Uncertainty-Induced Reallocations, Innovation, and Growth

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Abstract

Focusing on both micro and aggregate U.S. data, we document the existence of a significant link between aggregate uncertainty, capital markets valuations and reallocation of resources away from risky R&D-intensive capital. This link is important because a decrease in the aggregate share of R&D-oriented investments forecasts lower medium-term growth. We study a two-sector model in which one sector features relatively risky R&D-intensive capital essential to sustain growth, whereas the other sector features safer capital that is not innovation-intensive. Our model accounts for our novel empirical evidence obtained from both capital markets and aggregate data. We identify an important role for uncertainty shocks as they feature a first-order negative impact on medium-term growth and welfare.

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1 Introduction

It is well documented that periods of high uncertainty are often associated with sizable and prolonged economic slowdowns during which both private consumption and aggregate investment decline. Less is known, however, on the impact of uncertainty shocks on the composition of the shares of sectoral output and investments. In this manuscript, we study the reallocation motives that uncertainty shocks generate across different investment opportunities and we connect them to growth. Our focus is on the interplay of uncertainty shocks and innovative capital.

Specifically, focusing on US microeconomic data from Compustat, we look at a cross section of firms sorted according to their R&D intensity. We document that R&D-intensive firms are more affected by volatility shocks than non-innovative firms. That is, volatility shocks are more disruptive for innovation-oriented firms both in terms of market valuation and contraction in their investments. According to the data, when uncertainty increases there exists a relative reallocation effect that penalizes investments in R&D-intensive firms, that is, it shifts more resources towards less R&D-intensive firms. These findings hold also at longer horizons and hence are not limited to short-lived business cycle fluctuations.

Furthermore, these results apply also to measures of innovation output such as number and value of new patents. In addition, our results hold also when we consider broader measures of investment in innovation capital (SG&A expenditures) and when we use aggregate data from the National Income Product Accounts (NIPA) tables. Both micro and aggregate data support our main findings, that is, uncertainty shocks are a first-order concern for growth because they are more disruptive for innovation-oriented capital.

We rationalize these findings in a novel macro-finance model in which we propose a *capital markets-based* perspective of the effects of uncertainty shocks. Our setting features two sectors and a volatility-averse investor. One sector features high R&D-intensity and promotes endogenous growth by the creation of patents that grant risky monopolistic rents. The second sector takes the productivity frontier as given and does not innovate. As in the data, innovation capital is more exposed to volatility shocks and is riskier than tangible capital. With state-non-separable recursive preferences, the representative investor features aversion to volatility shocks and an increase in uncertainty prompts a reallocation of resources toward the safer sector at the cost of reducing innovation and growth.

Specifically, our economy comprises goods produced in sectors with different intensities of innovation capital. The High-R&D sector (henceforth, H-sector) uses tangible capital and promotes growth by accumulating patents that pay off monopolistic rents. Hence the value of assets in the H-sector depends on both the present value of its tangible capital marginal productivity and the present value of future monopoly rents (Hayashi 1982). With recursive preferences, the latter component is extremely sensitive to uncertainty shocks, implying that even small increases in uncertainty can depress the market value of intangible capital, that is, the stock of patents in the economy. Under our benchmark calibration, the marginal product of tangible capital is relatively smooth and tangible capital is less risky than intangible capital. This is consistent with the data, as R&D-intensive firms pay higher returns than other firms.

In contrast to the H-sector, the second sector (henceforth, L-sector) does not innovate and does not feature monopoly power. Hence capital in the L-sector is less risky because its value is not affected by monopolistic rents. This setting explains the reallocation motives in our model: when uncertainty increases, the monopoly rent channel induces the representative agent to both reduce drastically R&D investment and cut down tangible capital in the H-sector. Simultaneously, more resources are allocated toward safer capital in the L-sector. At the equilibrium, the L-sector offers a strong hedge against volatility shocks and represents a safer asset.

This model explains reallocation flows across sectors with different levels of innovation intensity, but it also has predictions on total aggregate investment. Specifically, it predicts that when volatility increases the household saving rate increases as well. In a closed economy, by market clearing, the total investment rate must increase as well, in contrast to what observed in both micro and aggregate data on private investments (among others, see Fernandez-Villaverde et al. 2011 and Bloom et al. 2018). Given this observation, in a second step of our study, we explore the allocation of resources outside of the domestic US private sector. We find that current account adjustments are small and explain a marginal share of the gap between private savings and private investment dynamics. The government sector, instead, has an important role as most of government expenditure relates to both wages and investment, that is, two important inputs of production.

Using standard VAR empirical techniques, we show the existence of a significant positive link between uncertainty and reallocation toward the government sector. Since the Bureau of Economic Analysis (BEA) provides a decomposition of government investment into tangible and intangible components, we are able to measure R&D intensity of government investments. Our data suggest that the US government has a low R&D intensity when compared to the firms in Compustat. Hence our additional empirical findings are consistent with the idea that volatility shocks are very disruptive because they prompt a reallocation toward safer, but not innovative, capital.

We modify our benchmark model and replace the L-sector with a government sector that hires labor and buys investment goods according to empirically-driven exogenous rules. The government takes the productivity frontier as given and uses these inputs to provide output that is allocated to the Household. In this setting, we think of the H-sector as representing the aggregate private sector in the US economy.

This configuration of our model reconciles the observed pattern of savings and private investments, in contrast to prior studies. When uncertainty spikes upward, our representative agent reduces private consumption in order to increase her precautionary savings. Simultaneously, private investment declines because the increase in private savings is dominated by a reduction of government savings, or, equivalently, by the higher government expenditure in investment goods. As in the data, a reallocation of investment goods toward the government sector forecasts lower future productivity growth. In contrast, a reallocation of resources toward private R&D forecasts higher long-term growth.

Related literature. Our manuscript contributes to a recently growing literature that studies the real effects of uncertainty shocks (see, among others, Aghion and Banerjee 2005; Bloom et al. 2007; Bloom 2009; Barrero et al. 2017; Di Tella 2017 and Alfaro et al. 2018). We find that higher productivity volatility is associated with a relevant reallocation away from R&D and a decline in future growth. A general equilibrium model with endogenous growth and multiple sectors suggests that these features of the data may result from aversion to volatility shocks. More broadly, our framework generates persistent growth stagnation through a channel which is both distinct and complementary to that in Kozeniauskas et al. (2019) and Bloom et al. (2019).

Our analysis relates to the recent literature examining the role of uncertainty both in the data and in economic models (see, among others, Jones et al. 2005; Justiniano and Primiceri 2008; Basu and Bundick 2017; Gilchrist et al. 2014; Jurado et al. 2015; Berger et al. 2018; Ludvigson et al. 2018; Kozeniauskas et al. 2018 and Bloom et al. 2018). We contribute with our attention to the reallocation of resources across sectors with different riskiness and growth prospects. Earlier work (see, among others, Romer 1990, Comin and Gertler 2006 and Kung and Schmid 2015) has not explored the implications of sectoral reallocation on growth and risk. In this paper, we develop a novel model with heterogenous sectors featuring a positive link between their riskiness and innovation-intensity. This setting accounts for our novel evidence on reallocation, innovation and growth. Furthermore, in the spirit of Bloom et al. (2020), we empirically document that uncertainty shocks are disruptive for several measures of innovation output, not just for R&D investment. Both at the firm and the industry level, when uncertainty spikes, innovation-intensive firms contract relatively more in term of future number patents, future patent value (Kogan et al. 2017), and future growth opportunities.

Barro et al. (2017) study the role of government assets in an endowment economy with heterogenous agents. Fernandez-Villaverde et al. (2015) focus on the real effect of uncertainty shocks to unproductive government expenditure and distortionary taxation in a neoclassical model with exogenous growth. We differ from these studies for (i) our novel empirical evidence on investment reallocation; (ii) our focus on productive government expenditure; and (iii) our focus on R&D and endogenous growth.

Belo et al. (2013) and Belo and Yu (2013) examine the effects of government investment and spending on asset prices. Our work complements their findings and highlights a new trade off between growth and government capital in times of higher uncertainty. Both our empirical focus on heterogenous forms of capital and endogenous growth, and our attention to priced uncertainty shocks are distinct from the work of Baxter and King (1993).

According to the first-best equilibrium in our model, it is optimal for the government to expand its size in bad times. This outcome is broadly consistent with that of new Keynesian models (for a recent example, see Christiano et al. 2011), but it is obtained by taking a capital markets view of the economy and without adopting nominal frictions (Basu and Bundick 2017, Fernandez-Villaverde et al. 2011). We differ from this literature for our risk-based approach and for our attention to the trade off between long-term growth and government size. In our setting with recursive preferences, uncertainty shocks are also discount rate shocks and hence our findings are consistent with those in Comin et al. (2017).[1]

In our model, we abstract away from uncertainty stemming from government policy (see, for example, Pastor and Veronesi 2012, 2013; Kelly et al. 2013; Fernandez-Villaverde et al. 2015; and Baker et al. 2016), the nature of government financing risk (see, among others, Lustig et al. 2008 and Berndt et al. 2012), the impact of distortionary taxation on innovation (among others, see Akcigit et al. 2018) and the role of government subsidies to relax credit constraints to innovators (among others, see Howell 2017). Our results do not contradict the existing evidence on the positive role of government support to innovation, they rather point out the existence of a relevant reallocation motive. Furthermore, in our empirical investigation we control for credit conditions. For a comprehensive study of the early endogenous

[1]We note that Futagami et al. (1993) have been the first one to consider an endogenous growth model with productive government capital. We differ from their study in many dimensions, the most important being that we focus on a stochastic environment with time-varying uncertainty and recursive preferences.

growth models, see Jones (1995).

In the next section, we show our main empirical evidence. Section 3 describes the model and its calibration. We summarize our main results in section 4. Section 5 concludes.

2 Empirical Evidence

In this section, we show our main empirical findings. We start by looking at micro-data from Compustat and study the link between uncertainty and relative investment across private firms sorted according to their innovation intensity. We find that uncertainty shocks produce (i) a reallocation of investment away from R&D-intensive firms; (ii) a decline in patent output; and (iii) a more severe reduction in the growth opportunities of R&D-intensive firms. Importantly, we document the existence of a positive link between the relative size of innovation capital and aggregate long-run productivity growth, implying that our uncertainty-induced reallocation is a leading indicator of future slow growth.

In a second step, we look at aggregate data from the NIPA tables in order to make sure that our reallocation result is a broad phenomenon that affects the entire economy as opposed to the limited subset of Compustat firms. Another advantage of working with aggregate data is that they allow us to proceed with a VAR analysis that is informative on the dynamics, e.g., the persistence of capital reallocation. In this step, we confirm that our reallocation results in the aftermath of volatility shocks are confirmed even using economy-wide aggregates.

When assessing reallocation in the cross section of R&D-sorted firms, we use either a broad market-based measure of uncertainty such as integrated stock market returns volatility ($iVol$), or the economic policy uncertainty (EPU) measure proposed by Baker et al. (2016). We use integrated volatility as benchmark because it has two relevant advantages: (i) it is easy to compute (see Appendix A for details), and (ii) it is available on long samples. On the other hand, we agree with Berger et al. (2018) on the fact that realized equity volatility is not a perfect measure of uncertainty.

We also document that most of the dynamics that we detect are active along medium- and long-run cycles. This observation motivates us in using a medium-scale model with endogenous growth suitable to study medium- and long-run responses to volatility shocks.

2.1 Reallocation Across Innovation-sorted Firms

Using quarterly accounting data from Compustat over the sample 1972:Q1-2016:Q4, we compute firm-level R&D intensity measured as the ratio of R&D expenses to total assets.

Table 1: Excess Returns in R&D-sorted Portfolios

$$r^{ex}_{i,t} = \overline{r}^{ex}_i + \beta_{z,i} e_{z,t} + \beta_{vol,i} e_{vol,t} + \epsilon_{i,t}$$

	High-R&D	Low-R&D	HML-R&D
$\overline{r}^{ex}_i$	10.64***	3.59***	7.05***
	(4.35)	(2.10)	(2.99)
$\beta_{z,i}$	13.73***	5.39***	8.34***
	(2.38)	(1.60)	(2.26)
$\beta_{vol,i}$	-18.30***	-11.19***	-7.10***
	(3.35)	(1.42)	(2.60)
R^2	0.17	0.21	0.08

Notes: Our sample starts in 1972 and ends in 2016. Returns are annualized, multiplied by 100, equal-weighted, and unlevered. The High (Low) portfolio includes the top (bottom) 20% of R&D intensity-sorted firms and accounts for about 10% of total market capitalization. HML-R&D refers to a portfolio long in the High-R&D portfolio and short in the Low-R&D portfolio. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. We model both $ivol$ and z as AR(1) processes and denote their innovations as $e_{vol,t}$ and $e_{z,t}$, respectively. Standard errors in parentheses are Newey-West adjusted.

This measurement is common in the empirical literature about R&D firms and prevents our analysis from being driven by a small set of very large firms. We use CRSP data for equity returns. Our results hold also in a post-1975 subsample completely based on the most recent Financial Accounting Standards Board (FASB) accounting standards about R&D activity expenses.

Portfolio-level results. In each calendar year, we form five portfolios with equal number of firms sorted according to their previous year's R&D intensity. The resulting composition of our portfolios is consistent with prior studies and is summarized in table B1 in Appendix B. This table shows also standard summary statistics for our portfolios. We are interested in studying both return dynamics and the subsequent investment adjustments of the firms in our portfolios upon the realization of adverse uncertainty shocks.

In table 1, we show informative summary statistics about returns for a subset of our portfolio returns. HML-R&D refers to the difference in behavior of the variables of interest across the High- and Low-R&D intensity portfolios. We interpret the figures for HML-R&D as being specific to innovative firms, as they are in excess of those observed for regular firms. Consistent with Chan et al. (2001), we observe a higher risk premium on R&D-intensive firms with respect to both levered and unlevered returns.

In order to formally test that the returns of innovative firms are more exposed to economic uncertainty than those of non-innovative firms, we proceed as follows. We regress the aggregate price-dividend ratio, pd_t, on $iVol_t$ and take the residual of this regression, z_t, as a factor capturing shocks that affect the level of the market activity and are orthogonal to uncertainty fluctuations. We model both $iVol$ and z as AR(1) processes and denote their innovations as $e_{vol,t}$ and $e_{z,t}$, respectively.[2] We then estimate the following standard time-series regression,

$$r_{i,t}^{ex} = \overline{r}_i^{ex} + \beta_{z,i} e_{z,t} + \beta_{vol,i} e_{vol,t} + \epsilon_{i,t},$$

where the left hand side refers to annualized unlevered excess returns, $e_{z,t}$ controls for shocks to the level of economic conditions, and $e_{vol,t}$ captures unexpected volatility shocks. Our results confirm that the market value of innovative firms is more exposed to both level and volatility shocks. When the market price of risk of level shocks is positive and that of volatility is negative, our estimated betas unambiguously imply that High-R&D firms must be riskier than Low-R&D firms under no-arbitrage.

Through the lens of our innovation-driven model, these results suggest that uncertainty shocks depress the market value of the rents associated to patents and hence they should discourage innovation-oriented investments. Given this observation, we now turn our attention to the study of investment dynamics by estimating jointly the following set of predictive regressions,

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\frac{R\&D_i}{Assets_i}}\right) ivol_t + \beta_z z_t + cntrl_t^i, \quad i = 1, ..., N, \tag{1}$$

where the left hand side refers to the h-periods ahead future investment growth rate, z_t controls for shocks to the level of economic conditions, and N captures the size of our cross section. We also consider additional control variables grouped in $cntrl_t^i$. The literature has already shown that $\beta_0 < 0$, i.e., uncertainty shocks have a negative impact on investment. We are interested in whether high R&D-intensity firms are more exposed than low R&D-intensity firms, that is, $\beta_{rnd} < 0$.

In table 2, we show our results when we focus on a cross section of five R&D intensity-sorted portfolios. We account for financial shocks, or equivalently, credit tightness, by adding the 10-year Baa credit spread as an additional control. Across different horizons, our results confirm that high R&D firms reduce more significantly both total investment and R&D intensity in the aftermath of an adverse volatility shock. According to a formal Wald test on the joint hypothesis $H_0 : \beta_{rnd} = \beta_0 = 0$, we reject the null, i.e., we find support about

[2] In the appendix, we show that our results are unchanged if we use an AR(2) representation (see table B2 and B3).

Table 2: Reallocation across R&D-sorted Portfolios

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{R\&D}{Assets}}i\right) ivol_t + \beta_z z_t + cntrl_t^i \qquad i = 1, ..., 5$$

Horiz. (years)	$\Delta Inv.(\%)$ β_{rnd}	$\Delta Inv.(\%)$ Wald	$\Delta Inv.(\%)$ R^2 Loss	$\Delta\frac{R\&D}{Assets}(p.p.)$ β_{rnd}	$\Delta\frac{R\&D}{Assets}(p.p.)$ Wald	$\Delta\frac{R\&D}{Assets}(p.p.)$ R^2 Loss
h=3	-1.80**	19.10	-52	-0.89***	12.96	-21
	(1.03)	[0.000]		(0.35)	[0.002]	
h=4	-1.54**	17.38	-33	-0.76**	11.18	-15
	(0.84)	[0.000]		(0.35)	[0.004]	
h=5	-1.35**	13.22	-22	-0.84**	8.70	-13
	(0.72)	[0.001]		(0.38)	[0.013]	

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using five portfolios sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the joint hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Asset$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. We control for the 10-year Baa credit spread.

uncertainty being a statistically relevant driver of investment dynamics. This effect is relevant also in terms of R^2. Over a three-year horizon, for example, removing our uncertainty measure from our predictive regressions let the R^2 for investment growth and R&D intensity decline by 52% and 21%, respectively.

For the sake of economic interpretation of the magnitude of our results, we note that the average R&D intensity in our High (Low) R&D-intensity portfolio is 16% (0.00%). This implies that when $iVol$ is two standard deviations away from its mean, the cumulative drop in relative investment across the High and Low portfolios over a 3-year horizon is 11.5%.[3] Similarly, over the same horizon the relative variation in R&D intensity is -1.9 percentage points.

Firm-level results. In table 3, we focus on a more granular cross section in which we assess β_{rnd} using firm-level data. In this case, we adopt firm-level fixed effects and account for time-variation in the firm-level R&D intensity characteristic. We continue to control for

[3] In our data, we use annualized percent integrated vol and hence $StD(iVol) = 6.67$. The cumulative growth rate of investment is annualized, therefore we derive our result by computing $h \cdot \beta_{rnd} \cdot (.16 - .00) \cdot 2StD(iVol)$ for each horizon, h. For the R&D intensity measure, we have $\beta_{rnd} \cdot (.16 - .00) \cdot 2StD(iVol)$.

Table 3: Reallocation across R&D-sorted Firms

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{R\&D}{Assets}}_{i,t}\right) ivol_t + \beta_z z_t + cntrl_t^i \quad i = 1, ..., N$$

Horiz. (years)		$\Delta Inv.(\%)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-2.56***	-3.94***	-5.39***	-4.56***	-3.34***	-2.61***
		(0.91)	(0.72)	(0.62)	(0.46)	(0.36)	(0.28)
	Wald	28.61	56.24	93.40	127.86	143.05	177.69
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-49	-46	-54	-54	-64	-64
h=5	β_{rnd}	-2.96***	-3.80***	-4.45***	-3.71***	-2.40***	-1.69***
		(0.71)	(0.56)	(0.48)	(0.36)	(0.28)	(0.22)
	Wald	25.72	43.31	74.67	90.84	77.80	98.27
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-42	-34	-44	-40	-43	-42
		$\Delta\frac{R\&D}{Assets}(p.p.)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-1.04***	-1.20***	-1.21***	-1.28***	-1.41***	-1.59***
		(0.11)	(0.09)	(0.09)	(0.09)	(0.10)	(0.10)
	Wald	43.49	85.34	90.23	104.21	115.08	145.41
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-90	-94	-93	-93	-88	-84
h=5	β_{rnd}	-1.39***	-1.67***	-1.80***	-1.82***	-2.00***	-2.05***
		(0.16)	(0.12)	(0.11)	(0.12)	(0.11)	(0.11)
	Wald	38.00	102.32	128.24	145.54	174.70	180.94
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-85	-92	-91	-92	-90	-84
		N=96	N=196	N=273	N=395	N=624	N=1020

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using a cross section of firms sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The firm-level R&D intensity average, $R\&D/Assets_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

credit conditions by using 10-year Baa credit spread and we also consider firm-level variables including Tobin's Q, a standard proxy for growth opportunities, as well as profitability. Our goal is to test whether volatility shocks are statistically and economically relevant, even after we account for key firm characteristics used to predict investment growth. In our first pass, we consider the subgroup of firms that is in our sample for the entire 1972-2016 period in order to ensure a balanced panel. We also consider larger subgroups of firms that are present in our sample for smaller fractions of our entire time span.

Across different groups and time horizons, our results suggest that volatility shocks are particularly disruptive for innovative firms. Equivalently, investment contractions originated by adverse volatility shocks come with important reallocations away from firms with innovation capital.

This result is significant both in terms of R^2 variations due to the elimination of our volatility measure, and according to a formal Wald test for $H_0 : \beta_{rnd} = \beta_0 = 0$. When $iVol$ is two standard deviations away from its mean, these results imply a cumulative drop in relative investment across the High and Low portfolios for $h = 3$ ranging from 7.5% to 21%, depending on whether we use a balanced cross section or not. The cumulative relative decline in R&D intensity ranges instead from 1.0 to 3.5 percentage points. Hence our reallocation magnitudes are relevant both when using portfolio-level and firm-level data.

We continue our investigation of firm-level data by looking at innovation outcomes. In table 4, we focus on both number and value of patents, as well as future growth opportunities as captured by future Tobin's Q. For the sake of brevity, we focus only on forecasting regressions over a 5-year horizon.

Patent grants is from Autor et al. (2020); our measure of value of patents is from Kogan et al. (2017) and is deflated using the CPI. In this case, we additionally require that the dependent variable must be non-missing for at least 10 years because patent-based measures are missing often in our sample. Our results confirm that volatility shocks are disruptive for innovation output. When $iVol$ is two standard deviations away from its mean, these results imply a cumulative drop in relative patent grants (real patent value) across the High and Low portfolios ranging from 7.3% to 20.9% (3.2% to 20.6%) over a 5-year horizon.

Robustness of our results. In Appendix B, table B4 shows that our results are confirmed also when we run our firm-level regressions replacing the Baker et al. (2016) measure of economic policy uncertainty (EPU). This finding is important because it broadens the relevance of our reallocation evidence since EPU is a distinct and broader type of uncertainty relative to productivity uncertainty. Table B5 confirms that our firm-level results are nearly unchanged when including time fixed effects. Similar results apply to our portfolio-level data.

Using R&D data prompts two natural questions. First, how do the results change if we offer a broader measure of intangible investment such as SG&A for which we have less frequent zero-entries? This measure comprises advertising, R&D, nonproductive expenses and

Table 4: Innovation Outcomes across R&D-sorted Firms

$$\Delta[\cdot]_{i,t\rightarrow t+5} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{R\&D}{Assets}}_{i,t}\right) ivol_t + \beta_z z_t + cntrl_t^i \quad i = 1, ..., N$$

Innovation Outcome		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
Patent Grants	β_{rnd}	-1.49	-1.66*	-2.21**	-1.54**	-1.60***	-1.88***
		(1.48)	(1.15)	(0.96)	(0.80)	(0.58)	(0.44)
	Wald	33.97	38.81	41.14	46.11	56.34	78.85
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-39	-37	-42	-41	-45	-41
Real Patent Val.	β_{rnd}	-0.66	-0.94	-2.09*	-2.29**	-1.81**	-1.85***
		(2.36)	(1.67)	(1.35)	(1.11)	(0.81)	(0.66)
	Wald	64.74	104.09	117.69	135.75	163.87	225.48
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-50	-48	-49	-44	-41	-39
Tobin's Q	β_{rnd}	-2.37***	-1.33**	-1.34***	-0.76**	-0.26	-0.11
		(0.84)	(0.61)	(0.53)	(0.40)	(0.30)	(0.22)
	Wald	10.04	21.89	31.83	63.44	102.39	122.61
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-4	-4	-5	-6	-8	-8

Notes: Our data sample starts in 1972 and ends in 2016. Cumulative growth rates for innovation outcomes are annualized. Patent values are deflated using the CPI. Our sources are detailed in Appendix A. All estimates are obtained through GMM using a cross section of firms sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The firm-level R&D intensity average, $\overline{R\&D/Assets}_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

is an important determinant in the cross section of equity returns (Eisfeldt and Papanikolaou 2013). In table B6, we show that our main results are preserved when we use SG&A as opposed to only R&D.

Second, how do the results change when we treat missing R&D data entries as zeros? Table B7 shows that our results continue to be significant even when we treat missing R&D observations as zeros.

In addition to both our portfolio- and firm-level analyses, we also conduct our empirical investigation at the industry level. Specifically, we aggregate our firms in to the Fama-French 49 industries. In table B8, we show that our results are similar to those obtained with firm-level data. We note that these results hold regardless of whether we include missing R&D as zeros or exclude them.

Table 5: Reallocation and Growth

Horizon (n):	5 year	7 year	10 year
$k^H_{R\&D}/k^{Compustat}_{tot}$	0.29***	0.29***	0.31***
	(0.08)	(0.05)	(0.04)
Adj R^2	0.34	0.48	0.43
Adj R^2 w/o	0.19	0.18	-0.17
$I^H_{R\&D}/I^{Compustat}_{tot}$	0.11***	0.11***	0.12***
	(0.03)	(0.02)	(0.02)
Adj R^2	0.32	0.42	0.28
Adj R^2 w/o	0.19	0.18	-0.17
Credit Tightness Control: Baa10y	Yes	Yes	Yes
Predicting Factors	Yes	Yes	Yes

Notes: This table reports the estimated coefficients $\hat{b}^n$ for the following regression:

$$\frac{1}{n}\Delta a_{t|t+n} = \alpha + b^n \cdot CapitalMeasures_t + b^n_c \cdot controls_t + resid_t,$$

where n is expressed in years. $k^H_{R\&D,t}/k^{Compustat}_{tot,t}$ refers to total assets in our High-R&D portfolio relative to total assets in our Compustat cross section focused on firms with non-missing R&D data. The investment ratio is constructed in a similar way using investment flows, as opposed to capital stock data. The other forecasting variables included in the regressions are: US Treasury bond yields with maturity from 1 to 6 years; inflation; price-dividends ratio; iVol; and Baa corporate yield minus 10y Treasury bond yield. Our sources are detailed in Appendix A. Our annual sample starts in 1972 and ends in 2016. Confidence intervals are HAC-adjusted. One, two, and three asterisks denote 10%, 5%, and 1% significance, respectively. Adj R^2 w/o refers to a regression in which we eliminate our $CapitalMeasures$ variables.

Reallocation and aggregate growth. Through the lens of models of endogenous growth in which R&D intensity is a leading indicator of medium-term growth and innovation, these results suggest that uncertainty shocks may anticipate periods of prolonged sluggish innovation and growth. Given this consideration, we investigate whether the relative composition of capital matters for future productivity growth by formally estimating the following forecasting regression:

$$\frac{1}{n}\Delta a_{t,t+n} \quad = \quad \mu + b^n CapitalMeasures_t + b^n_c Cntrls_t + resid_t \tag{2}$$

where $\Delta a_{t,t+n}$ is the n-year-ahead cumulative aggregate productivity growth at time t, $CapitalMeasures$ refers to different measures of capital composition that we detail in what follows, and $controls$ comprises a broad set of macro-financial leading indicators that accounts also for credit conditions (US Treasury bond yields with maturity from 1 to 6 years; inflation; price-dividends ratio; iVol; Baa corporate yield minus 10y Treasury bond yield).

In table 5, we show the existence of a positive correlation between the share of innovative capital and future long-term growth. This result applies to both capital stocks and investment flows. Importantly, across both panels, adding variables related to the innovation-based composition of capital increases substantially our adjusted R^2s, implying that our findings are statistically important. Since we control for many other well-known leading indicators that account also for credit conditions, we consider our R^2 improvements as very sizeable. To better interpret the economic relevance of this result, we notice that if our capital measure in the top panel increases by two standard deviations, cumulated productivity growth should decline by 3.5% over five years. We consider this economic effect as very relevant.[4]

In what follows, we broaden our analysis by looking at aggregate data.

2.2 Aggregate Data

In this section, we use aggregate data to support further our main findings, that is, uncertainty shocks are a first-order concern for growth because they are more disruptive for innovation-capital. This exercise has three main objectives. First, prove that our results apply to a broad cross section of firms, not only to those in Compustat. Second, we can study the propagation over time of volatility shocks onto aggregate variables. Third, we can obtain guidance on how to calibrate our production-based equilibrium model.

2.3 A VAR Analysis: Aggregate reallocation and Innovation.

We use a VAR analysis in order to determine the joint dynamics of level and volatility shocks as well as capital reallocation. We perform this exercise using both realized equity market volatility and an ex-ante measure of productivity growth volatility. This exercise has at least three relevant goals. First of all, we show that our reallocation results are robust to using either an equity-based or a macroeconomic measure of uncertainty. Second, our findings are not affected by the Berger et al. (2018) critique, as they hold also when we adopt an ex-ante measure of fundamental uncertainty. Third, by quantifying the empirical role of productivity volatility shocks, we obtain relevant guidance on both the calibration of our aggregate model and the assessment of its fit of the US aggregate data.

iVol-based results. We start by estimating the following VAR(1):

$$Y_{t+1} = \mu_Y + \Phi Y_t + \Omega Cntrl_t + \Sigma u_{t+1}, \tag{3}$$

[4] Since the standard deviation of our capital measure is 0.012, the cumulative decline is $2 \times 0.012 \times 0.28 \times 5$.

with

$$Y_t = \begin{bmatrix} a_t & iVol_t & \log X_t \end{bmatrix}, \tag{4}$$

and

$$X_t \in \{I_{R\&D,t}, I_{p,t}, Y_{p,t}\}.$$

The variables a_t, and $iVol_t$ denote log-productivity and integrated volatility for stock market returns, respectively. By including productivity, we control for shocks to the level of economic activity and isolate the role of uncertainty shocks on the last variable of the VAR which refers to economic aggregates such as private output (Y_p), private R&D investment ($I_{R\&D}$), and total private investment (I_p). The vector $Cntrl_t$ comprises other aggregate variables, such as, for example, the 10-year Baa credit spread. We focus on the sample period from 1972 to be consistent with the empirical evidence presented elsewhere in this section. We address the robustness of our results at the end of this section.

Throughout this study, we do not need to take a stand on causality across uncertainty and level shocks (for a further discussion of this point, see Berger et al. 2018). We identify impulse responses through a lower diagonal Cholesky decomposition and point out that their pattern does not change whether level shocks or volatility shocks are ranked first. For the purpose of our analysis, both methods produce similar orthogonalized level and volatility shocks.

Using our estimated VAR, we trace the response of investment and output aggregates to both productivity and volatility shocks in figure 1. A positive productivity shock boosts output as well as all forms of investment and it has a stronger effect on private investment. This result is reassuring as it is consistent with previous empirical evidence.

Uncertainty shocks, instead, are associated to both production and private investment contractions. The contraction in private investments is common across both fixed assets and innovation-oriented assets. Most importantly, uncertainty shocks produce adjustments quantitatively as relevant as those due to level shocks. Furthermore, the role of uncertainty shocks is still significative and evident if we follow Comin and Gertler (2006) and Hodrick and Prescott (1997) in filtering the data to extract medium cycle and business cycle fluctuations, respectively. As shown in figure 2, the effects of volatility shocks are disruptive at all frequencies, although to a lesser extent at business cycle frequency. This observation motivates our attention toward a medium scale endogenous growth model able to explain medium- and long-run dynamics.

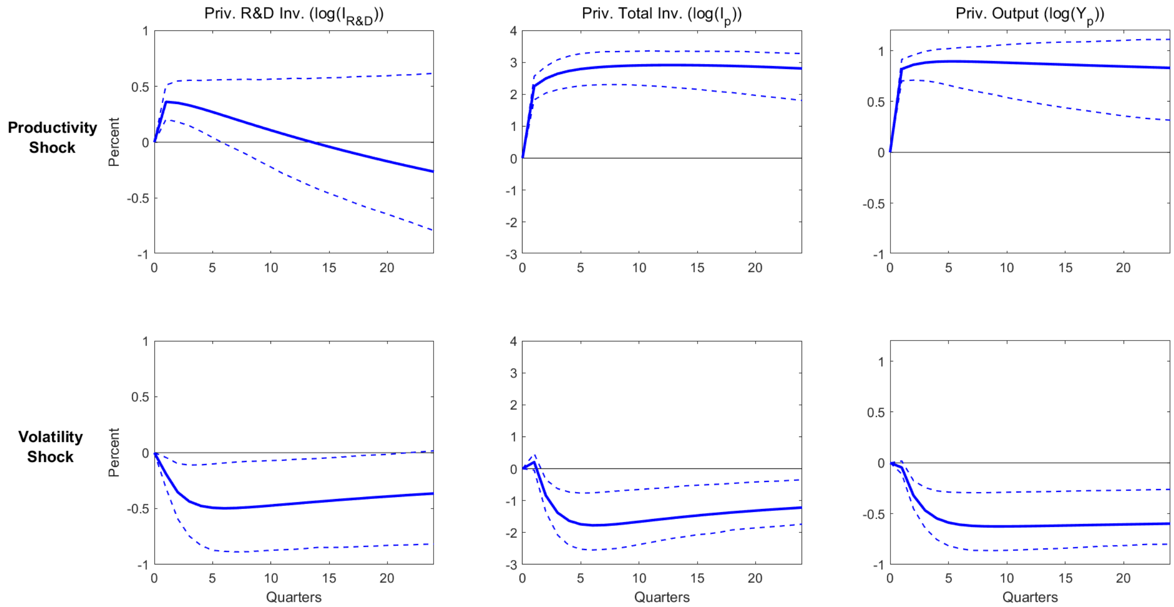


Fig. 1. **Aggregate Capital Reallocation in a VAR with iVol.** This figure shows the response to both productivity growth shocks and volatility shocks of gross private R&D investment ($I_{R\&D}$), total gross private investment (I_p), and private output (Y_p). All results are based on the VAR specified in equations (3)–(4), in which we use stock market integrated volatility to measure uncertainty. We control for the 10-year Baa credit spread. Our series are quarterly (1972:Q1-2016:Q4) and in log units (see Appendix A). Confidence intervals are HAC-adjusted.

Productivity uncertainty (PU). Integrated stock market volatility results from many different economic phenomena that are not solely related to uncertainty shocks. As an example, integrated volatility may be driven by sentiment shocks, or time-varying market frictions. In order to (i) focus on a fundamental measure of economy activity uncertainty, and (ii) address Berger et al. (2018)'s concerns related to the use of realized volatility, we extract ex-ante time-varying volatility from productivity growth using a standard predictive-factors approach that we detail in Appendix C.

We proceed as before by estimating the VAR specified in equation (3) using the following modified vector of variables

$$Y_{,t} = \begin{bmatrix} a_t & x_t & vol_t & \log X_t \end{bmatrix}, \tag{5}$$

where a_t controls for short-run realized growth shocks, the productivity long-run component x_t controls for growth news shocks, vol_t refers to our measure of productivity uncertainty,

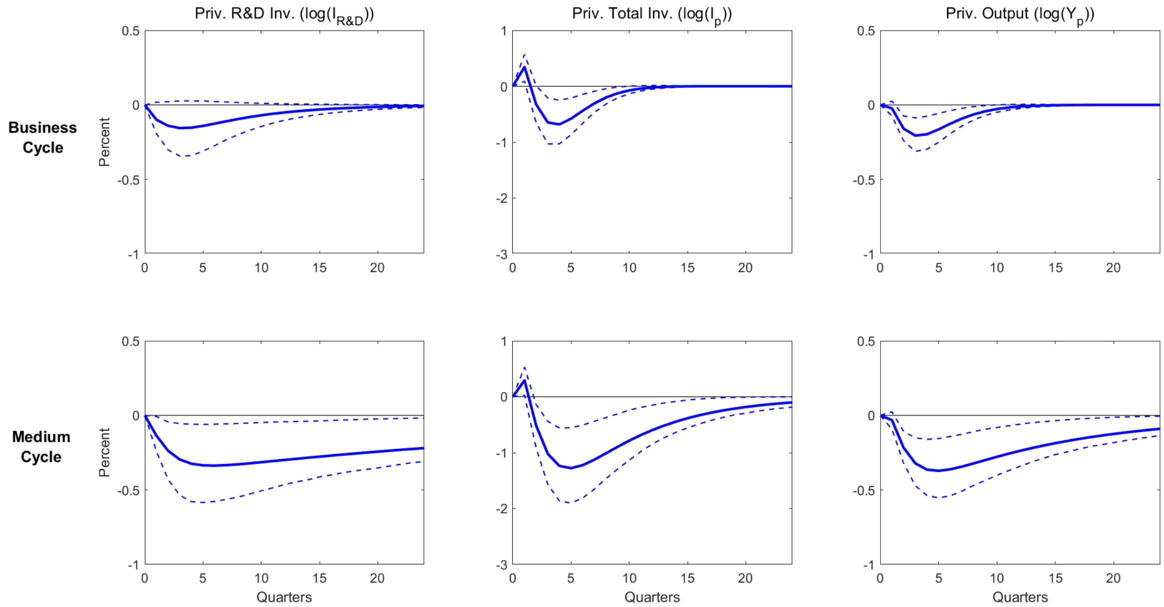


Fig. 2. **Aggregate Capital Reallocation and Cycles.** This figure shows the response to volatility shocks of gross private R&D investment ($I_{R\&D}$), total gross private investment (I_p), and private output (Y_p). All results are based on the VAR specified in equations (3)–(4), in which we use stock market integrated volatility to measure uncertainty. We control for the 10-year Baa credit spread. In the top (bottom) panels, all series are HP (band-pass) filtered and in log units over the quarterly sample 1972:Q1–2016:Q4 (see Appendix A). Confidence intervals are HAC-adjusted.

and X_t is defined as in equation (4).

We depict our main results in figure 3. The role of productivity level shocks is similar to that obtained using market volatility and can be found in Appendix B, figure B2. As in the case of integrated volatility, productivity-based uncertainty shocks promote a strong and persistent decline in private output as well as in private fixed investment and R&D. In this case, the decline in private R&D is as severe as that of fixed investment. We note that our aggregate results are not in contradiction with our reallocation evidence from Compustat. At the firm level, adverse volatility shocks cause a stronger investment contraction in innovation-oriented firms. Since innovation-oriented investments are concentrated among a relatively limited number of firms, when looking at aggregate data, i.e., at all firms, the reduction in total fixed investment is as severe as that of R&D.

Similarly to what we have done with micro data, we study the implications of adverse productivity shocks on two measures of innovation outcome, namely number and value of patents. The aggregate value of patents is obtained using the Kogan et al. (2017) method-

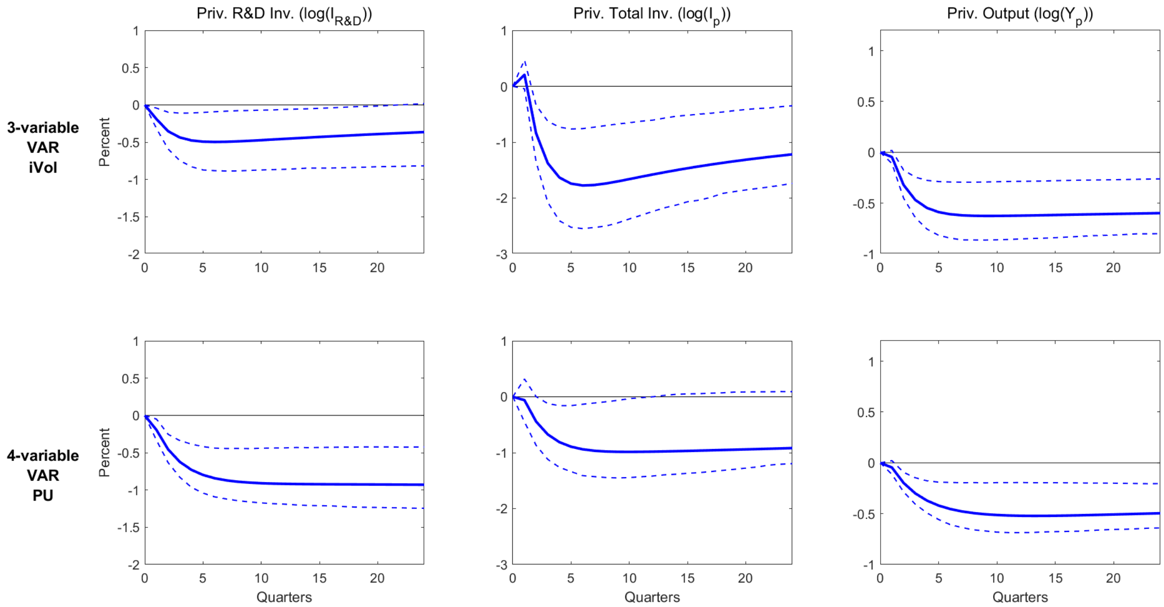


Fig. 3. **Capital Reallocation and Productivity Uncertainty.** The bottom part of this figure shows the response to shocks to productivity-based volatility of gross private R&D investment ($I_{R\&D}$), total gross private investment (I_p), and private output (Y_p). All results are based on the VAR specified in equations (3)–(5). In the top panels, we use stock market integrated volatility to measure uncertainty. We control for the 10-year Baa credit spread. Our sources are detailed in Appendix A, our quarterly sample is 1972:Q1–2016:Q4. Confidence intervals are HAC-adjusted.

ology. In figure 4, we confirm that uncertainty shocks are followed by severe contractions in innovation outcomes, not just in innovation inputs.

Realized volatility shocks. We have also replicated our analysis replacing our measure of expected volatility with the residual of the following equation,

$$|\Delta a_{t+1} - \mu - x_t| = b_0^v + b_v F_t + resid_{t+1}.$$

In this case, we find no significant reallocation, implying that what really matters for investment flows is the extent of expected long-term uncertainty.

Robustness. In Appendix B, we show that our results are unchanged when using (i) the full available sample period starting from 1961 or (ii) the post-1982 sample period, i.e., a subsample in which macroeconomic aggregates feauture a more stable dynamic pattern. These results apply to both the VAR in which we use iVol (figure B1) and that in which we use our measure of produtivity uncertainty (figure B2). In figure B3, we show that the disruptive effect of productivity uncertainty shocks can be fully captured only when we look

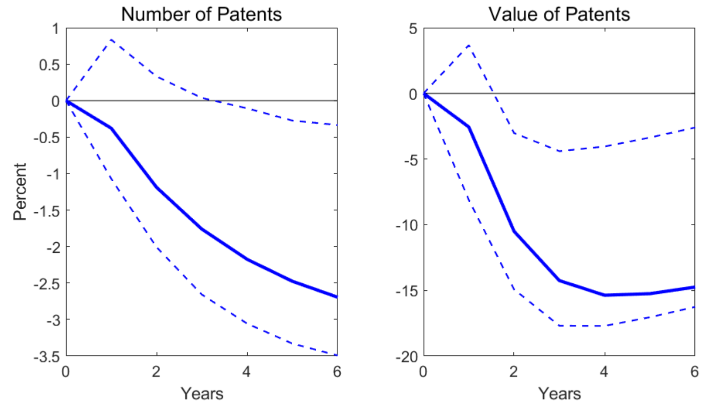


Fig. 4. **Innovation Output in a VAR with Productivity Uncertainty.** This figure shows the response to adverse shocks to productivity volatility of both aggregate number and value of patents. All results are based on the VAR specified in equations (3) and (5), in which we use productivity volatility to measure uncertainty. We control for the 10-year baa credit spread. Our quarterly series (1972:Q1–2016:Q4) are in log levels. Confidence intervals are HAC-adjusted.

at medium and long-run cycles, i.e., at relatively low frequencies. In figure B4, we show that our results remain unchanged if we estimate a VAR(2).

Reallocation and growth. We take seriously our equation (2) and re-estimate it using aggregate measures of R&D capital relative to aggregate fixed capital as reported in the NIPA tables. We summarize our results in table 6.

Our regressions confirm all of the results obtained from micro data. Namely, innovation capital is a very important leading indicator of growth both from a statistical and an economic perspective. To better interpret the economic relevance of this result, we notice that if our capital measure in the top panel decreases by two standard deviations, cumulated productivity growth should decline by 3.6% over five years.[5] This empirical evidence motivates and supports the production-based model that we describe in the next section.

3 R&D-Based Two-Sector Model

We start by describing the representative household problem and then describe our two production sectors. Inspired by our empirical analysis, the model features two sectors with unique features. One sector extracts risky monopolistic rents from patents and hence has an incentive to innovate and promote endogenous growth. We denote it as the H-sector,

[5]Since the standard deviation of our capital measure is 0.002, the cumulative decline is $2 \times 0.002 \times 1.79 \times 5$.

Table 6: **Aggregate Reallocation and Growth**

Horizon (n):	5 year	7 year	10 year
$K_{R\&D}/K_{Fixed}$	1.79***	1.52***	1.96***
	(0.60)	(0.39)	(0.39)
Adj R^2	0.40	0.44	0.40
Adj R^2 w/o	0.19	0.18	-0.17
$I_{R\&D}/I_{Fixed}$	0.42**	0.47***	0.39***
	(0.23)	(0.15)	(0.15)
Adj R^2	0.23	0.28	-0.08
Adj R^2 w/o	0.19	0.18	-0.17
Credit Tightness Control: Baa10y	Yes	Yes	Yes
Predicting Factors	Yes	Yes	Yes

Notes: This table is obtained as table 5, except that it uses aggregate capital measures from the NIPA tables. $K_{R\&D,t}/K_{Fixed,t}$ refers to aggregate R&D capital stock relative to total private fixed capital. The other ratio is constructed in a similar way using investment flows, as opposed to capital stock data. Our annual sample starts in 1972 and ends in 2016. Our sources are detailed in Appendix A.

i.e., the high-R&D sector and we think of it as relatively riskier, consistent with our asset markets empirical evidence.

The other sector is perfectly competitive, does not innovate and it features safer cash-flows. We denote this sector as the L-sector and identify it as a relatively safer investment. The goal of this model is to propose a capital market-based mechanism that links in equilibrium uncertainty shocks, reallocation, innovation, and growth, as in our empirical investigation. Specifically, this model highlights a novel mechanism through which uncertainty shocks can reduce growth: when investors price uncertainty shocks, an increase in uncertainty prompts a rebalancing of their portfolios toward safer assets. Since the H-sector is riskier than the L-sector, when uncertainty increases, the innovation-oriented sector shrinks and growth is subdued for a long time.

3.1 Household problem

The objective of the representative agent is to maximize her utility

$$U_t = \left[(1-\delta)\tilde{C}_t^{1-\frac{1}{\psi}} + \delta\Big(E_t\big[U_{t+1}^{1-\gamma}\big]\Big)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}$$

where the consumption bundle $\tilde{C}_t$ is

$$\tilde{C}_t = C_t - \bar{\omega}_{l,H} SL_t \frac{(L_{H,t} + \bar{\omega}_{l,L} L_{L,t})^{\omega_l}}{\omega_l},$$

in which C_t denotes the consumption of the final good, $L_{H,t}$ is the labor supply in the High-R&D sector, and $L_{L,t}$ is the labor supply in Low-R&D sector (i.e., zero-R&D sector). To ensure balance growth with Greenwood et al. (1988) preferences, we introduce an exogenous preference shock process, SL_t, cointegrated with productivity. Specifically, we define:

$$sla_t := \frac{SL_t}{A_t}$$

and assume

$$sla_t = (1 - \theta_{sla})\mu + (1 - \theta_{sla})(sla_{t-1} - \Delta a_t),$$

in which we set $\theta_{sla} \approx 0$ so that SL_t mimics an exogenous linear trend.

The budget constraint of the representative household is:

$$C_t + Z_t^L V_{L,t}^{ex} + Z_t^H V_{H,t}^{ex} = Z_{t-1}^L (V_{L,t}^{ex} + D_{L,t}) + Z_{t-1}^H (V_{H,t}^{ex} + D_{H,t}) + (w_{H,t} L_{H,t} + w_{L,t} L_{L,t})/\widetilde{p}_t,$$

where $Z_t^{\cdot} \in [0\,1]$ represents the percentage ownership in each sector; $V_{\cdot,t}^{ex}$ is the ex-dividends value of capital in each sector; $D_{\cdot,t}$ denotes the payout in each sector; and $w_{\cdot,t}$ is the wage paid in each sector. As described in detail in the next section, $\widetilde{p}_t$ is the relative price of the final good with respect to the numeraire, i.e., the good produced in our Low-R&D sector. This change of unit is required because all variables are expressed in terms of the final good.

Optimality. The optimal investment strategy implies that:

$$V_{s,t}^{ex} = E_t \left[M_{t+1} (V_{s,t+1}^{ex} + D_{s,t+1}) \right], \quad s \in \{L, H\},$$

where M_{t+1} is the stochastic discount factor (SDF) of the agent in final consumption units. In what follows, we often use the SDF in numeraire units,

$$M_{t+1}^L \equiv M_{t+1} \frac{\widetilde{p}_t}{\widetilde{p}_{t+1}}.$$

The optimal supply of labor in our two sectors implies:

$$w_{H,t}/\widetilde{p}_t = \bar{\omega}_{l,H} SL_t \left(L_{H,t} + \bar{\omega}_{l,L} L_{L,t}\right)^{\omega_l - 1}$$

and

$$w_{L,t}/\widetilde{p}_t = \bar{\omega}_{l,L} \cdot w_{H,t}/\widetilde{p}_t.$$

3.2 Final good producer

The final good in the economy is a bundle of the H-sector good, $Y_{H,t}$, and the L-sector good, $Y_{L,t}$:

$$Y_t = \left[\omega Y_{H,t}^{1-\frac{1}{\tau}} + (1-\omega) Y_{L,t}^{1-\frac{1}{\tau}}\right]^{\frac{1}{1-1/\tau}}.$$

The elasticity of substitution between these two goods is determined by τ. The relative demand of the private good with respect to the public good is also determined by ω. We assume the existence of a competitive producer that solves the following profit maximization problem taking prices as given:

$$\max_{Y_{H,t}, Y_{L,t}} \widetilde{p}_t Y_t - p_t Y_{H,t} - Y_{L,t},$$

where the price of the L-sector good is normalized to one (numeraire). Optimality implies:

$$\frac{\omega}{1-\omega}\left(\frac{Y_{H,t}}{Y_{L,t}}\right)^{-\frac{1}{\tau}} = p_t.$$

The relative price of the final good with respect to the numeraire good:

$$\widetilde{p}_t \equiv \frac{\partial Y_{L,t}}{\partial Y_t} = \frac{1}{1-\omega}\left(\frac{Y_{L,t}}{Y_t}\right)^{\frac{1}{\tau}}.$$

3.3 High-R&D Sector

H-sector final good. The production function of the H-sector final good is:

$$F_{H,t} = (K_{H,t}^{\alpha_H}(\Omega_{H,t} L_{H,t})^{1-\alpha_H})^{1-\xi} G_t^{\xi},$$

where the composite bundle G_t is defined as

$$G_t \equiv \left[\int_0^{N_t} X_{i,t}^{\nu}\, di\right]^{\frac{1}{\nu}},$$

$X_{i,t}$ is the quantity of the intermediate good $i \in [0, N_t]$, and N_t is the total mass of intermediate good varieties. Since each intermediate good requires a patent, N_t also measures the total mass of patents in use at date t.

The exogenous stationary process of productivity is $\Omega_{H,t} = e^{a_{H,t}}$, where $a_{H,t}$ follows an AR(1) process subject to volatility shocks, $\epsilon_{v,t}$:

$$\begin{aligned} a_{H,t} &= (1-\rho)\bar{a} + \rho a_{p,t-1} + e^{v_{t-1}}\sigma\epsilon_{a,t} \\ v_t &= \rho_v v_{t-1} + \sigma_v \epsilon_{\sigma,t} + \beta_{v,a}\epsilon_{a,t} \\ \epsilon_{\omega,t}, \epsilon_{\sigma,t} &\sim i.i.d. N(0,1). \end{aligned}$$

The parameter $\beta_{v,a}$ captures contemporaneous correlation across level and volatility shocks. In what follows, we refer to level shocks as short-run shocks, as they determine most of the variance of the growth dynamics over the short-run. Over longer horizon, capital reallocation is the main driver of growth.

The final producer in the H-sector has monopolistic power as determined by the demand elasticity τ. We assume that this private firm buys production inputs (investment goods $I_{H,t}$, labor $L_{H,t}$, and intermediate goods $X_{i,t}$) in a competitive way, that is, by taking their price as given. Hence the problem of the private firm is as follows:

$$\begin{aligned} V_{H,t} &= \max_{\substack{L_{H,t}, I_{H,t}, Y_{H,t} \\ K_{H,t+1}, X_{i,t}}} \underbrace{\frac{\omega}{1-\omega}\left(\frac{Y_{H,t}}{Y_{L,t}}\right)^{-\frac{1}{\tau}}}_{p_t} Y_{H,t} - \widetilde{p}_t I_{H,t} - w_{H,t} L_{H,t} \\ &\quad -p_t\left[\int_0^{N_t} P_{i,t} X_{i,t}\, di\right] + E_t[M^L_{t+1} V_{p,t+1}] \end{aligned}$$

subject to

$$\begin{aligned} Y_{H,t} &\leq F_{H,t} && (\lambda_{H,t}) \\ K_{H,t+1} &\leq \left(1 - \delta + \Gamma_{H,t}\left(\frac{I_{H,t}}{K_{H,t}}\right)\right) K_{H,t} && (q_{H,t}), \end{aligned}$$

where $\lambda_{H,t}$ is the shadow marginal cost, and $q_{H,t}$ is the shadow value of private capital. The adjustment cost function is specified as in Jermann (1998):

$$\Gamma_{H,t} = \frac{\alpha_{H,1}}{1-\frac{1}{\xi_H}}\left(\frac{I_{H,t}}{K_{H,t}}\right)^{1-\frac{1}{\xi_H}} + \alpha_{H,0},$$

and the optimality condition with respect to $I_{H,t}$ pins down the marginal value of private capital:

$$q_{H,t} = \frac{\widetilde{p}_t}{\Gamma'_{H,t}}. \tag{6}$$

The optimal demand of labor implies

$$\frac{w_{H,t}}{p_t} = (1 - 1/\tau)(1 - \alpha_H)(1 - \xi)\frac{Y_{H,t}}{L_{H,t}},$$

and the optimal condition with respect to $K_{H,t+1}$ is:

$$q_{H,t} = E_t\left[M^L_{t+1} p_{H,t+1}\right] \tag{7}$$

where $p_{H,t+1} = \frac{\partial V_{H,t+1}}{\partial K_{H,t+1}}$ and by envelope theorem we have

$$\frac{\partial V_{H,t}}{\partial K_{H,t}} = (1 - 1/\tau)p_t\alpha_H(1 - \xi)\frac{Y_{H,t}}{K_{H,t}} + \left(1 - \delta + \Gamma_{H,t} - \frac{I_{H,t}}{K_{H,t}}\Gamma'_{H,t}\right) q_{H,t}. \tag{8}$$

The optimal demand of $X_{i,t}$ implies

$$P_{i,t} = \left(1 - 1/\tau + 1/\tau\frac{\int_0^{N_t} P_{i,t}X_{i,t}\,di}{Y_{H,t}}\right)\frac{\partial Y_{H,t}}{\partial X_{i,t}}$$

where

$$\frac{\partial Y_{H,t}}{\partial X_{i,t}} = \xi(K_{H,t}^{\alpha_H}(\Omega_{H,t}L_{H,t})^{1-\alpha_H})^{1-\xi}\left[\int_0^{N_t} X_{i,t}^{\nu}\,di\right]^{\frac{\xi}{\nu}-1} X_{i,t}^{\nu-1}.$$

Intermediate Goods. Intermediate good producers can generate one unit of their own good by buying one unit of the private good at market price p_t. They have monopoly power and choose $P_{i,t}$ to maximize their profits, $\Pi_{i,t}$, each period:

$$\max_{P_{i,t}} \Pi_{i,t} \equiv \max_{P_{i,t}} \quad p_t \cdot \left[P_{i,t} \cdot X_{i,t}(P_{i,t}) - X_{i,t}(P_{i,t})\right].$$

Since $\Pi_{i,t}$ is measured in L-good units, the value $V_{i,t}$ of owning exclusive rights to produce intermediate good i is:

$$V_{i,t} = \Pi_{i,t} + (1 - \phi)E_t[M^L_{t+1}V_{i,t+1}], \tag{9}$$

where ϕ is the probability that a patent becomes obsolete. Because of symmetry, at the equilibrium all patents have the same value and we drop the i index, i.e., $V_{i,t} = V_t$.

Aggregation. In our symmetric equilibrium,

$$
\begin{aligned}
P_{i,t} &\equiv P_t = \frac{1}{\nu}, \qquad (10)\\
X_{i,t} &\equiv X_t = \left(\xi\nu\frac{1-1/\tau}{1-\xi/\tau}\left(K_{H,t}^{\alpha_H}(\Omega_{H,t}L_{H,t})^{1-\alpha_H}\right)^{1-\xi}N_t^{\frac{\xi}{\nu}-1}\right)^{\frac{1}{1-\xi}}
\end{aligned}
$$

Under the restriction $\alpha + \frac{\frac{\xi}{\nu}-\xi}{1-\xi} = 1$, the production function of private good sector can be written as:

$$Y_{H,t} = Z_{H,t}K_{H,t}^{\alpha}L_{H,t}^{1-\alpha} \qquad (11)$$

where

$$Z_{H,t} \equiv \overline{A}(\Omega_{H,t}N_t)^{1-\alpha}, \qquad (12)$$

is an endogenous productivity process that grows with the stock of patents N_t, and whose initial level, $\overline{A} \equiv \left(\xi\nu\frac{1-1/\tau}{1-\xi/\tau}\right)^{\frac{\xi}{(1-\xi)}}$, depends on the extent of competition as determined by the elasticities τ and ξ. Similarly to the process $\Omega_{H,t}$, measured productivity Z_t features time-varying volatility.

Innovators. Innovators develop new patents that are sold to the intermediate good producers in a competitive way. As a result, at the equilibrium, the price of a new patent is $V_{i,t}$. The law of motion of the intangible capital stock, N_t, is specified as follows:

$$N_{t+1} = \vartheta_t S_t + (1-\phi)N_t,$$

where S_t denotes R&D expenditures (in terms of the final good) and ϑ_t represents the productivity of the R&D sector that is taken as exogenous by the R&D sector. In the spirit of Comin and Gertler (2006), we assume that this technology coefficient involves a congestion externality effect

$$\vartheta_t = \chi\left(\frac{N_t}{S_t}\right)^{1-\eta},$$

where $\chi > 0$ is a scale parameter and $\eta \in [0,1]$ is the elasticity of new patents with respect to R&D. This specification captures the notion that concepts already discovered make it easier to come up with new ideas, $\partial\vartheta/\partial N > 0$, and that R&D investment has decreasing marginal returns, $\partial\vartheta/\partial S < 0$.[6] The free-entry condition in the R&D sector implies that

$$E_t[M_{t+1}^L V_{t+1}](N_{t+1} - (1-\phi)N_t) = \widetilde{p}_t S_t,$$

that is, the expected revenue from selling new patents must equal the incurred costs, or equivalently,

$$\frac{\widetilde{p}_t}{\vartheta_t} = E_t[M_{t+1}^L V_{t+1}]. \tag{13}$$

3.4 Low-R&D Sector.

The firm in the L-sector produces its own good in a competitive fashion. This firm uses labor, $L_{L,t}$, and final goods to accumulate government capital, $K_{L,t}$, and it solves the following dynamic problem:

$$V_{L,t} = \max_{Y_{L,t}, K_{L,t+1}, L_{L,t}, I_{L,t}} Y_{L,t} - \widetilde{p}_t I_{L,t} - w_{L,t} L_{L,t} + E_t[M_{t+1}^L V_{L,t+1}]$$

subject to

$$\begin{aligned} Y_{L,t} &\leq F_{L,t} = \chi_L Z_{H,t} K_{L,t}^{\alpha_L} L_{L,t}^{1-\alpha_L} && (\lambda_{L,t}) \\ K_{L,t+1} &\leq \left(1 - \delta + \Gamma_{L,t}\left(\frac{I_{L,t}}{K_{L,t}}\right)\right) K_{L,t} && (q_{L,t}), \end{aligned}$$

where $\lambda_{L,t}$ is the shadow marginal cost of the L-sector good, $q_{L,t}$ is the shadow value of the L-sector capital, and the parameter $\chi_L \leq 1$ captures a gap in the level of productivity across the two sectors. The adjustment cost function is defined as follows,

$$\Gamma_{L,t}\left(\frac{I_{L,t}}{K_{L,t}}\right) = \frac{\alpha_{L,1}}{1 - \frac{1}{\xi_L}} \left(\frac{I_{L,t}}{K_{L,t}} + 1\right)^{1 - \frac{1}{\xi_L}} + \alpha_{L,0},$$

and allows for reversibility of investment in this sector.[7] This assumption captures the ability of the H-sector to use infrastructure generated by L-sector and prevents limitations to reallocation cross sectors.

[6] Similarly, this congestion externality can be thought of as giving rise to adjustment costs to investment in intangible capital, that is, R&D. Absent the congestion externality, the marginal value of capital is fixed and excess returns are excessively smooth.

[7] The constant $\alpha_{L,0}$ is set so that at the deterministic steady state $\frac{I_L}{K_L} = \Gamma_L$. The coefficient $\alpha_{L,1}$ is set so that at the deterministic steady state $\Gamma_L' = 1$.

The optimality condition with respect to Y_L implies that

$$\lambda_{L,t} \equiv 1,$$

i.e., the marginal cost must be equal to the price of the good. As a result, the optimal demand of labor implies

$$w_{L,t} = F_{g,L_g,t}, \tag{14}$$

The optimality condition with respect to $I_{L,t}$ pins down the marginal value of capital in the L-sector:

$$q_{L,t} = \frac{\widetilde{p}_t}{\Gamma'_{L,t}}, \tag{15}$$

where $\widetilde{p}_t$ accounts for the fact that investment is made using the final good. The optimality with respect to $K_{L,t+1}$ implies

$$q_{L,t} = E_t \left[M^L_{t+1} p_{L,t} \right] \tag{16}$$

$$p_{L,t} = \frac{\partial V_{L,t+1}}{\partial K_{L,t+1}} = F_{L,K_L,t+1} + \left(1 - \delta + \Gamma_{L,t+1} - \frac{I_{L,t+1}}{K_{L,t+1}} \Gamma'_{L,t+1} \right) q_{L,t+1}. \tag{17}$$

Under this specification, the L-sector does not produce innovations, but it has access to the same production function of the private sector when we set $\chi_L = 1$ and $\alpha_L = \alpha_H$. Under this assumption, the L-sector differs from the H-sector only in that it does not hold risky innovation capital.

3.5 Cross Section of Returns

In our economy, we have three different capital stocks and two sectors. The gross return of tangible capital in the H-sector is

$$R^H_{K,t+1} = \frac{p_{H,t+1}}{q_{H,t}}$$

where q_H and $p_{H,t+1}$ are defined in equation (7)–(8). The gross return on innovation capital is

$$R_{S,t+1} = \frac{V_{t+1}}{V^{ex}_t}$$

where $V_t^{ex} := \tilde{p}_t/\vartheta_t$, consistent with equation (13). Given this notation, we define

$$HML - R\&D_t := R_{S,t+1} - R^H_{K,t+1}.$$

The gross return in the H-sector, R^H, is simply the value-weighted average of the returns of the two capital stocks, that is,

$$R_{H,t+1} = \frac{V^{ex}_{H,t+1} + D_{H,t+1}}{V^{ex}_{H,t}} = wg_t R^H_{K,t+1} + (1 - wg_t) R_{S,t+1}$$

with $wg_t := \frac{q_{H,t} K_{H,t+1}}{q_{H,t} K_{H,t+1} + V_t^{ex} S_{t+1}}$. The return in the L-sector can be computed as follows,

$$R_{L,t+1} = \frac{V^{ex}_{L,t+1} + D_{L,t+1}}{V^{ex}_{L,t}} = \frac{p_{L,t+1}}{q_{L,t}},$$

where q_L and p_L are defined in equations (15)–(17).

3.6 Calibration and Solution Method

Like in all medium-scale DSGE models, our calibration is a multidimensional object in which each moment is affected by many parameters simultaneously. Taken this into consideration, in table 7 we report our quarterly calibration and, for each parameter, the most relevant companion targeted moment. For some parameters, we report direct estimates from our data.

Before proceeding in further detail, we clarify the spirit of our calibration. We set many parameters to be common across sectors in order to highlight the role of risk heterogeneity stemming solely from different degrees of innovation-intensity. At this stage, we see our calibration as instructive about the key mechanism that accounts for our empirical findings. In this section, we map the H-sector to the aggregate domestic private sector in the US data and remain agnostic on the appropriate empirical counterpart of the L-sector. In section 4.2, we take a targeted stand on the L-sector and show that the government sector is able to match several key features of the data.

The preference parameters are set in the spirit of the long-run risk literature (Bansal and Yaron 2004). The parameter γ measures relative risk aversion with respect to a static gamble (Epstein and Zin 1989). Our effective relative risk aversion with respect to the consumption commodity, however, is lower because of the labor margin (Swanson (2012, 2018)). We set this parameter so that the average annual excess return in the High-R&D sector is greater

Table 7: **Benchmark Calibration**

Description		Value	Estimate/ Moment
Preferences			
Static Relative Risk Aversion	(γ)	12	$E\left[r_{H,ex}^{LEV}\right]$
Intertemporal Elasticity of Substitution	(ψ)	2	$\sigma(\Delta c)/\sigma(\Delta y)$
Subjective Discount Rate	(β)	$0.98^{1/4}$	$E\left[r^f\right]$
Labor Elasticity	(ω_l)	1.5	$V(L^{TOT})$
Labor Allocation	$(\bar{\omega}_l/\bar{\omega}_H)$	1.03	$E(L_H/L^{TOT})$
Cointegration Labor Preference Shock	(θ_{sla})	0.1	$V_t(SL_{t+1})$
Final Good Aggregator			
H-Good Bias	(ω)	0.8	$E[Y_H/Y]$
Elasticity of Substitution across goods	(τ)	5	$E(MR/Y)$
Production			
Tangible Capital Share	(α)	0.3	$E[r_K K/Y]$
Capital Depreciation Rate	(δ)	0.06/4	$E[I/Y]$
Adjustment Cost Elasticity	(ξ_s)	5	$\sigma(\Delta i_{tot})/\sigma(\Delta y)$
Intangible Capital Share	(ξ)	0.49	$E[r_S S/Y]$
Innovation			
Intangible Capital Congestion, Scale Param.	(χ)	0.122	$E[\Delta c]$
Intangible Capital Congestion, Elasticity	(η)	0.80	$E[HML_{R\&D}]$
Patent Death Rate	(ϕ)	0.04	$E[S/Y]$
Productivity			
Productivity Persistence	(ρ)	0.98	$ACF_1[\Delta y]$
Productivity Volatility	(σ)	0.032/2	$V[\Delta y]$
Relative Log-Volatility Persistence	(ρ_v)	0.74	0.73 (0.17)
Volatility of Relative Log-Volatility	(σ_v)	0.15	0.14 (0.06)
Relative Log-Volatility Short Run Exposure	$(\beta_{v,a})$	-3.5	-3.5 (2.25)
Average Prod. Gap	(χ_L)	65%	–

Notes: This table reports our benchmark quarterly calibration. Across sectors, the following restrictions apply: $\alpha_H = \alpha_L = \alpha$; $\delta_n = \delta_s = \delta$; $\xi_H = \xi_L = \xi_s$. MR/Y denotes the GDP share for total monopolistic rents. In the data, the H-sector corresponds to the US domestic private sector.

than 5%. The parameter ψ affects the intertemporal elasticity of substitution and it is set in order to have smooth consumption. The subjective discount rate enables us to target the average risk-free rate. We set the ratio $\bar{\omega}_L/\bar{\omega}_H = 1.03$ so that the relative amount of labor in the two sectors is about the same. We choose this calibration because our study intends to highlight the role of capital markets. Hence we try to make sectors as similar as possible aside from the riskiness of their capital profiles.

In the bundle that aggregates goods across sectors, the weight ω is chosen to target an economy in which the safe sector is relatively small. In the last section of this manuscript, we argue that in the data the L-sector can be disciplined by looking at the government sector and we provide further discussion about this parameter. The elasticity of substitution τ is set to have a total profit share comparable to the data. Since most of the rents are obtained within the innovation sector, τ is set to a very high level so that H-goods and L-goods are almost perfect substitutes.

Both the tangible capital income share, α, and the depreciation rate, δ, of tangible capital are set to the same values across sectors. We choose numbers standard in the real business cycle literature that targets the US economy (among others, see Croce 2014). We also set the elasticity of the adjustment cost functions to be the same across sectors, $\xi_L = \xi_H$, and choose a value that let both investment and tangible capital returns be enough volatile.

The parameters that determine the innovation activity are set as follows. The scaling parameter χ is set to have an annual average growth rate of 2.10% (in our data, the point estimate is 2.06% with a standard error of 0.69%). The elasticity of the congestion function, η, is the main driver of the risk premium associated to the innovation sector. We set the death rate of patents, ϕ, so that the GDP share of intangible investment is similar to that of tangible investment in the H-sector (Corrado et al. 2005).

The exogenous productivity process is calibrated to match key properties of our quarterly measured productivity. Specifically, we set the persistence of our stationary exogenous productivity, ρ, so that growth rate of output in our economy are as autocorrelated as in the data. Specifically, $ACF_1(\Delta y)$ in our model is 0.37 and in the data we have a point estimate with this exact value. The volatility of the level shocks to productivity, σ, is set to replicate the volatility of aggregate output.

Within the model, there is a nearly one-to-one pass-through of vol shocks from the exogenous productivity process to the measured one. Hence we can use our estimates to discipline these parameters. Both the persistence, ρ_v, and the magnitude of time-varying volatility, σ_v, are consistent with our confidence intervals reported in table C1. The parameter $\beta_{v,a}$ accounts for the negative correlation between relative volatility and short-run shocks and is set according to the data to -3.5. In untabulated sensitivity analysis, we find that this parameter plays no crucial role.

The productivity level in the L-sector is assumed to be lower than that in the H-sector. In the section 4.2, we argue that the L-sector can be disciplined by looking at the government sector and provide further support for a value of 65%, consistent with the data provided in the NIPA tables.

In table D1 (see Appendix D.1), we provide sensitivity analysis with respect to our preference parameters ψ and γ, as well as our congestion function parameters, χ and η. We also vary the rescaling parameter χ_L. This exercise supports our chosen values. We conclude this section by specifying that the model is solved with a third-order perturbation method. Moments from this model are obtained by simulating our pruned solution.

4 Results

In this section, we use our model to study the relevance of both recursive preferences and volatility shocks to generate the reallocation observed in the US capital markets. At first, we do not take a stand on the origin of the L-sector and focus solely on the High-innovation sector. Specifically, we look at tangible and intangible investment *within* the H-sector . In a second step, in section 4.2, we look at aggregate data and provide novel evidence about the government sector being a strong receiver of productive resources when volatility increases.

4.1 The Core of the Model

Responses. With respect to a positive level shock, our model behaves similarly to a standard production economy model, as private consumption, total labor, investments, and output simultaneously expand. For the sake of brevity, we depict these responses in Appendix D.1 (see figure D1). In figure 5, we depict only the predictions of our model with respect to volatility shocks. In contrast to a positive level shock, a positive (i.e., adverse) volatility shock produces a contraction in economic activity in the H-sector. Output, fixed investment, labor and R&D investments simultaneously decline in the innovation-oriented sector. This result is novel and it requires a detailed explanation.

Because of precautionary saving motives, the representative investor finds it optimal to increase total savings. In a one-sector economy, a volatility shock would produce an investment boom and more innovation. In our setting, however, our representative investor features aversion to volatility shocks and hence it finds it optimal to reallocate resources toward forms of capital that are less exposed to volatility. Since both the marginal product of tangible capital and the monopolistic rents generated through intangible capital are very exposed to volatility, the household reallocates resources toward capital in the L-sector. As a result, the model reproduces a contraction in both the size and the value of capital in the H-sector. Capital in the L-sector, in contrast, appreciates. Hence a strategy long in the

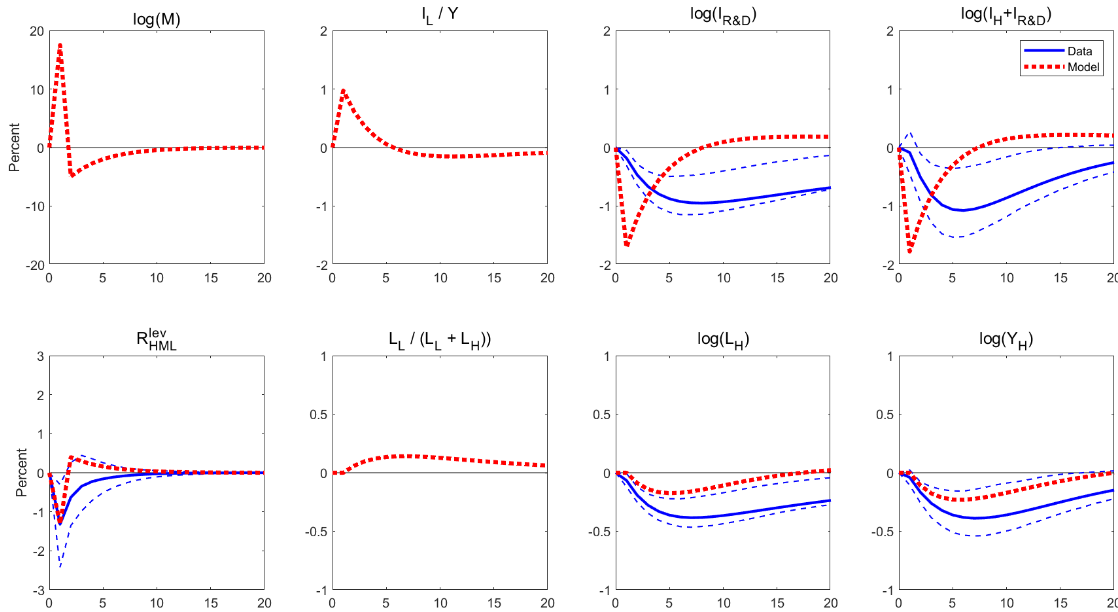


Fig. 5. **Model versus VAR.** This figure shows the response to productivity volatility shocks of the SDF (M); the excess return of the $HML - R\&D$ strategy; the ratio of gross investment in the L-sector to output (I_L/Y) and the share of labor allocated to the L-sector ($L_L/(L_L + L_H)$); gross R&D investment ($I_{R\&D}$) and total investment in the H-sector ($I_H + I_{R\&D}$); output (Y_H) as wells as labor (L_H) in the H-sector. The VAR-implied responses are obtained after passband-filtering our data and are consistent with those in figure 2. The model-implied responses are obtain from our benchmark quarterly specification.

H-sector and short in the L-sector produces a loss in a high marginal utility state and it carries a relevant risk premium, consistent with our data.

In figure 5, we also compare the responses of our H-sector with those of our VAR. A data-driven analysis of the responses in the L-sector is discussed in section 4.2. In this specific step, we focus on medium cycle fluctuations, in the spirit of Comin and Gertler (2006), and compare dynamics in our H-sector to those in the US private sector. Absent cross-sector reallocation costs, the model predicts a reallocation away from the private sector that is more pronounced but also less long-lived than in the data. We speculate that a friction that features time-to-reallocate could solve this problem. Overall, however, our model captures well the empirical pattern of output and labor in the H-sector. In addition, our macro-finance model features a response for the $HML - R\&D$ factor that lays within our VAR confidence intervals.

Given these supportive results, in what follows we focus on a wide set of moments obtained by simulating the model.

Table 8: Main Moments

Moment	Data Est.	Data St.Err.	Model
$\sigma(\Delta y)$ (%)	4.65	(0.89)	4.78
$\sigma(\Delta c)/\sigma(\Delta y)$	0.62	(0.07)	0.66
$\sigma(\Delta i_{tot})/\sigma(\Delta y)$	2.16	(0.13)	1.74
$\sigma(\Delta I_{R\&D})$ (%)	9.69	(2.00)	7.97
$E\left[(I_H + I_{R\&D})/Y\right]$ (%)	15.18	(0.77)	31.96
$\sigma((I_H + I_{R\&D})/Y)$ (%)	3.46	(0.67)	2.69
$\rho(\Delta c, \Delta \ln(I_H + I_{R\&D}))$	0.80	(0.05)	0.79
$\sigma(\Delta w_H L_H)$ (%)	6.96	(1.63)	4.09
$\sigma(\Delta L_H)$ (%)	2.75	(0.38)	3.08
$E\left[r_{H,ex}^{LEV}\right]$ (%)	5.57	(2.04)	5.21
$\sigma(r_{H,ex}^{LEV})$ (%)	19.64	(2.07)	15.11
$E\left[\textit{HML-R\&D}^{LEV}\right]$ (%)	5.43	(2.95)	2.88
$E\left[r^f\right]$ (%)	0.32	(0.64)	0.73
$\sigma(r^f)$ (%)	3.69	(0.66)	0.88
$\overline{\beta}$ for $\Delta z_{t\|t+40}/40$ using $I_{R\&D}/I_{Fixed}$	0.39	-0.15	0.46

Notes: Empirical moments are computed using annual data from 1929 to 2016. All data sources are discussed in Appendix A. Numbers in parentheses are standard errors adjusted for heteroscedasticity. The entries for the model are obtained by averaging the results across simulated small samples. Our solution is pruned. Our baseline calibration is detailed in table 7. The coefficient $\bar{\beta}$ refers to the slope of a predictive regression featuring 10-year average productivity growth, $\Delta \ln Z_{H,t|t+40}/40$, at the left-hand side and $I_{R\&D}/I_{Fixed}$ as predictor (see table 6 for its data counterpart).

Simulated moments. To better assess the performance of our model, in table 8 we show a comprehensive list of moments generated through simulations. In this step of our analysis, we abstract away from the L-sector and post-pone this discussion to section 4.2. The top portion of the table shows standard moments for macroeconomic aggregates. Our model matches very well all these well known figures, except the average share of total private investment which is too high due to intangible investment. We note, however, that McGrattan and Prescott (2009) and Corrado et al. (2006) argue that the BEA data may underestimate the extent of intangible investments and hence we regard our model output as plausible. Turning our attention to labor, we note that our model produces moments for both labor and labor income that are within our empirical confidence intervals.

In the bottom portion of this table, we show that in equilibrium the H-sector is risky as it requires an annualized levered premium of 5.21%. Most importantly, our model produces a sizeable average additional excess return required to hold R&D capital, $E[HML - R\&D]$.

This result obtains because the present value of patent rents is very sensitive to volatility shocks, meaning that it declines substantially when volatility increases. In addition, in this state of the world the household prefers to reallocate resources toward safer forms of capital. In the presence of adjustment costs, the reallocation amplifies the fall in the market value of R&D capital and makes the implied risk-premium higher. As a result, periods of higher volatility are associated with a contraction in innovative investments and medium-run growth.

In order to quantify the model-implied connection between capital reallocation and growth, we estimate the following regression using quarterly simulated data:[8]

$$\frac{1}{40}\Delta \log Z_{H,t|t+40} = \beta_0 + \bar{\beta}\frac{S_t}{I_{H,t}} + resid.$$

We have two remarks. First, this is not a moment that we directly target in our calibration. Second, this specification mimics as closely as possible what we do in the data and produces a positive coefficient, as in our empirical investigation.The magnitude of this coefficient is close to that in the data and hence we see this result as very supportive of our framework.

Key elements. In what follows, we discuss in more detail the role played by different elements in our model. We do so by removing one element at the time from our benchmark calibration and compare the most relevant changes in our simulated moments of interest. Since our goal is to highlight the marginal relevance of each element, we do not recalibrate the entire model. For comparability, in some of these experiments we adjust slightly the scale parameter for intangible capital congestion, χ, in order to avoid major changes in average growth. For the sake of brevity, we discuss the most relevant changes to the moments simulated across different settings in Appendix D.1 (see table D2).

The role of the L-sector. Without accounting for investment in the L-sector, the model produces counterfactual results on the reallocation of resources upon the arrival of volatility news shocks. As shown in figure 6, in this case the agent finds it optimal to increase R&D investment in order to slowly increase growth and compensate for the higher level of volatility. First of all, this is not consistent with our VAR evidence. Furthermore, because of adjustment costs, this reallocation implies an appreciation of R&D capital and a stronger depreciation of tangible capital when volatility increases. These results are not consistent with what we document in table 2, as innovation intensive firms are not a hedge against volatility shocks. Hence the L-sector is essential to absorb the excess of savings in the economy.

[8] Given our quarterly calibration, we use a forward looking average of productivity growth over 40 quarters, that is, a 10-year horizon.

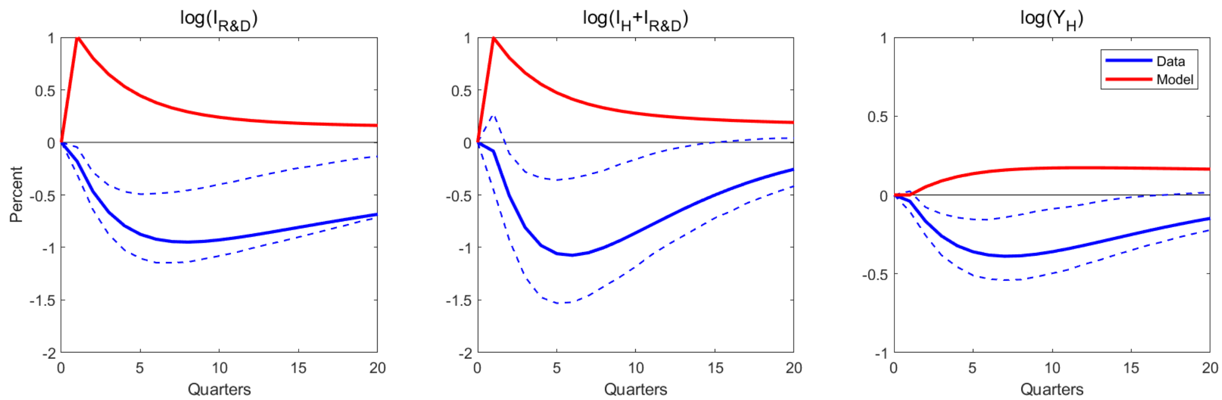


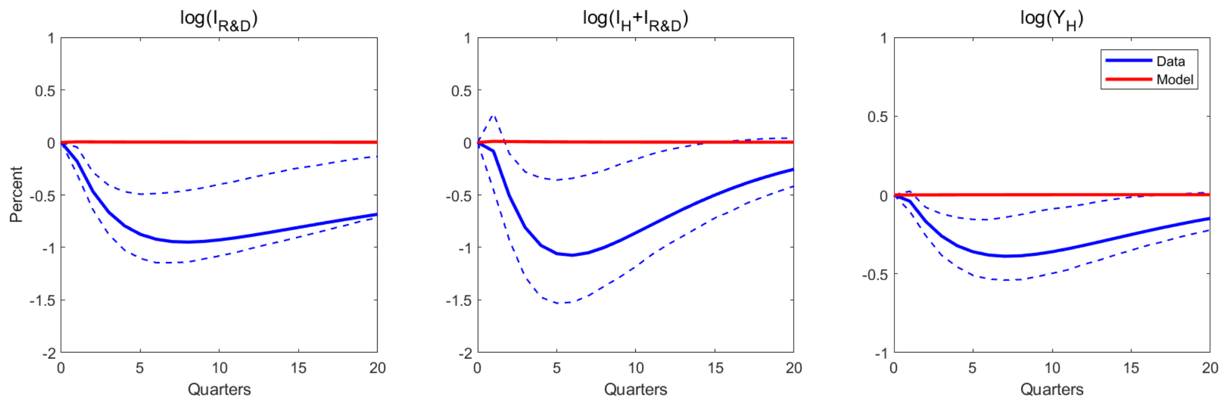
Fig. 6. **The Model without L-sector versus VAR.** This figure shows the response to volatility shocks to productivity of detrended R&D investment ($I_{R\&D}$), total gross investment ($I_H + I_{R\&D}$) and output output (Y_H). The VAR-implied responses are obtained as in figure 2. The model-implied responses are obtain from our benchmark quarterly specification without the L-sector ($\omega = 1$).

The role of preferences. Given our benchmark calibration, we can remove aversion to volatility shocks by either setting the relative risk aversion to 1/2 or by setting the intertemporal elasticity of substitution (IES) to 1/12. In figure 7 (top panels), we show that when we lower our relative risk aversion to 1/2, the model produces no reallocation with respect to vol shocks. Consistent with this finding, in table D2 we document that investments flows become smoother. Not surprisingly, the model-implied risk premia decline substantially as news shocks are no longer separately priced. All other macroeconomic moments remain almost unchanged given that we are keeping fixed the IES.

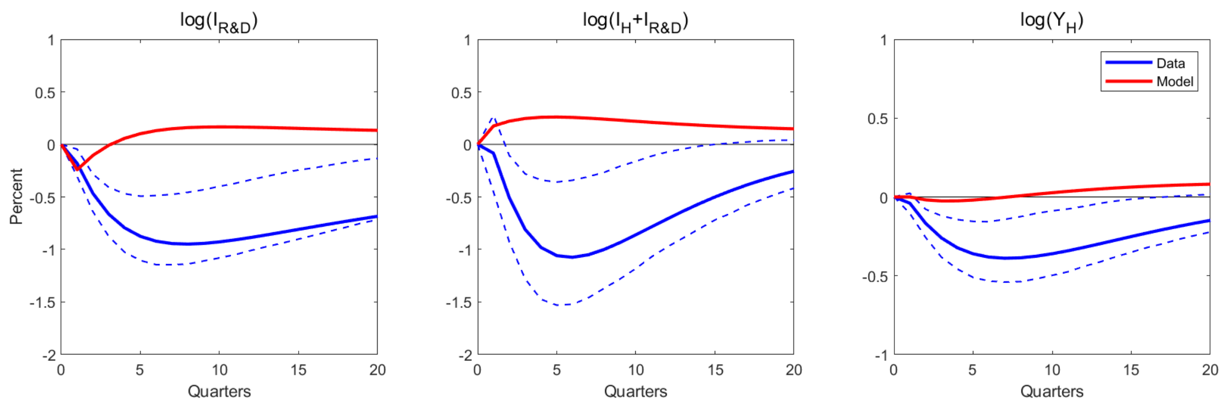
In contrast, increasing the risk aversion to 12 implies a much lower IES. In this case, we face both excessively low risk premia and excessively high risk-free rate, at odds with the data. On the macroeconomic side, all second moments depart substantially from the data. Furthermore, the reallocation with respect to vol shocks goes in the opposite direction of that found in the data (figure 7, bottom panels) and prescribes an increase in economic activity across all horizons.

The role of volatility shocks. Removing time varying volatility produces several intuitive and yet relevant results. First of all, output growth volatility declines by 26 basis points per year, i.e., a relevant amount in a setting with three different capital stocks and endogenous labor. This moderation is even more evident when we focus on the volatility of R&D investment. These moments are reported in table D2, Appendix D.1 .

Additional remarks. Our model explains reallocation flows across sectors with different level of innovation intensity, but it also has predictions on total aggregate investment. Specifically, it predicts that when volatility increases the household saving rate increases as well.



(a) RRA=1/2



(b) RRA=12

Fig. 7. **The Model with CRRA.** This figure shows the response to volatility shocks to productivity of detrended R&D investment ($I_{R\&D}$), total gross investment ($I_H + I_{R\&D}$) and output output (Y_H). The VAR-implied responses are obtained as in figure 2. The model-implied responses are obtain from our benchmark quarterly specification with RRA set to either 1/2 or 12.

In a closed economy, by market clearing, the total investment rate must increase as well, in contrast to what observed in both micro and aggregate data on private investments (among others, see Fernandez-Villaverde et al. 2011 and Bloom et al. 2018). Given this observation, in the next section we explore allocation of resources *outside* of the domestic US private sector.

4.2 Looking for the L-Sector: the Role of the Government.

In the previous section, we studied equilibrium responses in our multi-sector economy that are instructive of the salient properties that the L-sector should feature. In this section, we look for a sector that can fit the key characteristics of our L-sector in our model. Specifically, we are looking for a sector that: (i) expands labor and investment in times of high uncertainty; and (ii) whose capital provides safety in states with higher volatility.

In what follows, we argue that aggregate data from the US economy point to the government sector as an important absorber of resources in times of high volatility. We formally estimate US government hiring and investment policies and note that they replicate the instructive impulse responses derived in the previous section for the L-sector. This reinterpretation of our two-sector model sheds new light on the gap between private investment and savings already documented in prior work (among others, Bloom et al. (2018) and Basu and Bundick (2017)).

National Accounting. The equation for the expenditure approach to gross national product (GNP) implies

$$Y = C_p + I_p + CA + G \tag{18}$$

where Y is GNP, C_p is private consumption, I_p is gross domestic private investment, CA refers to the current account, and G is total government consumption expenditures, C_g, plus government gross investment, I_g. As a result, the following must hold:

$$\left(1 - \frac{CA}{Y}\right) - \frac{C}{Y} - \frac{I_p}{Y} = \frac{I_g}{Y} + \frac{C_g}{Y}. \tag{19}$$

The left-hand side of this equation measures private savings in excess of private investment needs relative to output. The right-hand side of this equation clarifies that explaining this excess of savings requires a non-private sector. We note also that the evidence that we discuss below applies whether we include or exclude the current account from the accounting identity presented above simply because net international flows represent a smaller share of national output.

In the left panel of figure 8, we show that periods of high uncertainty are usually associated with an increase in the private saving share and a decline in private investment. In a one-sector economy this gap must be identically zero and private investment must increase when savings increase. In the data, this gap is substantial and is bridged by the government. The right panel of figure 8 clarifies that most of the government expenditure is related to inputs of production, namely, wages and government investment.

Government capital: data and stylized facts. Government capital data are reported in the NIPA tables, according to criteria described in Bureau of Economic Analysis (2014). Examples of expenditures included in our government capital measure are provided in Appendix E, table E1. We include both tangible and intangible investment both at the federal and the

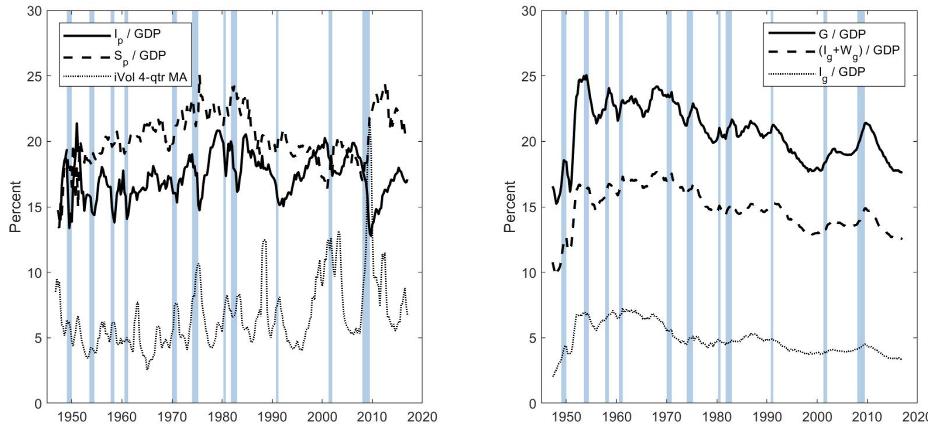


Fig. 8: Savings, Investment, and Government Expenditure

Notes: S_p and I_p refer to quarterly gross private savings (NIPA table 5.1) and private investment (NIPA table 1.1.5), respectively. G denotes the sum of government consumption expenditures and gross investment (NIPA table 1.1.5). Gross government investment and wages are denoted by I_g and W_g, respectively (NIPA table 3.9.5 and 3.10.5). All series are depicted as shares of GDP. The dotted line in the left panel refers to a four-quarter moving average of the equity market integrated volatility. Our sources are detailed in Appendix A.

local level. Our data are consistent with other sources explored by Aschauer (1988), Boskin et al. (1989), Peterson (1990), and Kamps (2004), and include both in-house investment and purchases from the private sectors.

From a business cycle point of view, government capital is important in at least three dimensions: (i) it is sizeable, as it is on average about one third of private capital; (ii) government investment is an important margin during the cycle as its growth rate is approximately 1.5 times more volatile than that of private investment; and (iii) the correlation of private and government investment growth is negative, implying that government capital is associated to important cross-sector reallocation during the cycle (see Appendix E, figure E1).More specifically, during periods of economic stress government investment becomes relatively more prominent than private investments.

In addition, we find it important to document that the R&D intensity of the government is moderate compared to that of many firms in our cross section (see table 9). As a result, aggregate data confirm at the sector-level what we have documented using firm-level microdata: there is a negative connection between R&D intensity and capital reallocation during periods of high uncertainty.[9]

[9] In Appendix E, we document additional related facts on government investments. We show that the reallocation patterns are much more pronounced for government investment than for government expenditure.

Table 9: Characteristics of Government Sector

Panel A: R&D-Intensity across Sectors

Firm-level quintiles (Compustat)				Govt
20%	40%	60%	80%	Avg
0.17	1.98	5.40	11.70	1.59
(0.02)	(0.10)	(0.33)	(0.75)	(0.04)

Panel B: Reallocation and Growth

Horizon (n):	5 year	7 year	10 year
I_g/I_{tot}	-0.15	-0.18*	-0.28**
	(0.12)	(0.11)	(0.14)
Adj R^2	0.19	0.21	0.04
Adj R^2 w/o	0.19	0.18	-0.17
Credit Tightness Control: Baa10y	Yes	Yes	Yes
Predicting Factors	Yes	Yes	Yes

Notes: In panel A, we sort Compustat firms with non-missing R&D expense according to their innovation intensity and form 5 groups. We report our average R&D-to-Assets ratio quintiles as well as the average value for the government sector. Panel B is obtained similarly to table 6. Our capital measure is government investment relative to total investment as measured in the NIPA tables ($I_{g,t}/I_{tot,t}$). The sample ranges from 1972 to 2016. Our sources are detailed in Appendix A.

In order to formally test whether private investments are substituted by government investments when volatility increases, we proceed with a VAR investigation that can inform us on the duration of these reallocations. In figure 9, we confirm that an adverse orthogonalized volatility shock produces a reallocation of aggregate resources away from the private sector and from R&D. Simultaneously, these resources are increased in the government sector. In the bottom panel of table 9, we also show that this reallocation is a leading indicator of slow growth. In table E3 (Appendix E), we show that total investment increases with respect to an adverse orthogonalized volatility shock, as in the model.

Given the similarities between these empirical responses and the ones from our benchmark model (see figure 5), in this section we think of the government as a productive entity that offers capital devoted to activities that are safer but not growth-enhancing. In what follows, we replace the label 'H-sector' (L-sector) with 'P-sector' (G-sector), i.e., we think of the aggregate private sector as innovation-intensive as opposed to the government sector.

The modified model (EGI). Since it is not common to model the government as a competitive sector, we modify our benchmark model by removing the first order conditions described

Equivalently, these dynamics are a distinct phenomenon when compared to non-investment government expenditure (see Appendix E, figure E2). We also present evidence of relevant reallocation of both capital flows and employment toward government capital in periods of high uncertainty (see Appendix E, table E2).

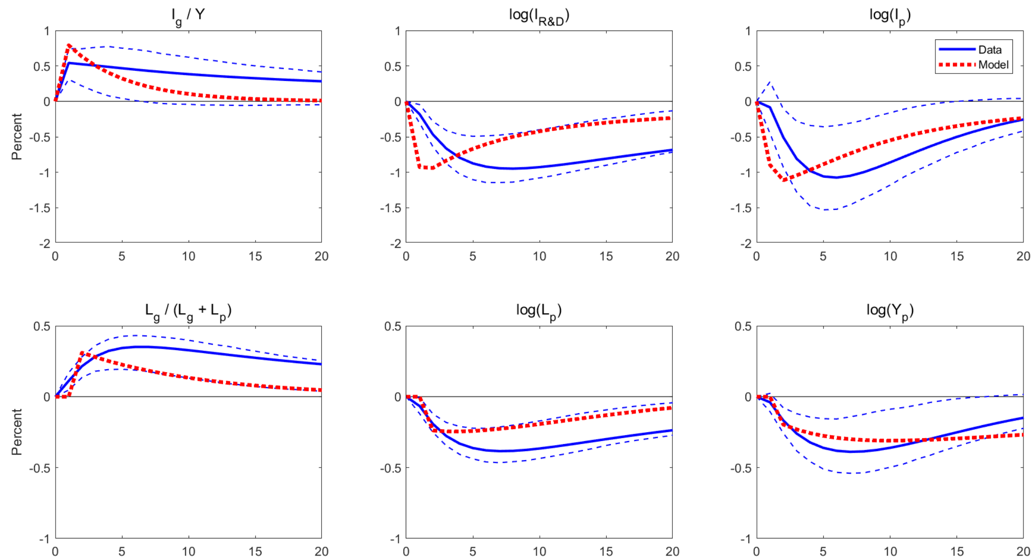


Fig. 9. **The Model with G-sector versus VAR.** This figure shows the response to volatility shocks to productivity of detrended R&D investment ($I_{R\&D}$), total private gross investment ($I_p = I_H + I_{R\&D}$), private output (Y_p), private labor (L_p), as well as the share of labor (L_g) and investment (I_g) of the government sector. The VAR-implied responses are obtained from the specification in equation (3) and (5). The model-implied responses are obtained from our quarterly specification with the government sector (EGI specification, equation (20)).

in equations (14)–(17) and we replace them with the following simple exogenous rules:

$$\begin{aligned}\frac{I_{g,t}}{Y_t} &= (1-\rho_{I_g})\frac{\bar{I}_g}{Y} + \rho_{I_g}\frac{I_{g,t-1}}{Y_{t-1}} + b_{i_g,a}\epsilon_{a,t} + b_{i_g,v}\epsilon_{v,t} \qquad (20)\\ \frac{L_{g,t}}{L_t} &= (1-\rho_{L_g})\frac{\bar{L}_g}{L} + \rho_{L_g}\frac{L_{g,t-1}}{L_{t-1}} + b_{l_g,a}\epsilon_{a,t} + b_{l_g,v}\epsilon_{v,t-1}.\end{aligned}$$

We refer to this configuration as exogenous government investment (EGI). For the sake of simplicity and parsimony, we choose to have univariate processes for both the government investment to output share and the labor share. These representations enable us to capture the main features of the path of inputs allocated to the government sector with a parsimonious calibration. We report our calibration and the aggregate moments from this model formulation in Appendix D.2 and table 10, respectively. Here we note that according to the NIPA tables, there exists a productivity gap between the private and the government sector of about 35%. This justifies our calibration for χ_L.

Turning our attention to the output of the model, we list several successes. First of all, the impulse responses to a volatility shock feature both a significant reallocation away from

Table 10: **Main Moments with Exog. Gov. Investment (EGI)**

Moment	Data		Benchmark	EGI
	Est.	St.Err.		
$\sigma(\Delta y)$ (%)	4.65	(0.89)	4.78	4.97
$\sigma(\Delta c)/\sigma(\Delta y)$	0.62	(0.07)	0.66	0.72
$\sigma(\Delta i_{tot})/\sigma(\Delta y)$	2.16	(0.13)	1.74	1.81
$\sigma(\Delta I_{R\&D})$ (%)	9.69	(2.00)	7.97	8.30
$E\left[(I_p + I_{R\&D})/Y\right]$ (%)	15.18	(0.77)	31.96	29.12
$\sigma((I_p + I_{R\&D})/Y)$ (%)	3.46	(0.67)	2.69	4.64
$\rho(\Delta c, \Delta \ln(I_p + I_{R\&D}))$	0.80	(0.05)	0.79	0.77
$\sigma(\Delta w_p L_p)$ (%)	6.96	(1.63)	4.09	4.00
$\sigma(\Delta L_p)$ (%)	2.75	(0.38)	3.08	3.32
$E\left[I_g/Y\right]$ (%)	5.31	(0.50)	6.94	5.10
$\sigma(I_g/Y)$ (%)	2.54	(0.99)	1.85	0.85
$E\left[\frac{K_g}{K_p+K_g}\right]$ (%)	24.85	(0.86)	36.90	33.21
$\sigma(\Delta w_g L_g)$ (%)	9.91	(3.33)	3.98	3.09
$\sigma(\Delta L_g)$ (%)	2.86	(0.75)	2.99	2.53
$\rho(\Delta L_p, \Delta L_g)$	0.39	(0.19)	0.95	0.70
$E\left[r_{p,ex}^{LEV}\right]$ (%)	5.57	(2.04)	5.21	7.86
$\sigma(r_{p,ex}^{LEV})$ (%)	19.64	(2.07)	15.11	15.17
$E\left[r_{g,ex}\right]$ (%)	1.20	(0.33)	-0.04	0.38
$\sigma(r_{g,ex})$ (%)	2.97	(0.24)	0.18	1.32
$E\left[\mathit{HML\text{-}R\&D}^{LEV}\right]$ (%)	5.43	(2.95)	2.88	3.31
$E\left[r^f\right]$ (%)	0.32	(0.64)	0.73	1.51
$\sigma(r^f)$ (%)	3.69	(0.66)	0.88	1.30
$\overline{\beta}$ for $\Delta z_{t\|t+40}/40$ using $I_{R\&D}/I_{Fixed}$	0.39	(0.15)	0.46	0.01
$\overline{\beta}$ for $\Delta z_{t\|t+40}/40$ using I_g/I_{tot}	-0.28	(0.14)	-0.02	-0.03

Notes: Empirical moments are computed using annual data from 1929 to 2016. All data sources are discussed in Appendix A. Numbers in parentheses are standard errors adjusted for heteroscedasticity. The entries for the model are obtained by averaging the results across simulated small samples. Our solution is pruned. Our baseline calibration is detailed in table 7. The calibration for the EGI specification is in Appendix D.2 . The coefficient $\bar{\beta}$ refers to the slope of a predictive regression featuring 10-year average productivity growth, $\Delta \ln Z_{H,t|t+40}/40$, at the left-hand side and either $I_{R\&D}/I_{Fixed}$ or I_g/I_{tot} as predictor. In this table, we use the labels H-sector and P-sector (L-sector and G-sector) interchangeably. The excess return of government capital is measured through the excess returns of government debt (Hall and Sargent 2011).

innovation capital and slow future growth (see figure 9). This model conforms very well with the cross section of returns that we observe in the data and it predicts the government capital is safer than other forms of capital. Within the model, when adverse volatility shocks materialize, the shadow value of government capital increases thus producing a positive capital gain in a high-marginal utility state. The insurance premium that government capital grants against volatility shocks generates a relative low risk-premium.

If the government has access to lump-sum taxation, the financing structure of the govern-

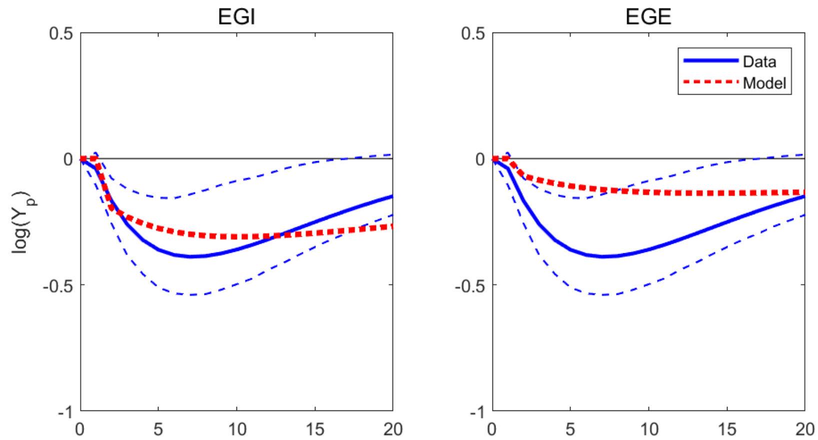


Fig. 10. **Government Capital vs Expenditure.** This figure shows the response to volatility shocks to productivity of detrended private output (Y_p). The left panel refers to a setting with Exogenous Government Investment (EGI). In the right panel, the model features Exogenous Government Expenditure (EGE) since $\delta_G = 0$. The VAR-implied responses are obtained from the specification in equations (3) and (5).

ment is not identified and we can find a market economy equilibrium in which the returns on government capital are equal to the returns on outstanding public debt and can be measured as in Hall and Sargent (2011). In this case, the model is consistent with the empirical observation that government liabilities, that is, US treasury bonds, appreciate in high-volatility states. We also note that this configuration of the model is qualitatively consistent with our predictability results on medium-term productivity growth.

Capital vs expenditure. Given our modified model, we look at the case in which $\delta_G =$ 100%, i.e., investment fully depreciates within a period. Similarly to Fernandez-Villaverde et al. (2011), this is a scenario in which the gap between private savings and private investments is bridged by productive exogenous government expenditure (EGE) and there is no government capital accumulation.

Absent the intertemporal margin associated to safer government capital, the representative agent finds it optimal to accumulate more capital in the private sector. As a result, the decline in the private sector is quantitatively limited compared to our EGI specification, as shown if figure 10. In our model with safer government capital, the decline of output lays within our empirical confidence intervals and it is actually as persistent as seen in our VAR estimated in levels. Equivalently, an EGI configuration is able to explain a severe contraction that goes beyond medium-cycle frequencies, in contrast to the EGE specification.

First-best. In Appendix D.3, we solve the first-best version of our model. In this setting, the planner overcomes the distortions related to both monopoly power (static distortion) and congestion (intertemporal distortion). Even at the first-best, a reallocation toward safer assets is optimal and it comes with a prolonged output decline. In contrast to our market economy equilibrium, the first-best equilibrium features a less pronounced reallocation. Due to the absence of risky monopolistic rents in the High-R&D sector, the High-R&D sector is safer than under the market equilibrium ($E\left[r_{H,ex}^{LEV}\right]$ declines) and the reallocation across sectors is relatively less beneficial in the aftermath of a volatility shock. On the other hand, we find it important to notice that the reallocation away from risky innovation-capital is welfare enhancing also from the planner's point of view.

Summary. Our reallocation results are robust to whether we think of the L-sector as a competitive one or a government-driven one. To the extent to which there exists a sector with (i) lower innovation intensity, and (ii) lower riskiness, high uncertainty states feature a relative contraction in innovation capital and hence growth.

5 Conclusions

We propose a novel capital market-based view of economic slowdowns associated to high-uncertainty periods. Specifically, focusing on U.S. micro data we show the existence of a significant positive link between uncertainty and investment reallocation away from risky innovation-oriented stocks. Furthermore, we confirm these dynamics in aggregate data. Our empirical tests suggest that this reallocation is a leading indicator of both slower innovation output and sluggish long-run output growth.

We rationalize these novel empirical findings in a production economy in which (a) the representative agent has an explicit fear toward uncertainty; and (b) there exists a well-defined cross section of assets featuring both risky innovation capital and safer capital that does not participate to the innovation activity. During periods of high uncertainty, innovation capital is perceived as extremely risky, as the present value of monopoly rents associated to patents is highly exposed to uncertainty shocks. With recursive preferences, there is a motive to reallocate resources away from risky activities and invest more in safer capital stocks. This reallocation generates a medium-run decline in growth broadly consistent with the data.

Future work should focus on the interplay between government investment and distortionary taxation. Furthermore, since government capital is related to uncertainty, it should be used to explain the cross-section of equity returns. It will be extremely useful to integrate our methodology with that in Aghion and Banerjee (2005).

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ON-LINE APPENDIX, NOT FOR PUBLICATION

Appendix A: Data Description

In what follows, we detail the sources of our data by grouping them in major groups.

A.1Aggregate Data

The national income and product accounts (NIPAs) are a set of economic accounts produced by the Bureau of Economic Analysis (BEA). See Bureau of Economic Analysis (2014) for more underlying details on the construction of the data series.

Government Investment. Data are from the NIPA table 3.1. The quarterly data are seasonally adjusted. Government gross investment consists of spending by both general government and government enterprises for fixed assets that benefit the public or that assist government agencies in their productive activities. Put another way, government gross investment is a measure of the additions to, and replacements of, the stock of government-owned fixed assets. It consists of investment by both general government and government enterprises in structures (such as highways and schools), in equipment (such as military hardware), and in intellectual property products (software and research and development), and it includes own-account investment by government. See Bureau of Economic Analysis (2014) for more details.

Real Government Investment. Data are from the NIPA table 3.9.1. Units are percent change from the previous period. The quarterly data are seasonally adjusted. Quarterly (annual) data in chained 2012 dollars are not available prior to 2002 (1967), so we use the avaiable percent change data series to construct the series of levels.

Government Expenditures and Investment. Data are from the NIPA table 1.1.5. The quarterly data are seasonally adjusted. Compared to government investment, this data series also includes government expenditures.

Private Investment. Fixed private investment data are from the NIPA table 1.1.5. The quarterly data are seasonally adjusted. See Bureau of Economic Analysis (2014) for more details.

Real Private Investment. Data are from the NIPA table 1.1.6. Units are billions of chained 2012 dollars. The quarterly data are seasonally adjusted.

Private Research and Development (R&D) Investment. Data are from the NIPA table 1.5.5. Units are billions of dollars. The quarterly data are seasonally adjusted.

Real Private Research and Development (R&D) Investment. The nominal series values are converted to real values using the implied deflator between the reported nominal private gross investment (NIPA table 1.1.5) and real private gross investment (NIPA table 1.1.6).

Personal Consumption Expenditures. Data are from the NIPA table 1.1.5. The quarterly data are seasonally adjusted.

Real Personal Consumption Expenditures. Data are from NIPA table 1.1.6. Units are billions of chained 2012 dollars. The quarterly data are seasonally adjusted.

Gross Domestic Product. Data are from the NIPA table 1.1.5. The quarterly data are seasonally adjusted.

Private Sector Output. Data are "Gross value added: GDP: Business" from the BEA. The quarterly data are seasonally adjusted.

Real Private Sector Output. Data are "Real gross value added: GDP: Business" from the BEA. Units are billions of chained 2012 dollars. The quarterly data are seasonally adjusted.

Gross Private Saving. Gross private saving data are from the NIPA table 5.1. The quarterly data are seasonally adjusted. See Bureau of Economic Analysis (2014) for more details.

Gross Government Wages. Compensation of general government employees data are from the NIPA table 3.10.5. The quarterly data are seasonally adjusted. See Bureau of Economic Analysis (2014) for more details.

Total Factor Productivity Growth. Business sector TFP data are from the Federal Reserve Bank of San Francisco.

Government and Private Capital. Capital stock data are from the NIPA table 5.10. We use the data series for fixed assets (structures, equipment, and intellectual property products) and thus our total capital stock ($K_g + K_g$) does not include inventories. Capital stocks are accumulated totals computed from gross investment, consumption of fixed capital, and other adjustments. See Bureau of Economic Analysis (2014) for more details.

Employment. Data are from the US Bureau of Labor Statistics (BLS). Private employment is measured as all the seasonally adjusted number of employees in all private industries. Government employment is measured as the seasonally adjusted number of employees across all levels of government.

Price Index and Inflation. We use the "All items in U.S. city average, all urban consumers, not seasonally adjusted" price index downloaded from the US Bureau of Labor Statistics (BLS) website. This price index is used both to deflate nominal data from Compustat and to compute inflation.

Integrated Volatility. We compute our quarterly integrated volatility measure as $\sqrt{66 \times \frac{1}{N} \sum_{i=1}^{N} \left(r_{m,i} - r_{f,i}\right)^2}$ where N is the number of daily observations in a given quarter and $r_{m,i} - r_{f,i}$ is the market excess return for a given day. Market excess return data were downloaded from the Kenneth R. French Data Library.

Price-Dividend Ratio. Price and dividend data are from Robert Shiller's website (`http://www.econ.yale.edu/~shiller/data.htm`). These monthly data series begin in 1871. We compute a quarterly price-dividend ratio data series by dividing the third month's price by the sum of dividends in each quarter. See the website for more details on the underlying data construction.

Treasury Zero-Coupon Yields. Data are from the Federal Reserve website (`http://www.federalreserve.gov/pubs/feds/2006/200628/200628abs.html`). We aggregate the raw daily data to a quarterly frequency by taking the average within each quarter. The resulting quarterly data series begins in 1961:Q2. See Gurkaynak et al. (2007) for details on the computation of the underlying daily data series.

Government Bond Returns. Data are from Ibbotson Associates. Returns are available for one-month Treasury bills (TBILL), intermediate-term bonds (ITGOVBD), and long-term government bonds (LTGOVBD).

Moody's Seasoned Baa Corporate Bond Yield Spreads. Data are from the Federal Reserve Bank of St. Louis.. The spreads are computed relative to the 10-year Treasury Constant Maturity. The quarterly data series are computed as the quarterly averages of the monthly series.

Economic Policy Uncertainty. Data are downloaded from the Economic Policy Uncertainty website (`http://www.policyuncertainty.com/us_historical.html`). We use the US Historical News-Based Policy Index. This index is constructed based on the results from keyword searches for terms related to economic and policy uncertainty in major US newspapers. See Baker et al. (2016) for more details.

Patent Counts. Data are from the US Patent and Trademark Office. Annual patent applications are for utility patents, U.S. origin.

Patent Market Values. Data are the sum of the annual firm-level values. See "Patent Market Values" within the Micro Data section for more details.

A.2 Micro Data

Compustat. Our full sample includes all firm-year observations with a non-missing value for total assets (AT) and keeps only firms incorporated in the USA (fic==“USA”). The annual data begin in 1950. Investment is the sum of capital expenditures (CAPX) and R&D expense (XRD). Nominal values are converted to real using our chosen CPI index. The CPI index is available at a monthly frequency and it is merged onto the Compustat data using the month of the end-of-period date (DATADATE). We also merge on monthly data from CRSP using DATADATE. We identify industry using the standard industry classification code (SIC). Variables used to compute measures of profitability are total revenue (REVT), and operating income before depreciation (OIBDP). Total debt used in measures of leverage is the sum of total long-term debt (DLTT) and debt in current liabilities (DLC). Book equity, which is used to compute book leverage, is defined as the sum of the book value of stockholders' equity (CEQ) and balance sheet deferred taxes (TXDITC) less the book value of preferred stock (PST).

CRSP. Market equity data are from the Center for Research in Security Prices (CRSP). Before merging with Compustat, we filter observations to keep ordinary common shares (SHRCD$\in (10, 11)$) that trade on the main US exchanges (EXCHCD$\in (1, 2, 3)$). We merge the CRSP data with our Compustat sample using the linked company code (LPERMNO) that connects the CRSP company code (PERMNO) with the Compustat company code (GVKEY). The CRSP/Compustat Merged linking table is provided by CRSP. Market capitalization in thousands is computed as the product of the absolute value of close price (PRC) and shares outstanding (SHROUT) divided by 10^6.

Patent Counts. Annual patent grants by Compustat GVKEY are downloaded from David Dorn's website (`https://www.ddorn.net/data.htm`). See Autor et al. (2020) for more details.

Patent Market Values. Annual total dollar value of innovation based on stock market in millions of USD by CRSP permanent security-level identification number (PERMNO) are downloaded from Amit Seru's website (`https://aseru.people.stanford.edu/data-and-discussions`). See Kogan et al. (2017) for more details.

Appendix B: Additional Empirical Results

In this section we report additional results to support our reallocation findings. Table B1 shows the composition of our top and bottom R&D intensity-sorted portfolios.

Table B1: R&D Intensity Portfolios

Panel A: Top 10 Industries in R&D-Sorted Portfolios

Low-R&D		High-R&D	
Category	% Count	Category	% Count
Eating Places	9.9	Biological Pds, Ex Diagnstics	12.3
Variety Stores	3.2	Prepackaged Software	11.0
Grocery Stores	3.2	Pharmaceutical Preparations	10.5
Crude Petroleum and Natural Gas	3.0	Semiconductor, Related Device	5.7
Women's Clothing Stores	2.8	Electromedical Apparatus	3.5
Misc Amusement and Rec Service	2.8	In Vitro, In Vivo Diagnostics	3.4
Department Stores	2.5	Cmp Integrated Sys Design	2.9
Family Clothing Stores	2.2	Computer Communications Equip	2.9
Misc Shopping Goods Stores	1.9	Tele and Telegraph Apparatus	2.8
Catalog and Mail-Order Houses	1.9	Computer Software and Services	2.7
Total	33.5	Total	57.8

Panel B: R&D-sorted Portfolios Summary Statistics

	High-R&D	Low-R&D
	Portfolio returns	
Mean	19.13	13.71
Standard deviation	32.60	21.67
Sample size (number of months)	540	540
	Portfolio characteristics	
Market capital share	11.94	11.15
R&D/Assets	16.89	0.04
Revenue/Assets	102.34	155.51
Book leverage	44.20	59.88
Average number of firms	439	445

Notes: Panel A shows the top-10 industries in our baseline high and low R&D-sorted portfolios. We count SIC codes across time and firms in each portfolio and report the most frequent industries within each portfolio. Panel B reports summary statistics for our portfolios. Returns are equal-weighted and presented in annualized percentages. The average market capital share, R&D/assets, sales/assets, and leverage are presented in percentages. R&D/assets is defined as annual research & development expenses divided by total assets and is used as our benchmark measure of R&D intensity. Sales/assets is defined as annual net sales divided by total assets. Leverage is expressed as a fraction of total assets.

Table B2: AR Regression Results for $iVol$ and z

	iVol	iVol	z	z
Lag iVol	0.669***	0.560***		
	(8.80)	(4.99)		
Lag 2 iVol		0.162**		
		(2.12)		
Lag z			0.985***	1.047***
			(115.69)	(8.53)
Lag 2 z				-0.063
				(-0.52)
R^2	0.447	0.461	0.970	0.970
Durbin-Watson	2.215	2.028	1.875	1.985

Notes: Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. Our quarterly data sample starts in 1972 and ends in 2016.

Table B2 features a comparison across an AR(1) and an AR(2) specification for both $iVol$ and z_t. The Durbin-Watson test statistic is close to 2 for both estimates, hence an AR(1) is a good approximation of the dynamics. Table B3 shows that our benchmark results are unchanged when we compute $e_{vol,t}$ using an AR(2).

Table B3: Excess Returns in R&D-sorted Portfolios (AR 2)

	$r^{ex}_{i,t} = \overline{r}^{ex}_i + \beta_{z,i} e_{z,t} + \beta_{vol,i} e_{vol,t} + \epsilon_{i,t}$		
	High-R&D	Low-R&D	HML-R&D
$\overline{r}^{ex}_i$	10.64***	3.59**	7.05***
	(4.35)	(2.10)	(2.99)
$\beta_{z,i}$	13.52***	5.26***	8.26***
	(2.41)	(1.53)	(2.28)
$\beta_{vol,i}$	-19.71***	-12.06***	-7.65***
	(3.07)	(1.33)	(2.46)
R^2	0.19	0.24	0.09

Notes: Our sample starts in 1972 and ends in 2016. Returns are annualized, multiplied by 100, equal-weighted, and unlevered. The High (Low) portfolio includes the top (bottom) 20% of R&D intensity-sorted firms and accounts for about 10% of total market capitalization. HML-R&D refers to a portfolio long in the High-R&D portfolio and short in the Low-R&D portfolio. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. We model $ivol$ and z as AR(1) and AR(2) processes, respectively, and denote their innovations as $e_{vol,t}$ and $e_{z,t}$, respectively. Standard errors in parentheses are Newey-West adjusted.

Table B4: Reallocation across R&D-sorted Firms (EPU)

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\frac{R\&D}{Assets}}_{i,t}\right) EPU_t + \beta_z z_t + cntrl_t^i \quad i = 1,...,N$$

Horiz. (years)		$\Delta Inv.(\%)$			$\Delta\frac{R\&D}{Assets}(p.p.)$		
		Balanced	$\geq 90\%T$	$\geq 80\%T$	Balanced	$\geq 90\%T$	$\geq 80\%T$
h=3	β_{rnd}	-0.41***	-0.72***	-0.90***	-0.18***	-0.24***	-0.31***
		(0.14)	(0.17)	(0.13)	(0.02)	(0.02)	(0.05)
	Wald	27.00	55.92	85.91	59.78	105.67	42.89
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-11	-13	-22	-75	-80	-79
h=4	β_{rnd}	-0.39***	-0.59***	-0.83***	-0.23***	-0.30***	-0.42***
		(0.14)	(0.14)	(0.13)	(0.03)	(0.03)	(0.06)
	Wald	34.76	57.80	80.26	60.06	116.50	53.04
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-12	-12	-22	-72	-79	-81
h=5	β_{rnd}	-0.42***	-0.53***	-0.71***	-0.27***	-0.36***	-0.50***
		(0.13)	(0.12)	(0.13)	(0.04)	(0.03)	(0.06)
	Wald	31.08	47.83	61.49	60.40	146.56	66.58
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-11	-9	-18	-71	-77	-78
		N=96	N=196	N=273	N=96	N=196	N=273

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using a cross section of firms sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms (EPU and $EPU \cdot R\&D/Assets$) from our benchmark specification. The price-dividend level is denoted as pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta EPU_t + z_t$. The firm-level R&D intensity average, $\overline{R\&D/Assets}_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

Table B4 shows that our firm-level reallocation results apply also when we use a broad measure of uncertainty such as the EPU measure by Baker et al. (2016).

In table B5, we show that our firm-level results are unchanged when we include a time fixed effect.

Table B5: Reallocation across R&D-sorted Firms (Fixed Effect)

		$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \alpha_t + \beta_{rnd}\overline{\frac{R\&D}{Assets}}_{i,t} iVol_t + cntrl_t^i \quad i = 1,...,N$					
Horiz.		$\Delta Inv.(\%)$			$\Delta\frac{R\&D}{Assets}(p.p.)$		
(years)		Balanced	$\geq 90\%T$	$\geq 80\%T$	Balanced	$\geq 90\%T$	$\geq 80\%T$
h=3	β_{rnd}	-3.14***	-4.81***	-5.69***	-1.25***	-1.44***	-1.88***
		(1.07)	(0.91)	(0.77)	(0.15)	(0.15)	(0.28)
	Wald	9.40	28.70	54.72	72.23	94.85	46.39
		[0.009]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-3	-5	-10	-56	-66	-69
h=4	β_{rnd}	-3.06***	-4.43***	-5.44***	-1.41***	-1.66***	-2.55***
		(0.93)	(0.82)	(0.76)	(0.18)	(0.18)	(0.39)
	Wald	16.60	36.47	59.53	75.86	100.30	46.69
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-4	-6	-12	-52	-63	-72
h=5	β_{rnd}	-3.06***	-4.20***	-4.94***	-1.60***	-1.94***	-3.01***
		(0.85)	(0.75)	(0.78)	(0.21)	(0.18)	(0.43)
	Wald	28.70	42.04	43.24	64.91	126.35	50.35
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-5	-6	-12	-50	-62	-69
		N=96	N=196	N=273	N=96	N=196	N=273

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using a cross section of firms sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The firm-level R&D intensity average, $R\&D/Assets_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate conditions by including a time fixed effect.

Table B6: Reallocation across SG&A-sorted Firms

$$\Delta[\cdot]_{i,t\rightarrow t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{SG\&A}{Assets}}_{i,t}\right) ivol_t + \beta_z z_t + cntrl_t^i \quad i = 1, ..., N$$

Horiz. (years)		$\Delta SG\&A(\%)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-0.10	-0.20***	-0.33***	-0.43***	-0.50***	-0.46***
		(0.08)	(0.06)	(0.05)	(0.05)	(0.04)	(0.04)
	Wald	15.38	47.35	75.01	127.06	193.77	249.77
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-37	-39	-40	-43	-42	-36
h=5	β_{rnd}	-0.06	-0.14***	-0.24***	-0.32***	-0.35***	-0.29***
		(0.07)	(0.05)	(0.04)	(0.04)	(0.04)	(0.03)
	Wald	12.30	34.53	58.09	96.72	138.49	182.48
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-24	-31	-34	-38	-38	-32
		$\Delta\frac{R\&D}{Assets}(p.p.)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-0.03***	-0.03***	-0.04***	-0.05***	-0.06***	-0.09***
		(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
	Wald	13.25	19.27	25.43	32.58	61.19	104.16
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-44	-51	-58	-71	-75	-73
h=5	β_{rnd}	-0.04***	-0.04***	-0.05***	-0.07***	-0.10***	-0.14***
		(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
	Wald	15.28	25.28	38.12	52.01	91.33	149.67
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-29	-43	-54	-69	-79	-74
		N=194	N=413	N=548	N=787	N=1233	N=1915

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative SG&A growth rates, $\Delta SG\&A$, are annualized. All estimates are obtained through GMM using a cross section of firms sorted on SG&A intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The firm-level R&D intensity average, $\overline{R\&D/Assets}_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

In table B6, we show that our main results are preserved when we use SG&A as opposed to only R&D.

Table B7: Reallocation across R&D-sorted Firms (Missing R&D included)

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{R\&D}{Assets}}_{i,t}\right) ivol_t + \beta_z z_t + cntrl_t^i \quad i = 1, ..., N$$

Horiz. (years)		$\Delta Inv.(\%)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-6.53***	-6.61***	-7.09***	-6.59***	-5.97***	-5.50***
		(0.61)	(0.48)	(0.45)	(0.42)	(0.38)	(0.31)
	Wald	97.11	151.69	194.94	212.79	211.40	284.80
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-76	-80	-86	-84	-87	-88
h=5	β_{rnd}	-5.06***	-5.00***	-5.48***	-4.94***	-4.31***	-3.93***
		(0.46)	(0.37)	(0.33)	(0.28)	(0.29)	(0.23)
	Wald	91.95	124.93	173.58	197.21	156.86	218.92
		[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-75	-69	-81	-78	-78	-79
		$\Delta\frac{R\&D}{Assets}(p.p.)$					
		Balanced	$\geq 90\%T$	$\geq 80\%T$	$\geq 70\%T$	$\geq 60\%T$	$\geq 50\%T$
h=3	β_{rnd}	-0.23***	-0.23***	-0.36***	-0.68***	-0.93***	-1.09***
		(0.04)	(0.04)	(0.09)	(0.19)	(0.18)	(0.15)
	Wald	15.66	18.01	10.69	6.37	12.95	27.71
		[0.000]	[0.000]	[0.000]	[0.002]	[0.000]	[0.000]
	R^2 Loss	-91	-97	-78	-77	-84	-87
h=5	β_{rnd}	-0.29***	-0.39***	-0.54***	-0.56***	-0.97***	-1.34***
		(0.05)	(0.13)	(0.16)	(0.21)	(0.19)	(0.15)
	Wald	17.10	7.05	5.73	4.53	14.22	37.87
		[0.000]	[0.001]	[0.003]	[0.011]	[0.000]	[0.000]
	R^2 Loss	-91	-98	-87	-88	-88	-90
		N=285	N=559	N=723	N=1025	N=1602	N=2410

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using a cross section of firms sorted on R&D intensity where missing values are considered to be zero. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as $ivol$ and pd. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The firm-level R&D intensity average, $\overline{R\&D/Assets}_{i,t}$, is computed over 3-year subsamples. We control for firm-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

In table B7, we show that our main results are preserved when we treat missing R&D data as zeros.

We aggregate the firms from our firm-level analysis to the Fama-French 49 industry level. In table B8, we show that our results are qualitatively, and often quantitatively, similar to

Table B8: Reallocation across R&D-sorted Industries

$$\Delta[\cdot]_{i,t\to t+h} = \alpha_i + \left(\beta_0 + \beta_{rnd}\overline{\tfrac{R\&D}{Assets}}_{i,t}\right) ivol_t + \beta_z z_t + cntrl_t^i \quad i = 1, ..., N$$

Horiz. (years)		$\Delta Inv.(\%)$		$\Delta \frac{R\&D}{Assets}(p.p.)$	
		Non-Missing R&D	Missing R&D = 0	Non-Missing R&D	Missing R&D = 0
h=3	β_{rnd}	-2.37	-1.71*	-1.23***	-0.67***
		(2.18)	(1.20)	(0.27)	(0.16)
	Wald	24.42	43.42	12.28	17.92
		[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-36	-67	-60	-49
h=5	β_{rnd}	-4.10**	-2.38***	-2.02***	-0.98***
		(1.77)	(0.93)	(0.47)	(0.25)
	Wald	13.32	24.36	10.32	12.50
		[0.000]	[0.000]	[0.000]	[0.000]
	R^2 Loss	-31	-38	-53	-42
		N=49	N=49	N=49	N=49

Notes: Our quarterly data sample starts in 1972 and ends in 2016. Cumulative investment growth rates, $\Delta Inv.$, are annualized. All estimates are obtained through GMM using a cross section of industries sorted on R&D intensity. Numbers in parentheses are Newey-West adjusted standard errors. We test $H_0 : \beta_{rnd} \geq 0$ and denote a significance level of 1%, 5%, and 10% with ***, **, *, respectively. We also test the join hypothesis $H_0 : \beta_0 = \beta_{rnd} = 0$ and report the implied Wald test (p-value in square brackets) as well as the associated reduction in R^2 from removing all volatility terms ($iVol$ and $iVol \cdot R\&D/Assets$) from our benchmark specification. Integrated return volatility and price-dividend level are denoted as *ivol* and *pd*. The variable z_t is the residual from the regression $pd_t = \alpha_{pd} + \beta ivol_t + z_t$. The industry-level R&D intensity average, $R\&D/Assets_{i,t}$, is computed over 3-year subsamples. Industries are defined as in Fama and French (1997) (N=49). We control for industry-level Tobin's Q and cash flow profitability, as well as for aggregate credit conditions as measured by the 10-year Baa credit spread ($cntrl_t^i$).

those obtained with firm-level data. We note that these results hold regardless of whether we include missing R&D as zeros or exclude them. We consider our new industry-level results as further validation of our original findings.

The VAR analysis in section 2.3 focuses on the sample period from 1972 to be consistent with our Compustat-based empirical evidence. However, it is possible to run this analysis from 1961 given the availability of the underlying data. In figures B1 and B2, we show the equivalent of figure 1 using either all available data or a post-1982 sample. Figure B1 and B2 differ from each other because the former is based on our iVol measure whereas the latter is obtained by our productivity-based measure of uncertainty. Both figures support our main results.

Figure B3 shows that the disruptive effect of productivity uncertainty shocks can be fully captured only when we look at medium and long-run cycles, i.e., at relatively low frequencies.

In figure B4, we show that our results remain unchanged if we estimate a VAR(2).

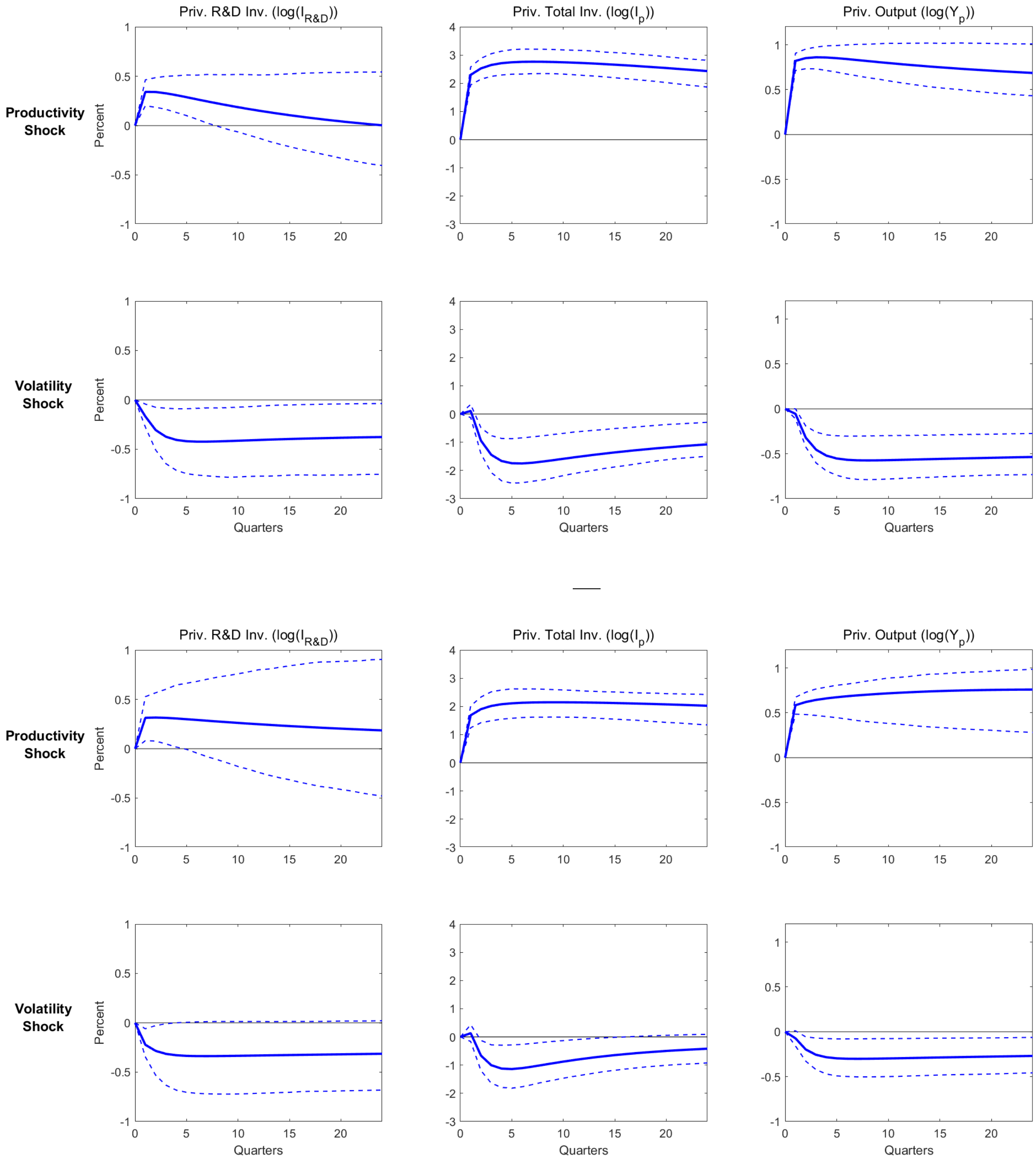


Fig. B1. **Aggregate Capital Reallocation in a VAR with iVol.** This figure shows the response to both productivity growth shocks and volatility shocks of total gross private investment (I_p); gross private R&D investment ($I_{R\&D}$), and private output (Y_p). All results are based on the VAR specified in equations (3)–(4), in which we use stock market integrated volatility to measure uncertainty. We control for the 10-year Baa credit spread. All series are in log-levels units. Our sources are detailed in Appendix A. In the top (bottom) portion of the figure, our quarterly sample starts in 1961 (1982) and ends in 2016. Confidence intervals are HAC-adjusted.

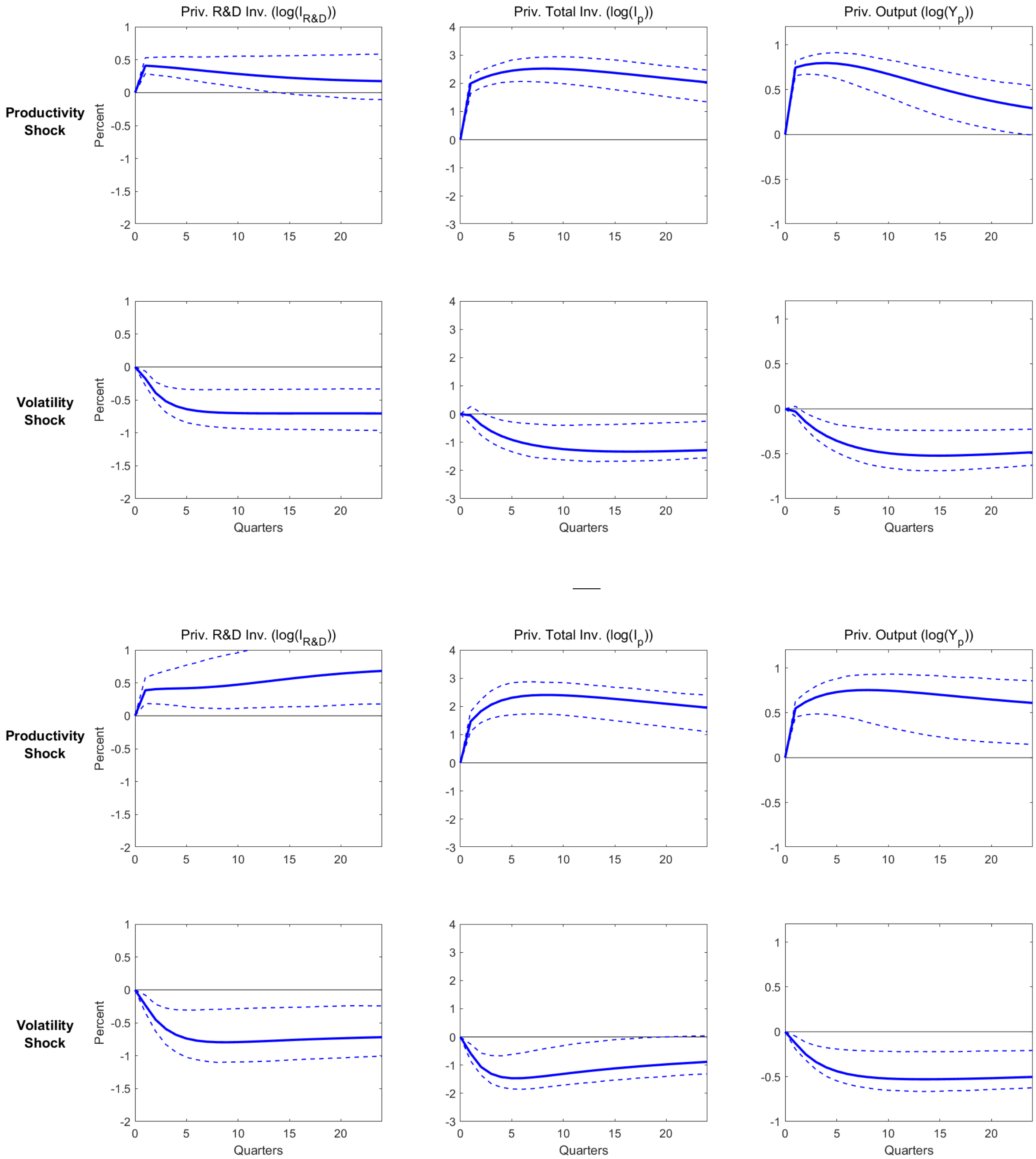


Fig. B2. **Capital Reallocation in a VAR with Productivity Uncertainty.** This figure shows the response to both productivity growth shocks and shocks to productivity volatility of total gross private investment (I_p); gross private R&D investment ($I_{R\&D}$), and private output (Y_p). All results are based on the VAR specified in equations (3) and (5), in which we use productivity volatility to measure uncertainty. We control for the 10-year baa credit spread. All series are in log units. Our sources are detailed in Appendix A. In the top (bottom) portion, our quarterly sample starts in 1961 (1982) and ends in 2016. Confidence intervals are HAC-adjusted.

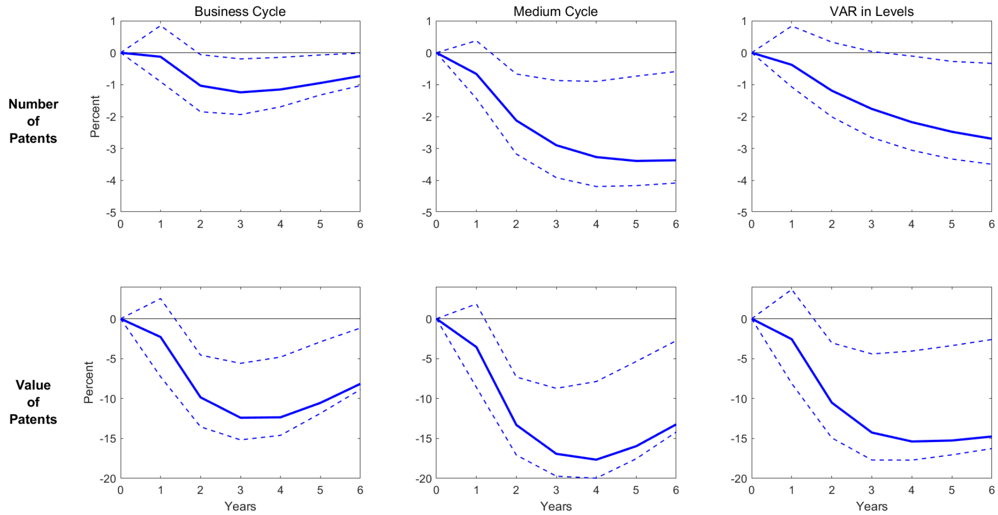


Fig. B3. **Innovation Output in a VAR with Productivity Uncertainty.** This figure shows the response to adverse shocks to productivity volatility of both aggregate number and value of patents. All results are based on the VAR specified in equations (3) and (5), in which we use productivity volatility to measure uncertainty. We control for the 10-year baa credit spread. Business (Medium) Cycle responses are obtained by HP (passband) filtering our data over the quarterly sample 1972:Q1–2016:Q4 (see Appendix A). Confidence intervals are HAC-adjusted.

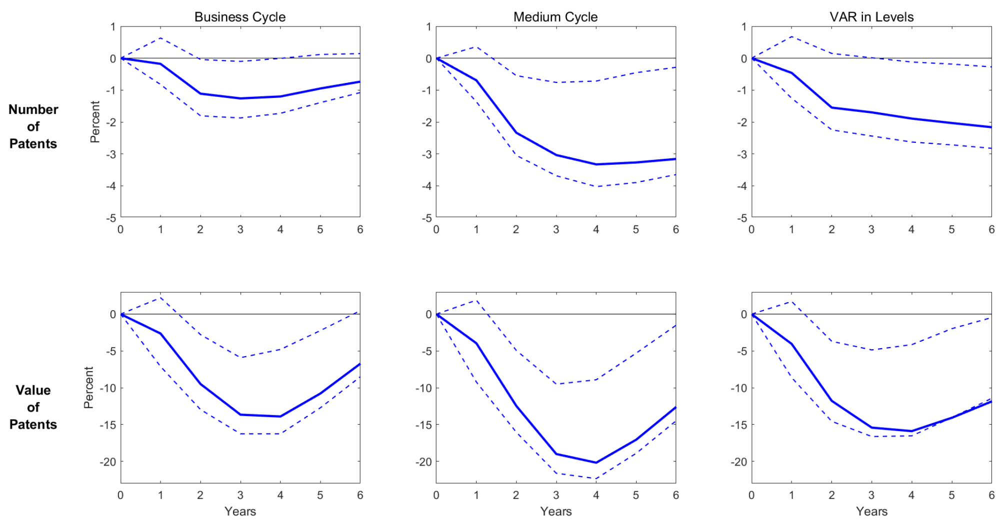


Fig. B4. **Innovation Outcomes in a VAR(2) with iVol.** This figure shows the response to productivity volatility shocks of number of patents and value of patents. All results are based on the VAR specified in equations (3)–(5), in which we use stock market integrated volatility to measure uncertainty, except that we include two lags of the endogenous variables instead of one. We control for the 10-year Baa credit spread. In the the first column, series are HP filtered and in log units. In the second column, series are band-pass filtered and in log units. In the third column, series are simply in log units including productivity. Our sources are detailed in Appendix A. Our annual sample for number of patents (value of patents) starts in 1963 (1961) and ends in 2016 (2010). Confidence intervals are HAC-adjusted.

Appendix C: Ex-ante Productivity Volatility

In the spirit of Bansal and Shaliastovich (2013), we form the following array of forecasting variables

$$F_t = [y_t(1),\ y_t(2),\ ...y_t(6),\ inf_t,\ pd_t,\ iVol_t], \tag{C1}$$

where $y(m)$ is the yield of a US Treasury bond with maturity m, inf denotes inflation, and pd refers to the price-dividend ratio. We extract expected volatility from productivity growth (Δa) by estimating the following equation

$$\Delta a_{t+1} = \mu + x_t + \exp^{\log(vol_t)} \epsilon_{a,t+1} \tag{C2}$$

$$x_t = b_x F_t \tag{C3}$$

where x_t captures the conditional mean of productivity and expected volatility is specified by the following projection:

$$\log vol_t = b_0^v + b_v F_t. \tag{C4}$$

We estimate these equations and report summary results for quarterly data in table C1. A standard Wald test rejects the null hypothesis that there is no predictability in productivity volatility. Our volatility measure is persistent and volatile.

We show our fitted volatility processes in figure C1 and make two remarks. First, our

Table C1: Productivity Uncertainty

Data	Persistence of Log-Vol. (ρ_v)	Volatility of Log-Vol. (σ_v)	Wald Test ($H_0 : b_v^i = 0\ \forall i$)
Quarterly	0.73	0.14	20.95
	(0.17)	(0.06)	[0.01]

Notes: This table reports results from estimating the system of equations (C2)–(C4) augmented with the following representation for log-volatility

$$\log(vol_t) = c_v + \rho_v \log(vol_{t-1}) + \sigma_v \epsilon_{v,t} + b_{v|sr}\epsilon_{a,t} + b_{v|lr}\epsilon_{x,t},$$

in which $\epsilon_{v,t}$ refers to a standardized volatility-specific shocks, as we control for both short-run productivity shocks ($\epsilon_{a,t}$) and growth news shocks ($\epsilon_{x,t}$). Growth news shocks are extracted by estimating $x_t = \rho_x x_{t-1} + \epsilon_{x,t}$. Numbers in parentheses are Newey-West adjusted standard errors. Numbers in square brackets are p-values for the null hypotheses that productivity volatility is constant ($H_0 : b_v^i = 0\ \forall i = 1, ..., 9$).

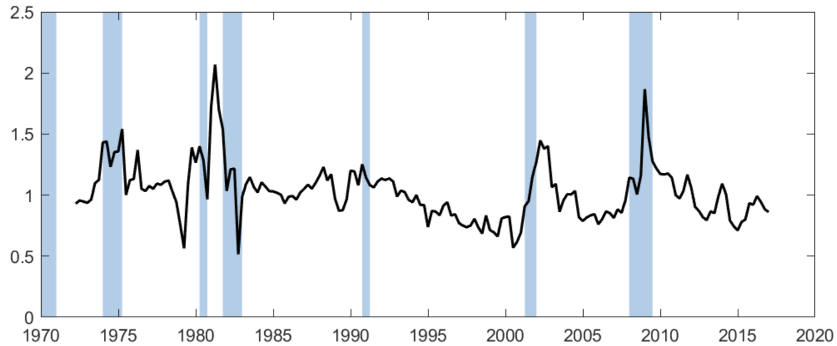


Fig. C1. **Productivity uncertainty**. This figure shows conditional volatility of productivity growth. We recover this measures by estimating the system of equation (C2)–(C4) by GMM. Our sources are detailed in Appendix A. Our sample starts in 1972 and ends in 2016.

estimates replicate the time-pattern documented in the literature for other macro quantities, as we capture both the post-1980 Great Moderation and the subsequent turbulence period. Second, productivity volatility is countercyclical. The contemporaneous exposure of vol shocks to short-run productivity shocks (in table C1, we denote it as $b_{v|sr}$) is estimated to be -3.5, with a standard error of 2.25.

Appendix D: Additional Model Results

D.1 Sensitivity Analysis for Main Model

Sensitivity analysis for our calibration. In table D1, we report sensitivity analysis with respect to key parameters for preferences and technology. We focus on the moments that change the most.

Table D1: Sensitivity Analysis

		Preferences				Intangible Capital Congestion					
		Risk Aversion		IES		Scale		Elasticity		Avg. Prod. Gap	
Moment	Benchmark	$\gamma = 10.8$	$\gamma = 13.2$	$\psi = 1.8$	$\psi = 2.2$	$\chi = 0.110$	$\chi = 0.130$	$\eta = 0.72$	$\eta = 0.88$	$\chi_L = 0.60$	$\chi_L = 0.70$
$\sigma(\Delta c)/\sigma(\Delta y)$	0.66	0.66	0.67	0.68	0.65	0.63	0.70	0.72	0.62	0.66	0.66
$\sigma((I_H + I_{R\&D})/Y)$ (%)	2.69	2.62	2.76	2.36	3.06	1.97	3.32	3.08	2.36	2.74	2.64
$\rho(\Delta c, \Delta \ln(I_H + I_{R\&D}))$	0.79	0.80	0.78	0.85	0.71	0.86	0.73	0.77	0.79	0.79	0.79
$\sigma(I_L/Y)$ (%)	1.85	1.80	1.90	1.68	2.04	1.97	1.83	1.55	2.30	1.81	1.90
$E\left[r^{LEV}_{H,ex}\right]$ (%)	5.21	4.94	5.42	4.89	5.55	4.27	6.72	7.65	4.41	5.23	5.19
$E\left[HML\text{-}R\&D^{LEV}\right]$ (%)	2.88	1.82	4.25	1.67	4.81	-0.98	12.26	9.67	-1.70	2.82	2.95
$E\left[r^f\right]$ (%)	0.73	0.85	0.61	0.81	0.66	0.01	1.38	1.69	-0.07	0.71	0.74
$\sigma(r^f)$ (%)	0.88	0.88	0.87	0.89	0.88	0.65	1.11	1.09	0.75	0.88	0.88

Notes: The entries for the model are obtained by repetitions of small samples. Our baseline calibration is detailed in table 7.

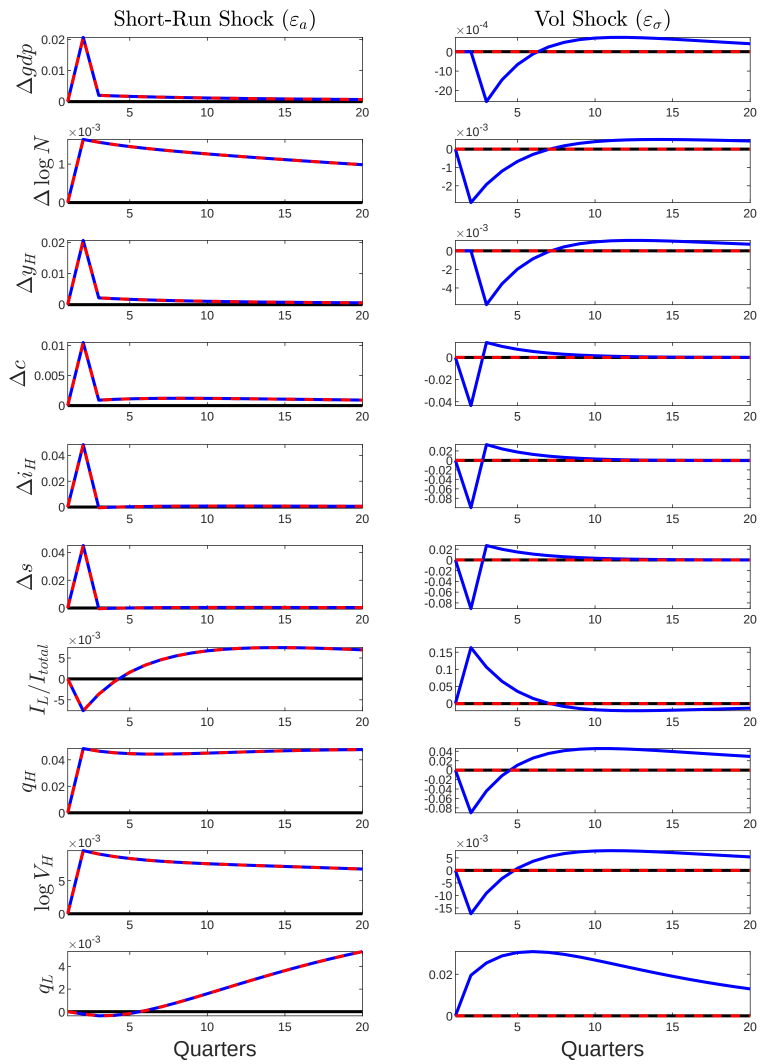


Fig. D1. **Impulse Responses.** This figure shows percentage deviations from steady state. Our benchmark calibration is reported in table 7. The dashed line refers to the model with no time-varying volatility ($\sigma_v = 0$).

Responses. In figure D1, we depict the response of variables of interest to both productivity level shocks and volatility shocks. We note several points. First, with respect to a positive level shock, our model behaves similarly to a standard production economy model, as private consumption, total labor, private investments, and output simultaneously expand.

On the asset pricing side, the higher level of productivity increases the value of both intangible (V_t) and tangible ($q_{H,t}$) capital in the H-sector. Since at the equilibrium there is a reallocation away from the H-sector (I_H/I_{tot} declines) for 5 quarters, the shadow value of the L-sector capital ($q_{L,t}$) declines as well.

In contrast to a positive level shock, a positive (i.e., adverse) volatility shock produces a contraction in economic activity and promotes a reallocation toward safer capital. Both consumption and investment fall and the value of both tangible and intangible capital decline. Because of aversion to volatility shocks, the representative agent finds it optimal to reallocate resources toward forms of capital that are less exposed to volatility. Since both the marginal product of tangible capital and the monopolistic rents generated through intangible capital are very exposed to volatility, the household reallocates resources toward capital in the L-sector causing it to appreciate.

Simulated moments and key model elements. In what follows, we discuss in more detail the role played by different elements in our model. We do so by removing one element at the time from our benchmark calibration and compare the most relevant changes in our simulated moments of interest. Since our goal is to highlight the marginal relevance of each element, we do not recalibrate the entire model and discuss only the subset of moments that change significantly across different settings in table D2 (see appendix D.1). For comparability, we adjust slightly the scale parameter for intangible capital congestion, χ, to maintain average growth unchanged.

D.2 Calibration for EGI

The system of equation (20) reported in main text is calibrated as follows:

$$\begin{aligned} \frac{I_{g,t}}{Y_t} &= (1-0.8)0.04 + 0.8\frac{I_{g,t-1}}{Y_{t-1}} - 0.04\epsilon_{a,t} + 0.04\epsilon_{v,t} \\ \frac{L_{g,t}}{L_t} &= (1-0.9)0.5 + 0.9\frac{L_{g,t-1}}{L_{t-1}} - 0.2\epsilon_{a,t} + 0.015\epsilon_{v,t-1}. \end{aligned} \tag{D5}$$

In order to be consistent with the data on the risk-free rate, we set the quarterly subjective discount factor to 0.9911. In order to match both the aggregate equity premium and the riskiness of R&D capital, we set $\eta = 0.76$ and increase the adjustment cost elasticity cost to 8.

D.3 First Best Analysis

The problem. The social planner maximizes the agent's utility:

$$U_t = \left[(1-\delta)\tilde{C}_t^{1-\frac{1}{\psi}} + \delta\Big(E_t\big[U_{t+1}^{1-\gamma}\big]\Big)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{D6}$$

Table D2: Key Elements in the Model

Moment	Data Est.	Data St.Err.	Model	Altern. Model
				No L-Sector ($\omega = 1$)
$E\left[(I_H + I_{R\&D})/Y\right]$ (%)	15.18	(0.77)	31.96	51.27
$E\left[r^{LEV}_{H,ex}\right]$ (%)	5.57	(2.04)	5.21	11.72
$E\left[\textit{HML-R\&D}^{LEV}\right]$ (%)	5.43	(2.95)	2.88	16.09
				No Vol ($\sigma_v = 0$)
$\sigma(\Delta y)$ (%)	4.65	(0.89)	4.78	4.48
$\sigma(\Delta I_{R\&D})$ (%)	9.69	(2.00)	7.97	7.35
$E\left[r^{LEV}_{H,ex}\right]$ (%)	5.57	(2.04)	5.21	5.00
				CRRA ($\gamma = 1/2$)
$\sigma(\Delta i_{tot})/\sigma(\Delta y)$	2.16	(0.13)	1.74	1.61
$\sigma(\Delta I_{R\&D})$ (%)	9.69	(2.00)	7.97	7.15
$E\left[r^{LEV}_{H,ex}\right]$ (%)	5.57	(2.04)	5.21	1.05
$E\left[\textit{HML-R\&D}^{LEV}\right]$ (%)	5.43	(2.95)	2.88	0.28
				CRRA ($\gamma = 12$)
$\sigma(\Delta y)$ (%)	4.65	(0.89)	4.78	3.86
$\sigma(\Delta c)/\sigma(\Delta y)$	0.62	(0.07)	0.66	1.08
$\sigma(\Delta i_{tot})/\sigma(\Delta y)$	2.16	(0.13)	1.74	0.82
$\sigma(\Delta I_{R\&D})$ (%)	9.69	(2.00)	7.97	3.73
$E\left[(I_H + I_{R\&D})/Y\right]$ (%)	15.18	(0.77)	31.96	18.86
$\sigma((I_H + I_{R\&D})/Y)$ (%)	3.46	(0.67)	2.69	0.32
$\rho(\Delta c, \Delta \ln(I_H + I_{R\&D}))$	0.80	(0.05)	0.79	0.99
$E\left[r^{LEV}_{H,ex}\right]$ (%)	5.57	(2.04)	5.21	2.31
$E\left[\textit{HML-R\&D}^{LEV}\right]$ (%)	5.43	(2.95)	2.88	-0.22
$E\left[r^f\right]$ (%)	0.32	(0.64)	0.73	11.38

Notes: Empirical moments are computed using annual data from 1929 to 2014. All data sources are discussed in Appendix A. Numbers in parentheses are standard errors adjusted for heteroscedasticity. The entries for the model are obtained by repetitions of small samples. Our baseline calibration is detailed in table 7.

where consumption bundle $\tilde{C}_t$

$$\tilde{C}_t = C_t - \bar{\omega}_{l,H} SL_t \frac{(L_{H,t} + \bar{\omega}_{l,L} L_{L,t})^{\omega_l}}{\omega_l}, \qquad \text{(D7)}$$

subject to:

- The resource constraint:

$$Y_t = \left[\omega_t \tilde{Y}_{H,t}^{1-\frac{1}{\tau}} + (1-\omega_t) Y_{L,t}^{1-\frac{1}{\tau}}\right]^{\frac{1}{1-1/\tau}} \geq C_t + I_{H,t} + I_{L,t} + S_t. \qquad (\lambda_{c,t}) \qquad \text{(D8)}$$

where $\tilde{Y}_{H,t} = Y_{H,t} - N_t X_t$

- The H-sector production function:

$$Y_{H,t} = (K_{H,t}^{\alpha}(\Omega_{H,t}L_{H,t})^{1-\alpha})^{1-\xi}\left[\left[\int_0^{N_t} X_{i,t}^{\nu}\, di\right]^{\frac{1}{\nu}}\right]^{\xi}, \tag{D9}$$

- The L-sector output:

$$Y_{L,t} = \chi_L Z_{H,t} K_{L,t}^{\alpha} L_{L,t}^{1-\alpha} \tag{D10}$$

- The law of motion of the intangible capital stock N_t, to innovation as follows:

$$N_{t+1} \leq \vartheta_t S_t + (1-\phi)N_t, \qquad (q_{n,t}) \tag{D11}$$

where $\vartheta_t = \chi\left(\frac{N_t}{S_t}\right)^{1-\eta}$.

- The law of motion of private capital:

$$K_{H,t+1} \leq \left(1-\delta+\Gamma_{H,t}\left(\frac{I_{H,t}}{K_{H,t}}\right)\right)K_{H,t} \qquad (q_{H,t}) \tag{D12}$$

- The law of motion of capital in the L-sector:

$$K_{L,t+1} \leq \left(1-\delta+\Gamma_{L,t}\left(\frac{I_{L,t}}{K_{L,t}}\right)\right)K_{L,t} \qquad (q_{L,t}),$$

Optimality. The optimal condition w.r.t C_t:

$$U_t^C = \lambda_{c,t} \tag{D13}$$

The optimal condition w.r.t $I_{H,t}$:

$$\Gamma'_{H,t} q_{H,t} = \lambda_{c,t} \tag{D14}$$

The optimal condition w.r.t. $K_{H,t+1}$:

$$q_{H,t} = E\left[U_t^U \frac{\partial U_{t+1}}{\partial K_{H,t+1}}\right] \tag{D15}$$

The envelope condition w.r.t. $K_{H,t}$ is:

$$\frac{\partial U_t}{\partial K_{H,t}} = \lambda_{c,t}\frac{\partial Y_t}{\partial Y_{H,t}}\frac{\partial Y_{H,t}}{\partial K_{H,t}} + \left(1-\delta+\Gamma_{H,t}-\frac{I_{H,t}}{K_{H,t}}\Gamma'_{H,t}\right)q_{H,t} \tag{D16}$$

The optimal condition w.r.t. $I_{L,t}$:

$$\Gamma'_{L,t} q_{L,t} = \lambda_{c,t} \tag{D17}$$

The optimal condition w.r.t. $K_{L,t+1}$:

$$q_{L,t} = E\left[U_t^U \frac{\partial U_{t+1}}{\partial K_{L,t+1}}\right] \tag{D18}$$

The envelope condition w.r.t. $K_{L,t}$ is:

$$\frac{\partial U_t}{\partial K_{L,t}} = \lambda_{c,t} \frac{\partial Y_t}{\partial Y_{L,t}} \frac{\partial Y_{L,t}}{\partial K_{L,t}} + \left(1 - \delta + \Gamma_{L,t} - \frac{I_{L,t}}{K_{L,t}} \Gamma'_{L,t}\right) q_{L,t} \tag{D19}$$

The optimal condition w.r.t. S_t:

$$\vartheta_t + \frac{\partial \vartheta_t}{\partial S_t} S_t = \frac{\lambda_{c,t}}{q_{n,t}} \tag{D20}$$

The optimal condition w.r.t. N_{t+1}:

$$q_{n,t} = E\left[U_t^U \frac{\partial U_{t+1}}{\partial N_{t+1}}\right] \tag{D21}$$

The envelope condition w.r.t. N_t is:

$$\frac{\partial U_t}{\partial N_t} = \lambda_{c,t} \left[\frac{\partial Y_t}{\partial Y_{H,t}} \frac{\partial Y_{H,t}}{\partial N_t} + \frac{\partial Y_t}{\partial Y_{L,t}} \frac{\partial Y_{L,t}}{\partial N_t} - \frac{\partial Y_t}{\partial Y_{H,t}} \frac{\partial N_t X_t}{N_t}\right] + \left(\frac{\partial \vartheta_t}{\partial N_t} S_t + (1 - \phi)\right) q_{n,t} \tag{D22}$$

The optimal condition w.r.t. $X_{i,t}$:

$$\frac{\partial Y_{H,t}}{\partial X_{i,t}} = 1 \tag{D23}$$

The optimal condition w.r.t. $L_{H,t}$:

$$\frac{\partial Y_t}{\partial Y_{H,t}} \frac{\partial Y_{H,t}}{\partial L_{H,t}} = \bar{\omega}_{l,H} SL_t \left(L_{H,t} + \bar{\omega}_{l,L} L_{L,t}\right)^{\omega_l - 1} \tag{D24}$$

The optimal condition w.r.t. $L_{L,t}$:

$$\frac{\partial Y_t}{\partial Y_{L,t}} \frac{\partial Y_{L,t}}{\partial L_{L,t}} = \bar{\omega}_{l,H} \bar{\omega}_{l,L} SL_t \left(L_{H,t} + \bar{\omega}_{l,L} L_{L,t}\right)^{\omega_l - 1}. \tag{D25}$$

Additional derivations. Note that:

$$\frac{\partial Y_t}{\partial Y_{H,t}} = \omega_t \left(\frac{Y_t}{Y_{H,t}}\right)^{\frac{1}{\tau}} = P_{H,t} \tag{D26}$$

$$\frac{\partial Y_t}{\partial Y_{L,t}} = (1-\omega_t)\left(\frac{Y_t}{Y_{L,t}}\right)^{\frac{1}{\tau}} = P_{L,t} \tag{D27}$$

$$X_t = \left(\frac{\xi}{\mu_p}\left(K_{H,t}^{\alpha}(\Omega_{H,t}L_{H,t})^{1-\alpha}\right)^{1-\xi} N_t^{\frac{\xi}{\nu}-1}\right)^{\frac{1}{1-\xi}} = \overline{A}^{\frac{1}{\xi}} K_{H,t}^{\alpha}(\Omega_{H,t}L_{H,t})^{1-\alpha} N_t^{-\alpha} \tag{D28}$$

$$\frac{\partial N_t X_t}{N_t} = (1-\alpha)\overline{A}^{\frac{1}{\xi}} K_{H,t}^{\alpha}(\Omega_{H,t}L_{H,t})^{1-\alpha} N_t^{-\alpha} = (1-\alpha)\frac{Y_{H,t}}{N_t}\overline{A}^{\frac{1}{\xi}-1} \tag{D29}$$

We can show that under the parameteric restriction $\alpha + \frac{\frac{\xi}{\nu}-\xi}{1-\xi} = 1$, the production function of private good sector can be written as:

$$Y_{H,t} = Z_{H,t} K_{H,t}^{\alpha} L_{H,t}^{1-\alpha} \tag{D30}$$

where

$$Z_{H,t} \equiv \overline{A}(\Omega_{H,t}N_t)^{1-\alpha}, \quad \overline{A} \equiv \left(\frac{\xi}{\mu_p}\right)^{\frac{\xi}{(1-\xi)}}, \tag{D31}$$

Hence we have:

$$\frac{\partial Y_{H,t}}{\partial N_t} = (1-\frac{\xi}{\nu})\frac{Y_{H,t}}{N_t}, \tag{D32}$$

and

$$\frac{\partial Y_{L,t}}{\partial N_t} = (1-\alpha)\frac{Y_{L,t}}{N_t}, \tag{D33}$$

and

$$\frac{\partial \vartheta_t}{\partial N_t} S_t = \vartheta_t(1-\eta)\frac{S_t}{N_t}. \tag{D34}$$

Calibration challenges. It is very well known that it is difficult to grant the existence of a deterministic steady state at the first-best when starting from the same configuration of the market economy. Equivalently, the set of parameters for which both the first-best and the market economy deterministic steady states exist is often empty. This problem is related to the fix-point that must be solved in order to find the endogenous steady state growth in the economy and that often has no bounded solution if the model is calibrated so that the second-best matches the data. Note that we are referring to the pure existence of a deterministic steady state: adding considerations about dynamic stability (Blanchard-Khan conditions) would make the set of suitable calibrations empty more frequently.

Given these considerations, it should not be surprising that under our benchmark calibration the first-best deterministic steady state is not bounded and a proper welfare comparison cannot be made. A possible way to interpret this result is that at the first best there is an infinite incentive to save-and-invest in order to take advantage of an infinite growth rate. As a result, a government intervention that removes both monopoly power and congestion externalizes can produce infinite welfare benefits even in a deterministic economy (deterministic steady state).

Given these considerations, we run our welfare analysis using a different calibration strategy and ask the following question: given a calibration for which the first-best steady state values of consumption and investment aggregates are acceptable, would our documented reallocation be optimal also at the first-best? If so, would it be more or less pronounced than under the second-best?

In order to address these questions, we look for minimal changes to our benchmark calibration that grant a dynamically stable first-best equilibrium. Specifically, we decrease the parameter that determines the average productivity of R&D investments ($\chi = 0.05$), and increase the labor supply elasticity ($\omega_l = 9$) to make sure that the Blanchard-Khan conditions hold. In what follows, we compare the dynamics under this first-best with those obtained under the market economy. For the sake of comparability, this market-based equilibrium differs from that in the main body of our manuscript because we set $\omega_l = 9$ instead of targeting the value of 1.5.

Results. We report a comparison of key simulated moments in table D3. Some moments are comparable across equilibria, others are inherently different. We also compare impulse responses to volatility shocks in figures D2 and D3.

We observe two relevant results. First, even at the first-best, a reallocation toward safer assets is optimal and it comes with a prolonged output decline. Second, at the first-best, the reallocation is less pronounced than under the market economy equilibrium.

One of the reasons for which the reallocation is less pronounced under the first-best is the absence of risky monopolistic rents in the High-R&D sector. Hence the High-R&D sector is safer than under the market economy equilibrium ($E\left[r_{H,ex}^{LEV}\right]$ declines) and the reallocation across sectors is relatively less beneficial. On the other hand, we find it important to notice that the reallocation away from risky innovation-capital is welfare enhancing also from the planner's point of view.

Table D3: Main Moments

	Benchmark $\omega_l = 9$	First Best
$\sigma(\Delta c)/\sigma(\Delta y)$	0.63	0.32
$\sigma(\Delta i_{tot})/\sigma(\Delta y)$	1.90	1.47
$\sigma(\Delta I_{R\&D})$ (%)	3.62	3.37
$E\left[(I_H + I_{R\&D})/Y\right]$ (%)	29.19	65.92
$\sigma((I_H + I_{R\&D})/Y)$ (%)	1.18	1.51
$\rho(\Delta c, \Delta \ln(I_H + I_{R\&D}))$	0.87	-0.19
$E\left[I_L/Y\right]$ (%)	4.56	3.03
$\sigma(I_L/Y)$ (%)	0.78	0.53
$E\left[\frac{K_L}{K_H+K_L}\right]$ (%)	29.90	19.56
$E\left[r^{LEV}_{H,ex}\right]$ (%)	2.40	1.14
$\sigma(r^{LEV}_{H,ex})$ (%)	13.58	13.33
$\sigma(r_{L,ex})$ (%)	0.08	0.04
$\sigma(r^f)$ (%)	0.39	0.43
$\overline{\beta}$ for $\Delta z_{t\|t+40}/40$ using $I_{R\&D}/I_{Fixed}$	0.88	0.57
$\overline{\beta}$ for $\Delta z_{t\|t+40}/40$ using I_g/I_{tot}	-0.19	-0.26

Notes: The entries for the model are obtained by repetitions of samples. The column Benchmark refers to the two-sector market economy (second-best). Our baseline calibration is detailed in table 7. We depart from our benchmark calibration by setting $\omega_l = 9$ for both configurations, and $\chi = 0.05$ for the first-best.

Appendix E: Government Capital Data

Table E1 reports some of the BEA components and categories regarding government investment, and figure E1 depicts the time series of both government investment and capital to their private counterparts.

In figure figure E2(a), we show that there is something unique about government investment that goes above and beyond the countercyclical behavior of total government expenditure. During recession periods, government expenditure increases relative to total private expenditure (i.e., gross private investment plus consumption) mainly through the public investment channel.

This dynamic behavior has been even more pronounced during the Great Recession, with almost no sign of reversal three years after the beginning of the recession (figure E2(b)).

Given this observation, in table E2 we show reallocation effects across the government and the private sector. We note that periods of elevated uncertainty are associated to a reallocation of both capital and labor from the private to the government sector. Aggregate

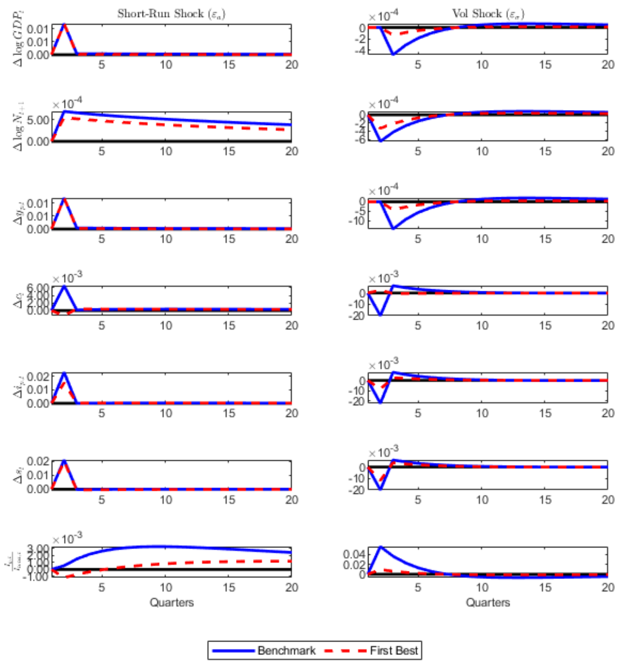


Fig. D2. **Benchmark vs First Best: Aggregate Dynamics.** This figure shows responses under both the first- and second-best.

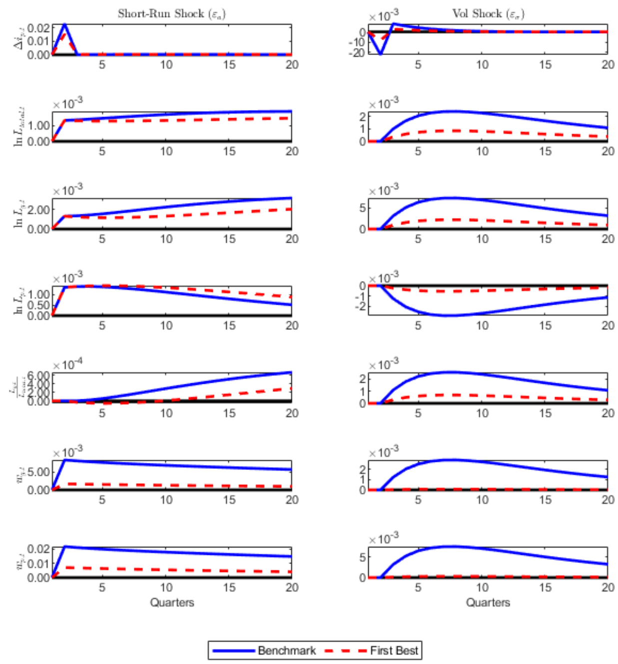
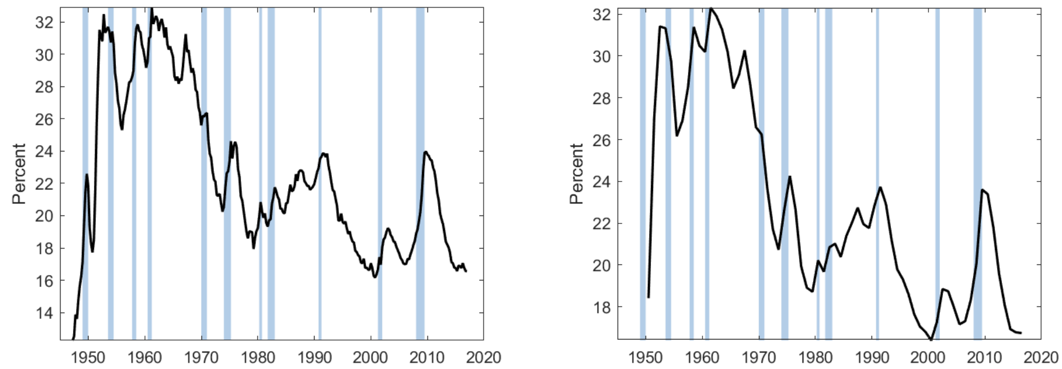


Fig. D3. **Benchmark vs First Best: Labor and Wages.** This figure shows responses under both the first- and second-best.

Table E1: Components of Government Gross Investment

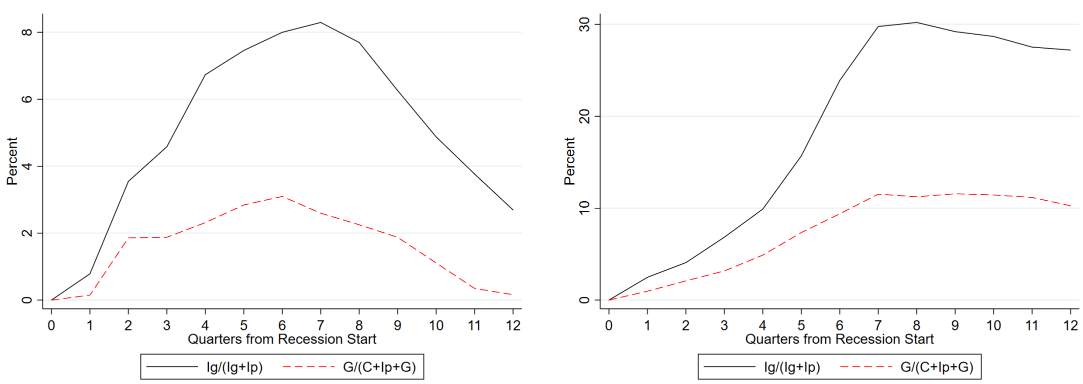
Component	Categories
Structures	Buildings (residential, industrial, educational, hospital, and other)
	Highways and streets
	Sewer systems
	Water systems
	...
Equipment	Vehicles
	Electronics
	...
Intellectual property products	Software
	R&D

Notes: Component breakdown as seen in NIPA table 3.9.5. Examples are from Bureau of Economic Analysis (2014).



(a) Relative Gross Investment $\left(\frac{I_g}{I_g+I_p}\right)$ (b) Relative Capital Stock $\left(\frac{K_g}{K_g+K_p}\right)$

Fig. E1. **Government Capital and Economic Fluctuations.** The left panel shows quarterly gross government investment (I_g) as a share of total domestic investment ($I_g + I_p$), which also includes private gross investment (I_p). The right panel shows the annual stock of government capital (K_g) as a share of the total domestic stock of capital ($K_g + K_p$), which also includes the private capital stock (K_p). Our data sources are detailed in Appendix A. For examples of government investment see table E1.



(a) Average post-1950 (b) Great Recession

Fig. E2. **Reallocation During Recessions.** In the left panel, we report the average path of the variables of interest across the latest 10 NBER recessions starting from 1950. Time $t = 1$ is the first quarter of the recession. The right panel focuses on the Great Recession only (2007:Q4–2009:Q2). Total federal expenditure is denoted by G. Total private expenditure is the sum of private consumption (C) and gross private investment (I_p). The subcomponet of government expenditure associated to gross government investment is denoted as I_g. Our data sources are detailed in Appendix A.

Table E2: Reallocation across Priv. and Gov. Sectors

Variable	Private	Government	PMG
	Full Sample (T=540)		
$\Delta Investment(\%)$	2.3	1.4	0.9
$\Delta R\&D/Assets(p.p.)$	0.05	0.08	-0.02
$\Delta Empl.(\%)$	1.6	1.2	0.5
	Top-20% iVol Periods (T=114)		
$\Delta Investment(\%)$	-1.2	1.1	-2.3
$\Delta R\&D/Assets(p.p.)$	0.04	0.08	-0.04
$\Delta Empl.(\%)$	-0.1	0.9	-1.0

Notes: Our sample starts in 1972 and ends in 2016. $\Delta Investment$ and $\Delta Empl$ denote the forward-looking real growth rate of total investment and number of employees, respectively (source: BEA and BLS). $\Delta R\&D/Assets(p.p.)$ refers to the forward-looking change in R&D expense divided by assets over the same time-horizon in percentage points (source: BEA). The panel 'Top-20% iVol Periods' refers to months (T) in which integrated US equity returns volatility has been in its historical top-20^{th} percentile.

data confirm that uncertainty reduces the R&D investment intensity in the private sector, whereas the government one remains unchanged. In table E3, we show that total investment increases with uncertainty shocks, as opposed to private investment.

Table E3: Contemporaneous Response to a 1-Std. Dev. Shock

Variable	Medium Cycle	Levels
	Panel A: Positive TFP Shock	
Total Investment ($\log(I_p + I_g)$)	1.49	1.61
	$[1.21, 1.71]$	$[1.32, 1.85]$
Private Investment ($\log(I_p)$)	1.74	1.89
	$[1.37, 2.04]$	$[1.51, 2.22]$
Government Investment ($\log(I_g)$)	0.65	0.63
	$[0.38, 0.90]$	$[0.37, 0.88]$
	Panel B: Adverse Volatility Shock	
Total Investment ($\log(I_p + I_g)$)	0.05	0.07
	$[-0.23, 0.33]$	$[-0.24, 0.37]$
Private Investment ($\log(I_p)$)	-0.08	-0.06
	$[-0.45, 0.27]$	$[-0.45, 0.32]$
Government Investment ($\log(I_g)$)	0.49	0.48
	$[0.24, 0.70]$	$[0.24, 0.71]$

Notes: This table shows the point estimate and confidence interval for the contemporaneous response of total, private, and government investment to an uncertainty shock. These estimates are obtained from our 4-variable VAR with productivity volatility as our uncertainty measure (equations (3) and (5)). The column Medium Cycle uses band-pass filtered data.