Collateral Quality and House Prices[‡]

Jing Zhou

February 2022

Abstract

This paper studies the effects of collateral quality shocks on house prices, the price-rent ratio, and the macroeconomy in a dynamic general equilibrium model with housing collateral. Collateral quality is not as perfect as is typically assumed in the existing literature on collateral constraints, and collateral quality shocks can simultaneously explain the salient features of the joint dynamics of house prices, the price-rent ratio, and output observed in the data. Moreover, depending on whether or not information about collateral quality is produced, there exist two lending regimes; endogenous switching between these two regimes also reinforces the patterns. I estimate this model using Bayesian methods and identify a conspicuous endogenous regime switch at the onset of the Great Recession. The results show that collateral quality shocks and the associated regime switch account for approximately half of the variations in house prices and the price-rent ratio during the housing boom and bust of the 2000s.

Keywords: collateral quality, endogenous regime switching, house prices, price-rent ratio, collateral constraint

JEL Classification: E22, E27, E32, E44

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[*]I thank Pengfei Wang, Jianjun Miao, David Cook, Juanyi (Jenny) Xu, Jiandong Ju, Yang Lu, Zhiwei Xu, Feng Dong, Tong Xu, Chang Ma, Sihao Chen, and Shi Qiu for helpful discussions. I have also benefited from comments from the the seminar and conference participants at the Hong Kong University of Science and Technology, Tsinghua University, Shanghai Jiao Tong University, Fudan University, Shanghai University of Finance and Economics, Southwestern University of Finance and Economics, the 2021 and 2017 China Meeting of the Econometric Society, and the 47th Money, Macro, and Finance Research Group Annual Conference. Finally, I acknowledge financial support from the Shanghai Institute of International Finance and Economics. All errors are my own.

[†]Department of Economics, Fudan University, 600 Guoquan Road, Shanghai, China 200433. Tel.: (+86)21-6465-2989. Email: jingzhou@fudan.edu.cn.

1 Introduction

The Great Recession and the associated housing boom-bust cycle have drawn intensive attention to the dramatic movements in house prices. As depicted in Figure 1, house prices in the U.S. show three salient patterns: (i) house prices and the price-rent ratio are highly volatile, about twice as volatile as output; (ii) movements in house prices and the price-rent ratio are strikingly close, implying that house prices are much more volatile than rents; and (iii) house prices and the price-rent ratio are both procyclical. These patterns are not unique to the U.S., according to Knoll (2016), but have been documented for 14 advanced economies.

Many studies attempt to decipher house price fluctuations, mostly from the perspective of housing collateral.[1] Existing studies on collateral constraints typically assume that collateral quality is perfect, which means that collateral is always as valuable as it was previously assessed to be. This further implies that with a properly adjusted loan-to-value ratio ("haircut"), lenders barely suffer losses in the event of default (Kiyotaki and Moore, 1997). In reality, however, this is not the case. The Financial Crisis Inquiry Report (2011) stated that for over 20% of households with debt backed by real estate, the value of their collateralized real estate did not offset the debt owed. As of February 2010, almost half of the commercial real estate loans could not be compensated for by the market value of the underlying property.

This imperfect quality of collateral, among other factors, lies at the root of the housing boom and bust. This missing factor raises an unanswered question in the literature: how important are fluctuations in collateral quality in explaining the joint dynamics of output, house prices, and the price-rent ratio observed in the data? If yes, to what extent?

To answer these questions, I consider collateral quality in an otherwise standard real business cycle model with housing collateral. I find that collateral quality shocks can simultaneously explain the three patterns described above. Moreover, fluctuations in collateral quality lead to endogenous switching between two lending regimes: one with symmetric ignorance of collateral quality by both borrowers and lenders, and the other with symmetric awareness of

[1] Davis and Nieuwerburgh (2015), Guerrieri and Uhlig (2016), and Piazzesi and Schneider (2016) provide excellent surveys on this topic.

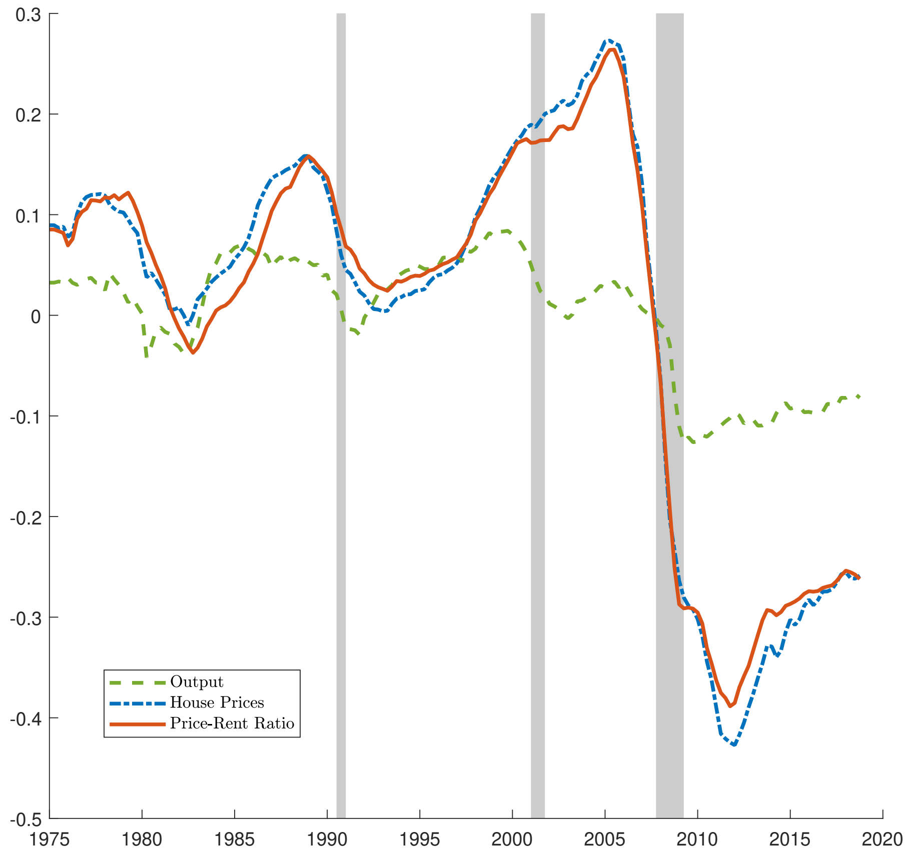

Figure 1: U.S. experience with output, house prices and the price-rent ratio. The data sources for output and rents are the National Income & Product Accounts (NIPA). House prices are obtained from the Federal Reserve, CoreLogic data, and the National Association of Realtors data collected by the Bank for International Settlements. The y-axis indicates the deviation of the logarithm of a variable from its trend. The detrending method is described in Section 4.1.

collateral quality. Endogenous regime switching also reinforces these three patterns. Quantitatively, when I confront this model with the U.S. aggregate data using Bayesian methods, I find that collateral quality shocks and the associated regime switch are responsible for about half of the variations in house prices and the price-rent ratio during the housing boom and bust of the 2000s.

My model is an infinite-horizon real business cycle model with housing collateral in a dynamic stochastic general equilibrium (DSGE) framework. Houses have intrinsic value as housing for households and collateral for entrepreneurs. Entrepreneurs own houses and face idiosyncratic investment efficiency shocks. They use their houses as collateral to obtain external financing for investment. The key friction in this paper is that houses are of two types: "good" and "risky." Good houses always have positive intrinsic value, whereas risky houses have positive intrinsic value only with a certain probability and zero value with the remaining probability. In other words, risky houses turn out to be good with a certain probability at the end of each period. I refer to this probability as *collateral quality* and to shocks to collateral quality as *collateral quality shocks*.[2]

A feature of this model is that buyers and sellers in the housing market have symmetric information, in that they both know the types of houses (good or risky) and collateral quality, but not which specific unit of risky houses is good. This feature distinguishes the current study from previous studies on asset quality from the perspective of information asymmetry. This assumption is motivated by the notion that policy makers are more concerned with a lack of clear and precise information on underlying asset quality than with information asymmetry. As Hughes (2010) reported in the Financial Times, "*Much of regulators' efforts have been focused on pushing issuers to provide more information on individual loans ... However... this is not a top priority and they were in fact more concerned with developing methods to analyze and compare cash flow data across different deals.*" According to the report, the lack of information is due to the huge costs of information collection and the prohibitively high levels of sophistication and skills required to process and analyze that information. As a result, lenders either have to spend resources to acquire information or make decisions based on coarse information.

Two lending regimes arise because of the above-mentioned information friction. In one regime, which is called the Information Sensitive (IS) regime, a lender pays an information acquisition cost to learn the exact quality of risky collateral and provides loans based on the acquired information. This regime features *symmetric awareness* of collateral quality. In the other regime, which is called the Information Insensitive (II) regime, a lender incurs no informa-

[2] Empirical evidence shows that collateral quality varies over time as a result of either institutional or technological changes in financial markets (see Keys, Mukherjee, Seru, and Vig, 2010; Becker, Bos, and Roszbach, 2020). I take these fluctuations in collateral quality as given throughout this paper.

tion acquisition cost and provides loans based on the appraised value of risky collateral. This regime features *symmetric ignorance* about collateral quality. An endogenous regime switch then occurs as an equilibrium outcome depending on collateral quality. When collateral quality is above an endogenous threshold, a lender chooses the II regime to avoid the information acquisition cost; when collateral quality is below this threshold, only the IS regime is feasible. I further show that for the same risky collateral, liquidity provision without information production is greater than with information production.

I show that collateral quality shocks can simultaneously rationalize the three patterns of house prices summarized in the first paragraph. The transmission mechanism is as follows. Endogenous house prices in this model are the sum of the expected present values of future rents augmented by an additional factor: the liquidity premium. Rents are the "dividends" of houses and reflect a representative household's marginal utility of housing (in terms of good houses), whereas the liquidity premium appears with collateral constraints as houses can be used as collateral to relax credit constraints and provide liquidity.[3]

For patterns (i) and (ii), compared with the existing literature where collateral quality is perfect, the presence of collateral quality shocks directly disturbs the price of risky houses, but only affects a representative household's marginal utility of housing through the equilibrium effect. Collateral quality shocks thus have a stronger impact on house prices than on rents. Moreover, the liquidity premium of risky houses depends on the equilibrium lending regime, which in turn depends on collateral quality. That is, relative to rents, regime switching triggered by collateral quality shocks further strengthens the impact of collateral quality shocks on house prices. These effects explain patterns (i) and (ii).

For pattern (iii), a positive collateral quality shock causes an increase in house prices and the price-rent ratio, consequently expanding aggregate liquidity provision by risky collateral. Abundant aggregate liquidity then encourages investment and output and vice versa. Thus, collateral quality shocks generate positive comovements between house prices, the price-rent ratio, and output. Furthermore, as noted earlier, when collateral quality is high, the II regime

[3] Favara and Imbs (2015) and Zevelev (2021) provide empirical evidence for this.

without information production is more likely to occur. Liquidity provision in this regime is even greater than that in the IS regime for the same risky collateral. In other words, the lending regime further exacerbates expansions and contractions in aggregate liquidity, strengthening the comovement between house prices, the price-rent ratio, and output. These effects explain pattern (iii).

To quantify the role of collateral quality shocks, I confront this endogenous regime-switching model with aggregate U.S. data from 1975Q1 to 2019Q4 using Bayesian methods. This model contains six aggregate shocks: collateral quality shocks, productivity shocks, housing demand shocks, labor supply shocks, financial shocks, and aggregate investment-specific technology (IST) shocks. The collateral quality shocks are estimated to be about 5 times as large as productivity shocks by unconditional standard deviations. With the estimated parameters, the model-simulated real business cycle moments match their data counterparts well. When I shut down collateral quality shocks in the model, the volatilities of house prices and the price-rent ratio fall by up to half. Furthermore, the historical decomposition shows that collateral quality shocks account for about half of the variations in house prices and the price-rent ratio during the housing boom and bust of the 2000s.

Notably, the estimation endogenously detects a conspicuous regime switch at the onset of the Great Recession in the sample. To see the role of regime switching per se, I also estimate a reference model in which I shut down the endogenous regime switching. The log marginal density of the data indicates that the benchmark model outperforms the reference model. By comparing the historical paths of the key macroeconomic variables generated by the benchmark and reference models, I can see that the identified regime switch causes non-negligible additional declines in house prices, the price-rent ratio, investment, and consumption, corroborating the amplification effect of regime switching on house price and macroeconomic volatility.

When comparing the role of collateral quality shocks with that of other shocks in the model, interestingly, I find that housing demand shocks and financial shocks, which are also discussed in the literature, play quantitatively minor roles in determining house price dynamics. The role of housing demand shocks is related to pattern (ii), which has proven challenging to be explained using a standard asset pricing approach (Kiyotaki, Michaelides, and Nikolov, 2011). According to the standard approach, house (asset) prices can be viewed as the sum of the expected present values of future rents (dividends), in which case house prices should move at similar rates to rents. However, this contradicts pattern (ii). Consequently, if housing demand shock is the dominant factor for housing boom-bust cycles, then the price-rent ratio can neither be volatile nor follow house prices closely. For financial shocks, I regard them as shocks to financial tightness that are orthogonal to the value of collateral. An increase in financial tightness reduces financing for investment and output, but increases the demand for collateral, which in turn drives up house prices. Financial shocks thus lead to a negative correlation between house prices and output, which contradicts pattern (iii).

The above counterfactual implications limit the estimated sizes of housing demand shocks and financial shocks. As a result, these shocks do not have significant quantitative effects on the simulated real business cycle moments or the historical decomposition of house prices and the price-rent ratio during the housing boom and bust of the 2000s. These results appear to be in contrast to those of previous studies, such as Iacoviello (2005), Liu, Wang, and Zha (2013) and Guerrieri and Iacoviello (2017). This discrepancy, however, is not surprising because these studies do not simultaneously consider the dynamics of house prices and the price-rent ratio. Therefore, the attempt to reproduce all of these patterns together reveals the quantitative importance of collateral quality shocks to house price business cycles.

Literature review First, this paper contributes to the burgeoning literature on house price fluctuations and their macroeconomic consequences. Iacoviello (2005), Iacoviello and Neri (2010), Liu, Wang, and Zha (2013), and Guerrieri and Iacoviello (2017), for example, examine housing collateral and primarily attribute excessive house price volatility to widespread changes in housing demand. However, these studies do not take the price-rent ratio into consideration.

A number of studies attempt to simultaneously explain house prices and the price-rent ratio (or rents) from various perspectives. Prominent examples include Kiyotaki, Michaelides, and Nikolov (2011), Sommer, Sullivan, and Verbrugge (2013), Favilukis, Ludvigson, and Nieuwerburgh (2017), Justiniano, Primiceri, and Tambalotti (2019),Garriga, Manuelli, and Peralta-Alva (2019), Kaplan, Mitman, and Violante (2020), Miao, Wang, and Zha (2020), Liu, Wang, and Zha (2021), and Greenwald and Guren (2021), among others. In particular, Miao, Wang, and Zha (2020) highlight discount factor shocks and Liu, Wang, and Zha (2021) point to credit supply shocks and provide a theoretical microfoundation for housing demand shocks. These two papers argue that the high volatility of the price-rent ratio is driven by the liquidity premium provided by housing collateral. Like these papers, my paper also emphasizes the role of the liquidity premium, but I focus on how the liquidity premium is affected by collateral quality shocks, which receives little attention in the literature. I also show the endogenous interaction between liquidity premium and the endogenous regime switching helps explain house prices and the price-rent ratio. While Favilukis, Ludvigson, and Nieuwerburgh (2017) and Justiniano, Primiceri, and Tambalotti (2019) both argue for variations in credit market conditions, my study decomposes these variations into variations in collateral value and orthogonal variations in financial tightness. The former is shown to be important for the dynamics of the price-rent ratio, whereas the latter plays a minor role. In addition, Garriga, Manuelli, and Peralta-Alva (2019) and Greenwald and Guren (2021) reconcile the disconnect between house prices and rents by underscoring the roles of segmentation in asset markets and segmentation between borrowers' and savers' housing stocks, respectively. Complementing these studies, the current paper examines this issue from the perspective of collateral quality and investigates the endogenous interactions between collateral quality, house prices, and the price-rent ratio.

Second, this paper relates to studies on information opacity in financial markets and its consequences. A closely related work is Gorton and Ordoñez (2014), which also studies the two types of debt contracts, the IS debt contract and II debt contract, and demonstrates how information opacity on collateral quality leads to both a credit boom and a subsequent collateral crisis. Asriyan, Laeven, and Martin (2021) also show that withholding information production can endogenize credit booms. Although these works and mine all deal with information opacity in financial markets, my work differs in two crucial aspects. First, collateral value in their works is exogenously given, whereas collateral value in my work is endogenously determined, because I want to understand the endogenous effects of collateral quality on collateral value. Second, their studies are generally qualitative and aim at theoretical illustrations. Building on these studies, my work takes one step further and quantitatively assesses whether and how shifts in collateral quality and the associated information friction can help account for the data. Like my work, there are other studies showing that information opacity can enhance asset liquidity in normal times but exacerbates market collapses in times of crisis (e.g., Pagano and Volpin, 2012; Hanson and Sunderam, 2013; Dang, Gorton, Holmström, and Ordoñez, 2017).

In this strand of research, my paper differs from studies on asset quality conducted from the perspective of asymmetric information and asset quality, such as Kurlat (2013), Guerrieri and Shimer (2014), Bigio (2015), and Asriyan, Fuchs, and Green (2019). In these studies, adverse selection creates a shadow cost for liquidity provision, whereas in my study, house buyers and sellers have symmetric information and I study the endogenous switching between symmetric ignorance and symmetric awareness.

Finally, this paper is related to the literature on endogenous regime switching in an economy. Mendoza (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2019), Benigno, Foerster, Otrok, and Rebucci (2020), among others, study endogenous regime switching tied with occasionally binding borrowing constraints. Recent research turns its focus to the regime switching linked to endogenous information production. This line of research is exemplified by Gorton and Ordoñez (2014), Gorton and Ordoñez (2019), Asriyan, Laeven, and Martin (2021) and Glasserman, Mamaysky, and Shen (2021). My paper complements this line of research by evaluating the role of asset quality and regime switching in a quantitative framework.

The remainder of this paper proceeds as follows. Section 2 describes the setup of the benchmark model. In Section 3, I solve the model and characterize the competitive equilibrium. I examine the model quantitatively in Section 4 and delve deeper into the transmission mechanisms in Section 5. Section 6 concludes the paper.

2 Model

I introduce collateral quality into a standard real business cycle model with housing collateral. A representative household is a family consisting of four types of members: workers, bankers, capital producers, and entrepreneurs. Each type is of a unit mass. Workers supply labor to production. Bankers provide loans to entrepreneurs who own houses, invest in capital, and produce consumption goods. Capital producers make new capital goods and sell them to entrepreneurs. There are three types of goods in this economy: consumption goods, capital goods, and houses.

2.1 Collateral quality

As the other parts of the model are standard, I first elaborate collateral quality and lending regimes and then briefly describe the other parts. Houses in this economy are owned by entrepreneurs and rented randomly to all households. The key friction is that there are two types of houses: good houses, denoted by $\overline{H}_t$, each unit of which provides positive utility to households, and risky houses, denoted by H_t, each unit of which is good with probability $\eta_t \in (0, 1)$ and bad with probability $1 - \eta_t$. Bad houses provide zero utility and can be thought of as lemons or toxic assets.[4] I refer to η_t as a *collateral quality shock*. It follows an AR(1) process $\ln(\eta_t) = (1 - \rho_\eta)\ln(\eta) + \rho_\eta \ln(\eta_{t-1}) + \sigma_\eta \varepsilon_{\eta t}$, where η is the unconditional mean of η_t, $\rho_\eta \in (-1, 1)$ measures persistence, and $\sigma_\eta > 0$ measures the standard deviation. $\varepsilon_{\eta t}$ is an independent and identically distributed (IID) standard normal random variable.

At the beginning of a period, all agents in this economy know the types of all houses, i.e., good or risky, and collateral quality η_t, but not which specific unit of risky houses is good. As households rent houses at random in each period, the probability of a household happening to rent a house belonging to its own entrepreneur is almost zero. At the end of a period, tenants pay a rental rate $\overline{R}_t$ for good houses and R_t for risky houses. Tenants cannot disclose

[4] In a more general setup, instead of assuming that bad houses provide no utility to households, we could assume that bad houses provide positive but lower utility to households than good houses.

information about the quality of houses to the housing and rental markets. I also assume no learning occurs. Therefore, agents in the housing market have symmetric information about the quality of risky houses.

Here, η_t should be not be interpreted in the narrow sense of construction quality (Stroebel, 2016), but in the broad sense of the quality of an asset in terms of its ability (probability) to generate dividend (rent) streams.

The timing of events in period t is as follows. (1) At the beginning of period t, all of the shocks are realized and households rent houses. Agents know the quality of risky houses η_t without knowing which specific unit is good. (2) Entrepreneurs who decide to invest borrow from bankers. (3) Entrepreneurs invest, produce, and repay their intratemporal loans. (4) Good and risky houses are traded. (5) Households receive income from all of their members and pay rent.

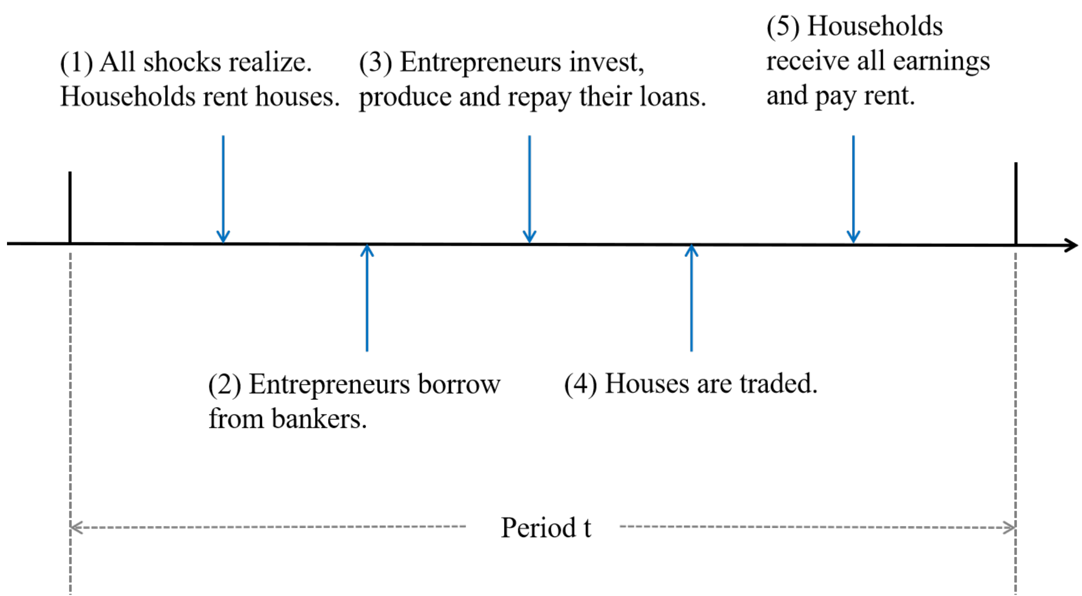

Figure 2: Timeline

2.2 Banker and lending regimes

To simplify the algebra, I follow Gorton and Ordoñez (2014) and consider only intratemporal loans.[5] Bankers operate in a competitive market and provide collateralized loans to entrepreneurs. On the one hand, a banker must earn zero profit in equilibrium given competition. For a banker, the benefit of lending, i.e., loan repayments minus the loan amount, must equal the cost of lending. Since loans are intratemporal, the cost of lending here is an information cost, if any. A banker can choose to learn the true quality of a unit of houses at an information acquisition cost, which is a fraction $\gamma \in (0, 1)$ of its appraised value. On the other hand, the borrower's repayment should always equal the true value of collateral. If the loan repayment exceeds the value of collateral, then the borrower will default. The banker then seizes the collateral and sells it on the housing market. In the following period, the borrower can re-enter the credit market without penalty. If the loan repayment is less than the value of collateral, then the borrower simply collateralizes fewer houses. Hence, in equilibrium, the loan repayment is always equal to the true value of collateral.

Let us consider the amount of credit supported by a unit of collateral. Let the prices of good and risky houses be $\overline{P}_t$ and P_t, respectively. For a unit of good houses, a banker has no incentive to investigate its quality and the loan amount for it is naturally $\overline{P}_t$. For a unit of risky houses, once its information is acquired, the information immediately goes public. If a house is identified as bad, then it is no longer traded and automatically disappears from the housing and rental markets. If the information is not produced, then the house type remains risky.

Depending on whether or not the information is produced, there are two types of debt contracts: the IS and II debt. These two types of debt are similar to those discussed in Gorton and Ordoñez (2014). However, my focus here is quite different from theirs. In their study, collateral value is exogenous, while I endogenize collateral value and investigate how it interacts with collateral quality and lending types.

[5] While enabling loans to be intertemporal complicates the algebra, it does not fundamentally change the insights of the model.

Information Sensitive Debt A lender pays the information acquisition cost and learns the true quality of collateral. If the collateral is good, then the debt contract is signed and the borrower receives a loan, denoted by $\widetilde{P}_t$. The above no-arbitrage argument posits that the borrower always repays $\overline{P}_t$, so the benefit to the lender is $\overline{P}_t - \widetilde{P}_t$. The cost of lending is the information acquisition cost $\gamma\overline{P}_t$. If the collateral is bad, then the contract is not signed. Both the benefit and cost to the lender are zero. Since the credit market is competitive, a banker is indifferent between lending or not, i.e.,

$$\eta_t(\overline{P}_t - \widetilde{P}_t) = \gamma\eta_t\overline{P}_t,$$

where the right-hand side of the equation is the cost of lending and the left-hand side is the expected benefit. Therefore, the credit per unit of risky collateral is given by $\widetilde{P}_t = (1-\gamma)\overline{P}_t$.

Information Insensitive Debt With this type of contract, lenders do not acquire information about the true quality of collateral and extend loans based on the appraised value. A lender's participation constraint becomes

$$P_t - \widetilde{P}_t \;\;=\;\; 0,$$

where the cost of lending is simply zero and the benefit of lending is $P_t - \widetilde{P}_t$, because the lender always makes a loan of $\widetilde{P}_t$ and the borrower always pays back P_t following the no-arbitrage argument. It then follows that the credit per unit of risky collateral is given by $\widetilde{P}_t = P_t$.

Lending regime The II debt contract, however, is not implemented unless the lender has no incentive to deviate. The lender may deviate if she finds it more profitable to secretly pay the information acquisition cost, discover the true quality of collateral, and lend only against good collateral. In other words, this type of lending is only feasible if secret information production is not profitable, i.e.,

$$\eta_t(\overline{P}_t - \widetilde{P}_t) \;\; \leq \;\; \gamma P_t.$$

The left-hand side of the above condition is the expected gain of the lender from behaving as if the lender honors the II contract when the collateral is good, while the right-hand side of the condition is the cost of information production. Thus, the following lemma is straightforward.

Lemma 1 *The II debt contract can be implemented in equilibrium only if*

$$\eta_t \left(\frac{\overline{P}_t}{P_t} - 1 \right) \;\; \leq \;\; \gamma. \tag{1}$$

If condition (1) does not hold, the equilibrium lending type can only be IS. The above lemma describes a necessary condition for the type II to be implemented. To check if (1) is sufficient, I use the following lemma.

Lemma 2 *If condition* (1) *holds, then*

$$P_t \geq (1-\gamma)\eta_t \overline{P}_t.$$

Proof. Please see Appendix A.1. ■

Lemma 2 states that when both lending types are feasible, an II debt contract allows for a larger loan than an IS contract for a given unit of collateral. In this case, competition in the credit market will make the II debt contract the only type in equilibrium. Consequently, there are two lending regimes: the *II regime* and *IS regime*, and the equilibrium regime is determined by the following proposition:

Proposition 1 *The II debt contract is implemented in equilibrium if and only if condition* (1) *holds.*

This proposition gives the necessary and sufficient conditions for the equilibrium regime, i.e., it is the II regime whenever condition (1) holds and is the IS regime otherwise. Therefore, the collateral value $\widetilde{P}_t$ is given by

$$\widetilde{P}_t = \begin{cases} P_t, & \text{if condition (1) holds;} \\ (1-\gamma)\overline{P}_t, & \text{if condition (1) does not hold.} \end{cases} \tag{2}$$

Note that house prices $\overline{P}_t$ and P_t are endogenously determined in the general equilibrium (in Section 3.1). Thus, Proposition 1 describes an *endogenous regime switching* condition. Using this proposition, I demonstrate in Section 5.4 that the model identifies an endogenous regime switch in U.S. data.

Evolution of houses Assume that the total supply of houses is inelastic and normalized to 1,[6] i.e.,

$$\overline{H}_t + H_t = 1. \tag{3}$$

Assume that all houses depreciate at a rate $\delta_h \in (0,1)$. In each period, new good houses $\overline{H}_{nt}$ and new risky houses H_{nt} emerge to compensate for the exiting portion of houses. The new houses are universal to all entrepreneurs and taken as given by them. I use the superscript "I" to label the II regime and "S" to label the IS regime. In the II regime, I assume that the proportion of the two types of new houses is the same as the existing ones, i.e., $\overline{H}^I_{nt}/H^I_{nt} = \overline{H}_{t-1}/H_{t-1}$. Then the good and risky houses evolve as

$$\overline{H}_t = (1-\delta_h)\overline{H}_{t-1} + \overline{H}^I_{nt}, \tag{4}$$

$$H_t = (1-\delta_h)H_{t-1} + H^I_{nt}; \tag{5}$$

In the IS regime, I assume that no new risky houses enter the market, i.e., $H^S_{nt} = 0$, and new good houses $\overline{H}^S_{nt}$ are pinned down by (3), (6), and (7). The good and risky houses then evolve

[6] Saiz (2010) and Gyourko, Saiz, and Summers (2008) empirically document that housing supply is limited, largely due to a limited supply of land. As a result, movements in house prices are dominated by movements in land prices rather than construction costs, as shown in Davis and Heathcote (2007) and Knoll, Schularick, and Steger (2017).

as

$$\overline{H}_t = (1-\delta_h)\left[\overline{H}_{t-1} + \eta_t H_{t-1}[1-\mathcal{F}(\epsilon_t^*)]\right] + \overline{H}_{nt}^S, \tag{6}$$

$$H_t = (1-\delta_h)H_{t-1}\mathcal{F}(\epsilon_t^*). \tag{7}$$

2.3 Household

I now describe households, entrepreneurs, and capital producers whose problems are common in standard business cycle models with housing collateral. In each period, a representative household consumes the following composite of consumption goods C_t and effective housing services $\overline{H}_t + \eta_t H_t$ (measured by good houses),

$$X_t \equiv \left[(1-\psi_{ht})C_t^{\frac{\chi-1}{\chi}} + \psi_{ht}\left[\exp(gt)\left(\overline{H}_t + \eta_t H_t\right)\right]^{\frac{\chi-1}{\chi}}\right]^{\frac{\chi}{\chi-1}}, \tag{8}$$

where $\chi > 1$ governs the elasticity of substitution between consumption goods and housing services, and $\psi_{ht} \in (0,1)$ measures the utility weight on housing services, which reflects a housing demand shock, as in Iacoviello (2005) and Liu, Wang, and Zha (2013). The housing demand shock follows an AR(1) process, $\ln(\psi_{ht}) = (1-\rho_h)\ln(\psi_h) + \rho_h \ln(\psi_{ht-1}) + \sigma_h \varepsilon_{ht}$, where persistence $\rho_h \in (-1,1)$ and standard deviation $\sigma_h > 0$. ε_{ht} is an IID standard normal random variable. The economy grows at a constant rate $g > 0$, and the term $\exp(gt)$ in (8) ensures a balanced growth path when housing supply is fixed.

Workers in this family provide labor N_t in each period. The household maximizes its lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[X_t - \omega X_{t-1} - \psi_{nt}\exp(gt)\frac{N_t^{1+\nu}}{1+\nu}\right]^{1-\kappa}}{1-\kappa}, \tag{9}$$

where $\beta \in (0,1)$ is the discount factor, $\kappa > 0$ measures the curvature of the period utility function, $\omega \in (0,1)$ represents habit formation, and $\nu > 0$ captures the inverse of the Frisch elasticity of labor supply. The utility weight on labor ψ_{nt} indicates a labor supply shock. It follows an AR(1) process $\ln(\psi_{nt}) = (1-\rho_n)\ln(\psi_n) + \rho_n \ln(\psi_{nt-1}) + \sigma_n \varepsilon_{nt}$ with persistence $\rho_n \in (-1, 1)$ and standard deviation $\sigma_n > 0$. ε_{nt} is an IID standard normal random variable.

The household pools labor income $W_t N_t$ and dividends D_t^e from entrepreneurs, D_t^b from bankers, and D_t^k from capital producers and distributes them equally to all members. Thus, the household's budget constraint is given by

$$C_t + \overline{R}_t \overline{H}_t + R_t H_t \quad \leq \quad W_t N_t + D_t^e + D_t^b + D_t^k. \tag{10}$$

where W_t is the real wage rate. In summary, a household maximizes (9) by choosing appropriate $\{C_t\}_{t=0}^{\infty}$, $\{N_t\}_{t=0}^{\infty}$, $\{\overline{H}_t\}_{t=0}^{\infty}$, and $\{H_t\}_{t=0}^{\infty}$, subject to constraints (8) and (10). The household's optimal decisions are given in Appendix A.2.

2.4 Entrepreneur

An entrepreneur indexed by j uses capital and labor as inputs and produces consumption goods Y_{jt} with a Cobb-Douglas production function

$$Y_{jt} \quad = \quad K_{jt-1}^{\alpha}(A_t N_{jt})^{1-\alpha}, \tag{11}$$

where A_t, K_{jt-1}, and N_{jt} represent aggregate productivity, capital input, and labor input, respectively. $\alpha \in (0, 1)$ is the share of capital in production. Let $A_t = \exp(gt)a_t$, where a_t is a transitory productivity shock. Assume that a_t follows an AR(1) process $\ln(a_t) = \rho_a \ln(a_{t-1}) + \sigma_a \varepsilon_{at}$, with persistence $\rho_a \in (-1, 1)$ and standard deviation $\sigma_a > 0$. ε_{at} is an IID standard normal random variable.

At the beginning of period t, each entrepreneur receives an idiosyncratic investment efficiency shock ϵ_{jt}. With this shock, the capital stock of entrepreneur j evolves as

$$K_{jt} = (1-\delta)K_{jt-1} + \epsilon_{jt} I_{jt}. \tag{12}$$

Here, ϵ_{jt} is randomly drawn from a distribution with the cumulative distribution function $\mathcal{F}(\epsilon)$. For tractability, I assume that ϵ_{jt} is IID across entrepreneurs and periods, and that investment is irreversible, i.e., $I_{jt} \geq 0$.

When an entrepreneur observes ϵ_{jt}, she has not yet received her sales revenue in period t and therefore has to rely on external financing for her investment. She can borrow from bankers with her houses $\overline{H}_{jt-1}$ and H_{jt-1} as collateral.[7] Therefore, entrepreneur j faces the collateral constraint

$$P_{kt}I_{jt} \leq \theta_t(\overline{P}_t\overline{H}_{jt-1} + \widetilde{P}_t H_{jt-1}), \tag{13}$$

where P_{kt} is the price of capital goods, $\overline{P}_t$ is the price of good houses, and $\widetilde{P}_t$ is the endogenous value per unit of risky houses, which is given by (2). θ_t is the maximum loan-to-value ratio and represents a financial shock as in Jermann and Quadrini (2012). It captures shocks to the credit market that are orthogonal to the value of collateral and follows an AR(1) process $\ln(\theta_t) = (1 - \rho_\theta)\ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \sigma_\theta \varepsilon_{\theta t}$, with persistence $\rho_\theta \in (-1, 1)$ and standard deviation $\sigma_\theta > 0$. $\varepsilon_{\theta t}$ is an IID standard normal random variable. Entrepreneur j maximizes the sum of the expected present values of dividend payments subject to (11), (12), and (13), the details of which are presented in Appendix A.3 .

2.5 Capital producer

A representative capital producer takes consumption goods as inputs and produces new capital goods subject to an adjustment cost. The capital producer sells the new capital goods in a competitive market at price P_{kt} and chooses $\{I_t\}_{t=0}^{\infty}$ to solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ P_{kt}I_t - \left[1 + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right] \frac{I_t}{Z_t} \right\}, \tag{14}$$

[7] Including physical capital as collateral does not materially change the results. I view houses as assets distinct from physical capital, with collateral value orthogonal to the entrepreneurs' transformation efficiency from investment to capital stock. This distinction is of course stark but helps distinguish the effects of fluctuations in collateral value from those in productivity (a_t shocks) and IST (ϵ_{jt} and Z_t shocks).

where Λ_t stands for the representative household's marginal utility of consumption goods in period t, $\Omega > 0$ denotes the adjustment cost, and Z_t represents an aggregate IST shock (Greenwood, Hercowitz, and Krusell, 1997). This shock affects the aggregate technology of transforming investment into physical capital and follows an AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \sigma_z \varepsilon_{zt}$, with persistence $\rho_z \in (-1, 1)$ and standard deviation $\sigma_z > 0$. ε_{zt} is an IID standard normal random variable. The optimal investment is given in Appendix A.4.

3 Equilibrium Characterization

In this section, I derive endogenous house prices and examine how they depend on collateral quality shocks and lending regimes. Following this, I characterize the competitive equilibrium.

3.1 Endogenous house prices

Let Q_t denote Tobin's (marginal) Q and R_{kt} denote the marginal product of capital. I use the indicator variable $\mathbf{1}_t^S$ to denote the equilibrium lending regime in period t, as determined by Proposition 1, i.e.,

$$\mathbf{1}_t^S = \begin{cases} 0, & \text{if II regime;} \\ 1, & \text{if IS regime.} \end{cases} \tag{15}$$

The following proposition outlines an entrepreneur's solution and the equilibrium house prices.

Proposition 2 *(i) Denote $\epsilon_t^* \equiv P_{kt}/Q_t \in (\epsilon_{min}, \epsilon_{max})$. When $\epsilon_{jt} \geq \epsilon_t^*$, entrepreneur j collateralizes all of her houses and makes a real investment*

$$I_{jt} = \frac{\theta_t}{P_{kt}} \left\{ \overline{P}_t \overline{H}_{jt-1} + \left[\mathbf{1}_t^S (1-\gamma) \eta_t \overline{P}_t + \left(1 - \mathbf{1}_t^S\right) P_t \right] H_{jt-1} \right\}. \tag{16}$$

When $\epsilon_{jt} < \epsilon_t^*$, entrepreneur j makes no real investment, i.e., $I_{jt} = 0$, and does not need to borrow. In equilibrium, all entrepreneurs are willing to hold any feasible amounts of $\overline{H}_{jt}$ and H_{jt}.

(ii) Tobin's Q and house prices in equilibrium satisfy

$$Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} \right], \tag{17}$$

$$\overline{P}_t = \underbrace{\overline{R}_t}_{rent} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[\underbrace{(1-\delta_h)\overline{P}_{t+1}}_{resale} + \underbrace{\theta_{t+1} \overline{P}_{t+1} \int_{\epsilon^*_{t+1}}^{\epsilon_{max}} \left(\frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon)}_{liquidity\ premium} \right], \tag{18}$$

$$\begin{aligned} P_t = {} & \underbrace{R_t}_{rent} + \beta(1-\delta_h) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \underbrace{\left\{ P_{t+1} \mathcal{F}(\epsilon^*_{t+1}) + \left[\mathbf{1}^S_{t+1} (1-\gamma) \eta_{t+1} \overline{P}_{t+1} + (1-\mathbf{1}^S_{t+1}) P_{t+1} \right] \left[1 - \mathcal{F}(\epsilon^*_{t+1}) \right] \right\}}_{resale} \\ & + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \underbrace{\theta_{t+1} \left\{ \left[\mathbf{1}^S_{t+1} (1-\gamma) \eta_{t+1} \overline{P}_{t+1} + (1-\mathbf{1}^S_{t+1}) P_{t+1} \right] \int_{\epsilon^*_{t+1}}^{\epsilon_{max}} \left(\frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon) \right\}}_{liquidity\ premium}, \end{aligned} \tag{19}$$

and the usual transversality conditions.

Proof: Please see Appendix B.1.

Part (i) of Proposition 2 describes an entrepreneur's investment decision. There exists a threshold ϵ^*_t for the idiosyncratic investment efficiency shock ϵ_{jt}. When the entrepreneur receives a favorable investment efficiency shock, i.e., $\epsilon_{jt} \geq \epsilon^*_t$, she finds it profitable to invest as much as possible, so she collateralizes all of her houses and exhausts her borrowing limit, as indicated by equation (16). When she receives an unfavorable investment efficiency shock, i.e., $\epsilon_{jt} < \epsilon^*_t$, she finds it unprofitable to invest and therefore does not borrow.

The main interest of this paper is in the two asset pricing equations, (18) and (19). In equilibrium, the marginal cost of purchasing one more unit of good (risky) houses, $\overline{P}_t$ (P_t), is equal to the marginal benefit of purchasing one more unit of good (risky) houses. For good houses, the benefit has three components: rent captured by $\overline{R}_t$, resale value captured by $(1-\delta_h)\overline{P}_t$ (net of depreciation), and the *liquidity premium* captured by $\theta_{t+1}\overline{P}_{t+1} \int_{\epsilon^*_{t+1}}^{\epsilon_{max}} \left(\frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon)$. While the first two terms are common in a standard user cost model, the last term appears only with collateral constraints. I label the last term the "liquidity premium," which can be interpreted as follows. When $\epsilon_{jt+1} \geq \epsilon^*_{t+1}$ in period $t+1$, the entrepreneur collateralizes all of her houses to finance real investment. Each unit of good houses supports a loan of $\theta_{t+1}\overline{P}_{t+1}$, which generates $\theta_{t+1}\overline{P}_{t+1}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)$ units of profit. Therefore, by serving as collateral, each unit of good houses generates $\theta_{t+1}\overline{P}_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)d\mathcal{F}(\epsilon)$ units of expected profit. When $\epsilon_{jt+1} < \epsilon^*_{t+1}$, the entrepreneur does not invest and the borrowing constraint is not binding. As a result, the liquidity premium reflects the option value of houses expanding the entrepreneur's borrowing limit if needed.

The liquidity premium of risky houses in equation (19) is interpreted analogously, but depends on the lending regime. If period $t+1$ is under the II regime, each unit of risky houses supports a loan of $\theta_{t+1}P_{t+1}$ and generates expected profit $\theta_{t+1}P_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)d\mathcal{F}(\epsilon)$; if period $t+1$ is under the IS regime, the expected profit becomes $(1-\gamma)\theta_{t+1}\eta_{t+1}\overline{P}_{t+1}$ $\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)d\mathcal{F}(\epsilon)$.

3.2 Liquidity and lending regimes

Using the two pricing equations above, I further prove that for the same risky collateral, allowing information opacity can sustain greater liquidity than mandating information transparency. Formally,

Proposition 3 *(i) Assume that there exists a deterministic steady state. Then $P_t \geq (1-\gamma)\eta_t\overline{P}_t$ around the steady state with equality if $\gamma = 0$.*

(ii) When collateral quality η_t is close to 1, the equilibrium regime is the II regime. Assume that γ is positive but not so large that the IS regime is possible, and that there are no other shocks. When collateral quality η_t is close to 1, the equilibrium regime is the II regime if $\eta_t > \eta^*_t$ and the IS regime otherwise, where η^*_t is endogenously determined.

Proof: Please see Appendix B.2.

Part (i) of this proposition differs from Lemma 2 in that the statement here holds for both regimes and not just for the II regime. It tells us that the market value of risky houses without information production is always higher than when information is produced. The underlying rationale is as follows. The intrinsic value of houses in this model has two components: providing housing services to households and providing a liquidity premium to entrepreneurs. Given the former component being equal, saving the information acquisition cost is equivalent to increasing the latter component, which in turn increases higher house prices. This intuition holds regardless of the current or steady-state regime. When there is no information acquisition cost, the gap between the endogenous value of risky collateral in the two regimes disappears.

Part (ii) of Proposition 3 demonstrates the endogenous lending regime enhances aggregate liquidity provision which varies with collateral quality shocks. Given that the total supply of houses is inelastic, aggregate liquidity is mainly dominated by house prices with the loan-to-value ratio θ_t fixed. When η_t is high, risky houses are likely to be good, implying high house prices and hence abundant aggregate liquidity. At the same time, bankers are unlikely to acquire costly information about collateral quality. According to Part (i) of Proposition 3, aggregate liquidity backed by risky houses in the II regime is even higher than that without considering endogenous lending regimes. Conversely, when η_t is low, the price of risky houses is low, leading to low liquidity provision. Moreover, in this case, bankers have a strong incentive to investigate the true quality of risky houses, and the IS regime is likely to be triggered. By part (i) of Proposition 3, aggregate liquidity provision supported by risky houses in this regime is further reduced. Therefore, along with collateral quality shocks, the lending regime reinforces liquidity provision by risky collateral.

3.3 Competitive equilibrium

Let $Y_t \equiv \int_0^1 Y_{jt} dj$ and $K_t \equiv \int_0^1 K_{jt} dj$ denote aggregate output and capital stock, respectively. A competitive equilibrium consists of sequences of aggregate quantities $\{C_t, I_t, N_t, Y_t, \overline{H}_t, H_t, K_t\}$ and prices $\{Q_t, W_t, R_{kt}, \overline{R}_t, R_t, \overline{P}_t, P_t, P_{kt}, \Lambda_t\}$ such that

(i) Households, entrepreneurs, bankers, and capital producers optimize.

(ii) The markets for labor, capital, and consumption goods all clear such that $N_t = \int_0^1 N_{jt} dj$,

$I_t = \int_0^1 I_{jt} dj$, and

$$Y_t = C_t + \left[1 + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2\right]\frac{I_t}{Z_t} + \mathbf{1}_t^S \gamma\eta_t \overline{P}_t H_t[1 - \mathcal{F}(\epsilon_t^*)], \tag{20}$$

where $\gamma\eta_t \overline{P}_t H_t[1 - \mathcal{F}(\epsilon_t^*)]$ is the total expenditure for information acquisition in the IS regime.

(iii) The markets for houses clear, such that $\overline{H}_t = \int_0^1 \overline{H}_{jt} dj$ and $H_t = \int_0^1 H_{jt} dj$.

(iv) The aggregate capital stock evolves as $K_t = (1-\delta)K_{t-1} + \int_0^1 \epsilon_{jt} I_{jt} dj$.

(v) The two types of houses evolve, as described by equations (4) to (7).

I relegate the full equilibrium equations of the competitive equilibrium to Appendix C. It should be noted that despite the presence of two regimes, given exogenous shocks and aggregate states, the equilibrium path is unique. This allows us to solve the deterministic steady state and local dynamics as for a standard real business cycle model. In the quantitative analysis of Section 4, I choose the II regime as the steady-state (default) regime and numerically verify the existence of a unique steady state of which the local dynamics are on a saddle path.

4 Estimation

I detrend the equilibrium system and present the detrended version in Appendix D. I then estimate the model using Bayesian methods (An and Schorfheide, 2007). A major obstacle to estimating such a model is that global solutions with endogenous regime switching are computationally costly and hinder likelihood-based estimation. I employ the Occ-Bin toolbox developed by Guerrieri and Iacoviello (2017), which effectively estimates a piecewise linear version of a nonlinear model, enabling the efficient estimation of an endogenous regime switching model. Atkinson, Richter, and Throckmorton (2020) further show that the estimation results of this toolbox are sufficiently accurate.[8]

[8] Atkinson, Richter, and Throckmorton (2020) compare the accuracy of two estimation methods. The first method estimates a fully nonlinear model using a particle filter and the second method estimates a piecewise linear version of the nonlinear model using the Occ-Bin toolbox. The authors find that these two methods produce similar estimates. Using a second-order perturbation method, Benigno, Foerster, Otrok, and Rebucci (2020) also estimate a DSGE model with an occasionally binding constraint.

4.1 Parameter estimates

I divide the parameters into two categories. The first category contains parameters that I calibrate to some stylized facts or to standard values from the literature. The second category contains parameters that I estimate using quarterly data on the U.S. macroeconomy and house prices from 1975Q1 to 2019Q4.

Calibration I set the growth rate $g = 0.005$ to match the average annual growth rate of real GDP per capita at 2% according to the U.S. National Income & Product Accounts (NIPA). β is set to 0.995, implying a steady-state annual real interest rate of 2%. I use standard values from the real business cycle literature for $\kappa = 2$, $\alpha = 0.36$, and $\delta = 0.03$. I set $\nu = 6.5$, which lies within the empirical range in related macroeconomic and microeconomic studies (Chetty et al., 2011). Set the utility weight on labor ψ_n such that the steady-state labor is equal to 1/3. Set $\epsilon_{min} = (\xi - 1)/\xi$ such that $\mathbb{E}(\epsilon) = 1$. I choose ψ_h to replicate the imputed rental of owner-occupied non-farm housing at 10% of personal consumption annual expenditure based on NIPA. The house depreciation rate δ_h is set to match an annual housing stock depreciation rate of 1.5%, which is also estimated from NIPA (Kaplan, Mitman, and Violante, 2019; Garriga, Manuelli, and Peralta-Alva, 2019). The share of good houses in the default regime is set to 0.2, which corresponds to the share of non-securitized mortgage loans in all mortgage origination during the housing boom of the 2000s (The Financial Crisis Inquiry Report, 2011) because all kinds of securitized mortgage loans were risky during the crisis. I choose the value of average collateral quality η to match the average pre-crisis charge-off rate on real estate loans of all commercial banks, as reported by the U.S. Board of Governors of the Federal Reserve System. This value ensures that the default regime is the II regime, that is, condition (1) holds in the steady state. All of the calibrated parameters are listed in Table 1.

Estimation The model contains six aggregate shocks: productivity shocks, housing demand shocks, labor supply shocks, collateral quality shocks, financial shocks, and aggregate IST shocks. These shocks match the six quarterly time series of the data: real consumption per

Table 1: Calibrated Parameters

Parameter	Value	Description
g	0.005	Average quarterly growth rate of aggregate productivity
β	0.995	Discount factor
κ	2	Intertemporal elasticity of substitution
α	0.36	Share of capital in production
δ	0.03	Capital depreciation rate
δ_h	0.004	House depreciation rate
ψ_h	0.1	Utility weight on housing services
ν	6.5	Inverse of the Frisch elasticity
ξ	5.6	Shape of the distribution of idiosyncratic investment efficiency
$\overline{H}$	0.2	Fraction of good houses
η	0.974	Average collateral quality

capita, real investment per capita, hours worked per capita, real house prices, the price-rent ratio, and the National Financial Conditions Index (NFCI). Except for the NFCI, all of these observables are in logarithms. All of the time series used in the estimation are demeaned.

Consumption is measured by the sum of non-housing services and non-durable goods. Investment is measured by the sum of private investment in software, equipment, structures, residential investment, and expenditure on durable goods. Consumption and investment are divided by the GDP deflator. All of these data are obtained from NIPA. Hours worked are measured by the hours of all persons in the non-farm business sector. To scale by population, I use the quarterly averages of the civilian non-institutional population. The last two variables are obtained from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis.

I use real and nominal house prices constructed by the Bank for International Settlements, based on data from the Federal Reserve, CoreLogic data, and the National Association of Realtors.[9] Rents are owner-equivalent imputed rents reported by NIPA.[10] The price-rent ratio is measured by nominal house prices divided by nominal rents.

[9] My results are also qualitatively and quantitatively robust to other sources of house prices such as the Federal Housing Finance Agency and Case Shiller Home Price Indexes.

[10] According to Favilukis, Ludvigson, and Nieuwerburgh (2017), the correlation between renters' rent and owners' equivalent rent is very high.

I follow Bigio (2015) and split the sample into two segments for detrending.[11] For the pre-crisis sample, I use a one-sided Hodrick-Prescott (HP) filter with a smoothing parameter of 100,000 to detrend the time series (except for the NFCI, which does not have a trend). Unlike the two-sided HP filter, this filter is unaffected by the correlation between current and subsequent observations and is suitable for estimation (Stock and Watson, 1999; Guerrieri and Iacoviello, 2017). For the sample that begins with the Great Recession, I use a linear trend. Figure 1 plots the detrended output, house prices, and the price-rent ratio.

I use the NFCI as a proxy for the overall condition of financial markets in the U.S. The NFCI of Chicago Federal Reserve is a weighted average of various financial activity variables and a comprehensive index of financial conditions in the U.S. Positive NFCI values are historically associated with tighter-than-average financial conditions, whereas negative values are historically associated with looser-than-average financial conditions. To map this index into the model, I adopt the method used by Miao, Wang, and Xu (2015) and define the following measurement equation

$$\text{NFCI}_t = -F_1 \left[\ln(\theta_t) - \ln(\theta)\right] - F_2 \left[\ln(P_{ht}) - \ln(P_h)\right], \tag{21}$$

where $F_1 > 0$ and $F_2 > 0$ are the estimated coefficients. This measurement equation is motivated by the collateral constraint, which indicates that the ease of external financing varies along two orthogonal dimensions: financial tightness θ_t and collateral value P_{ht}. The latter is the house price index in the model defined below and P_h is the steady-state value of P_{ht}. Either a relaxation in θ_t or an increase in P_{ht} reduces the measured NFCI.

The house price index in the model is defined as the weighted average of the prices of good and risky houses, i.e.,

$$P_{ht} \equiv \overline{P}_t \cdot \frac{\overline{H}_t}{\overline{H}_t + H_t} + P_t \cdot \frac{H_t}{\overline{H}_t + H_t}. \tag{22}$$

Similarly, the price-rent ratio in the model is the weighted average of the price-rent ratios of

[11] According to Bigio (2015), detrending the full sample universally will pull down the pre-Great Recession trend for output, leading to a positive cycle at the beginning of the recession.

different types of houses, i.e.,

$$\left(\frac{P}{R}\right)_{ht} \equiv \frac{\overline{P}_t}{\overline{R}_t} \cdot \frac{\overline{H}_t}{\overline{H}_t + H_t} + \left[\frac{P_t}{R_t} \cdot \eta_t + 0 \cdot (1 - \eta_t)\right] \frac{H_t}{\overline{H}_t + H_t} = \frac{P_{ht}}{MU_{ht}}, \tag{23}$$

where MU_{ht} is a representative household's marginal utility of housing. This definition is consistent with that in Favilukis, Ludvigson, and Nieuwerburgh (2017).

Priors and posteriors The parameters I estimate include $\{F_1, F_2, \omega, \chi, \Omega, \theta, \gamma\}$, and those governing the stochastic processes of shocks $\{\rho_a, \sigma_a, \rho_h, \sigma_h, \rho_n, \sigma_n, \rho_\eta, \sigma_\eta, \rho_\theta, \sigma_\theta, \rho_z, \sigma_z\}$. I set the prior distributions of these estimated parameters to be close to those used in Smets and Wouters (2007), Liu, Wang, and Zha (2013), and Miao, Wang, and Xu (2015), which cover the majority of empirical estimates in the literature. I set a rather diffuse prior for the elasticity between consumption goods and housing services χ because its range is dispersed in related studies.[12]

In Table 2, I report the priors and posteriors of all estimated parameters. I compute the means and 5th and 95th percentiles of the posterior distributions using the Metropolis-Hastings algorithm with 50,000 draws. I have verified that the estimates of these parameters are robust and insensitive to the prior distributions.

I use the posterior mode as the parameter value for all the following results. The posterior mode of habit formation ω is estimated to be 0.14, which is close to the estimated value reported in Miao, Wang, and Zha (2020). The posterior mode of χ is 19.6. This large elasticity of substitution between consumption and housing services is primarily driven by the smooth dynamics of rents in the data. With this estimate, the model-simulated expenditure on housing services as a share of total household expenditure is fairly stable, which is consistent with the stylized fact reported in Davis and Nieuwerburgh (2015) and Piazzesi and Schneider (2016). The parameter for the capital adjustment cost Ω is estimated to be 0.15, which is similar to the

[12] Flavin and Nakagawa (2008) estimate an elasticity of less than 0.2. Davis and Nieuwerburgh (2015) report this elasticity to be around one. Based on empirical evidence in Piazzesi, Schneider, and Tuzel (2007), this elasticity should be greater than unity. According to Davis and Martin (2005), the macro-based estimate of this elasticity is significantly greater than unity.

Table 2: Estimated Parameters

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	S.D.	Mode	Mean	5%	95%
F_1	Gamma	2	2	1.655	3.731	1.605	4.859
F_2	Gamma	2	2	0.129	0.154	0.021	0.318
ω	Beta	0.5	0.2	0.138	0.130	0.060	0.204
χ	Gamma	10	5	19.644	19.164	17.900	19.942
Ω	Gamma	2	2	0.150	0.129	0.060	0.215
θ	Beta	0.5	0.2	0.896	0.738	0.529	0.903
γ	Gamma	0.05	0.02	0.0439	0.0624	0.0436	0.0858
ρ_a	Beta	0.5	0.2	0.970	0.969	0.958	0.978
ρ_h	Beta	0.5	0.2	0.974	0.977	0.954	0.994
ρ_n	Beta	0.5	0.2	0.945	0.951	0.934	0.968
ρ_η	Beta	0.5	0.2	0.986	0.983	0.976	0.990
ρ_θ	Beta	0.5	0.2	0.988	0.933	0.881	0.985
ρ_z	Beta	0.5	0.2	0.961	0.964	0.947	0.979
σ_a (%)	Inv. Gamma	1	Inf	1.281	1.364	1.246	1.488
σ_h (%)	Inv. Gamma	1	Inf	0.625	0.656	0.600	0.722
σ_n (%)	Inv. Gamma	1	Inf	7.448	7.457	6.797	8.161
σ_η (%)	Inv. Gamma	1	Inf	4.282	4.394	3.879	4.959
σ_θ (%)	Inv. Gamma	1	Inf	2.151	1.056	0.689	2.231
σ_z (%)	Inv. Gamma	1	Inf	0.880	0.861	0.756	0.983

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 50,000 draws.

value estimated by related studies (Christiano, Eichenbaum, and Evans, 2005; Liu, Wang, and Zha, 2013; Miao, Wang, and Xu, 2015). The average financial condition θ is estimated to be 0.896, which is in line with the average loan-to-value ratio reported in Favilukis, Ludvigson, and Nieuwerburgh (2017), Guerrieri and Iacoviello (2017), and Garriga, Manuelli, and Peralta-Alva (2019). The estimated value of the information acquisition cost γ is 4.4%. Although there is no widely accepted empirical measure for this variable, the estimated value here is small compared with monitoring costs used in previous studies (Carlstrom and Fuerst, 1997; Christiano, Motto, and Rostagno, 2014).

Among the six aggregate shocks, collateral quality shocks show the second-largest standard deviation at the posterior mode, the unconditional standard deviation being about 5 times as large as that of productivity shocks. Interestingly, the estimated housing demand shocks have the smallest standard deviation, which is different from the findings reported in Iacoviello (2005), Liu, Wang, and Zha (2013), and Guerrieri and Iacoviello (2017).

4.2 Business cycle moments

I simulate the model using the calibrated and estimated parameters and compare the model-generated business cycle moments with those obtained from the data. The second and third columns of Table 3 summarize these moments. All of the variables are in logarithms and are HP filtered with a smoothing parameter of 1,600.

The model does a good job at delivering real business cycle moments compared with their counterparts in the data. In particular, the model- and data-implied house prices and price-rent ratios are close and about twice as volatile as output. Both house prices and the price-rent ratio are positively correlated with output. They are all consistent with patterns (i) - (iii) illustrated in Figure 1. The other real business cycle moments are also matched reasonably well. One drawback is that the first-order autocorrelations of house prices and the price-rent ratio are smaller than the data. However, this drawback is not due to my consideration of collateral quality shocks or regime switching, because when I exclude collateral quality shocks or regime switching, this moment still remains low, as shown in the fourth and fifth columns.

To see the contribution from fluctuations in collateral quality and from regime switches separately, in the fourth column I report the moments from a reference model in which I shut down the regime switching and set the II regime to always be the equilibrium regime. While other moments do not change substantially, housing market volatility falls by a quarter. When I further shut down collateral quality shocks in the fifth column, housing market volatility falls by half. This result reveals that collateral quality shocks and the associated regime switches are both important for exaggerating house price volatility and to roughly the same extent.

I also control for housing demand shocks and financial shocks and report the results in the

Table 3: Real Business Cycle Moments

Moment	Data	Benchmark	No regime switching	No quality shock	No housing demand shock	No financial shock
			Standard deviation			
output (%)	1.75	1.47	1.46	1.46	1.47	1.47
consumption	0.49	0.69	0.69	0.68	0.69	0.69
investment	2.25	2.49	2.46	2.44	2.48	2.48
labor	0.99	0.96	0.96	0.96	0.96	0.96
house price	1.78	1.84	1.37	0.88	1.83	1.83
price-rent ratio	1.86	1.98	1.51	0.92	1.94	1.97
			Correlation with output			
consumption	0.83	0.96	0.96	0.97	0.96	0.96
investment	0.97	0.83	0.85	0.86	0.83	0.83
labor	0.76	0.69	0.69	0.69	0.69	0.69
house price	0.55	0.40	0.54	0.84	0.40	0.40
price-rent ratio	0.57	0.35	0.47	0.77	0.36	0.35
			Autocorrelation			
output	0.90	0.73	0.72	0.72	0.73	0.73
consumption	0.83	0.73	0.73	0.73	0.73	0.73
investment	0.91	0.74	0.74	0.74	0.74	0.74
labor	0.93	0.71	0.71	0.71	0.71	0.71
house price	0.96	0.66	0.65	0.59	0.66	0.66
price-rent ratio	0.96	0.67	0.67	0.61	0.66	0.67

Note: (i) All of the variables are in logs and are HP filtered with a smoothing parameter of 1,600. I simulate the model for 15,000 periods and drop the first 5,000 periods. I then compute sample moments accordingly. I run the simulation 1,000 times and report the sample average.
(ii) Row 1 under "Standard deviation" denotes the standard deviation of output in percentage points. Rows 2 to 6 denote the standard deviations of consumption, investment,labor, house prices, and the price-rent ratio relative to that of output, respectively.
(iii) Column 2 displays the real business cycle moments from the data. Column 3 displays the moments from the benchmark model. Column 4 displays the moments from the reference model (without regime switching).
(iv) Columns 5 to 7 display the moments from variants for which I shut down collateral quality shocks, housing demand shocks, and financial shocks, respectively.

sixth and seventh columns, respectively. Eliminating either of these two types of shocks has little quantitative effect on the moments, indicating the limited roles of these shocks in this model. I discuss this result in greater depth in Section 5.2.

4.3 Impulse response

I now use impulse response functions to illustrate how collateral quality shocks account for the data patterns. I hit the economy with a two-standard-deviation negative quality shock, which is large enough to trigger a regime switch from II to IS, and plot the impulse responses in Figure 3. All else being equal, the solid line in each panel represents the impulse response of the benchmark model and the dashed line represents the impulse response of the reference model.

Following the negative quality shock, house prices decrease, which reduces external financing and hence investment. It follows that output and labor decrease. A negative quality shock also implies a decrease in good houses, which slightly raises a representative household's marginal utility of housing. This results in a decrease in the price-rent ratio of a similar magnitude as for that of house prices and significantly greater than for that of output. These responses confirm the data patterns. Consumption also falls due to the complementarity between consumption goods and housing services.

Clearly, as the regime switch causes abrupt changes and exacerbates the volatilities of the key variables, the responses of the benchmark model are much stronger than those of the reference model. As collateral quality recovers, the economy eventually returns to its default regime.

Moreover, impulse responses to collateral quality shocks exhibit asymmetries, as hinted by Figure 3. First, for a small negative shock that does not trigger a regime switch, the responses resemble the dashed lines. For a large negative shock that is sufficient to trigger a regime switch, macroeconomic volatility abruptly and sharply increases. Second, for a large positive collateral quality shock, condition (1) is never violated and no regime switch occurs. As a result, the adverse impact of a large negative quality shock can be greater than that of an equally large positive shock.

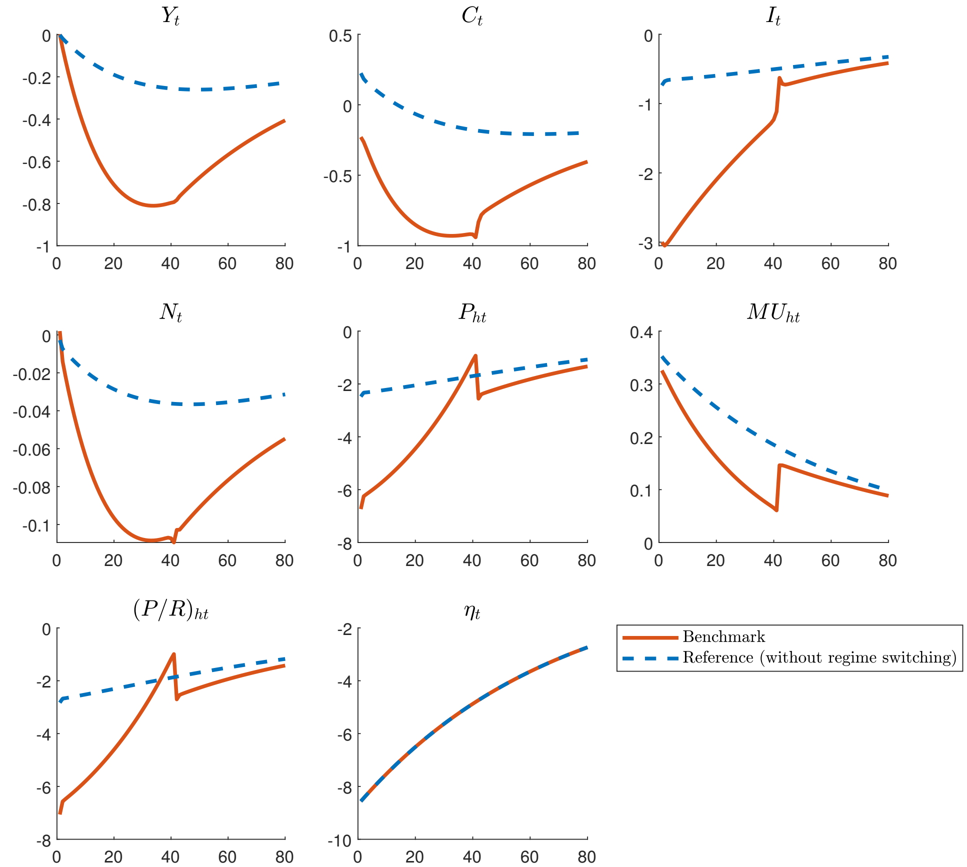

Figure 3: Impulse responses to a negative two-standard-deviation collateral quality shock. The solid lines stand for the benchmark model and the dashed lines for the reference model (without regime switching). All of the variables are detrended and expressed as percentage deviations from their steady-state values.

5 Discussion

This section provides deeper insights into the model by exploring various aspects of its performance. I first illustrate why collateral quality shocks can rationalize the salient features of the data, and then analyze the role of other relevant shocks and examine the relative importance of the different shocks using historical decomposition. I next identify the endogenous regime switch in the estimated sample and finally, check the validity of financial tightness predicted by the model.

5.1 Transmission mechanism

Among the six types of estimated shocks, productivity shocks, labor supply shocks, and IST shocks are commonly considered in the real business cycle literature, while housing demand shocks, financial shocks, and collateral quality shocks are specifically relevant to this study. I now compare the implications of the latter three shocks and illustrate why collateral quality shocks outperform the other two types of shocks in rationalizing the patterns observed in the data.

From a theoretical point of view, the standard approach to pricing houses treats houses as assets that generate rents as dividends. House prices are therefore equal to the sum of the expected present values of future rents, which are a representative household's marginal utility of housing (Favilukis, Ludvigson, and Nieuwerburgh, 2017). Thus, using the standard asset pricing approach, house prices can be expressed as

$$P_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} (1-\delta_h)^{\tau} MU_{ht+\tau}. \tag{24}$$

This channel is known as the "*house rent channel*."[13] Nonetheless, if this channel is dominant in determining house prices, then there a puzzle arises: house prices and rents should move at roughly similar rates, or house prices should not be significantly more volatile than rents, which contradicts the stylized fact that house prices are significantly more volatile than rents.

In this model, asset pricing equation (19) emphasizes another channel that is associated with collateral quality. To see that, I rewrite (19) as

$$P_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\Lambda_{t+\tau}}{\Lambda_t} \left(\Pi_{n=0}^{\tau} \Gamma_{t+n}\right) \eta_{t+\tau} MU_{ht+\tau}, \tag{25}$$

[13] Aside from rents, the only other variable that can move house prices through this channel is the discount factor, which Miao, Wang, and Zha (2020) harness to explain movements in the price-rent ratio of commercial real estate.

where

$$\Gamma_t = (1-\delta_h)\mathcal{F}(\epsilon_t^*) + (1-\delta_h)\left[\mathbf{1}_t^S(1-\gamma)\eta_t\frac{\overline{P}_t}{P_t} + (1-\mathbf{1}_t^S)\right]\left[1-\mathcal{F}(\epsilon_{t+1}^*)\right]$$
$$+\theta_t\left[\mathbf{1}_t^S(1-\gamma)\eta_t\frac{\overline{P}_t}{P_t} + (1-\mathbf{1}_t^S)\right]\int_{\epsilon_t^*}^{\epsilon_{max}}\left(\frac{Q_t}{P_{kt}}\epsilon - 1\right)d\mathcal{F}(\epsilon). \tag{26}$$

Equation (25) explains why collateral quality shocks affect house prices more than rents. First, compared with (24), the presence of collateral quality shocks η_t in (25) directly affects the price of risky houses P_t and consequently, the house price index P_{ht} (defined in equation (22)), but affects a representative household's marginal utility of housing MU_{ht} only through the equilibrium effect. Second, the term Γ_t contains both the resale value and liquidity premium. They both depend on the equilibrium lending regime, which in turn is disturbed by collateral quality shocks, as reflected by $\left[\mathbf{1}_t^S(1-\gamma)\eta_t\frac{\overline{P}_t}{P_t} + (1-\mathbf{1}_t^S)\right]$. As a result, absent in previous studies where collateral quality is perfect, collateral quality shocks have a stronger impact on house prices than on rents, thereby amplifying fluctuations in the price-rent ratio. They help resolve the puzzle and explain patterns (i) and (ii) in Figure 1.[14]

Regarding pattern (iii), as (25) shows, high collateral quality corresponds to high house prices. This leads to an expansion of aggregate liquidity, which in turn stimulates investment, and hence, output and vice versa. Furthermore, as Proposition 3 demonstrates, the II regime is generally associated with high collateral quality, whereas the IS regime is generally associated with low collateral quality. Variations in the lending regime further exaggerate the expansions and contractions of aggregate liquidity. These two effects cause comovements between house prices, the price-rent ratio, and output, which explains pattern (iii).

[14] Greenwald and Guren (2021) find that segmentation between borrowers' and savers' housing stocks is key to explaining the price-rent ratio and ownership in the data. The authors empirically document that the tenure supply curve ("the relative price schedules at which landlords are willing to supply owned relative rented housing at a given amount of total housing supply") is much closer to perfect inelasticity than to perfect elasticity, suggesting that the reality is much closer to the full segmentation scenario. This model does not consider ownership, so the supply curve here is perfectly inelastic, which is consistent with the finding in Greenwald and Guren (2021).

5.2 Comparison of shocks

Through the lens of the transmission mechanism discussed above, I now analyze the implications of housing demand shocks and financial shocks, both of which have received much attention in the literature.

Housing demand shock Figure 4 plots the impulse responses to a negative two-standard-deviation housing demand shock. When housing demand decreases, rents decrease, in turn causing house prices to decrease. Because the housing demand shock affects house prices mainly through rent streams, and rents in future periods are expected to recover, the decrease in house prices is smaller than that in rents in the impact period. This implies that the price-rent ratio increases counterfactually. In other words, housing demand shocks cannot generate a positive correlation between house prices and the price-rent ratio.

The counterfactual implication explain why housing demand shocks are estimated to be small. Although this result appears to contrast Iacoviello (2005), Liu, Wang, and Zha (2013), and Guerrieri and Iacoviello (2017), the discrepancy is not surprising as these studies do not consider the price-rent ratio. Liu, Wang, and Zha (2021) provide a microeconomic foundation for housing demand shocks in a theoretical framework. They argue that variations in the liquidity premium, instead of the reduced form of housing demand shocks, can reconcile the disconnect between house prices and rents; this is consistent the transmission mechanism here. Therefore, the simultaneous consideration of house prices and the price-rent ratio is important to quantitatively underpinning the sources of house price fluctuations.

Financial shock Figure 5 illustrates the impulse responses to a negative two-standard-deviation financial shock. Given the value of collateral, the negative financial shock tightens credit constraints and reduces investment and output. However, as financial conditions tighten, the demand for collateral increases. As a result, house prices rise, implying that they are countercyclical. This counterfactual implication explains the limited role of financial shocks in this model. This implication arises because financial shocks capture changes in the credit market that are

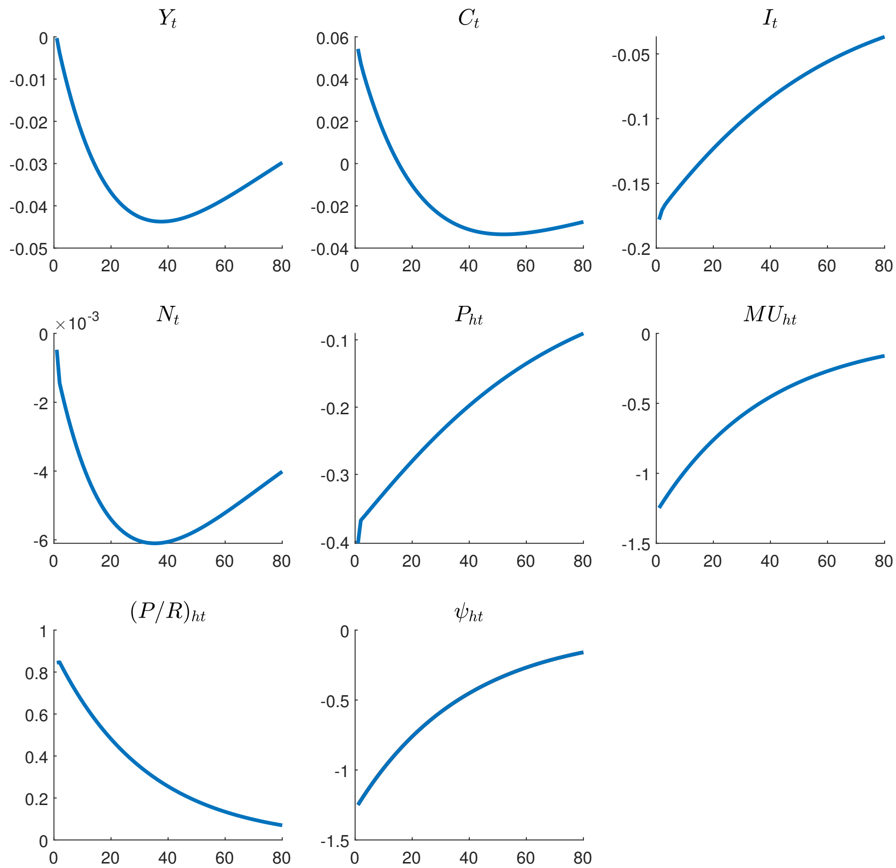

Figure 4: Impulse responses to a negative two-standard-deviation housing demand shock. All of the variables are detrended and expressed as percentage deviations from their steady-state values.

orthogonal to the value of collateral, which helps distinguish financial shocks from collateral quality shocks in this model.

This result echoes a number of recent studies, such as Kiyotaki, Michaelides, and Nikolov (2011), Sommer, Sullivan, and Verbrugge (2013), Justiniano, Primiceri, and Tambalotti (2015, 2019), and Kaplan, Mitman, and Violante (2019), all of which find that changes in credit conditions alone have limited effects on house prices with various models. However, one should not interpret the result here as negating the importance of changes in collateralized debt to the Great Recession. In the current study, changes in collateralized debt are decomposed into changes in collateral value and orthogonal changes in financial tightness (loan-to-value ratios).

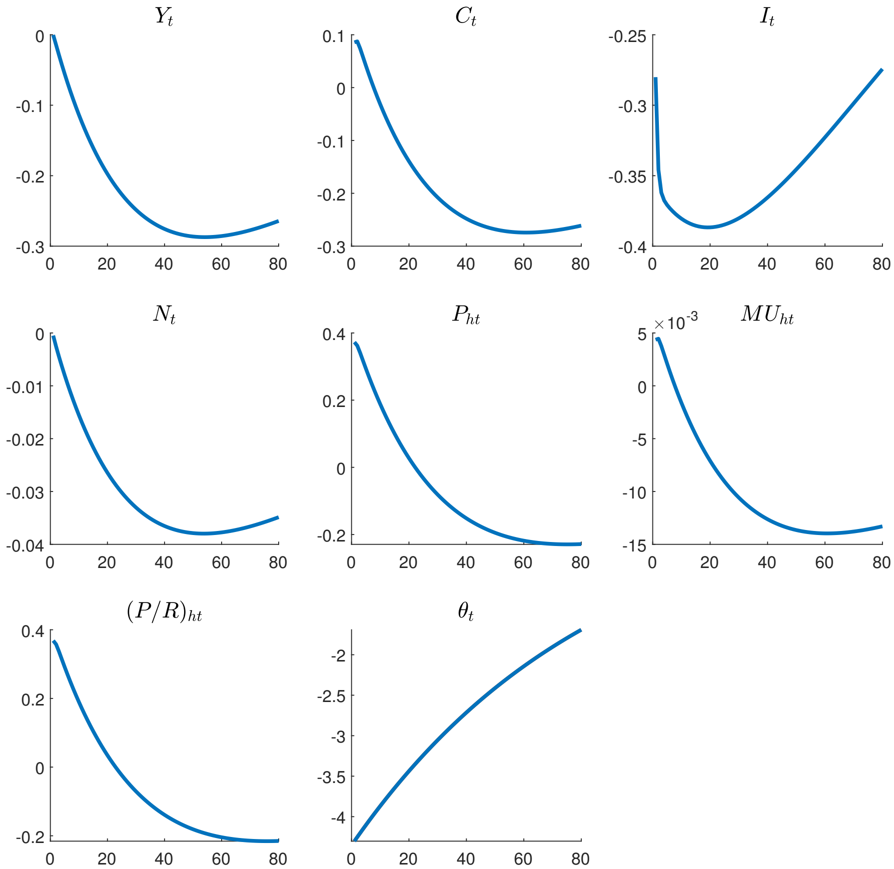

Figure 5: Impulse responses to a negative two-standard-deviation financial shock. All of the variables are detrended and expressed as percentage deviations from their steady-state values.

The result shows that the former plays a more important role than the latter in terms of explaining house price fluctuations.[15]

One caveat is that the Occ-Bin toolbox used estimates a linear model with a Kalman filter. Therefore, it cannot accommodate nonlinear features such as risk premia which, as Favilukis,

[15]As analyzed above, neither housing demand shocks nor financial shocks *alone* are likely to dominate the dynamics of output, house prices, and the price-rent ratio on their own. Here, I intuitively explain why these two types of shocks are unlikely to explain the patterns *jointly*. If these two types of shocks can jointly explain the patterns, then the magnitudes of the shocks would be jointly determined such that their counterfactual effects are offset by each other. However, as house prices and the price-rent ratio track each other closely in the data (Figure 1), a feature that goes strongly against the implication of housing demand shocks, the housing demand shocks must be small. It immediately follows that financial shocks cannot be significant either, as otherwise housing demand shocks would not be sufficient to reverse the counterfactual impact of financial shocks. For this reason, the dynamics in the data are unlikely to be attributed to housing demand shocks, financial shocks, or a combination of the two.

Ludvigson, and Nieuwerburgh (2017) demonstrate, is a key factor of how financial shocks affect house price fluctuations. I also do not include rich heterogeneity among agents, which the existing literature (e.g., Favilukis, Ludvigson, and Nieuwerburgh, 2017) view as another key factor that contributes to the housing boom-bust cycle. These ingredients are ignored in this model.

5.3 Historical decomposition

In this section, I assess the relative importance of the various shocks in accounting for the macroeconomy's historical path, especially during the Great Recession. Figure 6 presents the decomposition of house prices, the price-rent ratio, and investment in the estimated model. The shocks are marginalized in the following order: (i) collateral quality shocks, (ii) productivity shocks, (iii) housing demand and financial shocks, (iv) labor supply shocks, and (v) aggregate IST shocks. The height of a single-color column represents the marginal contribution of the corresponding shocks to a variable in a period, with the marginal contribution of all shocks adding up to the observed time series.

The decomposition shows that collateral quality shocks explain the largest share of movements in house prices and the price-rent ratio, as well as a conspicuous deterioration in collateral quality from the housing boom to the bust of the 2000s. During the housing boom, collateral quality shocks accounted for 50% to 60% of the variations in house prices and the price-rent ratio caused by all of the shocks in the model. They were also responsible for 20% to 25% of the variations in investment. During the Great Recession, taking 2009Q2 as an example, collateral quality shocks accounted for 19% out of the 28% decline in house prices, 21% out of the 29% decline in the price-rent ratio, and 6% out of the 29% decline in investment, respectively. Overall, collateral quality shocks contributed more than half of the variations in house prices and the price-rent ratio and roughly one fifth of the variations in investment during the housing boom and bust.

Productivity shocks also matter for the crisis, consistent with the quantitative findings in

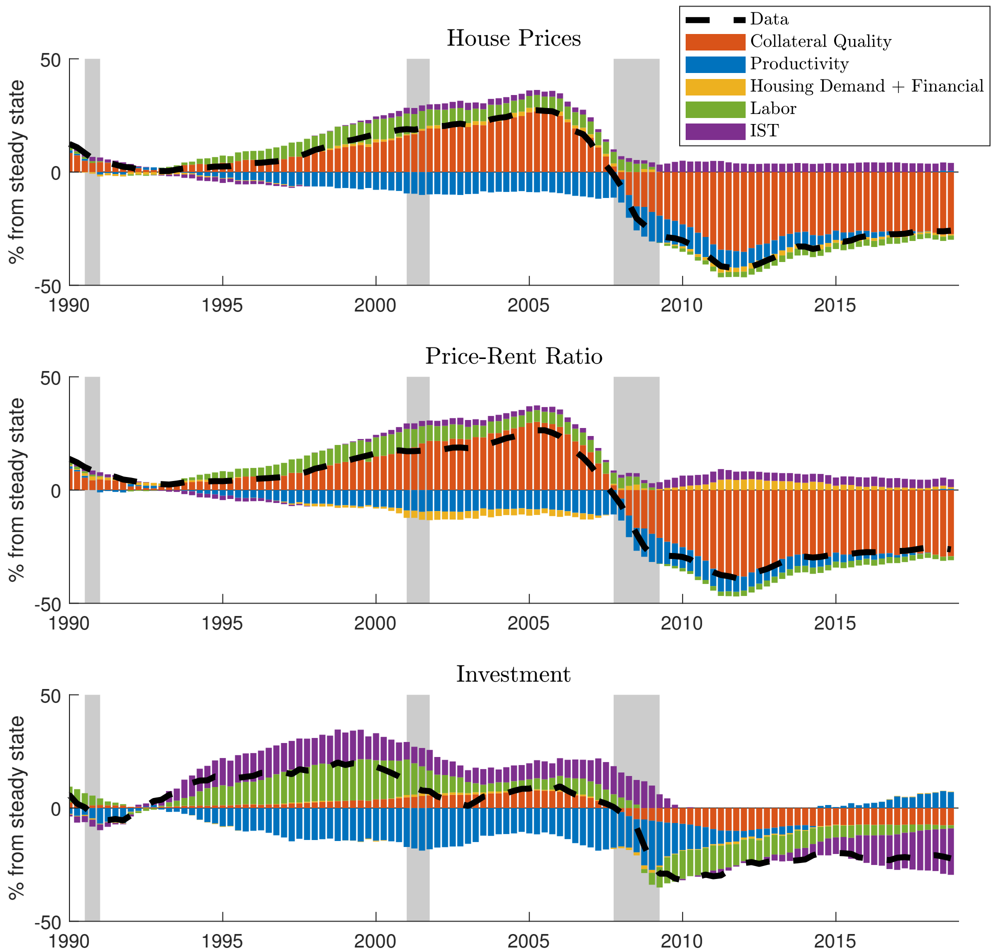

Figure 6: Historical decomposition of house prices, the price-rent ratio, and investment in the estimated model. All of the series are shown as deviations from their steady-state values. Shaded areas indicate recessions determined by the National Bureau of Economic Research (NBER).

Bigio (2015) and Favilukis, Ludvigson, and Nieuwerburgh (2017). The remaining variations in investment are attributed to IST and labor supply shocks. The contribution of housing demand and financial shocks is minor, thus reaffirming our discussion in the previous section.

5.4 The role of regime switching

To evaluate the role of regime switching per se, I estimate the reference model that excludes regime switching. The reference and benchmark models share the same calibrated parameter values and prior distributions for the same set of estimated parameters, making two models equally likely a priori. I report the estimated parameters of the reference model in Table 4 and compare the fitness of the two models in Table 5. The log marginal density of the data for the benchmark and reference models is 3,012.8 and 2,983.3, respectively, indicating that the data favor the benchmark model over the reference model.

Table 4: Estimated Parameters of the Reference Model

Parameter	Prior Distribution			Posterior Distribution			
	Distr.	Mean	S.D.	Mode	Mean	5%	95%
F_1	Gamma	2	2	1.506	2.330	1.574	3.184
F_2	Gamma	2	2	0.158	0.081	0.053	0.205
ω	Beta	0.5	0.2	0.182	0.138	0.076	0.207
χ	Gamma	10	5	19.394	19.136	18.052	19.931
Ω	Gamma	2	2	0.140	0.110	0.051	0.179
θ	Beta	0.5	0.2	0.895	0.837	0.669	0.965
γ	Gamma	0.05	0.02	0.0210	0.0290	0.0223	0.0402
ρ_a	Beta	0.5	0.2	0.979	0.974	0.965	0.982
ρ_h	Beta	0.5	0.2	0.993	0.987	0.976	0.997
ρ_n	Beta	0.5	0.2	0.963	0.953	0.936	0.967
ρ_η	Beta	0.5	0.2	0.994	0.985	0.980	0.991
ρ_θ	Beta	0.5	0.2	0.995	0.924	0.846	0.991
ρ_z	Beta	0.5	0.2	0.973	0.969	0.954	0.983
σ_a (%)	Inv. Gamma	1	Inf	1.256	1.350	1.228	1.471
σ_h (%)	Inv. Gamma	1	Inf	0.643	0.670	0.614	0.732
σ_n (%)	Inv. Gamma	1	Inf	7.116	7.469	6.819	8.221
σ_η (%)	Inv. Gamma	1	Inf	4.242	5.151	4.506	5.803
σ_θ (%)	Inv. Gamma	1	Inf	2.276	1.566	1.060	2.259
σ_z (%)	Inv. Gamma	1	Inf	0.973	0.876	0.773	0.998

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 50,000 draws.

Figure 3 also offers a support for the superior performance of the benchmark model. Without regime switching, the reference model relies on larger shocks to fit the declines observed during the Great Recession, resulting in a lower marginal density relative to the benchmark model.

Table 5: Comparison of model fitness

	log marginal density of the data	log posterior likelihood (at the posterior mode)
Benchmark	3,023.0	3,110.1
Reference	2,993.1	3,083.9

Note: The reference model denotes the model for which I shut down regime switching and set the default regime as the equilibrium regime.

I feed the estimated sequences of shocks into these two models to examine whether the model can identify an endogenous regime switch in the estimated sample. Figure 7 plots the paths, with the solid lines, dashed lines, and "plus" signs denoting the benchmark model, reference model, and data, respectively.

The fitted lines of the benchmark model in all of the panels precisely track the actual series as designed. The lines from the benchmark and reference models coincide until the beginning of the Great Recession and diverge thereafter. This divergence marks an endogenous regime switch in the sample. Moreover, the divergence between the solid and dashed lines suggests that had the regime switch not occurred, house prices would not have decreased as much as they did. Analogously, the price-rent ratio, investment, and consumption would have been higher in the absence of the regime switch.

The gaps between the solid and dashed lines measure the contribution of the regime switch per se to the declines in these variables. As of 2014Q3, when the U.S. ended QE3, the regime switch caused an additional 3.5% decline in investment, a 1.2% decline in consumption, a 12.4% decline in house prices, and a 12.3% decline in the price-rent ratio. These gaps are considerable compared to the declines in these variables throughout the entire recession (2007Q4 to 2009Q2), where were 29%, 5%, 27%, and 27%, respectively. In summary, the regime switch per se had a non-negligible adverse impact on the macroeconomy during the crisis.

Figure 7: Historical paths implied by the models and data. The solid lines stand for the benchmark model, the dashed lines for the reference model (without regime switching), and the dotted-dashed lines for the data. Shaded areas indicate recessions determined by NBER.

5.5 Model-predicted financial tightness

As discussed in Section 5.2, so far the literature has not reached a consensus on the effectiveness of financial shocks in explaining the credit boom-bust cycle. This study speaks to this literature by decomposing changes in the credit market into two orthogonal dimensions: changes in collateral value (i.e., $\overline{P}_t$ and $\tilde{P}_t$ in (13)) and changes in financial tightness (i.e., θ_t in (13)). Although I previously focused on the first dimension, I now examine whether my estimated model provides a reasonable prediction for the second dimension. Specifically, I check whether the model generates a reasonable historical path of financial tightness θ_t. For the counterpart of financial tightness in the data, I do not use the NFCI because (i) the NFCI is not a pure measure of financial tightness. The construction of the NFCI includes not only financial tightness indexes (corresponding to θ_t in the model), but also house price indexes (corresponding to P_{ht} in the model), as revealed by the measurement equation (21). (ii) The NFCI is already designed to be fitted in the estimation. Instead, I use the net percentage of domestic banks tightening standards for commercial real estate loans released by the Senior Loan Officer Survey of the U.S. Board of Governors of the Federal Reserve System. This indicator is a direct measure of financial tightness and is not purposely targeted in the estimation.

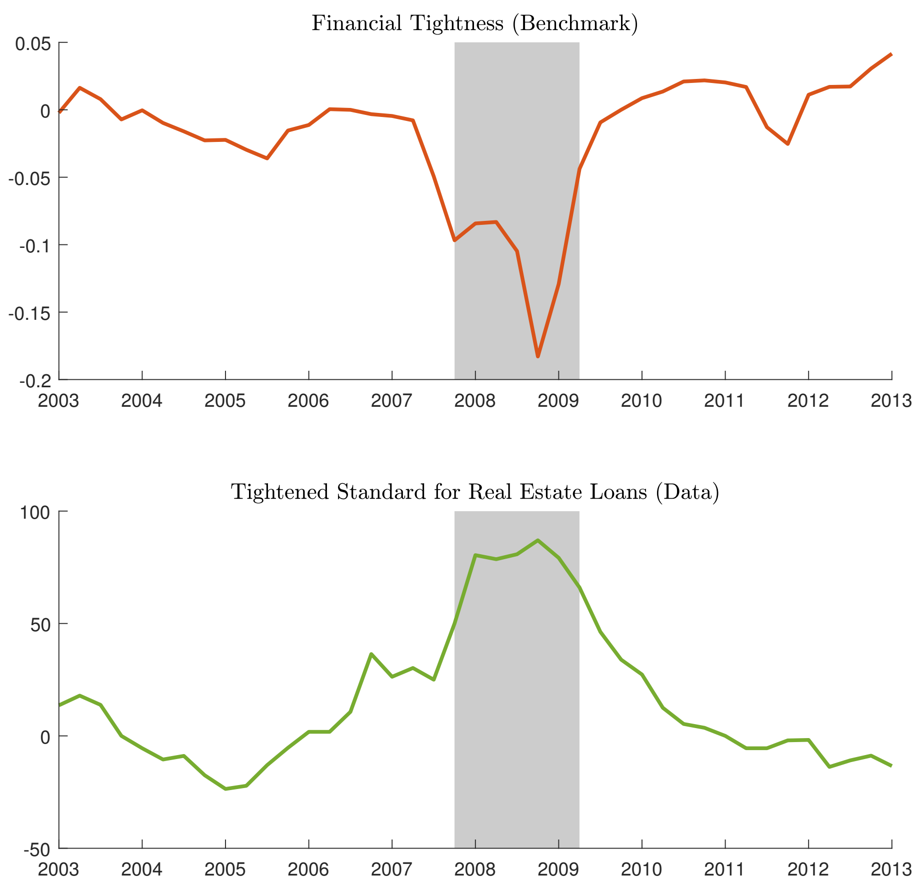

Figure 8: Comparison of financial tightness implied by the model and data. Financial tightness in the data is the net percentage of domestic banks tightening standards for commercial real estate loans released by the Senior Loan Officer Survey from the U.S. Board of Governors of the Federal Reserve System. Shaded areas indicate recessions determined by NBER.

The model-implied historical path well replicates the dynamics of financial tightness in the data. When financial conditions tighten, the data proxy is high and should correspond to a low value of financial shock θ_t. Therefore, financial tightness in the model and data is expected to move in the opposite direction, and Figure 8 verified this. Credit conditions were loose both before and after the crisis, but where tightened during the crisis. This additional performance test indicates that the limited role of financial shocks in this model is not due to an invalid prediction of financial tightness by the model. On the contrary, the financial tightness predicted by the model fits the credit boom-bust cycle well.

6 Conclusion

The main friction I consider in this paper is that collateral quality can be imperfect. Participants in financial markets cannot freely observe or assess the true quality of collateral. They must either spend resources to obtain information on collateral quality or make decisions based on coarse information. As a result, in my model, two lending regimes with endogenous switching emerge, depending on whether lenders are induced to pay an information acquisition cost and learn the precise quality of risky collateral.

Focusing on real estate collateral, I find that collateral quality shocks and associated regime switching can explain all of the three salient facts regarding the joint dynamics of house prices, the price-rent ratio, and output: The dynamics of house prices and the price-rent ratio are very similar, are significantly more volatile than output, and are procyclical over the business cycle. In particular, they help resolve the puzzle of the high volatility of the price-rent ratio. Based on the model estimation using Bayesian methods, I find that collateral quality shocks account for more than half of the variations in house prices and the price-rent ratio. Furthermore, the model identifies an endogenous regime switch at the onset of the Great Recession in the estimated sample, and the data favor the benchmark model over the reference model without regime switching. In conclusion, my study underscores the importance of collateral quality fluctuations in understanding housing market business cycles.

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A Appendix

A.1 Proof of Lemma 2

By condition (1),

$$\frac{P_t}{\overline{P}_t} > \frac{\eta_t}{\gamma + \eta_t} > \frac{\eta_t}{\gamma + 1} > (1-\gamma)\eta_t.$$

Q.E.D.

A.2 A Household's Problem

Let Λ_t be the Lagrangian multiplier for (10) and the optimal decisions on C_t, N_t, $\overline{H}_t$ and H_t satisfy

$$\begin{aligned}
\Lambda_t &= (1-\psi_{ht}) X_t^{\frac{1}{\chi}} C_t^{-\frac{1}{\chi}} \left[\left(X_t - \omega X_{t-1} - \psi_{nt} \exp(gt) \frac{N_t^{1+\nu}}{1+\nu} \right)^{-\kappa} \right. \\
&\quad \left. -\beta\omega \mathbb{E}_t \left(X_{t+1} - \omega X_t - \psi_{nt+1} \exp(gt+g) \frac{N_{t+1}^{1+\nu}}{1+\nu} \right)^{-\kappa} \right], \quad (27) \\
W_t \Lambda_t &= \left(X_t - \omega X_{t-1} - \psi_{nt} \exp(gt) \frac{N_t^{1+\nu}}{1+\nu} \right)^{-\kappa} \exp(gt) \psi_{nt} N_t^{\nu}, \quad (28) \\
(1-\psi_{ht}) \overline{R}_t C_t^{-\frac{1}{\chi}} &= \psi_{ht} [\exp(gt)]^{\frac{\chi-1}{\chi}} \left(\overline{H}_t + \eta_t H_t \right)^{-\frac{1}{\chi}}, \quad (29) \\
R_t &= \eta_t \overline{R}_t. \quad (30)
\end{aligned}$$

A.3 An Entrepreneur's Problem

I start with entrepreneur j's choice of labor input. This choice is a static problem as follows

$$\max_{N_{jt}} \quad K_{jt-1}^{\alpha} (A_t N_{jt})^{1-\alpha} - W_t N_{jt}. \quad (31)$$

The maximum of the above objective is $R_{kt}K_{jt-1}$, where R_{kt} is the marginal product of capital and equal to

$$R_{kt} = \alpha \left[\frac{(1-\alpha)A_t}{W_t} \right]^{\frac{1-\alpha}{\alpha}}. \tag{32}$$

I then study the entrepreneur's dynamic problem. Let $\overline{H}^{'}_{jt} \in [0, \overline{H}_{jt-1}]$ and $H^{'}_{jt} \in [0, H_{jt-1}]$ denote the good and risky houses collateralized by the entrepreneur, respectively. Regardless of the amount of $\overline{H}^{'}_{jt}$, the entrepreneur is always left with a value of $(1-\delta_h)\overline{P}_t\overline{H}^{'}_{jt} + (1-\delta_h)\overline{P}_t(\overline{H}_{jt-1} - \overline{H}^{'}_{jt}) = (1-\delta_h)\overline{P}_t\overline{H}_{jt-1}$ for her good houses after repaying her loan. Similarly, in the II regime, the entrepreneur is always left with a value of $(1-\delta_h)P_tH^{'}_{jt} + (1-\delta_h)P_t(H_{jt-1} - H^{'}_{jt}) = (1-\delta_h)P_tH_{jt-1}$ for her risky houses. In the IS regime, after repaying her loan, the entrepreneur is left with a value of $(1-\delta_h)(1-\gamma)\eta_t\overline{P}_tH^{'}_{jt}$ for her collateralized risky houses and a value of $(1-\delta_h)P_t(H_{jt-1} - H^{'}_{jt})$ for her uncollateralized houses.

The entrepreneur's dividends are her income from capital $R_{kt}K_{jt-1}$ net of her expenditure on investment $P_{kt}I_{jt}$, rental income $\overline{R}_t\overline{H}_{jt} + R_tH_{jt}$, and net income from house trading $\overline{P}_t\left[(1-\delta_h)\overline{H}_{jt-1} + \overline{H}_{nt} - \overline{H}_{jt}\right]$ and $\mathbf{1}^S_t(1-\delta_h)\left[(1-\gamma)\eta_t\overline{P}_tH^{'}_{jt} + P_t(H_{jt-1} - H^{'}_{jt})\right] + (1-\mathbf{1}^S_t)(1-\delta_h)P_tH_{jt-1} + P_tH_{nt} - P_tH_{jt}$. Here, $\overline{H}_{nt}$ and H_{nt} are new good and risky houses that emerge in each period, respectively. They are taken as given by the individual entrepreneur and described in the subsection "Evolution of houses" in Section 2.2.

The entrepreneur's dividends D_{jt} are then given by

$$\begin{aligned} D_{jt} = & \underbrace{R_{kt}K_{jt-1}}_{\text{capital income}} - \underbrace{P_{kt}I_{jt}}_{\text{investment}} + \underbrace{\overline{R}_t\overline{H}_{jt} + R_tH_{jt}}_{\text{rental income}} + \underbrace{\overline{P}_t\left[(1-\delta_h)\overline{H}_{jt-1} + \overline{H}_{nt} - \overline{H}_{jt}\right]}_{\text{good house trading}} \\ & \underbrace{+\mathbf{1}^S_t(1-\delta_h)\left[(1-\gamma)\eta_t\overline{P}_tH^{'}_{jt} + P_t(H_{jt-1} - H^{'}_{jt})\right] + \left(1-\mathbf{1}^S_t\right)(1-\delta_h)P_tH_{jt-1} + P_tH_{nt} - P_tH_{jt}}_{\text{risky house trading}}. \end{aligned} \tag{33}$$

The entrepreneur chooses $\left\{I_{jt}\right\}^{\infty}_{t=0}$, $\left\{K_{jt}\right\}^{\infty}_{t=0}$, $\left\{\overline{H}^{'}_{jt}\right\}^{\infty}_{t=0}$, $\left\{H^{'}_{jt}\right\}^{\infty}_{t=0}$, $\left\{\overline{H}_{jt}\right\}^{\infty}_{t=0}$, and $\left\{H_{jt}\right\}^{\infty}_{t=0}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{t} \beta^t \frac{\Lambda_t}{\Lambda_0} D_{jt} \tag{34}$$

subject to (2), (12), (13), (15), (32), (33), $0 \leq \overline{H}'_{jt} \leq \overline{H}_{jt-1}$ and $0 \leq H'_{jt} \leq H_{jt-1}$.[16]

A.4 A Capital Producer's Problem

The optimal investment I_t satisfies

$$\begin{aligned} Z_t P_{kt} &= 1 + \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)^2 + \Omega\left(\frac{I_t}{I_{t-1}} - \exp(g)\right)\frac{I_t}{I_{t-1}} \\ &\quad -\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\Omega\left(\frac{I_{t+1}}{I_t} - \exp(g)\right)\frac{Z_t}{Z_{t+1}}\left(\frac{I_{t+1}}{I_t}\right)^2. \end{aligned} \tag{35}$$

[16] Modeling collateralized lending can also be transformed to outright house sales. The insights from the model hold with either interpretation. Bigio (2015) presents an equivalence result between these two ways of modeling.

B Proofs of Propositions

B.1 Proof of Proposition 2

Given the problem described in Appendix A.3, I write the dynamic programming of entrepreneur j as follows

$$V_t\left(\epsilon_{jt}, K_{jt-1}, \overline{H}_{jt-1}, H_{jt-1}\right) = \max_{\substack{\left\{I_{jt}, K_{jt}, \overline{H}_{jt}^{'},\right. \\ \left. H_{jt}^{'}, \overline{H}_{jt}, H_{jt}\right\}}} D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(\epsilon_{jt+1}, K_{jt}, \overline{H}_{jt}, H_{jt}\right), \tag{36}$$

subject to (2), (12), (13), (15), (32), (33), $0 \leq \overline{H}_{jt}^{'} \leq \overline{H}_{jt-1}$ and $0 \leq H_{jt}^{\prime} \leq H_{jt-1}$.

Conjecture that the above value function takes the following form

$$V_t(\epsilon_{jt}, K_{jt-1}, \overline{H}_{jt-1}, H_{jt-1}) = \Phi_{Kt}(\epsilon_{jt})K_{jt-1} + \Phi_{\overline{H}t}(\epsilon_{jt})\overline{H}_{jt-1} + \Phi_{Ht}(\epsilon_{jt})H_{jt-1} + \Phi_t, \tag{37}$$

where $\Phi_{Kt}(\epsilon_{jt})$, $\Phi_{\overline{H}t}(\epsilon_{jt})$, $\Phi_{Ht}(\epsilon_{jt})$ and Φ_t are coefficients to be determined.

By definition, Tobin's Q satisfies

$$Q_t \equiv \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{t+1}(\epsilon_{jt+1}, K_{jt}, \overline{H}_{jt}, H_{jt})}{\partial K_{jt}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{min}}^{\epsilon_{max}} \Phi_{Kt+1}(\epsilon) d\mathcal{F}(\epsilon). \tag{38}$$

Also conjecture that

$$\overline{P}_t = \overline{R}_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{min}}^{\epsilon_{max}} \Phi_{\overline{H}t+1}(\epsilon) d\mathcal{F}(\epsilon), \tag{39}$$

$$P_t = R_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{min}}^{\epsilon_{max}} \Phi_{Ht+1}(\epsilon) d\mathcal{F}(\epsilon). \tag{40}$$

Substituting (12), (33), (38), (39) and (40) into the right-hand side of the Bellman equation

(36), I obtain

$$
\begin{aligned}
& D_{jt}+\beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}V_{t+1}\left(\epsilon_{jt+1},K_{jt},\overline{H}_{jt},H_{jt}\right)\\
= \; & R_{kt}K_{jt-1}-P_{kt}I_{jt}+\overline{R}_t\overline{H}_{jt}+R_tH_{jt}+\overline{P}_t\left[(1-\delta_h)\overline{H}_{jt-1}-\overline{H}_{jt}\right]\\
& +\left\{\mathbf{1}_t^S(1-\delta_h)\left[(1-\gamma)\eta_t\overline{P}_tH_{jt}^{'}+P_t(H_{jt-1}-H_{jt}^{'})\right]+\left(1-\mathbf{1}_t^S\right)(1-\delta_h)P_tH_{jt-1}-P_tH_{jt}\right\}\\
& +(1-\delta)Q_tK_{jt-1}+\epsilon_{jt}Q_tI_{jt}+\left(\overline{P}_t-\overline{R}_t\right)\overline{H}_{jt}+\left(P_t-R_t\right)H_{jt}+\Phi_t\\
= \; & R_{kt}K_{jt-1}+(1-\delta_h)\overline{P}_t\overline{H}_{jt-1}+\mathbf{1}_t^S(1-\delta_h)\left[(1-\gamma)\eta_t\overline{P}_tH_{jt-1}^{'}+P_t(H_{jt-1}-H_{jt-1}^{'})\right]\\
& +\left(1-\mathbf{1}_t^S\right)(1-\delta_h)P_tH_{jt-1}+(1-\delta)Q_tK_{jt-1}+(\epsilon_{jt}Q_t-P_{kt})I_{jt}+\Phi_t,
\end{aligned}
$$

where Φ_t absorbs the terms containing $\overline{H}_{nt}$ and H_{nt}.

When $\epsilon_{jt}<P_{kt}/Q_t$, the entrepreneur finds it not profitable to invest, so she does not borrow, i.e., $I_{jt}=\overline{H}_{jt}^{'}=H_{jt}^{'}=0$. When $\epsilon_{jt}\geq P_{kt}/Q_t$, the entrepreneur finds it profitable to invest as much as possible and therefore exhausts her borrowing limit, i.e.,

$$
\begin{aligned}
& \overline{H}_{jt-1}^{'}=\overline{H}_{jt-1},\;\; H_{jt-1}^{'}=H_{jt-1},\\
& P_{kt}I_{jt}=\theta_t\left[\overline{P}_t\overline{H}_{jt-1}+\mathbf{1}_t^S(1-\gamma)\eta_t\overline{P}_tH_{jt-1}+\left(1-\mathbf{1}_t^S\right)P_tH_{jt-1}\right],
\end{aligned}
$$

where investment I_{jt} is pinned down by (2) and (13). I then obtain (16) in Proposition 2. At the equilibrium house prices $\overline{P}_t$ and P_t, the entrepreneur is indifferent between purchasing and selling both types of houses.

Substituting the above decisions into the Bellman equation, I obtain

$$
\begin{aligned}
& V_t(\epsilon_{jt},K_{jt-1},\overline{H}_{jt-1},H_{jt-1})\\
= & \begin{cases}
R_{kt}K_{jt-1}+(1-\delta)Q_tK_{jt-1}+(1-\delta_h)\overline{P}_t\overline{H}_{jt-1}+\Phi_t \\
+\mathbf{1}_t^S(1-\delta_h)(1-\gamma)\eta_t\overline{P}_tH_{jt-1}+(1-\mathbf{1}_t^S)(1-\delta_h)P_tH_{jt-1} \\
+\theta_t\left(\frac{Q_t}{P_{kt}}\epsilon_{jt}-1\right)\left[\overline{P}_t\overline{H}_{jt-1}+\mathbf{1}_t^S(1-\gamma)\eta_t\overline{P}_tH_{jt-1}+\left(1-\mathbf{1}_t^S\right)P_tH_{jt-1}\right], & \text{if } \epsilon_{jt}\geq\epsilon_t^*;\\
R_{kt}K_{jt-1}+(1-\delta)Q_tK_{jt-1}+(1-\delta_h)\overline{P}_t\overline{H}_{jt-1}+(1-\delta_h)P_tH_{jt-1}+\Phi_t, & \text{if } \epsilon_{jt}<\epsilon_t^*.
\end{cases}
\end{aligned}
$$

Matching coefficients $\Phi_{Kt}(\epsilon_{jt})$, $\Phi_{\overline{H}t}(\epsilon_{jt})$, and $\Phi_{Ht}(\epsilon_{jt})$ in the above equation and equation (37), and making use of equations (38), (39), and (40) yield (17), (18), and (19) in Proposition 2. Q.E.D.

B.2 Proof of Proposition 3

Part (i) By Lemma 2 and equation (19), I obtain

$$\begin{aligned} P_t \;\geq\;& (1-\gamma)\eta_t\overline{R}_t + \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\Big\{(1-\delta_h)(1-\gamma)\eta_{t+1}\overline{P}_{t+1}\mathcal{F}(\epsilon^*_{t+1}) \\ &+(1-\delta_h)\left[\mathbf{1}^S_{t+1}(1-\gamma)\eta_{t+1}\overline{P}_{t+1} + (1-\mathbf{1}^S_{t+1})(1-\gamma)\eta_{t+1}\overline{P}_{t+1}\right]\left[1-\mathcal{F}(\epsilon^*_{t+1})\right] \\ &+\theta_{t+1}\left[\mathbf{1}^S_{t+1}(1-\gamma)\eta_{t+1}\overline{P}_{t+1} + (1-\mathbf{1}^S_{t+1})(1-\gamma)\eta_{t+1}\overline{P}_{t+1}\right]\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)d\mathcal{F}(\epsilon)\Big\} \\ =\;& (1-\gamma)\eta_t\overline{R}_t + \beta(1-\gamma)\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\eta_{t+1}\overline{P}_{t+1}\left[1-\delta_h+\theta_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)\right]. \quad (41) \end{aligned}$$

Detrending the above inequality (the detrending rule is given in Appendix D) leads to

$$p_t \geq (1-\gamma)\left\{\eta_t\overline{r}_t + \beta\exp[(1-\kappa)g]\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\eta_{t+1}\overline{p}_{t+1}\left[1-\delta_h+\theta_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)\right]\right\}. \quad (42)$$

Let $\widehat{m}_t$ denote the percentage deviation of a variable m_t around its steady-state value m. The log-linearized version of the above inequality around the steady state is

$$\begin{aligned} \widehat{p}_t \;\geq\;& \frac{\overline{r}}{\overline{p}}(\widehat{\eta}_t+\widehat{r}_t) + \frac{\overline{p}-\overline{r}}{\overline{p}}\mathbb{E}_t\widehat{\eta}_{t+1} + \frac{\overline{p}-\overline{r}}{\overline{p}}\left\{\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\overline{p}_{t+1}\left[1-\delta_h+\theta_{t+1}\widehat{\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)\right]\right\} \\ =\;& \widehat{\eta}_t + \frac{\overline{r}}{\overline{p}}\widehat{r}_t + \frac{\overline{p}-\overline{r}}{\overline{p}}\left\{\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\overline{p}_{t+1}\left[1-\delta_h+\theta_{t+1}\widehat{\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)\right]\right\}, \quad (43) \end{aligned}$$

where I use $\widehat{\eta}_t = \mathbb{E}_t\widehat{\eta}_{t+1}$ implied by the stochastic process of η_t.

Equation (18) is detrended as

$$\overline{p}_t = \overline{r}_t + \beta\exp[(1-\kappa)g]\mathbb{E}_t\frac{\lambda_{t+1}}{\lambda_t}\overline{p}_{t+1}\left[1-\delta_h+\theta_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)\right]. \quad (44)$$

Log-linearizing it around the steady state yields

$$\widehat{\overline{p}}_t = \frac{\overline{r}}{\overline{p}}\widehat{r}_t + \frac{\overline{p}-\overline{r}}{\overline{p}}\left\{\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\overline{p}_{t+1}\left[1-\delta_h+\widehat{\theta_{t+1}\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon-1\right)d\mathcal{F}(\epsilon)}\right]\right\}. \quad (45)$$

Combining (43) and (45), I have

$$\widehat{p}_t \geq \widehat{\eta}_t + \widehat{\overline{p}}_t. \quad (46)$$

Next, I show that $p \geq (1-\gamma)\eta\overline{p}$ in the deterministic steady state. If the steady-state regime is the II regime, then the steady-state versions of (18) and (19) become

$$\overline{p} = \overline{r} + \beta\exp[(1-\kappa)g]\overline{p}\left[1-\delta_h+\theta\int_{\epsilon^*}^{\epsilon_{max}}\left(\frac{Q}{P_k}\epsilon-1\right)d\mathcal{F}(\epsilon)\right], \quad (47)$$

$$p = \eta\overline{r} + \beta\exp[(1-\kappa)g]p\left[1-\delta_h+\theta\int_{\epsilon^*}^{\epsilon_{max}}\left(\frac{Q}{P_k}\epsilon-1\right)d\mathcal{F}(\epsilon)\right], \quad (48)$$

comparing which yields $p = \eta\overline{p}$.

If the steady-state regime is the IS regime, then the steady-state version of (42) becomes

$$\begin{aligned} p &\geq \eta\overline{r} + \beta(1-\gamma)\exp[(1-\kappa)g]\eta\overline{p}\left[1-\delta_h+\theta\int_{\epsilon^*}^{\epsilon_{max}}\left(\frac{Q}{P_k}\epsilon-1\right)d\mathcal{F}(\epsilon)\right] \\ &= (1-\gamma)\eta\overline{r} + \beta(1-\gamma)\exp[(1-\kappa)g]\eta\overline{p}\left[1-\delta_h+\theta\int_{\epsilon^*}^{\epsilon_{max}}\left(\frac{Q}{P_k}\epsilon-1\right)d\mathcal{F}(\epsilon)\right] \\ &= (1-\gamma)\eta\overline{p}, \end{aligned} \quad (49)$$

where the last equality is due to (47).

Combining (46) and (49) generates $p_t \geq (1-\gamma)\eta_t\overline{p}_t$ or $P_t \geq (1-\gamma)\eta_t\overline{P}_t$. When $\gamma = 0$, all inequalities above become equalities and then $P_t = \eta_t\overline{P}_t$.

Part (ii) When $\eta_t \to 1$, $P_t \to \overline{P}_t$, the left-hand side of condition (1) approaches zero, implying that condition (1) holds. Therefore, given that $\gamma > 0$, when $\eta_t \to 1$, the equilibrium regime is the II regime.

On the one hand, as γ is positive and the IS regime is possible, there must be some value of $\eta_t \in (0,1)$ for which the left-hand side of condition (1) is greater than γ, such that the IS regime occurs. On the other hand, as shown above, when η_t is close to 1, the left-hand side of condition (1) is smaller than γ such that the II regime occurs. Therefore, by the Intermediate Value Theorem, there must be some value of η_t^* such that condition (1) holds with equality, so that when $\eta_t > \eta_t^*$, condition (1) holds, and when $\eta_t < \eta_t^*$, condition (1) does not hold. Q.E.D.

C Equilibrium System

Proposition 4 *The equilibrium system is given by equations (17), (18), (19), (20), (27), (28), (29), (30), (32), (35), (4) and (5) for the II regime, (6) and (7) for the IS regime, $\epsilon_t^* = P_{kt}/Q_t$, and*

$$I_t = \frac{\theta_t}{P_{kt}} \left[\overline{P}_t \overline{H}_{t-1} + \mathbf{1}_t^S (1-\gamma)\eta_t \overline{P}_t H_{t-1} + \left(1 - \mathbf{1}_t^S\right) P_t H_{t-1}\right] \left[1 - \mathcal{F}(\epsilon_t^*)\right], \tag{50}$$

$$K_t = (1-\delta)K_{t-1} + \frac{\theta_t}{P_{kt}} \left[\overline{P}_t \overline{H}_{t-1} + \mathbf{1}_t^S (1-\gamma)\eta_t \overline{P}_t H_{t-1} + \left(1 - \mathbf{1}_t^S\right) P_t H_{t-1}\right] \int_{\epsilon_t^*}^{\epsilon_{max}} \epsilon d\mathcal{F}(\epsilon), \tag{51}$$

$$N_t = \left[\frac{(1-\alpha)A_t^{1-\alpha}}{W_t}\right]^{\frac{1}{\alpha}} K_{t-1}, \tag{52}$$

$$Y_t = K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha}, \tag{53}$$

for the endogenous variables $\{C_t, I_t, N_t, Y_t, \overline{H}_t, H_t, K_t, W_t, Q_t, R_{kt}, \overline{R}_t, R_t, \overline{P}_t, P_t, P_{kt}, \Lambda_t, \epsilon_t^*\}$. The usual transversality conditions hold.

I have already derived equations (17), (18), and (19) in Proposition 2, equations (27), (28), (29), and (30) in Appendix A.2, equation (32) in Appendix A.3, and equation (35) in Appendix A.4. Now I derive equation (20) and equations (50) to (53). First, I use the decision rule in Proposition 2 and the Law of Large Numbers to derive aggregate investment,

$$\begin{aligned} I_t &= \int_0^1 I_{jt} dj \\ &= \int_0^1 \mathbf{1}(\epsilon_{jt} \geq \epsilon_t^*) \frac{\theta_t}{P_{kt}} \left[\overline{P}_t \overline{H}_{jt-1} + \mathbf{1}_t^S (1-\gamma)\eta_t \overline{P}_t H_{jt-1} + \left(1 - \mathbf{1}_t^S\right) P_t H_{jt-1}\right] dj + 0 \cdot \int_0^1 \mathbf{1}(\epsilon_{jt} < \epsilon_t^*) dj \\ &= \frac{\theta_t}{P_{kt}} \left[\overline{P}_t \overline{H}_{t-1} + \mathbf{1}_t^S (1-\gamma)\eta_t \overline{P}_t H_{t-1} + \left(1 - \mathbf{1}_t^S\right) P_t H_{t-1}\right] \left[1 - \mathcal{F}(\epsilon_t^*)\right], \end{aligned}$$

where the last equality is due to the fact that ϵ_{jt} is IID across entrepreneurs. I obtain (50).

Similarly, I derive the evolution of aggregate capital stock as

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + \int_0^1 \epsilon_{jt} I_{jt} dj \\ &= (1-\delta)K_{t-1} + \frac{\theta_t}{P_{kt}} \left[\overline{P}_t \overline{H}_{t-1} + \mathbf{1}_t^S (1-\gamma)\eta_t \overline{P}_t H_{t-1} + \left(1 - \mathbf{1}_t^S\right) P_t H_{t-1} \right] \int_{\epsilon_t^*}^{\epsilon_{max}} \epsilon d\mathcal{F}(\epsilon), \end{aligned}$$

which is (51).

The entrepreneur's labor demand problem (31) gives

$$N_{jt} = \left[\frac{(1-\alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{jt-1}. \tag{54}$$

The labor market clearing condition implies that

$$N_t = \int_0^1 N_{jt} dj = \left[\frac{(1-\alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{t-1}, \tag{55}$$

which is (52).

Substituting (54) into production function (11), I derive aggregate output

$$Y_t = \int_0^1 Y_{jt} dj = \int_0^1 K_{jt-1}^{\alpha} A_t^{1-\alpha} \left[\frac{(1-\alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1-\alpha}{\alpha}} K_{jt-1}^{1-\alpha} dj = K_{t-1}^{\alpha} (A_t N_t)^{1-\alpha},$$

which is (53).

Substituting the flow-of-funds constraints of entrepreneurs, bankers and capital producers, $D_t^e = \int_0^1 D_{jt} dj$, $D_t^b = 0$, and $D_t^k = P_{kt} I_t - \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] \frac{I_t}{Z_t}$ into (10), I obtain

$$C_t + \left[1 + \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] \frac{I_t}{Z_t} + \mathbf{1}_t^S \gamma \eta_t \overline{P}_t H_t [1 - \mathcal{F}(\epsilon_t^*)] = Y_t,$$

which is (20). Q.E.D.

D Detrended Equilibrium System

I verify that the equilibrium variables ϵ_t^*, Q_t, R_{kt}, P_{kt}, N_t, $\overline{H}_t$, and H_t do not have trends. All the other equilibrium variables in Proposition 4 grow around the balanced growth path at rate g except for Λ_t. Letting $\Lambda_t = \lambda_t \exp(-\kappa g t)$ and any other growing variable $M_t = m_t \exp(gt)$, I detrend all of the conditions in Proposition 4 and obtain the following system:

$$
\begin{aligned}
\lambda_t &= (1-\psi_{ht}) x_t^{\frac{1}{\chi}} c_t^{-\frac{1}{\chi}} \left[\left(x_t - \omega \frac{x_{t-1}}{\exp(g)} - \psi_{nt} \frac{N_t^{1+\nu}}{1+\nu} \right)^{-\kappa} \right. \\
&\left. -\beta\omega \exp(-\kappa g) \mathbb{E}_t \left(x_{t+1} - \omega \frac{x_t}{\exp(g)} - \psi_{nt+1} \frac{N_{t+1}^{1+\nu}}{1+\nu} \right)^{-\kappa} \right], \qquad (56)
\end{aligned}
$$

$$
w_t \lambda_t = \left(x_t - \omega \frac{x_{t-1}}{\exp(g)} - \psi_{nt} \frac{N_t^{1+\nu}}{1+\nu} \right)^{-\kappa} \psi_{nt} N_t^{\nu}, \qquad (57)
$$

$$
(1-\psi_{ht}) \overline{r}_t c_t^{-\frac{1}{\chi}} = \psi_{ht} \left(\overline{H}_t + \eta_t H_t \right)^{-\frac{1}{\chi}}, \qquad (58)
$$

$$
r_t = \eta_t \overline{r}_t, \qquad (59)
$$

$$
\begin{aligned}
Z_t P_{kt} &= 1 + \frac{\Omega}{2} \exp(2g) \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 + \Omega \exp(2g) \left(\frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \qquad (60) \\
&- \beta \exp[(3-\kappa)g] \Omega \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left(\frac{i_{t+1}}{i_t} - 1 \right) \frac{Z_t}{Z_{t+1}} \left(\frac{i_{t+1}}{i_t} \right)^2, \qquad (61)
\end{aligned}
$$

$$
R_{kt} = \alpha \left[\frac{(1-\alpha) a_t}{w_t} \right]^{\frac{1-\alpha}{\alpha}}, \qquad (62)
$$

$$
y_t = c_t + \left[1 + \frac{\Omega}{2} \exp(2g) \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] \frac{i_t}{Z_t} + \mathbf{1}_t^S \gamma \eta_t \overline{p}_t H_t \left[1 - \mathcal{F}(\epsilon_t^*) \right], \qquad (63)
$$

$$
Q_t = \beta \exp(-\kappa g) \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[R_{kt+1} + (1-\delta) Q_{t+1} \right], \qquad (64)
$$

$$
\overline{p}_t = \overline{r}_t + \beta \exp[(1-\kappa)g] \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \overline{p}_{t+1} \left[1 - \delta_h + \theta_{t+1} \int_{\epsilon_{t+1}^*}^{\epsilon_{max}} \left(\frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon) \right], \qquad (65)
$$

$$
\begin{aligned}
p_t &= r_t + \beta \exp[(1-\kappa)g]\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\Big\{(1-\delta_h)p_{t+1}\mathcal{F}(\epsilon^*_{t+1}) \\
&\quad +(1-\delta_h)\left[\mathbf{1}^S_{t+1}(1-\gamma)\eta_{t+1}\overline{p}_{t+1} + (1-\mathbf{1}^S_{t+1})p_{t+1}\right]\left[1-\mathcal{F}(\epsilon^*_{t+1})\right] \\
&\quad +\theta_{t+1}\left[\mathbf{1}^S_{t+1}(1-\gamma)\eta_{t+1}\overline{p}_{t+1} + (1-\mathbf{1}^S_{t+1})p_{t+1}\right]\int_{\epsilon^*_{t+1}}^{\epsilon_{max}}\left(\frac{Q_{t+1}}{P_{kt+1}}\epsilon - 1\right)d\mathcal{F}(\epsilon)\Big\}, \quad (66) \\
\epsilon^*_t &= \frac{P_{kt}}{Q_t}, \quad (67) \\
i_t &= \frac{\theta_t}{P_{kt}}\left[\overline{p}_t\overline{H}_{t-1} + \mathbf{1}^S_t(1-\gamma)\eta_t\overline{p}_t H_{t-1} + \left(1-\mathbf{1}^S_t\right)p_t H_{t-1}\right]\left[1-\mathcal{F}(\epsilon^*_t)\right], \quad (68) \\
k_t &= (1-\delta)\frac{k_{t-1}}{\exp(g)} \\
&\quad +\frac{\theta_t}{P_{kt}}\left[\overline{p}_t\overline{H}_{t-1} + \mathbf{1}^S_t(1-\gamma)\eta_t\overline{p}_t H_{t-1} + \left(1-\mathbf{1}^S_t\right)p_t H_{t-1}\right]\int_{\epsilon^*_t}^{\epsilon_{max}}\epsilon d\mathcal{F}(\epsilon), \quad (69) \\
N_t &= \left[\frac{(1-\alpha)a_t^{1-\alpha}}{w_t}\right]^{\frac{1}{\alpha}}\frac{k_{t-1}}{\exp(g)}, \quad (70) \\
y_t &= \left[\frac{k_{t-1}}{\exp(g)}\right]^{\alpha}(a_t N_t)^{1-\alpha}, \quad (71)
\end{aligned}
$$

(4) and (5) for the II regime, and (6) and (7) for the IS regime.