Uncertainty, Liquidity Constraint, and Entrepreneurship

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This Version: February, 2022

Abstract

How do time-varying uncertainties affect entrepreneurship and consequently the aggregate economy? Based on a pan-European firm-level dataset, we find that the startups born in industries facing high uncertainties are smaller, less productive, and exhibit slower post-entry dynamics. The perverse effects of uncertainty on entrepreneurial activities are more pronounced when liquidity constraints are tighter. We estimate a dynamic occupation choice model with financial frictions and time-varying uncertainties to reconcile the empirical facts. Our quantitative results indicate that the productivity selection mechanism among new entrepreneurs is an important factor propagating uncertainty shocks to the real economy.

Keywords: Entrepreneurship, Time-varying uncertainty, Financial frictions, Productivity distortions

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1 Introduction

Entrepreneurship is a key determinant of economic growth. Entrepreneurial activities, by nature, are risky. However, the importance of the uncertainty in shaping entrepreneurship with firm-level frictions remains under-investigated. How does the uncertainty affect the nature of entrepreneurship and the entrepreneurial growth dynamics? What is the long-run impact of the uncertainty shock on the aggregate economy through the channel of entrepreneurship? Whether the uncertainty shock induces the post-crisis slow recovery? This paper provides empirical evidence and quantitative analysis to answer these questions.

Intuitively, when individuals seek to startup their new business, uncertainty is one of their primary considerations. A rise in uncertainty may change the expected entry value and influence individuals' entrepreneurial decisions. The impacts of uncertainty may vary across different types of individuals since the entry value depends upon the idiosyncratic characteristics when they are facing market frictions. As shown later, a rise in uncertainty may reduce the entry value of those entrepreneurs of high ability and may raise that of low ability ones. As a result, a higher uncertainty leads to productivity distortions by crowding in low-productivity entrants and crowding out those with high productivity. This distortionary effect on firm-level productivity might cause a short-run recession and a long-run slow recovery in the aggregate economy. This paper aims to quantitatively document the extent to which the channel mentioned above can propagate the disturbances of the uncertainty shock to the aggregate economy.

Using the European firm-level dataset, Orbis-Amadeus, we study the entrepreneurial dynamics under the uncertainty shock. Our empirical analysis suggests that the average size of startups is negatively correlated with the industry-level uncertainty measured by the dispersion of firm-level total factor productivity (TFP) shocks. The startups who enter the market during the high-uncertainty episodes tend to be less productive and have significantly slower post-entry growth dynamics than those startups entering in the normal time. The negative relationship between the uncertainty and the entrepreneurial dynamics is more pronounced when the financial condition becomes tighter. This suggests that the financial constraint exacerbates the productivity distortion among startups caused by the uncertainty shock.

Based on the empirical evidence, we construct a rich quantitative framework to analyze the productivity distortion channel through which the uncertainty may affect the entrepreneurship and post-entry growth dynamics. Our quantitative model features heterogeneous households and entrepreneurs. Households in the model are essentially Aiyagari-type workers with heterogeneous abilities (working efficiencies). The workers supply labor to entrepreneurs and obtain wage income. In each period, the worker makes occupation choices between being a worker or starting up a new business (i.e., being an entrepreneur) by paying a fixed cost. Once the business is successfully set up, the worker's ability becomes the new entrepreneur's innate ability. The entrepreneur's productivity contains a transitory component and a permanent component. The transitory productivity follows an AR(1) process with an initial value equal to the entrepreneur's innate talent and with an idiosyncratic productivity shock. The standard deviation of the idiosyncratic productivity shock is time-varying, reflecting the uncertainty facing the entrepreneur. The permanent productivity follows a technology upgrading process, with the upgrading probability positively depending on the entrepreneur's innate talent. The entrepreneur produces by combining labor and physical capital. She finances the physical capital investment through internal cash and external loan. The loan is constrained in the form of a liquidity constraint. The high-productivity entrepreneurs demand more external funds to pursue high profitability, resulting in a binding borrowing constraint. The low-productivity entrepreneurs do not rely on external finance due to a low capital demand, leaving the borrowing constraint non-binding. The above optimal decision rules suggest that the entrepreneur's value function is a concave function of productivity shock in the high-productivity regime and a convex function in the low-productivity regime. An increase in productivity uncertainty raises the expected value of being an entrepreneur for those workers with low ability and suppresses the expected value of entrepreneurship for those with high ability. As a result, the uncertainty shock renders *productivity selection* effects among startups by inducing relatively more low-ability individuals to start new businesses. Since the permanent component of the productivity positively correlates with the entrepreneur's innate talent, the startups who enter the market during the high uncertainty periods tend to have much slower growth dynamics than those in the normal time.

We calibrate the quantitative model using the European firm-level dataset by fitting the model-implied moments with their empirical counterparts. The model can tightly match the non-targeted firm-level moments observed in the data. Based on the calibrated model, we then quantify the consequences of the uncertainty shock on the entrepreneurial dynamics and the aggregate economy. Our quantitative exercise shows that a one-standard-deviation increase in uncertainty causes an 8%-30% reduction in the average size of startups. The uncertainty shock decreases aggregate production and drives slow-recovery dynamics. Therefore, the uncertainty shock renders an economic slowdown in the short run and sluggish output growth in the long run. The productivity selection mechanism is a crucial determinant of the aggregate dynamics under an uncertainty shock. Regarding the micro-level firm dynamics, we find that the firms born in the high uncertainty periods have significantly slower growth dynamics than those born in the low uncertainty periods. The divergence of the post-entry growth dynamics between two types of firms is largely mitigated when the productivity selection mechanism is muted. We further conduct a counterfactual analysis by varying the parameter value of borrowing constraints. The quantitative results show that the adverse impact of uncertainty on entrepreneurship becomes more pronounced in the case of tighter borrowing constraints than in the baseline model. Therefore, borrowing constraints strengthen the productivity selection mechanism in amplifying the adverse impacts of uncertainty shock on entrepreneurial activities and the aggregate economy.

Related Literature Our paper is directly related to several strands of literature. First, we contribute to the literature on entrepreneurship under market frictions. In their seminal work, Evans and Jovanovic (1989) empirically estimate the entrepreneurial decisions under liquidity constraints based on a static entrepreneurial choice model. Their analysis suggests that liquidity constraints are crucial to entrepreneurial decisions since the potential entrants with financially binding constraints must bear most of the entrepreneurial risks and thus are reluctant to start business. Quadrini (2000) studies how entrepreneurship affects household savings and wealth distribution in a dynamic general equilibrium model of endogenous entrepreneurial choices. The paper finds that the level of asset holdings is important in the agents' to undertake entrepreneurial activities due to the presence of borrowing constraints. Hamilton (2000) empirically studies the determinants of earnings differentials between paid employment and self-employment. The paper finds that most entrepreneurs have lower initial earnings and earnings dynamics than paid work, suggesting that the nonpecuniary benefit is essential for entrepreneurship. Paulson et al. (2006) incorporates the financial frictions of limited liability and moral hazard into a structural occupation choice model. Using Thailand household data, they estimate the model and identify the moral hazard as the dominate source of financial frictions in affecting entrepreneurial decisions. Cagetti and De Nardi (2006) construct and calibrate a quantitative model with endogenous occupational choices in which borrowing constraints are a key determinant of entrepreneurial decisions and the aggregate implications. They find that tighter borrowing constraints reduce average firm size, aggregate capital, and the fraction of entrepreneurs. Moreira (2016) uses micro-level data to document a new evidence that businesses born during recessionary periods start on a smaller scale and remain smaller over their entire life-cycle due to the selection mechanism at entry and demand-side channel. Unlike the above papers, we focus on the impact of time-varying uncertainty on entrepreneurship and the amplification channel of financial frictions. Our empirical and quantitative analysis uncovers a novel mechanism through which uncertainty and the financial constraint jointly play a role.

Second, our paper is closely related to the recent literature studying how uncertainty shocks affect potential entrepreneurial activities. Vereshchagina and Hopenhayn (2009) study entrepreneurial risk-taking behaviors in a dynamic occupational choice model. In their model, the rich with sufficient funds choose to be entrepreneurs. The entrepreneurial risks encourage entrepreneurship and risk-taking due to the non-convexity of value function. Wang et al. (2012) build a unified incomplete market entrepreneurial model with uninsurable risks and liquidity constraints. They study the interdependence between entrepreneurial choices and financial decisions. Their analysis suggests that the precautionary motive and financial frictions depress the entrepreneurs' economic activities, and the option to accumulate wealth is critical for entrepreneurship. Herranz et al. (2015) study how risk aversion affects entrepreneurs' endogenous production and financial decisions. In a dynamic incomplete market model, they find that more risk-averse entrepreneurs run smaller, more highly leveraged firms with more negative equity than their less risk-averse counterparts. Our paper contributes to this strand of literature by empirically and theoretically showing how uncertainty induces asymmetric effects on entrepreneurship through the productivity selection mechanism and its impact on the firm dynamics and the aggregate economy.

Finally, our paper contributes to vibrant literature that investigates the impacts of firm-level uncertainty. Bloom (2009) and Bloom et al. (2018) empirically document that the uncertainty shocks have a significant effect on the aggregate economy. They construct a dynamic heterogeneous firm model with non-convex capital and labor adjustment costs to show that the wait-and-see effect provides a crucial channel to propagate uncertainty shocks. Bachmann et al. (2013) identify uncertainty shock through the dispersion of investment rates, and they find that the estimated uncertainty shock is not the primary source of aggregate fluctuations in a standard heterogeneous firm model with investment lumpiness. Gilchrist et al. (2014) construct a quantitative model with both investment irreversibility and financial frictions. Their results show that the credit channel is more important for understanding the impact of uncertainty shocks. Dyrda (2015) documents asymmetric responses across age groups of U.S. firms to idiosyncratic uncertainty shocks. In this paper, the financial constraint is endogenously derived from a dynamic contract between financial intermediaries and entrepreneurs. An increase in idiosyncratic uncertainty tightens young firms' financial constraints, making them suffer more and eventually leading to an economic recession. Alfaro et al. (2018) study the impact of uncertainty shock in a model with real and financial frictions. They show that financial frictions amplify the adverse effects of uncertainty shocks on investment and hiring. An uncertainty shock associated with financial frictions induces the standard real-options effects on investment and hiring and leads firms to hoard cash and further cut investment and hiring. Arellano et al. (2019) study the aggregate impact of firm-level uncertainty in a heterogeneous firm model with borrowing constraint. An uncertainty shock causes aggregate output and labor declines since the individual firm's production is risky, and the uncertainty is uninsurable due to financial frictions. Berger et al. (2019) identify uncertainty shocks that are orthogonal to the current volatility shock and find these shocks have no impact on the aggregate economy. Their quantitative model in which fundamental shocks are skewed left can explain the facts. They find that it is the realization of volatility, rather than uncertainty about the future, induces economic recession. Unlike the above-mentioned quantitative models that focus on the channel of real frictions, our paper highlights the productivity selection effects of uncertainty on entrepreneurial activities and the firm dynamics.

2 Empirical Analysis

2.1 Data

We use the firm-level dataset, Orbis-Amadeus, with the sample periods from 1999 to 2015.[1] We focus on European countries because the company reporting is compulsory for all types of firms. For our purpose, the dataset has several advantages. First, the observations are at administrative level, including business registers. This information enables us to identify those entrants. Second, 99% of the firms in the dataset are of private ownership, whose primary financing source is bank loans. This implies shrinking of credit supply from the banking sector transmit *directly* to the tightening of external financing. Third, the dataset provides a narrowly defined industry classification for firms. Its 4-digit NACE (Rev 2) code has a comparable level of disaggregation level with the 6-digit NAICS code.We focus on the manufacturing industries with NACE codes from 1000 to 3999. Following the cleaning procedure in Gopinath et al. (2017), we obtain approximately 14,000 industry-country-year cells. Our sample contains approximately 40,000 entrants that report a positive size during the sample periods, with a median entry rate of 4% at industry level. In Appendix A, we provide detailed description regarding the data construction and summary statistics.

We complement the firm-level data with the bank-level cross-country dataset, BankScope, and the Macroprudential Policies Evaluation Database, MaPPED, for the European Union (EU) countries. The BankScope dataset contains the bank balance-sheet information covering the years of 1998 to 2015. The MaPPED dataset lists macroprudential policies from 1995 to 2014. It covers almost 1,700 policies including regulations on capital requirements, risk weights, and credit limits, from national central banks as well as other banking supervisory authorities (See Appendix B for details). One advantage of using policies with a macroprudential nature is that these instruments is very likely to have a significant impact on the whole banking system,

[1] Orbis-Amadeus dataset is compiled by the Bureau van Dijk Electronic Publishing (BvD). We use the "Orbis Historical" database directly provided by the BvD (link). The original data can be downloaded from the providers' website, or offline disks, requiring effort to compile. Kalemli-Ozcan et al. (2015) provides a careful and detailed description of the downloading and processing procedure in this case.

which provide crucial sources of external financing for startups.

2.2 Uncertainty and Entrepreneurship

2.2.1 Entrants' Size

Our empirical facts are related to the link between the uncertainty and the entrepreneurship (entrants). We follow the method in Bloom et al. (2018) to construct the indicator of within-industry uncertainty. This measurement is the overtime change in the dispersion (standard deviation) of TFP shocks across firms within each industry-country-year cell, reflecting the firm's ex-ante exposure to the industry-level uncertainty (Baker et al., 2016). For the entrepreneurship, we focus on the entrants' size, which is measured by the number of employees.

We conduct ordinary linear regression (OLS) to study the link between the within-industry uncertainty and the entrants' size. The baseline regression specification is

$$y_{j,i,c,t} = \beta_0 + \beta_1 \times \Delta\sigma_{i,c,t-1} + \alpha_{c,t} + \varepsilon_{j,i,c,t}. \tag{1}$$

The dependent variable $y_{j,i,c,t}$ is the logarithm of the size for firm j in industry i of country c at year t, normalized by the average value of the incumbents continuously present in the data in the corresponding industry-country-year cell. The independent variable $\Delta\sigma_{i,c,t-1}$ is the change of within-industry standard deviation of TFP shocks in industry i of country c between the year $t-1$ and the year $t-2$. The term $\alpha_{c,t}$ captures the country-year fixed effects. This implies we are looking at the same country-year, and therefore the recovered coefficient, β_1, represents the cross industry elasticity of uncertainty on startup size.

Column 1 of Table 1 shows that the entrant has significantly smaller size when the within-industry standard deviation of TFP shocks is higher. One unit (around 4 standard deviations) increase of the within-industry standard deviation of TFP shocks is associated with 12.6% reduction of the size for entrants in the next year. The result changes little when we control the average growth rate of loan and the average sales growth within the corresponding industry (Column 2 of Table 1).

We conduct a range of robustness checks around the baseline specification, and the negative association between uncertainty and startup size remains. In the second column, we replace startup size with the non-normalized log-number of employment. In column 3, we add industry level control including sales and loan growth. In Column 4, we proxy uncertainty by the more conservative inter-quartile ratio (IQR) of the TFP shocks within the industry-country-year cell. In Columns 5 and 6, we conduct the regressions on the pre-crisis (2000-2007) and the post-crisis (2008-2015) sub-samples. The significant coefficient obtained for normal-times sample is particularly interesting, as it suggests the negative relationship is not a crisis phenomena. This means that other confounding factors such as declining demand and credit are not likely to be independent driving force for the negative correlation. Nevertheless, we do observe the estimates of β_1 are more negative for the post-crisis periods than the pre-crisis periods. This implies that demand or financial factors may play an amplification role. In Columns 7 and 8, we use the amount of the tangible assets and the total assets, respectively, to measure firm size. The regressions yield a significantly negative link between the within-industry uncertainty and the entrants' size. Column 9 further regresses the entrants' productivity on the baseline measure of uncertainty. The significant negative sign implies a higher uncertainty is associated with lower average value of entrants' productivities, implying that the uncertainty may have selection effects on the entrants with different productivities. The adverse impact of the uncertainty on the entrants' productivities conforms to the dampening effect of uncertainty on the entrants' size.

Table 1: Uncertainty and Entrants' Size

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Specification:	Baseline	Abs	More Ctrls	IQR	Pre-crisis	Post-crisis	Tangible assets	Total assets	Productivity
$\Delta\sigma_{i,c,t-1}$	-0.126^{***}	-0.069^{**}	-0.091^{**}	-0.149^{**}	-0.114^{*}	-0.305^{***}	-0.264^{**}	-0.138^{***}	-0.067^{***}
	(0.033)	(0.031)	(0.042)	(0.066)	(0.054)	(0.085)	(0.063)	(0.027)	(0.018)
Country-Year FEs	yes	yes	yes	yes	yes	yes	yes	yes	yes
$\mathbf{R}^2$	0.059	0.053	0.101	0.059	0.091	0.034	0.101	0.103	0.104
Observations	52,098	52,644	44,886	52,098	21,577	19,764	40,007	57,729	39,352
Years	2000-2015	2000-2015	2000-2015	2000-2015	2000-2007	2008-2015	2000-2015	2000-2015	2000-2015

Notes: This table reports the regression results under different specifications. The full sample covers the years from 2000 to 2015. Each regression includes the country-year fixed effect. The dependent variable is the entrants' size. The independent variable of interest is the change of the within-industry uncertainty. The uncertainty indicator ($\sigma_{i,c,t-1}$) in all columns (except Column 3) is the standard deviation of TFP shocks within each industry-country-year cell. Column 1 is the baseline regression that estimates (1). Column 2 introduces two controls, including the average growth rate of the loan and the average sales growth within each industry-country-year cell, into the baseline regressions. Column 3 uses the inter-quartile ratio (IQR) of TFP shocks within each industry-country-year cell to measure the uncertainty. Columns 4 and 5 conduct the baseline regressions using pre-crisis and post-crisis subsamples. Columns 6 and 7 replace the dependent variable, employment, with the tangible assets and the total assets, respectively, to measure the entrants' size. Column 8 regresses the entrants' productivity on the uncertainty within each industry-country-year cell. t values are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

2.2.2 Post-entry Growth

In the previous section, we have established the fact that startups in industries with high uncertainty are smaller. A natural question that follows is whether the size difference persists. To this end, we study the relationship between the uncertainty and the post-entry dynamics of entrants. We regress the post-entry growth rate of size (measured by the number of employees) for each entrant on the standard deviation of TFP shocks within the industry-country-year cell. In each regression of Table 2, we focus on the cohort of entrants and compute the average growth rate of the number of employees for each entrant between its entry year and x years after its entry. Table 2 reports the estimation results for the average growth of size for each entrant over its entry year (age 0) to subsequential x years later (age x), where $x = \{3, 5, 7, 10\}$. The table shows that a firm that starts up at the higher uncertainty period tends to have a significantly lower growth after its entry. One unit (around 4 stdandard deviations) increase in the within-industry standard deviation of TFP shocks is associated with a 8.42% reduction in the average growth of size for new firms during their entry year and three years after (Column 1). The adverse effects become gradually weakened but remain significantly negative when we span the post-entry years from three to five, seven, and ten, subsequently (Columns 2-4).

In addition, these regression results seem to suggest the effect of uncertainties on startup size travels through the productivity channel. If the size difference is driven by demand or financial factors, i.e. startups are small because either shrinking industrial demand or credit supply, then one should not be able to observe the negative correlation to persist over firm's life-cycle, given that these factors reverts in the short run. We consolidate the first set of empirical findings into the following stylized fact.

Fact 1 *Firms starts smaller in industries with increasing uncertainties, and remains smaller over their life-cycles.*

2.2.3 Role of External Financing Conditions

This section shows the negative correlation between uncertainty and entrepreneurship depends importantly on the variations of the external financing conditions faced by the entrants. We establish this fact by exploiting the dataset, MaPPED, and we construct the indicator for the external financing conditions regarding the supply bank credits. MaPPED contains a comprehensive list of regulation policies on credit institutions for the member countries under the ECB mandate from 1995 to 2014. We focus on the banking system since 99% of the firms in our sample are of private ownership, whose primary external financing source is the bank credit.

Table 2: Uncertainty and Post-entry Growth

	(1)	(2)	(3)	(4)
Post-entry years:	$[0,3]$	$[0,5]$	$[0,7]$	$[0,10]$
$\Delta\sigma_{i,c,t-1}$	-0.0842^{**}	-0.0579^{**}	-0.0301^{***}	-0.0257^{*}
	(0.0297)	(0.0175)	(0.00187)	(0.0114)
Country-Year FEs	yes	yes	yes	yes
Controls	yes	yes	yes	yes
$\mathbf{R}^2$	0.053	0.039	0.023	0.025
Observations	1,899	1,930	1,931	19,31
Years	2000-2015	2000-2015	2000-2015	2000-2015

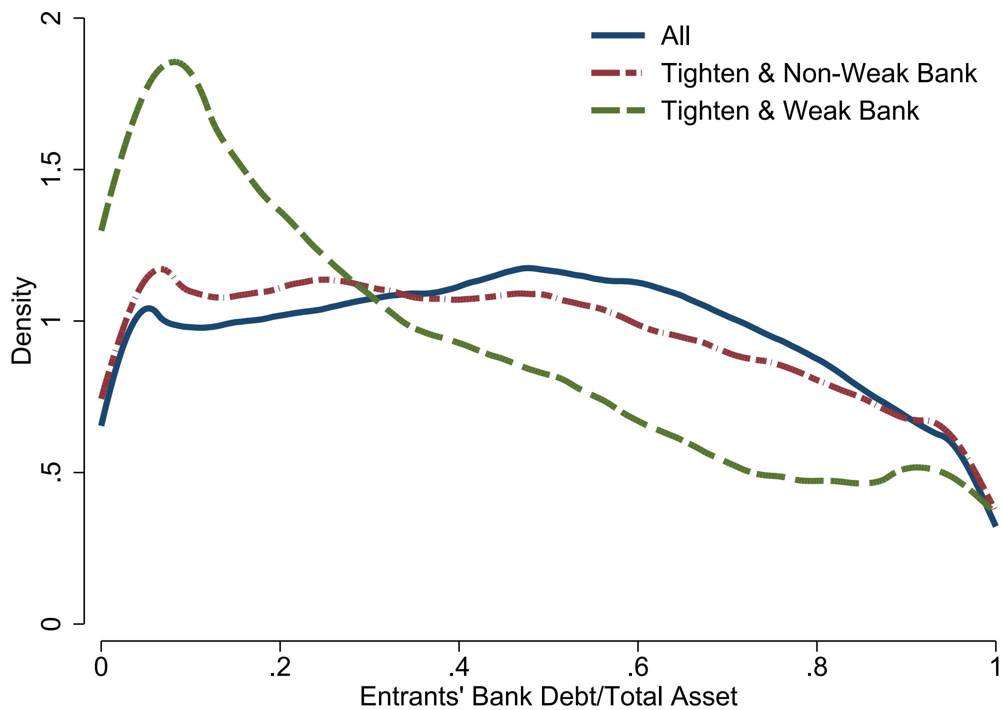
Notes: This table reports the results for the regressions of the uncertainty on the post-entry growth with different specifications. The full sample covers the years from 2000 to 2015. Each regression includes the country-year fixed effect and controls. The dependent variable is the average growth rate of number of employees for new firms over their entry year and x $(=\{3,5,7,10\})$ years after. The independent variable of interest is the change of the within-industry uncertainty. The uncertainty indicator ($\sigma_{i,c,t-1}$) in all columns is the standard deviation of TFP shocks within each industry-country-year cell. The controls include the average growth rate of loan and the average sales growth within the corresponding industry. Columns 1 to 4 correspond to the post-entry years from zero to three, five, seven, and ten, respectively. t values are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

The regulations on credit institutions (mostly banks) may directly affect the entrants' financing condition through the bank lending channel.

We first document the relationship between the credit policy tightening and entrants' external financing condition. A tightened credit policy may cause a more significant adverse impact on banks with a lower level of capitalization, leading to a tighter external financing condition facing individual firms. To see this, we introduce a dummy variable of a tightening credit policy regime, $Tighten_{c,t}$, for country c in year t, which equals one if a macro-prudential regulation is implemented, and zero otherwise. We use a dummy variable of weak bank, $Weak_{c,t}$, to characterize the strength of country c's banking system in year t. It equals one if the average core Tier 1 ratio, defined as the banks' core equity over risk-adjusted total asset, across banks in this country's belongs to the bottom 10% in year t in the full sample, and zero otherwise. According to Peek and Rosengren (2005), Hanson et al. (2011), and Blattner et al. (2018), a low value of core Tier 1 ratio measures the weakness of a banking sector.

Figure 1 plots the distribution of entrants' leverage under the subsamples with various scenarios. The entrant's leverage is the ratio between the bank loan and the total asset. The green (or red) line represents the distribution of entrants' leverage in the country with a weak (or non-weak) banking system and in a tightened credit condition. The blue line is the distribution of entrants' leverage ratio for the whole sample. Figure 1 shows that under a tightened credit

Figure 1: Entrants' Leverage and Credit Condition



Notes: This figure plots distributions of entrants' leverage for countries with different strengths of banking systems under the tightening credit policy. The leverage of each entrant is defined as the ratio between its bank loan and total asset. "Tighten" corresponds to the case where the credit condition dummy $Tighten_{c,t}$ equals 1. "Weak bank" corresponds to the case where the bank strength dummy $Weak_{c,t}$ equals 1. The green (or red) line represents the distribution of entrants' leverage in the subsample with tightened credit condition and weak (or non-weak) banking system. The blue line represents the distribution in the full sample..

condition, the entrants' leverage ratios in countries with a weak banking system (green line) concentrate significantly more on the left tail of the distribution, compared to that in the whole sample (blue line). The contractionary effect caused by a tightened credit condition is much weaker for those entrants in countries with a relatively strong banking system (red line).[2]

We now study the impact of the external financing condition on the relationship between uncertainty and entrepreneurship. We introduce the interaction terms $\Delta\sigma_{i,c,t-1} \times Tight_{c,t}$ and $\Delta\sigma_{i,c,t-1} \times Tighten_{c,t} \times Weak_{c,t}$ to the regression equation (1). Column 1 in Table 3 conducts the baseline regression where the dependent variable is the entrants' size measured by the number of employees (normalized by the industry average). The result shows that the estimated coefficient of the triple term $\Delta\sigma_{i,c,t-1} \times Tight_{c,t} \times Weak_{c,t}$ is -0.761 at 1% level of significance. The negative value of the coefficient implies that the tightening of the external financing condition

[2]We further regress the entrants' leverage ratio on $Tighten_{c,t}$ and $Tighten_{c,t} \times Weak_{c,t}$. The coefficient of $Tighten_{c,t}$ is -0.0238 and insignificant. Whereas, the coefficient of $Tighten_{c,t} \times Weak_{c,t}$ is -0.150 with a significance at 1%. The estimation results confirms the pattern in Figure 1.

leads to a more negative relationship between the within-industry uncertainty and the entrants' size. Moreover, the magnitude of the coefficient of $\Delta\sigma_{i,c,t-1} \times Tight_{c,t} \times Weak_{c,t}$ is much larger than that of the uncertainty term $\Delta\sigma_{i,c,t-1}$ (-0.102). This result indicates that the external financing condition plays as a major channel through which a higher level of uncertainty is associated with smaller entrants. Column 2 replace the left hand side with the non-normalized number of employment. Columns 3-4 splits the samples by the pre-crisis and post crisis period. Columns 5-8 conduct similar estimations where the dependent variable is the size growth for different spans of post-entry years, analogous to Table 2. The four columns show that when the credit condition is tightened in a country with a weak banking system, a firm starts up at the higher uncertainty period tends to grow significantly slower after its entry. This result indicates that the slower post-entry growth dynamics of the entrants born at the higher uncertainty periods are mainly driven by the tighter external financing condition. We now have the second stylized facts.

Table 3: Uncertainty and Entrepreneurship under a Tightened Financing Condition

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Specifications:	Baseline	Abs	Pre-crisis	Post-crisis	Post-entry years			
					$[0,3]$	$[0,5]$	$[0,7]$	$[0,10]$
$\Delta\sigma_{i,c,t-1}$	-0.102^{***}	-0.130	-0.0936^{*}	-0.1575^{***}	0.0176	-0.00583	-0.00482	-0.0202
	(0.0228)	(0.120)	(0.0453)	(0.0241)	(0.0804)	(0.0386)	(0.0287)	(0.0198)
$\Delta\sigma_{i,c,t-1} \times Tight_{c,t}$	-0.00631	0.2224	-0.112^{*}	0.0428	0.0609	0.0659	-0.00109	0.0322
	(0.160)	(0.143)	(0.0588)	(0.1648)	(0.125)	(0.110)	(0.0821)	(0.0557)
$\Delta\sigma_{i,c,t-1} \times Tighten_{c,t} \times Weak_{c,t}$	-0.761^{***}	-0.9331^{*}	-0.434^{***}	-1.0941^{***}	-0.137	-0.292^{**}	-0.251^{**}	-0.233^{***}
	(0.223)	(0.4463)	(0.0704)	(0.2109)	(0.155)	(0.110)	(0.0835)	(0.0549)
Country-year FEs	yes	yes	yes	yes	yes	yes	yes	yes
Controls	yes	yes	yes	yes	yes	yes	yes	yes
$\mathbf{R}^2$	0.050	0.108	0.067	0.063	0.060	0.043	0.026	0.026
Observations	41,001	41,238	12,479	26,922	1,899	1,930	1,931	1,931
Years	2000-2015	2008-2015	2000-2007	2008-2015	2000-2015	2000-2015	2000-2015	2000-2015

Notes: This table reports the results of regressions considering the external financing condition. The full sample covers the years from 2000 to 2015. Each regression includes the country-year fixed effect and two controls including the average growth rate of the loan and the average sales growth within each industry-country-year cell. The dependent variable is the entrants' size in Columns 1-4, and the post-entry growth of size in Columns 5-8. The independent variable of interest is the triple term $\Delta\sigma_{i,c,t-1} \times Tighten_{c,t} \times Weak_{c,t}$, where $Tighten_{c,t}$ and $Weak_{c,t}$ take the same definitions as those in Figure 1. Columns 1-4 take similar specifications (except two interaction terms) as their counterparts in Columns 1-4 of Table 1. Columns 5-8 take the similar specifications (except two interaction terms) as those in Columns 1-4 of Table 2. t values are presented in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively.

Fact 2 *The impact of uncertainty in Fact 1 is larger when external financing condition tightens.*

3 The Model

The economy has infinite horizons. At the beginning of date t, there is a continuum of entrepreneurs of mass m_t and workers of mass $1 - m_t$. A constant fraction μ of agents born as workers and the same fraction of agents (entrepreneurs and workers) exit. We interpret newborn workers as the offspring of the exiting agents. Following Cagetti and De Nardi (2006), we assume that when an agent exits, his assets are bequeathed to his offspring as an endowment. Workers in the economy are potential entrants for the production sector. In each period, an individual worker decides to stay as a worker or to enter the market being an entrepreneur. We first describe the optimization problem of entrepreneurs, who are the incumbents in the production sector, and then discuss the workers' occupational decisions.

3.1 Entrepreneurs

We index entrepreneurs with their ages. A new-entry entrepreneur (entrant) has an age of zero. In date t, an entrepreneur with age τ enters the market in date $t - \tau$. There are two types of assets in the economy, physical capital k and risk-free bond b. The risk-free bond offers a time-invariant interest rate $r > 0$.

In each date t, the entrepreneur of age τ uses physical capital k_t^τ and labor n_t^τ to produce final goods y_t^τ. The production function follows a Cobb-Douglas form with decreasing returns to scale

$$y_t^\tau = a_t^\tau \left(k_t^\tau\right)^\alpha \left(n_t^\tau\right)^\gamma, \tag{2}$$

where $\alpha > 0$, $\gamma > 0$ and $\alpha + \gamma < 1$. a_t^τ is the idiosyncratic productivity that contains a transitory component z_t^τ and a permanent component e_t^τ,

$$\log a_t^\tau = \log z_t^\tau + \log e_t^\tau. \tag{3}$$

For transitory productivity component, we follow Bloom et al. (2018) and assume that the transitory component z_t^τ follows an AR(1) process with time-varying volatility (note the term $-\frac{\sigma_{t-1}^2}{2}$ preserves the conditional mean of z_t, that is, $\mathbf{E}[z_t|z_{t-1}] = z_{t-1}^{\rho_z}$.)

$$\log z_t^\tau = -\frac{\sigma_{t-1}^2}{2} + \rho_z \log z_{t-1}^{\tau-1} + \sigma_{t-1}\varepsilon_t^\tau, \tag{4}$$

where the idiosyncratic shock ε_t^τ is independently distributed with cumulative density function (CDF), $\mathbf{N}(0,1)$; σ_t is an idiosyncratic uncertainty shock, following a two-state Markov chain with high (or low) state, σ_H (or σ_L), and a 2×2 transition matrix Π. For an entering entrepreneur with age zero ($\tau = 0$) in date t, we set the lag of the productivity z_{t-1}^{0} to be the value of ability in the last period $t-1$, x_{t-1}, when he was a worker, i.e.,

$$z_{t-1}^{0} = x_{t-1}. \tag{5}$$

We will provide more details about x_{t-1} in the next section.

We borrow the idea from Klette and Kortum (2004) to specify the dynamics of the permanent component of productivity e_t^{τ}. For an entrepreneur of age τ, her permanent component of the productivity, e_t^{τ}, follows a stochastic process. We assume this process depends on the ability in the period when he (as a worker) decides to be an entrepreneur. In particular, the permanent productivity e_t^{τ} of an entrepreneur with age τ has a discrete support from the lowest state $e(1)$ to the highest one $e(N)$, i.e., $e_t \in \{e(1), \ldots, e(N)\}$. Over the life-cycle, the entrepreneur's permanent productivity follows the process below. A new entrepreneur starts with the lowest state of the permanent productivity $e(1)$ and climbs the technology ladder in a stochastic way. In period t, for an incumbent entrepreneur of age τ with productivity $e_t^{\tau} = e(n)$, where $n \in \{1, 2, \ldots, N\}$, her next-period permanent productivity $e_{t+1}^{\tau+1}$ is determined as

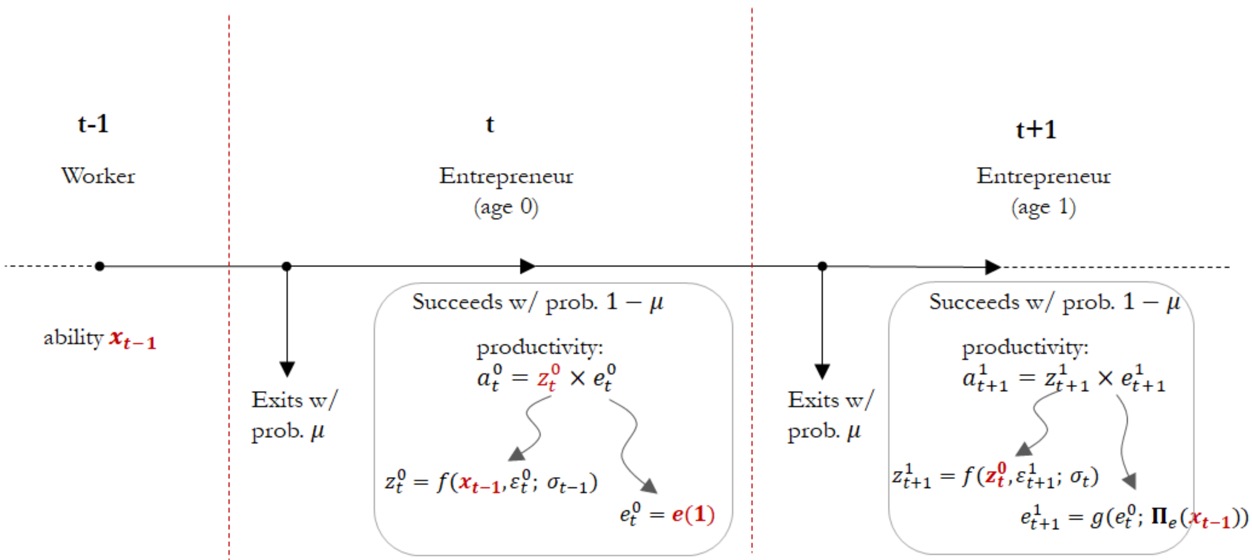
$$e_{t+1}^{\tau+1} = \begin{cases} e(n) & \text{w. prob } p(x) \\ e(n+1) & \text{w. prob } 1 - p(x) \end{cases}. \tag{6}$$

x in the last equation is the entrepreneur's innate ability when she is a worker deciding to be an entrepreneur. For instance, an entrepreneur of age τ in period t has an innate ability $x_{t-\tau-1}$ when she was a worker. Last equation indicates that the entrepreneur's innate ability x determines the likelihood of productivity upgrading. We further specify the probability $p(x)$ take a Logit form

$$p(x) = \chi \frac{\exp(\nu x)}{1 + \exp(\nu x)}. \tag{7}$$

The parameter $\chi < 1$ governs the maximum speed of productivity upgrading and ν measures the extent to which the entrepreneur's innate ability affects the speed of upgrading. This specification is parsimonious, yet it delivers a degree of flexibility in terms of capturing the pattern of post-entry growth observed in the data. In the quantitative exercise, we choose $N = 5$,

Figure 2: Evolution of entrepreneur's productivity



Notes: This graph demonstrates how a new-born entrepreneur's productivity a_t^0 evolves over time. The workers in date $t-1$ are those who decide to start their entrepreneurship in date t. z_t^0 is the transitory component of the productivity, which follows an AR(1) process described in (4). e_t^0 is the permanent component of the productivity that follows a stochastic technology-ladder process described in (6), whose transition probability matrix depends on the worker's ability in date $t-1$.

implying a transition matrix satisfies

$$\Pi_e(x) = \begin{bmatrix} 1-p(x) & p(x) & 0 & 0 & 0 \\ 0 & 1-p(x) & p(x) & 0 & 0 \\ 0 & 0 & 1-p(x) & p(x) & 0 \\ 0 & 0 & 0 & 1-p(x) & p(x) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Note that when entrepreneurs achieve the highest level of permanent productivity $e(N)$, she would keep the same state steadily with probability one. Figure 2 gives a graphic illustration for the evolution of an entrant's overall productivity a_t^τ.

We now specify the entrepreneur's optimization problem. After the realization of the idiosyncratic productivity shock ε_t^τ, the entrepreneur chooses the optimal labor hiring n_t^τ at the real wage rate w_t to solve the profit maximization problem

$$\pi_t\left(k_t^\tau, a_t^\tau\right) = \max_{n_t^\tau \geq 0} \left\{ a_t^\tau \left(k_t^\tau\right)^\alpha \left(n_t^\tau\right)^\gamma - w_t n_t^\tau \right\} \tag{8}$$

In date t, the budget constraint for an entrepreneur with age τ is

$$c_t^{\tau} + k_{t+1}^{\tau+1} + b_{t+1}^{\tau+1} = \pi_t \left(k_t^{\tau}, a_t^{\tau}\right) + (1-\delta)\, k_t^{\tau} + (1+r)\, b_t^{\tau}, \tag{9}$$

where k_t^{τ} and b_t^{τ} are the asset holdings determined in the last period; δ is the depreciation rate; r is a constant risk-free rate. Besides, we assume that the entrepreneur faces a borrowing constraint

$$b_{t+1}^{\tau+1} \geq -\theta k_{t+1}^{\tau+1}, \tag{10}$$

where the parameter $\theta \in (0,1)$ captures the tightness of the financial constraint.

The entrepreneur has a utility function $u\left(c_t^{\tau}\right)$ that satisfies $u^{'}(c) > 0$ and $u^{''}(c) < 0$. Let $\mathbf{s}_t^{\tau} = \{k_t^{\tau}, b_t^{\tau}, z_t^{\tau}, e_t^{\tau}, \sigma_{t-1}, x\}$ denote the individual state of entrepreneur with age τ. $V_t\left(\mathbf{s}_t^{\tau}\right)$ denote the value function of an entrepreneur of age τ with individual state vector $\mathbf{s}_t^{\tau}$.[3] The entrepreneur chooses consumption c_t^{τ} and next-period asset holdings k_{t+1}^{τ} and b_{t+1}^{τ} to solve the following Bellman equation

$$V_t\left(\mathbf{s}_t^{\tau}\right) = \max_{\left\{c_t^{\tau}, k_{t+1}^{\tau}, b_{t+1}^{\tau}\right\}} u\left(c_t^{\tau}\right) + \beta(1-\mu) \int \mathbf{E}_t \left[V_{t+1}\left(\mathbf{s}_{t+1}^{\tau+1}\right) | e_t^{\tau}\right] d\mathbf{F}\left(z_{t+1}^{\tau+1} | z_t^{\tau}, \sigma_t\right), \tag{11}$$

subject to the budget constraint (9) and the borrowing constraint (10). In the last equation, $\beta \in (0,1)$ is the discount rate; μ is the exogenous exit rate; $\mathbf{F}\left(z_{t+1}^{\tau+1} | z_t^{\tau}, \sigma_t\right)$ denotes the conditional CDF of $z_{t+1}^{\tau+1}$ based on the AR(1) process (4) with uncertainty σ_t; $\mathbf{E}_t\left[V_{t+1}\left(\mathbf{s}_{t+1}^{\tau+1}\right) | e_t^{\tau}\right]$ is the expected value of V_{t+1} conditional on e_t^{τ}, where the expectation operator is taken on $e_{t+1}^{\tau+1}$.

We remark on the specification of entrepreneurs as follows. Overall, the setting follows the standard incomplete market environment with collateral constraints (e.g. Angeletos (2007) and among others). The crucial difference is how startups draw their initial productivities, and how such productivities evolves overtime. The standard Hopenhayn-type of model assumes away the effect of ex-ante heterogeneity for startups, and therefore mutes the productivity selection channel. We extend the standard model by assuming that certain entrants-specific ability, which determines both the initial and post-entry productivities. The former serves the purpose of reconciling the initial size difference and the latter helps to explain post entry dynamics. Under such specifications, when heightened uncertainty makes high ability startups reluctant to enter, we should be able to see not only smaller initial size but also slowed dynamics.

[3]We use the subscript t to summarize all the aggregate variables faced by individual entrepreneurs.

3.2 Workers

Workers are heterogeneous in terms of their abilities. Each worker is endowed by an idiosyncratic ability x_t that follows an AR(1) process

$$\log x_t = \rho_x \log x_{t-1} + \sigma_x u_t, \tag{12}$$

where $\rho_x \in (0,1)$; $\sigma_x > 0$ is the time-invariant standard deviation of the idiosyncratic ability shocks; u_t is independently distributed with CDF $\mathbf{N}(0,1)$. Let $\mathbf{G}(x_t|x_{t-1})$ denotes the CDF of x_t conditional on x_{t-1} based on the above AR(1) process.

Workers save or borrow through risk-free bonds. At the beginning of each date t, after the realization of idiosyncratic ability shock u_t, a worker decides consumption c_t^w and bond holdings b_{t+1}^w, and inelastically provides one unit of labor. Then, the worker decides whether to be an entrepreneur or stay as a worker in the next period. If the worker decides to be an entrepreneur, there incurs a fixed entry cost $\xi > 0$. We assume that the occupation choice from being a worker to an entrepreneur is irreversible. That is, once a worker becomes an entrepreneur, he cannot switch back. Let $\mathbf{s}_t^w = \{b_t^w, x_t, \sigma_t\}$ denote the individual state vector for a worker. The budget constraint is given by

$$c_t^w + b_{t+1}^w = (1+r)\, b_t^w + w_t x_t - \xi o_t, \tag{13}$$

where o_t indicates the occupation choice, equal to one if being an entrepreneur and zero for staying as a worker. Besides, we impose an ad hoc borrowing constraint

$$b_{t+1}^w \geq -b_{\max}^w, \tag{14}$$

where $b_{\max}^w > 0$ is an exogenous borrowing limit.[4]

The workers's utility function $u(c_t^w)$ takes the same form as that of entrepreneurs. Let $W_t(\mathbf{s}_t^w)$ denote the value function of a worker with individual state vector $\mathbf{s}_t^w$ in date t. In the beginning of date t, the worker chooses consumption c_t^w and bond holdings b_{t+1}^w to solve the Bellman equation

$$W_t(\mathbf{s}_t^w) = \max_{\{c_t^w, b_{t+1}^w\}} u(c_t^w) + \beta(1-\mu)\, \tilde{W}_t\left(b_{t+1}^w, x_t, \sigma_t\right), \tag{15}$$

subject to budget constraint (13) and borrowing constraint (14). In the above equation, β and μ are the discount rate and the exit rate, whose values are the same as those of entrepreneurs;

[4] The value of $b_{\max}^w$ should not exceed the natural borrowing limit to avoid the default risks of those workers who borrow.

$\tilde{W}_t\left(b^w_{t+1}, x_t, \sigma_t\right)$ is the continuation value, which is obtained after the worker's occupation choice. The continuation value depends on the bond holdings b^w_{t+1} for next period and the worker's ability x_t. This is because the former will affect the beginning-of-period wealth available to the agent in date $t+1$ and the latter will determine the working efficiency (if staying as a worker) or the productivity (if being an entrepreneur).

If the worker in date t decides to start the entrepreneurship in next period $t+1$, she will take the bond holdings b^w_{t+1} (or debt outstanding if $b^w_{t+1} < 0$) with her. We assume that for a startup, the physical capital stock is zero, i.e., $k^0_{t+1} = 0$. As we discussed in the entrepreneur's problem, the productivity of a new-entry entrepreneur satisfies

$$\log a^0_{t+1} = \log z^0_{t+1} + \log e^0_{t+1}. \tag{16}$$

The transitory productivity z^0_{t+1} in the last equation satisfies the AR(1) process defined in (4) with the lag term equals to x_t, which is the ability of this worker in date t. The permanent component e^0_{t+1} is the lowest grid on the technology-ladder, which the follows a stochastic process (6) with transition probability $p(x_t)$ increasing in the potential entrant's (worker) ability x_t. The above setup implies that the productivity of new-entry entrepreneur a^0_{t+1} is increasing in the potential entrant's (worker) ability x_t.The positive relationship between the startup productivity and the worker's ability is crucial to create selection effects of uncertainty on the entrants' productivity.

According to the above specification, the individual state vector for a new-entry entrepreneur in date $t+1$ is $\mathbf{s}^0_{t+1} = \{0, b^w_{t+1}, z^0_{t+1}, e^0_{t+1}, \sigma_t, x_t\}$, where z^0_{t+1} satisfies (4) and e^0_{t+1} satisfies (6). The optimal occupation decision o_t solves the following discrete choice problem

$$\tilde{W}_t\left(b^w_{t+1}, x_t, \sigma_t\right) = \max_{\{o_t\}} \left\{ \int V_{t+1}\left(\mathbf{s}^0_{t+1}\right) d\mathbf{F}\left(z^0_{t+1} | x_t, \sigma_t\right), \int \mathbf{E}_t\left[W_{t+1}\left(\mathbf{s}^w_{t+1}\right) | \sigma_t\right] d\mathbf{G}\left(x_{t+1} | x_t\right) \right\}. \tag{17}$$

3.3 Aggregation and Equilibrium

Let $\mathbf{S}^e_t = \{\mathbf{s}^\tau_t\}^\infty_{\tau=0}$ be the individual state vector for the entrepreneurs in the economy and Φ^e_t be the CDF of $\mathbf{S}^e_t$. Define the aggregate capital K_t, bond holdings B^e_t, labor hiring N_t, consumption C^e_t and output Y_t in the entrepreneur sector as $J_t = \int j^\tau_t d\Phi^e_t$, for $j = \{k, b, n, c, y\}$. Let Φ^w_t denote the CDF of individual state of workers $\mathbf{s}^w_t$. The aggregate bond holdings B^w_t and consumption C^w_t in the worker sector are given by $B^w_t = \int b^w_t d\Phi^w_t$ and $C^w_t = \int c^w_t d\Phi^w_t$.

Let $O_t = \int o_t d\Phi^w_t$ denote the mass of the workers who decide to be entrepreneur in date

$t+1$. The total mass of worker at the beginning of date $t+1$ satisfies

$$m_{t+1} = (1-\mu)(1-O_t)\, m_t + \mu. \tag{18}$$

As each worker inelastically provides one unit of labor, the aggregate supply of effective labor in date t is $N_t^w = m_t \int x_t d\Phi_t^w$.

The total mass of entrepreneur at the beginning of date $t+1$ is $(1-\mu)(1-m_t)+(1-\mu)\, O_t m_t$, which equals to $1-m_{t+1}$. That is, the population of agents in our economy remains one unit measure over time.

A competitive equilibrium consists of allocations and prices such that (i) taking the prices as given, the allocations solve the optimizing problems of the household and the firms; and (ii) the prices clear the markets for physical capital, labor, bond, and final goods.

The labor market clearing condition implies $N_t = N_t^w$. For the bond market, the total demand is $B_t = B_t^e + B_t^w$. We consider a small open economy, i.e., there is an exogenous supply of bond such that the risk-free rate is fixed at r over time. Final goods market clearing condition implies a resource constraint

$$C_t^w + C_t^e + K_{t+1} - (1-\delta)\, K_t + \xi O_t + B_{t+1} - (1+r)\, B_t = Y_t, \tag{19}$$

where $K_{t+1} - (1-\delta)\, K_t$ is the aggregate real investment; ξO_t is the total expenditure on the entry cost; $B_{t+1} - (1+r)\, B_t$ is the current account balance.

4 A Simple Illustration of the Mechanism

The productivity selection effects induced by the uncertainty shock is novel in our model. To illustrate this mechanism, we consider a static partial equilibrium model. All the prices in the economy are exogenous. This model is a simplification of our baseline dynamic model. To highlight our channel, the model does not consider the standard wait-and-see effects. The economy is populated with unit measure of agents, who are potential entrants. Each agent has an ability x and is endowed with a net worth b^w. The variable b^w corresponds the bond holdings in the beginning of each period, b_t^w, in the baseline model. We assume the state vector (x, b^w) follows a joint distribution $\Phi(b^w, x)$. The agent has a utility function $u(c) = \log(c)$. She decides whether to be an entrepreneur or a worker. If being a worker, the agent inelastically provides one unit of labor at real wage rate w and obtains wage income wx. Let $W(b^w, x)$ denote the value function for a worker with net worth b^w and ability x, which satisfies $W(b^w, x) = \log(wx + b^w)$.

Analogous to the baseline model, starting an entrepreneurship incurs a fixed cost $\xi > 0$ measured by the dis-utility. The productivity of the entrepreneur is $a(x,z) = xz$, where the productivity shock z is independently distributed with CDF $\mathbf{F}(z;\sigma)$ on the support $[z_{\min}, z_{\max}]$ with standard deviation σ. The parameter σ corresponds to the uncertainty shock in the baseline model. We assume the shock z is realized after the agent's occupation choice.

Let $V(b^w, x, z)$ denote the value function for an entrepreneur with net worth b^w and ability x, who draws a productivity shock z after the entry. Define the expectation of $V(b^w, x, z)$ as $\bar{V}(b^w, x; \sigma) = \int_{z_{\min}}^{z_{\max}} V(b^w, x, z)\, d\mathbf{F}(z;\sigma)$. Then, the expected net value of being an entrepreneur is $\bar{V}(b^w, x; \sigma) - \xi$, which depends on the uncertainty σ.

For an agent with net worth b^w and ability x, her occupation decision solves the following discrete choice problem

$$\max\left\{W(b^w, x), \bar{V}(b^w, x; \sigma) - \xi\right\}. \tag{20}$$

It is straightforward that given the individual state (b^w, x), the change of uncertainty σ would affect the agents' occupation choices through the change of $\bar{V}(b^w, x; \sigma)$. This in turn influences the average production efficiency in the entrepreneur sector, since the mean of the productivity $a(x,z)$ depends on the distribution of entrants' abilities. Whereas, the value of being a work $W(b^w, x)$ does not respond to the change of uncertainty.

We now discuss how the uncertainty affects $\bar{V}(b^w, x; \sigma)$. The entrepreneur with a state vector (b^w, x, z) chooses the amount of physical capital k, risk-free bond (or debt if negative) b and labor hiring n to solve the profit maximization problem

$$\pi(b^w, x, z) = \max_{\{k,b,n\}} a(x,z)\, k^{\alpha} n^{\gamma} - wn + (1-\delta)k + (1+r)b, \tag{21}$$

subject to a flow-of-funds constraint $k + b = b^w$ and a borrowing constraint $b \geq -\theta k$. In the end of the date, the entrepreneur consumes the profit, resulting in a value function $V(b^w, x, z) = \log(\pi(b^w, x, z))$. The optimal decisions are summarized by the following proposition.

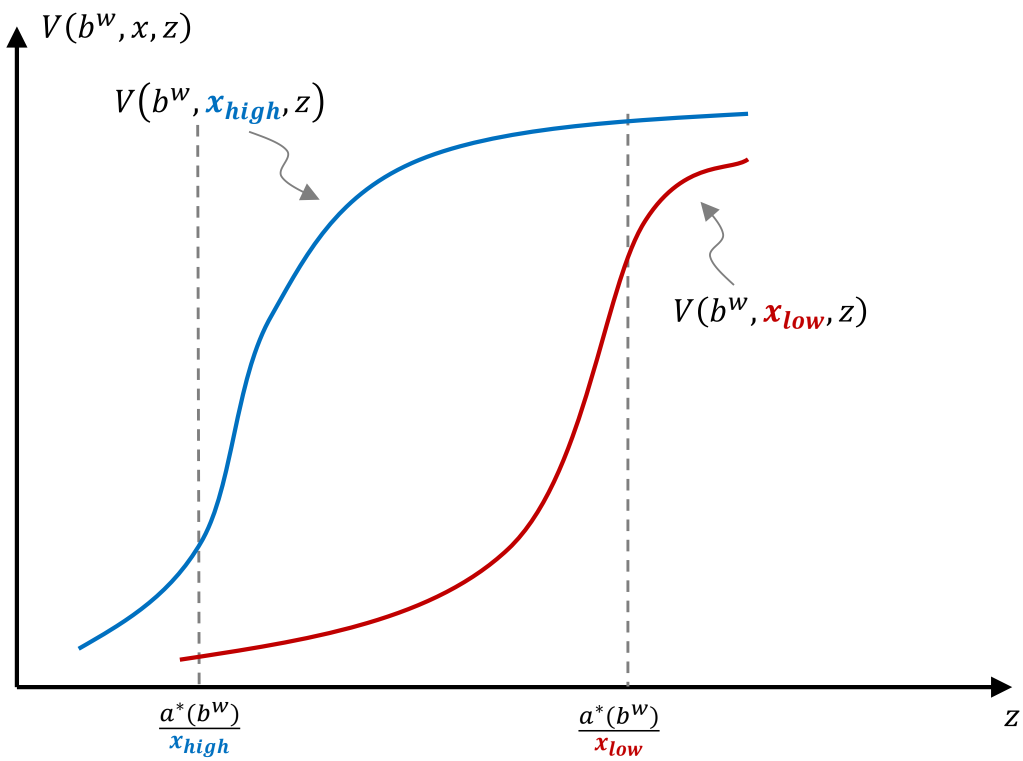
Proposition 1 *Given the net worth b^w, the optimal capital decision $k(b^w, x, z)$ follows a trigger strategy*

$$k(b^w, x, z) = \begin{cases} \left[\frac{a(x,z)}{a^*(b^w)}\right]^{\frac{1}{1-\alpha-\gamma}} \bar{k}(b^w) & \text{if } z < \frac{a^*(b^w)}{x} \\ \bar{k}(b^w) & \text{if } z \geq \frac{a^*(b^w)}{x} \end{cases}, \tag{22}$$

where $\bar{k}(b^w) = \frac{1}{1-\theta} b^w$ is the maximum of capital the entrepreneur can hold under the borrowing constraint, and the productivity cutoff $a^*(b^w)$ satisfies

$$a^*(b^w) = \left(\frac{\alpha}{r+\delta}\right)^{\gamma-1} \left(\frac{\gamma}{w}\right)^{-\gamma} \bar{k}(b^w)^{1-\alpha-\gamma}. \tag{23}$$

Figure 3: Value functions for entrepreneurs with different abilities



Notes: This graph demonstrates the value of being an entrepreneurs for agents with different abilities x and a fixed net worth b^w. The x-axis indicates the idiosyncratic productivity z. The blue line depicts the case where the agent has a high ability, and the red line is for the case of low ability.

Proof. See Appendix C. ■

Equation (23) implies that the cutoff $a^*(b^w)$ is strictly increasing in the net worth b^w. That is, given the ability x, it is less likely for an entrepreneur with a higher net worth to be financially constrained. Since $a = xz$, from an ex-ante perspective, the probability that an entrepreneur with ability x has a binding borrowing constraint is $\mathbf{F}\left(\frac{a^*(b^w)}{x}\right)$. Therefore, a lower ability implies that the entrepreneur is less likely to be financially constrained.

Under the optimal capital decision, it is straightforward to obtain the profit function $\pi(b^w, x, z)$ and the value function $V(b^w, x, z)$. The following proposition states the shape of value function regarding the idiosyncratic productivity z.

Proposition 2 *Assume that* $\frac{(\alpha+\gamma)(1-\theta)\alpha}{(1-\alpha-\gamma)^2} > 1$ *and* $1 - \theta(1+r) < \delta$. *Given the state* (b^w, x)*, the value function* $V(b^w, x, z)$ *is strictly convex in* z *for* $z < \frac{a^*(b^w)}{x}$ *and strictly concave in* z *for* $z \geq \frac{a^*(b^w)}{x}$. *Therefore, the expected value of being an entrepreneur* $\bar{V}(b^w, x; \sigma) - \xi$ *is decreasing (increasing) in the uncertainty* σ *for a sufficiently large (small) ability* x.

Proof. See Appendix D. ■

The productivity $a(x,z) = xz$ implies that the entrepreneur's productivity is disturbed by the idiosyncratic shock z around the ability x. For an agent with a sufficiently low ability x, the value function takes a convex shape in z. The red line in Figure 3 depicts this case. The Jensen's inequality implies that the expected value of being entrepreneur $\bar{V}(b^w, x; \sigma)$ increases with the uncertainty σ. The positive effect of uncertainty on the entry value reflects the standard Oi-Hartman-Abel effect.

In contrast, for an agent with a sufficiently high ability x, the value function takes a concave shape in z, which decreases with the uncertainty σ. The blue line in Figure 3 depicts this case. The intuition is as follows. A high ability agent is more likely to be financially constrained for a good draw of z, resulting in a limited increase in the value of being entrepreneur. However, for a bad draw of z, the declines of capital input and the production are not affected by the financial constraint, which implies the magnitude in reduction is even larger. As a result, the uncertainty depresses the expected value of being entrepreneur, $\bar{V}(b^w, x; \sigma) - \xi$, for those agents with high abilities.

The above mechanism indicates that when a positive uncertainty shock hits the economy (σ increases), those productive but financially constrained agents are more likely to be a worker since $\bar{V}(b^w, x; \sigma) - \xi$ declines. While, for those less productive agents, more of them tend to start their business because $\bar{V}(b^w, x; \sigma) - \xi$ increases. Therefore, the asymmetric impacts of uncertainty on the entry value shift the composition of entrants' productivity towards a less-efficient end along the extensive margin, resulting in a sullying effect of uncertainty shocks through the channel of firm entry.

5 Quantitative Implications

This section presents the quantitative implications of the baseline dynamic model. We first estimate the model-specific structural parameters using the Simulated Method of Moments (SMM). We then show the model fit of the targeted and non-targeted moments constructed directly from the ORBIS-Amadeus dataset. We conduct simulation exercises based on the estimated model by introducing an uncertainty shock that follows an AR(1) process. We finally isolate the productivity-selection channel of an uncertainty shock by estimating an otherwise standard Hopenhayn-type firm dynamics model, where both the initial productivity and the post-entry growth are independent of the entering worker's ability.

5.1 Calibration and Estimation

The worker sector in the baseline model follows a standard setup of the Aiyagari-Bewley-Huggett incomplete market model. The entrepreneur sector follows a recent strand of literature that emphasizes the uninsured idiosyncratic risk of entrepreneurship, e.g., Angeletos (2007). The novel part of our model is how entrepreneurs' innate ability (or talent) determines the entrants' size and their post-entry dynamics. On the firm entry part, our model deviates from the classical Hopenhayn model in the following two regards. First, the entrant's initial production efficiency is positively related to its ability like that in Cagetti and De Nardi (2006). This assumption is necessary for capturing the dispersion of entrants' size under the disturbances of idiosyncratic uncertainties. Second, our model incorporates a mechanism that indicates how workers' ability plays a role in determining entrants' post-entry growth dynamics. This mechanism is crucial to understanding the empirically observed pattern of post-entry growth during high uncertain periods. In what follows, we first demonstrate how we build in the second mechanism.

Model Solution We solve the model using the Value Function Iteration (VFI) approach. An individual worker has three state variables, including the savings, ability, and the productivity uncertainty, $\{b_t^w, x_t, \sigma_t\}$. We include the uncertainty σ_t as a state variable of a worker because it affects the expected value of being an entrepreneur. For an individual entrepreneur in age-τ cohort, she has six state variables, consisting of capital stock, bond holdings, permanent productivity, transitory productivity, innate ability that determines the post-entry dynamics of the productivity, and the productivity uncertainty, $\{k_t^\tau, b_t^\tau, z_t^\tau, e_t^\tau, x, \sigma_t\}$. We discretize the space of state variables including 50 grids for k_t^τ and b_t^τ, 5 grids for z_t^τ, e_t^τ and x, and 2 grids for σ_t. Our discretization yields 500 grids for workers and 625,000 grids for entrepreneurs. Given the structural parameter values, we solve the model by using the standard VFI approach with linear interpolation on the off-grid points. We also parallelize the computation to speed up the whole solution procedure.

Calibration and Estimation We group the structural parameters in our model into two subsets. The parameters in the first subset are standard in the literature. We directly assign their values according to the calibration in the existing literature. The parameters in the second subset are model-specific or critical to the model's dynamics. We estimate these parameters by fitting the model simulated moments to those constructed from the Orbis-Amadeus dataset.

The first subset of parameters include the risk-free rate r, discount rate β, input shares α and γ, collateral parameter θ, and parameters in the worker's ability shock process, ρ_x and σ_x. We follow Gopinath et al. (2017) to set $r = 0.02$ and $\beta = 0.98$. We follow Basu and Fernald (1997) to pin down the parameter values of α and γ such that $\alpha + \gamma = 0.85$ and the capital income share of 1/3. We calibrate the parameter in the collateral constraint θ to be 0.8 according to the calibration in Midrigan and Xu (2014). Following the calibration in Storesletten et al. (2004), we set the persistence of worker's ability shock $\rho_x = 0.95$ and the standard deviation $\sigma_x = 0.078$. Table 4 summarizes the above parameterizations.

Table 4: Parameter Values

Parameters		Baseline	No Selection
Calibration			
Risk-free rate	r	0.0200	
Discount rate	β	0.9800	
Capital depreciation	δ	0.0600	
Capital share	α	0.2805	
Labor share	γ	0.5695	
Collateral parameter	θ	0.8000	
Persistence in ability	ρ_x	0.9500	
Std of ability shock	σ_x	0.0780	
Estimation			
Fixed entry cost	f	1.4644	2.0018
		(0.1057)	(0.1708)
Exit rate	μ	0.0673	0.0686
		(0.0007)	(0.0020)
Persistence of z shock	ρ_z	0.8631	0.8639
		(0.0389)	(0.0599)
Std of z shock	σ_L	0.1199	0.1156
		(0.0101)	(0.0035)
Parameter in transition prob. of e_t	χ	0.8595	0.8886
		(0.0153)	(0.0804)
Parameter in transition prob. of e_t	ν	3.9281	0.0000
		(0.1310)	(0.0000)
Gap between prod. states	$\log(\frac{e(N)}{e(1)})$	0.5733	0.7246
		(0.0184)	(0.0140)
Worker's borrowing limit	$b^w_{\max}$	1.2453	1.2846
		(0.0478)	(0.0085)

Notes: The upper panel reports the parameter values based on the calibration. The bottom panel reports the parameter values based on the estimation. The numbers in parentheses are standard errors. The column of baseline corresponds to the estimation based on the baseline model. The column of *no selection* corresponds to the estimation based on the model where the post-entry productivity is irrelevant to the worker's (as an entrant) ability. In the control model, the initial value of transitory component z_t^0 and the transition probability of technology upgrading p are independent with the ability x_{t-1}. In both estimations, we use the same SMM approach and an identical set of moments.

The second subset of parameters are those to be estimated, including the exit rate μ, the fixed start-up cost f, the worker's borrowing limit $b^w_{\max}$, parameters in the process of transitory productivity ρ_z and σ_L, parameters in the probability of transition in the permanent productivity process χ and ν, and the gap between the highest state of permanent productivity and the lowest state, $\log\left(\frac{e(N)}{e(1)}\right)$. To estimate these eight parameters, we employ eight moments such as the average firm age, the average size of entrants measured by number of employees, the average value of leverage ratio for entrants, the average entry rate, the autocorrelation of sales growth and the dispersion of the innovation of sales growth process for mature firms[5], the average post-entry growth rate of sales and the dispersion of growth rate of sales for entering firms. Appendix xx provides more details about the construction of targeted moments. The information of firm age, size, and leverage of entrants help to identify λ, f and $b^w_{\max}$. The moments regarding the incumbent firms reflect the post-entry dynamics, which may help to identify those parameters in the process of permanent productivity, such as χ, ν and $\log\left(\frac{e(N)}{e(1)}\right)$. The remaining moments are used to identify the parameters in the process of transitory productivity, including ρ_z and σ_L .

We use simulated method of moments (SMM) to estimate the above eight parameters by solving the following minimization problem

$$\hat{\Theta} = \arg\min_{\Theta} \left[\mathbf{M}(\Theta) - \mathbf{M}^d\right]' \hat{\mathbf{W}} \left[\mathbf{M}(\Theta) - \mathbf{M}^d\right], \tag{24}$$

where $\mathbf{M}(\Theta)$ is the vertically-stacked moment vector generated after the model is solved, and $\mathbf{M}^d$ is the counterpart in the data. The diagonal matrix $\hat{\mathbf{W}}$ weights each moments. Following Adda and Cooper (2003), the elements in the weighting matrix, $\hat{\mathbf{W}}$, are computed from the inverse of the variance-covariance matrix of data moments based on bootstrapped samples from the firm-level data.

The bottom panel in Table 4 shows the estimation result. For the baseline model, the fixed entry cost f is 1.46, which is approximately 1.7 times as large as the median worker's wage. The exit rate of entrepreneurs μ is 6.7%, which matches the average firm age of around 13. The persistence of transitory productivity ρ_x is 0.86. The standard deviation of transitory productivity innovation σ_L is 0.12, which is consistent with the value in the literature like Midrigan and Xu (2014). The gap between the highest and the lowest states of permanent productivity $\log\left(\frac{e(N)}{e(1)}\right)$ is 0.57, implying the top 20th percentile is about 30% more efficient than the median. The estimation of χ implies an the median probability of productivity upgrading of 0.43, and the magnitude of ν implies that an increase initial ability of startup by 1% may lead to on

[5]The mature firms are incumbent firms with age no less than ten years. These firms have relatively stationary sale growth process.

average 1.8% increase in the probability of technological upgrading on a year-by-year basis. The standard errors reported in parentheses of Table 4 indicate that the parameters are well identified in the baseline estimation.

Table 5 reports the model-simulated moments and compares them with the counterparts in the data. The upper panel shows the moments targeted in the estimation, and the bottom panel shows the results for non-targeted moments. The first column indicates that the targeted moments simulated in the baseline model tightly fit those in the data. The baseline model explains reasonably well for a set of stylized facts regarding the firm dynamics, such as sales dynamics for mature firms, post-entry sales growth for entrants, and the size of entrants relative to incumbents. Whereas, for the control model where the entrants' productivity processes are irrelevant to their innate ability, the model fails to replicate the observed relative size of entrants. The control model predicts a much smaller size of entrants than that observed in the data. One possible reason is that the entering value for those low-ability entrepreneurs becomes larger since the post-entry productivity is irrelevant to their low ability, resulting in more low-ability entrepreneurs starting their business. For the non-targeted moments, the baseline model closely fits the data. The baseline model generates higher dispersions for the firm-level variables than those in the control model. This is due to the introduction of the post-entry growth structure as specified in (6). Since the permanent component of productivity positively depends on the entrepreneur's innate ability, the firm-level variables across different age cohorts become more dispersed associated with the technology upgrading. In contrast to the control model, the baseline model predicts relatively larger shares of activities such as output, employment, and capital of young firms (with age 1-5), and faster growth dynamics of output, employment, and capital for young firms.

5.2 The Impact of Uncertainty Shocks

We now quantitatively document the impact of uncertainty shock on the entrepreneurship and the firm dynamics. Following the specification in Bloom et al. (2018), we model the uncertainty shock as an unanticipated increase of σ_t to σ_H for all firms. Afterwards, the idiosyncratic uncertainty σ_t follows a two-state Markov process with transition matrix Σ_σ for σ_L and σ_H, and eventually converges to a stationary distribution. In the quantitative exercises, we set the low state of uncertainty $\sigma_L = 0.12$ as shown in Table 4. We then set the high state of uncertainty $\sigma_H = 2\sigma_L$. We follow Bloom et al. (2018) to specify the transition matrix between two states L and H as

$$\Sigma_\sigma = \begin{bmatrix} 0.9082 & 0.0918 \\ 0.2012 & 0.7988 \end{bmatrix}.$$

Table 5: Model-simulated moments versus data

Moments	*baseline*	*control*	*data*
Targeted moments			
Entry rate	0.0632	0.0687	0.0666
Persistence of sales growth	0.9474	0.9304	0.9022
SD of sales growth innovation	0.5761	0.5479	0.5558
Firm age	12.8385	12.5745	13.2697
Post-entry sales growth	0.1367	0.2450	0.1268
SD of post-entry sales growth	0.5066	0.5228	0.4958
Entrant's leverage ratio	-0.4855	-0.4427	-0.5696
Rel. size of entrants to incumbents	-1.3025	-3.0819	-1.2229
Non-targeted moments			
SD of employment growth	0.6235	0.5180	0.3410
SD of capital growth	0.7325	0.4790	0.6100
SD of employment	2.0004	1.6736	1.1580
SD of capital	2.2429	1.4651	1.9440
1-year autocorr. employment	0.9607	0.9301	0.9480
5-year autocorr. employment	0.7078	0.5923	0.7950
1-year autocorr. capital	0.9639	0.9327	0.9510
5-year autocorr. capital	0.8333	0.6497	0.8520
Share of producers, age 1-5	0.3164	0.3536	0.3440
Share of sales, age 1-5	0.0983	0.0306	0.2100
Share of employment, age 1-5	0.0983	0.0306	0.0960
Share of capital, age 1-5	0.1095	0.0463	0.2490
Rel. sales growth: age 1-5 v.s. 6+	0.2058	0.1840	0.1880
Rel. employment growth: age 1-5 v.s. 6+	0.2058	0.1840	0.1130
Rel. capital growth: age 1-5 v.s. 6+	0.1561	0.0815	0.1720

Notes: The upper panel reports the results for the targeted moments in the estimation. The bottom panel reports the results for the non-targeted moments. The column of *baseline* corresponds to the simulation based on the baseline model. The column of *control* corresponds to the estimation based on the model where the post-entry productivity is irrelevant to the worker's (as an entrant) ability. In the control model, the initial value of transitory component z_t^0 and the transition probability of technology upgrading p are independent with the ability x_{t-1}. The column of *data* corresponds to the moments directly constructed from the data.

Dynamic Responses Figure 4 reports the dynamics of aggregate variables in response to a positive uncertainty shock. We normalize the variable in each panel (except the upper left one) by the level in the initial steady state. The left panel in the first line shows the dynamics of uncertainty, which is measured by the average value of σ_t across entrepreneurs on a period-by-period basis. As the uncertainty shock is pre-determined, this figure shows that the dispersion of innovations to the transitory productivity increases at the beginning of period $t = 0$. The right panel in the first line plots the average size of entrants with age 1, i.e., $\tau = 1$, in each period. For the entrant's size, we focus on the entrepreneurs with age 1 (the second period since this entrepreneur starts her business). Our model takes the new entrants one period to accumulate physical capital and produce.[6] The figure shows that an increase in uncertainty induces an 8% drop of the average size (measured as the number of employees) of age-1 entrepreneurs in period 1, followed by a further 30% reduction for the age-1 entrepreneurs in period 2. The adverse effect of uncertainty shock on the firm size in period 1 is mild and smaller than in period 2. The reason is that the age-1 group in period 1 corresponds to the workers who decide the entrepreneurship in period $t = -1$ (one period before the arrival of uncertainty shock). Thus, the uncertainty shock does not cause a productivity selection effect on the age-1 cohort in period 1. The mild drop in these entrepreneurs' size merely reflects the adverse impact of uncertainty on the physical capital investment due to higher investment risks. For age-1 entrepreneurs in period 2, the uncertainty shock arriving in period 0 affects their entrepreneurial decisions, thus causes productivity selection effects on these entrepreneurs. As a consequence, these entrants' size experiences a significantly large drop in period 2 when they start to hire labor and produce.

The second line in Figure 4 reports the average productivity of new entrants with age 0. The left panel presents the transitory productivity z_t^0, and the right one is for the permanent productivity e_t^0. These two panels indicate the selection effect of uncertainty shock on the entrants' productivity. In response to a positive uncertainty shock, the transitory and permanent productivities for new entrants born in period 1 decline considerably. The reason is that the entrepreneurship of new entrants (age-0) in period 1 was decided in period 0 when they were workers. A positive uncertainty shock asymmetrically affects the entry decisions of workers with heterogeneous abilities, resulting in a productivity selection effect discussed in Section 4. Note that there is no productivity selection effect on the new entrants in period 0 since their entry decisions were made in period $t = -1$ when the uncertainty shock has not arrived yet.

The uncertainty shock also reduces the entry rate due to the standard "wait-and-see" effect, as shown in the left panel of the third line in Figure 4. We define the entry rate as the share of new entrants with age 0 to the total mass of entrepreneurs. According to this definition, the entry rate starts to respond in period 1 since the uncertainty shock affects the entry decision of workers in period 0, who becomes age-0 entrepreneurs in period 1. The entry rate overshoots after period 6 because the mass of workers near the entry cutoff accumulates. When the uncertainty shock eventually vanishes, these workers enter the market. The right panel in the third line of Figure 4 shows the dynamics of average bond holdings. The entrepreneurs tend to hold more risk-free bonds in response to an increase in the uncertainty because of the

[6] That is, for an age-0 entrepreneur who starts her business in period 0, her physical capital is zero. In that period, she decides the physical capital level and then produces in the next period when she is age 1.

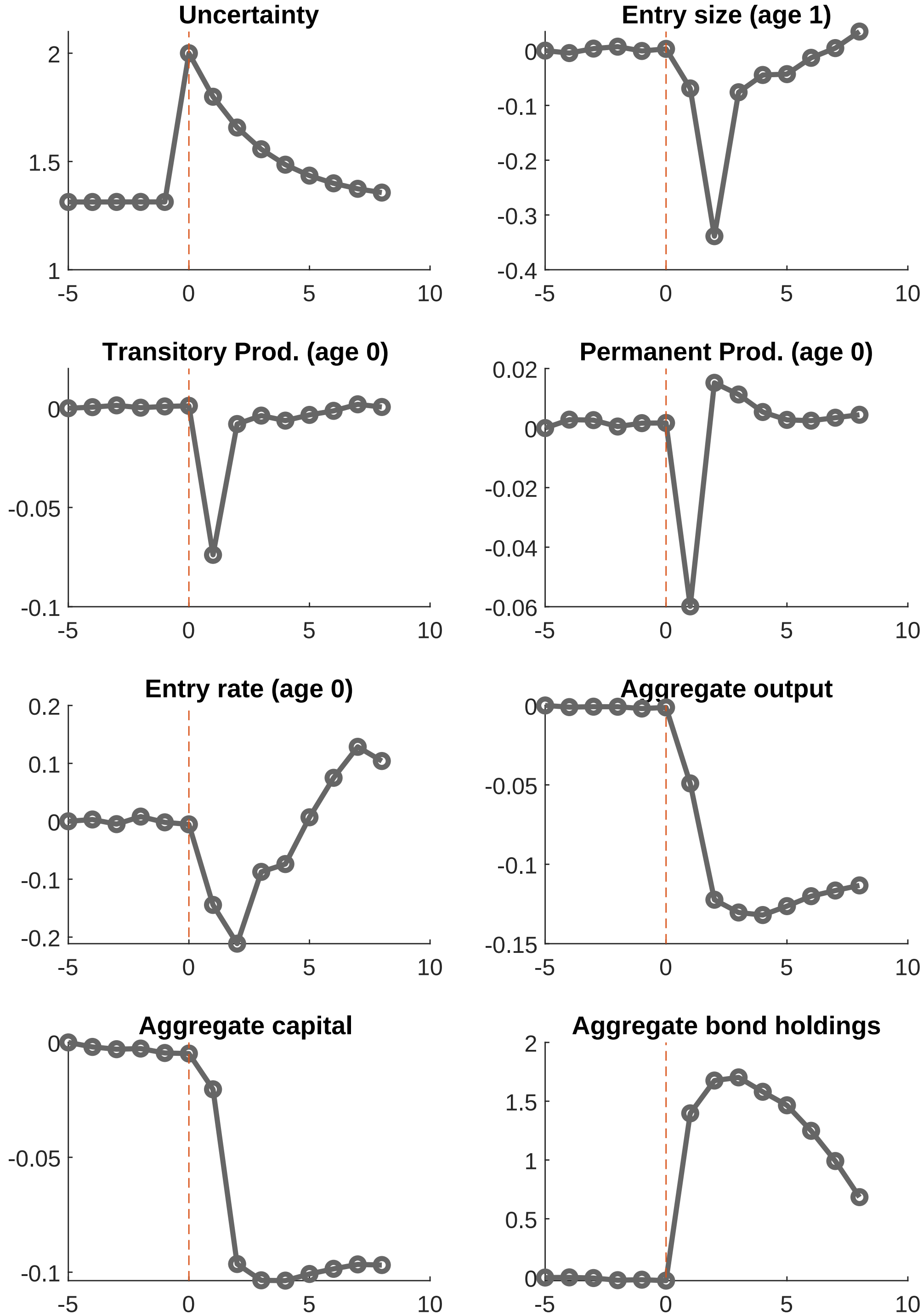
precautionary motive.

The last line in Figure 4 reports the impact of uncertainty on the aggregate output and physical capital stock. An increase in uncertainty decreases aggregate production, and the dynamics of output present a slow recovery pattern. The aggregate capital stock has similar dynamics. Therefore, the uncertainty shock renders a persistent negative impact on the aggregate economy. The productivity selection mechanism primarily drives the persistence in the aggregate dynamics. The permanent component of the productivity e_t follows a technological upgrading process shown in (6). An increase in uncertainty causes more low-ability workers to start their entrepreneurship. These new entrants have a much lower productivity growth than those in the normal episodes, resulting in persistent effects of an uncertainty shock on the aggregate economy.

Post-entry dynamics The productivity selection mechanism implies that a higher uncertainty leads to relatively more low-ability workers than high-ability ones starting their entrepreneurship, resulting in a lower average efficiency across entrants. Since the entrant's post-entry productivity positively correlates with her innate ability, the entrants born in high uncertainty episodes would have slower growth dynamics than those born in regular time. The quantitative exercise simulates 100,000 startup firms for high uncertainty and low uncertainty episodes and tracks each startup for ten years. The high uncertainty group of startups is those born when the uncertainty shock arrives. The low uncertainty group is those born on the date without the uncertainty shock. We then compute the average firm size measured by the number of employees against their age for both groups. The left panel in Figure 5 shows that the initial size differs in two groups, reflecting the productivity selection effect for entrants caused by the uncertainty shock. The gap of firm size between two groups diverges by age and becomes stable after age equals 7. The divergence in the size gap is because the post-entry productivity positively correlates with the initial ability of the entrepreneur. The gap becomes stable because the entrepreneur's productivity reaches the highest state in the technology upgrading process after a sufficiently long time.

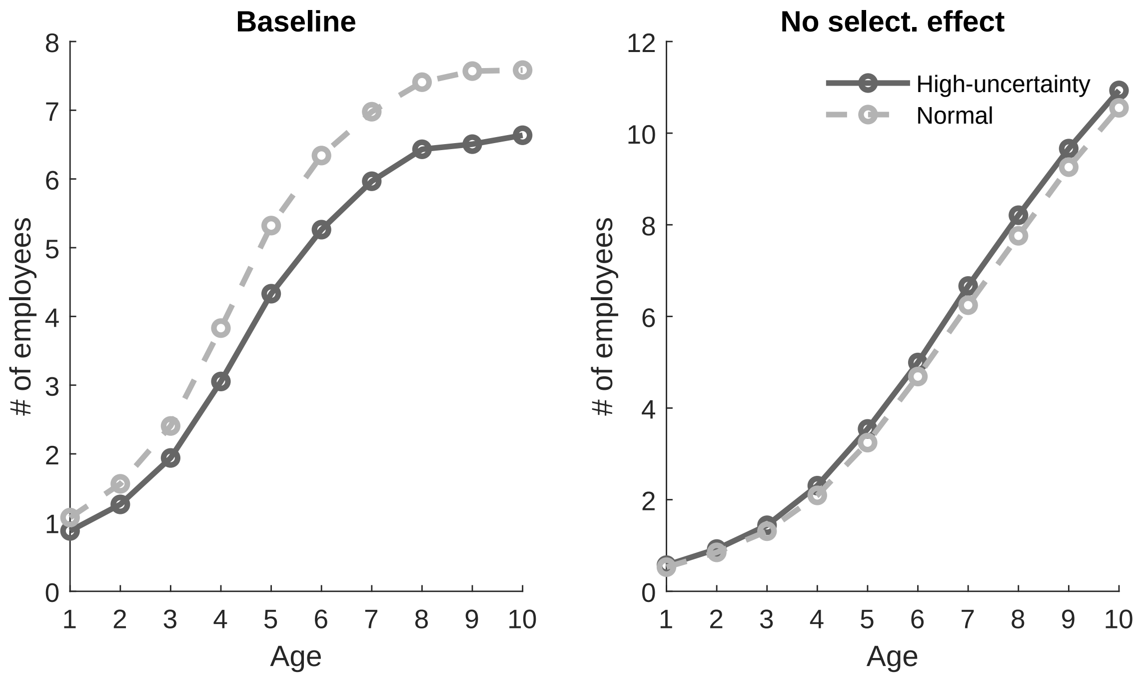
In the right panel of Figure 5, we mute the productivity selection channel by assuming that the entrepreneurs' productivity processes are irrelevant to their innate ability when they were workers. The figure shows that uncertainty does not reduce the firm size. The firms established in the high uncertainty episodes have a relatively larger size than those in the low uncertainty episodes. However, the differences in post-entry dynamics between the two scenarios are visibly minor. In the absence of a productivity selection channel, the wait-and-see effect leads entrepreneurs with a larger size to enter the market due to the fixed entry cost. As a result, firms established in the high uncertainty episodes have more considerable post-entry

Figure 4: Dynamics in response to an increase in uncertainty



Notes: This figure plots the dynamics of variables in response to a positive uncertainty shock that arrives in period 0 (red dashed line). The economy stays in the initial steady state for periods -5 to -1 before the uncertainty shock hits the economy. All the variables, except for the uncertainty, are in log term and are normalized by the level in the initial steady state. The uncertainty is the average value of the uncertainty indicators (1 for σ_L and 2 for σ_H) across firms. The entry size variable corresponds to the entrepreneurs with age 1 since the age-0 entrepreneurs do not produce. The productivity indicators correspond to the new entrants (i.e., firms with age 0). The entry rate is defined as the share of the mass of age-0 entrepreneurs to the total mass of firms.

Figure 5: Post-entry size under an uncertainty shock

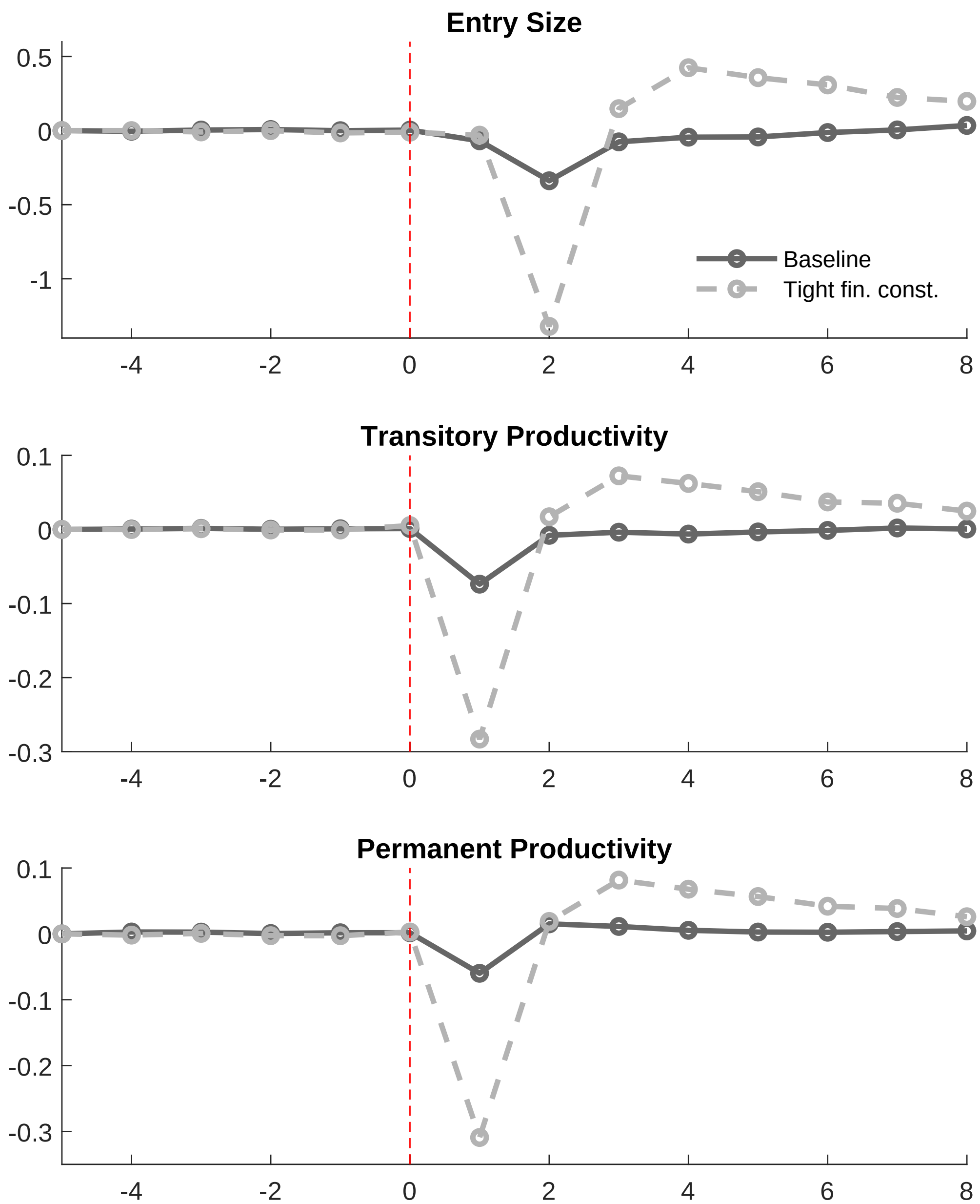


Notes: This figure plots the post-entry size against the firm age in response to a positive uncertainty shock. The solid lines represent the baseline model, and the dashed lines are for the model where the productivity selection channel is muted. In the baseline model, the post-entry productivity is positively correlated with the entrepreneur's ability. In the control model, the post-entry productivity and the entrepreneur's ability are independent. The "high-uncertainty" corresponds to the scenario where the firms were born on the date when the uncertainty shock hits the economy. The "normal" corresponds to the scenario where there is no uncertainty shock. We simulate 100,000 new entrants in each case and track them for ten years and compute the average size across firms in each age.

size dynamics than those in the low uncertainty episodes.

Role of financial constraint The analysis from the static model indicates that the entrepreneur's borrowing constraint plays a crucial role in the productivity selection mechanism. We now conduct a counterfactual analysis by varying the value of the parameter governing the borrowing constraint θ. Figure 6 compares the productivity and size dynamics for entrants in the baseline model where $\theta = 0.8$ with those in the control model with a tighter borrowing constraint where $\theta = 0.5$. The figure shows that both the transitory and permanent productivities experience a larger reduction in the control model than that in the control model, implying that the financial constraint magnifies the productivity selection effects on startups. The adverse impact on the entrants' size is more pronounced in the control model than those in the baseline model. The above results conform the empirical findings shown in Table 3.

Figure 6: Dynamics under a tighter financial constraint



Notes: This figure plots the dynamics of firm size of entrants and entrants' productivities in response to a positive uncertainty shock for the baseline model (solid lines) and the model with a tighter financial constraint (dashed lines). In the baseline model, the financial constraint parameter $\theta = 0.8$, and in the control model $\theta = 0.5$. Each variable is in the log term and the vertical axis indicates the percentage deviation from the initial steady state (0.01 = 1%). The firm size is measured by the number of employees. The transitory and permanent productivities correspond to $\log(z_t)$ and $\log(e_t)$, respectively. A positive uncertainty shock arrives in period 0 (red dashed line). The economy stays in the initial steady state for periods -5 to -1 before the uncertainty shock hits the economy. The entry size corresponds to the entrepreneurs with age 1 since the age-0 entrepreneurs do not produce. The productivity indicators correspond to the new entrants (i.e., firms with age 0).

6 Conclusion

We study how uncertainty influences entrepreneurial activities and the aggregate consequences. We document the empirical relationship between time-varying uncertainties and entrepreneurial activities using a European firm-level dataset. A rise in uncertainty reduces the average size of startups and their productivities. The entrepreneurs who start their new businesses during the high-uncertainty time have slower growth dynamics than those in the low-uncertainty time. These facts indicate that uncertainty renders productivity selection among agents when they undertake entrepreneurship, and the perverse effects remain persistent by affecting the entrepreneurial growth dynamics. After considering financial factors, we find that the negative relationship between uncertainty and entrepreneurial activities becomes more pronounced in a tightened external financing condition.

We build a dynamic entrepreneurial choice model with uninsurable idiosyncratic uncertainty and borrowing constraints. The model features heterogeneous workers who make occupation decisions to be paid employment or undertake entrepreneurship. After the business is successfully set up, the worker's ability transforms to the entrepreneur's innate talent. The agents with high ability who undertake entrepreneurship face restrictive borrowing constraints as they demand more capital investment. On the contrary, the agents with low ability are not financially constrained in their entrepreneurial activities due to weak capital demand. The value function of being an entrepreneur is convex in the productivity for those low-ability agents and concave for those high-ability ones. As a result, when the uncertainty of future productivity is high, low-ability agents have stronger incentive than high-ability ones to enter the market, resulting in productivity distortions among startups. Since permanent productivity positively correlates with the entrepreneur's innate talent, an uncertainty shock causes slow enterpreneurial growth dynamics. Our calibrated model can match the empirical facts quite well. The quantitative exercise on the aggregate analysis predicts that a higher uncertainty may lead to a short-run economic slowdown and long-run sluggish output dynamics.

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Appendix

A Firm Level Data

Data Construction To maximize data coverage, we append ORBIS historical files with the Amadeus dataset. The ORBIS historical dataset combines all the offline version of ORBIS, with the oldest version from October 2005. Later versions, 07/2006, 10/2007, 08/2008, 10/2009, 10/2010, 10/2011, 10/2012, are added and merged by the data provider. The AMADEUS dataset is directly downloaded from the Wharton Research Data Services (WRDS). Following Gopinath et al. (2017), we combine their unique Bureau Van Dijk Index (BvD ID). If the index appears in both datasets, we keep the one from the ORBIS historical file to get a longer span. For our purpose, the ORBIS historical product have several advantages. First, it allows us to make comparison through time, using financial older than 10 years, which is the maxim passable for their online version. Second, the product makes sure BvD ID number are compatible with the corresponding version of ORBIS so that one do not need to worry about the ID change overtime. Third, Bureau Van Dijk has checked the link of historical data to all firms, which allows us to clearly identify if a firm enters/exiting from operation. Fourth, the industry classification has been made consistent across different vintages. Its most detailed classification: 4 digit NACE (Statistical classification of economic activities in the European Communities), has around 230 sub-sectors within the manufacturing industry, with a comparable detailedness to 6 digit NAICS (North American Industry Classification Codes), which have around 270 sub manufacturing sectors.

Firm Coverage We keep dataset from 1999-2015, NAICS Rev-2 code from 1000 to 3399 (manufacturing), and we only keep unconsolidated accounts. After the same cleansing method in Gopinath et al. (2017), we obtain dataset that covers 813,710 manufacturing firms (4,531,777 firm-year observations). Our sectoral statistics are at iso2×industry×year level, and is computed on the set of firms that exists in the panel throughout. In doing so, we only keep firms whose legal status is "Active", and we drop firms that change its main operating sector in any calendar year. Table A.1 reports firm coverage on the "small" (1-19 employee), "medium-sized" (20-249 employee), and the "large" (250+ employee). The table replicates the corresponding one in Gopinath et al. (2017).

Productivity Measures and Constructing Cross Industry Uncertainty Proxy We proceed to productivity estimation. Following the standard practice, we deflate firm nominal sales, value added, wage bills, using an output price deflator. Since firm level price is not observed in our

Table A.1: Firm Coverage (year==2006)

iso2	size	employment	staff	output
DE	1-19 employee	0.0079	0.0058	0.0149
DE	20-249 employees	0.3517	0.3167	0.3329
DE	250+ employees	0.6404	0.6775	0.6523
ES	1-19 employee	0.3350	0.3037	0.2738
ES	20-249 employees	0.6102	0.6310	0.6485
ES	250+ employees	0.0548	0.0652	0.0776
FR	1-19 employee	0.1151	0.1232	0.0673
FR	20-249 employees	0.2483	0.2066	0.1851
FR	250+ employees	0.6366	0.6702	0.7476
IT	1-19 employee	0.1640	0.1466	0.1700
IT	20-249 employees	0.6580	0.6545	0.6403
IT	250+ employees	0.1780	0.1988	0.1897
NO	1-19 employee	0.1706	0.3760	0.6304
NO	20-249 employees	0.2545	0.1149	0.0760
NO	250+ employees	0.5748	0.5091	0.2937
PT	1-19 employee	0.2771	0.2231	0.1973
PT	20-249 employees	0.6138	0.6336	0.6433
PT	250+ employees	0.1092	0.1433	0.1594

dataset, we use the gross output price deflators from the EUROSTAT at the two-digit industry level. We measure the capital stock as the value with the price of investment goods. As in Gopinath et al. (2017), we use country specific price of investment from the World Development Indicators. Fixed assets include both tangible and intangible fixed assets. Our results change very little when we exclude firm intangibles. To construct sector level variables (MPK dispersion, loan growth, etc), we drop any country-sector-year pair with less than 2 firms that report non negative values for value added, employment, material cost, as well as cost of employee to obtain an meaningful sectoral level aggregation result.

To estimate firm level productivity, we follow the Wooldridge (2009) adaption of the standard Levinsohn and Petrin (2003) method. The estimation is by two digit nace pairs and we conduct the production function estimation separately for each country. This allows for the possibility of potentially different capital and labor share for different countries. The following table reports the estimated capital and. Our estimation gives a median markup value of around 0.15. To construct the firm level productivity shock for the regression analysis, we follow Bloom et al. (2018) to run the following regression

$$\log(A_{jt}) = \rho \log(A_{j,t-1}) + i_j + \lambda_{ct} + \varepsilon_{jt}, \quad \text{(A.1)}$$

in which we regress the current period log productivity for firm j on its lagged value, controlling for firm and country×year fixed effect. The residual ε_{jt} is thus the productivity shock.

Summary Statistics To get a better sense of the regression variables, we report the summary statistics regarding sector size, startup size and productivity, and sectoral variables including growth and dispersion measures. For completeness, we list their 1, 25, 50, 75, and 99th percentiles. The sector size is the number of firms within each industry-country-year cell. In our working dataset, the smallest cell contains only 3 firm-year observations and the median is 71. Firm size is also small on average, with the median number of employment being around 5 and the top 1% reaches to around 250. Row 3 to 6 lists the relative size and efficiency measures of startups relative to the incumbents within their corresponding industry-country-year cell. For a given sector, the startups are substantially smaller than the incumbents, except only a few, and the measured productivity is also lower. The average of industrial uncertainty is 0.17, which is roughly consistent with the the U.S. Census dataset in Bloom et al. (2018). The last two rows show sectoral loan growth is around 0.6% and the sales growth is around 2%.

Table A.2: Summary Statistics

	p1	p25	p50	p75	p99
sector size	3	24	71	193	1234
(ln) empl	.6931472	1.098612	1.609438	2.302585	5.459586
(ln) rel. TFP	-2.421461	-1.122176	-.6967445	-.2641941	1.72268
(ln) rel. empl	-5.502424	-2.998236	-2.336303	-1.591726	.9590153
(ln) rel. tfas	-8.844154	-4.970454	-3.523456	-2.183138	1.692107
(ln) rel. toas	-9.062955	-4.676758	-3.501091	-2.378995	1.183255
(std) tfp residual	.0584827	.1407547	.1782524	.2714179	.5716038
(iqr) tfp residual	.0591874	.1469769	.1841145	.2900693	.7314428
loan growth	-1.557337	-.1917261	.0065252	.2163166	1.422855
sales growth	-.6363955	-.0829047	.0224595	.1140251	.7901689

B Banks and Macroprudential Policies

Our data for banks are from the Bankscope data, which is a cross-country dataset on banks' balance sheet information. It is also provided by Bureau Van Dijk, and it spans across 190 countries from 1998 to 2015. Its financial information includes total asset, size, loans to private sector, various measures of leverage, profitability, amount of loans outstanding, as well as their interbank positions. The dataset has been harmonized by its provider so that international comparison is suitable. We use data from 1999-2015, consistent with the span of firm financial dataset. We only keep banks that continuously report positive employment, total asset, tangible fixed asset, total loans.

Credit institution regulations from the European Central Bank (ECB). It provides details of macroprudential (or similar) policy actions taken in the European Union since 1995. It was complied by the staff from each central bank in the ECB mandate area. According to ECB, the design of this dataset is to provide empirical assessment for macroprudential policies. The reference is Budnik and Kleibl (2018), which details the constructions and coverages of the policies.

C Proof of Proposition 1

The optimal labor decision implies a labor demand $n = \left[\frac{\gamma}{w} a\left(x, z\right)\right]^{\frac{1}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}}$, resulting in a capital income $\left(1-\gamma\right)\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} a^{\frac{1}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}}$. Replacing b with the flow-of-funds constraint, the profit can be expressed as

$$\pi = \left(1-\gamma\right)\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}} a^{\frac{1}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}} - \left(r+\delta\right)k + \left(1+r\right)b^{w}, \tag{C.1}$$

subject to borrowing constraint $k \leq \frac{1}{1-\theta} b^{w}$.

If $a \leq a^{*}\left(b^{w}\right)$, the interior optimal k can be achieve, satisfying

$$k = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} a^{\frac{1}{1-\alpha-\gamma}}, \tag{C.2}$$

where the cutoff of productivity $a^{*}\left(b^{w}\right)$ satisfies

$$\left(\frac{\alpha}{r+\delta}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}} \left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \left(a^{*}\right)^{\frac{1}{1-\alpha-\gamma}} = \frac{1}{1-\theta} b^{w}. \tag{C.3}$$

The last equation indicates that $a^{*}\left(b^{w}\right)$ is strictly increasing in the net worth b^{w}. That is, it is less likely for an entrepreneur with a higher net worth to be financial constraint. Since $a = xz$, from an ex-ante perspective, the probability that an entrepreneur with ability x has a binding borrowing constraint is $\mathbf{F}\left(\frac{a^*(b^w)}{x}\right)$. Therefore, a lower ability implies that less likely to be financially constrained.

If $a > v^*(b^w)$, the borrowing constraint is binding. The optimal capital satisfies $k = \frac{1}{1-\theta} b^w$. Therefore, the optimal capital decision can be summarized as

$$k\left(b^w, x, z\right) = \min\left\{\left[\frac{a}{a^*\left(b^w\right)}\right]^{\frac{1}{1-\alpha-\gamma}}, 1\right\} \bar{k}\left(b^w\right), \tag{C.4}$$

where $\bar{k}\left(b^w\right) = \frac{1}{1-\theta} b^w$ is the maximum of capital the entrepreneur can hold under the borrowing constraint and the productivity cutoff $a^*\left(b^w\right)$ is determined by (C.3). Q.E.D.

D Proof of Proposition 2

Under the optimal capital decision derived in Proposition 1, the profit function $\pi\left(b^w, x, z\right)$ is given by

$$\pi\left(b^w, x, z\right) = \begin{cases} \varpi_1 x^{\frac{1}{1-\alpha-\gamma}} z^{\frac{1}{1-\alpha-\gamma}} + (1+r)\, b^w & \text{if } z < \frac{a^*(b^w)}{x} \\ \varpi_2 x^{\frac{1}{1-\gamma}} z^{\frac{1}{1-\gamma}} + \frac{1-\delta-\theta(1+r)}{1-\theta} b^w & \text{if } z \geq \frac{a^*(b^w)}{x} \end{cases},$$

where $\varpi_1 = (1-\alpha-\gamma)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}$ and $\varpi_2 = (1-\gamma)\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}}\left(\frac{1}{1-\theta}\right)^{\frac{\alpha}{1-\gamma}}\left(b^w\right)^{\frac{\alpha}{1-\gamma}}$. The value function $V\left(b^w, x, z\right) = \log\left(\pi\left(b^w, x, z\right)\right)$.

We now prove that under a sufficient condition $\frac{(\alpha+\gamma)(1-\theta)\alpha}{(1-\alpha-\gamma)^2} > 1$, the value function $V\left(b^w, x, z\right)$ is strictly convex in z for $z \leq \frac{a^*(b^w)}{x}$. The value function in this case is

$$V\left(b^w, x, z\right) = \log\left[\varpi_1 x^{\frac{1}{1-\alpha-\gamma}} z^{\frac{1}{1-\alpha-\gamma}} + (1+r)\, b^w\right].$$

Given the wealth level a_i, we derive the partial derivatives as

$$\begin{aligned} \frac{\partial V\left(b^w, x, z\right)}{\partial z} &= \frac{x}{1-\alpha-\gamma} \frac{\varpi_1 (xz)^{\frac{\alpha+\gamma}{1-\alpha-\gamma}}}{\varpi_1 (xz)^{\frac{1}{1-\alpha-\gamma}} + (1+r)\, b^w} > 0, \\ \frac{\partial^2 V\left(b^w, x, z\right)}{\partial z^2} &= \frac{\varpi_1 x^2}{1-\alpha-\gamma} \frac{(xz)^{2\frac{\alpha+\gamma}{1-\alpha-\gamma}}}{\left[\varpi_1 (xz)^{\frac{1}{1-\alpha-\gamma}} + (1+r)\, b^w\right]^2} \left[\frac{\alpha+\gamma}{1-\alpha-\gamma} (xz)^{\frac{-1}{1-\alpha-\gamma}} (1+r)\, b^w - \varpi_1\right]. \end{aligned}$$

The sign of $\frac{\partial^2 V(b^w,x,z)}{\partial z^2}$ depends on the value of the term $\frac{\alpha+\gamma}{1-\alpha-\gamma}(xz)^{\frac{-1}{1-\alpha-\gamma}}(1+r)\, b^w - \varpi_1$. Replacing b^w with the equation that defines $a^*(b^w)$, i.e., $b^w = (1-\theta)\left(\frac{\alpha}{r+\delta}\right)^{\frac{1-\gamma}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}}(a^*)^{\frac{1}{1-\alpha-\gamma}}$, yields

$$\begin{aligned}\frac{\alpha+\gamma}{1-\alpha-\gamma}(xz)^{\frac{-1}{1-\alpha-\gamma}}(1+r)\,b^w - \varpi_1 &= (1-\alpha-\gamma)\left(\frac{\alpha}{r+\delta}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\alpha-\gamma}} \\ &\quad\times\left[\frac{(\alpha+\gamma)(1-\theta)\,\alpha}{(1-\alpha-\gamma)^2}\left(\frac{a^*}{a}\right)^{\frac{1}{1-\alpha-\gamma}}-1\right],\end{aligned}$$

which is strictly positive for any $a < a^*$ under the condition $\frac{(\alpha+\gamma)(1-\theta)\alpha}{(1-\alpha-\gamma)^2} > 1$.

We now prove that the value function $V(b^w, x, z)$ is strictly concave in z for $z > \frac{a^*(b^w)}{x}$. In this case, the value function takes the form

$$V(b^w, x, z) = \log\left[\varpi_2 x^{\frac{1}{1-\gamma}} z^{\frac{1}{1-\gamma}} + \frac{1-\delta-\theta(1+r)}{1-\theta} b^w\right],$$

where $\varpi_2 = (1-\gamma)\left(\frac{\gamma}{w}\right)^{\frac{\gamma}{1-\gamma}}$. Given the wealth level b^w, we derive the partial derivatives as

$$\begin{aligned}\frac{\partial V(b^w,x,z)}{\partial z} &= \frac{\frac{x}{1-\gamma}\varpi_2\left(\frac{1}{1-\theta}b^w\right)^{\frac{\alpha}{1-\gamma}}(xz)^{\frac{\gamma}{1-\gamma}}}{\varpi_2\left(\frac{1}{1-\theta}b^w\right)^{\frac{\alpha}{1-\gamma}}(xz)^{\frac{1}{1-\gamma}}+\frac{1-\delta-\theta(1+r)}{1-\theta}b^w}, \\ \frac{\partial^2 V(b^w,x,z)}{\partial z^2} &= \frac{\varpi_2}{1-\gamma}\left(\frac{b^w}{1-\theta}\right)^{\frac{\alpha}{1-\gamma}}(xz)^{\frac{2\gamma}{1-\gamma}}x^2\frac{-\varpi_2\left(\frac{b^w}{1-\theta}\right)^{\frac{\alpha}{1-\gamma}}+\frac{\gamma}{1-\gamma}(xz)^{\frac{-1}{1-\gamma}}\frac{1-\delta-\theta(1+r)}{1-\theta}b^w}{\left[\varpi_2\left(\frac{b^w}{1-\theta}\right)^{\frac{\alpha}{1-\gamma}}(xz)^{\frac{1}{1-\gamma}}+\frac{1-\delta-\theta(1+r)}{1-\theta}b^w\right]^2}.\end{aligned}$$

It is straightforward to show that $\frac{\partial^2 V(b^w,x,z)}{\partial z^2} < 0$ under the condition $1-\delta-\theta(1+r) < 0$. Therefore $V(b^w, x, z)$ is strictly concave in z when $z > \frac{a^*(b^w)}{x}$.

For a sufficiently large x such that $\frac{a^*(b^w)}{x} < z_{\min}$, the value function would be strictly concave for any $z \in [z_{\min}, z_{\max}]$, implying that the expected value of $V(b^w, x, z)$, $\bar{V}(b^w, x; \sigma) = \int V(b^w, x, z)\, d\mathbf{F}(z; \sigma)$ decreases with the uncertainty σ due to the property of Jensen's inequality. Similarly, for a sufficiently low x such that $\frac{a^*(b^w)}{x} > z_{\max}$, the value function would be strictly convex for any $z \in [z_{\min}, z_{\max}]$, resulting that $\bar{V}(b^w, x; \sigma)$ increases with the uncertainty σ. Q.E.D.

E SMM Procedure

Let $\Theta = \{\theta_1, \ldots, \theta_m\}$ represents parameters, and let $\mathbf{M} = \{\mathbf{m}_1, \ldots, \mathbf{m}_k\}$ represents moments. We numerically compute the following matrix containing derivatives of moments with respect to changes in parameters,

$$\frac{\partial \mathbf{M}}{\partial \Theta} = \begin{pmatrix} \frac{\partial \mathbf{m}_1}{\partial \theta_1} & \frac{\partial \mathbf{m}_1}{\partial \theta_m} \\ \cdots & \cdots \\ \cdots & \cdots \\ \frac{\partial \mathbf{m}_k}{\partial \theta_1} & \frac{\partial \mathbf{m}_k}{\partial \theta_m} \end{pmatrix}_{k \times m} \tag{E.1}$$

Then the standard error vector is given by

$$\mathbf{\Sigma}_{m \times 1} = \sqrt{\text{DiagInv}\left[\left(\frac{\partial \mathbf{M}}{\partial \Theta}\right)'_{m \times k} \hat{\mathbf{W}}_{k \times k} \left(\frac{\partial \mathbf{M}}{\partial \Theta}\right)_{k \times m}\right]}, \tag{E.2}$$

where

$$\hat{\mathbf{W}} = \begin{pmatrix} \frac{1}{\hat{\sigma}_1^2} & \\ \cdots & \cdots \\ \cdots & \cdots \\ & \frac{1}{\hat{\sigma}_k^2} \end{pmatrix}_{k \times k} \tag{E.3}$$

represents the weighting matrix for data. $\hat{\sigma}_k$ is the bootstrapped standard error for data moments.