Delay and Learning in Coordination*

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Abstract

This paper studies a coordination game with incomplete information and the option to delay. The delay option enables the agents to observe a binary signal depending on whether the early actions surpasses a threshold. The anticipation of information incentivizes agents to wait and free-ride on the information, whereas acting early can help to generate good news, and consequently induce more coordination. Relying on this trade-off, we find the unique monotone equilibrium, which features the inability of the delay option in facilitating coordination. If the waiting period is sufficiently short or the interest rate is sufficiently low, the delay option induces more inaction and makes coordination more difficult to achieve, as compared with the static case. We further discuss the theoretical implications for understanding the impact of monetary policy on economic recovery and how the accessibility of information affects financial stability.

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1 Introduction

A speculative attack against a currency, which is equivalent to a run on a bank with limited reserves, has been taken as a central exhibit for a coordination game of strategic complementarity with under common knowledge, two equilibria , no run or a run that leads to the depletion of the reserves (Obstfeld 1996; Diamond and Dybvig (1983)). This multiplicity has been resolved by the removal of the assumption of common knowledge, following the pathbreaking work of Carlsson and Van Damme (1993). The application of that method to a stylized one period model by Morris and Shin (1998) is not satisfactory, however. Speculative attacks or games of coordination do not take place as a one period game but they unfold in a process of learning and contagion that either leads to an outburst or dies out in a fizzle.

The assumption of a single period imposes an extraordinary amount on coordination on agents who play that simultaneous game. That assumption provides to agents a power to coordinate that is grossly overstated compared to real situation. Interestingly, when numerical values are plugged in, the model of Morris and Shin, which admittedly is highly stylized, may have implausible policy implication because of this coordination power that is implicitly given to the speculating agents. As explained later in the paper, in the example of a speculative attack against a currency or a private bank the cost or running against the bank is very small when compared to the payoff of a successful attack. Within the model, that implies that the bank should keep a level of reserves that is nearly equal to the mass of the speculators. A private bank should have a policy that is near full reserve banking. We will show here that the introduction of a second period, with the choice between running now or in the next period, can dramatically alter these policy implications.

In a game with strategic complementarity and a large number of agents, the possibility of delay introduces two new effects. The first one is that the expectation of information, by increasing the incentive for waiting, reduces the level of activity so much that there is less coordination. In the second effect, the actions in the first period provides sufficient information for good news that induce more activity in the second period and hence more coordination. We will argue that in an extension of the canonical Obstfeld-Morris-Shin model, the first effect is likely to predominate.

It is a common remark that the level of a speculative attack in its first phase, against a currency of a government has an impact on its unfolding (Lohmann, 1994). In the model of this paper that level is observed through a discrete filter: at the end of the first period, the remaining players observe a binary variable Y that is equal to 1 when the mass of activity ("investment") in the first period is greater than some set value γ. When this activity is smaller than γ, $Y = 0$.

This setting provides a sharp separation between the two effects that are critical in the dynamic game. When γ is higher than some critical value γ_0, which is inversely related to the cost of investment and the discount factor, we have the first effect. When γ is lower than that critical value, the second effect is at work. Let us sketch the argument.

Following related works, let us assume a continuum of agents, of mass one, each with an option to make a fixed size investment, at a fixed cost c, $(0 < c < 1)$, in one of two periods. Coordination is successful when, at the end of the game, the mass of investment, X, is greater than $1 - \theta$, where θ is the "fundamental". When coordination is successful, each agent who has "invested" receives a gross payoff of 1. The payoff of investment in the second period is reduced by a discount factor. In a one period setting, the multiplicity of equilibria that arises under common knowledge when $0 < \theta < 1$, is resolved when the common knowledge assumption is replaced by the imperfect information of individuals about the fundamental θ. As in these previous works, we will assume that this imperfect information takes the form of individual signals $s_i = \theta + \sigma\epsilon_i$, where ϵ_i is a noise.

Assume that the strategy in the first period is monotone: for some $\hat{s}_1$, agents with a signal $s_i \geq \hat{s}_1$ invest without delay. The opportunity cost of delay is the expected payoff multiplied by the rate of discount. The benefit of delay arises from avoiding to pay the investment cost in the first period when coordination eventually fails. For the marginal agent with signal $\hat{s}_1$, the two are equal. For an agent with a higher value of s, the opportunity cost is higher since he is more confident that the fundamental θ has a high value, and that coordination will succeed.

The sub-game in period 2 depends on the observation of Y at the end of the first period. Assume that if $Y = 0$, which is bad news about the fundamental, there is no more investment (a property that will be proven in the analysis). When $Y = 1$ more agents invest in period 2. There are two possibilities.

When the parameter γ is higher than γ_0, the observation $Y = 1$ takes place only if aggregate investment in the first period is high. $Y = 1$ is a signal of a high value of the fundamental (that is necessary to generate that investment). Since both investment and fundamental are high, the first period investment is sufficient to generate coordination. All remaining players, having learned that, invest in the second period, but they are like free riders on the early investors. Coordination is based only on the first period investors, who have a signal higher than $\hat{s}_1$, a value that is higher than the strategy of the one period game. Because the set of signals for investment in the first period is smaller than in the static game, and agents in the second period are irrelevant for achieving a successful coordination, that success requires a higher value of the fundamental than in the static game. The option for delays makes the success of coordination less likely.

The second case takes place when $\gamma < \gamma_0$. Here, a successful coordination requires the contribution of some investors in the second period. The expectation of that contribution increases the aggregate investment in the first period (which remains smaller than in the one period context). The observation $Y = 0$ reveals that the fundamental is really low (since it cannot generate a investment greater than γ even when agents do less delay and γ is relatively low). Therefore, the observation $Y = 1$ provides a strong boost to the expectations of the remaining players. They invest for a signal above $\hat{s}_2$ (with, obviously, $\hat{s}_2 < \hat{s}_1$), which is lower than the cutoff point in the one period model. Consequently, the minimum value of the fundamentals with coordination is lower than in the one period model. The option for delay facilitates coordination.

The critical value γ_0 depends on the discount rate. Since the financial market moves fast and speculative attacks take place in a relatively short time, the discount rate can be very low in these applications. The condition $\gamma > \gamma_0$ holds true for any γ if the discount rate is sufficiently low. When the discount rate is low, the opportunity cost of delay is low, agents delay more, and therefore $Y = 1$ is unlikely to occur. Despite the strong incentive for delay, as we have discussed, the time-2 investments made by "free riders" does not determine the coordination success. Therefore, coordination is much more difficult in a fast moving financial market with a low discount rate.

The strong impact of the option for delay can be illustrated by the application of the model to the stylized description of a speculative attack that has been used by Morris and Shin. Assume that the cost of speculation is 10 percent of the gross payoff after a successful attack. If the crisis takes two weeks, a long time for a crisis, and the annual interest rate is 24 percent, a very high value (example of Sweden**), $\gamma_0 = 0.09$. Suppose now that agents notice the bank run when at least 10 percent of them run to the bank. The case $\gamma > \gamma_0$ applies. We will see that in this example, the bank should keep an amount of reserves that is no more than 10 percent of the deposits (instead of the 90 percent ratio of the static case).

Applying our theory to the investment game, the dynamic model also provides a novel channel through which a policy of low interest rate can work against stimulating investment and economic recovery. In a low interest rate environment, the discount rate is low and therefore the investors have a higher incentive of waiting for more information. As discussed, the inaction in the early period makes the "good news" $Y = 1$ more difficult to be generated and, therefore, makes the coordination on investing more difficult to achieve.

Related Literature

This paper belongs to the extensive literature on coordination games. The primary focus of this study is to understand how the delay option changes the coordinating behavior, and, in this sense, it is different from the existing studies (e.g., Carlsson and Van Damme (1993), Morris and Shin (1998)) on equilibrium selection which models coordination as a static game. Similar to the global game literature, in our model, the agents have incomplete information about the fundamental, and they are endowed with heterogeneous private information. However, our theory finds that the delay option can significantly change the equilibrium play, which challenges the existing understanding of equilibrium selection (built on heterogeneous private beliefs) in coordination games.

The impact of a naturally embedded delay option with observational learning has been studied in games without strategic interaction (e.g., Chamley and Gale (1994), Gul and Lundholm (1995) and Fajgelbaum, Schaal and Taschereau-Dumouchel (2017)). We investigate this impact in a dynamic coordination game in which the fundamental is time-invariant, all agents can only attack the regime once and collect payoffs at the end of the game. This dynamic setting with endogenous timing choice differentiates our study different from the many existing studies on dynamic coordination (e.g., among others, see Chamley (2003), Chassang (2010), Frankel and Pauzner (2000), Angeletos, Hellwig and Pavan (2007), Basak and Zhou (2020)). In addition, we consider a dynamic coordination problem among a large amount of agents so that investment by one agent cannot trigger investment of her fellow agents either through signaling and/or via complementarities, which is studied by Gale (1995) and Corsetti et al. (2004).

Another distinctive feature of our dynamic structure is that agents can learn an endogenous binary public signal that provides imperfect information about the historical actions. Introducing public information into global games usually breaks the unique equilibrium characterization in global game literature, no matter it is purely exogenous (Hellwig (2002)), or a market price which serves as a noisy public signal aggregating private information (Hellwig, Mukherji and Tsyvinski (2006)), or a public observation of the past history (Chamley (1999), Angeletos and Werning (2006)). Interestingly, when the public signal is produced through the endogenous timing choices, our theory demonstrates the uniqueness of monotone equilibrium.

As the realization of the public signal and its informativeness rely on the past irreversible actions, our model features information externality. That differentiates our work with Kováč and Steiner (2013) and Dasgupta (2007). Kováč and Steiner (2013) focus on an environmental with an option of delay for another private noisy information about the fundamental while the past actions are unobservable; Dasgupta (2007) considers delay with private noisy information about the past actions. In both papers, agents learn privately and the amount of information revealed is independent of the past action. In our model, by design, the public signal Y depends on agents' actions. This endogenous public signal update agents' belief about θ in a truncated way and its informativeness of the public signal is determined by the likelihood of early action in equilibrium. This effect which is at the root of the results of Vives (1993, 1995), is essential in the present paper.

In sharp contrast to Dasgupta (2007), our study demonstrates the inability of delay option in promoting coordination, which is robust to the the accessibility of information (as long as the delay cost is sufficiently small). This finding is largely consistent with experimental evidence provided by Jin, Zhou and Brandenburger (2021).[1]

Outline The remainder of the paper is organized as follows. Section 2 presents our model and we solve the unique equilibrium for any given parameter choice in Section 3. In Section 4, we discuss how the learning opportunity and the delay cost affects the impact of the delay option on facilitating coordination. Policy implications are discussed in Section 5 and Section 6 concludes.

[1]Jin, Zhou and Brandenburger (2021) study the option of delay in a complete-information coordination game with no delay cost. They provide experimental evidence to show that the delay option favors coordination on the reversible action (no invest) but impairs coordination on the irreversible action, and a theory (with agents who hold other-regarding preferences) built on iterated weak dominance to explain this finding. The experimental evidence is largely consistent with our theory by taking the delay cost varnishingly small.

2 Model Setup

We augment a canonical global game of regime change by adding a delay option (and an opportunity of learning associated with this option). The model has natural applications to currency attacks (see Morris and Shin (1998), Angeletos, Hellwig and Pavan (2007)) and political regime change (see Chamley (1999)).

Players and Actions There is a unit mass of risk neutral agents, indexed by $i \in [0,1]$. They discount future payoff by $\delta \in (0,1)$. Each agent has an option to make a fixed size investment.[2]

Delay Option The model has two periods, $t = 1, 2$. Each agent i can choose to invest or not at $t = 1$; and if not, then choose to invest or not at $t = 2$. Investment is assumed *irreversible*. Therefore, if an agent choose to invest, we write agent i's time of investment as $t_i \in \{1, 2\}$; otherwise, we write $t_i = \emptyset$.

Outcome of Coordination The "fundamental" of the economy is denoted by θ. In our primary example, θ represents the difficulty for investment success.[3] By a standard abuse of notation, the mass of agents who invest is the same as the mass of investment. Formally, we write the mass of investment at $t \in \{1, 2\}$ as $X_t = \int_{i \in [0,1]} \mathbb{1}\{t_i = t\} \mathrm{d}i$. Whether coordination is successful depends on the the fundamental and the *aggregate investment* $X := X_1 + X_2$ at the end of $t = 2$. Successful coordination is achieved if and only if $X \geq 1 - \theta$.

Payoffs Agents receive their payoff at the end of $t = 2$. The payoff from not investing is normalized to 0. The present value of payoff from investing at $t_i \in \{1, 2\}$ captures *strategy complementarity* and *costly delay*, and it is defined by the canonical form of a coordination game:

$$U(t_i, \theta, X) = \delta^{t_i - 1}(Z - c) \text{ with } Z = \begin{cases} 1, & X \geq 1 - \theta, \\ 0, & \text{if } X < 1 - \theta. \end{cases} \tag{1}$$

[2]Depending on the application, one can interpret investment as speculative attacks against a central bank, or withdrawal from a bank, or attacks against an existing political regime.

[3]In other applications, θ may represent the weakness of a central bank (with reserves $1 - \theta$), or of a political regime.

In words, the investment return $Z = 1$ is realized only when the coordination is successful, that is, when sufficiently many other investors choose to invest and/or the fundamental θ is sufficiently strong. The investment opportunity is profitable as the cost of investment $c \in (0, 1)$. Delayed investment is costly since if the coordination is successful coordination, the payoff from investing earlier is strictly higher –i.e., $1 - c > \delta(1 - c)$.

Exogenous Information As in many studies since Carlsson and Van Damme (1993), we assume that agents have imperfect information on θ. In detail, they have a common prior of θ, which is diffuse (uniformly distributed) over the real line.[4] In addition, each agent i has a conditional independent and identical noisy private information, that is,

$$s_i = \theta + \sigma\epsilon_i, \;\; \epsilon_i \sim F(\cdot).$$

The parameter $\sigma > 0$ scales the noisiness of the private signal. Later, we will discuss the limiting case where $\sigma \to 0+$. We make the following assumption about the distribution of the noise term $F : (-\infty, +\infty) \to [0, 1]$.

Assumption 1. *The distribution of individual signals has a mean zero and a variance one, i.e.,* $\mathbb{E}_F(\epsilon_i) = 0$, *and is symmetric:* $F(-x) = 1 - F(x)$. *In addition, it is continuous, strict increasing and log-concave.*

Under this assumption, an agent with private signal s_i has a subjective posterior distribution on θ with a mean s_i and a cumulative distribution function $F(\frac{\theta - s_i}{\sigma})$. On the other side, for any give θ, the mass of agents who receive private signal greater than s is $1 - F(\frac{s-\theta}{\sigma})$. By symmetry of $F(\cdot)$, $1 - F(\frac{s-\theta}{\sigma}) = F(\frac{\theta - s}{\sigma})$. The uniform distribution $U[-a, a]$ ($a > 0$) and standard normal distribution satisfy Assumption 1.

Endogenous Information A distinctive feature of the model is that agents observe at the end of the first period a public signal $Y \in \{0, 1\}$. It is a piece of history-dependent information, which increases with the level of investment in the first period (X_1). For some fixed parameter $\gamma \in (0, 1)$, the information generation process is a step function

[4] This assumption could be replaced by a uniform distribution over a sufficiently large range. For example, we can assume the information environment as follows. Agents get the private information $s_i = \theta + \sigma\epsilon_i$, in which $\epsilon_i \in [-\frac{1}{2}, \frac{1}{2}]$ and follows a distribution $\Psi(\cdot)$, whereas the common prior is $\theta \in U[\underline{\theta}, \bar{\theta}]$ with $\bar{\theta} > 1 + \sigma$ and $\underline{\theta} < -\sigma$.

defined as follows.[5]

$$Y = \begin{cases} 0, \text{ if } X_1 < \gamma, \\ 1, \text{ if } X_1 \geq \gamma. \end{cases} \tag{2}$$

This step function is motivated by discrete policies, *e.g.*, the raising of the Central Bank's discount rate to defend the currency, the minimum size of a demonstration to create a news event. Apart from this public signal Y and private signal s_i, we assume agents cannot receive any other information regarding the fundamental θ or the history X_1.

The discrete assumption is much more than an "expedient." It corresponds to a plausible case. The parameter γ plays a central role in the analysis, which measures how easy the "good news" can be generated and observed. When γ takes a relatively high value, an observation $Y = 1$ will be relatively unlikely and therefore very informative for good news (high θ).

Monotone Strategies Throughout the paper, we will restrict our attention to monotone strategies and use the *Perfect Bayesian Equilibrium in monotone strategies* as our solution concept. If an agent plays a *monotone strategy*, then, at any history (i.e., $Y = 0, 1$ at $t = 2$, or no history at $t = 1$), she invests at signal s_i' if she invests at any $s_i < s_i'$; she will not invest at s_i'' if she does not invest at some $s_i > s_i''$. Following this definition, we can write a monotone strategy as $(\hat{s}_1, \hat{s}_2^0, \hat{s}_2^1)$, that is, the agent would invest at $t = 1$ if $s_i \geq \hat{s}_1$, and chooses to wait if $s_i < \hat{s}_1$, and then invest after $Y = 0$ (or $Y = 1$) at $t = 2$ if and only if $s_i \in [\hat{s}_2^0, \hat{s}_1)$ (or $s_i \in [\hat{s}_2^1, \hat{s}_1)$). As investment is irreversible in this dynamic setting, the information thresholds at $t = 2$ –i.e., $\hat{s}_2^0$ and $\hat{s}_2^1$, must be (weakly) lower than $\hat{s}_1$.

3 Solving the Model

3.1 Preliminaries

Before we proceed to solve for the equilibrium, let us first consider the static benchmark model in the absence of the delay option and the opportunity of learning Y.

[5]Clearly, when $\gamma = 0$, it is not possible to see $Y = 0$; and when $\gamma = 1$, $Y = 1$ is impossible. Therefore, in these extreme cases, the public signal Y cannot be informative. To avoid this trivial cases, we consider $\gamma \in (0, 1)$.

Static Benchmark

Suppose that agents invest if and only if $s_i \geq \hat{s}$ for some $\hat{s}$. Then, for any given θ, the aggregate investment is $X(\theta, \hat{s}) = 1 - F(\frac{\hat{s}-\theta}{\sigma})$. From the definition in (1), coordination is successful if and only if $\theta \geq F(\frac{\hat{s}-\theta}{\sigma})$.

To facilitate our analysis, let us define a function $\Theta(s)$ by the following equation

$$\Theta(s) = F(\frac{s - \Theta(s)}{\sigma}). \tag{3}$$

This definition is proper as, clearly, for any s, there exists a unique solution of $\Theta(s)$. In addition, by definition, $\Theta(s)$ is strictly increasing in s. Therefore, we can write the condition for successful coordination simply as $\theta \geq \Theta(\hat{s})$.

Now, consider the marginal agent with signal $\hat{s}$, the probability that coordination succeeds is $\mathbb{P}(\theta \geq \Theta(\hat{s})|\hat{s})$. Given the diffuse prior on θ, this probability is equal to $1 - F(\frac{\Theta(\hat{s})-\hat{s}}{\sigma}) = F(\frac{\hat{s}-\Theta(\hat{s})}{\sigma})$, which by definition of $\Theta(\cdot)$ in (3) is equal to $\Theta(\hat{s})$. The function $\Theta(\hat{s})$ has therefore a *double* interpretation. In the static setting with monotone strategy, $\Theta(\hat{s})$ measures both the minimum value of θ for a successful coordination and, for the marginal agent with $\hat{s}$, the probability that coordination is successful.

Lemma 1 (Static Benchmark)**.** *In the unique equilibrium, agents invest if and only if $s_i \geq s_0 = \Theta^{-1}(\theta_0) = \theta_0 + \sigma F^{-1}(\theta_0)$. Based on this equilibrium, coordination is successful if and only if $\theta \geq \theta_0 = c$.*

Lemma 1 presents a well-known result in the global game literature, which is re-stated here to serve as a comparison benchmark.[6] Following our discussion, to understand this result, note that in equilibrium, the marginal agent with signal $\hat{s}$ must be indifferent between investing and not investing, that is, $\Theta(\hat{s}) \cdot 1 - c = 0$. Therefore, $\theta_0 = c$ and $s_0 = \Theta^{-1}(\theta_0)$.

History-dependent Binary Signal Y

Back to the dynamic setting where each agent has the delay option, this option is valuable only when it is associated with some opportunity of learning additional information. Otherwise, as delayed investment is costly, no one will take the delay option, and, therefore, the dynamic game will be reduced to a static one. In our model, the binary public signal Y serves this role and incentivizes (some) agents not to act earlier.

[6] See, for example, among others, Morris and Shin (2003).

For the Bayesian agents who move at $t = 2$, the meaning of the public signal Y depends on the play of the agents at $t = 1$. To see this, consider that each agent's investment set $t = 1$ is $[\hat{s}_1, +\infty)$. Under that strategy, the aggregate investment at $t = 1$ $X_1(\theta, \hat{s}_1) = F(\frac{\theta - \hat{s}_1}{\sigma}) \geq \gamma$ occurs if and only if $\theta \geq \Theta_\gamma(\hat{s}_1)$, where $\Theta_\gamma(\cdot)$ is defined as

$$\Theta_\gamma(s) = s + \sigma F^{-1}(\gamma). \tag{4}$$

Accordingly, $Y = 1$ (see (2)) essentially means that $\theta \geq \Theta_\gamma(\hat{s}_1)$, which makes agents more optimistic about the fundamental θ, whereas $Y = 0$ is a piece of negative news indicating $\theta < \Theta_\gamma(\hat{s}_1)$.[7] When $\hat{s}_1$ increases (or agents are more reluctant to invest at $t = 1$), $Y = 1$ is less likely to be generated (as $\Theta_\gamma(\hat{s}_1)$ increases). Interestingly, for the same reason, when it is generated, $Y = 1$ becomes a stronger piece of information. Likewise, the parameter γ controls the likelihood of $Y = 1$ and affects the informativeness of this positive news. As $\Theta_\gamma(\hat{s}_1)$ increases with γ for any given $\hat{s}_1$, with a higher γ, $Y = 1$ is more rare ex-ante but it is more informative ex-post.

Moreover, given all others choose to invest iff $s_i \geq \hat{s}_1$, the marginal agent with private signal $\hat{s}_1$ always expects to see $Y = 1$ with probability $1 - \gamma$. This result relies on the so-called *Laplacian* beliefs (about the aggregate action) in the global game literature, that is, the marginal agent holds the belief that the aggregate investment X_1 follows a uniform distribution over $[0, 1]$. The properties of $\Theta_\gamma(.)$ are summarized in the following Lemma.

Lemma 2 (Endogenous Learning)**.** *If all agents take the monotone strategy $\hat{s}_1$ at $t = 1$,*

1. *the public signal $Y = 1$ ($Y = 0$) is generated when $\theta \geq \Theta_\gamma(\hat{s}_1)$ ($\theta < \Theta_\gamma(\hat{s}_1)$).*

2. *$\Theta_\gamma(\hat{s}_1)$ increases with $\hat{s}_1$ and γ.*

3. *In addition, for the marginal agent with $s_i = \hat{s}_1$, $\mathbb{P}(Y = 1|\hat{s}_1) = \mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) = 1 - \gamma$.*

Thus far, we have discussed how the ex-ante likelihood and the ex-post informativeness of message $Y = 1$ are determined by the time-1 investment strategy $\hat{s}_1$ as well as the information parameter γ. Next, we are going to consider what agents would do after receiving the public binary signal (if they have chosen to wait at $t = 1$). We will show that the strategy of waiting and then investing after $Y = 0$, i.e., $\hat{s}_2^0 < \hat{s}_1$, cannot support any equilibrium with monotone strategies.

[7] It is worth noting that, as $\theta \in (-\infty, +\infty)$, for any possible $\hat{s}_1$, both signals $Y = 1$ and $Y = 0$ will be on path, so that in any possible equilibrium agents can always apply the Bayes rule to determine their posterior beliefs about θ.

No Investing after $Y = 0$

Since the public signal Y is binary, for any agent who chooses to wait, she cannot invest at $t = 2$ both after the observations $Y = 1$ and $Y = 0$. Such a strategy would be equivalent to delay with a commitment to invest in period 2, without regard to the realization of Y. Such a strategy yields a payoff that is equal to the discounted value of the first period investment. As such, this strategy is strictly dominated by investing at $t = 1$, which saves the delay cost of investing. See Chamley and Gale (1994) and Gul and Lundholm (1995) for a similar argument in a different economic environment where the delay option is available. The following Lemma applies this logic to the case where agents play monotone strategies.

Lemma 3. *Any monotone strategy $(\hat{s}_1, \hat{s}_2^0, \hat{s}_2^1)$ that can possibly constitute an equilibrium satisfies $\max\{\hat{s}_2^0, \hat{s}_2^1\} = \hat{s}_1$.*

The basic intuition behind this Lemma should be clear. If $\max\{\hat{s}_2^0, \hat{s}_2^1\} < \hat{s}_1$, then an agent with $s_i \in (\max\{\hat{s}_2^0, \hat{s}_2^1\}, \hat{s}_1)$ will wait at $t = 1$ and then invest at $t = 2$ regardless of Y, which contradicts with sequential rationality. Lemma 3 puts a restriction on the set of possible monotone strategies that can constitute an equilibrium. Following this result, in any possible equilibrium, either $\hat{s}_2^0 = \hat{s}_1$ or $\hat{s}_2^1 = \hat{s}_1$. The following Lemma further demonstrates that former case (i.e., $\hat{s}_2^0 = \hat{s}_1$) must hold true in equilibrium with monotone strategies. In words, there does not exist any equilibrium in which an agent who waits at $t = 1$ and then invest at $t = 2$ only after receiving the negative news $Y = 0$.

Lemma 4 (No Investing after $Y = 0$)**.** *There exists no monotone equilibrium where $\hat{s}_2^0 < \hat{s}_1$.*

This result can be understood intuitively. First of all, recall that $Y = 1$ delivers a positive news about fundamental θ, whereas $Y = 0$ is a piece of negative news. If, in equilibrium, agents who have waited choose to invest only after $Y = 0$, then the negative news (i.e., $\theta < \Theta_\gamma(\hat{s}_1)$) cannot be "too bad." This is because, if $\theta < 0$, coordination success can never happen regardless of what agents do. If $\Theta_\gamma(\hat{s}_1)$ is small and close to 0, then the chance of success is slim, and, thus, no one would invest after $Y = 0$. In fact, as we prove in the Appendix, to have agents to invest after $Y = 0$, $\Theta_\gamma(\hat{s}_1)$ has to be greater than the fundamental threshold in the static benchmark $\theta_0 = c$. However, this condition makes $Y = 1$ a very optimistic piece of news, and therefore, agents would be more willing to invest following it (as compared with the negative news $Y = 0$). Formally, we show that, under the condition $\Theta_\gamma(\hat{s}_1) > c$, the marginal agent with $\hat{s}_1$ always chooses to invest at

$t = 2$ after $Y = 1$. As such, $\hat{s}_2^0 < \hat{s}_1$ must imply $\hat{s}_2^1 < \hat{s}_1$, which contradicts with Lemma 3. For that reason, equilibrium with $\hat{s}_2^0 < \hat{s}_1$ does not exist.

All Possible Equilibrium and Fundamental Cutoff $\hat{\theta}$

By Lemma 3 and 4, in all possible equilibrium with monotone strategies, any agent who chooses to wait at $t = 1$, if she chooses to invest at $t = 2$, she only does that after $Y = 1$; that is, $\hat{s}_2^0 = \hat{s}_1$ and $\hat{s}_2 := \hat{s}_2^1 \leq \hat{s}_1$.[8] Next, we consider any possible strategy $(\hat{s}_1, \hat{s}_2)$ and try to understand the outcome of the dynamic coordination game if all agents take such a strategy; that is, the set of all fundamentals θ that can lead to a coordination success.

Note that it is possible that the coordination success is already achieved based on time-1 investment, i.e., $\theta + X_1 \geq 1$, which is independent of any additional investment happens after $t = 1$. This occurs if and only if $\theta \geq \Theta(\hat{s}_1)$. Since $Y = 1$ indicates $\theta \geq \Theta_\gamma(\hat{s}_1)$, if the condition $\Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$ holds, then coordination never fails after $Y = 1$, regardless of what agents would do after $Y = 1$ (or the value of $\hat{s}_2$). In this case, $\hat{\theta} = \Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$. If we can find an equilibrium in this fashion, then $Y = 1$ predicts the coordination success and coordination may even succeed after $Y = 0$ (when $\theta \in [\hat{\theta}, \Theta_\gamma(\hat{s}_1))$).

In contrast, if $\hat{s}_1$ satisfies $\Theta(\hat{s}_1) > \Theta_\gamma(\hat{s}_1)$, then, as there is no investment after $Y = 0$, coordination success never occurs after $Y = 0$ because this negative news indicates $\theta < \Theta_\gamma(\hat{s}_1) < \Theta(\hat{s}_1)$. In this case, $Y = 1$ (or $\theta \geq \Theta_\gamma(\hat{s}_1)$) may not predict coordination success. Coordination success occurs only after $Y = 1$ (i.e., $\theta \geq \Theta_\gamma(\hat{s}_1)$) and when the fundamental $\theta \geq \Theta(\hat{s}_2)$. (Recall that, given $Y = 1$, all agents with $s_i \geq \hat{s}_2$ will invest by the end of $t = 2$.) Therefore, there still exists a fundamental cutoff $\hat{\theta}$, above which coordination will succeed, and $\hat{\theta} = \max\{\Theta_\gamma(\hat{s}_1), \Theta(\hat{s}_2)\}$.

Therefore, there exist two possible types of equilibrium, which depends on the whether or not $\Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$. The following Lemma summarizes the how the fundamental cutoff $\hat{\theta}$ is determined and how that depends on the time-1 investment strategy $\hat{s}_1$.

Lemma 5. *If all agents adopt the strategy $(\hat{s}_1, \hat{s}_2)$,*

1. *the fundamental cutoff $\hat{\theta}$ is*

$$\hat{\theta} = \mathbb{1}\{\Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)\}\Theta(\hat{s}_1) + \mathbb{1}\{\Theta(\hat{s}_1) > \Theta_\gamma(\hat{s}_1)\} \max\{\Theta(\hat{s}_2), \Theta_\gamma(\hat{s}_1)\}. \quad (5)$$

2. *If $\hat{s}_1 \geq s_\gamma$ in which,*

$$s_\gamma \equiv 1 - \gamma + \sigma F^{-1}(1 - \gamma). \quad (6)$$

[8] For simplicity, we replace $\hat{s}_2^1$ by $\hat{s}_2$ from now on.

then the fundamental cutoff $\hat{\theta} = \Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$ *and coordination never fails after* $Y = 1$.

3. *If* $\hat{s}_1 < s_\gamma$*, then* $\Theta(\hat{s}_1) > \Theta_\gamma(\hat{s}_1)$ *and the fundamental cutoff* $\hat{\theta} = \max\{\Theta(\hat{s}_2), \Theta_\gamma(\hat{s}_1)\}$. *In addition, coordination never succeeds after* $Y = 0$.

Proof. Given $\hat{s}_2^0 = \hat{s}_1$ and $\hat{s}_2 = \hat{s}_2^1 \leq \hat{s}_1$ (Lemma 4), the determination of the fundamental cutoff $\hat{\theta}$ in (5) follows immediately from the above discussions.

To compare $\Theta_\gamma(.)$ and $\Theta(.)$, recall that $\Theta(\hat{s}_1) + \sigma F^{-1}(\Theta(\hat{s}_1)) = \hat{s}_1$ (see (3)) and $\Theta_\gamma(\hat{s}_1) + \sigma F^{-1}(1-\gamma) = \hat{s}_1$ (see (4)). Then, the difference is

$$\Theta(\hat{s}_1) - \Theta_\gamma(\hat{s}_1) = \sigma\left[F^{-1}(1-\gamma) - F^{-1}(\Theta(\hat{s}_1)).\right] \tag{7}$$

It is easy to check that $\Theta(\hat{s}_1) = \Theta_\gamma(\hat{s}_1)$ has the unique solution $\hat{s}_1 = s_\gamma = \Theta^{-1}(1-\gamma) = 1 - \gamma + \sigma F^{-1}(1-\gamma)$. Because both $\Theta(.)$ and $F^{-1}(.)$ are increasing functions, $\Theta(\hat{s}_1) - \Theta_\gamma(\hat{s}_1)$ is strictly decreasing in $\hat{s}_1$. Therefore, $\Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$ if and only if $\hat{s}_1 \geq s_\gamma$. The rest of the proof follows the determination of $\hat{\theta}$ in (5). □

A critical threshold $\hat{s}_1 = s_\gamma$

Next, we focus our attention on a critical threshold $\hat{s}_1 = s_\gamma$, which equates $\Theta(\hat{s}_1) = \Theta_\gamma(\hat{s}_1)$. In words, given no one invests after $Y = 0$ in any equilibrium, no success can be achieved after $Y = 0$ (for any $\theta < \Theta_\gamma(s_\gamma) = \Theta(s_\gamma)$); and the success of coordination is already achieved based on time-1 investment X_1 as long as $Y = 1$ is generated (since $\theta \geq \Theta_\gamma(s_\gamma) = \Theta(s_\gamma)$). Therefore, this time-1 investment strategy makes the public signal $Y = 1$ a perfect indicator of coordination success; that is, successful coordination occurs after and only after $Y = 1$ (regardless of $\hat{s}_2$).

Suppose that all other agents choose $\hat{s}_1 = s_\gamma$, then waiting and then investing after $Y = 1$ does not miss any opportunity for successful coordination. The expected payoff difference for the marginal agent endowing with $s_i = s_\gamma$ is

$$\begin{aligned} H(\gamma) \equiv& \mathbb{P}(\theta \geq \Theta(s_\gamma)|s_\gamma) - c - \delta\mathbb{P}(Y = 1|s_\gamma)(1-c) \\ =& 1 - \gamma - c - \delta(1-\gamma)(1-c). \end{aligned} \tag{8}$$

The following Lemma summarizes some useful properties of function H.

Lemma 6. $H(\gamma)$ *decreases with* γ *and* $H(\gamma) = 0$ *if and only if* $\gamma = \gamma_0 := \frac{(1-\delta)(1-c)}{1-\delta(1-c)}$.

Proof. Since $H(\gamma) = (1-\delta)(1-c) - [1 - \delta(1-c)]\gamma$, it is decreasing in γ as $1 - \delta(1-c) > 0$. $H(\gamma) = 0$ has a unique solution $\gamma_0 = \frac{(1-\delta)(1-c)}{1-\delta(1-c)} \in (0, 1-c)$. □

In this special case with $\hat{s}_1 = s_\gamma$, an agent investing early pays the investment cost c irrespective of Y. Conditional on $Y = 1$, coordination is successful and investing early yields a payoff of 1, whereas waiting and investing after $Y = 1$ yields a discounted payoff $\delta(1-c)$. Therefore, the expected benefit from investing earlier is $(1-\delta(1-c))$ times the probability of successful coordination, which equates $1-\gamma$ for the marginal agent. Therefore, any increases in γ reduces the probability of success and, in turn, makes investing early less attractive. Lemma 6 says that once γ increases and surpasses γ_0, then the marginal agent would strictly prefer to wait since $H(\gamma) < 0$.

3.2 Equilibrium

With all these preparations, we are ready to solve for equilibrium. Since $\hat{s}_2^0 = \hat{s}_1$ must hold in any equilibrium, an equilibrium can be characterized by $(\hat{s}_1, \hat{s}_2)$. The monotone strategy $(\hat{s}_1, \hat{s}_2)$ can be formally described as follows: agents invest at $t = 1$ if $s_i \geq \hat{s}_1$; agents choose to wait at $t = 1$ and then invest at $t = 2$ only after observing $Y = 1$ if $s_i \in [\hat{s}_2, \hat{s}_1)$; and agents never invest if $s_i < \hat{s}_2$. Following the Bayes rule, the public signal $Y = 1$ ($Y = 0$) implies $\theta \geq \Theta_\gamma(\hat{s}_1)$ ($\theta < \Theta_\gamma(\hat{s}_1)$) in equilibrium. The equilibrium fundamental threshold that determines the coordination outcome is defined as θ^*; that is, successful coordination is achieved if and only if $\theta \geq \theta^*$.

3.2.1 Waiting for the good news: $\gamma > \gamma_0$

The next Proposition presents the unique equilibrium under the parameter condition $\gamma > \gamma_0$.[9]

Proposition 1. *When $\gamma > \gamma_0$, there exists a unique equilibrium ($\hat{s}_1 = s_1^*, \hat{s}_2 = -\infty$), where $s_1^* = \Theta^{-1}(c + \delta(1-\gamma)(1-c))$. Based on this equilibrium, coordination is successful if and only if $\theta \geq \theta^* = c + \delta(1-\gamma)(1-c)$.*

To understand this result intuitively, let us start with the case in which $\hat{s}_1 = s_\gamma$. Recall that when $\gamma > \gamma_0$, if all other agents choose $\hat{s}_1 = s_\gamma$, the marginal agent would prefer to wait and then invest after $Y = 1$ since the expected payoff difference $H(\gamma) < 0$ (Lemma 6). Because of this deviation, in equilibrium, we must have $\hat{s}_1 > s_\gamma$. Interestingly, this deviation does not change the fact that coordination is a guaranteed success after $Y = 1$

[9] It is worth noting that, by definition of γ_0, if $\gamma \geq 1-c$, then the condition $\gamma > 1-\gamma_0$ holds true regardless of the discount factor $\delta \in (0,1)$. In contrast, this condition relies on the parameter δ if $\gamma < 1-c$.

since $\Theta_\gamma(\hat{s}_1) > \Theta(\hat{s}_1)$ (Lemma 5) and therefore, if such an equilibrium exists, agents whoever have waited at $t = 1$ will invest at $t = 2$ following $Y = 1$ regardless of their private information –i.e., $\hat{s}_2 = -\infty$. However, although waiting and investing after $Y = 1$ is a guaranteed success, agents who wait are going to miss some opportunity of investing and receiving a high payoff since there is some coordination success following $Y = 0$ even without time-2 investment (when $\theta \in [\Theta(\hat{s}_1), \Theta_\gamma(\hat{s}_1))$. In this way, the deviation stops at the equilibrium cutoff s_1^*.

Recall that when $\hat{s}_1 > s_\gamma$, successful coordination purely relies on the time-1 investment and, thus, the equilibrium fundamental threshold $\theta^* = \Theta(s_1^*)$. To see how the equilibrium threshold s_1^* is determined, note that, in equilibrium, the marginal agent with s_1^* is indifferent between investing early and waiting given that others are taking s_1^*; that is

$$\mathbb{P}(\theta \geq \Theta(s_1^*)|s_1^*) - c - \delta\mathbb{P}(Y = 1|s_1^*)(1 - c) = \Theta(s_1^*) - c - \delta(1 - \gamma)(1 - c) = 0 \quad (9)$$

That immediately solves for the equilibrium threshold s_1^*, and it is easy to verify that $s_1^* > s_\gamma$ given $\gamma > \gamma_0$.[10] In the Appendix, we further check that, when all other players take the equilibrium strategy, investing early is a strictly better (worse) choice than waiting and then investing after $Y = 1$ whenever $s_i > s_1^*$ ($s_i < s_1^*$) so that $(\hat{s}_1 = s_1^*, \hat{s}_2 = -\infty)$ is indeed an equilibrium.

To understand why there does not exist any equilibrium with $\hat{s}_1 \leq s_\gamma$, let us consider any of such $\hat{s}_1$ as an candidate equilibrium time-1 investment strategy and suppose the equilibrium fundamental cutoff is $\hat{\theta}$. Under the condition $\hat{s}_1 \leq s_\gamma$, we know that $\Theta(\hat{s}_1) \geq \Theta_\gamma(\hat{s}_1)$ (Lemma 5). As there is no investment after $Y = 0$, coordination cannot be successful for any $\theta < \Theta_\gamma(\hat{s}_1)$. Therefore, regardless of the equilibrium $\hat{s}_2$, we know that the equilibrium fundamental cutoff is such that $\hat{\theta} \geq \Theta_\gamma(\hat{s}_1)$. In this case, the expected payoff difference for the marginal agent is

$$\begin{aligned} &\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - c - \delta\left(\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) \times 1 - \mathbb{P}(Y = 1|\hat{s}_1)c\right) \\ \leq &\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) - c - \delta\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1)(1 - c) = H(\gamma) < 0, \end{aligned}$$

which implies that, for any $\hat{s}_1 \leq s_\gamma$, the marginal agent always prefers to wait. Therefore, such equilibrium cannot exists and the monotone equilibrium characterized in Proposition 1 is indeed the unique equilibrium under the parameter condition $\gamma > \gamma_0$.

[10] Note that based on the monotonicity of $\Theta(.)$, this is equivalent to $\Theta(s_1^*) = c + \delta(1 - \gamma)(1 - c) > \Theta(s_\gamma) = 1 - \gamma$, which holds true under $\gamma > \gamma_0$.

Notably, in the unique equilibrium, the binary public signal Y perfectly coordinates the actions of the agents who have waited – all investing after $Y = 1$ and no investing after $Y = 0$ — and successful coordination always occur following $Y = 1$. However, as agents are reluctant to invest early (i.e., $s_1^* > s_\gamma$), this excessive waiting makes the positive news $Y = 1$ very unlikely to arrive. Moreover, since the coordination success is determined by the time-1 investment, the excessive waiting and the resulting inaction at $t = 1$ make coordination success unlikely.

3.2.2 Investing to generate the good news: $\gamma < \gamma_0$

Consider the case in which $\hat{s}_1 = s_\gamma$ under the parameter condition $\gamma < \gamma_0$. In this case, as $H(\gamma) > 0$ (see Lemma 6), the marginal agent with $\hat{s}_1$ would deviate to investing early. As a result, we would expect that the equilibrium $\hat{s}_1 < s_\gamma$.

Formally, we can show that there does not exist any equilibrium with $\hat{s}_1 \geq s_\gamma$. To see this, consider any $\hat{s}_1 \geq s_\gamma$. Under such a time-1 investment strategy, by Lemma 5, $Y = 1$ predicts coordination success and, thus, in any possible equilibrium, $\hat{s}_2 = -\infty$. Therefore, the fundamental cutoff $\hat{\theta} \leq \Theta_\gamma(\hat{s}_1)$. If the cutoff $\hat{s}_1$ can support an equilibrium, then the marginal agent with $\hat{s}_1$ should be indifferent between investing at $t = 1$ and waiting and then investing at $t = 2$ after $Y = 1$. However, the expected payoff difference for the marginal agent is strictly positive since

$$\begin{aligned}
&\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - c - \delta\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1)(1-c) \\
\geq &\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) - c - \delta\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1)(1-c) \\
= &H(\gamma) > 0.
\end{aligned}$$

As a result, we cannot construct and equilibrium with $\hat{s}_1 \geq s_\gamma$. The following Proposition demonstrates the unique equilibrium under $\gamma < \gamma_0$. In this equilibrium, $s_1^* < s_\gamma$ and $s_2^* > -\infty$.

Proposition 2. *When $\gamma < \gamma_0$, there exists a unique equilibrium $(\hat{s}_1 = s_1^*, \hat{s}_2 = s_2^*)$ with*

$$s_1^* = \sigma F^{-1}\left(\frac{c\left[1-\delta(1-\gamma)\right]}{1-\delta}\right) + \theta^* \quad \text{and} \quad s_2^* = \Theta^{-1}(\theta^*) \tag{10}$$

where the fundamental threshold $\theta^* \in (0, c)$ uniquely solves

$$G(\theta^*) := \frac{\theta^*}{F\left(F^{-1}(\theta^*) - F^{-1}\left(\frac{c[1-\delta(1-\gamma)]}{1-\delta}\right) - F^{-1}(\gamma)\right)} = c. \tag{11}$$

The construction of the equilibrium can be understood as follows. First, as discuss, any equilibrium with monotone strategies under $\gamma < \gamma_0$ must admit $\hat{s}_1 < s_\gamma$. As agents are willing to invest at $t = 1$, the positive signal $Y = 1$ is more likely to be generated. Moreover, given that $\hat{s}_1 < s_\gamma$, coordination success may not occur after $Y = 1$ since $\Theta_\gamma(\hat{s}_1) < \Theta(\hat{s}_1)$ (Lemma 5). As there is no invest following $Y = 0$ in any possible equilibrium, the condition $\Theta_\gamma(\hat{s}_1) < \Theta(\hat{s}_1)$ implies no coordination success after $Y = 0$. Therefore, though it is not guaranteed, all successful coordination occurs after $Y = 1$, which is determined by the new investment X_2 happened after $Y = 1$ instead of the time-1 investment X_1. That is, coordination is successful only when $\theta \geq \Theta_\gamma(\hat{s}_1)$ and $\theta + \mathbb{P}(s_i \geq \hat{s}_1|\theta) + \mathbb{P}(\hat{s}_2 \leq s_i < \hat{s}_1|\theta) \geq 1$, or equivalently,

$$\theta \geq \hat{\theta} = \max\{\Theta(\hat{s}_2), \Theta_\gamma(\hat{s}_1)\}. \tag{12}$$

For the marginal agent with $\hat{s}_2$, he is indifferent between waiting and then investing after $Y = 1$ and not investing, i.e.,

$$\delta\mathbb{P}(Y = 1|\hat{s}_2)\left[\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_2, \theta \geq \Theta_\gamma(\hat{s}_1)) \times 1 - c\right] = 0 \tag{13}$$

It is worth noting that the public signal $Y = 1$, though it does not guarantee the success of investment, it boosts the agents' belief as it indicates that $\theta \geq \Theta_\gamma(\hat{s}_1)$ in equilibrium. Therefore, this information makes $s_i \in (\hat{s}_2, \hat{s}_1)$, who are not so sure about the investment success at $t = 1$, more confident about $\theta \geq \hat{\theta}$. Therefore, they choose to invest after $Y = 1$ at $t = 2$.

Furthermore, the marginal agent with $\hat{s}_1$ is indifferent between investing at $t = 1$ and waiting and then investing at $t = 2$ after $Y = 1$; that is,

$$\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - c = \delta\left[\mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) - c\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1)\right] \tag{14}$$

In the Appendix, we show that $(\theta^*, \hat{s}_1^*, \hat{s}_2^*)$ as presented in Proposition 2 is the unique set of solution for $(\hat{\theta}, \hat{s}_1, \hat{s}_2)$ which satisfy equations (12), (13) and (14). The condition $\gamma < \gamma_0$ ensures that, in equilibrium, $\theta^* = \Theta(s_2^*) > \Theta_\gamma(s_1^*)$,[11] which means investment may fail after $Y = 1$ when $\theta \in [\Theta_\gamma(s_1^*), \theta^*)$ and $Y = 1$ cannot serve a perfect predict for coordination success. We further show that for any $s_i > s_1^*$, agents strictly prefer to investing at $t = 1$; for any $s_i \in (s_2^*, s_1^*)$, agents strictly prefer to wait and then invest at

[11]Technically, $\Theta_\gamma(s_1^*) = \theta^* + \sigma F^{-1}(\gamma) + \sigma F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta})$. Under the condition $\gamma < \gamma_0$, $\Theta_\gamma(s_1^*) < \theta^*$ holds true since it can be proved that $\sigma F^{-1}(\gamma) + \sigma F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta}) < 0$ (see the proof of Proposition 2 in the Appendix for details).

$t = 2$ after $Y = 1$; and, for any $s_i < s_2^*$, agents strictly prefer not to invest. In this way, we show that (s_1^*, s_2^*) constitutes an equilibrium for the case in which $\gamma < \gamma_0$.

4 Does the delay option facilitate coordination?

Whether coordination takes place or not depends on the value of the fundamental θ. Facilitating coordination means expanding the range of the fundamental's values for which coordination takes place. Of course, if $\theta < 0$ coordination cannot take place. Likewise, if $\theta > 1$, the regime's fall is independent of individual actions. The only values of interest are between 0 and 1, for which under perfect information, there are two equilibria with and without coordination. As in most models of coordination, coordination takes place here when θ is greater than some value, say, θ^*. Facilitating coordination is equivalent to lowering θ^*. In the static benchmark with no option for delay, we know $\theta^* = \theta_0 = c$ (Lemma 1).

The option of delay, as considered in our model, enables agents to wait and then make a more informative investment decision based on new information available to them. Therefore, whether the delay option could facilitate coordination depends on the learning technology. Below, from this perspective, we discuss how the impact of delay option depends on the information structure.

4.1 Endogenous Public Learning

Our model studies an arguably realistic information structure in which the agents who choose to wait observe a public signal which offers imperfect information about historical activities. Only with sufficiently many actions, will the good news $Y = 1$ be generated. A critical feature of the learning technology in the present paper is the informativeness of the public signal $Y = 0, 1$ depends on the action taken by the agents at $t = 1$. The equilibrium strategy $\hat{s}_1$ determines how agents interpret the public signal $Y = 1$ ($Y = 0$) as $\theta \geq \Theta_\gamma(\hat{s}_1)$ ($\Theta < \Theta_\gamma(\hat{s}_1)$). Hypothetically, if no agent invests at $t = 1$ (i.e., $\hat{s}_1 \to +\infty$), then the signal $Y = 1$ would be super informative as $\Theta_\gamma(\hat{s}_1) \to +\infty$ (see (4)); if all agents invest at $t = 1$ (i.e., $\hat{s}_1 \to -\infty$), then the signal $Y = 0$ becomes very informative as $\Theta_\gamma(\hat{s}_1) \to -\infty$. The parameter γ governs the informativeness of the public signal Y, and, thus, plays a critical role in shaping the equilibrium.

Existing studies (e.g., Kováč and Steiner (2013) and Dasgupta (2007)) on the impact of delay option on coordination focus on private learning. Notably, the information agents wait for is a noisy signal about the fundamental θ. The availability and informativeness of this private signal does not rely on the time-1 investment strategy $\hat{s}_1$. To see this clearly, in Dasgupta (2007), the information is $y_i = \Phi^{-1}(X_1) + \tau\eta_i$, where τ is the scale of the noisiness and η_i is a conditionally independent noise following a standard normal distribution. In equilibrium, agents know the time-1 strategy $\hat{s}_1$, and, therefore, y_i becomes a noisy signal about θ, whose informativeness does not rely on $\hat{s}_1$, i.e., $y_i = \Phi^{-1}\left(\Phi(\frac{\hat{s}_1-\theta}{\sigma})\right) + \tau\eta_i = \frac{\hat{s}_1-\theta}{\sigma} + \tau\eta_i$.

We believe that how agents learn is affected by other agents' play is a critical feature that should be incorporated in a model of dynamic coordination with the option of delay. Because of this critical difference in the learning technology, our model features quite different equilibrium prediction (as compared with Dasgupta (2007)) and highlights the possibility that delay can hinder coordination.[12]

In our model, the parameter γ governs how much information agents can learn by waiting. This parameter affects availability and informativeness of the good signal $Y = 1$ – the only signal that may persuade more agents to invest at $t = 2$ – is determined in equilibrium. Given any time-1 investment strategy $\hat{s}_1$, When γ increases, the availability of the good news ex-ante reduces whereas it becomes more informative ex-post since $\Theta_\gamma(\hat{s}_1)$ increases with γ. Relying on this trade-off, our theory is able to identify a clear condition – whether γ is greater or less than γ_0, which determines whether or not the delay option facilitates coordination.[13]

Proposition 3. *Based on the unique equilibrium with monotone strategies, $\theta^*(\gamma) < \theta_0$ when $\gamma < \gamma_0$ and $\theta^*(\gamma) > \theta_0$ when $\gamma > \gamma_0$.*

Proof. When $\gamma > \gamma_0$, by Proposition 1, $\theta^* = c + \delta(1-\gamma)(1-c) > \theta_0 = c$; while, when $\gamma < \gamma_0$, by Proposition 2, $\theta^* < c = \theta_0$. (For the proof of $\theta^* < c$, see the proof of Proposition 2 in the Appendix.) □

Next, we discuss this result based on the unique equilibrium solved in Proposition 1 and 2. Let us first consider the case in which $\gamma \to 0+$. In this case, the game is asymp-

[12] It is worth noting that, when comparing the dynamic setting with delay option to the static model, Dasgupta (2007) focuses on the limiting case where the noise of private information varnishes. Delay option is found to be promoting coordination on investment in that limiting case. However, our result on the impact of delay option does not depend on the noisiness of the private information.

[13] For illustration purpose, we introduce the notation $\theta^*(\gamma)$ to denote the fundamental cutoff θ^* for any given parameter γ.

totically equivalent to the static game since the probability of $Y = 1$ is close to 1 and this observation is not informative (since $\Theta_\gamma(\hat{s}_1) = -\infty$ when $\gamma \to 0+$ for any $\hat{s}_1$). As γ increases from zero, $Y = 1$ becomes informative (since $\Theta_\gamma(s_1^*) > -\infty$). Therefore, some agents who chooses to wait would invest at $t = 2$ as they becomes more optimistic about θ after learning $Y = 1$. As long as $\gamma < \gamma_0$, all coordination successes occur after $Y = 1$. Successful coordination (at least some of them) relies on time-2 investment (Proposition 2). Compared with the static case, since there will be some additional investments at $t = 2$ for any $\theta \geq \Theta_\gamma(s_1^*)$, such time-2 investment, in turn, incentivizes more early investment because of strategic complementarity. In this way, the delay option, together with the observability of Y, facilitate coordination.

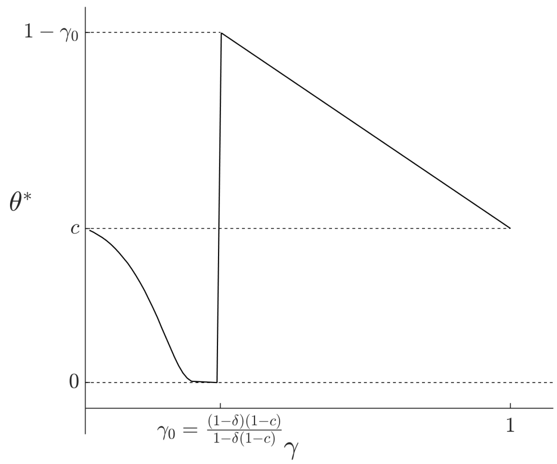
In the range of $\gamma \in (0, \gamma_0)$, when γ increases, $Y = 1$ becomes a more informative signal that predicts the coordination success. Formally, in equilibrium, the difference between the cutoff $\Theta_\gamma(s_1^*)$ and the successful cutoff $\theta^* = \Theta(s_2^*)$ shrinks when γ increases.[14] Therefore, $Y = 1$ becomes a more effective signal predicting the coordination success (although it is not a perfect predictor). Consequently, there will be more investment after $Y = 1$, which incentivizes more time-1 investment through complementarity and, thus, increases the occurrence of coordination success. In the limiting case where $\gamma \to \gamma_0-$, the difference between $\Theta_\gamma(s_1^*)$ and θ^* disappears, both of which converges to 0. Therefore, $Y = 1$ almost surely predicts the coordination success, and more importantly, the chance of successful coordination is maximized.

When $\gamma = \gamma_0$, $Y = 1$ becomes a perfect predictor of coordination success; that is, successful coordination occurs and only occurs after $Y = 1$. As in the limiting case of $\gamma \to \gamma_0-$, there exists an equilibrium in which $\Theta(s_1^*) > \Theta(s_2^*) = \Theta_\gamma(s_1^*) = 0$. Recall that, as long as $Y = 1$ perfectly predicts coordination success, the marginal agent's payoff difference is $H(\gamma_0) = 0$ in this special case with $\gamma = \gamma_0$ for any possible $\hat{s}_1$ (Lemma 6). Therefore, we can find multiple equilibria in which the agents at $t = 2$ always invest after $Y = 1$ (given the perfect predictability of $Y = 1$), i.e., $s_2^* = -\infty$, as long as coordination success is possible ($\Theta_\gamma(s_1^*) \geq 0$) and it relies on time-2 investment, i.e., $\Theta(s_1^*) \leq \Theta_\gamma(s_1^*)$.

When γ further increases and surpasses γ_0, the equilibrium changes dramatically. As γ is sufficiently large, the good news becomes so informative that coordination always succeeds after $Y = 1$. Agents have strong incentive to wait and "free ride" this information to secure an investment success. Notably, coordination success is purely dependent

[14]The difference is $\sigma\left(F^{-1}(1-\gamma) - F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta})\right)$. It decreases with γ and is strictly positive when $\gamma < \gamma_0$. When $\gamma \to \gamma_0-$, this difference converges to 0.

Figure 1: Equilibrium θ^* and Parameter γ



Note: We adopt the standard normal distribution for F and the values of other parameters are $\delta = 0.8$, $c = 0.3$.

on time-1 investment, but all agents who have waited would follow $Y = 1$ to invest regardless of their private information (Proposition 1). This incentive is maximized in the limiting case where $\gamma \to \gamma_0+$. In this limiting case, coordination occurs and only occurs after $Y = 1$, and therefore, waiting and then investing only after $Y = 1$ becomes very appealing. As the coordination success is determined by the early investment, the excessive incentive of waiting is going against successful coordination. That is why θ^* achieves its highest value in the limiting case.

When γ increases from γ_0, $Y = 1$ becomes rare and, therefore, the opportunity of successful investment after waiting is less likely to arise. As such, the delay option becomes less attractive and more agents choose to invest early, thereby increasing the likelihood of coordination success. In the limit of $\gamma \to 1-$, it is almost impossible to have $Y = 1$ (since γ is nearly equal to the mass of agents). Asymptotically, the game is the same as the one period game, and, thus, θ^* tends to $\theta_0 = c$.

Figure 1 decipts a numerical example of how the equilibrium θ^* changes with the parameter γ, confirming the above-discussed intuition. Based on the solved unique equilibrium, the following Corollary demonstrates how θ^* changes the parameter γ, proving the comparative statics result theoretically. In this Corollary, we also solve for all possible equilibrium for the special case where $\gamma = \gamma_0$.

Corollary 1. *Consider any $\gamma \in (0, 1]$.*

1. *$\theta^*(\gamma) \in (0, c)$ decreases with γ when $\gamma \in (0, \gamma_0)$. In addition, $\lim_{\gamma \downarrow 0} \theta^*(\gamma) = c$, and $\lim_{\gamma \uparrow \gamma_0} \theta^*(\gamma) = 0$.*

2. *$\theta^*(\gamma) \in (c, 1-\gamma)$ decreases with γ when $\gamma \in (\gamma_0, 1)$. In addition, $\lim_{\gamma \downarrow \gamma_0} \theta^*(\gamma) = 1 - \gamma_0$ and $\lim_{\gamma \uparrow 1} \theta^*(\gamma) = c$.*

3. *If $\gamma = \gamma_0$, multiple monotone equilibria arise. Any $s_1^* \in [\sigma F^{-1}(1-\gamma_0), s_{\gamma_0}]$ and $s_2^* = -\infty$ can constitute an equilibrium. In any equilibrium, investment is successful if and only if $\Upsilon = 1$ and $\theta^* = \Theta_\gamma(s_1^*)$.*

4.2 Discount Rate and Delay Cost

Another critical feature of our dynamic setting is that investing (or attacking) is irreversible and delayed investment is costly. The delay cost is governed by the discount factor δ. The delay cost determines the value of delay option. When $\delta = 0$, no one will exercise the delay option, and, thus, the dynamic model is reduced to a static one.

The discount factor can be written as $\delta = e^{-r\tau}$ where r is the discount rate per unit of time and τ the time length of a period. Recall that the parameter condition which determines whether the delay option can facilitate coordination is $\gamma < \gamma_0 = \frac{(1-\delta)(1-c)}{1-\delta(1-c)}$. Since $\gamma_0 \in (0, 1-c)$, the delay option does not help coordination for any $\gamma \geq 1 - c$ regardless of the discount factor $\delta \in (0, 1)$. However, when $\gamma < 1 - c$, the above condition under which the delay option helps can be interpreted as $\delta > \delta_0 := \frac{1-c-\gamma}{(1-c)(1-\gamma)}$.

Proposition 4. *Under the condition that $\gamma < 1 - c$,*

1. *if $\delta < \delta_0$, the unique equilibrium (s_1^*, s_2^*) is as described in Proposition 1 and, thus, delay option makes coordination more difficult, i.e., $\theta^* > \theta_0$;*

2. *if $\delta > \delta_0$, the unique equilibrium (s_1^*, s_2^*) is as described in Proposition 2, and, thus, delay option facilitates coordination, i.e., $\theta^* < \theta_0$.*

Proof. This follows immediately from Proposition 1 and 2. □

How the discount factor δ affects the ultimate coordination outcome can be understood intuitively. With a higher δ, delay becomes less costly, which results in excessive waiting. As more agents tend to wait and then only invest after $Y = 1$, successful coordination becomes more difficult. As long as $\delta > \delta_0$, the delay option (with a sufficiently lower delay cost) cannot facilitate coordination.

The worst case scenario occurs when $\delta \to 1-$, under which the fundamental cutoff θ^* reaches its maximum $c + (1-\gamma)(1-c)$. In the limit, delay is asymptotically costless and the incentive for waiting reaches its maximum. Interestingly, even in this case, not everyone will choose to wait because waiting and then only investing after $Y = 1$ will miss some opportunities of successful investment (since $\Theta_\gamma(s_1^*) > \Theta(s_1^*)$) as compared with investing early.

Recall that the discount factor $\delta = e^{-r\tau}$. The period length of waiting for information, or τ, may vary across different markets, and the interest rate r is subject to monetary policy interventions. Therefore, the discussion on how discount rate changes coordination outcome may have important implications for more specific applications and policy interventions, which we will discuss later.

4.3 Equilibrium Selection and Welfare

Coordination games are famous for admitting multiple (pure strategy) equilibria. This causes the difficulty in predicting the coordinating behavior and generating reasonable policy implications. Carlsson and Van Damme (1993) introduce the a perturbation of each individual's private information to relax the common knowledge assumption, and, show that this gives rise to a unique equilibrium selection which features the risk-dominant equilibrium. Morris and Shin (1998) apply the global game approach to the application of currency attacks and bank runs. However, the equilibrium selection is known to be highly sensitive to the belief environment (Rubinstein (1989), Weinstein and Yildiz (2007), etc.)

A standard context for a coordination game is a bank run (Diamond-Dybvig) or a central bank (Obstfeld). With regard to these applications, we believe that these crises do not take place in a unique period, on which, and this is an important coordination requirement, agents coordinate. Multiple periods is a necessary assumption for the analysis of such crises (Chamley, 2003). From this perspective, our model, building on the heterogeneous private information setting which follows the global game approach, presents an interesting observation of how the realistic dynamic structure can change the equilibrium selection.

Static Coordination If modeled as a static game, when individuals almost surely know θ, i.e., $\sigma \to 0$, the unique equilibrium predicts that when $\theta > \theta_0 = c$, all agents will choose to invest (or attack), and the investment (or attack) will be successful; when $\theta < c$, all agents will choose to not invest (or not attack), and any investment (or attack) is unsuccessful.

Dynamic Coordination with Delay In sharp contrast, if coordination is modeled as a dynamic game with irreversible attacks and opportunity of public learning, then, as we have shown, the model features quite different equilibrium selection. Note that when $\sigma \to 0$, all agents would have almost the same private information and, generically, their actions will be perfectly coordinated. As an equilibrium outcome, X_1 is either 0 or 1, so that no one will invest at $t = 2$.[15]

As discussed, the equilibrium selection depends on the characteristics associated with the delay option. If $\gamma > \gamma_0$,[16] all agents choose to invest (or attack) at $t = 1$ if $\theta > \theta^* = c + (1 - \gamma)(1 - c)$ and choose not to invest (attack) when $\theta < \theta^*$. In this case, as $\theta^* > \theta_0 = c$, the equilibrium selection features, from an ex-ante point of view, less investment (or irreversible attacks) and lower incidence of coordination on investing, as compared with the static setting.

If $\gamma < \gamma_0$, agents coordinate perfectly on investing (or attacking) at $t = 1$ if $\theta > \theta^*$ ($\theta^* < \theta_0$ is determined by (11)); and they coordination perfectly on not investing (not attacking) if $\theta < \theta^*$. Since $\theta^* < \theta_0$, this equilibrium selection features more investing/attacking and a higher incidence of coordination on investing/attacks.

Application to the Financial Market To understand how the dynamic setting and the resulting equilibrium selection can make a difference, let us consider the example of coordination in financial market, in which market participants and, especially, financial institutions react quickly. That implies, the period length of waiting for the public information or τ, is short, thereby inducing a high discount factor δ. The present model highlights that short periods, or fast reactions, can stabilize a financial institution that faces multiple equilibria.[17]

[15] To see this, for any $\gamma \in (0, 1)$, if $X_1 = 0$, then $Y = 0$ and, thus, there is no investment at $t = 2$. Otherwise, if $X_1 = 1$, since all agents have invested at $t = 1$, no one invests at $t = 2$.

[16] Or, equivalently, we have $\delta > \delta_0$ when $\gamma < 1 - c$ or any $\delta \in (0, 1)$ when $\gamma \geq 1 - c$.

[17] It is worth noting that the following discussion does not rely on the assumption $\delta \to 0$. As shown in Proposition 1 and 2, the fundamental cutoff θ^* does not rely on parameter σ.

When parameter values are plugged into a static model, the implication for policy is not plausible. Consider that agents are choosing whether or not to attack a financial institution (instead of invest). Attacking is irreversible and it succeeds if and only if the bank's reserve $1 - \theta < X$.

The payoff from coordination corresponds to the rate of a devaluation, say, more than 10 percent. The cost is the interest rate of the speculators during the crisis. If the crisis takes two weeks, a long time for a crisis, and the annual interest rate (as the borrowing cost) is 24 percent(example of Sweden**), the cost of speculation is 10 percent of the potential gain. For the present model, with a payoff normalized at 1, $c = 0.1$. As predicted by the static coordination, for any value of $\theta > \theta^* = c = 0.1$, agents will coordinate on attacking and, as a result, the bank fails. In other terms, the bank, in order to defend any attack, has to keep 90 percent of its deposits in cash.

The present model shows that when agents have the choice of not running and waiting for the observation that there is a run (with $Y = 1$), the bank may keep a much smaller amount of reserves to be safe. Suppose that agents notice the bank run when at least 10 percent of them run on the bank early, i.e., $\gamma = 0.1$, and the discount factor $\delta = 0.99$. Taking the same numerical values as in the above example of the static game ($c = 0.1$), we have $\gamma_0 = 0.0826 < \gamma$. (The value of γ_0 is not very sensitive to the plausible values of the discount rate δ as δ is very close to 1). The amount of reserves that the bank should keep is given by equation by Proposition 1: $\theta^* = c + \delta(1-\gamma)(1-c) = 0.911$. Therefore, a reserve of only less than 10 percent of the deposits can make a bank safe (instead of 90 percent in the static case).

Ex-ante Welfare Apart from the likelihood of investment success, under certain scenarios, a policy maker may also care about the ex-ante welfare of all agents. It is worth noting that, when $\gamma > \gamma_0$, although coordination success is less likely, when $\theta \geq \Theta_\gamma(s_1^*)$ (or given $Y = 1$), all agents end up investing and getting a high payoff. In contrast, although coordination success is more likely when $\gamma < \gamma_0$, there always exist some agents (with $s_i < s_2^*$) miss the opportunity of investing when that is proved later to be the right choice (when $\theta \geq \theta^*$). Therefore, the welfare comparison of these two cases depends on the prior distribution of θ and the improper uniform prior distribution cannot allow us to make such comparison.

Alternatively, we can consider that the fundamental θ is drawn from a normal distribution θ is $N(\theta_p, \sigma_p^2)$, and by taking $\sigma_p \to +\infty$, this setting converges to our current setting. Moreover, as we have shown earlier, in the limiting case where the noise in private information is varnishingly small, i.e., $\sigma \to 0$, all agents' actions are perfectly coordinated – all agents invest at $t = 1$ if and only if coordination on investment is successful. Under this setting (first taking $\sigma_p \to +\infty$ and then taking $\sigma \to 0$), the ex-ante welfare $W(\gamma)$ can be evaluated by the fundamental cutoff θ^*. That is because, in this limiting case, all agents collect the payoff from successful investment, $1 - c$, when and only when $\theta \geq \theta^*$. As such, the ex-ante welfare increases when the cutoff θ^* decreases.

5 Policy Discussions

In what follows, we discuss the policy implication of our theory. In our discussion, we consider a policy maker who either only cares about the ex-ante probability of successful coordination, or only cares about the welfare of agents but only focuses on the above-mentioned limiting case. In either case, the policy maker judges this economy by the fundamental cutoff θ^*. Moreover, the policy maker cannot observe the true realization of θ, and, thus, the adopted policy (no matter it is about the interest rate r or the information threshold γ) cannot signal the true fundamental θ.

5.1 Inaction and Interest Rate r

Consider an economy experiencing a recession and the policy maker wants to stimulate investments for the economic recovery. However, the investment game features strategic complementarity; that is, whether investment will be successful or not depend on how many other investor opt in. Naturally, during the economic turmoil, the investors are uncertain about the fundamental of the economy, which also matter for the investment return. Another important feature that is highlighted in our model is that investors can always choose to wait for more information before pledging their capital. This economic environment can be perfectly mapped to our model. In reality, we often observe policy makers to reduce the interest rate r as a way to promote investments and boost recovery. To fulfill this goal, the nominal interest rate was even reduced to zero after the great recession and in recent years. How does such monetary policy affect the incentive of waiting and inaction, as well as the coordination on investment?

Our model is well suited to address these questions. Recall that, given any period length of waiting τ, the discount factor in our model $\delta = e^{-r\tau}$ increases when the interest rate r decreases. When $r \to 0$, δ converges to 1. Recall that when $\gamma \geq 1 - c$, the condition $\gamma > \gamma_0$ holds true regardless of δ (or r). Therefore, the equilibrium is specified in Proposition 1, and delay option always hurts coordination on investment. Given $\gamma \geq 1 - c$ and taking everything else the same, any decrease in the interest rate r create extra incentive for waiting, thereby increasing s_1^* and making coordination success less likely to occur. See Figure 2 (a) for a graphical illustration of $\theta^*(r)$.

Now consider the cases in which $\gamma < 1 - c$. Given this condition, whether or not the delay option can be helpful in promoting coordination depends on whether $\delta < \delta_0$, which, in turn, depends on whether $r > r_0 := -\frac{1}{\tau} \ln \delta_0$. Suppose the interest rate is reduced significantly from above r_0 to a value below r_0. This essentially makes $\delta > \delta_0$, and based on Proposition 4, this change in r makes the delay option counterproductive in promoting coordination on investment, and, therefore, such a policy is ineffective in stimulating the economy.

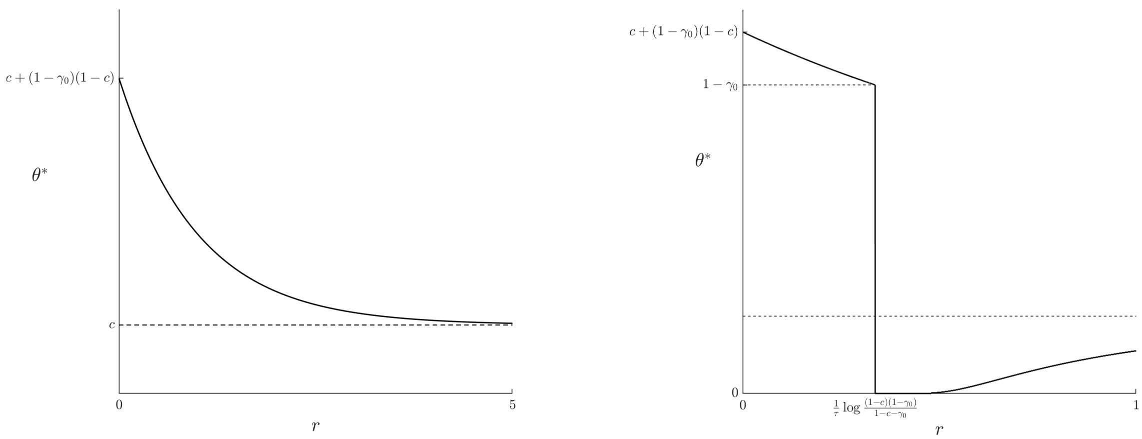
Interestingly, no matter $\gamma \geq 1 - c$ or not, when r decreases from a relative low value and becomes sufficiently close to 0, such a policy results in more inaction (or waiting) at $t = 1$, thereby reducing the chance of having the positive news $Y = 1$ and impairing aggregate investment. In fact, the worst case scenario arises at the zero lower bound when $r \to 0$, where the fundamental cutoff θ^* reaches its maximum. Figure 2(b) clearly demonstrates how the change in r affect the fundamental cutoff θ^* when $\gamma < 1 - c$.

Admittedly, lowering the interest rate may directly reduce the cost of investment c and this, in turn, helps boost investment. However, we are interested in how changes in interest rate would affect the incentive of waiting in our dynamic coordination model, and, in this way, changes the investment outcome. Notably, our model identifies a clean channel through which a lower interest rate may promote more inaction in the economic recovery, which makes such a monetary policy ineffective in stimulating the economic recovery. We believe this channel is important to be taken into account given that the macroeconomy always features strong complementarity, a dynamic structure with delay options and a low interest rate environment.

5.2 Accessibility of Information γ

Consider the cases in which the policy maker can influence the accessibility of information, which captures the minimum size of a demonstration to generate a news event and is governed by the parameter γ. When γ is close to 0, then the agents can easily easily

Figure 2: The equilibrium θ^* and the interest rate r



(a) Cases with $\gamma \geq 1 - c$

(b) Cases with $\gamma < 1 - c$

Note: We adopt standard normal distribution for F, $\tau = 1$, $c = 0.4$, $\gamma = 0.3$ for sub-figure (a), and $\gamma = 0.7$ for sub-figure (b).

observe $Y = 1$ as long as a small amount of (irreversible) actions occurs. Therefore, information is easily accessible. Otherwise, if γ is relatively large and close to 1, then by design, observing past actions become very difficult and, therefore, information is not accessible. Moreover, we consider the policy maker does not know θ so her policy that influences the accessibility of news event would not signal any information about θ.[18]

In some applications, for example, coordination on investment, the policy maker wants to promote the taking of the irreversible action. Therefore, she would influence accessibility of information, trying to reduce the fundamental cutoff θ^*. In this case, she would make the good news $Y = 1$ very easy to be generated; that is, it appears as long as a very small number of agents invest early (or $\gamma < \gamma_0$). It is worth noting that when the good news is easily accessible, it may not predict the ultimate success. However, only with high accessibility of information, the delay option together with the news Y can operate to make coordination success on investing more likely.

[18]An alternative and possibly more plausible assumption is that, although the policy maker understands θ much better than the agents, any policy or institution concerning the accessibility of information is designed ex-ante before θ is realized. Such policy and institution, e.g., enacted by law, cannot be modified easily and thus lasts for long periods of time.

In other application (e.g., bank runs), the policy maker would prevent the agents from taking the irreversible actions (or running on a bank). Recall that, in such applications, the adverse outcome occurs (or bank fails) as long as $1-\theta > 1-\theta^*$. Therefore, the policy maker wants to influence γ so as to increase θ^*. Based on our theory, this requires the "bad" news $Y=1$ occurs only if a significant share of agents have attacked (or ran on a bank). Such a bad news, only it is generated, perfectly predicts the policy maker's unfavorable outcome. [19]

This finding is largely close to what we observe in reality. Any little progress in a collective investment project (e.g., a fundraising event) will be reported, although such small progress cannot predict the success of the project. In contrast, the vulnerability of financial institutions or occurrence of existent attacks will not be revealed publicly until the failure of these institutions are doomed. By building a dynamic coordination model, out study provides some rationale for such design of information revelation. In more detail, from an ex-ante perspective, such design help increases the incidence of investment success as well as strengthen the resilience of financial institutions.

6 Conclusion

Coordination usually happens in a dynamic fashion and agents always have the option of delay for more information. In this paper, we construct a dynamic model of coordination with an option for delay. A binary public signal arises depending on the history of action. Interestingly, the good news — sufficiently many other agents have already pledged to this action — is more likely to be generated ex-ante only when it is less informative ex-post. (Both the availability and informativeness depend on $\Theta_\gamma(s_1^*)$, and, in turn, on equilibrium strategy $\hat{s}_1$.) We solve for the unique equilibrium in monotone strategies and observe that the option of delay may not facilitate coordination. When agents are inclined to wait for the good news instead of acting earlier, this makes the good news difficult to be produced, which, in turn, hinders successful coordination. This always happen if the delay cost is sufficiently low, either because of a small waiting period or a low interest

[19]It is worth pointing out that since the dynamic model is reduced to a static one in the limiting cases with $\gamma \to 0$ and $\gamma \to 1$. In those cases, the information $Y=1$ is produced with probability 1 and 0, and, thus, is useless. As such, the optimal choice for the policy maker, if we take the model seriously, is $\gamma \to \gamma_0-$ in the former case where the policy maker wants to minimize θ^*, and $\gamma \to \gamma_0+$ if the policy maker wants to maximize θ^*.

rate.

Theoretically, this study offers a simple and tractable dynamic model, which features option of delay (dynamic structure), privately informed agents with public learning about history (information environment), and strategic complementarity (payoff interaction). When the public information is generated by the past actions, our model proves the uniqueness of equilibrium holds true generically. Compared with static models, we construct some numerical example to demonstrate that this equilibrium selection provides a more plausible description of coordination problems in reality. With regard to policy implications, our dynamic model identifies a channel through which lowering the interest rate may encourage the inaction (or waiting) and slow the economic recovery.

Throughout the paper, we take the information generating process of the public signal Y exogenous. It can be thought of as the agent naturally have such learning opportunities. We briefly discuss the case in which a policy maker can influence the accessibility of information (or γ). It would be interesting to study how to provide additional information to facilitate coordination (given the exogenous source of public information).[20] In addition, we restrict our attention to monotone strategies throughout the paper, and the uniqueness of equilibrium relies on that restriction. The standard iterated elimination idea in global game literature cannot be applied to the dynamic setting with endogenous timing because of the lack of complementarity in the sense that if more agents choose to attack early, the incentive of attacking early might be reduced when the delay option is available. It will be interesting, although challenging, to study under what conditions, the equilibrium monotone strategies is uniquely rationalizable. We believe these are promising areas for future studies.

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[20] In a recent study, Basak and Zhou (2021) study the optimal information provision policy in a different dynamic coordination setup where agents who choose to wait may miss the opportunity of successful attack. They do not consider some exogenous source of public information as in the present paper but assume that the policy maker have perfect control of information flow overtime. The optimal policy found in Basak and Zhou (2021) requires the policy maker to have access to the fundamental θ, and thus, is different from the exogenous public signal Y considered in the present paper.

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Appendix

Proof of Lemma 1 This is a well-known result and the proof follows the discussions in the main text.

Proof of Lemma 2 Given all agents take the cutoff strategy $\hat{s}_1$ at $t = 1$, $X_1 = \mathbb{P}(s_i \geq \hat{s}_1|\theta) = F(\frac{\theta - \hat{s}_1}{\sigma})$. $X_1 \geq \gamma$ is equivalent to $F(\frac{\theta-\hat{s}_1}{\sigma}) \geq \gamma$, or $\theta \geq \Theta_\gamma(\hat{s}_1) = \hat{s}_1 + \sigma F^{-1}(\gamma)$. Therefore, $Y = 1$ if and only if $\theta \geq \Theta_\gamma(\hat{s}_1)$. It is easy to check that $\Theta_\gamma(\hat{s}_1)$ increases with $\hat{s}_1$. Furthermore, for an agent with $s_i = \hat{s}_1$,

$$\mathbb{P}(Y = 1|s_i = \hat{s}_1) = \mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) = F(\frac{\hat{s}_1 - \Theta_\gamma(\hat{s}_1)}{\sigma}) = F(\frac{\hat{s}_1 - \hat{s}_1 - \sigma F^{-1}(\gamma)}{\sigma}) = 1 - \gamma.$$

□

Proof of Lemma 3 By definition, $\hat{s}_2^0 \leq \hat{s}_1$ and $\hat{s}_2^1 \leq \hat{s}_1$. Therefore, $\max\{\hat{s}_2^0, \hat{s}_2^1\} \leq \hat{s}_1$. Consider a monotone strategy $(\hat{s}_1, \hat{s}_2^0, \hat{s}_2^1)$ that satisfies $\max\{\hat{s}_2^0, \hat{s}_2^1\} < \hat{s}_1$. Then, we can find $\tilde{s}$ and $\varepsilon > 0$ such that $(\tilde{s} - \varepsilon, \tilde{s} + \varepsilon) \in (\max\{\hat{s}_2^0, \hat{s}_2^1\}, \hat{s}_1)$. Under this strategy, the agent would wait and then invest at $t = 2$ regardless of Y when receiving any $s_i \in (\tilde{s} - \varepsilon, \tilde{s} + \varepsilon)$. Conditional on any s_i, if the expected payoff from investing is strictly positive, then this strategy is strictly dominated by investing at $t = 1$; otherwise, if the expected payoff is strictly negative, this is strictly dominated by not investing. Therefore, this strategy is not strictly dominated only if investing at $Y = 0$ and investing at $Y = 1$ both yield an expected payoff of 0. But this cannot hold true for all $s_i \in (\tilde{s} - \varepsilon, \tilde{s} + \varepsilon)$ since the probability of success after $Y = 1$ $\mathbb{P}(\theta \geq \hat{\theta}|s_i, \theta \geq \Theta_\gamma)$ varies with s_i for any threshold $\hat{\theta}$. This completes the proof. □

Proof of Lemma 4 Suppose there exists an equilibrium strategy $(\hat{s}_1, \hat{s}_2^0, \hat{s}_2^1)$ in which $\hat{s}_2^0 < \hat{s}_1$. Then, based on Lemma 3, $\hat{s}_2^1 = \hat{s}_1$ must holds, meaning that no one will choose to invest after $Y = 1$ but some agents with $s_i \in (\hat{s}_2^0, \hat{s}_1)$ would wait and then invest after $Y = 0$.

To have such an equilibrium, $\Theta(\hat{s}_2^0) < \Theta_\gamma(\hat{s}_1)$ must hold true; that is, successful coordination is possible following $Y = 0$. When this condition holds, successful coordination is achieved when $\theta \in (\Theta(\hat{s}_2^0), \Theta_\gamma(\hat{s}_1))$. Otherwise, no one would invest after $Y = 0$. Another condition that must hold is $\Theta(\hat{s}_1) > \Theta_\gamma(\hat{s}_1)$. Otherwise, if $\Theta(\hat{s}_1) \leq \Theta_\gamma(\hat{s}_1)$, then

$Y = 1$ is followed by coordination success and therefore, $\hat{s}_2^1 = -\infty$ and thus, $\hat{s}_2^1 = \hat{s}_1$ cannot hold true. In summary, if such an equilibrium exists, then the range of success is $[\Theta(\hat{s}_2^0), \Theta_\gamma(\hat{s}_1)) \cup [\Theta(\hat{s}_1), +\infty)$.

Next, consider the sub-game following $Y = 0$. The agent with signal $\hat{s}_2^0$ is indifferent between investing and not investing, i.e.,

$$\mathbb{P}\left(\theta \in [\Theta(\hat{s}_2^0), \Theta_\gamma(\hat{s}_1)) \middle| \theta < \Theta_\gamma(\hat{s}_1), \hat{s}_2^0\right) = \frac{F(\frac{\Theta_\gamma(\hat{s}_1)-\hat{s}_2^0}{\sigma}) - F(\frac{\Theta(\hat{s}_2^0)-\hat{s}_2^0}{\sigma})}{F(\frac{\Theta_\gamma(\hat{s}_1)-\hat{s}_2^0}{\sigma})} = c.$$

Therefore, we have

$$F(\frac{\Theta(\hat{s}_2^0) - \hat{s}_2^0}{\sigma}) = (1-c)F(\frac{\Theta_\gamma(\hat{s}_1) - \hat{s}_2^0}{\sigma}).$$

By definition of $\Theta(\cdot)$ (see (3)), we have

$$\Theta(\hat{s}_2^0) = 1 - F(\frac{\Theta(\hat{s}_2^0) - \hat{s}_2^0}{\sigma}) = 1 - (1-c)F(\frac{\Theta_\gamma(\hat{s}_1) - \hat{s}_2^0}{\sigma}) \geq c.$$

Therefore, from the above discussion, for such an equilibrium $\hat{s}_2^0 < \hat{s}_2^1 = \hat{s}_1$ to exist, we must have

$$\Theta(\hat{s}_1) > \Theta_\gamma(\hat{s}_1) > \Theta(\hat{s}_2^0) \geq c. \tag{15}$$

Now, consider an agent with $s_i = \hat{s}_1 - \varepsilon$ with $\varepsilon \downarrow 0$. To have such an equilibrium, in the sub-game starting from $Y = 1$, this agent does not invest, as she only invests after $Y = 0$. However, the ex-ante payoff from investing after $Y = 1$ for agent with $\hat{s}_1$ is

$$\mathbb{P}(\theta \geq \Theta(\hat{s}_1)|\hat{s}_1) \cdot 1 - \mathbb{P}(\theta \geq \Theta_\gamma(\hat{s}_1)|\hat{s}_1) \cdot c = \Theta(\hat{s}_1) - (1-\gamma)c.$$

This payoff is strictly positive as $\Theta(\hat{s}_1) > c$ (see (15)), implying that the agent with $s_i = \hat{s}_1-$ would strictly prefer to investing following $Y = 1$. That contradicts with the indifference condition for $\hat{s}_1$. Therefore, any equilibrium with $\hat{s}_2^0 < \hat{s}_2^1 = \hat{s}_1$ does not exists. □

Proof of Proposition 1 We first show that $(\hat{s}_1 = s_1^*, \hat{s}_2 = -\infty)$ is indeed an equilibrium when $\gamma > \gamma_0$. Consider agent i with s_i. Given other agents' strategy $(\hat{s}_1 = s_1^*, \hat{s}_2 = -\infty)$. Since $s_1^* > s_\gamma$, $\theta^* = \Theta(s_1^*) < \Theta_\gamma(s_1^*)$. The expected payoff from investing at $t = 1$ is

$$\mathbb{P}(\theta \geq \theta^*|s_i) - c = F(\frac{s_i - \theta^*}{\sigma}) - c,$$

whereas the expected payoff from waiting and then investing at $t = 2$ after $Y = 1$ is

$$\delta\mathbb{P}(\theta \geq \Theta_\gamma(s_1^*)|s_i)(1-c) = \delta(1-c)F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma}) \geq 0$$

Since this payoff is non-negative for any s_i, agent i will never choose to wait and then not invest after $Y = 1$, which yields a payoff of 0. Therefore, the expected payoff difference for agent i is

$$I(s_i; \hat{s}_1 = s_1^*, s_2^* = -\infty) := F(\frac{s_i - \Theta(s_1^*)}{\sigma}) - \delta(1-c)F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma}) - c$$

From the indifference condition (9), $I(s_1^*; s_1^*, -\infty) = 0$. To prove this is an equilibrium, we want to show that $I(s_i; s_1^*, -\infty) \geq 0$ if and only if $s_i \geq s_1^*$. This proof relies on the following property which is implied by the log-concavity of F.

A useful property based on the log-concavity of F

Lemma 7. *Given the distribution F is log-concave, for any $y > 0$, $\frac{F(x)}{F(x+y)}$ is strictly increasing in x.*

Proof.

$$\begin{aligned}\frac{\mathrm{d}\frac{F(x)}{F(x+y)}}{\mathrm{d}x} &= \frac{f(x)}{F(x+y)} - \frac{F(x)f(x+y)}{F^2(x+y)} \\ &= \frac{F(x)}{F(x+y)}\left(\frac{f(x)}{F(x)} - \frac{f(x+y)}{F(x+y)}\right) > 0\end{aligned}$$

□

Let us rewrite the payoff difference I as

$$I(s_i; s_1^*, -\infty) = F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})\left[\frac{F(\frac{s_i - \Theta(s_1^*)}{\sigma})}{F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})} - \delta(1-c) - \frac{c}{F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})}\right]$$

By Lemma 7, since $\Theta(s_1^*) \leq \Theta_\gamma(s_1^*)$, $\frac{F(\frac{s_i - \Theta(s_1^*)}{\sigma})}{F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})}$ is increasing in s_i. Moreover, $-\frac{c}{F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})}$ also increases with s_i. As such, for any $s_i > s_1^*$, we have

$$I(s_i; s_1^*, -\infty) > F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})\left[\frac{F(\frac{s_1^* - \Theta(s_1^*)}{\sigma})}{F(\frac{s_1^* - \Theta_\gamma(s_1^*)}{\sigma})} - \delta(1-c) - \frac{c}{F(\frac{s_1^* - \Theta_\gamma(s_1^*)}{\sigma})}\right] = 0,$$

and, for the same reason, for any $s_i < s_1^*$, $I(s_1; s_1^*, -\infty) < 0$. In this way, we show that strategy $(\hat{s}_1 = s_1^*, \hat{s}_2 = -\infty)$ is the best response of agent i, thereby proving that all agents taking this strategy can constitute an equilibrium.

Under the condition $\gamma > \gamma_0$, the uniqueness of this equilibrium is shown in the main text. Briefly, we show that there does not exist a monotone equilibrium with $\hat{s}_1 \leq s_\gamma$, and $(s_1^*, -\infty)$ is the unique equilibrium for $\hat{s}_1 > s_\gamma$. $\square$

Proof of Proposition 2 We first prove that there exists a unique $\theta^* \in (0, c)$, which solves (11). To show this, we can rewrite G as

$$G(\theta^*) := \frac{F\left(F^{-1}(\theta^*)\right)}{F\left(F^{-1}(\theta^*) - F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta}) - F^{-1}(\gamma)\right)}$$

Since $\gamma < \gamma_0$, it is easy to check that

$$1 - \gamma > \frac{c\,[1 - \delta(1-\gamma)]}{1-\delta},$$

which implies

$$-F^{-1}(\frac{c\,[1-\delta(1-\gamma)]}{1-\delta}) - F^{-1}(\gamma) = -F^{-1}(\frac{c\,[1-\delta(1-\gamma)]}{1-\delta}) + F^{-1}(1-\gamma) > 0.$$

Therefore, based on the log-concavity of F, G is strictly increasing in $F^{-1}(\theta^*)$ (see Lemma 7) and, thus, in θ^*. Moreover, it is easy to check that $G(\theta^*) \geq \theta^*$. As such, we know $\lim_{\theta^* \to 0} G(\theta^*) = 0 < c$, and $G(\theta^* = c) > c$ since $F^{-1}(c) - F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta}) - F^{-1}(\gamma) < +\infty$. Therefore, the solution to $G(\theta^*) = c$ is unique and $\theta^* \in (0, c)$. Furthermore, as we have shown, since $F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta}) + F^{-1}(\gamma) < 0$ under the condition $\gamma < \gamma_0$, we have

$$\Theta_\gamma(s_1^*) = \theta^* + F^{-1}(\frac{c\,[1-\delta(1-\gamma)]}{1-\delta}) + F^{-1}(\gamma) < \theta^*.$$

Next, to prove that (s_1^*, s_2^*) (as shown in (10)) is indeed an equilibrium, we further verify that given all other agents take this strategy, agent i's best response is to take the same strategy. That is, if $s_i < s_2^*$, agent i never invests; agent i choose to wait and then invest only after $Y = 1$ if $s_i \in [s_2^*, s_1^*)$; and agent i will invest eary if $s_i \geq s_1^*$.

For agent i with s_i, given other agents' strategy (s_1^*, s_2^*), the expected payoff difference between investing early and waiting and then investing (after $Y = 1$) is

$$
\begin{aligned}
&\mathbb{P}(\theta \geq \theta^*|s_i) - c - \delta\left[\mathbb{P}(\theta \geq \theta^*|s_i) - \mathbb{P}(\theta \geq \Theta_\gamma(s_1^*)|s_i)c\right] \\
=&(1-\delta)F(\frac{s_i - \theta^*}{\sigma}) + \delta c F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma}) - c \\
=&F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})\left((1-\delta)\frac{F(\frac{s_i-\theta^*}{\sigma})}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})} + \delta c - \frac{c}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})}\right)
\end{aligned}
$$

Since $\theta^* > \Theta_\gamma(s_1^*)$, by Lemma 7, $\frac{F(\frac{s_i-\theta^*}{\sigma})}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})}$ is strictly increasing in s_i. Since $-\frac{c}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})}$ also increases with s_i, we know that the expression

$$
(1-\delta)\frac{F(\frac{s_i-\theta^*}{\sigma})}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})} + \delta c - \frac{c}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})}
$$

is strict increasing in s_i. Moreover, based on the difference condition (14), this expression achieves 0 when $s_i = s_1^*$. As such, for any $s_i > s_1^*$, agent i strictly prefers to investing early; and for any $s_i < s_1^*$, agent i strictly prefers to waiting and then investing (after $Y = 1$).

Next, consider the payoff from waiting and then investing after $Y = 1$ is

$$
\delta\left[\mathbb{P}(\theta \geq \theta^*|s_i) - \mathbb{P}(\theta \geq \Theta_\gamma(s_1^*)|s_i)c\right] = \delta F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma})\left[\frac{F(\frac{s_i-\theta^*}{\sigma})}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})} - c\right].
$$

By Lemma 7 and $\theta^* > \Theta_\gamma(s_1^*)$, we know that $\frac{F(\frac{s_i-\theta^*}{\sigma})}{F(\frac{s_i-\Theta_\gamma(s_1^*)}{\sigma})} - c$ is increasing in s_i. Furthermore, based on the indifference condition (13), we know this expression obtains 0 when $s_i = s_2^*$. Therefore, we know that when $s_i > s_2^*$, given waiting at $t = 1$, investing after $Y = 1$ is strictly better off than not investing after $Y = 1$; and, if $s_i < s_2^*$, then no investing is the optimal choice.

Lastly, a sensible equilibrium requires

$$
s_1^* = \theta^* + \sigma F^{-1}(\frac{c\left[1-\delta(1-\gamma)\right]}{1-\delta}) > s_2^* = \theta^* + \sigma F^{-1}(\theta^*).
$$

So, it suffices to show that $\theta^* < \frac{c[1-\delta(1-\gamma)]}{1-\delta}$. Based on the monotonicity of $G(.)$, this is equivalent to

$$
G(\theta^*) = c < G(\frac{c\left[1-\delta(1-\gamma)\right]}{1-\delta}) = \frac{\frac{c[1-\delta(1-\gamma)]}{1-\delta}}{1-\gamma},
$$

which holds true since $\frac{1-\delta(1-\gamma)}{(1-\delta)(1-\gamma)} \in (0,1)$ for any δ and $\gamma \in (0,1)$.

To sum up, given all other agents take the monotone strategy (s_1^*, s_2^*), we have shown that for $s_i < s_2^*$, agent i strictly prefers no investing (waiting and investing after $Y = 1$ is better than investing early, but it yields a negative payoff); for $s_i \in (s_2^*, s_1^*)$, she strictly prefers waiting and then investing after $Y = 1$ (which yields a strictly positive payoff and is strictly better than investing early); and for $s_i > s_1^*$, she strictly prefers to invest early at $t = 1$, as compared with waiting and then investing after $Y = 1$ (which yields a strictly positive payoff). This completes the proof that the monotone strategy (s_1^*, s_2^*) can constitute an equilibrium.

We discuss the uniqueness of this equilibrium in the main text. Briefly, the uniqueness relies on the uniqueness of θ^* that satisfies $G(\theta^*) = c$ when considering $\hat{s}_1 < s_\gamma$, as well as non-existence of monotone equilibrium for any $\hat{s}_1 \geq s_\gamma$. □

Proof of Corollary 1

1. Consider $\gamma \in (0, \gamma_0)$. By Proposition 2, the fundamental cutoff θ^* uniquely solves $G(\theta^*) = c$. Since $-F^{-1}(\frac{c[1-\delta(1-\gamma)]}{1-\delta}) - F^{-1}(\gamma)$ decreases with γ, we know $G(\theta^*)$ increases with γ. Further, as $-F^{-1}(\frac{c[1-\delta(1-\gamma_0)]}{1-\delta}) - F^{-1}(\gamma_0) = 0$, this expression is strictly positive for any $\gamma \in (0, \gamma_0)$. As such, by Lemma 7, $G(\theta^*)$ is strictly increasing in θ^* for any fixed $\gamma \in (0, \gamma_0)$. Therefore, the unique solution θ^*, which makes $G(\theta^*) = c$ decreases with γ. Furthermore, for any $\theta^* \neq 0$, $\lim_{\gamma \downarrow 0} G(\theta^*) = \theta^*$ and therefore, $\lim_{\gamma \downarrow 0} \theta^*(\gamma) = c$.

   Now, consider any increasing sequence of $\{\gamma_k\}_{k \geq 1}$ that converges to γ_0. By definition of θ^*, we know that $G(\theta^*(\gamma_k), \gamma_k) = c$. As $\theta^*(\gamma)$ decreases with γ and $\theta^*(\gamma_k) \in [0,1]$ for all k, we know that the limit $\lim_{k\to\infty} \theta^*(\gamma_k)$ exists (denoted by η). Next, we prove that η must be 0. Suppose that $\eta > 0$. By continuity of G on θ^* and γ, we have

$$c = \lim_{k\to\infty} G(\theta^*(\gamma_k), \gamma_k) = G(\lim_{k\to\infty} \theta^*(\gamma_k), \lim_{k\to\infty} \gamma_k) = G(\eta, \gamma_0) = \frac{\eta}{F\left(F^{-1}(\eta)\right)} = 1.$$

   This is a contradiction, thereby proving $\eta = 0$.

2. In cases where $\gamma > \gamma_0$, all results follow immediately from Proposition 1 (since $\theta^*(\gamma) = c + \delta(1-\gamma)(1-c)$).

3. Lastly, we consider a special case in which $\gamma = \gamma_0$. First of all, we show that, in any possible equilibrium, the fundamental cutoff $\theta^* = \Theta_\gamma(s_1^*)$. Or equivalently, $Y = 1$ perfectly predicts the coordination success (i.e., no success after $Y = 0$ and coordination is successful as long as $Y = 1$), which requires $\Theta(\hat{s}_2) \leq \Theta_\gamma(\hat{s}_1) \leq \Theta(\hat{s}_1)$. This is because the fundamental cutoff $\hat{\theta}$ is determined by (12).

   To show this, suppose the fundamental cutoff $\hat{\theta} > \Theta_{\gamma_0}(\hat{s}_1)$ for some equilibrium $\hat{s}_1$. Then, for the marginal agent, the expected payoff difference between investing early and waiting and then investing after $Y = 1$ is strictly positively when $\gamma = \gamma_0$, regardless of $\hat{s}_1$; that is,

$$\begin{aligned}&\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - c - \delta\left(\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - \mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1)c\right)\\ <&(1-\delta)\mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1) - c + \delta\mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1)c\\ =&(1-\delta)(1-\gamma_0) - c + \delta(1-\gamma_0)c = 0.\end{aligned}$$

   Therefore, no such equilibrium can exist. Similarly, suppose $\hat{\theta} < \Theta_{\gamma_0}(\hat{s}_1)$, this difference for the marginal agent is always positive regardless of $\hat{s}_1$; that is,

$$\begin{aligned}&\mathbb{P}(\theta \geq \hat{\theta}|\hat{s}_1) - c - \delta\mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1)\,(1-c)\\ >&\mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1) - c - \delta\mathbb{P}(\theta \geq \Theta_{\gamma_0}(\hat{s}_1)|\hat{s}_1)\,(1-c)\\ =&(1-\gamma_0) - c - \delta(1-\gamma_0)(1-c) = 0.\end{aligned}$$

   In fact, it is easy to check that, under $\gamma = \gamma_0$, when $\hat{\theta} = \Theta_{\gamma_0}(\hat{s}_1)$, this difference is 0 regardless of $\hat{s}_1$.

   Next, as $\hat{\theta} = \Theta_{\gamma_0}(\hat{s}_1)$, $Y = 1$ predicts coordination success and thus, $\hat{s}_2 = -\infty$. We need to find the time-1 cutoff $\hat{s}_1$ which makes the condition $\Theta(\hat{s}_2) = 0 \leq \Theta_{\gamma_0}(\hat{s}_1) \leq \Theta(\hat{s}_1)$ holds true. (Only under this condition, $\hat{\theta} = \Theta_{\gamma_0}(\hat{s}_1)$.) By Lemma 5, we know that $\hat{s}_1 \leq s_{\gamma_0}$ is the sufficient and necessary condition for $\Theta_{\gamma_0}(\hat{s}_1) \leq \Theta(\hat{s}_1)$. In addition, as $\Theta_{\gamma_0}(\hat{s}_1) = \hat{s}_1 + \sigma F^{-1}(\gamma_0)$, $\Theta(\hat{s}_1 = -\infty) \leq \Theta_{\gamma_0}(\hat{s}_1)$ requires $\hat{s}_1 \geq \sigma F^{-1}(1-\gamma_0)$. Therefore, s_1^* can be any number between $\sigma F^{-1}(1-\gamma_0)$ and s_{γ_0} and $s_2^* = -\infty$. In equilibrium, $\theta^* = \Theta_\gamma(s_1^*)$.

   Given that all other agents take the strategy of any $s_1^* \in [\sigma F^{-1}(1-\gamma_0), s_{\gamma_0}]$, and $s_2^* = -\infty$, it is easy to check that the expected payoff difference

$$\mathbb{P}(\theta \geq \Theta_\gamma(s_1^*)|s_i) - c - \delta\mathbb{P}(\theta \geq \Theta_\gamma(s_1^*)|s_i)(1-c) = (1-\delta(1-c))F(\frac{s_i - \Theta_\gamma(s_1^*)}{\sigma}) - c$$

is strictly increasing in s_i, and, as we have shown, the difference obtains 0 when $s_i = s_1^*$. As such, for any $s_i > s_1^*$, the agent strictly prefers to investing at $t = 1$; and for $s_i < s_1^*$, the agent strictly prefers to waiting and then investing (after $Y = 1$). This completes the proof. $\square$