Asset Bubbles and Monetary Policy in Open Economies*

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Abstract

We present a theoretical framework to examine the role of asset bubbles in setting monetary policy in open economies. The framework features an intrinsically useless bubble asset that commands a liquidity premium for firms that face uninsurable idiosyncratic shocks to investment efficiency and financial constraints. Low foreign interest rate is conducive to the formation of bubbles. We show that while an interest rate rule that responds to bubble prices can stabilize output following foreign interest rate shocks, it will be much less effective following domestic productivity shocks.

Keywords: Asset bubbles; Monetary policy; Open economies

JEL Classification: E32, E44, E52, F41

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1 Introduction

In a globalized world, it is now well documented that countries' economic conditions (e.g., output, inflation, external balance, asset prices etc.) are not only driven by domestic shocks but also foreign shocks. How or whether monetary policy should react to asset prices have become a heated topic since the global financial crisis. Nevertheless, there is relatively scarce research on this important question in an open economy setting that takes into account both domestic and foreign factors. This paper presents a framework that embeds asset bubbles into an open economy New Keynesian model to study the role of asset bubbles in conducting monetary policies.

In the aftermath of the global financial crisis, some studies suggest that central banks should adopt the "lean-against-the-wind" policy to prevent bubbles from forming by raising interest rates. However, the "lean-against-the-wind" implicitly assumes that higher interest rates reduce asset bubbles. Galí (2014) theoretically challenges the relationship between interest rates and asset price bubbles behind conventional wisdom. In equilibrium, the bubble must grow at the rate of interest rates. If interest rates rise, it will push the bubble up. Galí and Gambetti (2015) and Blot et al. (2018) provide empirical supports for this. Dong et al. (2020) introduce financial intermediation and deposit reserve requirements into a rational bubble model and find that central banks should cut rates by welfare analysis. However, Hirano et al. (2017), André et al. (2018), and Miao et al. (2019) show that a "lean-against-the-wind" policy can effectively curb asset bubbles.

Most studies in this vein, however, consider closed economies. Although there are studies that deal with optimal monetary policy in open economies (e.g., Kollmann (2002); Gali and Monacelli (2005); Clarida (2014)), they do not consider the role of asset bubbles. We present a theoretical framework to examine the role of asset bubbles in setting monetary policy in open economies. The framework features an intrinsically useless bubble asset that commands a liquidity premium for firms that face uninsurable idiosyncratic shocks to investment efficiency and financial constraints.

Our study makes three contributions to the literature. First, Low foreign interest rate is conducive to the formation of bubbles. As foreign interest rate declines, there are capital inflows accompanied by real exchange rate appreciation, lending booms, and investment booms. The increased demand for domestic bonds reduces the domestic interest rate, thereby fueling a bubble.

Second, except for the interest rate channel, there is an additional asset price channel in the transmission mechanism of monetary policy. There is an amplification effect of asset bubbles to positive technology shocks and negative global interest rate shocks. Bubbles serve as a store of value and relax the borrowing constraint. These shocks increase demand for domestic goods and lower the domestic nominal interest rate. The lower bond yields stimulate demand for bubbles, resulting in a higher asset price. This raises the net worth of firms holding bubble assets, further stimulating demand for domestic goods and reducing the nominal interest rate. Such amplification effect brings about greater asset bubbles and more consumption, investment, and output.

Finally, we analyze the effectiveness of monetary policy in response to asset prices. We take different values for the coefficient before the asset price gap in the monetary policy rule as an example to analyze the impacts of monetary policy. We show that while an interest rate rule that responds to bubble prices can stabilize output following foreign interest rate shocks, it will be ineffective following domestic productivity shocks. Capital inflows induce higher asset prices. Anticipating higher interest rates in response to asset prices, firms desire to hold more bubble assets to provide liquidity. The expansion of asset bubbles crowds out investment substantially, reducing output volatility. Therefore, the "lean-against-the-wind" policy creates a trade-off between output stability and the risk of asset bubbles.

Literature Review. Our paper mainly relates to two strands of the literature. At first, our paper is related to the recent literature on asset bubbles in open economies (e.g., Caballero and Krishnamurthy (2006); Ventura (2012); Basco (2014); Martin and Ventura (2015); Miao et al. (2021)). Most studies typically adopt the overlapping-generations (OLG) framework, except for Miao et al. (2021). Like our paper, this literature emphasizes the importance of credit constraints for the emergence of asset bubbles, especially in the infinite horizon models. Our model differs from this literature in the addressed questions and modeling details. We introduce price stickiness and central bank to analyze the impacts of monetary policy, which is increasingly essential (Asriyan et al. (2021)).

The second strand of literature is related to recent research on monetary policy in response to asset bubbles. After Galí (2014) questions the effectiveness of the "lean-against-the-wind" policy theoretically, Galí and Gambetti (2015) tested it by using a structural vector autoregression model with time-varying coefficients (TVC-SVAR) and found that when monetary policy is assumed not to respond to current asset prices, rising interest rates will lead to an increase in stock prices. Blot et al. (2018) use principal component analysis and point out that the impact of monetary policy on asset bubbles is asymmetric, tight monetary policy cannot suppress bubbles, but loose monetary policy will prompt asset bubbles. In a DNK model, Dong et al. (2020) introduce financial intermediaries and deposit reserve requirements and find that monetary policy can affect the conditions of the existence of bubbles, and the optimal monetary policy is cutting interest rates in response to increasing asset prices. Furthermore, they point out that although monetary policy responds optimally to asset prices, it increases inflation volatility while reducing bubble volatility. However, the other studies support the "lean-against-the-wind" policy. Hirano et al. (2017) introduce price stickiness into a rational bubble model and find that regardless of the size of the bubble, a positive monetary policy shock (an increase in interest rates) would reduce output, investment, inflation, and asset price bubbles. Different from their results, we find the asset price increases in a positive monetary policy shock.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the existence conditions of bubbleless and bubbly equilibria. Section 4 shows the impulse responses to shocks and policy implications. Section 5 concludes the paper. The technical proofs are available in the Appendix.

2 The Model

We consider a small open economy consisting of domestic intermediate good firms, banks, capital good producers, households, retailers, and a central bank. We assume that intermediate good firms are ex ante identical but face idiosyncratic investment efficiency shocks, which determine the efficiency that a firm transformed capital goods into installed capital. To produce domestic intermediate goods, these firms hire labor, import foreign inputs, and invest in capital goods. They can borrow from (or lend to) domestic banks but are subject to credit constraints. They can also trade a bubble asset. Domestic banks play the role of financial intermediary by borrowing from foreign investors and lend in the domestic debt market. As in Schmitt-Grohé and Uribe (2003), we introduce external debt-elastic interest rate faced by domestic banks in cross-border borrowing. Retailers sell to both domestic and foreign markets with Calvo-type sticky price and a central bank sets monetary policy. Figure 1 illustrates the model structure.

2.1 Households

In each period, the income of households includes labor income, dividends from domestic banks, firms, retailers, and capital goods producers. The household consumes domestic final goods and trades firms' stocks. Assume households do not trade bonds or bubble assets.[1] The household's optimization problem is

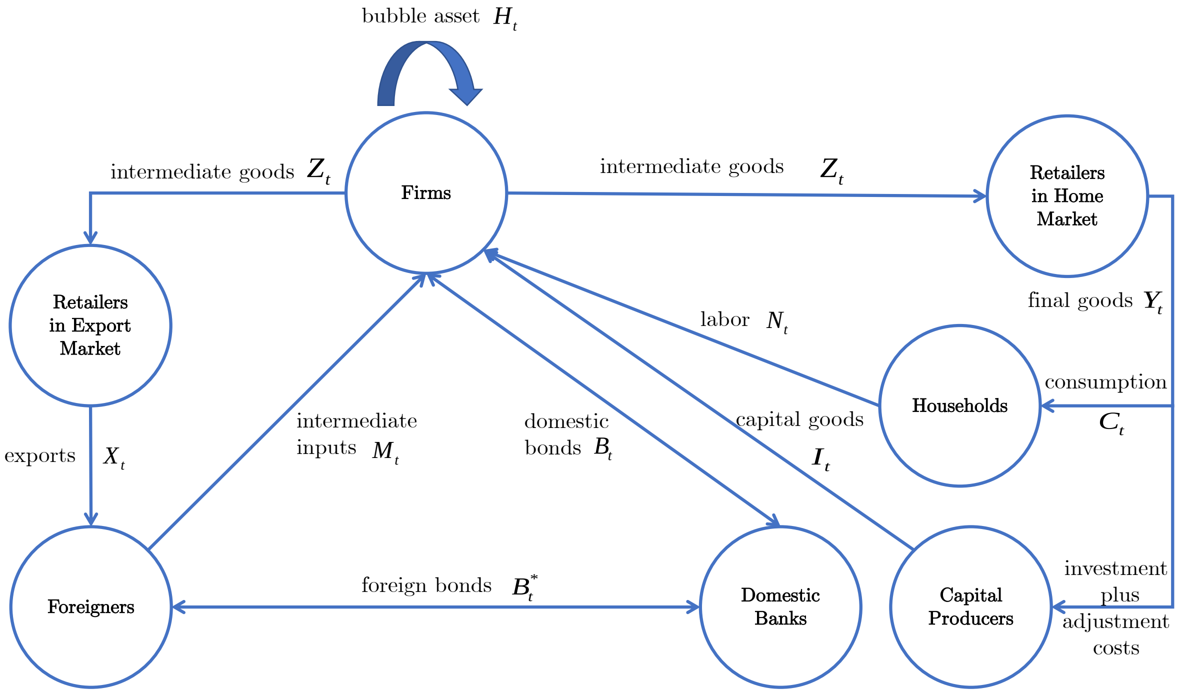
$$\max_{\psi_{j,t+1}, C_t, N_t} E_0 \sum_{t=0}^{\infty} \xi_t \beta^t \left[\ln \left(C_t - hC_{t-1} \right) - \kappa \frac{N_t^{1+\varphi}}{1+\varphi} \right],$$

subject to the budget constraints

$$\int \psi_{j,t+1} \left(V_{jt} - D_{jt} \right) dj + P_t C_t = W_t N_t + \int \psi_{jt} V_{jt} dj + D_t^b + D_t^k + D_t^r, \tag{1}$$

[1] In equilibrium, as the prices of bonds and bubbles involve liquidity premium for firms but not for households, the return of these assets are too low for households. Therefore, households won't hold them. See Appendix A.1 for the proof.

Figure 1: Model Structure



where D_t^r is the profit of all retailers (in domestic market and export market), ξ_t is an exogenous preference shock that follows

$$\ln \xi_t = \rho_\xi \ln \xi_{t-1} + \varepsilon_{\xi t},$$

where $\rho_\xi \in (0,1)$, and $\varepsilon_{\xi t}$ is an i.i.d. standard normal random variable.

The marginal utility is

$$\Lambda_t = \frac{1}{P_t}\left[\frac{\xi_t}{C_t - hC_{t-1}} - \beta h E_t \frac{\xi_{t+1}}{C_{t+1} - hC_t}\right], \tag{2}$$

and the labor supply is

$$\Lambda_t W_t = \kappa \xi_t N_t^\varphi.$$

2.2 Firms

There are a continuum of domestic intermediate good firms indexed by $j \in [0,1]$. The production function of firm j is

$$Z_{jt} = K_{jt-1}^\alpha \left(A_t N_{jt}\right)^{1-\alpha-\gamma} M_{jt}^\gamma, \alpha \in (0,1), \gamma \in (0,1), \alpha + \gamma \in (0,1),$$

where K_{jt-1}, A_t, N_{jt}, and M_{jt}, represent capital, productivity, labor and foreign inputs, respectively. Productivity A_t follows an AR(1) process

$$\ln A_t = \rho_A \ln A_{t-1} + \varepsilon_{At},$$

where $\rho_A \in (0,1)$, and ε_{At} is an i.i.d. standard normal random variable.

Firm j maximizes the profits by solving following static problem

$$\max_{N_{jt},M_{jt}} P_{zt} K_{jt-1}^{\alpha} \left(A_t N_{jt}\right)^{1-\alpha-\gamma} M_{jt}^{\gamma} - W_t N_{jt} - e_t P_{ft}^{*} M_{jt} = R_{kt} K_{jt-1}, \tag{3}$$

where P_{zt} is the price of domestic intermediate goods, W_t is nominal wage in local currency, and P_{ft}^{*} is the exogenous price of foreign inputs in foreign currency. One unit of foreign currency can be exchanged for e_t units of domestic currency in the spot market (e_t is called the nominal exchange rate in terms of direct quotation). We assume that the foreign input price follows an AR(1) process

$$\ln P_{ft}^{*} = \rho_{P_f^*} \ln P_{ft-1}^{*} + \varepsilon_{P_f^* t},$$

where $\rho_{P_f^*} \in (0,1)$, and $\varepsilon_{P_f^* t}$ is an i.i.d. standard normal random variable.

Due to the constant-returns-to-scale production function, we can show that the maximized objective of intermediate good firms is linear in capital and denoted as $R_{kt}K_{jt-1}$, where

$$R_{kt} = \alpha P_{zt}^{\frac{1}{\alpha}} A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{\gamma}{e_t P_{ft}^{*}}\right)^{\frac{\gamma}{\alpha}}. \tag{4}$$

The first-order conditions gives labor demand and foreign input demand

$$N_{jt} = P_{zt}^{\frac{1}{\alpha}} A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t P_{ft}^{*}}\right)^{\frac{\gamma}{\alpha}} K_{jt-1},$$

$$e_t P_{ft}^{*} M_{jt} = \gamma P_{zt} Z_{jt}.$$

We assume that the law of motion of firm j's capital follows

$$K_{jt} = (1-\delta) K_{jt-1} + \varepsilon_{jt} I_{jt}, \tag{5}$$

where $\delta \in (0,1)$ is the depreciation rate, and ε_{jt} represents a firm-specific investment efficiency shock. The shock is assumed to be independently and identically across firms and over time, whose cumulative distribution function is F (the density function is f) on support $[\varepsilon_{\min}, \varepsilon_{\max}] \subset [0,\infty)$ and investment is assumed to be irreversible such that $I_{jt} \geq 0$.

Firms can trade a one-period risk-free bond and a bubble asset with fixed aggregate supply. If firm j holds bonds $B_{jt} < (\geq) \, 0$, it implies the firm borrows (lends). We denote R_{ft} as domestic market nominal interest rate between periods t and $t+1$. Firms can borrow from domestic banks and use the value of existing capital as collateral. The borrowing constraint is

$$\frac{B_{jt}}{R_{ft}} \geq -\mu P_{kt} K_{jt-1}, \tag{6}$$

where $\mu \in (0,1)$ is a parameter that represents the extent to which capital can back firms' borrowing, and P_{kt} denotes the nominal price of the capital goods.

The bubble asset is intrinsically useless and we normalize its aggregate supply to 1. Firm j's holding of bubble asset in period t is H_{jt}. Assume firms cannot short the bubble asset, the short-sale constraint is

$$H_{jt} \geq 0. \tag{7}$$

The flow-of-funds constraint can be written as

$$D_{jt} = R_{kt} K_{jt-1} - P_{kt} I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_{ht}(H_{jt-1} - H_{jt}), \tag{8}$$

where D_{jt} denotes the dividends, and P_{ht} denotes the nominal price of the bubble asset.

We also impose a constraint that firm cannot issue new equity

$$D_{jt} \geq 0. \tag{9}$$

Let $V_{jt}(\cdot)$ denote the value function for firm j at time t, we can write firm's maximization problem using dynamic programming

$$V_{jt}\left(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}\right) = \max_{H_{jt}, I_{jt} \geq 0, B_{jt}} D_{jt} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right),$$

subject to above constraints (5), (6), (7), (8), and (9). Here, $\beta \in (0,1)$ is the subjective discount factor, and Λ_t denotes the marginal utility of households.

Define Tobin's (marginal) Q as

$$Q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right)}{\partial K_{jt}}.$$

Note that idiosyncratic investment efficiency shocks affects Tobin's Q, which is not equal to the price of capital goods P_{kt}.

We summarize the firm's optimal decisions in the following proposition.

Proposition 1 *(firm's optimal decisions) Denote* $\bar{\varepsilon}_t = \frac{P_{kt}}{Q_t} \in (\varepsilon_{\min}, \varepsilon_{\max})$.

1. *When* $\varepsilon_{jt} < \bar{\varepsilon}_t$*, the firm makes no investment and holds any amounts of bonds and bubble assets as long as (6), (7), and the following constraint hold*

$$R_{kt}K_{jt-1} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_{ht}(H_{jt-1} - H_{jt}) \geq 0.$$

2. *When* $\varepsilon_{jt} \geq \bar{\varepsilon}_t$*, the firm exhausts its borrowing limit and sells all bubble assets to make investment*

$$I_{jt} = \frac{1}{P_{kt}} \left[R_{kt}K_{jt-1} + \mu P_{kt}K_{jt-1} + B_{jt-1} + P_{ht}H_{jt-1} \right]. \tag{10}$$

3. *The Tobin's Q, the domestic interest rate, and the price of bubble asset satisfy*

$$Q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu P_{kt+1}) \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right], \tag{11}$$

$$\frac{1}{R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right], \tag{12}$$

$$P_{ht} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[P_{ht+1} \left(1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right) \right], \tag{13}$$

and the transversality conditions.

Proof. See Appendix A.1.

This proposition shows that there is critical cutoff $\bar{\varepsilon}_t = \frac{P_{kt}}{Q_t}$ for all firms. Only firms with high enough investment efficiency, $\varepsilon_{jt} \geq \bar{\varepsilon}_t$, are willing to exhaust its borrowing capacity and sell all their bubble assets to invest since each unit of local currency to invest can generate positive net profit $\left(\varepsilon_{jt}Q_t/P_{kt} - 1\right) \geq 0$.

Equations (11), (12), and (13) are the asset pricing equations for capital, bonds, and the bubble asset. Importantly, the integral terms represent the liquidity premium due to the idiosyncratic investment efficiency shocks and financial constraints, which are the key to existence of bubbles. In equation (13), the right-hand side is the expected marginal benefit of holding bubble asset in period $t+1$. In period $t+1$, if the firm obtains high investment effciency $\varepsilon_{jt+1} \geq \bar{\varepsilon}_{t+1}$, the bubble asset can be sold to finance investment and get positive net profits, which is reflected in the integral term.

2.3 Capital Producers

Representative capital good producers use domestic final goods as inputs to produce domestic capital goods and the production is subject to quadratic adjustment costs. Note P_{kt} denotes the nominal price of capital goods. The capital producer's objective is to maximize the discounted dividends

$$\max_{I_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^k,$$

where

$$D_t^k = P_{kt} I_t - \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] P_t I_t. \tag{14}$$

The first-order condition is given by

$$\begin{aligned} P_{kt} &= \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] P_t + \Omega_k P_t \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} \\ &\quad - \beta \Omega_k E_t \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \left(\frac{I_{t+1}}{I_t} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2. \end{aligned}$$

2.4 Domestic Banks

Domestic banks intermediate financial transactions internationally. R_{ft}^* denotes the nominal foreign interest rate between periods t and $t+1$ faced by domestic banks in international financial market.

Representative domestic banks solve the maximization problem

$$\max_{B_t^*, B_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} D_t^b,$$

subject to the flow-of-funds constraint

$$D_t^b = \frac{B_t^*}{R_{ft}^*} e_t - B_{t-1}^* e_t + \frac{B_t}{R_{ft}} - B_{t-1}, \tag{15}$$

where B_t^* denotes the nominal foreign bonds and B_t denotes the aggregate domestic bonds issued by firms, $B_t \equiv \int_0^1 B_{jt} dj$. $B_t^* > 0$ (≤ 0) indicates that domestic banks borrow from (lends to) foreigners. At period t, banks issue foreign bonds B_t^* at price $1/R_{ft}^*$ and converts into domestic bonds $B_t^* e_t / R_{ft}^*$ before lending funds to firms. The bank obtains the dividends after repaying the foreign bonds $B_{t-1}^* e_t$, lending to firms $\frac{B_t}{R_{ft}}$, and obtaining the repayment $-B_{t-1}$ from firms (note that $B_t < 0$ implies borrowing).

The first-order conditions are

$$\frac{e_t}{R_{ft}^*} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} e_{t+1},$$

$$\frac{1}{R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}.$$

From the above equations, we can get the uncovered interest parity (UIP) equation

$$\frac{R_{ft}}{R^*_{ft}} = E_t \frac{e_{t+1}}{e_t}. \tag{16}$$

In log-lineralized form, it is

$$\log\left(\frac{R_{ft}}{R_f}\right) - \log\left(\frac{R^*_{ft}}{R^*_f}\right) - \log\left(\frac{\pi_{e,t+1}}{\pi_e}\right) = 0.$$

Following Itskhoki and Mukhin (2021), we additionally introduce a UIP shock ω_t, in log-lineralized form,

$$\log\left(\frac{R_{ft}}{R_f}\right) - \log\left(\frac{R^*_{ft}}{R^*_f}\right) - \log\left(\frac{\pi_{e,t+1}}{\pi_e}\right) = \log\left(\omega_t\right).$$

It follows that the (uncovered) interest rate parity deviates from zero by the magnitude of the financial shock ω_t, which may have a number of origins explored in the macro-finance literature (see Cochrane (2017)). ω_t is exogenous and follows

$$\ln \omega_t = \rho_\omega \ln \omega_{t-1} + \varepsilon_{\omega,t},$$

where $\rho_\omega \in (0,1)$, and $\varepsilon_{\omega,t}$ is an i.i.d. standard normal random variable.

Furthermore, following Schmitt-Grohé and Uribe (2003), we assume that the international borrowing rate faced by domestic banks, R^*_{ft}, satisfies

$$R^*_{ft} = R^*_t + g\left(\frac{e_t B^*_t}{P_{zt} Z_t}\right), \tag{17}$$

where R^*_t denotes the global interest rate and $g(\cdot)$ is a country-specific interest rate premium, which is an increasing function of $\frac{e_t B^*_t}{P_{zt} Z_t}$, the ratio of external debt to intermediate good firms' aggregate output in domestic currency (abbreviated as external debt-output ratio),

$$g\left(\frac{e_t B^*_t}{P_{zt} Z_t}\right) = \Omega\left(\exp\left(\frac{e_t B^*_t}{P_{zt} Z_t} - \underline{B}^*\right) - 1\right),$$

where $\Omega > 1$, and $\underline{B}^*$ is the target level of the ratio of external debt to nominal output. The country premium is an increasing and convex function of deviations of actual debt-output ratio from the target).

Assume the global interest rate R_t^* is exogenous and satisfies

$$\ln R_t^* = (1-\rho_{R^*})\ln R^* + \rho_{R^*}\ln R_{t-1}^* + \varepsilon_{R^*t},$$

where $\rho_{R^*} \in (0,1)$, and ε_{R^*t} is an i.i.d. standard normal random variable.

2.5 Retailers

There is a continuum of retailers in the domestic market, indexed by $r \in [0,1]$. Retailers buy goods from intermediate good firms at price P_{zt}, package them, and sell them to domestic agents at nominal price P_{rt}. Each retailer acts as a monopolist.

The domestic final good, used for consumption and investment, is a constant elasticity of substitution (CES) aggregator of the retailers' output, denoted as Y_{rt}:

$$Y_t = \left[\int_0^1 (Y_{rt})^{\frac{\psi-1}{\psi}}\, dr\right]^{\frac{\psi}{\psi-1}},$$

where $\psi > 1$. Each retailer faces the demand function

$$Y_{rt} = Y_t \left[\frac{P_{rt}}{P_t}\right]^{-\psi}.$$

Following Calvo (1983), we assume each retailer can change their price optimally in period t with probability $1-\chi$. A retailer then chooses $\widetilde{P}_{rt}$ optimally to solve

$$\max_{\widetilde{P}_{rt}} E_t \sum_{i=0}^{\infty} \chi^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t}\left[\widetilde{P}_{rt} - P_{z,t+i}\right] Y_{r,t+i}, \tag{18}$$

The law of motion for the aggregate price level is

$$P_t = \left[\int_0^1 P_{rt}^{1-\psi} dr\right]^{\frac{1}{1-\psi}} = \left[(1-\chi)\left(\widetilde{P}_{rt}\right)^{1-\psi} + \chi\left(P_{t-1}\right)^{1-\psi}\right]^{\frac{1}{1-\psi}}, \tag{19}$$

and the optimal domestic price is

$$\widetilde{P}_{rt} = \frac{\psi}{\psi-1}\frac{E_t \sum_{i=0}^{\infty} \chi^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i} P_{z,t+i} P_{t+i}^{\psi}}{E_t \sum_{i=0}^{\infty} \chi^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i} P_{t+i}^{\psi}}.$$

Similarly, there is a continuum of retailers in the export market, indexed by $r^* \in [0,1]$. Retailers buy goods from intermediate good firms at price P_{zt}, package them, and sell them to foreign market at nominal price P_{rt}^* in foreign currency.

Assume the foreign market is large enough, so the home country takes the foreign demand function as given

$$Y_{rt}^{*} = Y_{t}^{*} \left(P_{rt}^{*}\right)^{-\psi^{*}}, \tag{20}$$

where $\psi^{*} > 1$, and Y_t^{*} denotes the foreign demand shock, which follows an AR(1) process

$$\ln Y_t^{*} = \rho_{Y^{*}} \ln Y_{t-1}^{*} + \varepsilon_{Y^{*}t},$$

where $\rho_{Y^{*}} \in (0,1)$, and $\varepsilon_{Y^{*}t}$ is an i.i.d. standard normal random variable.

Following Calvo (1983), we assume each retailer can change their price optimally in period t with probability $1-\chi^{*}$. A retailer then chooses $\widetilde{P}_{rt}^{*}$ optimally to solve

$$\max_{\widetilde{P}_{rt}^{*}} E_t \sum_{i=0}^{\infty} \chi^{*i} \beta^{i} \frac{\Lambda_{t+i}}{\Lambda_t} \left[e_{t+i} \widetilde{P}_{rt}^{*} - P_{z,t+i} \right] Y_{r,t+i}^{*}. \tag{21}$$

The optimal export price is

$$\widetilde{P}_{rt}^{*} = \frac{\psi^{*}}{\psi^{*}-1} \frac{E_t \sum_{i=0}^{\infty} \chi^{*i} \beta^{i} \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i}^{*} P_{z,t+i}}{E_t \sum_{i=0}^{\infty} \chi^{*i} \beta^{i} \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i}^{*} e_{t+i}}.$$

2.6 Central Bank

We focus on the traditional monetary policy rule. The central bank sets the nominal interest rate in response to the current inflation, output, and asset prices

$$\ln R_{ft} = \ln R_f + \theta_{\Pi} \ln \frac{\Pi_t}{\Pi} + \theta_z \ln \frac{Z_t}{Z} + \theta_p \ln \frac{P_{ht}}{P_h} + \ln \zeta_t. \tag{22}$$

ζ_t is an exogenous monetary policy shock that follows an AR(1) process

$$\ln \zeta_t = \rho_{\zeta} \ln \zeta_{t-1} + \varepsilon_{\zeta t},$$

where $\rho_{\zeta} \in (0,1)$, and $\varepsilon_{\zeta t}$ is an i.i.d. standard normal random variable. In baseline, we set $\theta_p = 0$, and the above rule is called the Taylor rule.

2.7 Competitive Equilibrium

Denote $M_t \equiv \int_0^1 M_{jt} dj$, $K_t \equiv \int_0^1 K_{jt} dj$ and $Z_t \equiv \int_0^1 Z_{jt} dj$. A competitive equilibrium consists of sequences of aggregate quantities $\{C_t, K_t, I_t, Y_t, B_t, B_t^{*}, H_t, M_t, X_t, N_t\}$, shareholdings $\{\psi_{j,t+1}\}$, intermediate goods $\{Y_{jt}\}$, and prices $\{W_t, P_t, P_t^{*}, P_{zt}, P_{kt}, P_{ht}, R_{kt}, R_{ft}, R_{ft}^{*}, e_t\}$ such that:

(i) Households, firms, capital goods producers, domestic banks and retailers optimize.

(ii) The markets for labor, the bubble asset, domestic capital goods, bonds, stocks, domestic consumption goods and domestic intermediate goods all clear so that

$$N_t = \int_0^1 N_{jt} dj, H_t = \int_0^1 H_{jt} dj = 1, I_t = \int_0^1 I_{jt} dj,$$

$$\frac{B_t^* e_t}{R_{ft}^*} + \frac{B_t}{R_{ft}} = 0, \psi_{j,t+1} = 1,$$

$$Y_t = C_t + \left[1 + \frac{\Omega_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t,$$

$$\begin{aligned} Z_t &\equiv \int_0^1 Z_{jt} dj = \int_0^1 Y_{rt} dr + \int_0^1 Y_{rt}^* dr^* \\ &= Y_t * \int_0^1 \left[\frac{P_{rt}}{P_t} \right]^{-\psi} dr + \int_0^1 Y_{rt}^* dr^*. \end{aligned}$$

Define $V_{rt} \equiv \int_0^1 [P_{rt}^*]^{-\psi^*} dr^*$, $V_{yt} \equiv \int_0^1 \left[\frac{P_{rt}}{P_t} \right]^{-\psi} dr$, and

$$X_t \equiv \int_0^1 Y_{rt}^* dr^* = Y_t^* * \int_0^1 [P_{rt}^*]^{-\psi^*} dr^* = Y_t^* V_{rt}, \tag{23}$$

then

$$Z_t = Y_t V_{yt} + X_t. \tag{24}$$

(iii) The law of motion of aggregate capital follows

$$K_t = (1 - \delta) K_{t-1} + \int_0^1 \varepsilon_{jt} I_{jt} dj.$$

The full characterization of the equilibrium system is shown in Appendix (B).

3 Steady-state Analysis

At first, we summarize some common key equations that are helpful for solving the steady state and deriving the existence conditions of bubbles.

In steady state, from the equation of asset pricing of bonds (12), the equilibrium domestic interest rate is a function of the cutoff $\bar{\varepsilon}$:

$$R_f(\bar{\varepsilon}) = \frac{1}{\beta \left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) dF(\varepsilon) \right]}, \tag{25}$$

and $\frac{\partial R_f(\bar{\varepsilon})}{\partial \bar{\varepsilon}} > 0$.

Then we use the equation of asset pricing of capital (11) to derive the steady state real rental rate of capital, which is a function of the cutoff $\bar{\varepsilon}$ as well:

$$\frac{R_k}{P_k}(\bar{\varepsilon}) = \frac{[1-\beta(1-\delta)] - \beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)}{\beta\left(\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right)}. \tag{26}$$

Lemma 1 *When $\mu > 0$ is sufficiently small, $\frac{\partial \frac{R_k}{P_k}(\bar{\varepsilon})}{\partial \bar{\varepsilon}} < 0$.*

Proof.

Suppose μ is sufficiently small in the rest of paper and this assumption ensures the unique solution of bubbleless equilibrium.

By the first-order condition of domestic banks (16) and domestic interest rate condition (17), we can derive the steasy-state debt-to-output ratio as

$$\frac{eB^*}{P_z Z}(\bar{\varepsilon}) = \ln\left(\frac{R_f(\bar{\varepsilon}) - R^*}{\Omega} + 1\right) + \underline{B}^*. \tag{27}$$

Then $\frac{eB^*}{P_z Z}$ is a function of $\bar{\varepsilon}$ and $\frac{\partial \frac{eB^*}{P_z Z}(\bar{\varepsilon})}{\partial \bar{\varepsilon}} = \frac{1}{\frac{R_f - R^*}{\Omega}+1}\frac{1}{\Omega}\frac{\partial R_f(\bar{\varepsilon})}{\partial \bar{\varepsilon}} > 0$.

Suppose the economy aims to hold balance of payment ($\underline{B}^* = 0$), when $R_f(\bar{\varepsilon})$ is higher (lower) than the global interest rate R^*, $\frac{eB^*}{P_z Z}$ is positive (negative), which means that there is net capital inflow (outflow) or capital account deficit (surplus). When $R_f(\bar{\varepsilon})$ decreases, saving abroad is more attractive, so capital outflow increases. Moreover, if the economy sets the target level of debt-to-output ratio, there is a floor level of global interest rate, $\underline{R}^*$. The country faces capital account deficit if $R_f(\bar{\varepsilon})$ is lower than $\underline{R}^*$.

We focus on two equilibria, one is the bubbleless (fundamental) equilibrium where $P_h = 0$, and the other is the bubbly equilibrium where $P_h > 0$. We use a subscript f (b) to denote a variable in a bubbleless (bubbly) equilibrium.

3.1 Bubbleless Equilibrium

The following assumption is required to ensure the existence of a bubbleless equilibrium.

Assumption 1 *1. Assume that*

$$\delta < \left[\frac{R_k}{P_k}(\varepsilon_{\min})\left(1 - \frac{1}{\alpha}\frac{eB^*}{P_z Z}(\varepsilon_{\min})\right) + \mu\right]E(\varepsilon),$$

where $\frac{R_k}{P_k}$ is given by (26) and $\frac{eB^*}{P_z Z}$ is given by (27).

We summarize the bubbleless equilibrium in the following proposition.

Proposition 2 *(bubbleless equilibrium) Suppose Assumption 1 hold, then there is a unique solution $\bar{\varepsilon}_f$ for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$ to the equation*

$$\delta = \left[\frac{R_k}{P_k}(\bar{\varepsilon})\left(1 - \frac{1}{\alpha}\frac{eB^*}{P_z Z}(\bar{\varepsilon})\right) + \mu\right]\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \tag{28}$$

Moreover, if

$$\begin{aligned} & 1 - \frac{\psi - 1}{\psi}\left[\alpha + \mu\alpha\left(\frac{R_k}{P_k}(\bar{\varepsilon}_f)\right)^{-1} - \frac{eB^*}{P_z Z}(\bar{\varepsilon}_f)\right]\left[1 - F(\bar{\varepsilon}_f)\right] \\ > \; & \frac{\psi^* - 1}{\psi^*}\left[\frac{eB^*}{P_z Z}(\bar{\varepsilon}_f)\left(1 - \frac{1}{R_f(\bar{\varepsilon}_f)}\right) + \gamma\right] > 0, \end{aligned} \tag{29}$$

then there is a unique bubbleless equilibrium.

Proof. See Appendix C.1.

A sufficiently small μ ensures that $\frac{R_k}{P_k}(\bar{\varepsilon})$ decreases in $\bar{\varepsilon}$, which allows us to use the intermediate value theorem to derive the unique solution of $\bar{\varepsilon}$ under the Assumption (1).[2] The steady-state value of other variables can be determined after getting $\bar{\varepsilon}$. In bubbleless equilibrium, since the export demand is exogenous and given by Equation (23), exports are positive. What's more, the consumption needs to be positive as well. Thus we derive the condition (29).

3.2 Bubbly Equilibrium

We summarize the bubbly equilibrium in the following proposition.

Proposition 3 *(bubbly equilibrium) Suppose there exists a bubbleless equilibrium with the investment cutoff $\bar{\varepsilon}_f$, which is characterized by Proposition 2. If there exists a bubbly equilibrium, then the following condition holds*

$$1 < \beta\left[1 + \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1\right)dF(\varepsilon)\right]. \tag{30}$$

Otherwise, if the above condition holds, the equation

$$1 = \beta\left[1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\left(\frac{\varepsilon}{\bar{\varepsilon}} - 1\right)dF(\varepsilon)\right]$$

[2] See Appendix A.3 for the proof.

has a unique solution $\bar{\varepsilon}_b$ for $\bar{\varepsilon} \in (\varepsilon_{\min}, \varepsilon_{\max})$. Moreover, if

$$1 - \frac{\psi - 1}{\psi}\left[\alpha + \mu\alpha\left(\frac{R_k}{P_k}(\bar{\varepsilon}_b)\right)^{-1} - \frac{eB^*}{P_z Z}(\bar{\varepsilon}_b) + \frac{P_h}{P_{zb} Z_b}\right]\left[1 - F(\bar{\varepsilon}_b)\right] \tag{31}$$
$$> \frac{\psi^* - 1}{\psi^*}\left[\frac{eB^*}{P_z Z}(\bar{\varepsilon}_b)\left(1 - \frac{1}{R_f(\bar{\varepsilon}_b)}\right) + \gamma\right] > 0,$$

where

$$\frac{P_h}{P_{zb} Z_b} = \left(\frac{R_k}{P_k}(\bar{\varepsilon}_b)\right)^{-1}\left[\frac{\delta\alpha}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - \mu\alpha\right] - \alpha + \frac{eB^*}{P_z Z}(\bar{\varepsilon}_b),$$

then there exists a unique bubbly equilibrium.

Proof. See Appendix C.2.

The key condition for a bubble existing is condition (30), which indicates that the marginal benefit of purchasing bubble assets (the right-hand side of the condition) should exceed the marginal cost (the left-hand side) so that the firm is willing to hold the bubble asset. The intuition of condition (31) is similar to the condition (29) in the bubbleless equilibrium.

It is worth noting that the global interest rate R^* affects $\bar{\varepsilon}_b$ through condition (30) by influencing $\bar{\varepsilon}_f$. We summarize the results in the following proposition.

Proposition 4 *The smaller the global interest rate R^* is, the more likely the bubble existence condition (30) holds, and thus the more likely a domestic bubble can emerge. In bubbly equilibrium, the investment cutoff $\bar{\varepsilon}_b$, the domestic interest rate R_{fb}, and the real capital rental rate $\frac{R_{kb}}{P_{kb}}$ are independent on R^* while the bubble-to-output ratio $\frac{P_h}{P_{zb} Z_b}$ decreases in R^*.*

Proof. See Appendix A.2.

Intuitively, as R^* declines, there are capital inflows accompanied by real exchange rate appreciation, lending booms, and investment booms. The increased demand for domestic bonds reduces the domestic interest rate, thereby fueling a bubble. On the contrary, when R^* is sufficiently high, firms have sufficiently liquidity by saving abroad to finance investment. Therefore, they have no demand for the bubble asset, and a bubble cannot emerge.

In addition, we can easily show that $\bar{\varepsilon}_b > \bar{\varepsilon}_f$, that is, the bubble would crowd out the investment in the extensive margin. In the intensive margin, the bubble raises the net worth of the firm holding bubble assets, crowding in more investment. Overall, the net effect of bubble on investment depend on which effects above dominates. Further, by (25), it's easy to show that $R_{fb} > R_{ff}$ because the nominal interest rate is an increasing function of $\bar{\varepsilon}$. The reason is that bubble assets also crowd out the demand for bonds, leading to lower bond prices and higher interest rate.

4 Quantitative Analysis

We focus on small open economy and regard foreign economy as exogenous, so we take foreign price as normalization to ensure the dynamic responses to shocks are based on the same steady-states. Then we normalize the domestic nominal prices and nominal debt by the nominal exchange rate to get the real prices and debt based in foreign currency. We use a lower case variable $x_t \equiv \frac{X_t}{e_t}$, and $\lambda_t \equiv \Lambda_t e_t$ to denote the real variables. For example, $p_{ht} \equiv \frac{P_{ht}}{e_t}$ means the price of bubble asset in foreign currency. We denote $\pi_t \equiv \frac{p_t}{p_{t-1}}$, and $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$ as the inflation of domestic price level and depreciation of domestic currency, respectively.

4.1 Calibration

The parameters are either calibrated or taken from the traditional literature. We assume the firm-specific investment efficiency shock follows a Pareto distribution with the CDF $F(\varepsilon) = 1 - \left(\frac{\varepsilon}{\varepsilon_{\min}}\right)^{-\eta}$ and set $\eta = 8$. We set $\varepsilon_{\min} = 1 - 1/\eta$ so that $E(\varepsilon) = 1$. We set the share of capital and foreign inputs $\alpha = 0.33$, $\gamma = 0.22$, the subjective discount factor $\beta = 0.99$, and depreciation rate $\delta = 0.05$, which are standard in the business cycle literature. For the utility weight on labor, we choose $\kappa = 16$ so that the bubbly steady-state number of hours worked equals $1/3$. We set the pledgeability parameter of capital value $\mu = 0.3$, the habit formation parameter $h = 0.53$, the adjustment cost of capital $\Omega_k = 0.09$, and debt-elastic interest rate parameter $\Omega = 0.13$, which are consistent with the estimates based on Mexican data in Dong et al. (2020). For price stickiness, we set $\chi = \chi^* = 0.75$ so that the duration of price adjustments is 4 quarters, and $\psi = \psi^* = 5$, implying that the markup in steady state is $\psi/(\psi - 1) = 1.25$. We set $\varphi = 2$ so that the Frisch elasticity is $1/2$. For simplicity, we set $\underline{B}^* = 0$ so that the economy aims to hold balance of payment.

For the parameter values of the interest rate rules, we follow the DNK literature (e.g., Bernanke et al. (1999), Gilchrist and Leahy (2002), Galí (2015) and Dong et al. (2020)) and set $\theta_\Pi = 1.5$, $\theta_Z = 0.125$ in the Taylor rule (22). We set the persistence parameters for the shocks as 0.9 except for monetary policy shock as 0.5. We set $R^* = 0.95$ so that in bubbly steady state, home country borrows from abroad.[3]

[3]In bubbly steady-state, $R_{fb} = 1$. As most small open economies are debtor countries, we set $R^* < R_{fb}$. Moreover, we can introduce growth to make $R_{fb} > 1$ such that $0 < R^* < R_{fb}$, which is more consistent with data while the main results in this paper do not change.

4.2 Impulse Responses

4.2.1 Technology shock

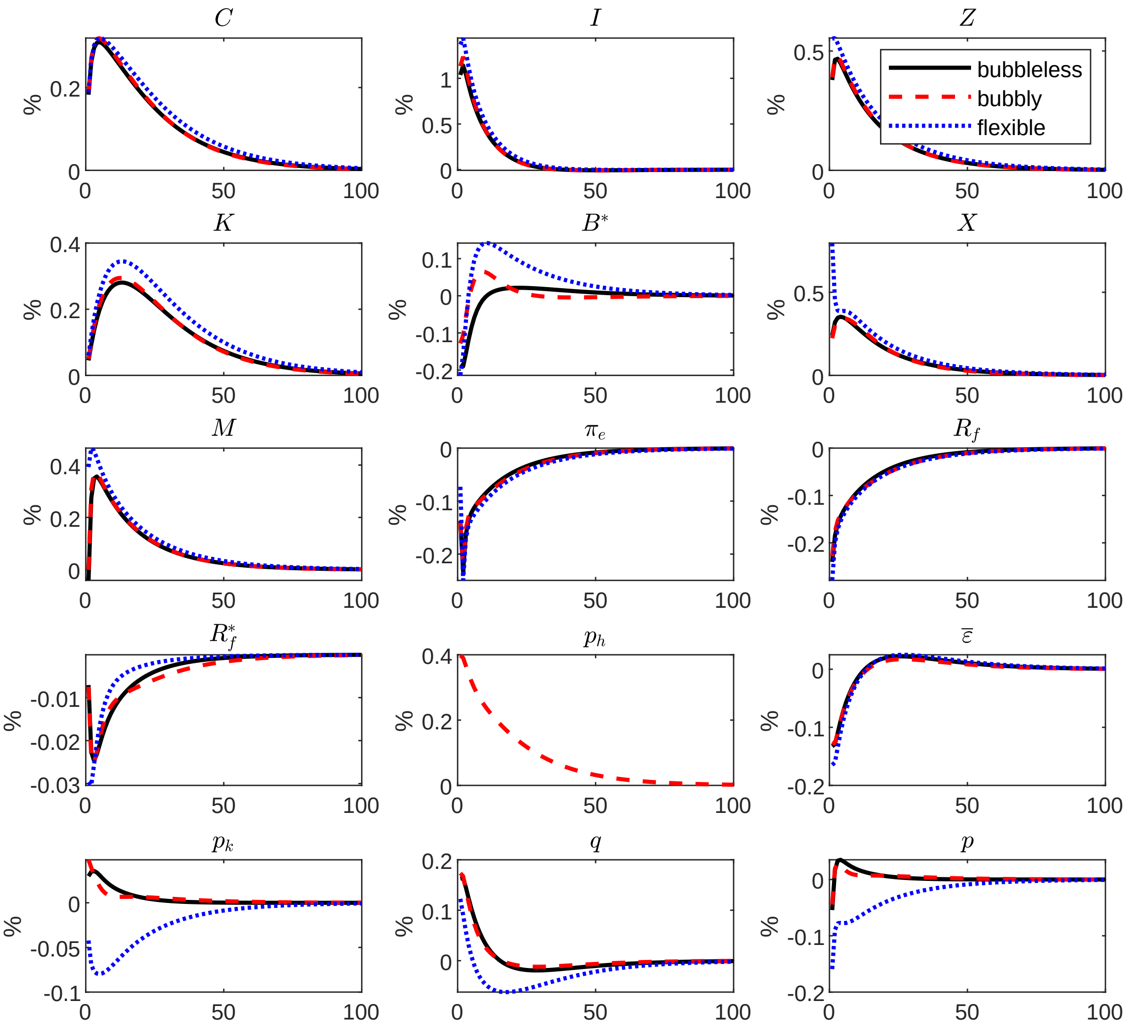
We start by analyzing the impacts of traditional technology shock. Figure 2 shows the impulse responses to a positive technology shock that raises ε_{At} by 1%. Here, $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. Note that the lower case variable x_t are expressed in foreign currency. We first use the bubbleless economy with flexible prices as the benchmark to illustrate the impact of exogenous technological shocks on the economy (the dotted lines in the figure). The increase in productivity reduces the marginal cost of domestic intermediate goods p_{zt}, and because all retailers can adjust their prices freely, the price of final goods p_t falls according to equation (19), which greatly stimulates domestic demand and exports by equations (2) and (20). The substantial increase in demand for domestic goods leads to an appreciation of the domestic currency, which boosts imports. The central bank needs to cut interest rates facing falling inflation. Moreover, a positive technological shock increases the marginal product of capital r_{kt}, and the consequent increase in the Tobin's Q q_t lowers the firm's investment threshold $\bar{\varepsilon}_t$. More firms invest increases the demand for domestic bonds, increasing the price of domestic bonds and reducing the domestic interest rate R_{ft}. The rise in consumption, investment, and exports leads to a substantial increase in total output, which in turn reduces the external debt-to-output ratio $\frac{B_t^*}{p_{zt}Z_t}$ and the debt-elastic interest rate R_{ft}^* faced by domestic banks. However, the decline of debt-elastic interest rate is far lower than the decline in domestic interest rate, so the opportunity cost of borrowing in the international market rises and external debt B_t^* falls. After introducing price stickiness (the solid lines in the figure), some retailers cannot adjust their prices, so the price of final goods is relatively higher, and the changes in consumption, investment, and total output are weak.

Next, we analyze the role of bubbles. In a bubbly economy, since bubble assets can serve as a store of value, they increase the firm's net worth and allow firms to invest more. The rising investment demand from positive technological shocks further stimulates demand for bubble assets, and bubble prices rise sharply. This further raises the net worth of firms, stimulates their consumption and investment. This generates an amplification effect of bubbles and leads to higher consumption, investment, capital, and output in a bubbly economy (the dashed lines in the figure).

4.2.2 Negative foreign interest rate shock

In a small open economy, the impacts of external shocks on domestic economic fluctuations is particularly essential. We consider a 1% negative global interest rate shock that reduces ε_{R^*t}

Figure 2: Impulse responses to a positive 1% technology shock



Note: We raise ε_{At} by 1%. The solid lines describe the bubbleless economy. The dashed lines describe the bubbly economy. The dotted lines describe the bubbleless economy with flexible prices (set $\chi = \chi^* = 0$). $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. $p_{ht} \equiv \frac{P_{ht}}{e_t}$ is the normalized asset price which means the price of bubble asset in foreign currency. All variables are expressed as percentage deviations from their nonstochastic steady state values.

by 1%, and the results are shown in Figure 3. In the bubbleless economy, when the global interest rate is suddenly lowered by 1%, according to the equation (17), the interest rate faced by domestic banks in cross-border borrowings R^*_{ft} drops directly, leading to an increase in the domestic foreign debt B^*_t. Such capital inflows stimulate domestic banks' demand for domestic bonds, which in turn reduces the domestic interest rate R_{ft}. Therefore, domestic demand expands, leading to an increase in consumption and investment. Due to the appreciation caused by capital inflows, exports fall and imports rise.[4] At the same time, a large inflow of foreign capital has greatly increased the demand for domestic goods, making the prices of domestic final goods p_t rise. Without sticky prices, domestic interest rate falls more, and consumption, investment, and output rise more.

In a bubbly economy, the negative global interest rate shock leads to capital inflows and lower domestic interest rates, which also implies lower bond yields, leading to higher demand for bubble assets and higher bubble prices. The net worth of firms holding bubble assets raises, further stimulating demand for domestic goods, and interest rates fall further. Such amplification effect brings about greater asset bubbles and more consumption and investment.

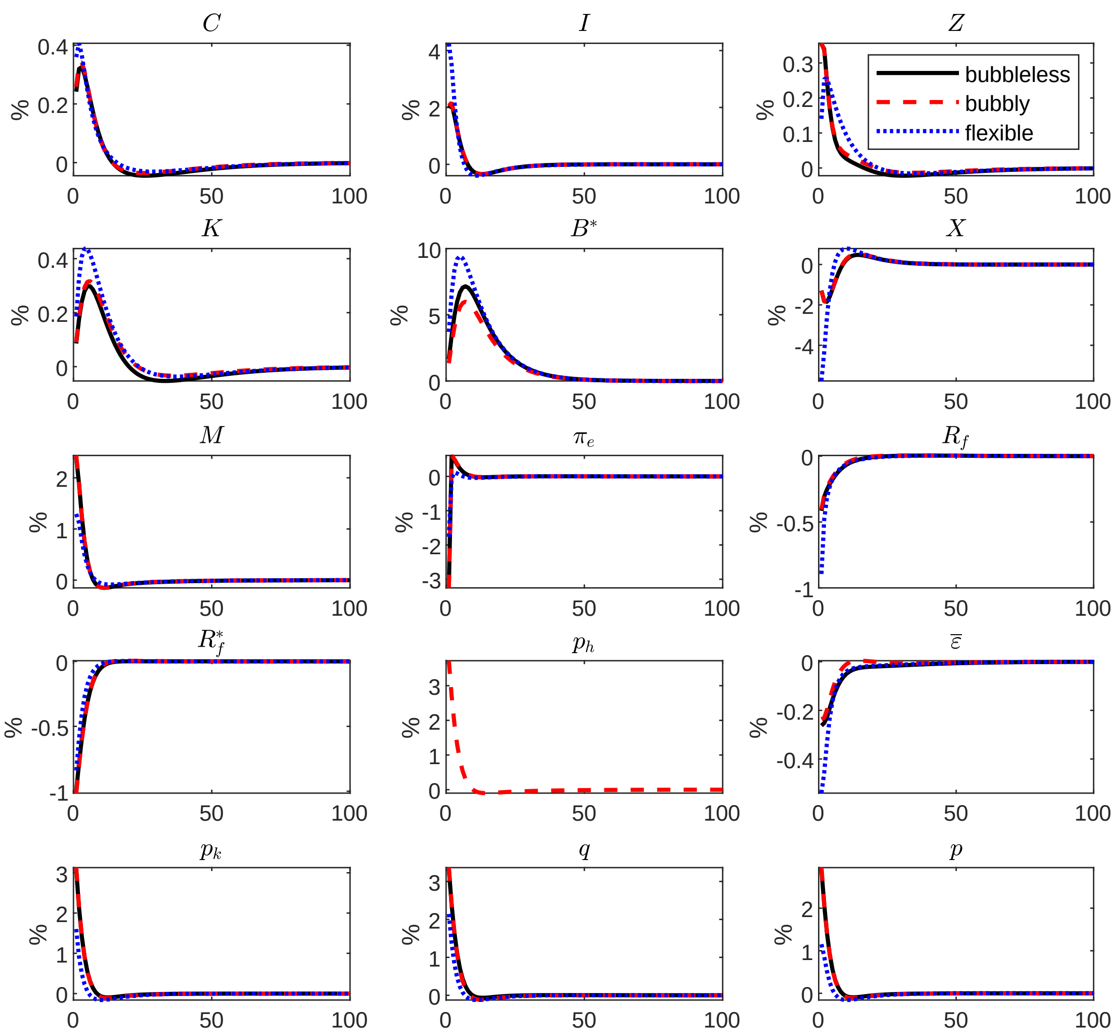
4.2.3 Negative Monetary Policy Shock

Next, we study the impacts of an expansionary monetary policy shock. Figure 4 is the impulse responses of a 1% negative monetary policy shock that reduces $\varepsilon_{\zeta t}$ by 1%. Since monetary policy is ineffective under flexible prices, we only show results without (solid lines) and with bubbles (dashed lines). After a negative monetary policy shock, the domestic nominal interest rate R_{ft} falls. The transmission mechanism of monetary policy is through the interest rate channel. Because of stick prices, the real interest rate decreases accordingly. On the one hand, the cost of borrowing for firms reduces, and more investment can be made through bonds. On the other hand, the return of households holding firm's equity decreases with the decline of interest rates. Therefore, consumption and investment increase, which further raises the price of intermediate goods p_{zt}, resulting in a decrease in exports. In addition, since the production of domestic intermediates requires foreign inputs, increased demand for domestic goods boosts imports. Although under the uncovered interest rate parity, the decline in domestic interest rates leads to a depreciation of the exchange rate, but the impact of depreciation on imports and exports is dominated by the increase in domestic demand, resulting in a decline in exports and an increase in imports. Facing a growing supply of domestic bonds, the domestic banks borrow more from abroad with higher debt-elastic interest rate.

Although both negative global interest rate shocks and negative domestic interest rate shocks lead to a decline in domestic interest rates, these two shocks have different effects on domes-

[4] While the appreciation of the local currency would depress exports, rising demand for domestic goods dominates the effect of appreciation.

Figure 3: Impulse responses to a negative 1% shock to the global interest rate



Note: We reduce ε_{R^*t} by 1%. The solid lines describe the bubbleless economy. The dashed lines describe the bubbly economy. The dotted lines describe the bubbleless economy with flexible prices (set $\chi = \chi^* = 0$). $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. $p_{ht} \equiv \frac{P_{ht}}{e_t}$ is the normalized asset price which means the price of bubble asset in foreign currency. All variables are expressed as percentage deviations from their nonstochastic steady state values.

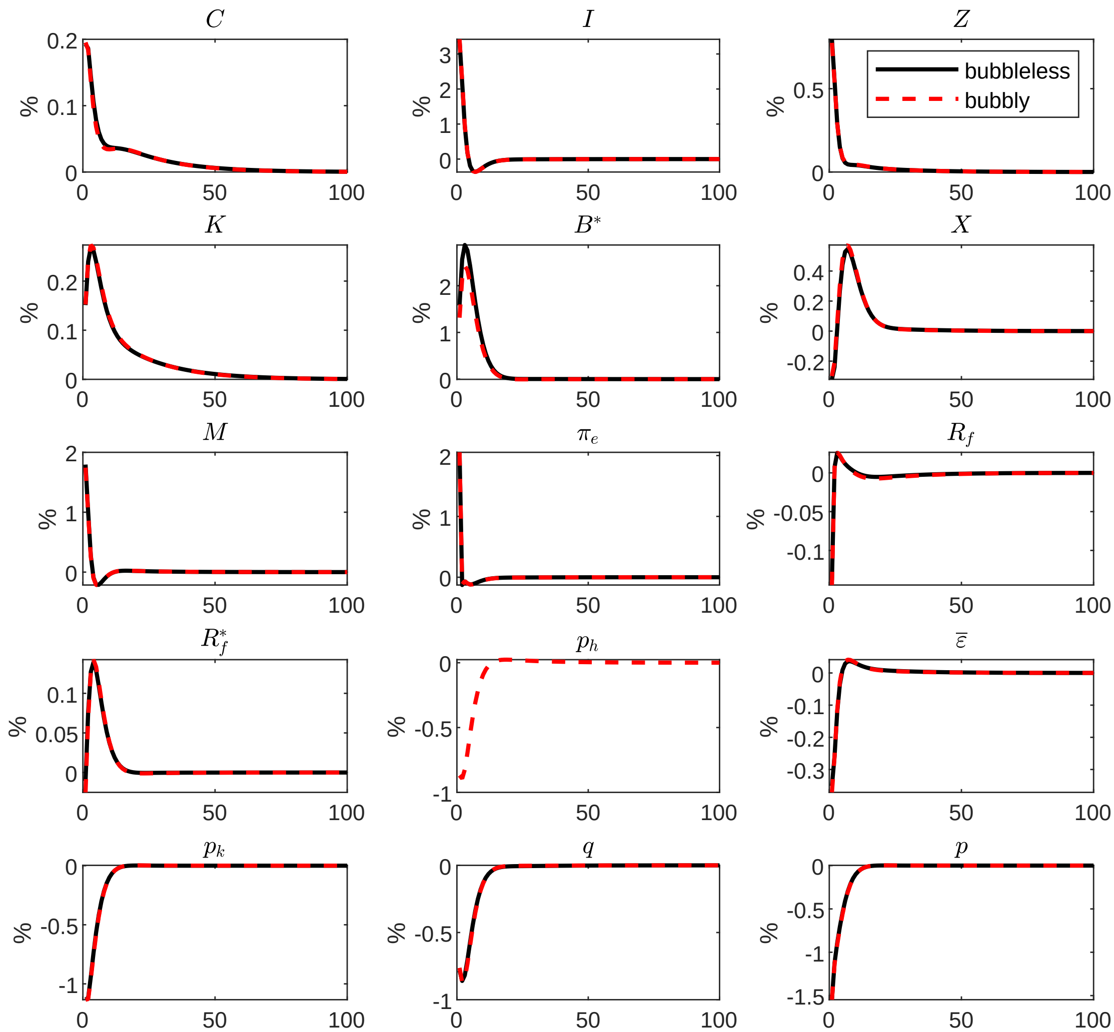
tic final goods prices and Tobin's Q, resulting in different responses of asset prices. After an expansionary monetary policy shock, the decline in the domestic interest rate stimulated the domestic economy substantially. Therefore, the supply of domestic goods exceeds the demand, and the prices of final goods in the domestic market p_t drop sharply, lowering the prices of capital goods p_{kt}. For firms, the sharp drop in investment costs lowers the firm's investment efficiency cutoff $\bar{\varepsilon}_t$. Although this stimulates investment in the extensive margin, the decline in Tobin's Q q_t weakens the investment demand of individual firms in the intensive margin. On the one hand, the dropped interest rate lowers the cost of borrowing, which in turn reduces firms' demand for bubble assets. On the other hand, the lowering of the investment efficiency cutoff makes more firms sell bubble assets to finance investment. Eventually, the asset prices fall.

4.3 Policy Implications

We further study whether monetary policy should respond to asset prices. In the monetary policy rule (22), we take $\theta_p = 0$ as the benchmark, and here we set $\theta_p = 0.1$ ($\theta_p = -0.1$) to analyze the impacts of central bank raising (reducing) nominal interest rate facing higher asset prices. Figures 5 and 6 show the impulse responses under a positive technology shock and a negative global interest rate shock, respectively. Under the technology shock, compared to the baseline, if the central bank raises interest rates in terms of rising asset prices ($\theta_p = 0.1$, the "lean-against-the-wind" policy, the dotted lines in the figure), anticipating an increase in the relative cost of investing through borrowing, the firm is willing to hold more bubble assets, which promote asset prices to rise. This enhances the crowding-in effect of the bubble. The net worth of firms holding bubble assets is higher than the baseline and domestic demand for domestic goods rises more, leading to a higher price of final goods p_t. This in turn raises the price of capital goods p_{kt} and the investment efficiency cutoff $\bar{\varepsilon}_t$, leading to less investment and output. Therefore, facing rising asset prices, the central bank raises interest rates would boost asset price bubbles.

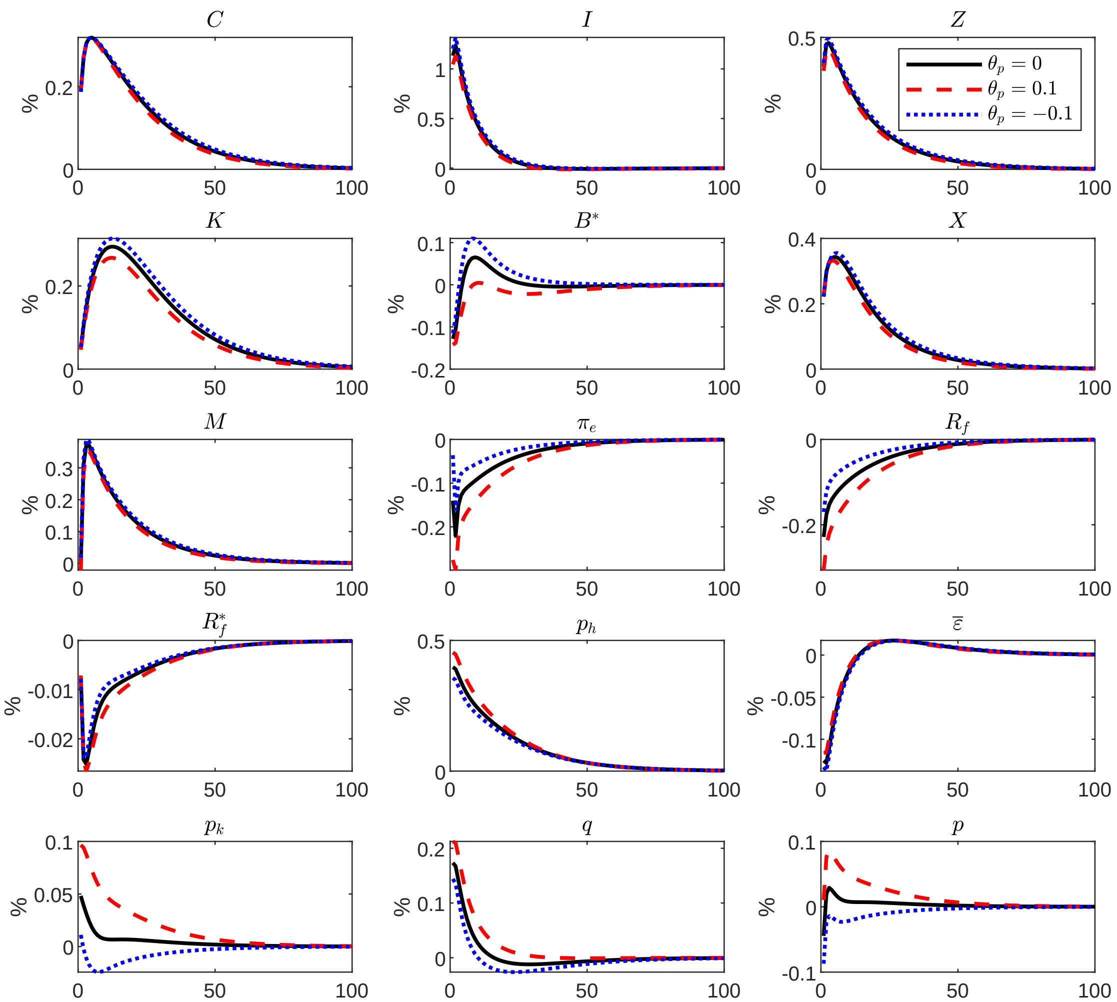
Under a negative global interest rate shock, the central bank raises interest rate when asset prices rise is more effective than under a positive technology shock in stabilizing output. This is because the asset prices rise more when global interest rate reduces 1% (4% > 0.5%). Anticipating higher interest rates in response to asset prices ($\theta_p = 0.1$, the dotted lines in the figure), firms desire to hold more bubble assets to provide liquidity. In this case, the capital flowing into the domestic country is not used for production as the asset bubbles crowd out investment substantially. Therefore, the "lean-against-the-wind" policy can stabilize the economy at the expense of rising asset prices.

Figure 4: Impulse responses to a negative 1% shock to the nominal domestic interest rate



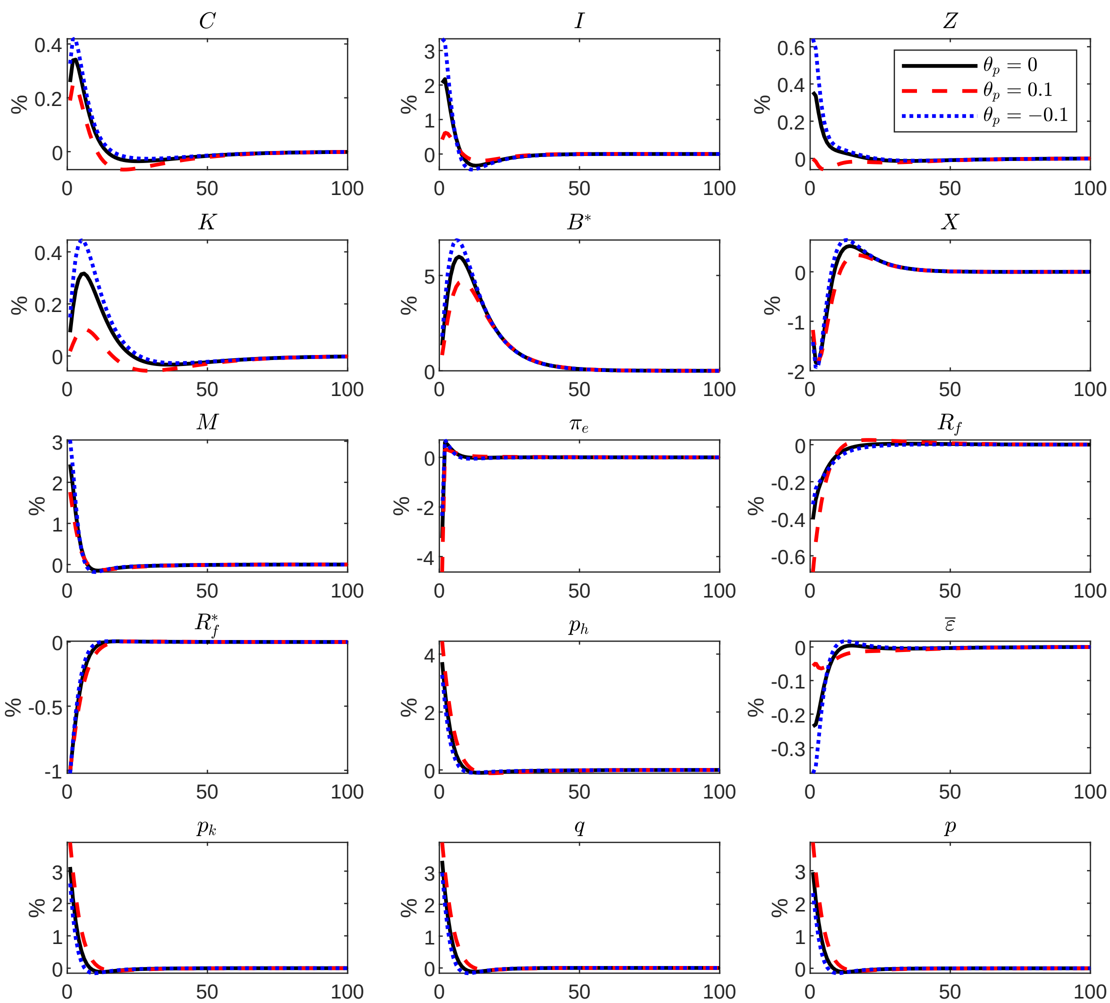
Note: We reduce $\varepsilon_{\zeta t}$ by 1%. The solid lines describe the bubbleless economy. The dashed lines describe the bubbly economy. Since monetary policy is ineffective under flexible prices, we only show results without and with bubbles. $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. $p_{ht} \equiv \frac{P_{ht}}{e_t}$ is the normalized asset price which means the price of bubble asset in foreign currency. All variables are expressed as percentage deviations from their nonstochastic steady state values.

Figure 5: Policy responses to a positive 1% technology shock



Note: We raise ε_{At} by 1%. The solid lines describes the baseline that we set $\theta_p = 0$ in the monetary policy rule (22). The dashed and dotted lines represent responses under the monetary policy rule with $\theta_p = 0.1$ and $\theta_p = -0.1$, respectively. $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. $p_{ht} \equiv \frac{P_{ht}}{e_t}$ is the normalized asset price which means the price of bubble asset in foreign currency. All variables are expressed as percentage deviations from their nonstochastic steady state values.

Figure 6: Policy responses to a negative 1% global interest rate shock



Note: We reduce ε_{R^*t} by 1%. The solid lines describes the baseline that we set $\theta_p = 0$ in the monetary policy rule (22). The dashed and dotted lines represent responses under the monetary policy rule with $\theta_p = 0.1$ and $\theta_p = -0.1$, respectively. $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$, indicates the degree of exchange rate depreciation, and an increase in π_{et} means a depreciation of the local currency. $p_{ht} \equiv \frac{P_{ht}}{e_t}$ is the normalized asset price which means the price of bubble asset in foreign currency. All variables are expressed as percentage deviations from their nonstochastic steady state values.

5 Conclusion

We present a theoretical framework to examine the role of asset bubbles in setting monetary policy in open economies. The framework features an intrinsically useless bubble asset that commands a liquidity premium for firms that face uninsurable idiosyncratic shocks to investment efficiency and financial constraints. Low foreign interest rate is conducive to the formation of bubbles, which induces lower domestic interest rate. Except for the traditional interest rate channel, there is an additional asset price channel in the transmission mechanism of monetary policy. There is an amplification effect of asset bubbles to positive technology shocks and negative global interest rate shocks. Bubbles serve as a store of value and relax the borrowing constraint. These shocks increase demand for domestic goods and lower the domestic nominal interest rate. The lower bond yields stimulate demand for bubbles, resulting in a higher asset price. This raises the net worth of firms holding bubble assets, further stimulating demand for domestic goods and reducing the nominal interest rate. Such amplification effect brings about greater asset bubbles and more consumption, investment, and output.

We show that while an interest rate rule that responds to bubble prices can stabilize output following foreign interest rate shocks, it will be much less effective following domestic productivity shocks. Capital inflows induce higher asset prices. Anticipating higher interest rates in response to asset prices, firms desire to hold more bubble assets to provide liquidity. The expansion of asset bubbles crowds out investment substantially, reducing output volatility. Therefore, the "lean-against-the-wind" policy creates a trade-off between output stability and the risk of asset bubbles.

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Appendix

A Proofs

A.1 Proposition 1

Firms' optimization problem is

$$V_{jt}\left(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}\right) = \max_{H_{jt}, I_{jt}\geq 0, B_{jt}} D_{jt} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right), \tag{A.1}$$

subject to

$$K_{jt} = (1-\delta) K_{jt-1} + \varepsilon_{jt} I_{jt},$$

$$\frac{B_{jt}}{R_{ft}} \geq -\mu P_{kt} K_{jt-1},$$

$$H_{jt} \geq 0,$$

$$D_{jt} = R_{kt} K_{jt-1} - P_{kt} I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_{ht}(H_{jt-1} - H_{jt}), \tag{A.2}$$

$$D_{jt} \geq 0.$$

Guess the value function is linear

$$V_{jt}\left(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}\right) = \phi_t^K\left(\varepsilon_{jt}\right) K_{jt-1} + \phi_t^H\left(\varepsilon_{jt}\right) H_{jt-1} + \phi_t^B\left(\varepsilon_{jt}\right) B_{jt-1},$$

where $\phi_t^K\left(\varepsilon_{jt}\right), \phi_t^H\left(\varepsilon_{jt}\right), \phi_t^B\left(\varepsilon_{jt}\right)$ are to be determined.

Define Tobin's (marginal) Q as

$$\begin{aligned} Q_t &= \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right)}{\partial K_{jt}} \\ &= \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^K(\varepsilon)\, dF(\varepsilon). \end{aligned}$$

Similarly, conjecture

$$P_{ht} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^H(\varepsilon)\, dF(\varepsilon),$$

and

$$\frac{1}{R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^B(\varepsilon)\, dF(\varepsilon).$$

From above equations and the law of motion of capital,

$$\beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right) = Q_t\left[(1-\delta) K_{jt-1} + \varepsilon_{jt} I_{jt}\right] + P_{ht} H_{jt} + \frac{B_{jt}}{R_{ft}}.$$

Plug in the flow-of-funds constraint (A.2), the right-hand side of the value function (A.1) can be written as

$$\begin{aligned}
& D_{jt} + \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{j,t+1}\left(\varepsilon_{jt+1}, K_{jt}, H_{jt}, B_{jt}\right) \\
= \;& R_{kt} K_{jt-1} - P_{kt} I_{jt} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_{ht}(H_{jt-1} - H_{jt}) + Q_t\left[(1-\delta) K_{jt-1} + \varepsilon_{jt} I_{jt}\right] + P_{ht} H_{jt} + \frac{B_{jt}}{R_{ft}} \\
= \;& R_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1} + (1-\delta) Q_t K_{jt-1} + \left(Q_t \varepsilon_{jt} - P_{kt}\right) I_{jt}.
\end{aligned}$$

When $Q_t \varepsilon_{jt} - P_{kt} < 0$, i.e., $\varepsilon_{jt} < \bar{\varepsilon}_t \equiv \frac{P_{kt}}{Q_t}$, the firm won't invest, $I_{jt} = 0$, because the marginal cost exceeds the marginal benefit of investment. Moreover, the firm is indifferent between buying bonds and bubble assets. Then the value function for non-investing firms is

$$V_{jt}\left(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}\right) = R_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1} + (1-\delta) Q_t K_{jt-1}. \tag{A.3}$$

When $\varepsilon_{jt} > \bar{\varepsilon}_t \equiv \frac{P_{kt}}{Q_t}$, the firm invests as much as possible. The firm will sell the bubble asset and exhaust the borrowing limit to invest, so $H_{jt} = 0$, $\frac{B_{jt}}{R_{ft}} = -\mu P_{kt} K_{jt-1}$. As the firm faces the equity constraint, $D_{jt} \geq 0$,

$$\begin{aligned}
P_{kt} I_{jt} &\leq R_{kt} K_{jt-1} - \frac{B_{jt}}{R_{ft}} + B_{jt-1} + P_{ht}(H_{jt-1} - H_{jt}) \\
&= R_{kt} K_{jt-1} + \mu P_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1}.
\end{aligned}$$

To maximize investment, the firm's investment is

$$I_{jt} = \frac{1}{P_{kt}}\left[R_{kt} K_{jt-1} + \mu P_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1}\right], \tag{A.4}$$

and $D_{jt} = 0$. Plug (A.4) into the Bellman equation, the value function for investing firms is

$$\begin{aligned}
V_{jt}\left(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}, B_{jt-1}\right) &= R_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1} + (1-\delta) Q_t K_{jt-1} \\
&\quad + \left(\frac{Q_t \varepsilon_{jt}}{P_{kt}} - 1\right)\left[R_{kt} K_{jt-1} + \mu P_{kt} K_{jt-1} + B_{jt-1} + P_{ht} H_{jt-1}\right].
\end{aligned} \tag{A.5}$$

Combine (A.3) and (A.5), we obtain the aggregate asset-pricing equations

$$Q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[R_{kt+1} + (1-\delta)\, Q_{t+1} + (R_{kt+1} + \mu P_{kt+1}) \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1\right) dF(\varepsilon)\right],$$

$$P_{ht} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[P_{ht+1}\left(1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1\right) dF(\varepsilon)\right)\right],$$

$$\frac{1}{R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}\left[1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1\right) dF(\varepsilon)\right].$$

As the capital, bonds and bubble assets (when its price is positive) can raise the firm's net worth, there exists liquidity premium $\int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1\right) dF(\varepsilon)$ for these assets. What's more, because of the liquidity premium, $P_{ht} > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} P_{ht+1}$, and $\frac{1}{R_{ft}} > \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t}$, so the returns of bonds and bubble assets to household in equilibrium are too low. Therefore, households won't hold bonds or bubble assets.

A.2 Proof of Proposition 4

When R^* rises, from (27), $\frac{eB^*}{P_z Z}(\bar{\varepsilon})$ decreases. Then the right-hand side of condition (28) increases in R^* and decreases in $\bar{\varepsilon}$. So $\bar{\varepsilon}_f$ increases in R^*. A smaller $\bar{\varepsilon}_f$ makes the condition (30) more likely to hold. What's more, when R^* is large enough, $\bar{\varepsilon}_f$ is sufficiently high and hence the bubble can't exist.

In Appendix C.2, the investment cutoff is independent on R^*. Thus R^* doesn't affect R_{fb}, $\frac{R_{kb}}{P_{kb}}$ by (25) and (26). From (C.3), $\frac{P_h}{P_{zb} Z_b}$ increases in $\frac{e_b B_b^*}{P_{zb} Z_b}$, which decreases in R^*. Therefore, $\frac{P_h}{P_{zb} Z_b}$ decreases in R^*.

A.3 Proof of Lemma 1

From the equation of asset pricing of capital (11), the steady state real rental rate of capital is a function of the cutoff $\bar{\varepsilon}$:

$$\frac{R_k}{P_k}(\bar{\varepsilon}) = \frac{[1-\beta(1-\delta)] - \beta\mu \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon})\, dF(\varepsilon)}{\beta\left(\bar{\varepsilon} + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} (\varepsilon - \bar{\varepsilon})\, dF(\varepsilon)\right)}.$$

Take a partial derivative of it, we obtain

$$
\begin{aligned}
\frac{\partial \frac{R_k}{P_k}(\bar{\varepsilon})}{\partial \bar{\varepsilon}} &= \left[\beta^2\left(\bar{\varepsilon}+\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right)^2\right]^{-1} * \\
&\quad \left[\begin{array}{c} -\beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(-1)\,dF(\varepsilon)\,\beta\left(\bar{\varepsilon}+\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right) \\ -\left[\left[1-\beta(1-\delta)\right]-\beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right]*\beta\left(1+\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(-1)\,dF(\varepsilon)\right) \end{array}\right] \\
&= \frac{\beta^2\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\varepsilon dF(\varepsilon)-\beta F(\bar{\varepsilon})\left[1-\beta(1-\delta)\right]}{\beta\left(\bar{\varepsilon}+\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right)^2} \\
&= \frac{\beta\mu\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}\varepsilon dF(\varepsilon)-\left[1-\beta(1-\delta)\right]F(\bar{\varepsilon})}{\beta\left(\bar{\varepsilon}+\int_{\bar{\varepsilon}}^{\varepsilon_{\max}}(\varepsilon-\bar{\varepsilon})\,dF(\varepsilon)\right)^2}.
\end{aligned}
$$

When $\mu = 0$, it is negative. So when μ is sufficiently small, it is also negative.

A.4 Net Export

From households' budget constraint, the definition of D_t^b, D_t^k, D_t^r, and market clearing conditions, we obtain

$$
\begin{aligned}
P_tC_t - \int D_{jt}dj &= W_tN_t + \frac{B_t^*}{R_{ft}^*}e_t - B_{t-1}^*e_t + \frac{B_t}{R_{ft}} - B_{t-1} + P_{kt}I_t \\
&\quad - \left[1+\frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]P_tI_t + D_t^r.
\end{aligned}
$$

Plug the budget constraint of firm into above equation, then

$$
\begin{aligned}
& P_tC_t - \left(R_{kt}K_{t-1} - P_{kt}I_t - \frac{B_t}{R_{ft}} + B_{t-1} + P_{ht}(H_{t-1}-H_t)\right) \\
= \; & W_tN_t + \frac{B_t^*}{R_{ft}^*}e_t - B_{t-1}^*e_t + \frac{B_t}{R_{ft}} - B_{t-1} + P_{kt}I_t - \left[1+\frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]P_tI_t + D_t^r.
\end{aligned}
$$

After some simple algebra, we can get

$$
P_tY_t - R_{kt}K_{t-1} - W_tN_t = \frac{B_t^*}{R_{ft}^*}e_t - B_{t-1}^*e_t + D_t^r.
$$

Then using the optimal condition of marginal product of capital and labor,

$$P_t Y_t - P_{zt}\left[Y_t * \int_0^1 \left[\frac{P_{rt}}{P_t}\right]^{-\psi} dr + X_t\right] + e_t P_{ft}^* M_t = \frac{B_t^*}{R_{ft}^*} e_t - B_{t-1}^* e_t + D_t^r.$$

The profits of all retailers are

$$\begin{aligned}
D_t^r &= \int_0^1 (P_{rt} - P_{zt})\, Y_{rt} dr + \int_0^1 (e_t P_{rt}^* - P_{zt})\, Y_{rt}^* dr^* \\
&= \int_0^1 (P_{rt} - P_{zt})\, Y_t \left[\frac{P_{rt}}{P_t}\right]^{-\psi} dr + \int_0^1 (e_t P_{rt}^* - P_{zt})\, Y_t^* \left(P_{rt}^*\right)^{-\psi^*} dr^* \\
&= Y_t P_t - P_{zt} Y_t \int_0^1 \left[\frac{P_{rt}}{P_t}\right]^{-\psi} dr + Y_t^* e_t \int_0^1 \left(P_{rt}^*\right)^{1-\psi} dr^* - Y_t^* P_{zt} \int_0^1 \left(P_{rt}^*\right)^{-\psi^*} dr^*,
\end{aligned}$$

then

$$Y_t^* e_t \int_0^1 \left(P_{rt}^*\right)^{1-\psi} dr^* - e_t P_{ft}^* M_t = B_{t-1}^* e_t - \frac{B_t^*}{R_{ft}^*} e_t.$$

Define $V_{pt} \equiv \int_0^1 \left(P_{rt}^*\right)^{1-\psi} dr^*$, then the net export in domestic currency is

$$Y_t^* e_t V_{pt} - e_t P_{ft}^* M_t = B_{t-1}^* e_t - \frac{B_t^*}{R_{ft}^*} e_t.$$

B Equilibrium System

The equilibrium system is given by 28 equations for 28 endogenous variables $\{C_t, K_t, I_t, Y_t, B_t, B_t^*, M_t, N_t, X_t, \bar{\varepsilon}_t, Z_t, R_{ft}, R_{ft}^*, V_{rt}, V_{pt}, V_{yt}, W_t, P_t, P_{kt}, P_{ht}, R_{kt}, Q_t, \Lambda_t, \widetilde{P}_{rt}^*, \widetilde{P}_{rt}^*, P_{zt}, \Pi_t, e_t\}$.

1. Foreign demand
$$X_t = Y_t^* * V_{rt}. \tag{B.1}$$

2. Price dispersion V_{rt}
$$V_{rt} = \chi^* V_{r,t-1} + (1-\chi^*)\left(\widetilde{P}_{rt}^*\right)^{-\psi^*}.$$

3. Intermediate good production
$$Z_t = K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha-\gamma} M_t^{\gamma}. \tag{B.2}$$

4. Demand for foreign intermediate goods
$$e_t P_{ft}^* M_t = \gamma P_{zt} Z_t. \tag{B.3}$$

5. Domestic consumption good market clearing

$$Y_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t. \tag{B.4}$$

6. Net nominal exports

$$Y_t^* e_t V_{pt} - e_t P_{ft}^* M_t = B_{t-1}^* e_t - \frac{B_t^*}{R_{ft}^*} e_t. \tag{B.5}$$

7. Price dispersion V_{pt}

$$V_{pt} = \chi^* V_{p,t-1} + (1 - \chi^*)\left(\widetilde{P}_{rt}^*\right)^{1-\psi^*}.$$

8. Optimal condition of domestic banks

$$\frac{R_{ft}}{R_{ft}^*} = E_t \frac{e_{t+1}}{e_t}.$$

9. Debt-elastic interest rate

$$R_{ft}^* = R_t^* + \Omega\left(\exp\left(\frac{e_t B_t^*}{P_{zt} Z_t} - \underline{B}^*\right) - 1\right).$$

10. Bond market clearing

$$\frac{B_t^* e_t}{R_{ft}^*} + \frac{B_t}{R_{ft}} = 0.$$

11. Optimal condition of capital producers

$$\begin{aligned} P_{kt} \quad = \quad & \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] P_t + \Omega_k P_t \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}} \\ & - \beta \Omega_k E_t \frac{\Lambda_{t+1}}{\Lambda_t} P_{t+1} \left(\frac{I_{t+1}}{I_t} - 1\right)\left(\frac{I_{t+1}}{I_t}\right)^2. \end{aligned}$$

12. Optimal condition of households

$$\Lambda_t = \frac{1}{P_t}\left[\frac{\xi_t}{C_t - hC_{t-1}} - \beta h E_t \frac{\xi_{t+1}}{C_{t+1} - hC_t}\right]. \tag{B.6}$$

13. Labor supply

$$\Lambda_t W_t = \kappa \xi_t N_t^{\varphi}. \tag{B.7}$$

14. Cutoff of investment efficiency

$$\bar{\varepsilon}_t = \frac{P_{kt}}{Q_t}.$$

15. Investment

$$I_t = \frac{1}{P_{kt}} \left[R_{kt} K_{t-1} + \mu P_{kt} K_{t-1} + B_{t-1} + P_{ht} \right] \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} dF(\varepsilon). \tag{B.8}$$

16. Asset-pricing of capital

$$Q_t = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} + (R_{kt+1} + \mu P_{kt+1}) \int_{\bar{\varepsilon}_t+1}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right].$$

17. Asset-pricing of bonds

$$\frac{1}{R_{ft}} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right].$$

18. Asset-pricing of bubble assets

$$P_{ht} = \beta E_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[P_{ht+1} \left(1 + \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right) \right]. \tag{B.9}$$

19. Law of motion for capital

$$K_t = (1-\delta) K_{t-1} + \frac{1}{P_{kt}} \left[R_{kt} K_{t-1} + \mu P_{kt} K_{t-1} + B_{t-1} + P_{ht} \right] \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \tag{B.10}$$

20. Labor demand

$$N_t = P_{zt}^{\frac{1}{\alpha}} A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{W_t} \right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{e_t P_{ft}^*} \right)^{\frac{\gamma}{\alpha}} K_{t-1}. \tag{B.11}$$

21. Marginal product of capital

$$R_{kt} = \alpha \frac{P_{zt} Z_t}{K_{t-1}}. \tag{B.12}$$

22. Inflation

$$\Pi_t = \frac{P_t}{P_{t-1}}.$$

23. Price of final goods

$$P_t = \left[(1-\chi) \left(\widetilde{P}_{rt} \right)^{1-\psi} + \chi \left(P_{t-1} \right)^{1-\psi} \right]^{\frac{1}{1-\psi}}.$$

24. Optimal domestic price

$$\widetilde{P}_{rt} = \frac{\psi}{\psi - 1} \frac{E_t \sum_{i=0}^{\infty} \chi^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i} P_{z,t+i} P_{t+i}^{\psi}}{E_t \sum_{i=0}^{\infty} \chi^i \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y_{t+i} P_{t+i}^{\psi}}.$$

25. Optimal foreign price

$$\widetilde{P}^*_{rt} = \frac{\psi^*}{\psi^* - 1} \frac{E_t \sum_{i=0}^{\infty} \chi^{*i} \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y^*_{t+i} P_{z,t+i}}{E_t \sum_{i=0}^{\infty} \chi^{*i} \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} Y^*_{t+i} e_{t+i}}.$$

26. Taylor rule

$$\ln R_{ft} = \ln R_f + \theta_\Pi \ln \frac{\Pi_t}{\Pi} + \theta_z \ln \frac{Z_t}{Z} + \theta_p \ln \frac{P_{ht}}{P_h} + \ln \zeta_t.$$

27. Intermediate good market clearing

$$Z_t = Y_t V_{yt} + X_t. \tag{B.13}$$

28. Price dispersion V_{yt}

$$V_{yt} = \chi \Pi_t^{\psi} V_{y,t-1} + (1 - \chi) \left[\frac{\widetilde{P}_{rt}}{P_t} \right]^{-\psi}.$$

C Steady State

C.1 Bubbleless Equilibrium

$P_h = 0$ for all t always satisfies

$$P_h = \beta \left[P_h \left(1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) dF(\varepsilon) \right) \right].$$

We solve the bubbleless equilibrium in the following steps.

At first, we solve the investment efficiency cutoff $\bar{\varepsilon}$. By law of motion of capital (B.10) and dividing both side by K, we get

$$\begin{aligned} \delta &= \frac{1}{P_k} \left[R_k + \mu P_k + \frac{B}{K} \right] \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) \\ &= \left[\frac{R_k}{P_k} (\bar{\varepsilon}) + \mu + \frac{B}{Z} \frac{Z}{K} \frac{1}{P_k} \right] \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \end{aligned}$$

As in steady state, $R_k = \alpha P_z \frac{Z}{K}$, and $eB^* = -B$, then

$$\begin{aligned} \delta &= \left[\frac{R_k}{P_k}(\bar{\varepsilon}) + \mu + \frac{B}{Z}\frac{R_k}{P_k}(\bar{\varepsilon})\frac{1}{\alpha P_z} \right] \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) \\ &= \left[\frac{R_k}{P_k}(\bar{\varepsilon}) \left(1 - \frac{1}{\alpha}\frac{eB^*}{P_z Z}(\bar{\varepsilon}) \right) + \mu \right] \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) . \end{aligned} \tag{C.1}$$

A sufficiently small μ ensures a unique solution to the Equation (C.1). As $\frac{R_k}{P_k}(\bar{\varepsilon})$ and $\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)$ decrease in $\bar{\varepsilon}$, and $\frac{eB^*}{P_z Z}(\bar{\varepsilon})$ increases in $\bar{\varepsilon}$, the right-hand side (RHS) of the Equation (C.1) decreases in $\bar{\varepsilon}$. What's more, the value of RHS is larger than δ when $\bar{\varepsilon} = \varepsilon_{\min}$ by Assumption (1) and is 0 when $\bar{\varepsilon} = \varepsilon_{\max}$. Therefore, we can get a unique solution $\bar{\varepsilon}_f \in (\varepsilon_{\min}, \varepsilon_{\max})$ to the Equation (C.1). Then we have $R_{ff} = R_f(\bar{\varepsilon}_f)$, $\frac{R_{kf}}{P_{kf}} = \frac{R_k}{P_k}(\bar{\varepsilon}_f)$, and $\frac{e_f B_f^*}{P_{zf} Z_f} = \frac{eB^*}{P_z Z}(\bar{\varepsilon}_f)$.

By (B.5), and in steady state, $R_f^* = R_f$, we obtain

$$Y^* V_p - P_f^* M = B^* \left(1 - \frac{1}{R_f(\bar{\varepsilon})} \right)$$

From (B.1), $Y^* V_p = X \widetilde{P}_r^* = X \frac{\psi^*}{\psi^* - 1} \frac{P_z}{e}$. Plugging it and (B.3) into above equation, we obtain the export-to-output ratio

$$\frac{X}{Z} = \frac{\psi^* - 1}{\psi^*} \left[\frac{eB^*}{P_z Z}(\bar{\varepsilon}) \left(1 - \frac{1}{R_f(\bar{\varepsilon})} \right) + \gamma \right]$$

In bubbleless equilibrium, from Equation (23), exports are positive, thus

$$\frac{X_f}{Z_f} = \frac{\psi^* - 1}{\psi^*} \left[\frac{eB^*}{P_z Z}(\bar{\varepsilon}_f) \left(1 - \frac{1}{R_f(\bar{\varepsilon}_f)} \right) + \gamma \right] > 0,$$

which derives the second inequality in condition (29).

Using (B.8), $eB^* = -B$, and dividing both side by Z, we obtain

$$\begin{aligned} \frac{I}{Z} &= \frac{1}{P_k} \left[R_k \frac{K}{Z} + \mu P_k \frac{K}{Z} + \frac{B}{Z} \right] \left[1 - F(\bar{\varepsilon}) \right] \\ &= \left[\frac{R_k}{P_k}(\bar{\varepsilon}) \frac{K}{Z} + \mu \frac{K}{Z} - \frac{B^*}{Z} \frac{e}{P_k} \right] \left[1 - F(\bar{\varepsilon}) \right] . \end{aligned}$$

Plugging $R_k = \alpha P_z \frac{Z}{K}$, and $P_k = \frac{\psi}{\psi - 1} P_z$ into above equation, the investment-to-output ratio is a function of $\bar{\varepsilon}$:

$$\frac{I}{Z} = \frac{\psi - 1}{\psi} \left[\alpha + \mu\alpha \left(\frac{R_k}{P_k}(\bar{\varepsilon}) \right)^{-1} - \frac{eB^*}{P_z Z}(\bar{\varepsilon}) \right] \left[1 - F(\bar{\varepsilon}) \right],$$

and in bubbleless equilibrium,

$$\frac{I_f}{Z_f} = \frac{\psi - 1}{\psi}\left[\alpha + \mu\alpha\left(\frac{R_k}{P_k}\left(\bar{\varepsilon}_f\right)\right)^{-1} - \frac{eB^*}{P_z Z}\left(\bar{\varepsilon}_f\right)\right]\left[1 - F\left(\bar{\varepsilon}_f\right)\right].$$

By (B.13) and (B.4),

$$C = Y - I = Y + X - I - X.$$

Then the consumption-to-output ratio is

$$\begin{aligned}
\frac{C_f}{Z_f} &= 1 - \frac{I_f}{Z_f} - \frac{X_f}{Z_f} \\
&= 1 - \frac{\psi - 1}{\psi}\left[\alpha + \mu\alpha\left(\frac{R_k}{P_k}\left(\bar{\varepsilon}_f\right)\right)^{-1} - \frac{eB^*}{P_z Z}\left(\bar{\varepsilon}_f\right)\right]\left[1 - F\left(\bar{\varepsilon}_f\right)\right] \\
&\quad - \frac{\psi^* - 1}{\psi^*}\left[\frac{eB^*}{P_z Z}\left(\bar{\varepsilon}_f\right)\left(1 - \frac{1}{R_f\left(\bar{\varepsilon}_f\right)}\right) + \gamma\right]
\end{aligned}$$

To ensure $C_f > 0$, $\frac{C_f}{Z_f} > 0$ derives the first inequality in condition (29).

By (B.6) and (B.7), we obtain

$$\begin{aligned}
\kappa\xi N^{1+\varphi} &= \Lambda W N \\
&= \frac{\xi\left(1 - \beta h\right)}{P\left(1 - h\right)C}\left(1 - \alpha - \gamma\right)P_z Z.
\end{aligned}$$

As $P = \frac{\psi}{\psi - 1} P_z$, we pin down N_f by

$$N_f = \left[\frac{\left(1 - \beta h\right)\left(1 - \alpha - \gamma\right)}{\left(1 - h\right)\kappa}\frac{\psi - 1}{\psi}\frac{Z}{C}\right]^{\frac{1}{1+\varphi}}.$$

Next, by (B.2), we obtain

$$Z = A\left(\frac{\psi}{\psi - 1}\right)^{\frac{-\alpha}{1-\alpha-\gamma}}\left(\alpha\left(\frac{R_k}{P_k}\left(\bar{\varepsilon}\right)\right)^{-1}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\gamma\left(\frac{e}{P_z}\right)^{-1}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N. \tag{C.2}$$

Combining (C.2) and (B.1) yields

$$\frac{X}{Z} = \frac{Y^*\left(\frac{\psi^*}{\psi^* - 1}\left(\frac{e}{P_z}\right)^{-1}\right)^{-\psi^*}}{A\left(\frac{\psi}{\psi - 1}\right)^{\frac{-\alpha}{1-\alpha-\gamma}}\left(\alpha\left(\frac{R_k}{P_k}\left(\bar{\varepsilon}\right)\right)^{-1}\right)^{\frac{\alpha}{1-\alpha-\gamma}}\left(\gamma\left(\frac{e}{P_z}\right)^{-1}\right)^{\frac{\gamma}{1-\alpha-\gamma}} N},$$

which leads to

$$\frac{e_f}{P_{zf}} = \left(\frac{X_f}{Z_f}\frac{AN_f}{Y^*}\right)^{\frac{1-\alpha-\gamma}{\psi^*\gamma}} \left(\frac{\psi}{\psi-1}\right)^{\frac{-\alpha}{\psi^*\gamma}} \left(\alpha\frac{P_{kf}}{R_{kf}}\right)^{\frac{\alpha}{\psi^*\gamma}} \gamma^{\frac{\gamma}{\psi^*\gamma}} \left(\frac{\psi^*}{\psi^*-1}\right)^{\frac{1-\alpha-\gamma}{\gamma}}.$$

By Equation (C.2), we can derive Z_f, and then B_f^*, B_f, I_f, C_f, X_f can be easily determined.

Using (B.12) and (B.3), we solve the steady state K_f and M_f. Then $\frac{W_f}{P_{zf}}$, and $\Lambda_f P_{zf}$ can be derived by (B.11), and (B.6), respectively. Finally, the relative price is normalized by nominal exchange rate e_t, then the steady state value of all variables can be solved.

C.2 Bubbly Equilibrium

At first, if the bubbly equilibrium exists, we want to compare the investment cutoff $\bar{\varepsilon}_b$ and $\bar{\varepsilon}_f$ and derive the condition (30). By (B.10) and dividing both side by Z, we can derive

$$\frac{P_h}{Z} = \frac{\delta\frac{K}{Z}P_k}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - R_k\frac{K}{Z} - \mu P_k\frac{K}{Z} - \frac{B}{Z}.$$

Plugging $\frac{K}{Z} = \frac{\alpha P_z}{R_k}$, and $\frac{B}{Z} = -\frac{eB^*}{Z}$ into above equation yields bubble-to-nominal-output ratio

$$\frac{P_h}{P_z Z} = \left(\frac{R_k}{P_k}(\bar{\varepsilon})\right)^{-1}\left[\frac{\delta\alpha}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - \mu\alpha\right] - \alpha + \frac{eB^*}{P_z Z}(\bar{\varepsilon}).$$

Define

$$G(\bar{\varepsilon}) \equiv \left(\frac{R_k}{P_k}(\bar{\varepsilon})\right)^{-1}\left[\frac{\delta\alpha}{\int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - \mu\alpha\right] - \alpha + \frac{eB^*}{P_z Z}(\bar{\varepsilon}).$$

Since $\frac{eB^*}{P_z Z}(\bar{\varepsilon})$ increases in $\bar{\varepsilon}$, $\frac{R_k}{P_k}(\bar{\varepsilon})$ and $\int_{\bar{\varepsilon}_{\bar{\varepsilon}}}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)$ decrease in $\bar{\varepsilon}$, we get $G'(\bar{\varepsilon}) > 0$.

In bubbly equilibrium, $P_h > 0$, then

$$G(\bar{\varepsilon}_b) = \frac{P_h}{P_{zb}Z_b} = \frac{P_{kb}}{R_{kb}}\left[\frac{\delta\alpha}{\int_{\bar{\varepsilon}_b}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - \mu\alpha\right] - \alpha + \frac{e_b B_b^*}{P_{zb}Z_b} > 0. \tag{C.3}$$

In the bubbleless equilibrium, from (B.10),

$$G(\bar{\varepsilon}_f) = \frac{P_{kf}}{R_{kf}}\left[\frac{\delta\alpha}{\int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} - \mu\alpha\right] - \alpha + \frac{e_f B_f^*}{P_{zf}Z_f} = 0.$$

Therefore, $\bar{\varepsilon}_b > \bar{\varepsilon}_f$. Then we have

$$1 < \beta \left[1 + \int_{\bar{\varepsilon}_f}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}_f} - 1 \right) dF(\varepsilon) \right].$$

Now suppose that (30) holds, we want to solve the bubbly equilibrium. In equilibrium, $P_h > 0$, then (B.9) yields

$$1 = \beta \left(1 + \int_{\bar{\varepsilon}}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\bar{\varepsilon}} - 1 \right) dF(\varepsilon) \right).$$

The right-hand side of the above equation decreases in $\bar{\varepsilon}$. When $\bar{\varepsilon} = \varepsilon_{\max}$, it equals to β, and when $\bar{\varepsilon} = \bar{\varepsilon}_f$, it is larger than 1 by (30). Then we can get the unique solution $\bar{\varepsilon}_b$ to the above equation for $\bar{\varepsilon} \in (\bar{\varepsilon}_f, \varepsilon_{\max})$. Then $\frac{R_{kb}}{P_{kb}}$, R_{fb}, $\frac{e_b B_b^*}{P_{zb} Z_b}$ are easy to derive.

By (B.8), the investment-to-output ratio is different from that in the bubbleless equilibrium because bubble asset can be used to finance investments, which is

$$\frac{I_b}{Z_b} = \frac{\psi - 1}{\psi} \left[\alpha + \mu\alpha \left(\frac{R_{kb}}{P_{kb}} \right)^{-1} - \frac{e_b B_b^*}{P_{zb} Z_b} + \frac{P_h}{P_{zb} Z_b} \right] [1 - F(\bar{\varepsilon}_b)].$$

Similar to the bubbleless equilibrium, we can solve the remaining variables. The export-to-output ratio and consumption-to-output ratio is

$$\frac{X_b}{Z_b} = \frac{\psi^* - 1}{\psi^*} \left[\frac{eB^*}{P_z Z} (\bar{\varepsilon}_b) \left(1 - \frac{1}{R_f(\bar{\varepsilon}_b)} \right) + \gamma \right],$$

$$\frac{C_b}{Z_b} = 1 - \frac{I_b}{Z_b} - \frac{X_b}{Z_b}.$$

To ensure the exports and consumption are positive, we derive the condition

$$\begin{aligned} & 1 - \frac{\psi - 1}{\psi} \left[\alpha + \mu\alpha \left(\frac{R_{kb}}{P_{kb}} \right)^{-1} - \frac{e_b B_b^*}{P_{zb} Z_b} + \frac{P_h}{P_{zb} Z_b} \right] [1 - F(\bar{\varepsilon}_b)] \\ > \quad & \frac{\psi^* - 1}{\psi^*} \left[\frac{eB^*}{P_z Z} (\bar{\varepsilon}_b) \left(1 - \frac{1}{R_f(\bar{\varepsilon}_b)} \right) + \gamma \right] > 0. \end{aligned}$$

The steps of solving steady-state value of other variables are similar to the bubbleless equilibrium.

D Normalized System

We focus on small open economy and regard foreign economy as exogenous, so we take foreign price as normalization to ensure the dynamic responses to shocks are based on the same steady-states. Then we normalize the domestic nominal prices and nominal debt by the nominal exchange rate to get the real prices and debt based in foreign currency. We use a lower case variable $x_t \equiv \frac{X_t}{e_t}$, and $\lambda_t \equiv \Lambda_t e_t$ to denote the real variables. We denote $\pi_t \equiv \frac{p_t}{p_{t-1}}$, and $\pi_{et} \equiv \frac{e_t}{e_{t-1}}$ as the inflation of domestic price level and depreciation of domestic currency, respectively.

The normalized bubbly equilibrium system can be described by 40 equations for 40 variables $\{C_t, K_t, I_t, Y_t, b_t, B_t^*, M_t, N_t, X_t, \bar{\varepsilon}_t, Z_t, R_{ft}, R_{ft}^*, V_{rt}, V_{pt}, V_{yt}, w_t, p_t, p_{kt}, p_{ht}, r_{kt}, q_t, \lambda_t, \widetilde{P}_{rt}, \widetilde{P}_{rt}^*, p_{zt}, \pi_t, \pi_{et}, f_{1t}, f_{2t}, F_{1t}^*, F_{2t}^*, LIQ_t, R_t^*, Y_t^*, \xi_t, A_t, \zeta_t, P_{ft}^*, \omega_t\}$

1. Foreign demand
$$X_t = Y_t^* * V_{rt}.$$

2. Price dispersion V_{rt}
$$V_{rt} = \chi^* V_{r,t-1} + (1-\chi^*) \left(\widetilde{P}_{rt}^*\right)^{-\psi^*}.$$

3. Intermediate good production
$$Z_t = K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha-\gamma} M_t^{\gamma}.$$

4. Demand for foreign intermediate goods
$$P_{ft}^* M_t = \gamma p_{zt} Z_t.$$

5. Domestic consumption good market clearing
$$Y_t = C_t + \left[1 + \frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t.$$

6. Net nominal exports
$$Y_t^* V_{pt} - P_{ft}^* M_t = B_{t-1}^* - \frac{B_t^*}{R_{ft}^*}.$$

7. Price dispersion V_{pt}
$$V_{pt} = \chi^* V_{p,t-1} + (1-\chi^*) \left(\widetilde{P}_{rt}^*\right)^{1-\psi^*}.$$

8. Optimal condition of domestic banks

$$\log\left(\frac{R_{ft}}{R_f}\right)-\log\left(\frac{R^*_{ft}}{R^*_f}\right)-\log\left(\frac{\pi_{e,t+1}}{\pi_e}\right)=\log\left(\omega_t\right).$$

9. Debt-elastic interest rate

$$R^*_{ft}=R^*_t+\Omega\left(\exp\left(\frac{B^*_t}{p_{zt}Z_t}-\underline{B}^*\right)-1\right).$$

10. Bond market clearing

$$\frac{B^*_t}{R^*_{ft}}+\frac{b_t}{R_{ft}}=0.$$

11. Optimal condition of capital producers

$$\begin{aligned}p_{kt} \quad = \quad & \left[1+\frac{\Omega_k}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2\right]p_t+\Omega_k p_t\left(\frac{I_t}{I_{t-1}}-1\right)\frac{I_t}{I_{t-1}}\\ & -\beta\Omega_k E_t\frac{\lambda_{t+1}}{\lambda_t}p_{t+1}\left(\frac{I_{t+1}}{I_t}-1\right)\left(\frac{I_{t+1}}{I_t}\right)^2.\end{aligned}$$

12. Optimal condition of households

$$\lambda_t=\frac{1}{p_t}\left[\frac{\xi_t}{C_t-hC_{t-1}}-\beta h E_t\frac{\xi_{t+1}}{C_{t+1}-hC_t}\right].$$

13. Labor supply

$$\lambda_t w_t=\kappa\xi_t N_t^{\varphi}.$$

14. Cutoff of investment efficiency

$$\bar{\varepsilon}_t=\frac{p_{kt}}{q_t}.$$

15. Investment

$$I_t=\frac{1}{p_{kt}}\left[r_{kt}K_{t-1}+\mu p_{kt}K_{t-1}+\frac{b_{t-1}}{\pi_{e,t}}+p_{ht}\right]\left(\frac{\bar{\varepsilon}_t}{\varepsilon_{\min}}\right)^{-\eta}.$$

16. Asset-pricing of capital

$$q_t=\beta E_t\frac{\lambda_{t+1}}{\lambda_t}\left[r_{kt+1}+\left(1-\delta\right)q_{t+1}+\left(r_{kt+1}+\mu p_{kt+1}\right)LIQ_{t+1}\right].$$

17. Asset-pricing of bonds
$$\frac{1}{R_{ft}} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{e,t+1}} \left[1 + LIQ_{t+1}\right].$$

18. Asset-pricing of bubble assets
$$p_{ht} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left[p_{ht+1}\left(1 + LIQ_{t+1}\right)\right].$$

19. Law of motion for capital
$$K_t = (1-\delta) K_{t-1} + \frac{\eta}{\eta - 1} I_t \bar{\varepsilon}_t.$$

20. Labor demand
$$N_t = p_{zt}^{\frac{1}{\alpha}} A_t^{\frac{1-\alpha-\gamma}{\alpha}} \left(\frac{1-\alpha-\gamma}{w_t}\right)^{\frac{1-\gamma}{\alpha}} \left(\frac{\gamma}{P_{ft}^*}\right)^{\frac{\gamma}{\alpha}} K_{t-1}.$$

21. Marginal product of capital
$$r_{kt} = \alpha \frac{p_{zt} Z_t}{K_{t-1}}.$$

22. Inflation
$$\pi_t = \frac{p_t}{p_{t-1}}.$$

23. Price of final goods
$$p_t = \left[(1-\chi)\left(\widetilde{p}_{rt}\right)^{1-\psi} + \chi \left(p_{t-1}\right)^{1-\psi} \pi_{e,t}^{\psi-1}\right]^{\frac{1}{1-\psi}}.$$

24. Optimal price
$$\widetilde{p}_{rt} \equiv \frac{\psi}{\psi - 1} \frac{f_{1t}}{f_{2t}}.$$

25. Auxiliary variable
$$f_{1t} = \lambda_t p_{zt} p_t^{\psi} Y_t + \chi \beta E_t f_{1,t+1} \pi_{e,t+1}^{\psi}.$$

26. Auxiliary variable
$$f_{2t} = \lambda_t p_t^{\psi} Y_t + \chi \beta E_t f_{2,t+1} \pi_{e,t+1}^{\psi-1}.$$

27. Optimal export price
$$\widetilde{P}_{rt}^* \equiv \frac{\psi^*}{\psi^* - 1} \frac{F_{1t}^*}{F_{2t}^*}.$$

28. Auxiliary variable

$$F_{1t}^{*} = \lambda_t p_{zt} Y_t^{*} + \chi^{*} \beta E_t F_{1,t+1}^{*}.$$

29. Auxiliary variable

$$F_{2t}^{*} = \lambda_t Y_t^{*} + \chi^{*} \beta E_t F_{2,t+1}^{*}.$$

30. Taylor rule

$$\ln R_{ft} = \ln R_f + \theta_\Pi \ln \frac{\pi_t}{\pi} + \theta_\Pi \frac{\pi_{e,t}}{\pi_e} + \theta_y \ln \frac{Z_t}{Z} + \theta_{P_h} \ln \frac{p_{ht}}{p_h} + \ln \zeta_t.$$

31. Intermediate good market clearing

$$Z_t = Y_t V_{yt} + X_t.$$

32. Price dispersion V_{yt}

$$V_{yt} = \chi \pi_t^{\psi} \pi_{et}^{\psi} V_{y,t-1} + (1-\chi) \left[\frac{\widetilde{p}_{rt}}{p_t} \right]^{-\psi}.$$

33. Liquidity premium

$$LIQ_t \equiv \int_{\bar{\varepsilon}_t}^{\varepsilon_{\max}} \left(\frac{\varepsilon q_t}{p_{kt}} - 1 \right) dF(\varepsilon) = \frac{1}{\eta - 1} \varepsilon_{\min}^{\eta} \bar{\varepsilon}_t^{-\eta}.$$

34. Foreign demand shock

$$\ln Y_t^{*} = \rho_{Y^*} \ln Y_{t-1}^{*} + \sigma_{Y^*} \varepsilon_{Y^* t}.$$

35. Nominal global interest rate shock

$$\ln R_t^{*} = (1 - \rho_{R^*}) \ln R^{*} + \rho_{R^*} \ln R_{t-1}^{*} + \sigma_{R^*} \varepsilon_{R^* t}.$$

36. Preference shock

$$\ln \xi_t = \rho_{\xi} \ln \xi_{t-1} + \sigma_{\xi} \varepsilon_{\xi t}.$$

37. Technology shock

$$\ln A_t = \rho_A \ln A_{t-1} + \sigma_A \varepsilon_{At}.$$

38. Monetary policy shock

$$\ln \zeta_t = \rho_{\zeta} \ln \zeta_{t-1} + \sigma_{\zeta} \varepsilon_{\zeta t}.$$

39. Foreign intermediate good price shock

$$\ln P^*_{ft} = \rho_{P^*_f} \ln P^*_{ft-1} + \sigma_{P^*_f} \varepsilon_{P^*_f t}.$$

40. UIP shock

$$\ln \omega_t = \rho_\omega \ln \omega_{t-1} + \sigma_\omega \varepsilon_{\omega,t}.$$