Automation, Market Concentration, and the Labor Share*

Hamid Firooz†
University of Rochester

Zheng Liu‡
Federal Reserve Bank of San Francisco

Yajie Wang§
University of Rochester

February 22, 2022

Abstract

During the past two decades, a rising share of production has been concentrated in a small number of superstar firms. We argue that the rise of automation technologies and the cross-sectional variation of robot use rates have contributed to the increases in industrial concentration. Motivated by empirical evidence, we build a general equilibrium model with heterogeneous firms, endogenous automation decisions, and variable markups. Firms choose between two types of technologies, one uses workers only and the other uses both workers and robots subject to an idiosyncratic fixed cost of robot operation. Larger and more productive firms are more likely to choose the automation technology. A decline in the cost of robot adoption increases the relative automation usage by large firms, raising their market share of sales. However, the employment share of large firms does not increase as much as the sales share because the expansion of large firms relies more on robots than on workers. Our calibrated model predicts a cross-sectional distribution of automation usage in line with firm-level data. The model also implies that a decline in automation costs reduces the labor income share and raises the average markup, both driven by between-firm reallocation, consistent with empirical evidence.

Keywords: Automation, market concentration, labor share, markup, reallocation, heterogeneous firms.

JEL Codes: E24, L11, O33.

*Very preliminary and incomplete. Please do not quote without the authors' permissions. We thank Mollie Pepper for excellent research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the Federal Reserve Bank of San Francisco or the Federal Reserve System.

†Email address: hfirooz@ur.rochester.edu

‡Email address: Zheng.Liu@sf.frb.org

§Email address: ywang273@ur.rochester.edu

1 Introduction

Industries have become increasingly concentrated, with each major sector in the US economy increasingly dominated by a small number of superstar firms. Autor et al. (2020) show that the rise of superstar firms has important consequences for the macro economy. In particular, it has contributed to the decline in the labor income share, since resources are reallocated to larger firms that use less labor-intensive technologies for production. It is less clear, however, what might explain the rise of the superstar firms. We argue that the rapid growth of industrial robots, and the rise of automation in general, since the early 2000s has played an important role in explaining the increases in market concentration, particularly in the manufacturing sector.

The connection between automation and concentration can be visualized from the time-series plots in Figure 1. The figure shows the average shares of sales and employment of the largest firms within four-digit manufacturing industries (Panel A).[1] The sales shares of the top 4 firms and the top 20 firms have increased steadily over time, especially after the early 2000s. The employment shares of those top firms have also increased, but at a slower pace. The rise in concentration coincides with the rise in automation, as shown in Panel B of the figure. Since the early 2000s, the relative price of robots has declined by nearly 40%, while the number of industrial robots per thousand US manufacturing employees has quadrupled.

The time-series correlation between automation and concentration is also present in cross-sectional data. We use Compustat firm-level data to construct industry concentration measures at the 2-digit industry level. We consider two alternative measures of concentration: top 1% firms' share and the Herfindahl–Hirschman Index (HHI), based on both sales and employment. To construct a measure for 2-digit industry-level robot density, we use the operation stocks of robots from the International Federation of Robotics (IFR) for each industry, combined with the manufacturing employment (or labor hours) in those industries from the Bureau of Labor Statistics. We obtain an unbalanced panel covering 13 industries for 12 years from 2007 to 2018.

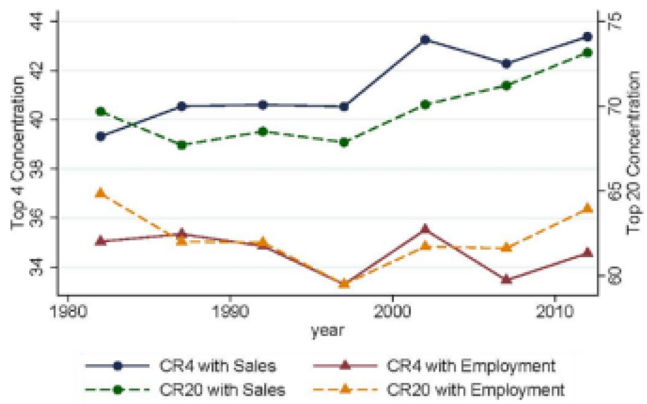
After controlling for industry and year fixed effects, we find that robot density is positively correlated with sales-based measures of industry concentration and the correlation is economically important and statistically significant. The correlation of robot density with employment-based concentration measures are also positive, although they are statistically insignificant and the magnitudes are much smaller than the correlations with sales-based measures. These cross-sectional correlations are consistent with the time-series patterns displayed in Figure 1.

The evidence suggests that automation may have led to concentration of production in a few superstar firms. To the extent that those superstar firms have lower labor shares

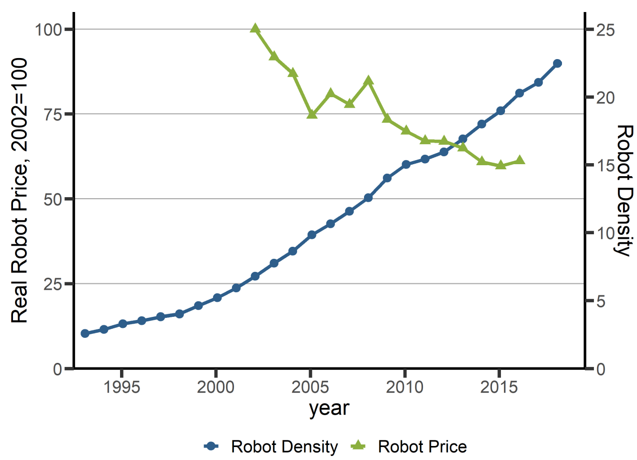
[1] The figure is taken from Figure IV in Autor et al. (2020) with permissions from the Oxford University Press (License Number 5241431011126).

Figure 1. Trends in industry concentration and automation.

(a) Panel A: Industry Concentration



(b) Panel B: Robot Price and Density



Note: Panel A is taken from Autor et al. (2020) and it shows the industry concentration measured by both the sales share and the employment share of the top 4 firms (the left scale) or the top 20 firms (the right scale) in a given industry. Panel B shows the unit value of newly shipped industrial robots deflated by the personal consumption expenditure price index (red line, left scale) and robot density measured by the operation stock of robots per thousand manufacturing workers (blue line, right scale). Both the series of robot price and the operation stock of industrial robots are taken from the International Federation of Robotics (IFR).

(Autor et al., 2020), an industry with greater exposures to automation should also have a lower labor share on average. This is indeed the case. We use an industry-level measure of the labor share constructed from the NBER-CES dataset and find that robot density is negatively and significantly correlated with the labor share at the 2-digit industry level.

To understand the connection between the rise of automation, market concentration, and the labor share, we construct a general equilibrium model featuring heterogeneous firms, variable markup, and endogenous automation decisions. Firms have access to two types of technologies for producing differentiated intermediate goods: one is the traditional technology that uses workers as the only input and the other is an automation technology that uses both workers and robots, with a constant elasticity of substitution. Operating the automation technology incurs a random fixed cost. Firms also face idiosyncratic and persistent productivity shocks. A firm's automation decision (i.e., whether to operate the traditional or the automation technology) depends on the realizations of the fixed cost relative to productivity. At a given fixed cost, larger firms are more likely to use robots in production because they have higher productivity and also higher market power and profits.

We calibrate the model parameters to match several moments in the data and in the empirical literature. To calibrate the fixed cost of operating the automation technology and the exogenous cost of new robot adoptions, we target the firm-weighted and the employment-weighted averages of robot adoption rates taken from the 2018 Annual Business Survey (ABS) conducted by the US Census Bureau (Zolas et al., 2020). The ABS covers a large and nationally representative sample of over 850,000 firms in all private non-farm business sectors. The calibrated model predicts that the automation probability increases with firm productivity and firm size, whereas the labor share decreases with firm productivity and size. Given a technology choice, higher productivity enables a firm to produce more output and to employ more workers (i.e., the firm is larger). With variable markup under the Kimball preferences (Kimball, 1995), larger firms face lower demand elasticities and charge higher markups. Thus, markup also increases with firm productivity.

An exogenous decline in the robot adoption costs increases the share of automating firms (the extensive margin) and it also benefits large firms who operate the automation technology (the intensive margin), raising the aggregate use rate of robots. As production is reallocated from small to large firms, market concentration measured by the sales share of top firms increases. Since large firms are more likely to use robots, which substitute for workers, the share of employment of the top firms does not increase as much as does the share of their sales. This model prediction is consistent with the empirical correlations of market concentration with automation shown in Figure 1 and also with our cross-sectional evidence. Furthermore, since larger firms have higher markups and lower labor shares, the between-firm reallocation triggered by a decline in automation costs reduces the average labor share and increases the average markup, consistent with the reallocation channel documented by Autor et al. (2020).

Our model does well in predicting the cross-sectional distribution of artificial intelligence (AI) usage observed in the firm-level data from the ABS. In particular, the usage of automation technologies is highly skewed towards large and high-productivity firms (Zolas et al., 2020), both in the model and in the data. Since this moment is not targeted in our calibration, the ability of the model to correctly predict the cross-sectional distribution of automation usage lends credence to the model's mechanism.

Our work is motivated by the empirical evidence on market concentration documented by Autor et al. (2020). Their study highlights an important between-firm reallocation channel that connects the rise in product market concentration with the fall in the labor share. If an industry becomes increasingly dominated by superstar firms, which have high markups and lower labor shares, then industries with a larger increase in concentration should also have larger declines in the labor share. They discuss a few potential drivers of the rise of superstar firms (what they call a "winner takes most" mechanism), such as greater market competition (e.g., through globalization) or scale-biased technological change related to intangible capital and information technology. Our work complements that of Autor et al. (2020) by providing a formal theoretical framework for understanding what drives the "winner takes most" mechanism. Our model suggests that the rise in automation during the past two decades may have contributed to the rise of superstar firms, since larger firms are more likely to pay the fixed cost of operating automation technologies. Consistent with firm-level data, our model predicts that the adoption of automation technologies is highly skewed toward large and high-productivity firms. Our model also predicts that a decline in robot adoption costs benefit disproportionately large and high-productivity firms, increasing market concentration and reducing the labor share. To the extent that robots substitute for workers, a decline in robot costs raises sales concentration more than employment concentration, consistent with both the time-series evidence in Autor et al. (2020) and our own cross-sectional evidence.

Our work is also related to the growing literature on the macroeconomic and distributional impacts of automation. Several studies examine the effects of automation on employment, wages, and labor productivity (Acemoglu and Restrepo, 2018, 2020; Graetz and Michaels, 2018; Arnoud, 2018; Leduc and Liu, 2019). Some other studies provide evidence that automation of routine jobs has contributed to wage inequality by displacing jobs in middle-skill occupations (Autor, Levy and Murnane, 2003; Autor, Dorn and Hanson, 2013; Jaimovich and Siu, 2020). Using French firm-level data, Acemoglu, Lelarge and Restrepo (2020) document evidence that robot adopters experienced significant declines in labor shares and increases in value added and productivity. The find that robot adoptions create a large impact on the labor share because adopters are larger and grow faster than their competitors, in line with our model's implications. Our paper contributes to this literature by providing a theoretical framework and documenting some empirical evidence that connect automation with market concentration and the labor share.

Table 1. Summary Statistics

	#obs	mean	min	p25	p50	p75	max	s.d.
robots/thousand employees	151	31.43	0.00140	0.301	2.892	11.33	419.9	89.24
robots/million hours	151	20.23	0.00108	0.215	2.213	8.393	243.5	53.16
top 1% share of sales	117	0.307	0.0916	0.222	0.304	0.366	0.765	0.129
top 1% share of employment	104	0.275	0.106	0.213	0.277	0.316	0.456	0.0795
HHI (sales)	137	1433.2	411.2	672.4	1443.2	2000.3	6194.0	787.9
HHI (employment)	135	1331.8	289.6	680.8	1483.4	1862.0	2651.9	676.6
labor share	151	0.292	0.135	0.237	0.299	0.354	0.440	0.0766

2 Empirical Evidence

The time series in Figure 1 suggest that the rise in market concentration in the manufacturing sector is correlated with the rise in automation during the past two decades. We now provide some cross-sectional evidence that market concentration is also correlated with automation at the 2-digit industry level.

To measure industry-level market concentration, we use the firm-level data from Compustat. For each industry, we construct a sales-based measure of concentration using the share of the largest 1% firms in terms of sales. We also construct an employment-based measure of concentration using the share of the largest 1% firms in terms employment. For robustness, we also consider the Herfindahl–Hirschman Index (HHI) for each industry based on sales and employment.[2] We measure the industry-level labor share using the NBER-CES Manufacturing Industry Dataset. For each industry, the labor share is measured by the ratio of payrolls to value added.

We use the IFR data to construct a measure of industry-level robot density. Specifically, for each 2-digit level industry, the robot density is measured by the operation stock of robots per thousand workers. We consider an alternative measure of industry-level robot density based on labor hours, which is measured by the operation stock of robots per million hours. The data of industry-level employment (PRODE) and labor hours (PRODH) are both taken from the NBER-CES Manufacturing Industry Dataset.[3]

We obtain an unbalanced panel with 13 industries covering the 12 years from 2007 to 2018.[4] Table 1 reports the summary statistics of variables.

[2]The HHI is defined as the sum of squared sales (or employment) shares across firms within the industry. When we compute the industry concentration measures, we require that each industry has at least 10 firms.

[3]The IFR uses the International Standard Industrial Classification (ISIC, Rev. 4) for industry classification, while NBER-CES and Compustat use the North American Industry Classification System (NAICS). We match the ISIC Rev. 4 industry codes with the NAICS2017US codes using the concordance table developed by the US Census Bureau.

[4]Prior to 2007, the IFR data on industrial robots at the 2-digit industry level are very limited. The codes for the 13 industries included in our sample are 10-12, 13-15, 16&31, 17-18, 19-22, 23, 24, 25, 26-27,

The table shows that robot density varies widely in our sample. For example, the interquartile range (IQR) of robots per thousand workers is about 11, which is about one-third of the sample mean. The standard deviation of robot density is also large (about three times the mean). These patterns reflect both within-industry changes in robot adoptions over time and across-industry heterogeneity in robot adoptions and the growth rates of robot use. Market concentration in our sample also displays large variations. For example, the sales share of the top 1% firms averages about 31%, with an IQR of about 14% and a standard deviation of 13%. The employment share of the top 1% firms averages about 28%, and it varies less than the sales share, with an IQR of about 10% and a standard deviation of about 8%. Market concentrations measured by the HHI of sales and employment display similar patterns as those measured by the top firms' market shares. The labor income share from the NBER-CES dataset is about 30%, which is lower than aggregate labor share, partly because our dataset covers the manufacturing sector, which is less labor intensive than service sectors.

To examine the correlation between market concentration and automation at the industry level, we estimate the empirical specification

$$log(Y_{jt}) = \beta_0 + \beta_1 \log(robot_{jt}) + \gamma_j + \delta_t + \varepsilon_{jt}. \tag{1}$$

Here, the dependent variable Y_{jt} is a measure of market concentration in industry j and year t (sales share of top 1% firms, employment share of top 1% firms, HHI based on sales, or HHI based on employment) or the labor income share. The key independent variable is $robot_{jt}$, which denotes the robot density in industry j and year t. In the regression, we control for the industry fixed effects (γ_j) and the year fixed effects (δ_t). The term ε_{jt} denotes the regression residual. The parameter β_0 is the intercept term.

The key parameter of interest is β_1, which measures the elasticity of market concentration (or labor share) with respect to robot density. For example, for the market concentration regression, if β_1 is estimated to be positive after controlling for industry fixed effects and time fixed effects, then it would suggest that an industry more exposed to automation than average (i.e., with a higher robot density) has also greater market concentration.

Table 2 shows the estimation results under the empirical specification (1), where robot density is measured by the number of robots per thousand workers ($robot/emp$). In the regression, each industry-level variable is weighted by the industry's sales in the initial year (i.e., 2007). The standard errors (i.e., the numbers in parentheses) are clustered at the industry level.

The regression results show that, controlling for industry and year fixed effects, robot density is positively correlated with market concentration. However, the correlations of robot density with sales-based concentration are different from those with employment-based measures. Column (1) of the table shows that, in an industry with robot density that is one percent higher than the mean, the sales share of the top 1% firms would be

28, 29, 30, D&E.

Table 2. Industry-Level Evidence

	(1)	(2)	(3)	(4)	(5)
	ln(top 1% share)		ln(HHI)		ln(labor share)
	sales	employment	sales	employment	
ln(robot/emp)	0.0737***	0.0135	0.0948**	0.0283	-0.0151*
	(0.0157)	(0.0504)	(0.0407)	(0.0299)	(0.00830)
Constant	-1.109***	-1.362***	6.978***	6.474***	-1.554***
	(0.0710)	(0.0674)	(0.101)	(0.0238)	(0.0167)
Observations	117	104	137	135	151
Industry FE	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Adjusted R^2	0.944	0.688	0.920	0.984	0.980

Standard errors in parentheses are clustered at the industry level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

about 7.4% higher than average or 2.3 percentage points higher than the average sales share (which is about 31%). This estimated coefficient is statistically significant at the one-percent confidence level. The correlation of robot density with the employment share of the top 1% firms, although positive, is small and statistically insignificant (Column (2)). A similar correlation pattern holds if we measure concentration using the HHI (see Columns (3) and (4)). These regression results from cross-sectional data corroborate well with the time-series correlations between automation and market concentration.

Table 2 further shows that robot density is negative correlated with the labor share. After controlling for industry and year fixed effects, a higher robot density is associated with a lower labor share, and the correlation is statistically significant (see Column (5)).

The cross-sectional correlations of robot density with market concentration and the labor share are robust to alternative measures of robot density, as shown in Table 3, where we measure robot density by the number of industrial robots per million hours (instead of per thousand workers).

Table 3. Industry-Level Evidence by Hours

	(1)	(2)	(3)	(4)	(5)
	ln(top 1% share)		ln(HHI)		ln(labor share)
	sales	employment	sales	employment	
ln(robot/hours)	0.0734***	0.0133	0.0961**	0.0284	-0.0143*
	(0.0156)	(0.0501)	(0.0410)	(0.0297)	(0.00800)
Constant	-1.086***	-1.358***	7.009***	6.483***	-1.559***
	(0.0731)	(0.0629)	(0.111)	(0.0244)	(0.0159)
Observations	117	104	137	135	151
Industry FE	✓	✓	✓	✓	✓
Year FE	✓	✓	✓	✓	✓
Adjusted R^2	0.943	0.688	0.921	0.984	0.980

Standard errors in parentheses are clustered at the industry level.

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

3 The Model

3.1 Households

The economy is populated by a large number of infinitely-lived identical households with a unit measure. The representative household has the utility function

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t\left[\ln C_t - \chi\frac{N_t^{1+\xi}}{1+\xi}\right], \tag{2}$$

where C_t denotes consumption, N_t denotes labor supply, $\beta \in (0,1)$ is a subjective discount factor, $\xi \geq 0$ is the inverse Frisch elasticity of labor supply, $\chi > 0$ is the weight on the disutility from working, and $\mathbb{E}$ is an expectation operator.

The household faces the sequence of budget constraints

$$C_t + Q_{at}I_{at} \leq W_t N_t + r_{at}A_t + \pi_t, \tag{3}$$

where I_{at} denotes the investment in automation (i.e., robot adoption), A_t denotes the beginning-of-period robot stock, Q_{at} denotes the relative price of robots, W_t denotes the real wage rate, and π_t denotes the share of profit from the firms that the household owns.

The stock of robots evolves according to the law of motion

$$A_{t+1} = (1-\delta_a)A_t + I_{at}, \tag{4}$$

where $\delta_a \in [0,1]$ denotes the robot depreciation rate.

The household takes the prices Q_{at}, W_t, and r_{at} as given, and maximizes the utility function (2) subject to the budget constraints (3) and the law of motion for robots (4). The optimizing consumption-leisure choice implies the labor supply equation

$$W_t = \chi N_t^{\xi} C_t. \tag{5}$$

The optimizing choice of robot adoptions implies that

$$Q_{at} = \mathbb{E}_t \frac{\beta C_t}{C_{t+1}} \left[r_{a,t+1} + Q_{a,t+1}(1-\delta_a) \right]. \tag{6}$$

3.2 Intermediate goods producers

There is a large number of monopolistically competitive firms with a unit measure. Each firm $j \in [0,1]$ has access to a constant-returns technology to produce a differentiated intermediate good with productivity $\phi_t(j)$. The production function is given by

$$y_t(j) = \phi_t(j) \left[\alpha_a A_t(j)^{\frac{\eta-1}{\eta}} + (1-\alpha_a) N_t(j)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{7}$$

where $y_t(j)$ denotes the firm's output; $A_t(j)$ and $N_t(j)$ denote the inputs of robots and workers, respectively; and $\phi_t(j)$ denotes an idiosyncratic productivity shock. The parameter η is the elasticity of substitution between robots and workers. The parameter α_a measures the relative importance of robot input in production.

The idiosyncratic productivity shock follows a stationary AR(1) process

$$\ln \phi_t(j) = \gamma \ln \phi_{t-1}(j) + \varepsilon_t(j), \quad \varepsilon_t(j) \sim N(0, \sigma_\phi^2) \tag{8}$$

where $\gamma \in (0,1)$ measures the persistence of the productivity shock, and $\sigma_\phi > 0$ denotes the standard deviation of the innovation.

To automate (i.e., to have $A_t(j) > 0$), firms have to pay a per-period fixed cost f_a, which is an i.i.d. draw from the Cumulative Distribution Function $F(\cdot)$. If the firm pays f_a, the firm hires both workers and robots from the representative household at the competitive wage rate W_t and robot rental rate r_{at}. Cost-minimizing leads to the conditional factor demand functions

$$r_{at} = \alpha_a \lambda_t(j) \phi_t(j)^{\frac{\eta-1}{\eta}} \left(\frac{y_t(j)}{A_t(j)} \right)^{\frac{1}{\eta}}, \tag{9}$$

$$W_t = (1-\alpha_a) \lambda_t(j) \phi_t(j)^{\frac{\eta-1}{\eta}} \left(\frac{y_t(j)}{N_t(j)} \right)^{\frac{1}{\eta}}, \tag{10}$$

where $\lambda_t(j)$ denotes the real marginal cost of production for firm j:

$$\lambda_t(j) = \frac{\left[\alpha_a^{\eta} r_{at}^{1-\eta} + (1-\alpha_a)^{\eta} W_t^{1-\eta} \right]^{\frac{1}{1-\eta}}}{\phi_t(j)} \tag{11}$$

If the firm does not pay f_a, the firm would hire workers only, i.e., $A_t(j) = 0$. In that case, the production function (7) delivers the input demand:

$$N_t(j) = \frac{y_t(j)}{\phi_t(j)}(1-\alpha_a)^{\frac{\eta}{1-\eta}} \tag{12}$$

The marginal cost of production in this case would be

$$\lambda_t(j) = \frac{(1-\alpha_a)^{\frac{\eta}{1-\eta}} W_t}{\phi_t(j)} \tag{13}$$

Note that for a given firm j, the marginal cost when adopting robots (equation (11)) is always smaller than the marginal cost when using labor only (equation (13)).

3.3 Final good producers

Final good producers make a composite homogeneous good out of the intermediate varieties and sell it to consumers in a perfectly competitive market. The final good Y_t is produced using a bundle of intermediate goods $y_t(j)$, according to the following commonly available Kimball aggregator:

$$\int_0^1 \Lambda(\frac{y_t(j)}{Y_t})dj = 1 \tag{14}$$

where the intermediate varieties are denoted by j. For ease of notation, we suppress the time subscript t in what follows.

3.4 Demand for intermediate goods

Denote relative production of firm j by $q(j) := \frac{y(j)}{Y}$. Given the prices for intermediate varieties $p(j)$, final good producers minimize their cost, which delivers the following demand for intermediate good j:

$$p(j) = \Lambda'(q(j))D \tag{15}$$

where D is the demand shifter

$$D = \left(\int \Lambda'(q(j))q(j)dj \right)^{-1} \tag{16}$$

We follow Klenow and Willis (2016) and assume:

$$\Lambda(q) = 1 + (\sigma - 1)\exp(\frac{1}{\varepsilon})\varepsilon^{\frac{\sigma}{\varepsilon}-1}[\Gamma(\frac{\sigma}{\varepsilon}, \frac{1}{\varepsilon}) - \Gamma(\frac{\sigma}{\varepsilon}, \frac{q^{\varepsilon/\sigma}}{\varepsilon})] \tag{17}$$

with $\sigma > 1$ and $\varepsilon \geq 0$ and $\Gamma(s, x)$ is the upper incomplete Gamma function

$$\Gamma(s, x) = \int_x^\infty v^{s-1}e^{-v}dv \tag{18}$$

With the specification for Λ, we get:

$$\Lambda'(q(j)) = \frac{\sigma - 1}{\sigma}\exp(\frac{1 - q(j)^{\frac{\varepsilon}{\sigma}}}{\varepsilon}) \tag{19}$$

which implies the following demand elasticity faced by firm j:

$$\sigma(q(j)) = -\frac{\Lambda'(q(j))}{\Lambda''(q(j))q(j)} = \sigma q(j)^{-\frac{\varepsilon}{\sigma}} \tag{20}$$

So the firm with relative production $q(j)$ charges the markup

$$\mu(j) = \frac{\sigma(q(j))}{\sigma(q(j)) - 1} \tag{21}$$

So larger firms have more market power, face lower demand elasticities, and charge higher markups.

Note that firms will not increase production to the point where demand elasticity gets less than one. This means that to keep demand elasticity more than one, the relative quantity of firm j is bounded above:

$$q(j) < \sigma^{\frac{\sigma}{\varepsilon}} \tag{22}$$

This implies that firms' output remains constant after some point, simply because firms avoid to get too big.

3.5 Automation decision

We label firm j using its productivity ϕ. After observing automation fixed cost f_a, the firm decides about its price $p(\phi)$, quantity $y(\phi)$, and inputs $N(\phi)$ and $A(\phi)$. The firm's profit if it automates would be

$$\Pi^a(\phi) = \max_{p(\phi),y(\phi),N(\phi),A(\phi)} \Big[p(\phi)y(\phi) - WN(\phi) - r_a A(\phi) - f_a\Big] \tag{23}$$

subject to equations (9), (10) and (15).

If using labor only, the firm's profit would be

$$\Pi^n(\phi) = \max_{p(\phi),y(\phi),N(\phi)} \Big[p(\phi)y(\phi) - WN(\phi)\Big] \tag{24}$$

subject to equations (12) and (15).

Let $\bar{f}_a(\phi) \equiv \Pi^a(\phi) - \Pi^n(\phi) + f_a$. Firms with productivity ϕ automate if and only if they draw an automation fixed cost less than $\bar{f}_a(\phi)$:

$$f_a < \bar{f}_a(\phi) \Longleftrightarrow \mathbb{I}_a(\phi, f_a) = 1 \tag{25}$$

where $\mathbb{I}_a(\cdot)$ is an indicator function showing the automation decision. Before firms draw their fixed cost of automation, the automation probability for a firm with productivity ϕ equals $F(\bar{f}_a(\phi))$.

Note that firms face a trade off when deciding whether to automate. On the one hand, firms need to pay a fixed cost f_a to automate. On the other hand, however, the marginal cost when adopting robots (equation (11)) is always smaller than the marginal cost when using labor only (equation (13)). Since larger firms charge higher markups and earn more profits, they are more likely to automate.

3.6 Markup variation with marginal cost

Since price is equal to markup times marginal cost, we can write equation (15) as:

$$\frac{\sigma q(j)^{\frac{-\varepsilon}{\sigma}}}{q(j)^{\frac{-\varepsilon}{\sigma}} - 1}\lambda(j) = \frac{\sigma - 1}{\sigma} \exp\left(\frac{1 - q(j)^{\frac{\varepsilon}{\sigma}}}{\varepsilon}\right) D \tag{26}$$

Take logs and differentiate this equation to get (assuming D is fixed):

$$\frac{dq(j)/q(j)}{d\lambda(j)/\lambda(j)} = \frac{-\sigma}{\frac{\varepsilon}{\sigma q(j)^{\frac{-\varepsilon}{\sigma}} - 1} + q(j)^{\frac{\varepsilon}{\sigma}}} \tag{27}$$

This elasticity is negative, and the absolute value of this elasticity is decreasing in $q(j)$, that is, in response to the same shock to marginal cost, larger firms' market shares respond less.

3.7 Stationary equilibrium

We focus on the stationary equilibrium of the model. In the equilibrium, the household and firms optimize, and the markets for the final good, intermediate goods, labor, and robots clear. The final good market clearing condition is given by

$$C + Q_a I_a + \int_\phi \int_0^{\bar{f}_a(\phi)} f_a \; dF(f_a) \; dG(\phi) = Y, \tag{28}$$

where $G(\cdot)$ is the ergodic distribution implied by the productivity process (8). Appendix A outlines the solution algorithm.

4 Model mechanism

Firms are heterogeneous along two dimensions: they face idiosyncratic shocks to both productivity (ϕ) and the fixed cost of operating the automation technology (f_a). The automation decision depends on the combination of the realizations of ϕ and f_a.

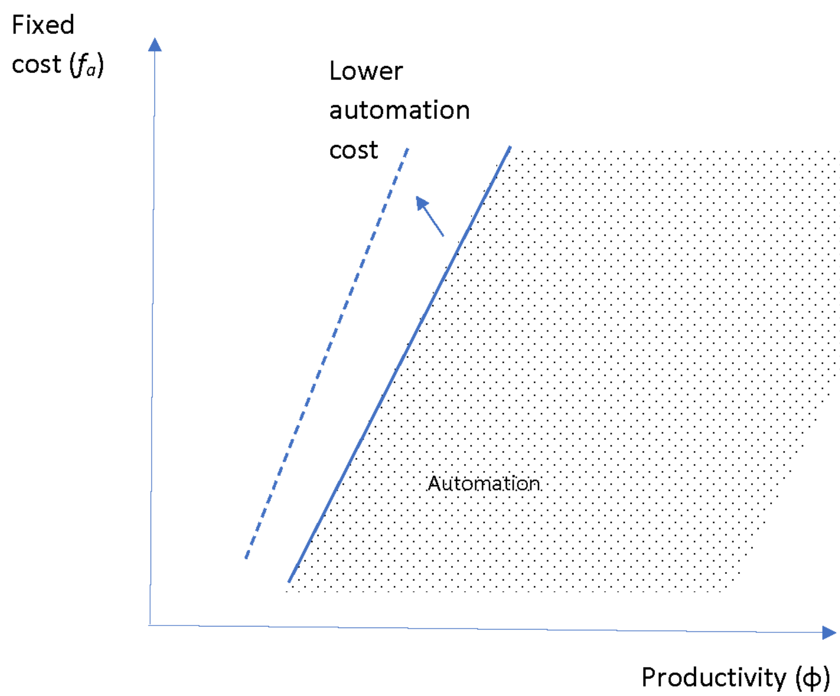
Figure 2 illustrates the automation decision rules. For any given productivity ϕ, a firm will choose to automate if the realized fixed cost is sufficiently low. Similarly, for any given fixed cost f_a, a firm will automate if the realized productivity is sufficiently high. There is an upward slowing line that separates the technology choices. To the right of the line (high ϕ and low f_a), firms use the automation technology and to the left of the line, they use the labor-only technology. Firms with combinations of ϕ and f_a on the upward-sloping line are indifferent between the two types of technologies.

The location of the indifference line is endogenous, depending on aggregate economic conditions. A decline in the relative price of robots (Q_a), for example, will lower the robot rental rate r_a, reducing the marginal cost of using the automation technology. This would shift the indifference curve up (from the solid to the dashed line), such that more firms would choose to automate (an extensive margin) and those firms already operating the automation technology would increase their use of robots (an intensive margin).

For a given technology choice (labor only or automation), a high productivity firm is also a large firm in terms of both employment and output. As illustrated in Figure 2, a high-productivity firm is also more likely to use robots at any given fixed cost. Thus, larger firms are more likely to operate the automation technology. The increased use of robots improves labor productivity, enabling those robot-using firms to become even larger and increasing the share of top firms in the product market (through the intensive margin). However, the decline in robot price also induce some less productive firms to switch from the labor-only technology to the automation technology (through the extensive margin), partially offsetting the increase in the share of sales of the top firms. The net effect of the decline in the robot price on sales concentration can be ambiguous, depending on the relative strength of the extensive vs. the intensive margin effects. Under our calibration, the intensive margin effect dominates, such that a lower robot price leads to higher concentration of sales in large firms.

A higher share of sales of the large firms does not directly translate into a higher share of employment of those firms. A decline in the price of robots improves labor productivity for firms that use robots, allowing those firms to expand. Since larger firms are more likely to use robots, they also benefit more from the declined rental costs of robots. Since robots substitute for workers, large firms can increase production without proportional increases in labor input. Thus, the share of employment of large firms increases by less than the share of sales. Furthermore, the increased use of robots reduces aggregate labor demand and also the labor income share. This is the key model mechanism to generate a positive correlation between automation and market concentration, but a negative correlation between automation and the labor share. The model mechanism also implies a larger cor-

Figure 2. Automation decision rules.



Note: This figure shows the automation decisions as a function of firm-level productivity (ϕ) and the fixed cost of operating the automation technology (f_a). Firms with (ϕ, f_a) to the lower-right of the solid line choose to automate (the shaded area) and those to the upper-left of the line choose to use the labor-only technology. A decline in the robot price shifts the indifference line upward (from the solid to the dashed line), inducing more use of the automation technology.

relation of automation with market concentration measured by the share of sales of top firms than that measured by the share of employment.

5 Calibration

Table 4 lists the parameters. The first group of parameters in Panel A are externally assigned, following the literature. The frequency of the model is quarterly, so the discount rate β equals 0.99, implying a 4% annual interest rate. The inverse Frisch elasticity ξ is 0.5 as in Rogerson and Wallenius (2009). We normalize the weight of disutility from working χ to one. The elasticity of substitution η is set to 3, following Eden and Gaggl (2018). We set the persistence of productivity shocks γ to 0.95, following Khan and Thomas (2008). As for the standard deviation of productivity shocks σ_ϕ, we follow Bloom et al. (2018). They find that the low standard deviation is 0.051 and the high value is 0.209. Also, the unconditional probability of low standard deviation is 68.7%. Therefore, the average standard deviation is 0.1. Next, we follow Edmond, Midrigan and Xu (2021) to set the Kimball elasticity σ to 10.86 and the super-elasticity ϵ/σ to 0.16. We use 0.02 as the robot depreciation rate δ, implying an 8% annual depreciation rate used by IFR. The weight of robots as the input in the production function is set to 0.465, following Eden and Gaggl (2018).

Panel B of Table 4 lists the parameters calibrated by moment matching. The relative price of robots Q_a mainly affects the fraction of firms that automate (i.e., firms using robots), which equals

$$\int_\phi F(\bar{f}_a(\phi))\ dG(\phi)$$

We use the fraction of firms that use robots in the ABS data from Zolas et al. (2020), which is 1.3%.

We assume the CDF of the automation fixed cost is a uniform distribution $U(0, F_a)$. The upper-bound of the this distribution, F_a, mainly affects the relation between firm size and automation decision. The idea is that as F_a gets larger, it becomes less likely for small firms to be able to cover the fixed cost of automation and as a result, employment share of firms that choose to automate rises. Therefore, to calibrate F_a we target the employment share of firms that automate, which in our model equals

$$\frac{\int_\phi F(\bar{f}_a(\phi))N(\phi)\ dG(\phi)}{\int_\phi N(\phi)\ dG(\phi)} \tag{29}$$

We set this data moment to 13.4%, from Zolas et al. (2020).

As Table 5 shows, the model closely matches these targeted moments. Moreover, to validate the model, we explore how the model performs in generating the distribution of

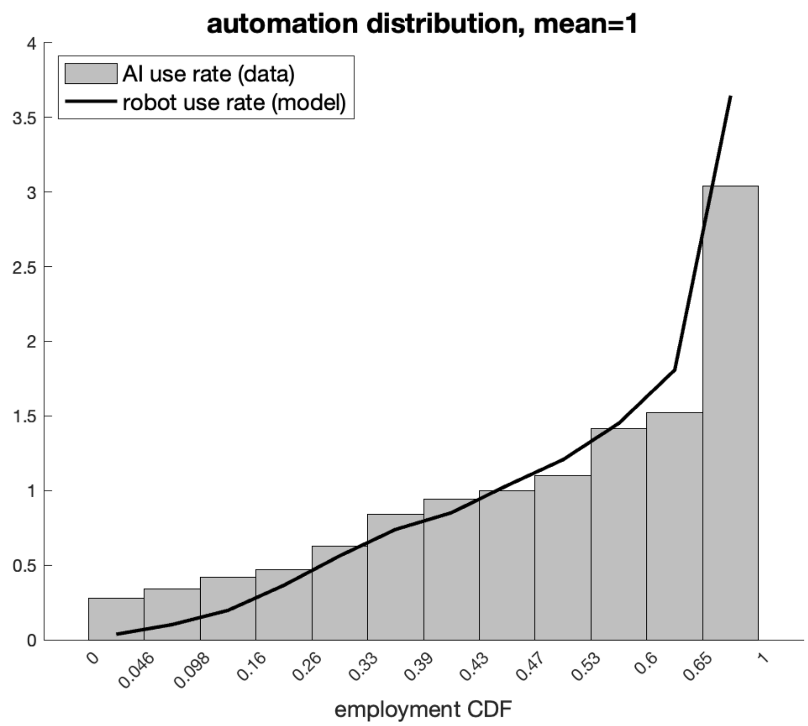
Table 4. Parameters

Parameter	**Notation**	**Value**	**Sources/Matched Moments**
		Panel A: Assigned Parameters	
Discount factor	β	0.99	4% annual interest rate
Inverse Frisch elasticity	ξ	0.5	Rogerson and Wallenius (2009)
Working disutility weight	χ	1	Normalization
Elasticity of substitution	η	3	Eden and Gaggl (2018)
Productivity persistence	γ	0.95	Khan and Thomas (2008)
Productivity standard deviation	σ_ϕ	0.1	Bloom et al. (2018)
Demand elasticity parameter	σ	10.86	Edmond, Midrigan and Xu (2021)
Super-elasticity	ϵ/σ	0.16	Edmond, Midrigan and Xu (2021)
Robot depreciation rate	δ_a	0.02	8% annual depreciation rate
Robot input weight	α_a	0.465	Eden and Gaggl (2018)
		Panel B: Parameters from Moment Matching	
Relative price of robots	Q_a	50.1	Fraction of firms that automate
Automation fixed cost	F_a	1.5	Employment share of firms that automate

Table 5. Matched Moments

Moments	**Data**	**Model**
Fraction of firms that automate	1.3%	1.2%
Employment share of firms that automate	13.4%	13.4%

Figure 3. Automation Distribution



Note: This figure plots the distribution of AI use rate (i.e., fraction of firms that use AI in their production) in the ABS data (from Zolas et al., 2020) and the fraction of firms with robots in the model.

automation in the ABS data. Figure 3 plots the distribution of AI use rate (i.e., fraction of firms that use AI in their production) in the data (from Zolas et al., 2020) and the fraction of firms with robots in the model.[5] The model closely matches this non-targeted distribution of robot use rates.

6 Quantitative Results

Figure 4 shows firms' decision rules conditional on the idiosyncratic productivity ϕ. We show the decision rules in our baseline calibration (black-solid line) and in a counterfactual in which the robot price Q_a falls by 50% (the dashed-red line). As figure 4 shows, more

[5]In Figure 8 of Zolas et al. (2020), they show the share of firms that use AI technologies for each size category, e.g., 1-4 employees, 5-9 employees, or 10,000+ employees. To make a fair comparison between the data and the model, we use the 2017 County Business Patterns and Economic Census to obtain the number of employees for each firm size category to compute the cumulative distribution function (CDF) of employment-based firm sizes in the data. Now we have the AI use rates with respect to the employment CDF in the data. Then we calculate the robot use rates with respect to the employment CDF in the model in the same way. In Figure 3, we normalize their mean to one for comparison.

productive firms are larger (i.e., hire more workers and have higher relative output $q(\phi)$). Since more productive firms are relatively larger, they have more market power and charge higher markups. Larger firms are more likely to automate since they are more profitable (i.e., have larger sales and higher markups), and as a result, are more likely to be able to cover the fixed cost of automation. Larger firms have lower labor shares for two reasons. First, these firms charge higher markups and therefore the share of labor compensation in total sales is larger.[6] This force is at play regardless of the automation decision. Second, larger firms are more likely to automate, and as a result have lower labor shares. This effect is working only for the firms that automate.

A fall in automation cost Q_a makes automation more attractive and increases automation probabilities. Since the reduction in Q_a acts like a positive productivity shock for firms with robots, firms that do not automate lose their market shares to those that do. As a result, number of workers $N(\phi)$ and relative size $q(\phi)$ fall (rise) for firms that do not (do) automate. Notice that a reduction in robot price activates two competing forces on the employment at the firms with robots. On the one hand, these firms substitute away from workers to robots, which tends to reduce employment at these firms. On the other hand, however, these firms gain market share and get larger, and therefore, will employ more workers. As figure 4 shows, the latter effect dominates and firms with robots employ more workers after the reduction in robot price. Finally, since firms with (without) robots get relatively larger (smaller) after the reduction in Q_a, they charge higher (lower) markups.

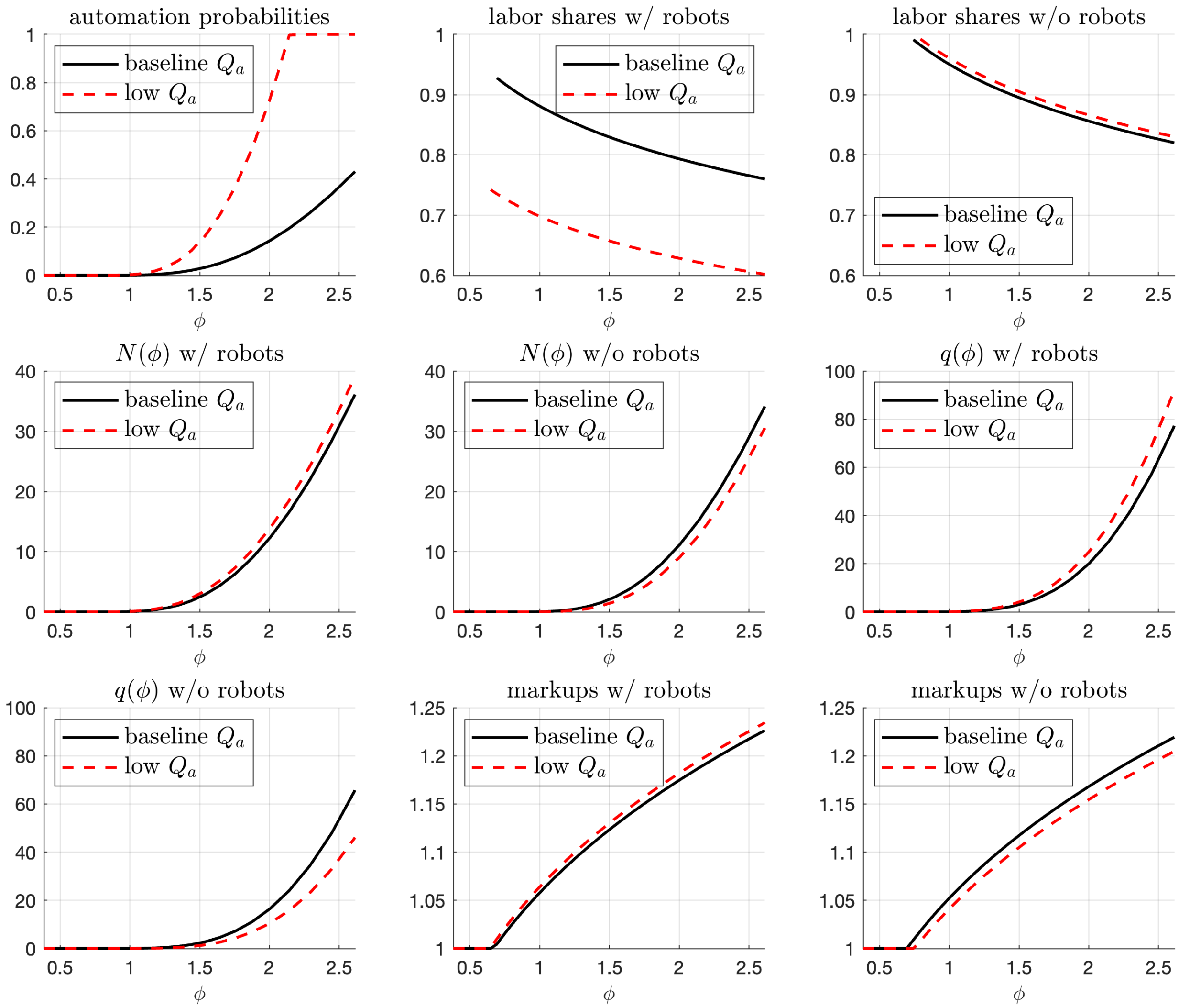
To analyze the effects of automation on aggregate variables in the economy, we perform the following counterfactual experiments: we raise and reduce by 50% the robot price Q_a from its baseline calibrated value (which is 50.1). Figure 5 shows the effects on fraction of firms that automate, share of top 1% firms, labor share, and average markup. As the robot price falls, more firms will find it profitable to adopt robots and the fraction of firms that automate rises. Since larger firms are more likely to automate, they benefit more than smaller firms from a reduction in the robot price. As a result, the product market gets more concentrated and the share of top 1% firms rises. Interestingly, the sales-share of top 1% of firms rises more than their employment-share, which is caused by two forces. First, the relative automation usage by large firms rises, and therefore their sales-share rises more than their employment-share. Second, larger firms that benefit from the reduction in robot price get relative larger and charge higher markups. These higher markups refrain increasing production and hiring. As Q_a falls, larger firms get larger, which causes the average markup (both sales- and cost-weighted) to increase.[7] Moreover, as Q_a falls, labor share in the economy falls not only because some firms substitute labor with robots, but also because of the increase in average markups.

To explore the role of variable markups in generating the results, we consider the constant-markup version of the model in which we replace the Kimball aggregator (14) with

[6] Note that since we do not have intermediate inputs in the model, sale and value added are equal.

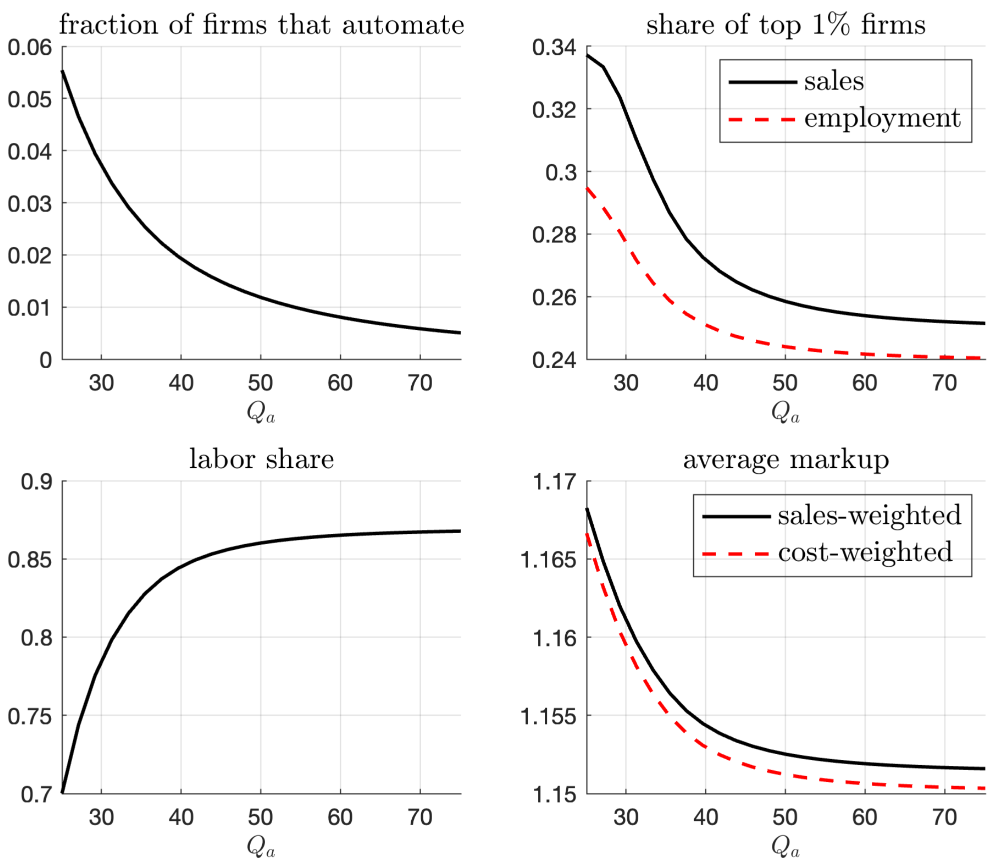
[7] To derive the cost-weighted average markup, we use total variable costs at each firm, as in Edmond, Midrigan and Xu (2021).

Figure 4. Firms' Decision Rules



Note: This figure shows firms' decision rules for the firms that automate (w/ robots) and those that do not automate (w/o robots). The solid-black lines are associated with our baseline calibration, while red-dashed lines show the results for a counterfactual in which robot price Q_a falls by 50%.

Figure 5. Aggregate Variables



Note: This figure shows the effects of counterfactual changes in the robot price Q_a on the fraction of firms that automate, share of top 1% firms, labor share, and average markup. We raise and reduce by 50% the robot price Q_a from its baseline calibrated value (which is 50.1).

a Constant Elasticity of Substitution (CES) aggregator. Appendix B oulines this model and its solution algorithm. We use a CES elasticity that generates the same cost-weighted average markup as in our baseline variable-markup model (i.e., the average markup equals 1.15). Our calibration shows that the CES model is not able to match the fraction of firms that automate and employment share of firms that automate at the same time. Indeed, while we match the former, the latter moment will be too high. This is because the distribution of firm size is too skewed to the right when we use the empirically estimated productivity process from Bloom et al. (2018) and CES production function at the same time.

In the CES model, we perform the same counterfactual exercise which increases/decreases the robot price by 50% from the baseline value of 50.1. Figure 6 shows the results. As this figure shows, aggregate variables respond less to a fall in robot price; As the robot price falls, the increase in the fraction of firms that automate as well as the decline in labor share are not as strong as they are in the variable-markup model. This is because unlike the variable-markup model, as firms get larger they do not charge higher markups in this CES model, and therefore the probability of being able to cover the automation cost does not rise as much as in the variable-markup model.

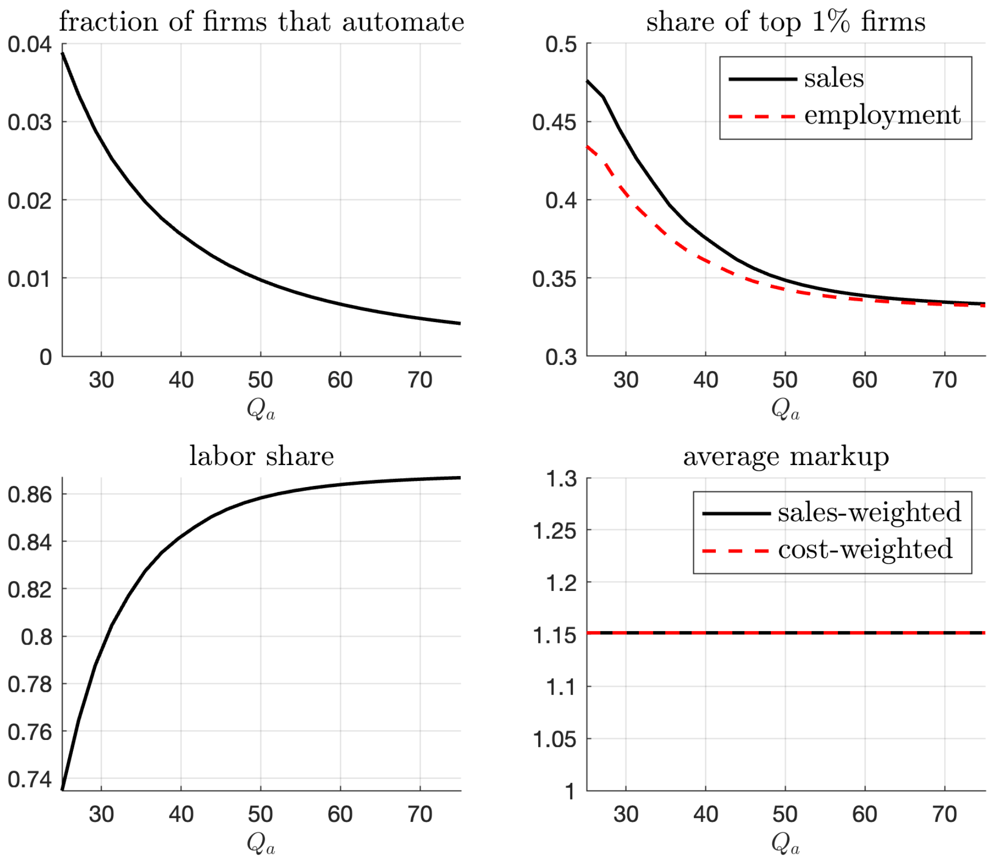
7 Conclusion

We study how increased automation may have contributed to the rise in industrial concentration and the decline in the labor share since the early 2000s. For this purpose, we build a general equilibrium model with heterogeneous firms, variable markup, and endogenous automation decisions. Our calibrated model does well in matching the cross-sectional distribution of automation usage observed in the firm-level data. The model implies that larger firms with higher productivity and higher markups are more likely to use the automation technology. A decline in automation costs (e.g., robot prices) intensifies the automation usage by large firms, improving labor productivity of those firms and enabling them to expand the market share of their production. Since robots substitute for workers, the decline in automation costs leads to a sharper increase in market concentration measured by sales than that measured by employment. Since large firms are more likely to operate the automation technology and robots can substitute for workers, the relative expansion of large firms leads to a decline in aggregate labor income share through a between-firm reallocation channel, consistent with empirical evidence.

References

Acemoglu, Daron, and Pascual Restrepo. 2018. "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review*, 108(6): 1488–1542.

Figure 6. Aggregate Variables (Case of CES)



Note: This figure shows the effects of counterfactual changes in the robot price Q_a in the CES version of the model on the fraction of firms that automate, share of top 1% firms, labor share, and average markup. We raise and reduce by 50% the robot price Q_a from its baseline calibrated value (which is 50.1).

Acemoglu, Daron, and Pascual Restrepo. 2020. "Robots and Jobs: Evidence from US Labor Markets." *Journal of Political Economy*, 128(6): 2188–2244.

Acemoglu, Daron, Claire Lelarge, and Pascual Restrepo. 2020. "Competing with Robots: Firm-Level Evidence from France." *AEA Papers and Proceedings*, 110: 383–88.

Arnoud, Antoine. 2018. "Automation Threat and Wage Bargaining." Unpublished Manuscript, Yale University.

Autor, David, David Dorn, Lawrence F Katz, Christina Patterson, and John Van Reenen. 2020. "The Fall of the Labor Share and the Rise of Superstar Firms*." *The Quarterly Journal of Economics*, 135(2): 645–709.

Autor, David H., David Dorn, and Gordon H. Hanson. 2013. "The China Syndrome: Local Labor Market Effects of Import Competition in the United States." *American Economic Review*, 103(6): 2121–2168.

Autor, David H., Frank Levy, and Richard J. Murnane. 2003. "The Skill Content of Recent Technological Change: An Empirical Exploration." *The Quarterly Journal of Economics*, 118(4): 1279–1333.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J Terry. 2018. "Really uncertain business cycles." *Econometrica*, 86(3): 1031–1065.

Eden, Maya, and Paul Gaggl. 2018. "On the welfare implications of automation." *Review of Economic Dynamics*, 29: 15–43.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu. 2021. "How costly are markups?"

Graetz, Georg, and Guy Michaels. 2018. "Robots at Work." *The Review of Economics and Statistics*, 107(5): 753–768.

Jaimovich, Nir, and Henry E. Siu. 2020. "Job Polarization and Jobless Recoveries." *The Review of Economics and Statistics*, 102(1): 129–147.

Khan, Aubhik, and Julia K Thomas. 2008. "Idiosyncratic shocks and the role of nonconvexities in plant and aggregate investment dynamics." *Econometrica*, 76(2): 395–436.

Kimball, Miles S. 1995. "The Quantitative Analytics of the Basic Neomonetarist Model." *Journal of Money, Credit and Banking*, 27(4): 1241–1277.

Klenow, Peter J, and Jonathan L Willis. 2016. "Real rigidities and nominal price changes." *Economica*, 83(331): 443–472.

Leduc, Sylvain, and Zheng Liu. 2019. "Robots or Workers? A Macro Analysis of Automation and Labor Markets." Federal Reserve Bank of San Francisco Working Paper No. 2019-17.

Rogerson, Richard, and Johanna Wallenius. 2009. "Micro and macro elasticities in a life cycle model with taxes." *Journal of Economic theory*, 144(6): 2277–2292.

Zolas, Nikolas, Zachary Kroff, Erik Brynjolfsson, Kristina McElheran, David N Beede, Cathy Buffington, Nathan Goldschlag, Lucia Foster, and Emin Dinlersoz. 2020. "Advanced Technologies Adoption and Use by U.S. Firms: Evidence from the Annual Business Survey." National Bureau of Economic Research Working Paper 28290.

Appendices

A Solution Algorithm

Out of iterations

1. Given the relative price of robots Q_a, the rental rate of robots r_a is determined by (6): $r_a = Q_a/\beta - Q_a(1-\delta_a)$.
2. The distribution of firms is the stationary distribution of the productivity process because there is no entry or exit.

There are three loops to solve the problem. Y loop is outside W loop. And W loop is outside q oop.

Y loop: use bisection to determine the aggregate final goods and other aggregate variables

1. Guess a Y.
2. Compute W and firms' relative production $q(j)$ in the W loop as explained below.
3. Given the equilibrium wage rate, compute other aggregate variables by finding Y using the bisection method:
   (a) Given the solved relative production $q(j)$, we have $y(j) = q(j)Y$.
   (b) Given the known r_a and W, compute the marginal costs $\lambda(j)$ by eq. (11) and (13), and we can get $A(j)$ and $N(j)$ from eq. (9) and (10).
   (c) The aggregate robot and labor demand are $A = \int A(j)d\nu$ and $N = \int N(j)d\nu$.
   (d) Consumption C is determined by (5).
   (e) The steady state I is from (4).
   (f) Compute Y^{new} using the resource constraint (28). Stop if Y converges.
      i. If $Y = Y^{\text{new}}$, Y and all other aggregate variables are found.
      ii. If $Y > Y^{\text{new}}$, lower Y. Go back to Step 1.
      iii. If $Y < Y^{\text{new}}$, increase Y. Go back to 1.

W loop: use bisection to determine the wage rate

1. Guess a wage W.
2. Compute firms' relative production $q(j)$ in the q loop as explained below.

3. Check whether the Kimball aggregator (14) holds.
   (a) If LHS = RHS, the wage rate is found and jump out of W loop to Y loop.
   (b) If LHS > RHS, increase the wage rate to lower $q(j)$ according to eq. (15). Go back to Step 2.
   (c) If LHS < RHS, lower the wage rate to raise $q(j)$ according to eq. (15). Go back to Step 2.

q loop: find the relative production

1. Given the factor prices r_{at} and W_t, the marginal cost of production is determined by eq. (11) with robots and (13) without robots.
2. Guess a demand shifter D.
3. Use eq. (15) to solve the market shares $q(\phi)$ for each ϕ with and without robots.
   (a) The right-hand side of (15) is a function of q by plugging in (19).
   (b) The price in the left-hand side is the marginal cost in (11) or (13) times the markup in (21), which is also a function of q.
   (c) Use the bisection method to solve for q in eq. (15).
4. Compute the automation decisions.
   (a) Compute $y(j) = q(j)Y$ with and without robots.
   (b) Compute the demand for $A(j)$ and $N(j)$ with and without robots from (9), (10), and (12).
   (c) Compute the profits with and without robots and thus get the automation cutoffs of f_a and thus the automation probabilities according to (**??**).
5. Given the automation decisions, compute D^{new} by (16). Stop if D converges. Otherwise, go back to Step 2 and repeat until D converges.
   (a) If $D = D^{\text{new}}$, D and $q(j)$ are found and jump out of q loop to W loop.
   (b) If $D > D^{\text{new}}$, lower D. Go back to Step 2.
   (c) If $D < D^{\text{new}}$, increase D. Go back to Step 2.

B The CES Model

Final good producers make a composite good out of the intermediate varieties and sell it to consumers. The final good Y is produced using a bundle of intermediate goods $y(j)$, according to the following commonly available CES aggregator:

$$Y = \left[\int_0^1 y(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}} \tag{30}$$

where the intermediate varieties are denoted by j.

Given the prices for intermediate varieties $p(j)$, final good producers minimize their cost, which gives the following demand for each intermediate good:

$$y(j) = \left(\frac{p(j)}{P} \right)^{-\sigma} Y \tag{31}$$

where P is the CES aggregate price index

$$P = \left[\int_0^1 p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \tag{32}$$

All firms charge the same Dixit-Stiglitz markup $\frac{\sigma}{\sigma-1}$:

$$p(j) = \frac{\sigma}{\sigma - 1} \lambda(j) \tag{33}$$

where the marginal cost $\lambda(j)$ defined in (11) and (13).

B.1 Solution Algorithm for the CES Model

Out of iterations

1. Given the relative price of robots Q_a, the rental rate of robots r_a is determined by (6): $r_a = Q_a/\beta - Q_a(1 - \delta_a)$.
2. The distribution of firms is the stationary distribution of the productivity process because there is no entry or exit.

There are two loops to solve the problem. Y loop is outside W loop. And there is no q loop for the CES case because it does not have the demand shifter D.

Y loop: use bisection to determine the aggregate final goods and other aggregate variables

1. Guess a Y.
2. Compute W and firms' output $y(j)$ in the W loop as explained below.
3. Given the equilibrium wage rate, compute other aggregate variables by finding Y using the bisection method:

(a) Given the known r_a and W, compute the marginal costs $\lambda(j)$ by eq. (11) and (13), and we can get $A(j)$ and $N(j)$ from eq. (9) and (10).

(b) The aggregate robot and labor demand are $A = \int A(j)d\nu$ and $N = \int N(j)d\nu$.

(c) Consumption C is determined by (5).

(d) The steady state I is from (4).

(e) Compute Y^{new} using the resource constraint (28). Stop if Y converges.

i. If $Y = Y^{\text{new}}$, Y and all other aggregate variables are found.

ii. If $Y > Y^{\text{new}}$, lower Y. Go back to Step 1.

iii. If $Y < Y^{\text{new}}$, increase Y. Go back to 1.

W loop: use bisection to determine the wage rate

1. Guess a wage W.

2. Given the factor prices r_{at} and W_t, the marginal cost of production is determined by eq. (11) with robots and (13) without robots. The prices with or without robots are from (33).

3. Compute $y(j)$ by the demand function (31) with and without robots.

4. Compute the automation decisions.

   (a) Compute the demand for $A(j)$ and $N(j)$ with and without robots from (9), (10), and (12).

   (b) Compute the profits with and without robots and thus get the automation cutoffs of f_a and thus the automation probabilities according to (**??**).

5. Check whether the price of final goods (32) equals one.

   (a) If $P = 1$, the wage rate is found and jump out of W loop to Y loop.

   (b) If $P > 1$, lower the wage rate to lower P. Go back to Step 2.

   (c) If $P < 1$, increase the wage rate to raise P. Go back to Step 2.