, as in Eisfeldt and Rampini (2009) and Rampini and Viswanathan (2013). ϵ_0 is the initial wealth that a representative household is endowed with.

Table IV

CORRELATION OF OUTPUT WITH REALLOCATION

Correlation of output with		**Size**			**WW index**		
Variables	**Aggregate**	**S**	**M**	**L**	**C**	**MC**	**UC**
$RA^{unadj.}$	0.77	0.59	0.56	0.76	0.59	0.56	0.73
$RA^{adj.}$	0.54	-0.05	0.16	0.53	0.11	0.06	0.56
$\frac{RA^{unadj.}}{AT_{-1}}$	0.69	0.46	0.38	0.67	0.49	0.39	0.65
$\frac{RA^{adj.}}{AT_{-1}}$	0.45	-0.01	0.12	0.45	0.01	0.02	0.45

This table presents the correlation of output with reallocation measures. Deviations from trend are computed using the Hodrick and Prescott (1997) filter (H-P filter). The first two rows focus on level (i.e., the natural logarithm of the level of each variable is used). The next two rows report turnover rates, defined as each variable divided by a measure of the total stock, in which we use lagged total assets. Unadjusted reallocation is defined as the sum of acquisitions and sales of property, plant and equipment. Adjusted reallocation is defined as the sum of unadjusted reallocation and leased capital change in each year. Output is the log GDP series obtained from the Federal Reserve Bank of St. Louis. Standard errors are corrected for heteroscedasticity and autocorrelation of the residuals à la Newey and West (1987) and are computed using a GMM approach adapted from the Hansen, Heaton, and Ogaki GAUSS programs. Size is defined by total assets, while the financial constraint level is classified by WW index, according to Whited and Wu (2006). We use "S", "M", and "L" to denote small, medium, and large firm groups, respectively. We use "UC", "MC", and "C" to denote unconstrained, mildly constrained, and constrained firm groups, respectively.

Table V

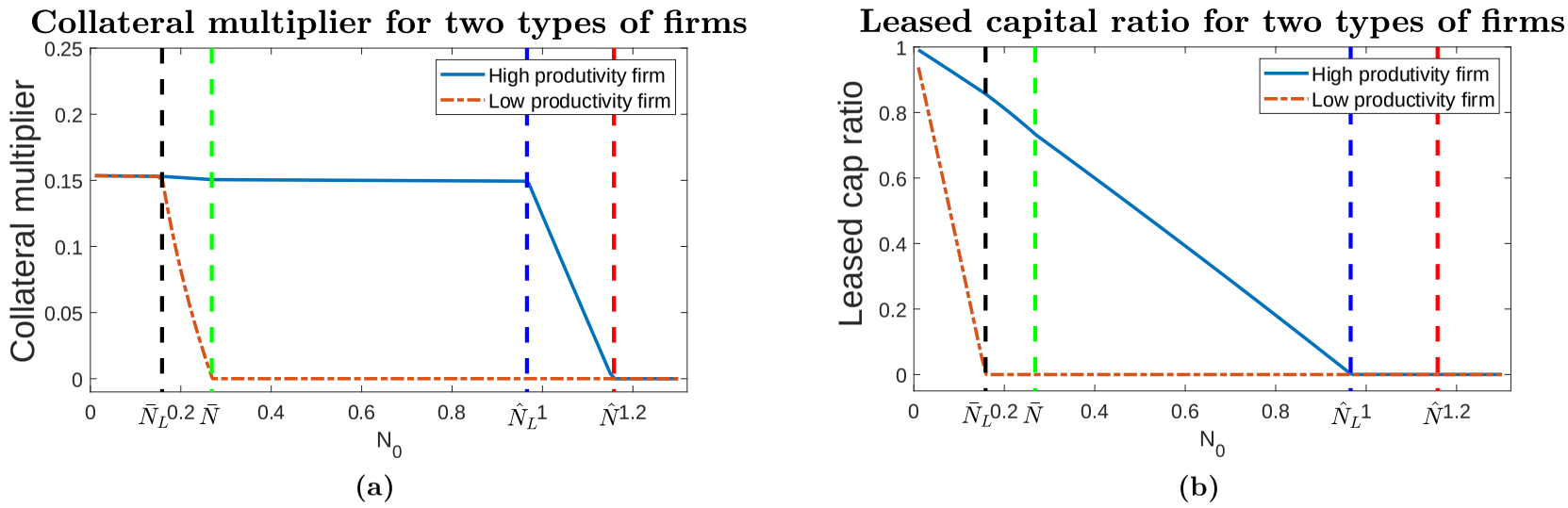
CORRELATION OF OUTPUT WITH MPK DISPERSION (UNADJUSTED V.S. ADJUSTED)

Correlation of output with		**Size**			**WW index**		
Variables	**Aggregate**	**S**	**M**	**L**	**C**	**MC**	**UC**
mpk dispersion- unadjusted	-0.55	-0.40	-0.42	-0.52	-0.41	-0.39	-0.46
[t]	-3.35	-2.03	-2.02	-3.53	-2.16	-1.97	-2.84
mpk dispersion- adjusted	-0.31	-0.15	-0.31	-0.29	-0.22	-0.22	-0.31
[t]	-1.36	-0.63	-1.28	-1.59	-0.96	-0.89	-1.55

This table presents the correlation of output with *mpk* ($\log(MPK)$) dispersion. Deviations from trend are computed using the Hodrick and Prescott (1997) filter (H-P filter). The time series of the unadjusted *mpk* dispersion is computed as the (equal weighted) cross-sectional variance of the unadjusted *mpk*, after controlling for industry and year fixed effect. The time series of the adjusted *mpk* dispersion is computed as the (equal weighted) cross-sectional variance of the adjusted *mpk*, after controlling for industry and year fixed effect. Output is the log GDP series obtained from the Federal Reserve Bank of St. Louis. Standard errors are corrected for heteroscedasticity and autocorrelation of the residuals à la Newey and West (1987) and are computed using a GMM approach adapted from the Hansen, Heaton, and Ogaki GAUSS programs. Size is defined by total assets, while the financial constraint level is classified by WW index, according to Whited and Wu (2006). We use "S", "M", and "L" to denote small, medium, and large firm groups, respectively. We use "UC", "MC", and "C" to denote unconstrained, mildly constrained, and constrained firm groups, respectively.

Figure II

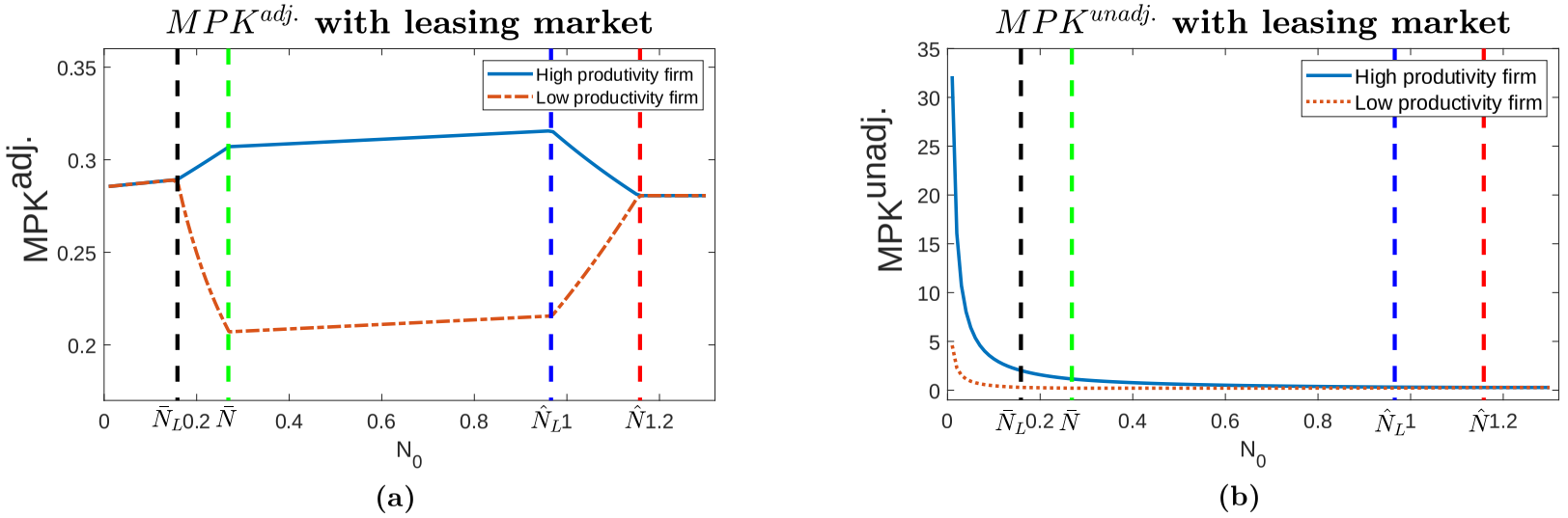
LAGRANGIAN MULTIPLIER AND LEASED CAPITAL RATIO



The left panel plots the lagrangian multiplier of the collateral constraint for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth. The right panel plots the leased capital ratio for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth.

Figure III

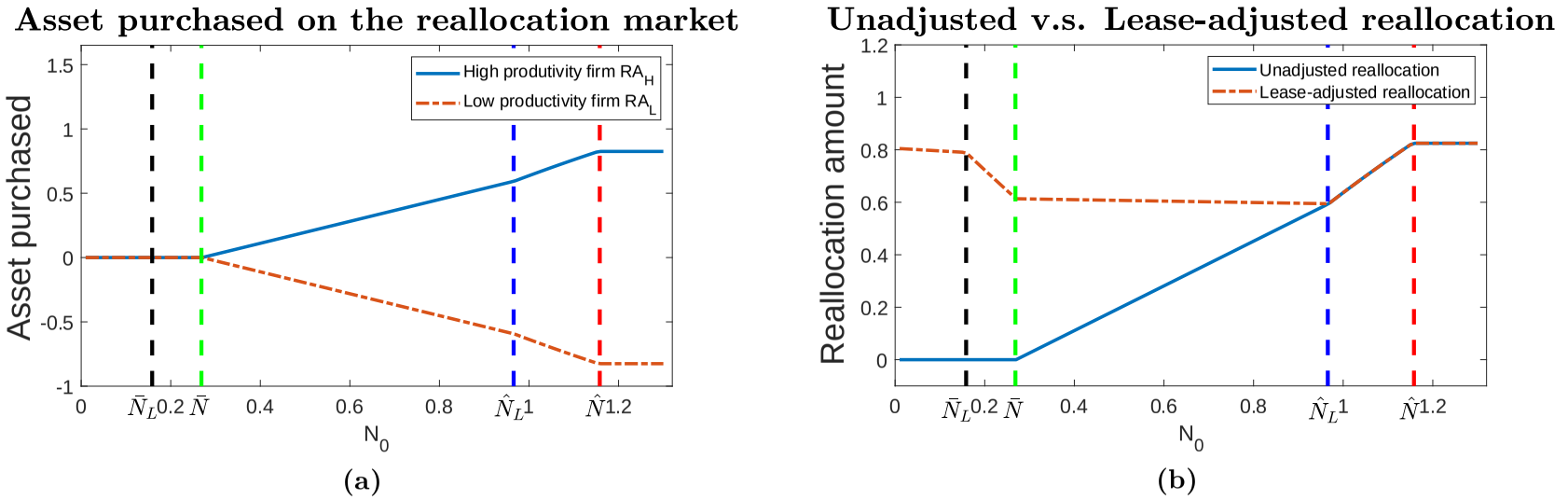
ADJUSTED AND UNADJUSTED MPK IN THE ECONOMY WITH THE RENTAL MARKET



The left panel plots the adjusted MPK for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth in our model economy. The right panel plots the unadjusted MPK for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth in our model economy.

Figure IV

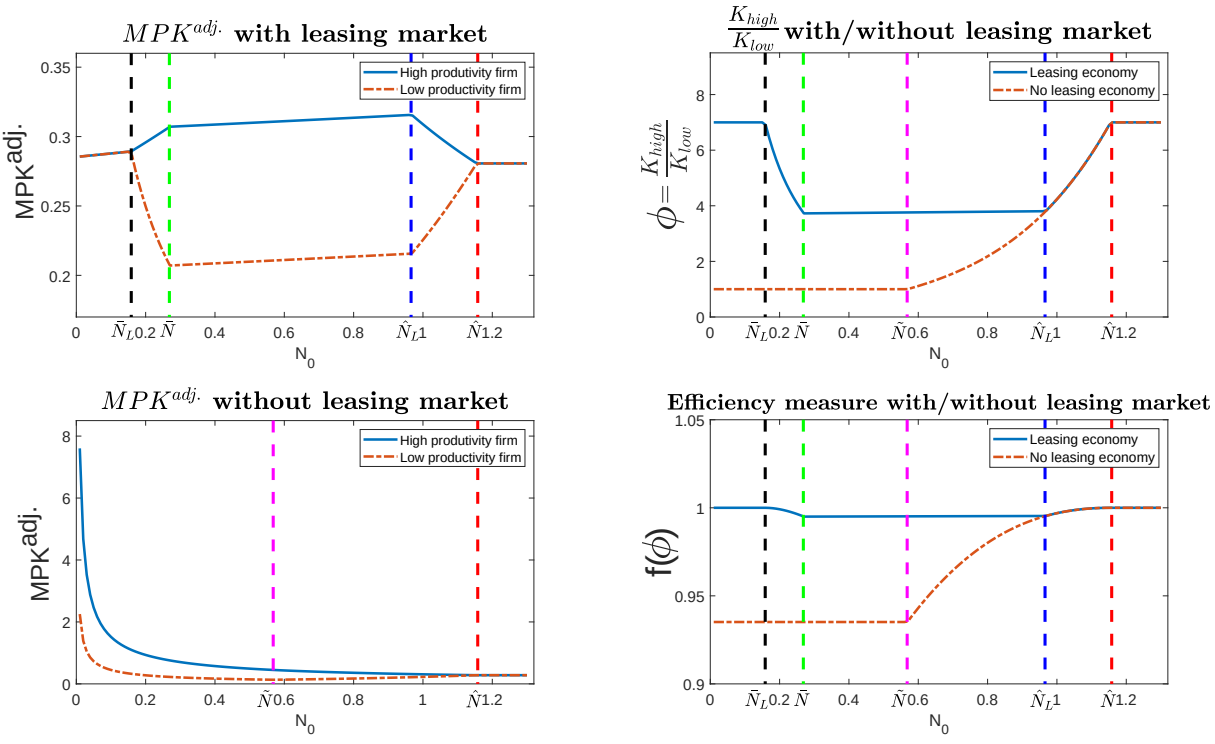
ASSET PURCHASED AND CAPITAL REALLOCATION (UNADJUSTED V.S. ADJUSTED)



The left panel plots the amount of asset purchased on the reallocation market for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth in our model economy. The right panel plots the unadjusted capital reallocation (blue) and adjusted capital reallocation (red) as a function of firms' initial wealth in our model economy. The unadjusted capital reallocation is defined as the total amount of asset purchased and sold on the reallocation market, while the adjusted capital reallocation explicitly considers leased capital as a source of capital reallocation.

Figure V

COUNTERFACTUAL ANALYSIS WITHOUT THE RENTAL MARKET

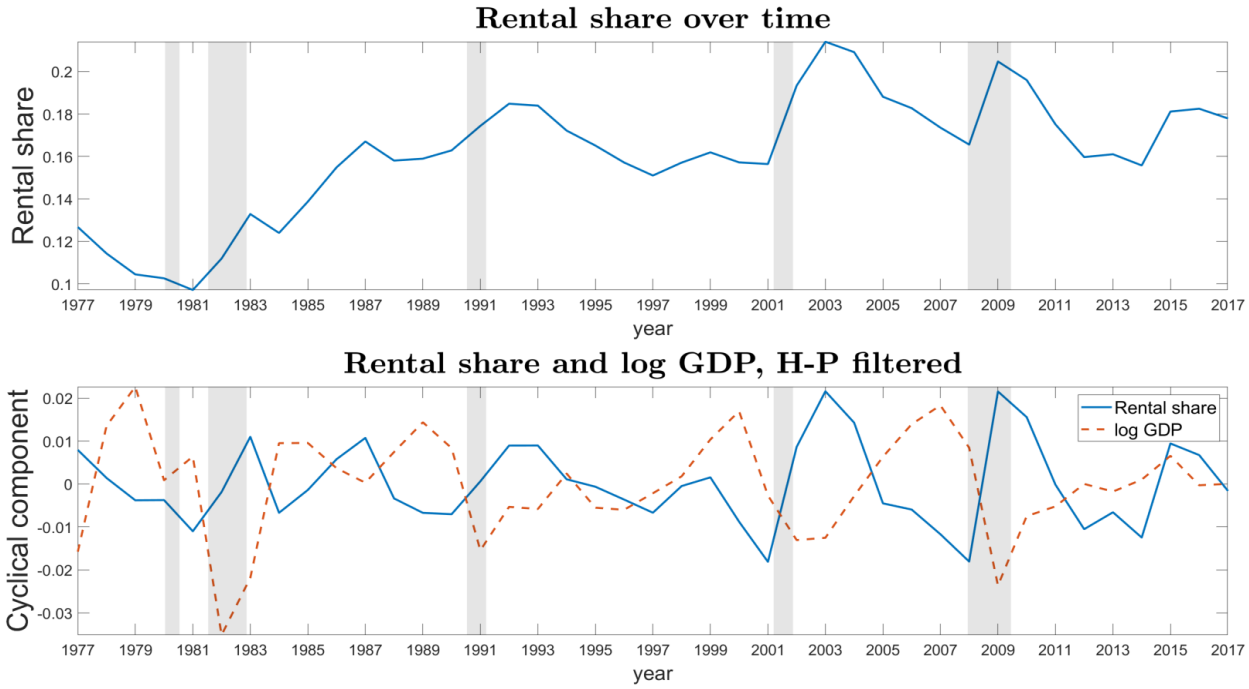


(a) **(b)**

The top left panel plots the adjusted MPK for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth in our model economy. The bottom left panel plots the adjusted MPK for high productivity firms (blue) and for low productivity firms (red) as a function of firms' initial wealth in the economy where the rental market is artificially shut down. The left right panel plots the ratio of total capital between high productivity and low productivity firms as a function of firms' initial wealth in our model economy (blue) and in the economy where the rental market is artificially shut down (red). The bottom right panel plots the efficiency measure as a function of firms' initial wealth in our model economy (blue) and in the economy where the rental market is artificially shut down (red).

Figure VI

OPERATING LEASE OVER THE BUSINESS CYCLE



This figure plots the leased capital ratio (cash-flow based, i.e., rental share) over the business cycle. The top panel plots the leased capital ratio over time. The bottom panel plots the cyclical component of Hodrick and Prescott (1997) (H-P) filtered ratio and log GDP. The blue line denotes the cyclical component of the ratio. The red line denotes the cyclical component of log GDP. The shaded areas denote NBER business cycle recessions.

Appendix

A Lagrangian

To facilitate discussion, we present the Lagrangian of a typical firm under our simplifying assumptions:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \max M_1 \left[\pi D_1^H + (1-\pi) D_1^L\right] \\
& + \pi\eta_{H0} \left[\; N_0 + B_0^H - K_1^o - qRA_1^H \; \right] \\
& + (1-\pi)\eta_{L0} \left[\; N_0 + B_0^L - K_1^o - qRA_1^L \; \right] \\
+ \quad \pi\eta_{H1} \quad & \left[\; p_H y_H - \tau_l (K_1^l)^H - R_f B_0^H - D_1^H + (1-\delta)(K_1^o + RA_1^H) - WL_H \; \right] \\
+ \quad (1-\pi)\eta_{L1} \quad & \left[\; p_L y_L - \tau_l (K_1^l)^L - R_f B_0^L - D_1^L + (1-\delta)(K_1^o + RA_1^L) - WL_L \; \right] \\
& + \pi\xi_{H0}\eta_{H0} \left[\theta(K_1^o + RA_1^H) - B_0^H\right] \\
& + (1-\pi)\xi_{L0}\eta_{L0} \left[\theta(K_1^o + RA_1^L) - B_0^L\right] \\
& + \pi\bar{\nu}_{H0}\eta_{H0} \left[K_1^o + RA_1^H\right] \\
& + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \left[K_1^o + RA_1^L\right] \\
& + \pi\underline{\nu}_{H0}\eta_{H0} \left[\left(K_1^l\right)^H\right] \\
& + (1-\pi)\underline{\nu}_{L0}\eta_{H0} \left[\left(K_1^l\right)^L\right] \\
& + \pi d_{H1} \left[\left(D_1\right)^H\right] \\
& + (1-\pi) d_{L1} \left[\left(D_1\right)^L\right]
\end{aligned}
$$

F.O.C.s:

$$[D_1^H] : \pi M_1 - \pi\eta_{H1} + \pi d_{H1} = 0 \tag{A1}$$

$$[D_1^L] : (1-\pi) M_1 - (1-\pi)\eta_{L1} + (1-\pi) d_{L1} = 0 \tag{A2}$$

$$
\begin{aligned}
[K_1^o] : \pi & \left[\alpha \frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right] \eta_{H1} \\
& + (1-\pi) \left[\alpha \frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right] \eta_{L1} \\
& - \pi\eta_{H0} - (1-\pi)\eta_{L0} \\
+ \theta\pi\xi_{H0}\eta_{H0} + \theta(1-\pi)\xi_{L0}\eta_{L0} & + \pi\bar{\nu}_{H0}\eta_{H0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0
\end{aligned} \tag{A3}
$$

$$[RA_1^H] : -q\pi\eta_{H0} + \pi\left[\frac{\alpha p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right]\eta_{H1} + \pi\theta\xi_{H0}\eta_{H0} + \pi\bar{\nu}_{H0}\eta_{H0} = 0 \quad \text{(A4)}$$

$$\begin{aligned}[RA_1^L] : -q(1-\pi)\eta_{L0} + (1-\pi)\left[\frac{\alpha p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right]\eta_{L1} \\ +(1-\pi)\theta\xi_{L0}\eta_{L0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0\end{aligned} \quad \text{(A5)}$$

$$[(K_1^l)^H] : \pi\alpha\frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H}\eta_{H1} - \pi\tau_l\eta_{H1} + \pi\underline{\nu}_{H0}\eta_{H0} = 0 \quad \text{(A6)}$$

$$[(K_1^l)^L] : (1-\pi)\alpha\frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L}\eta_{L1} - (1-\pi)\tau_l\eta_{L1} + (1-\pi)\underline{\nu}_{L0}\eta_{L0} = 0 \quad \text{(A7)}$$

$$[B_0^H] : \pi\eta_{H0} - \pi R_f\eta_{H1} - \pi\xi_{H0}\eta_{H0} = 0 \quad \text{(A8)}$$

$$[B_0^L] : (1-\pi)\eta_{L0} - (1-\pi)R_f\eta_{L1} - (1-\pi)\xi_{L0}\eta_{L0} = 0 \quad \text{(A9)}$$

$$[L_H] : \pi\,(1-\alpha)\,\frac{p_H y_H}{L_H} = \pi W \quad \text{(A10)}$$

$$[L_L] : (1-\pi)\,(1-\alpha)\,\frac{p_L y_L}{L_L} = (1-\pi)W \quad \text{(A11)}$$

where d_{L1} and d_{H1} must be zero, since D_1^L and D_1^H are sure to be positive. In our setup, firms must always have owned capital; hence, $\bar{\nu}_{L0}$ and $\bar{\nu}_{H0}$ must be 0. As suggested by this set of optimality conditions, the price q is 1.

B Propositions

B.1. Proposition 1

Let K_i denote the total amount of capital used by a firm after reallocation. Using the fact that $y_i = {p_i}^{-\eta}Y$, we can write:

$$p_i y_i = y_i^{1-\frac{1}{\eta}} Y^{\frac{1}{\eta}} = \left[z_1^i K_i^{\alpha} L_i^{1-\alpha}\right]^{1-\frac{1}{\eta}} Y^{\frac{1}{\eta}} \quad \text{(B12)}$$

Because firms are perfectly competitive, their decisions on capital don't affect their prices. The price is an equilibrium concept. Hence, we can write the (true) marginal product of capital as:

$$MPK_i = MPK_i^{(T)} = \alpha\left(z_1^i\right)^{1-\frac{1}{\eta}} K_i^{\alpha\left(1-\frac{1}{\eta}\right)-1} L_i^{(1-\alpha)\left(1-\frac{1}{\eta}\right)} Y^{\frac{1}{\eta}} \quad \text{(B13)}$$

and write the marginal product of labor as:

$$W = (1-\alpha)\left(z_1^i\right)^{1-\frac{1}{\eta}} K_i^{\alpha\left(1-\frac{1}{\eta}\right)} L_i^{(1-\alpha)\left(1-\frac{1}{\eta}\right)-1} Y^{\frac{1}{\eta}} \tag{B14}$$

This equation implies $L_i \propto [z_1^i K_i^\alpha]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}}$. Using the resource constraint, $\int L_i di = 1$, we can obtain:

$$L_i = \frac{[z_1^i K_i^\alpha]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}}}{\int \left[[z_1^i K_i^\alpha]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} \right] di} \tag{B15}$$

We denote $\bigtriangledown = \int \left[[z_1^i K_i^\alpha]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} \right] di$. Then we can write:

$$\begin{aligned} MPK_i^{(T)} &= \alpha \frac{p_i y_i}{K_i} \\ &= \alpha \left(z_1^i\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} K_i^{\frac{-1}{1+\alpha\eta-\alpha}} \bigtriangledown^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} \end{aligned} \tag{B16}$$

From FOCs in Lagrangian, we know $\frac{MPK_i^{(T)}}{W} = \frac{\alpha}{1-\alpha}\frac{L_i}{K_i}$. Therefore, for any individual, the total capital versus labor ratio is:

$$\frac{L_i}{K_i} = \frac{(1-\alpha)\, MPK_i^{(T)}}{\alpha W} \tag{B17}$$

We can then write L_i and K_i in terms of y_i using $y_i = A_1 z_1^i K_i^\alpha L_i^{1-\alpha}$:

$$K_i = \frac{y_i}{A_1 z_1^i}\left(\frac{\alpha W}{(1-\alpha)\, MPK_i^{(T)}}\right)^{1-\alpha} \tag{B18}$$

$$L_i = \frac{y_i}{A_1 z_1^i}\left(\frac{\alpha W}{(1-\alpha)\, MPK_i^{(T)}}\right)^{-\alpha} \tag{B19}$$

Using the demand function $y_i = p_i^{-\eta} Y$, we do an integration:

$$K - total = Y \int \frac{p_i^{-\eta}}{A_1 z_1^i}\left(\frac{\alpha W}{(1-\alpha)\, MPK_i^{(T)}}\right)^{1-\alpha} di \tag{B20}$$

$$L - total = Y \int \frac{p_i^{-\eta}}{A_1 z_1^i} \left(\frac{\alpha W}{(1-\alpha) MPK_i^{(T)}} \right)^{-\alpha} di \tag{B21}$$

Consequently,

$$(K - total)^{\alpha} (L - total)^{1-\alpha} = Y \left\{ \int \frac{p_i^{-\eta}}{A_1 z_1^i} \left(\frac{1}{MPK_i^{(T)}} \right)^{1-\alpha} di \right\}^{\alpha} \left\{ \int \frac{p_i^{-\eta}}{A_1 z_1^i} \left(\frac{1}{MPK_i^{(T)}} \right)^{-\alpha} di \right\}^{1-\alpha} \tag{B22}$$

This implies that TFP is:

$$TFP = \frac{1}{\left\{ \int \frac{p_i^{-\eta}}{A_1 z_1^i} \left(\frac{1}{MPK_i^{(T)}} \right)^{1-\alpha} di \right\}^{\alpha} \left\{ \int \frac{p_i^{-\eta}}{A_1 z_1^i} \left(\frac{1}{MPK_i^{(T)}} \right)^{-\alpha} di \right\}^{1-\alpha}} \tag{B23}$$

From now on, we simplify p_i. From the FOCs, we know:

$$\begin{cases} MPK_i^{(T)} K_i &= \alpha p_i y_i \\ \\ WL_i &= (1-\alpha) p_i y_i \end{cases}$$

Thus,

$$p_i y_i = MPK_i^{(T)} K_i + WL_i \tag{B24}$$

Also,

$$MPK_i^{(T)} K_i + WL_i = \frac{y_i}{A_1 z_1^i} \left(\frac{MPK_i^{(T)}}{\alpha} \right)^{\alpha} \left(\frac{W}{1-\alpha} \right)^{1-\alpha} \tag{B25}$$

As a result,

$$p_i = \frac{1}{A_1 z_1^i} \left(\frac{MPK_i^{(T)}}{\alpha} \right)^{\alpha} \left(\frac{W}{1-\alpha} \right)^{1-\alpha} \tag{B26}$$

Since the price of the final good is 1, we have (from the zero profit condition for final good producer) $1 = P = \left[\int_{[0,1]} p_i^{1-\eta} di \right]^{\frac{1}{1-\eta}}$. Motivated by this, we do another integration, which is:

$$1 = \left[\int_{[0,1]} \left\{ \frac{1}{A_1 z_1^i} \left(\frac{MPK_i^{(T)}}{\alpha} \right)^{\alpha} \left(\frac{W}{1-\alpha} \right)^{1-\alpha} \right\}^{1-\eta} di \right]^{\frac{1}{1-\eta}} \tag{B27}$$

and we can obtain:

$$\left(\frac{W}{1-\alpha}\right)^{(1-\alpha)} = \left[\int_{[0,1]} \left\{\frac{1}{A_1 z_1^i}\left(\frac{MPK_i^{(T)}}{\alpha}\right)^{\alpha}\right\}^{1-\eta} di\right]^{-\frac{1}{1-\eta}} \tag{B28}$$

Then we have (from Eq. (B26)):

$$p_i = \frac{\frac{1}{A_1 z_1^i}\left(\frac{MPK_i^{(T)}}{\alpha}\right)^{\alpha}}{\left[\int_{[0,1]} \left\{\frac{1}{A_1 z_1^i}\left(\frac{MPK_i^{(T)}}{\alpha}\right)^{\alpha}\right\}^{1-\eta} di\right]^{\frac{1}{1-\eta}}} \tag{B29}$$

Therefore, combining with Eq (B23), we can write TFP as:

$$TFP = \frac{\left[\int_{[0,1]} \left(\frac{1}{A_1 z_1^i}\right)^{1-\eta}\left(MPK_i^{(T)}\right)^{\alpha-\alpha\eta} di\right]^{\frac{1+\alpha\eta-\alpha}{\eta-1}}}{\left\{\int \left(\frac{1}{A_1 z_1^i}\right)^{1-\eta}\left(MPK_i^{(T)}\right)^{\alpha-1-\alpha\eta} di\right\}^{\alpha}} \tag{B30}$$

and this TFP is $f(\phi)$ in Proposition 1.

Finally, we apply Eq. (B16) and get:

$$\begin{aligned} f(\phi) = TFP \quad &= \quad \frac{\left\{\int \left\{\left(\frac{1}{A_1}\right)^{1-\eta}\left[z_1^i K_i^{\alpha}\right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\right\} di\right\}^{\frac{1+\alpha\eta-\alpha}{\eta-1}}}{\left\{\int \left(\frac{1}{A_1}\right)^{1-\eta} K_i di\right\}^{\alpha}} \\ & With\ A_1 = 1 \\ \Rightarrow f(\phi) \quad &= \quad \frac{\left\{\int \left\{\left[z_1^i K_i^{\alpha}\right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\right\} di\right\}^{\frac{1+\alpha\eta-\alpha}{\eta-1}}}{\left\{\int K_i di\right\}^{\alpha}} \end{aligned} \tag{B31}$$

Under the two period model, we define $\frac{K_H}{K_L} = \phi$. Then the optimal $\hat{\phi} = \left(\frac{z_H}{z_L}\right)^{\eta-1}$.

Now,

$$\begin{aligned}\bigtriangledown &= \int \left[\left[z_1^i K_i^\alpha \right]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} \right] di \\ &= \pi \left[z_H K_H^\alpha \right]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} + (1-\pi) \left[z_L K_L^\alpha \right]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}}\end{aligned}$$

Meanwhile,

$$\begin{aligned}MPK_i^{(T)} &= \alpha \left(z_1^i \right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \left(K_i \right)^{\frac{-1}{1+\alpha\eta-\alpha}} \bigtriangledown^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} \\ &= \alpha \left(z_1^i \right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \left(K_i \right)^{\frac{-1}{1+\alpha\eta-\alpha}} \\ &* \left\{ (1-\pi) \left[z_L K_L^\alpha \right]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} + \pi \left[z_H K_H^\alpha \right]^{\frac{1-\frac{1}{\eta}}{1-(1-\alpha)\left(1-\frac{1}{\eta}\right)}} \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} \\ \Rightarrow MPK_i^{(T)} &= \alpha \left(z_1^i \right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \left(K_i \right)^{\frac{-1}{1+\alpha\eta-\alpha}} \left\{ (1-\pi) + \pi \left[\phi^\alpha \left(\hat{\phi} \right)^{\frac{1}{\eta-1}} \right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} \\ &* \left[z_L K_L^\alpha \right]^{\left(1-\frac{\eta-1}{1+\alpha\eta-\alpha}\right)}\end{aligned}$$

From Eq. (B31):

$$\begin{aligned}f(\phi) = TFP &= \frac{\left\{ \int \left\{ \left[z_1^i K_i^\alpha \right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \right\} di \right\}^{\frac{1+\alpha\eta-\alpha}{\eta-1}}}{\left\{ \int K_i di \right\}^\alpha} \\ &= \frac{\left\{ (1-\pi) \left[z_L K_L^\alpha \right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}} + \pi \left[z_H K_H^\alpha \right]^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}}}{\left[(1-\pi) K_L + \pi K_H \right]^\alpha} \\ &= \left\{ (1-\pi) \left(\frac{z_L}{((1-\pi)+\pi\phi)^\alpha} \right)^{\frac{(\eta-1)}{\alpha\eta-\alpha+1}} + \pi \left(\frac{z_H \phi^\alpha}{((1-\pi)+\pi\phi)^\alpha} \right)^{\frac{(\eta-1)}{\alpha\eta-\alpha+1}} \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}}\end{aligned}$$

Under the normalization of two types of shocks:

$$(1-\pi) z_L^{\eta-1} + \pi z_H^{\eta-1} = 1$$

We have:

$$(1-\pi) z_L^{\eta-1} + \pi \left(\hat{\phi} z_L^{\eta-1} \right) = 1$$

Thus:

$$\begin{cases} z_L &= \left(\frac{1}{\pi\hat{\phi}+(1-\pi)}\right)^{\frac{1}{\eta-1}} \\ z_H &= \left(\frac{\hat{\phi}}{\pi\hat{\phi}+(1-\pi)}\right)^{\frac{1}{\eta-1}} \end{cases}$$

Hence, we replace z_L and use $K = \pi K_H + (1-\pi)K_L$, and we obtain:

$$\begin{aligned} MPK_L^{(T)} &= \alpha\left(\frac{1}{\pi\hat{\phi}+(1-\pi)}\right)^{\frac{1}{\eta-1}} K^{\alpha-1}\left[(1-\pi)+\pi\phi\right]\left[(1-\pi)+\pi\hat{\phi}\right]^{\frac{1}{\eta-1}} \\ &\quad * \left\{(1-\pi)+\pi\hat{\phi}^{\frac{1}{\alpha\eta-\alpha+1}}\phi^{\frac{\alpha(\eta-1)}{\alpha\eta-\alpha+1}}\right\}^{-1} f(\phi) \end{aligned}$$

$$\Rightarrow MPK_L^{(T)} = \alpha K^{\alpha-1}\left[(1-\pi)+\pi\phi\right]\left\{(1-\pi)+\pi\hat{\phi}^{\frac{1}{\alpha\eta-\alpha+1}}\phi^{\frac{\alpha(\eta-1)}{\alpha\eta-\alpha+1}}\right\}^{-1} f(\phi)$$

Similarly,

$$MPK_H^{(T)} = \hat{\phi}^{\frac{1}{1+\alpha\eta-\alpha}}\phi^{\frac{-1}{1+\alpha\eta-\alpha}} MPK_L^{(T)}$$

QED.

B.2. Proposition 2

To prove Proposition 2, we start with the case in which no firms are constrained. In this case, both types of firms choose optimally, and the economy achieves the first best outcome. Both types of firms have an equalized MPK, $R_{f,u} - 1 + \delta$.

We denote the capital requirement for high productivity and low productivity firms as $\widehat{K}_H$ and $\widehat{K}_L$, respectively. Then in this first best case, we have:

$$\alpha\left(z_1^H\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\left(\widehat{K}_H\right)^{\frac{-1}{1+\alpha\eta-\alpha}}\left\{\int\left[\left[z_1^i\left(\widehat{K}_i\right)^{\alpha}\right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}}\right]di\right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} = R_{f,u} - 1 + \delta$$

$$\alpha\left(z_1^L\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\left(\widehat{K}_L\right)^{\frac{-1}{1+\alpha\eta-\alpha}}\left\{\int\left[\left[z_1^i\left(\widehat{K}_i\right)^{\alpha}\right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}}\right]di\right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} = R_{f,u} - 1 + \delta$$

It is obvious that $\widehat{K}_H > \widehat{K}_L$. As firms' initial wealth N_0 drops, eventually they cannot optimally choose the desired capital level. Intuitively, high productivity firms will become constrained first since they require higher optimal capital. Therefore they require higher initial wealth.

To prove this, we start from a slightly different angle and assume that firms are not endowed with the same initial wealth N_0. Suppose that at initial wealth $\widehat{N}_H$, high productivity firms just become constrained. Similarly, at initial wealth $\widehat{N}_L$, low productivity firms just become constrained. Meanwhile, suppose both types of firms just become constrained at the same time in the same economy. We denote $\lambda = \frac{1}{1-\theta}$. Therefore,

$$\alpha \left(z_1^H\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \left(\lambda \widehat{N}_H\right)^{\frac{-1}{1+\alpha\eta-\alpha}} \left\{ \int \left[\left[z_1^i \left(\widehat{K}_i\right)^{\alpha} \right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}} \right] di \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} = R_{f,c} - 1 + \delta$$

$$\alpha \left(z_1^L\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} \left(\lambda \widehat{N}_L\right)^{\frac{-1}{1+\alpha\eta-\alpha}} \left\{ \int \left[\left[z_1^i \left(\widehat{K}_i\right)^{\alpha} \right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}} \right] di \right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1} = R_{f,c} - 1 + \delta$$

It implies that:

$$\widehat{N}_H > \widehat{N}_L$$

which means that, in the same economy, high productivity firms would require higher net worth to begin with, so that they just switch from being unconstrained to being constrained. Low productivity firms would require lower initial net worth.

We now revert back to the original case in which firms are given the same N_0. Following the above logic, we can clearly see that when N_0 decreases, high productivity firms naturally become constrained earlier than low productivity firms.

We denote this threshold as $\widehat{N}$. When $N_0 > \widehat{N}$, both types of firms are unconstrained. When $N_0 \leq \widehat{N}$, high productivity firms will be constrained while low productivity firms are still unconstrained.

As the initial wealth N_0 further drops, both types of firms will become constrained. We denote this threshold as $\overline{N}$.

B.3. Proposition 3

We denote the threshold that high productivity firms start to use leased capital as $\widehat{N}_L$, and the threshold that low productivity firms start to use leased capital as $\overline{N}_L$.

When firms use leased capital, their MPK is equal to the sum of the net interest rate, depreciation rate and monitoring cost. Following similar logic in the proof for Proposition 2, we know that high productivity firms will start to use leased capital earlier than low productivity firms. This is because high productivity firms always require higher initial wealth, and thus they will become sufficiently constrained to use leased capital earlier than

low productivity firms, when both types are given the same initial wealth. Consequently, $\widehat{N}_L > \overline{N}_L$.

From the user cost comparison in subsection 2, we know that only when firms become sufficiently constrained will they begin to lease. Hence, $\widehat{N} > \widehat{N}_L$ and $\overline{N} > \overline{N}_L$.

We next compare $\widehat{N}_L$ and $\overline{N}$. We again use the logic in the proof for Proposition 2. Suppose that firms' initial wealth are not the same. Meanwhile, we focus on the case in which high productivity firms just begin to lease capital and low productivity firms just become constrained. In this scenario, we denote the initial wealth requirement for high productivity firms as $\widehat{n}_L$, and denote the initial wealth requirement for low productivity firms as $\overline{n}$.

From the MPK formulas, we know:

$$\widehat{n}_L = \frac{1}{\lambda}\left(\frac{R_{f,lc} - 1 + \delta + h}{\alpha\left(z_1^H\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\left\{\int\left[\left[z_1^i\left(\widehat{K}_i\right)^{\alpha}\right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}}\right]di\right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1}}\right)^{\alpha-1-\alpha\eta}$$

$$\overline{n} = \frac{1}{\lambda}\left(\frac{R_{f,lc} - 1 + \delta}{\alpha\left(z_1^L\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}\left\{\int\left[\left[z_1^i\left(\widehat{K}_i\right)^{\alpha}\right]^{\frac{\eta-1}{\alpha\eta-\alpha+1}}\right]di\right\}^{\frac{\alpha\eta-\alpha+1}{(\eta-1)}-1}}\right)^{\alpha-1-\alpha\eta}$$

The comparison between $\widehat{n}_L$ and $\overline{n}$ can be reduced to:

$$\frac{R_{f,lc} - 1 + \delta + h}{\alpha\left(z_1^H\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}} \text{ versus } \frac{\left(R_{f,lc} - 1 + \delta\right)\left(\frac{z_H}{z_L}\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}}{\alpha\left(z_1^H\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}}}$$

and hence,

$$h \text{ versus } \left(R_{f,lc} - 1 + \delta\right)\left(\left(\frac{z_H}{z_L}\right)^{\frac{\eta-1}{1+\alpha\eta-\alpha}} - 1\right)$$

Based on our benchmark parameters (and calculated R_f), the former is smaller than the latter one. This suggests that only when high productivity firms are endowed with higher initial wealth will they lease capital at the same time when low productivity firms become constrained - i.e., $\widehat{n}_L > \overline{n}$.

Using this logic, we revert back to our original scenario when firms are given the same N_0. We can conclude that, as N_0 drops, high productivity firms will begin leasing earlier

than when low productivity firms become constrained. Therefore, $\widehat{N}_L > \overline{N}$.

C Alternative setups

C.1. Monopolistic competition

The first extension is the framework of monopolistic competition, consistent with Hsieh and Klenow (2009). We keep all else the same as in our baseline model, except that each firm now fully takes into account the impact of its production decision on price.

Setup

Final goods producer:

$$\max_{\{y_i\}} \left\{ Y - \int_{[0,1]} p_i y_i di \right\}_{Y=\left[\int_{[0,1]} y_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}}$$

Intermediate goods producer: For each firm, we specify the profit maximization problem as:

$$\begin{aligned}
&\max_{D_t^i, B_0^i, (K_1^l)^i, K_1^o, RA_1^i, p_i} E\left[\sum_{t=0}^{1} M_t D_t^i\right] \\
D_0^i + K_1^o + qRA_1^i &= N_0^i + B_0^i \\
D_1^i &= p_i y_i - \tau_l (K_1^l)^i - R_0 B_0^i + (1-\delta)(K_1^o + qRA_1^i) - WL_i \\
B_0^i &\leq \theta(K_1^o + qRA_1^i) \\
K_1^o + RA_1^i &\geq 0 \\
\left(K_1^l\right)^i &\geq 0 \\
D_t^i &\geq 0 \quad (t = 0, 1) \\
y_i &= A_1 z_1^i \left(K_1^o + RA_1^i + (K_1^l)^i\right)^{\alpha} L_i^{1-\alpha}
\end{aligned}$$

where $i = H, L$. Here firm i maximizes its discounted dividends by choosing the initial owned capital stock K_1^o, a state-contingent plan for capital reallocation RA_1^i, borrowing from household B_0^i, leased capital $(K_1^l)^i$, labor L_i, the price p_i for its output, and its dividend D_1^i, subject to the budget constraint, the collateral constraint, the inverse demand function, and the law of motion for dividend in period 1.

Household:

$$\max_{C_0,C_1,B_0,w_0^i,K^l} E\left[\sum_{t=0}^{1}\beta^t u\left(C_t\right)\right]$$

$$s.t.: C_0 + B_0 + \int w_0^i V_0^i di + K_1^l = \epsilon_0$$

$$\tau_l K_1^l + (1-\delta-h)K_1^l + R_0 B_0 + \int w_0^i (V_1^i + D_1^i) di + W = C_1$$

The market clearing conditions are:

$$C_0 + \int w_0^i V_0^i di + \int D_0^i di + \int K_1^o di + K_1^l = \epsilon_0 + \int N_{i,0} di;$$

$$\int p_i y_i di + \int (1-\delta)(K_1^o + qRA_1^i) di + (1-\delta-h)K_1^l = C_1;$$

$$B_0 = \int (B_0)^i di;$$

$$K_1^l = \int (K_1^l)^i di;$$

$$w_0^i = 1, for\ \ all\ \ i$$

$$\int RA_1^i di = 0$$

$$\int L_i di = 1$$

Lagrangian

Final goods producer:

$$\max_{\{y_i\}} \left\{ Y - \int_{[0,1]} p_i y_i di \right\} = \left[\int_{[0,1]} y_i^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}} - \int_{[0,1]} p_i y_i di$$

F.O.C. implies:

$$p_i = y_i^{-\frac{1}{\eta}} Y^{\frac{1}{\eta}}$$

We next present the Lagrangian of a typical firm under our simplifying assumptions:

$$
\begin{aligned}
\mathcal{L} \quad = \quad & \max M_1 \left[\pi D_1^H + (1-\pi) D_1^L\right] \\
& + \pi\eta_{H0} \left[\; N_0 + B_0^H - K_1^o - qRA_1^H \; \right] \\
& + (1-\pi)\eta_{L0} \left[\; N_0 + B_0^L - K_1^o - qRA_1^L \; \right] \\
+ \quad \pi\eta_{H1} \quad & \left[\; y_H^{1-\frac{1}{\eta}} Y^{\frac{1}{\eta}} - \tau_l (K_1^l)^H - R_f B_0^H - D_1^H + (1-\delta)(K_1^o + RA_1^H) - WL_H \; \right] \\
+ \quad (1-\pi)\eta_{L1} \quad & \left[\; y_L^{1-\frac{1}{\eta}} Y^{\frac{1}{\eta}} - \tau_l (K_1^l)^L - R_f B_0^L - D_1^L + (1-\delta)(K_1^o + RA_1^L) - WL_L \; \right] \\
& + \pi\xi_{H0}\eta_{H0} \left[\theta(K_1^o + RA_1^H) - B_0^H\right] \\
& + (1-\pi)\xi_{L0}\eta_{L0} \left[\theta(K_1^o + RA_1^L) - B_0^L\right] \\
& + \pi\bar{\nu}_{H0}\eta_{H0} \left[K_1^o + RA_1^H\right] \\
& + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \left[K_1^o + RA_1^L\right] \\
& + \pi\underline{\nu}_{H0}\eta_{H0} \left[\left(K_1^l\right)^H\right] \\
& + (1-\pi)\underline{\nu}_{L0}\eta_{H0} \left[\left(K_1^l\right)^L\right] \\
& + \pi d_{H1} \left[\left(D_1\right)^H\right] \\
& + (1-\pi) d_{L1} \left[\left(D_1\right)^L\right]
\end{aligned}
$$

F.O.C.s:

$$[D_1^H] : \pi M_1 - \pi\eta_{H1} + \pi d_{H1} = 0 \tag{C32}$$

$$[D_1^L] : (1-\pi) M_1 - (1-\pi)\eta_{L1} + (1-\pi) d_{L1} = 0 \tag{C33}$$

$$
\begin{aligned}
[K_1^o] : \pi & \left[(1-\frac{1}{\eta})\alpha \frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right]\eta_{H1} \\
& + (1-\pi)\left[(1-\frac{1}{\eta})\alpha \frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right]\eta_{L1} \\
& - \pi\eta_{H0} - (1-\pi)\eta_{L0} \\
+\theta\pi\xi_{H0}\eta_{H0} & + \theta(1-\pi)\xi_{L0}\eta_{L0} + \pi\bar{\nu}_{H0}\eta_{H0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0
\end{aligned} \tag{C34}
$$

$$[RA_1^H] : -q\pi\eta_{H0} + \pi\left[(1-\frac{1}{\eta})\frac{\alpha p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right]\eta_{H1} + \pi\theta\xi_{H0}\eta_{H0} + \pi\bar{\nu}_{H0}\eta_{H0} = 0 \tag{C35}$$

$$
\begin{aligned}
[RA_1^L] : -q(1-\pi)\eta_{L0} + (1-\pi)\left[(1-\frac{1}{\eta})\frac{\alpha p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right]\eta_{L1} & \\
+ (1-\pi)\theta\xi_{L0}\eta_{L0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0 &
\end{aligned} \tag{C36}
$$

$$[(K_1^l)^H] : \pi(1-\frac{1}{\eta})\alpha\frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H}\eta_{H1} - \pi\tau_l\eta_{H1} + \pi\underline{\nu}_{H0}\eta_{H0} = 0 \tag{C37}$$

$$[(K_1^l)^L] : (1-\pi)(1-\frac{1}{\eta})\alpha\frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L}\eta_{L1} - (1-\pi)\tau_l\eta_{L1} + (1-\pi)\underline{\nu}_{L0}\eta_{L0} = 0 \tag{C38}$$

$$[B_0^H] : \pi\eta_{H0} - \pi R_f\eta_{H1} - \pi\xi_{H0}\eta_{H0} = 0 \tag{C39}$$

$$[B_0^L] : (1-\pi)\eta_{L0} - (1-\pi)R_f\eta_{L1} - (1-\pi)\xi_{L0}\eta_{L0} = 0 \tag{C40}$$

$$[L_H] : \pi(1-\frac{1}{\eta})\,(1-\alpha)\,\frac{p_H y_H}{L_H} = \pi W \tag{C41}$$

$$[L_L] : (1-\pi)(1-\frac{1}{\eta})\,(1-\alpha)\,\frac{p_L y_L}{L_L} = (1-\pi)W \tag{C42}$$

where d_{L1} and d_{H1} must be zero since D_1^L and D_1^H must be positive. In our setup, firms must always have owned capital, meaning that $\bar{\nu}_{L0}$ and $\bar{\nu}_{H0}$ must be 0. As this set of optimality conditions suggests, the price q is 1.

MPK

In this framework, the adjusted **true** MPK is:

$$MPK^{adj.}_{mono.} = \left(1-\frac{1}{\eta}\right)\alpha\frac{p_i y_i}{K_1^o + RA_1^i + (K_1^l)^i} = \left(1-\frac{1}{\eta}\right)\alpha\frac{\textit{Value-Added}}{\textit{Total Capital}}$$

In monopolistic competition, $py - WL$ is equal to $\left(\frac{1}{\eta} + \alpha\frac{\eta-1}{\eta}\right)py$. This corresponds to operating income (ex rental expense) in Compustat. Thus, we can compute the value-added as $\frac{OIBDP+XRENT}{\frac{1}{\eta}+\alpha\left(1-\frac{1}{\eta}\right)}$. We can then compute the adjusted MPK as the ratio of operating income (ex rental expense) to total utilized capital, multiplied by a constant, which depends on α and η. Since our focus is within-industry variation of firm outcomes, α and η are homogeneous within a single sector. The within industry $\log(MPK)$ dispersion will not be affected by the constant consisting of α and η.

With respect to the numerator adjustment, in monopolistic competition, the numerator OIBDP in Chen and Song (2013) only subtracts the marginal cost of leased capital, with the monopolistic rents created by leased capital remaining in the numerator. We can see this from the following equation:

$$MPK^{C.\&S.} = \frac{\textit{OIBDP}}{\textit{Owned Capital}} = \frac{\textit{(OIBDP+XRENT) - XRENT}}{\textit{Owned Capital}} = \frac{\left(\frac{1}{\eta} + \alpha\left(1-\frac{1}{\eta}\right)\right)p_i y_i - \tau_l(K_1^l)^i}{K_1^o + RA_1^i}$$

where $p_i y_i$ is equal to $\frac{1}{\alpha\left(1-\frac{1}{\eta}\right)}\tau_l\left(K_1^o + RA_1^i + (K_1^l)^i\right)$ when a firm leases. This means that $MPK^{C.\&S.}$ is equal to:

$$\frac{\tau_l\left(K_1^o + RA_1^i + (K_1^l)^i\right)\frac{\frac{1}{\eta}+\alpha(1-\frac{1}{\eta})}{(1-\frac{1}{\eta})\alpha} - \tau_l(K_1^l)^i}{K_1^o + RA_1^i}$$

It is obvious that $MPK^{C.\&S.}$ is larger than the rental fee per unit τ_l, which is the true MPK when a firm uses leased capital. In this case, $MPK^{C.\&S.}$ varies across firms with different leased capital ratios. The correct adjustment should hence subtract an additional term in the numerator. That is, we should use the following as the adjusted numerator:

$$[OIBDP+XRENT]\frac{(1-\frac{1}{\eta})\alpha}{\frac{1}{\eta}+\alpha(1-\frac{1}{\eta})} - XRENT$$

C.2. Fixed cost

In our second extension, we consider the model with a fixed cost of leasing. The fixed cost represents any additional cost relative to using owned capital, which is not included in rental fees. For example, the extra decoration costs for leased items could be one potential source. For simplicity's sake, we model it in a reduced form f_i for each unit of leased capital.

Setup

Final goods producer:

$$\max_{\{y_i\}}\left\{Y - \int_{[0,1]} p_i y_i di\right\}_{Y=\left[\int_{[0,1]} y_i^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}}$$

Intermediate goods producer: For each firm, we specify the profit maximization problem as:

$$\begin{aligned}
&\max_{D_t^i, B_0^i, (K_1^l)^i, K_1^o, RA_1^i} E\left[\sum_{t=0}^{1} M_t D_t^i\right] \\
D_0^i + K_1^o + qRA_1^i &= N_0^i + B_0^i \\
D_1^i &= p_i y_i - (\tau_l + f_i)(K_1^l)^i - R_0 B_0^i + (1-\delta)(K_1^o + qRA_1^i) - WL_i \\
B_0^i &\leq \theta(K_1^o + qRA_1^i) \\
K_1^o + RA_1^i &\geq 0 \\
\left(K_1^l\right)^i &\geq 0 \\
D_t^i &\geq 0 \quad (t = 0, 1) \\
y_i &= A_1 z_1^i \left(K_1^o + RA_1^i + (K_1^l)^i\right)^{\alpha} L_i^{1-\alpha}
\end{aligned}$$

where $i = H, L$. Firm i's objective is to maximize the discounted dividends by choosing the initial owned capital stock K_1^o, a state-contingent plan for capital reallocation RA_1^i, borrowing from household B_0^i, leased capital $(K_1^l)^i$, labor L_i, and its dividend D_1^i, subject to the budget constraint, the collateral constraint, the inverse demand function, and the law of motion for dividend in period 1.

Household:

$$\begin{aligned}
&\max_{C_0, C_1, B_0, w_0^i, K^l} E\left[\sum_{t=0}^{1} \beta^t u\left(C_t\right)\right] \\
&s.t.: C_0 + B_0 + \int w_0^i V_0^i di + K_1^l = \epsilon_0 \\
\tau_l K_1^l + (1 - \delta - h) K_1^l + R_0 B_0 + \int w_0^i (V_1^i + D_1^i) di + W &= C_1
\end{aligned}$$

The market clearing conditions are:

$$C_0 + \int w_0^i V_0^i di + \int D_0^i di + \int K_1^o di + K_1^l = \epsilon_0 + \int N_{i,0} di;$$
$$\int p_i y_i di + \int (1-\delta)(K_1^o + qRA_1^i) di + (1-\delta-h)K_1^l - \int f_i (K_1^l)^i = C_1;$$
$$B_0 = \int (B_0)^i di;$$
$$K_1^l = \int (K_1^l)^i di;$$
$$w_0^i = 1, for\ \ all\ \ i$$
$$\int RA_1^i di = 0$$
$$\int L_i di = 1$$

Lagrangian

We present the Lagrangian of a typical firm under our simplifying assumptions:

$$\begin{aligned}
\mathcal{L} \quad = \quad & \max M_1 \left[\pi D_1^H + (1-\pi) D_1^L\right] \\
& + \pi \eta_{H0} \left[\ N_0 + B_0^H - K_1^o - qRA_1^H\ \right] \\
& + (1-\pi)\eta_{L0} \left[\ N_0 + B_0^L - K_1^o - qRA_1^L\ \right] \\
+ \quad \pi\eta_{H1} \quad & \left[\ p_H y_H - (\tau_l + f_H)(K_1^l)^H - R_f B_0^H - D_1^H + (1-\delta)(K_1^o + RA_1^H) - WL_H\ \right] \\
+ \quad (1-\pi)\eta_{L1} \quad & \left[\ p_L y_L - (\tau_l + f_L)(K_1^l)^L - R_f B_0^L - D_1^L + (1-\delta)(K_1^o + RA_1^L) - WL_L\ \right] \\
& + \pi \xi_{H0} \eta_{H0} \left[\theta(K_1^o + RA_1^H) - B_0^H\right] \\
& + (1-\pi)\xi_{L0}\eta_{L0} \left[\theta(K_1^o + RA_1^L) - B_0^L\right] \\
& + \pi \bar{\nu}_{H0} \eta_{H0} \left[K_1^o + RA_1^H\right] \\
& + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \left[K_1^o + RA_1^L\right] \\
& + \pi \underline{\nu}_{H0} \eta_{H0} \left[\left(K_1^l\right)^H\right] \\
& + (1-\pi)\underline{\nu}_{L0}\eta_{H0} \left[\left(K_1^l\right)^L\right] \\
& + \pi d_{H1} \left[(D_1)^H\right] \\
& + (1-\pi) d_{L1} \left[(D_1)^L\right]
\end{aligned}$$

F.O.C.s:

$$[D_1^H] : \pi M_1 - \pi\eta_{H1} + \pi d_{H1} = 0 \tag{C43}$$

$$[D_1^L] : (1-\pi)M_1 - (1-\pi)\eta_{L1} + (1-\pi)d_{L1} = 0 \tag{C44}$$

$$\begin{aligned}[K_1^o] : \pi \left[\alpha \frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right]\eta_{H1} \\ +(1-\pi)\left[(\alpha \frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right]\eta_{L1} \\ -\pi\eta_{H0} - (1-\pi)\eta_{L0} \\ +\theta\pi\xi_{H0}\eta_{H0} + \theta(1-\pi)\xi_{L0}\eta_{L0} + \pi\bar{\nu}_{H0}\eta_{H0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0\end{aligned} \tag{C45}$$

$$[RA_1^H] : -q\pi\eta_{H0} + \pi\left[\frac{\alpha p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H} + (1-\delta)\right]\eta_{H1} + \pi\theta\xi_{H0}\eta_{H0} + \pi\bar{\nu}_{H0}\eta_{H0} = 0 \tag{C46}$$

$$\begin{aligned}[RA_1^L] : -q(1-\pi)\eta_{L0} + (1-\pi)\left[\frac{\alpha p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L} + (1-\delta)\right]\eta_{L1} \\ +(1-\pi)\theta\xi_{L0}\eta_{L0} + (1-\pi)\bar{\nu}_{L0}\eta_{L0} \quad = \quad 0\end{aligned} \tag{C47}$$

$$[(K_1^l)^H] : \pi\alpha \frac{p_H y_H}{K_1^o + RA_1^H + (K_1^l)^H}\eta_{H1} - \pi(\tau_l + f_H)\eta_{H1} + \pi\underline{\nu}_{H0}\eta_{H0} = 0 \tag{C48}$$

$$[(K_1^l)^L] : (1-\pi)\alpha \frac{p_L y_L}{K_1^o + RA_1^L + (K_1^l)^L}\eta_{L1} - (1-\pi)(\tau_l + f_L)\eta_{L1} + (1-\pi)\underline{\nu}_{L0}\eta_{L0} = 0 \tag{C49}$$

$$[B_0^H] : \pi\eta_{H0} - \pi R_f \eta_{H1} - \pi\xi_{H0}\eta_{H0} = 0 \tag{C50}$$

$$[B_0^L] : (1-\pi)\eta_{L0} - (1-\pi)R_f\eta_{L1} - (1-\pi)\xi_{L0}\eta_{L0} = 0 \tag{C51}$$

$$[L_H] : \pi\,(1-\alpha)\,\frac{p_H y_H}{L_H} = \pi W \tag{C52}$$

$$[L_L] : (1-\pi)\,(1-\alpha)\,\frac{p_L y_L}{L_L} = (1-\pi)W \tag{C53}$$

where d_{L1} and d_{H1} must be zero since D_1^L and D_1^H must be positive. In our setup, firms must always have owned capital, meaning that $\bar{\nu}_{L0}$ and $\bar{\nu}_{H0}$ must be 0. As this set of optimality conditions suggests, the price q is 1.

MPK

The adjusted **true** MPK is:

$$MPK_{f.}^{adj.} = \alpha \frac{p_i y_i}{K_1^o + RA_1^i + (K_1^l)^i} = \alpha \frac{\textit{Value-Added}}{\textit{Total Capital}}$$

In the model with an additional fixed cost, $py - WL$ is equal to αpy. That is to say, operating income (ex rental expense) in Compustat corresponds to αpy. Hence, we can compute the adjusted mpk as the log difference between operating income and total utilized capital.

In this case, the numerator in Chen and Song (2013) is also biased, in the sense that fixed cost associated with leasing is still kept in the numerator:

$$MPK^{C.\&S.} = \frac{OIBDP}{Owned\ \ Capital} = \frac{(OIBDP+XRENT)\ -\ XRENT}{Owned\ \ Capital} = \frac{\alpha p_i y_i - \tau_l (K_1^l)^i}{K_1^o + RA_1^i}$$

where $\alpha p_i y_i$ can be replaced by $(\tau_l + f_i)\left(K_1^o + RA_1^i + (K_1^l)^i\right)$, rather than $\tau_l\left(K_1^o + RA_1^i + (K_1^l)^i\right)$. Therefore, the correct numerator in this specification should be:

$$\alpha p_i y_i - (\tau_l + f_i)\,(K_1^l)^i = OIBDP - f_i (K_1^l)^i$$

D Data construction

D.1. Data source

Our sample consists of firms in Compustat, available from WRDS. The sample period ranges from 1977 to 2017. We focus on firms with positive rental expenditure data (XRENT from Compustat), non-missing standard industrial classification (SIC) codes, and firms trading on NYSE, AMEX, and NASDAQ. We exclude utility firms that have four-digit SIC codes between 4900 and 4999, finance firms that have SIC codes between 6000 and 6999 (finance, insurance, trusts, and real estate sectors), as well as public administrative firms that have SIC codes between 9000 and 9999. We also explicitly drop industries that serve as lessors (i.e., SIC code 7377 and industries whose SIC begin with 735 and 751). We additionally eliminate firms that are not incorporated in the US and/or do not report in US dollars. Macroeconomic data are from the Federal Reserve Economic Data (FRED) maintained by Federal Reserve Bank in St. Louis.

D.2. Constructing leased capital

We adopt methods in the previous literature to measure leased capital. We define leased capital as eight times current rental payment, following Rampini and Viswanathan (2013) and Lim, Mann and Mihov (2017). We refer to this direct capitalized item as leased capital

(multiplier). This capitalization procedure infers rented capital from rental fees and the user cost of rented capital, in which the user cost is estimated from common figures on interest rate, depreciation rate, and monitoring cost (it implies a user cost of roughly 1/8). This capitalization process is also consistent with the common industry practice.

An alternative measure for leased capital uses a discounting method following Li, Whited and Wu (2016), which is equal to the present value of current and future lease commitments. We discount future lease commitments in years 1-5 (MRC1–MRC5) at the BAA bond rate. We similarly discount lease commitments beyond year 5 (MRCTA) by assuming that they are evenly spread out in years six to ten. The leased capital, then, is the sum of current rental payment and the present value of future lease commitments as calculated above, which we denote as leased capital (commitment).

We omit intangible capital due to the inherent problems with it not being an consistent measure of all intangible investments, valuation and depreciation. We use Property, Plant and Equipment - Total (Net), i.e., PPENT, to measure purchased tangible capital and further define leased capital ratio as leased capital divided by the sum of leased and owned capital. Leased capital ratio measures the proportion of total capital input in a firm's production obtained from leasing activity.

The rental share of each firm is defined as the percentage of rental fee accounts for in total expenditure (sum of capital expenditure and rental fee) for each year:

$$\text{Rental share} = \frac{\text{rental expenses}}{\text{rental expenses} + \text{capital expenditures}}$$

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