Fiscal and Monetary Policy Interactions in a Model with Low Interest Rates*

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Abstract

We provide a dynamic new Keynesian model in which entrepreneurs face uninsurable idiosyncratic investment risk and credit constraints. Government bonds provide liquidity service and raise net worth. Multiple steady states with positive values of public debt can be supported for a given permanent deficit-to-output ratio. The steady-state interest rates are less than economic growth and public debt contains a bubble component. We analyze the determinacy regions of policy parameter space and find that a large set of monetary and fiscal policy parameters in either regime M or regime F can achieve debt and inflation stability given persistent fiscal deficits.

Keywords: Fiscal and monetary policy, determinacy, interest rate, new Keynesian model, public debt, bubble

JEL Classifications: E12, E31, E32, E43, E44, E52, E62

*To be added later.

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1 Introduction

Our paper is motivated by two empirical observations as shown in Figure 1.[1] First, since 1980, nominal interest rates on US government bonds have steadily declined. They are lower than the US nominal GDP growth rates on average over 1950-2018 and also in each of the recent 10 years. According to current forecasts of GDP growth, this is expected to remain the case for the foreseeable future. Second, the US government has experienced fiscal deficits for many years, especially since early 2000s. The average of primary-deficits-to-GDP ratio over 1950-2019 is 0.28%. Moreover, public debt has risen since mid 1970. While it dropped in late 1990s, it started to rise again since 2000, reaching a peak of 70% of GDP in 2019. Similar patterns of declining safe rates and rising public debt for many other countries are documented by Rachel and Smith (2015) and Reinhart and Rogoff (2010).

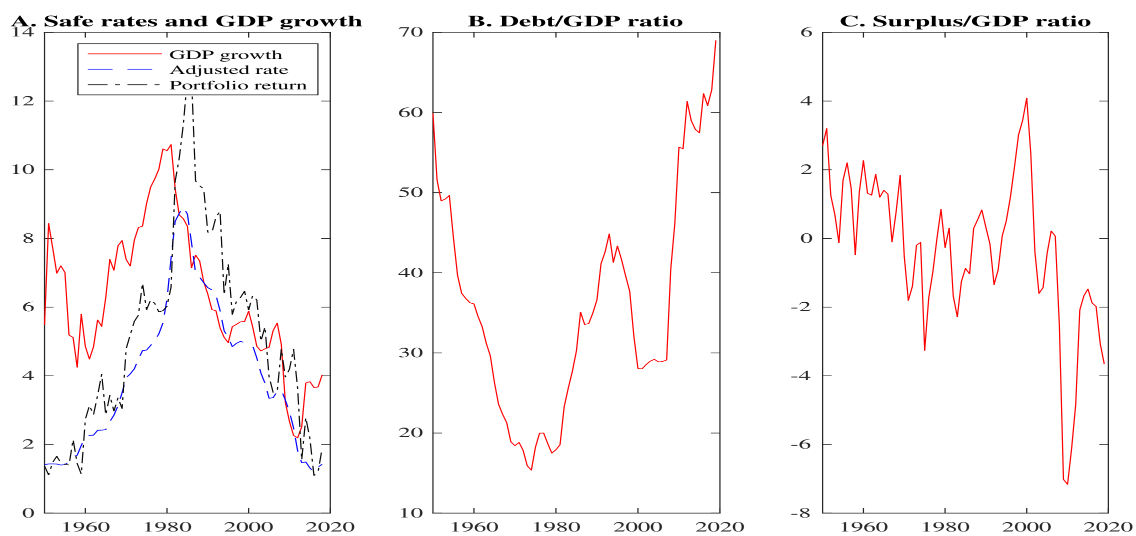

Figure 1: Nominal Safe rates and GDP growth, public-debt/GDP ratios, and primary-surpluses/GDP ratios. All vertical axes are in percentage.

Low interest rates and high public debt pose serious challenges to policy makers and academic researchers. In this paper we address the following positive questions: What are the implications of low interest rates for public debt policy? Can permanent primary deficits be sustained in the long run? What coordination of monetary and fiscal policy is needed to provide a nominal anchor and price stability? How does the economy respond to fiscal and monetary policy shocks?

To address these questions, we build a dynamic new Keynesian (DNK) model with financial frictions. Our critical assumption is that entrepreneurs face credit constraints and uninsurable

[1]In Panel A, the data of nominal GDP growth rates and tax adjusted safe rates over 1950-2018 are taken from Blanchard (2019). The data of nominal returns on the entire portfolio of US government bonds over 1950-2017 is taken from Hall et al. (2018). Because the data are quite volatile, we follow Rachel and Smith (2015) and plot the moving averages over the past 5 years in Panel A. The data for Panel B over 1950-2019 is taken from Hall et al. (2018). The data for Panel C over 1950-2019 is taken from Jiang et al. (2019).

idiosyncratic investment shocks (Kiyotaki and Moore (1997, 2019)). They can only trade one-period riskfree private and government bonds. The two types of bonds are perfect substitutes except that they are issued by different suppliers. In this case bonds provide liquidity service because they can raise owners' net worth and relax credit constraints. When the investment shock is sufficiently high, productive entrepreneurs sell bonds to finance real investment. Unproductive entrepreneurs are willing to buy bonds despite their low returns for precautionary reasons, because unproductive entrepreneurs anticipate that they may become productive in the future and need to finance real investment using bonds. The low interest rate on the bonds can support a positive value of government bonds even though these bonds are unbacked by taxes or even when they are rolled over to finance principal and interest payments as well as primary deficits.

We characterize the (nonstochastic) steady states of our detrended equilibrium system. We show that if the steady-state surplus is positive, then there is a unique steady state in which the real interest rate is higher than economic growth and the real value of public debt is equal to its fundamental value, i.e., the present value of future surpluses. Low interest rates are possible in the steady state only when the government runs permanent primary deficits or when there is zero deficit/surplus. There are multiple steady states for a given permanent primary-deficit-to-output ratio, if it is not too high. In this case all steady-state interest rates are less than the economic growth rate and all steady-state real values of public debt are positive. If the steady-state surplus/deficit is zero, then there are exactly two steady states. In one steady state, public debt has no value; and in the other, public debt has a positive value, which is a pure bubble. The multiplicity of steady states is generated by the debt Laffer curve that gives a non-monotonic relation between the total interest expense and the real interest rate. Such non-monotonicity is due to the positive relation between the interest rate and public debt. In our model with financial frictions, an increase in public debt reduces the liquidity premium and raises the real interest rate.

Under a reasonable calibration when the steady-state primary-deficit-to-GDP ratio is targeted at the average 0.28% of the US data over 1950-2019, our model delivers two steady states. The maximum sustainable deficit-to-GDP ratio is 0.30% and the associated public-debt-to-GDP ratio is 29.91%. We also find that, for the real version of our calibrated model, the low-interest-rate steady state is a saddle and the high-interest-rate steady state is a source. This also implies that there is a unique bounded (detrended) equilibrium around the former steady state, but there is no bounded equilibrium around the latter steady state. For our monetary model, the results are very different as the price level or inflation must also be determined in equilibrium.

We follow Leeper's (1991) approach by specifying feedback rules for monetary and fiscal policy and study what policy rules can produce a unique locally stable solution for both inflation and public debt. According to Leeper (1991), monetary policy is active if the interest rate rule satisfies the Taylor principle; otherwise, it is passive. Fiscal policy is passive if the government can raise enough taxes (primary surplus) to stabilize debt dynamics when public debt rises; otherwise it is active. The critical value for the fiscal policy response parameter is the steady-state interest expense.[2] Leeper (1991) argues that an active policy must be combined with a passive policy to achieve equilibrium determinacy. An active monetary policy and passive fiscal policy mix corresponds to the conventional case (regime M). An active fiscal policy and passive monetary policy mix (regime F) is associated with the fiscal theory of the price level (FTPL).

Relative to many studies in the literature, we find the following novel results:

1. There are three regions of the policy parameter space for each of the two steady states. These regions categorize local equilibrium determinacy around each steady state. The first region generates explosive solutions, the second region generates multiple bounded equilibria, and the third region generates a unique stable equilibrium. These regions are different for different steady states. Moreover, both active and passive monetary policies can achieve equilibrium determinacy, even if fiscal policy is passive.

2. The interest-rate-peg policy discussed in Woodford (2001) generates a unique local equilibrium around the high-interest-rate steady state, but generates sunspot equilibria around the low-interest-rate steady state.

3. The government can select a particular steady state by specifying the public debt target as that steady-state value. Then the deterministic equilibrium system in the absence of aggregate shocks will converge to the selected steady state along a saddle path. Thus a complete specification of fiscal policy must include both the debt target and the policy response coefficient.

4. The conventional regime M is associated with Ricardian equivalence and thus a shock to lump-sum taxes does not affect the real allocation and inflation. By contrast, Ricardian equivalence in our model does not hold in any regime due to financial frictions. Raising lump-sum taxes on credit-constrained households/workers reduces their consumption and raise their labor supply due to the wealth effect. Thus, this policy has a stimulative effect, though it is quantitatively is small.[3]

5. Except for the preceding case, the dynamic responses of the economy to monetary and fiscal shocks in the determinate regime M and regime F are qualitatively similar to those in the standard DNK models (e.g., Woodford (2003) and Kim (2003)). But the transmission channel is different especially in regime F. By the FTPL, the transmission of shocks relies on the

[2]Woodford (2003, p.312) calls a passive fiscal policy rule locally Ricardian and an active rule otherwise.

[3]If lump-sum taxes are levied on credit-constrained entrepreneurs, they will reduce investment and hence labor demand. This will generate a contractionary effect.

revaluation of public debt as a forward-looking solution using the present value government budget constraints. This argument fails because the fundamental value of public debt (the present value of future primary surpluses) may not be finite when the interest rate is less than economic growth. Our theory complements the FTPL by showing that the value of public debt contains a bubble component. We show that permanent deficits can be supported by rolling over public debt with a finite positive value due to self-fulfilling beliefs about its bubble component. Because public debt provides liquidity service to credit-constrained entrepreneurs, they are willing to trade public debt with low interest rates, generating a bubble in the debt. Thus, for the model with low interest rate in regime F, a shock to either interest rate or primary surplus affects the value of public debt through both its fundamental and bubble components.

Our results have some implications for the US and Japan experiences. First, Clarida, Gali and Gertler (2000) and Lubik and Schorfheide (2004) document evidence that the Fed interest rate policy was passive prior to 1980 and then it became active during the Volcker–Greenspan era. According to Leeper (1991), to ensure price determinacy, fiscal policy must be active prior to 1980 and must shift to be passive after 1980. Our model shows that both active and passive monetary policies can achieve price determinacy even if fiscal policy remains passive. Second, Japan has mostly run primary deficits since 1960s and with no primary surpluses in sight, but inflation has not risen much. This seems inconsistent with the FTPL as the present value of future deficits may not be well defined. Our model shows that public debt contains a bubble component,[4] which must be included in its valuation when the interest rate is lower than economic growth. Once taking into account this component, a large set of monetary and fiscal policy response parameters in either regime M or regime F can achieve stable debt and inflation dynamics given persistent fiscal deficits.

Related literature. Our paper is related to three strands of the literature. First, our paper is closely related to the recent literature on the implications of low interest rates for monetary and fiscal policies. Bullard and Russell (1999), Chalk (2000), and Blanchard (2019) study public debt policy based on the overlapping generations (OLG) model of Diamond (1965). In a dynamically inefficient economy, the government can rollover public debt at a low interest rate or run Ponzi schemes to support permanent deficits. Kaas (2016) studies similar questions in a model with infinite-lived agents, in which entrepreneurs are subject to uninsurable idiosyncratic productivity risk and credit constraints. Brunnermeier, Merkel and Sannikov (2020a) also study similar questions in a continuous-time model with uninsurable idiosyncratic capital return risk, but without credit constraints. Like us, Sims (2020) emphasizes the importance of the positive relation between public

[4] See Jiang et al. (2019) for evidence.

debt and the real interest rate. Unlike our paper, all these papers do not study the interactions of monetary and fiscal policies.

Bassetto and Cui (2018) revisit the implications of the FTPL under low interest rates which are generated by sources such as dynamic inefficiency, liquidity premium of public debt, or its favorable risk profile. They show that the interest-rate-peg policy may not pin down a unique equilibrium price level. Brunnermeier, Merkel and Sannikov (2020b) also revisit the FTPL and show that a particular fiscal policy can pin down a unique equilibrium price level. Like us, they emphasize that public debt contains a bubble component when interest rates are low. Both papers derive multiple steady states. Unlike these two papers, our paper considers general feedback rules for monetary and fiscal policies following Leeper (1991), analyzes determinacy regions of policy parameter space, and studies dynamic responses of the economy to monetary and fiscal policy shocks.

Similar to these papers and the early paper by Woodford (1990), we show that there are multiple steady states with low interest rates generated by the liquidity premium under incomplete markets and credit constraints and that public debt contains a bubble component under persistent primary deficits. The liquidity premium of public debt can also be generated in models with monetary search frictions (e.g., Berentsen and Waller (2018), Bassetto and Cui (2018), and Dominguez and Gomis-Porqueras (2019)). In these models monetary policy is nonneutral in the long run. By contrast, we adopt the DNK framework that is more amenable to quantitative analysis and Bayesian estimation (e.g., Lubik and Schorfheide (2004)).

Second, our paper is related to the literature on the fiscal and monetary policy interactions surveyed by Leeper and Leith (2016) and Canzoneri, Cumby and Diba (2010), and particularly related to the FTPL developed primarily by Leeper (1991), Woodford (1994), Sims (1994), and Cochrane (1998). This literature is too large for us to cite all relevant papers. We mention two closely related papers by Cui (2016) and Canzoneri et al. (2011). Cui (2016) studies a DNK model based on Kiyotaki and Moore (2019) with endogenous fluctuations in liquidity. Also using a DNK model, Canzoneri et al. (2011) argue that government bonds provide liquidity services and are imperfect substitutes for money. Both papers feature a unique steady state and derive three regions of policy parameter space similar to ours.

Third, our paper is related to the literature on asset bubbles surveyed by Miao (2014) and Martin and Ventura (2018). Asset bubbles can emerge in either dynamically inefficient OLG models (Tirole (1985)) or in models with infinitely-lived agents facing financial frictions or under incomplete markets. Our model is based on Miao and Wang (2012), Miao, Wang and Zhou (2015), Miao and Wang (2018), and Dong, Miao and Wang (2020), in which liquidity premium is important for the emergence of a bubble. Unlike these papers, we focus on the interactions of monetary and fiscal policies under low interest rates.

2 Model

In this section we present a DNK model with financial frictions. Consider an infinite-horizon economy consisting of households, firms, retailers, and a government (monetary authority). Assume that retailers are monopolistically competitive and their role is to introduce nominal price rigidities.

2.1 Households

There is a continuum of identical households of measure unity. The representative household is an extended family consisting of workers, entrepreneurs, and retailers. Each entrepreneur runs a firm and workers supply labor to firms. The family and firms can trade one-period riskfree private and government bonds. The two types of bonds are perfect substitutes except that the private bonds are in zero net supply and the government bonds are issued by the government only. Entrepreneurs and retailers hand in their dividends to households who are shareholders.

Each household chooses consumption $\{C_t\}$, labor supply $\{N_t\}$, and bond holdings $\{D_{ht}\}$ to maximize utility

$$\max_{\{C_t, D_{ht}, N_t\}} \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \psi N_t\right)\right], \tag{1}$$

subject to

$$C_t + D_{ht} = W_t N_t + \Upsilon_t + \frac{R_{t-1}}{\Pi_t} D_{ht-1} - T_t, \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount factor, W_t is the real wage, R_{t-1} is the nominal interest rate between periods $t-1$ and t, $\Pi_t = P_t/P_{t-1}$ is the gross inflation rate, Υ_t denotes total real dividends from entrepreneurs and retailers, and T_t denotes real lump-sum taxes. Here P_t denotes the aggregate price level in period t. Assume that households cannot borrow so that $D_{ht} \geq 0$ for all t.

The first-order conditions imply that

$$W_t = \frac{\psi}{\Lambda_t}, \tag{3}$$

$$1 \geq \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\Pi_{t+1}}, \text{ with equality when } D_{ht} > 0, \tag{4}$$

where $\Lambda_t = 1/C_t$ denotes the household marginal utility. We will show later that the household will not hold any bonds (i.e., $D_{ht} = 0$) in an equilibrium around the neighborhood of a steady state, because the real return on the bonds is too low.

2.2 Entrepreneurs

Each entrepreneur $j \in [0,1]$ runs a firm that combines labor N_{jt} and capital K_{jt-1} to produce an intermediate (wholesale) good j in period t according to the production function

$$Y_{jt} = K_{jt-1}^{\alpha} \left(A_t N_{jt}\right)^{1-\alpha}, \ \alpha \in (0,1),$$

where A_t denotes the labor-augmenting technology that grows at the rate g. For simplicity we assume that A_t is deterministic with $A_{-1} = 1$.

The entrepreneur sells wholesale goods to retailers at the real price p_{wt}. The static profit maximization problem yields

$$R_{kt}K_{jt-1} = \max_{N_{jt}} \; p_{wt}K_{jt-1}^{\alpha} \left(A_t N_{jt}\right)^{1-\alpha} - W_t N_{jt},$$

where

$$R_{kt} = \alpha \left(\frac{(1-\alpha) A_t}{W_t} \right)^{\frac{1-\alpha}{\alpha}} p_{wt}^{\frac{1}{\alpha}}, \tag{5}$$

and the first-order condition gives labor demand

$$W_t = (1-\alpha) p_{wt} A_t K_{jt-1}^{\alpha} \left(A_t N_{jt}\right)^{-\alpha}. \tag{6}$$

At the beginning of period t, the entrepreneur faces idiosyncratic investment-specific shock ε_{jt} and makes investment I_{jt} to increase his capital stock so that the law of motion for capital follows

$$K_{jt} = (1-\delta)K_{jt-1} + \varepsilon_{jt} I_{jt}, \tag{7}$$

where $\delta \in (0,1)$ represents the depreciation rate. Suppose that the cumulative distribution function of ε_{jt} is F and the density function is f on $[\varepsilon_{\min}, \varepsilon_{\max}] \subset (0,\infty)$ and ε_{jt} is independently and identically distributed across firms and over time. Assume that there is no insurance market against the idiosyncratic investment-specific shock and that investment is irreversible at the firm level so that $I_{jt} \geq 0$.

Entrepreneur j can hold B_{jt} units of private bonds and $D_{jt} \geq 0$ units of government bonds in terms of the consumption good. His flow-of-funds constraints are given by

$$C_{jt} + I_{jt} + B_{jt} + D_{jt} = R_{kt}K_{jt-1} + \frac{R_{t-1}}{\Pi_t} B_{jt-1} + \frac{R_{t-1}}{\Pi_t} D_{jt-1}, \tag{8}$$

where C_{jt} denotes real dividends. Entrepreneur j can use its capital as collateral to borrow and faces the following borrowing constraint due to imperfect contract enforcement:[5]

$$B_{jt} \geq -\mu K_{jt-1}, \;\; \mu \in [0,1). \tag{9}$$

Suppose that equity finance is so costly that the firm does not issue any new equity.[6] Thus we impose the constraint

$$C_{jt} \geq 0. \tag{10}$$

[5]Unlike Kiyotaki and Moore (1997), we do not use future capital as collateral. Using future capital as collateral will complicate algebra significantly without changing our key insights. See Caballero and Krishnamurthy (2006), Miao and Wang (2018), and Miao, Wang and Zhou (2015) for related discussions.

[6]Our key insights will not change as long as new equity issues are sufficiently limited (see Miao and Wang (2018) and Miao, Wang and Xu (2015)).

The entrepreneur's objective is to maximize the discounted present value of dividends. We can write his decision problem using dynamic programming

$$V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) = \max_{\{I_{jt}, D_{jt}, B_{jt}\}} C_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(K_{jt}, B_{jt}, D_{jt}, \varepsilon_{jt+1}\right), \tag{11}$$

subject to (7), (8), (9), and (10), where we have used the household's intertemporal marginal rate of substitution as the stochastic discount factor. Here $V_t(\cdot)$ denotes the value function.

Define Tobin's (marginal) Q as

$$q_t^k = \frac{\partial}{\partial K_{jt}} \mathbb{E}_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(K_{jt}, B_{jt}, D_{jt}, \varepsilon_{jt+1}\right).$$

The following proposition characterizes entrepreneur j's optimal decisions:

Proposition 1 *Suppose that $\varepsilon_t^* \equiv 1/q_t^k \in (\varepsilon_{\min}, \varepsilon_{\max})$ in an equilibrium. Then, for $\varepsilon_{jt} \geq \varepsilon_t^*$, we have $B_{jt} = -\mu K_{jt-1}$, $D_{jt} = 0$,*

$$I_{jt} = \left(R_{kt} + \mu\right) K_{jt-1} + \frac{R_{t-1}}{\Pi_t} D_{jt-1}; \tag{12}$$

and for $\varepsilon_{jt} < \varepsilon_t^*$, we have $I_{jt} = 0$, but B_{jt} and D_{jt} are indeterminate. Moreover, q_t^k and R_t satisfy

$$q_t^k = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} R_{kt+1}\left(1 + q_{t+1}^l\right) + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1}^k (1-\delta) + \beta \mu \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1}^l, \tag{13}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{R_t}{\Pi_{t+1}} \left(1 + q_{t+1}^l\right), \tag{14}$$

where

$$q_t^l \equiv \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \left(q_t^k \varepsilon - 1\right) dF(\varepsilon). \tag{15}$$

The transversality conditions hold

$$\lim_{i\to\infty} \mathbb{E}_t \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} q_{t+i}^k K_{jt+i} = \lim_{i\to\infty} \mathbb{E}_t \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} B_{jt+i} = \lim_{i\to\infty} \mathbb{E}_t \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} D_{jt+i} = 0. \tag{16}$$

This proposition shows that there is an investment cutoff ε_t^* such that entrepreneur j makes real investment if $\varepsilon_{jt} \geq \varepsilon_t^*$. The cutoff ε_t^* is equal to the inverse of Tobin's Q when the investment profit is exactly zero. The entrepreneur uses his internal funds $R_{kt}K_{jt-1}$, private debt μK_{jt-1}, and the principal and interest value of government bonds $R_{t-1}D_{jt-1}/\Pi_t$ to finance investment expenditures as shown in (12). If $\varepsilon_{jt} < \varepsilon_t^*$, he does not make real investment and buys bonds from other productive entrepreneurs. Because entrepreneurs are effectively risk neutral as shown in (11), they are indifferent between specific levels of bond holdings. Only aggregate level is determined in equilibrium by market clearing. Thus the interest rates on private and public bonds are the same and satisfy the asset pricing equation (14). Unlike the standard equation without financial frictions, there is a liquidity premium term q_{t+1}^l in (14).

Intuitively, both private and government bonds raise entrepreneurs' net worth and help them relax credit constraints. Purchasing one dollar of bonds today gives not only principal plus interest tomorrow, but also allows a productive, credit-constrained entrepreneur with $\varepsilon_{t+1}^j > \varepsilon_{t+1}^*$ to finance investment so that he makes $q_{t+1}^k \varepsilon_{t+1}^j - 1$ dollars of profits tomorrow. The integral term in (15) gives the expected profits generated by holding bonds. Define the real interest rate as

$$R_t^r = \left\{ \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left(1 + q_{t+1}^l \right) \right\}^{-1}.$$

Then the real interest rate is negatively related to the liquidity premium and will play an important role in our analysis.

Equation (13) is an asset-pricing equation for Tobin's Q. Unlike the standard equation without financial frictions, the liquidity premium term also appears in (13) because capital return raises an entrepreneur's net worth and also because capital is used as collateral in our model.

2.3 Retailers

Retailers are monopolistically competitive. In each period t they buy intermediate goods from entrepreneurs at the real price p_{wt} and sell good j at the nominal price P_{jt}. Intermediate goods are transformed into final goods according to the CES aggregator

$$Y_t = \left[\int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1. \tag{17}$$

Thus retailers face demand given by

$$Y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\sigma} Y_t, \tag{18}$$

where the price index is given by

$$P_t \equiv \left[\int_0^1 P_{jt}^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}}. \tag{19}$$

To introduce price stickiness, we assume that each retailer is free to change its price in any period only with probability $1 - \xi$, following Calvo (1983). To introduce trend inflation, we follow Erceg, Henderson and Levin (2000) and assume that whenever the retailer is not allowed to reset its price, its price is automatically increased at the steady-state inflation rate. The retailer selling good j chooses the nominal price P_{jt}^* in period t to maximize the discounted present value of real profits

$$\max_{P_t^*} \sum_{k=0}^{\infty} \xi^k \mathbb{E}_t \left[\frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left((1+\tau) \frac{\Pi^k P_{jt}^*}{P_{t+k}} - p_{w,t+k} \right) Y_{jt+k}^* \right], \tag{20}$$

subject to the demand curve

$$Y_{jt+k}^* = \left(\frac{\Pi^k P_{jt}^*}{P_{t+k}} \right)^{-\sigma} Y_{t+k}, \;\; k \geq 0, \tag{21}$$

where τ denotes the output subsidy and Π denotes the steady-state inflation target. We use the household intertemporal marginal rate of substitution as the stochastic discount factor because retailers must hand in all profits to households who are the shareholders.

The first-order condition gives the pricing rule

$$P_{jt}^* = P_t^* \equiv \frac{1}{1+\tau}\frac{\sigma}{\sigma-1}\frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\xi)^k \Lambda_{t+k} p_{w,t+k} P_{t+k}^{\sigma} Y_{t+k} (\Pi^k)^{-\sigma}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\xi)^k \Lambda_{t+k} P_{t+k}^{\sigma-1} (\Pi^k)^{1-\sigma} Y_{t+k}} \tag{22}$$

for all j.

We set $1+\tau = \sigma/(\sigma-1)$ to completely remove the distortion due to monopolistic competition. Let $p_t^* = P_t^*/P_t$. We can then write the pricing rule in a recursive form as follows

$$p_t^* = \frac{\Gamma_t^a}{\Gamma_t^b}, \tag{23}$$

where

$$\Gamma_t^a = \Lambda_t p_{wt} Y_t + \beta\xi \mathbb{E}_t \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\sigma} \Gamma_{t+1}^a, \tag{24}$$

$$\Gamma_t^b = \Lambda_t Y_t + \beta\xi \mathbb{E}_t \left(\frac{\Pi_{t+1}}{\Pi}\right)^{\sigma-1} \Gamma_{t+1}^b. \tag{25}$$

The aggregate price level satisfies

$$P_t = \left[\xi \left(\Pi P_{t-1}\right)^{1-\sigma} + (1-\xi)\left(P_t^*\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}},$$

or

$$1 = \left[\xi \left(\frac{\Pi}{\Pi_t}\right)^{1-\sigma} + (1-\xi)\, p_t^{*1-\sigma}\right]^{\frac{1}{1-\sigma}}. \tag{26}$$

2.4 Monetary and Fiscal Policies

The government issues one-period riskfree nominal debt $(P_t D_t)$, where D_t denotes real debt. The government budget constraint is given by

$$G_t + \Upsilon_{gt} + \frac{R_{t-1} D_{t-1}}{\Pi_t} = T_t + D_t, \tag{27}$$

where G_t denotes real government spending and Υ_{gt} denotes real output subsidy. Let $S_t \equiv T_t - G_t - \Upsilon_{gt}$ denote the real primary surplus. Then we rewrite (27) as

$$\frac{R_{t-1} D_{t-1}}{\Pi_t} = S_t + D_t. \tag{28}$$

Following Leeper (1991), suppose that the government adjusts real primary surplus in response to the real value of public debt. Because our model features long-run growth, we consider detrended policy rule as in Cui (2016):

$$s_t/y = s/y + \phi_s(d_{t-1} - d)/y + z_{st}, \tag{29}$$

where $s_t = S_t/A_t$, $d_t = D_t/A_t$, and $y_t = Y_t/A_t$, and the variables s, d, and y are the corresponding steady-state values. The parameter ϕ_s describes the strength of fiscal adjustment. The variable z_{st} follows an AR(1) process

$$z_{st} = \rho_s z_{st-1} + \epsilon_{st},$$

where $|\rho_s| < 1$ and ϵ_{st} is a normal white noise with mean zero and variance σ_s^2.

The monetary authority sets the nominal interest rate as a function of the current inflation rate:

$$R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(z_{mt}), \tag{30}$$

where R and Π denote the nominal interest rate and the inflation rate targets (steady-state values). The variable z_{mt} follows an AR(1) process

$$z_{mt} = \rho_m z_{mt-1} + \epsilon_{mt},$$

where $|\rho_m| < 1$ and ϵ_{mt} is a normal white noise with mean zero and variance σ_m^2. The parameter ϕ_m describes the strength of the interest rate adjustment in response to inflation. Assume that $\{z_{mt}\}$ and $\{z_{st}\}$ are independent processes.

2.5 Equilibrium

Equations (4) and (14) suggest that the interest rate is too low for the household to hold any government bonds so that $D_{ht} = 0$ in equilibrium. Thus the market-clearing conditions for private and government bonds are given by

$$\int B_{jt}dj = 0, \quad \int D_{jt}dj = D_t.$$

Define aggregate investment, aggregate labor, and aggregate capital as $I_t = \int I_{jt}dj$, $N_t = \int N_{jt}dj$, and $K_t = \int K_{jt}dj$. The labor demand condition (6) implies that all firms have the same capital-labor ratio and thus we have

$$W_t = (1-\alpha)p_{wt}A_t K_{t-1}^{\alpha} (A_t N_t)^{-\alpha}. \tag{31}$$

Using (31) to eliminate W_t in (5), we can show that the capital return is equal to the marginal product of capital

$$R_{kt} = \alpha p_{wt} K_{t-1}^{\alpha-1} (A_t N_t)^{1-\alpha}. \tag{32}$$

By Proposition 1 and the market-clearing conditions described above, we obtain aggregate investment as follows

$$I_t = \left((\mu + R_{kt}) K_{t-1} + \frac{R_{t-1}}{\Pi_t} D_{t-1} \right) (1 - F(\varepsilon_t^*)). \tag{33}$$

Intuitively, aggregate investment is financed by private debt μK_{t-1} and the net worth of high productivity entrepreneurs with $\varepsilon_{jt} \geq \varepsilon_t^*$. The latter consists of capital return $R_{kt} K_{t-1}$ and the value of government bonds $R_{t-1} D_{t-1} / \Pi_t$. Similarly, we can derive the aggregate capital stock from (7) as

$$K_t = (1-\delta) K_{t-1} + \left(\left(\mu + R_{kt}\right) K_{t-1} + \frac{R_{t-1}}{\Pi_t} D_{t-1} \right) \int_{\epsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \tag{34}$$

Aggregating (18) yields aggregate output as

$$Y_t = \frac{1}{\Delta_t} K_{t-1}^{\alpha} \left(A_t N_t\right)^{1-\alpha}, \tag{35}$$

where the price dispersion $\Delta_t = \int \left(P_{jt}/P_t\right)^{-\sigma} dj$ satisfies the following recursive condition

$$\Delta_t = (1-\xi) p_t^{*-\sigma} + \xi \left(\frac{\Pi}{\Pi_t}\right)^{-\sigma} \Delta_{t-1}, \tag{36}$$

with Δ_{-1} being exogenously given.

The aggregate resource constraint is given by

$$C_t + I_t + G_t = Y_t. \tag{37}$$

For simplicity, assume that government spending G_t is a fixed constant fraction G_y of output Y_t in each period.

Given initial conditions for predetermined variables $\{K_{t-1}, R_{t-1}, D_{t-1}, \Delta_{t-1}\}$ and a monetary and fiscal monetary policy mix with exogenous policy shocks $\{z_{mt}, z_{st}\}$, a competitive equilibrium consists of 20 variables $\{N_t, q_t^k, R_t, q_t^l, p_t^*, \Gamma_t^a, \Gamma_t^b, \Pi_t, D_t, S_t, p_{wt}, W_t, R_{kt}, I_t, K_t, Y_t, \Delta_t, C_t, \Lambda_t, \varepsilon_t^*\}$ satisfying a system of 20 equations (3), (13), (14), (15), (23), (24), (25), (26), (28), (29), (30), (31), (32), (33), (34), (35), (36), and (37), $\Lambda_t = 1/C_t$, and $\varepsilon_t^* = 1/q_t^k \in (\varepsilon_{\min}, \varepsilon_{\max})$.

2.6 Public Debt Valuation

In this subsection we briefly review the basic idea of the FTPL and its difficulty when the interest rate is lower than economic growth and/or when the government runs persistent primary deficits.

Following Cochrane (1998, 2020), we can use (14) to rewrite the government budget constraint (28) as

$$\frac{R_{t-1} D_{t-1}}{\Pi_t} = S_t + \frac{R_t D_t}{R_t} = S_t + \mathbb{E}_t \frac{\beta \Lambda_{t+1}}{\Lambda_t} \left(1 + q_{t+1}^l\right) \frac{R_t D_t}{\Pi_{t+1}},$$

Solving forward yields

$$\frac{D_{t-1} R_{t-1}}{\Pi_t} = S_t + \lim_{T \to \infty} \mathbb{E}_t \sum_{i=1}^{T} \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} S_{t+i} Q_{t+i}^l + \lim_{T \to \infty} \mathbb{E}_t \frac{\beta^{T+1} \Lambda_{t+1+T}}{\Lambda_t} \frac{D_{t+T} R_{t+T}}{\Pi_{t+1+T}} Q_{t+1+T}^l, \tag{38}$$

where we define the cumulated liquidity premium from period $t+1$ to $t+i$ as

$$Q_{t+i}^l = \prod_{j=1}^{i} \left(1 + q_{t+j}^l\right) > 1 \text{ for } i \geq 1.$$

The first two terms on the right-hand side of (38) represent the real fundamental value of public debt. The last term of (38) represents a bubble component.

In standard models without liquidity premium ($q_t^l = 0$ for all t), a bubble can be ruled out using the transversality condition (16). Specifically, aggregating (16) and using the market-clearing condition and (14), we obtain

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}_t \frac{\beta^{T+1}\Lambda_{t+1+T}}{\Lambda_t} \frac{D_{t+T}R_{t+T}}{\Pi_{t+1+T}} &= \lim_{T\to\infty} \mathbb{E}_t \frac{\beta^T \Lambda_{t+T}}{\Lambda_t} D_{t+T}\mathbb{E}_{t+T}\left[\frac{\beta\Lambda_{t+T+1}}{\Lambda_{t+T}}\frac{R_{t+T}}{\Pi_{t+1+T}}\right] \\ &= \lim_{T\to\infty} \mathbb{E}_t \frac{\beta^T\Lambda_{t+T}}{\Lambda_t} D_{t+T} = 0. \end{aligned} \tag{39}$$

Bassetto and Cui (2018) and Cochrane (2020) argue that low interest rate (less than economic growth) and persistent fiscal deficits on average can generate a finite fundamental value of public debt when agents are sufficiently risk averse or when risk is sufficiently high. Then the usual FTPL can ensure price determinacy. While the conventional monetary regime treats equation (38) as a constraint (implying that fiscal policy needs to adjust when the present value of future surpluses differs from the real value of debt), the FTPL views it as an equilibrium condition. Specifically, given a positive predetermined nominal value of debt $(P_{t-1}D_{t-1}R_{t-1})$, the current price P_t will adjust to ensure equation (38) holds. A shock to the current primary surplus does not have to lead to changes in future primary surpluses; instead, the price level can adjust to restore equality (38).

We argue that the usual FTPL fails or is incomplete when public debt contains a bubble component. In our model with financial frictions, transversality conditions cannot rule out a bubble due to liquidity premium. Specifically, (16) can only ensure the last equality of (39), but may not force the following limit (i.e., bubble) to zero

$$\lim_{T\to\infty} \mathbb{E}_t \frac{\beta^{T+1}\Lambda_{t+1+T}}{\Lambda_t}\frac{R_{t+T}}{\Pi_{t+1+T}} D_{t+T} Q^l_{t+1+T} = \lim_{T\to\infty} \mathbb{E}_t \frac{\beta^T \Lambda_{t+T}}{\Lambda_t} D_{t+T} Q^l_{t+T}.$$

Thus the bubble component in (38) must be taken into account in the valuation of public debt when applying the FTPL (see Brunnermeier, Merkel and Sannikov (2020a,b) for a similar point). For example, when primary surplus $S_t = 0$ for all t, equation (28) becomes

$$\frac{R_{t-1}D_{t-1}}{\Pi_t} = D_t. \tag{40}$$

The fundamental value of debt is zero, but debt can be rolled over and has a finite positive value as shown in the next section. As will also be discussed in the next section, the bubble component can even approach positive infinity when the interest rate is less than economic growth.

3 Steady States

Our model features long-run balanced growth. To study steady states and the dynamics around the balanced growth path, we first detrend the equilibrium system by using the transformation of

$x_t = X_t/A_t$ for any variable $X_t \in \{K_t,\ D_t,\ S_t,\ Y_t,\ W_t,\ C_t,\ I_t,\ G_t\}$. For the stochastic discounter factor, we denote $\lambda_t = A_t\Lambda_t$. The capital return R_{kt} does not have trend and has a constant steady state R_k. The detrended system is provided in Appendix B. Throughout the paper, we focus on steady states in which the primary surplus to output ratio s/y is an exogenous constant over time. We use variables without time subscripts to denote their steady-state values and present the steady-state system in Appendix C.

3.1 Investment Cutoff

The critical step to solve for a steady state is to derive the investment cutoff ε^*. Once it is determined, other variables can be easily derived as shown in Appendix C. Since monetary policy is neutral in the steady state, we only need to solve for real variables. We first derive the real interest rate $R^r \equiv R/\Pi$ and the capital return R_k. It follows from (14) and $q^k = 1/\varepsilon^*$ that

$$R^r = \frac{(1+g)/\beta}{1+\int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon/\varepsilon^* - 1)\, dF(\varepsilon)} \equiv R^r(\varepsilon^*). \tag{41}$$

Clearly, R^r is less than $(1+g)/\beta$ in the standard DNK model due to financial frictions. It can be checked that $R^r(\varepsilon^*)$ increases with ε^*. Intuitively, as the investment cutoff ε^* increases, more efficient firms make investment, Tobin's Q ($q^k = 1/\varepsilon^*$) declines, the liquidity premium declines, and the real interest rate rises.

Using (13) and $q^k = 1/\varepsilon^*$, we obtain

$$R_k = \frac{\left[(1+g)/\beta - (1-\delta)\right]/\varepsilon^* - \mu \int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon/\varepsilon^* - 1)\, dF(\varepsilon)}{1+\int_{\varepsilon^*}^{\varepsilon_{\max}} (\varepsilon/\varepsilon^* - 1)\, dF(\varepsilon)} \equiv R_k(\varepsilon^*). \tag{42}$$

The monotonicity of $R_k(\varepsilon^*)$ plays an important role in characterizing the steady-state equilibria.

Lemma 1 *For any $\mu > 0$, $R_k(\varepsilon^*)$ has a unique maximum at $\varepsilon_k \in (\varepsilon_{\min}, \varepsilon_{\max})$, which satisfies*

$$\mu \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) - (\beta^{-1}(1+g) - 1 + \delta)F(\varepsilon_k) = 0.$$

Moreover, $\partial R_k(\varepsilon^*)/\partial \varepsilon^* > 0$ for $\varepsilon^* \in [\varepsilon_{\min}, \varepsilon_k]$ and $\partial R_k(\varepsilon^*)/\partial \varepsilon^* < 0$ for $\varepsilon^* \in [\varepsilon_k, \varepsilon_{\max}]$. If $\mu = 0$, we have $\varepsilon_k = \varepsilon_{\min}$ and $\partial R_k(\varepsilon^*)/\partial \varepsilon^* < 0$ for $\varepsilon^* \in [\varepsilon_{\min}, \varepsilon_{\max}]$.

This lemma shows that R_k is not monotonic in ε^*. This is because an increase of ε^* has two opposing effects: It reduces the liquidity premium and hence raises R_k for a similar intuition discussed earlier. But it also reduces Tobin's Q and hence reduces R_k.

Next, dividing (34) by $k_t \equiv K_t/A_t$, we can derive the steady-state real value of government liabilities relative to capital as

$$\frac{R^r d}{k} = \frac{(g+\delta) - (\mu + R_k(\varepsilon^*)) \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)}{\int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} \equiv \Phi(\varepsilon^*). \tag{43}$$

The function $\Phi(\varepsilon^*)$ represents the maturity value of public debt (including both the principal and interest) relative to capital. By Lemma 1, $\Phi(\varepsilon^*)$ increases with ε^* on $[\varepsilon_k, \varepsilon_{\max}]$, but may not be monotonic on $[\varepsilon_{\min}, \varepsilon_{\max}]$.

Lemma 2 *There exists a unique solution $\varepsilon^* = \varepsilon_l \in (\varepsilon_k, \varepsilon_{\max})$ to the equation $\Phi(\varepsilon^*) = 0$. For a sufficiently small $\mu \geq 0$, we have $\Phi(\varepsilon^*) < 0$ on $\varepsilon^* \in [\varepsilon_{\min}, \varepsilon_k]$.*

This lemma shows that there is a unique cutoff ε_l such that the steady-state value $R^r d$ is equal to zero by (43). Since $R^r d \geq 0$, Lemma 2 and (43) show that any steady-state cutoff ε^* must satisfy $\varepsilon^* \geq \varepsilon_l > \varepsilon_k$ if $\mu \geq 0$ is sufficiently small. Otherwise, public debt has a negative value by (43). Throughout our analysis, we will maintain this assumption.

Now we derive the steady-state version of the government budget constraint (28) as

$$\left(\frac{R^r}{1+g} - 1\right)\frac{d}{y} = \frac{s}{y}. \tag{44}$$

The left side of equation (44) represents the interest payment of public debt relative to output and the right side represents the primary-surplus-to-output ratio. Rewrite the expression on the left side as

$$\left(\frac{R^r}{1+g} - 1\right)\frac{d}{y} = \frac{R^r - (1+g)}{R^r}\frac{R^r d}{k}\frac{k}{y}\frac{1}{1+g}. \tag{45}$$

It follows from (32) and (35) that $k/y = \alpha(1+g)/R_k$. Substituting this expression into (45) and using (41) and (43), we can rewrite (44) as

$$\Psi\left(\varepsilon^*\right) \equiv \frac{R^r(\varepsilon^*) - (1+g)}{R^r(\varepsilon^*)}\frac{\alpha}{R_k(\varepsilon^*)}\Phi(\varepsilon^*) = \frac{s}{y}. \tag{46}$$

For any exogenously given steady-state surplus-to-output ratio s/y, equation (46) determines the steady state cutoff ε^*.

Figure 2 illustrates the functions $R^r(\varepsilon^*)$, $R_k(\varepsilon^*)$, $\Phi(\varepsilon^*)$, and $\Psi\left(\varepsilon^*\right)$ for $\varepsilon_{\max} = \infty$. As shown in Panel B, a steady state investment cutoff is determined by the crossing point of the curve $\Psi\left(\varepsilon^*\right)$ and the horizontal line s/y. The steady-state $R^r d/k$ can be read from the curve $\Phi(\varepsilon^*)$ in Panel D. This curve crosses the horizontal axis at ε_l. For $R^r d/k \geq 0$, a steady state cutoff must be in the region ε_l and $[\varepsilon_l, \varepsilon_{\max}]$. In the next three subsections we consider three cases depending on the signs of s/y. We will show that there may be multiple steady states because $\Psi\left(\varepsilon^*\right)$ is not a monotonic function.

3.2 Government Debt as a Pure Bubble

We first consider the case in which $S_t = 0$ for all t. Then public debt is an unbacked asset like a pure bubble (Diamond (1965) and Tirole (1985)). Its fundamental value is zero. There exists a steady state in which the detrended debt has no value, i.e., $d = 0$. There may exist another

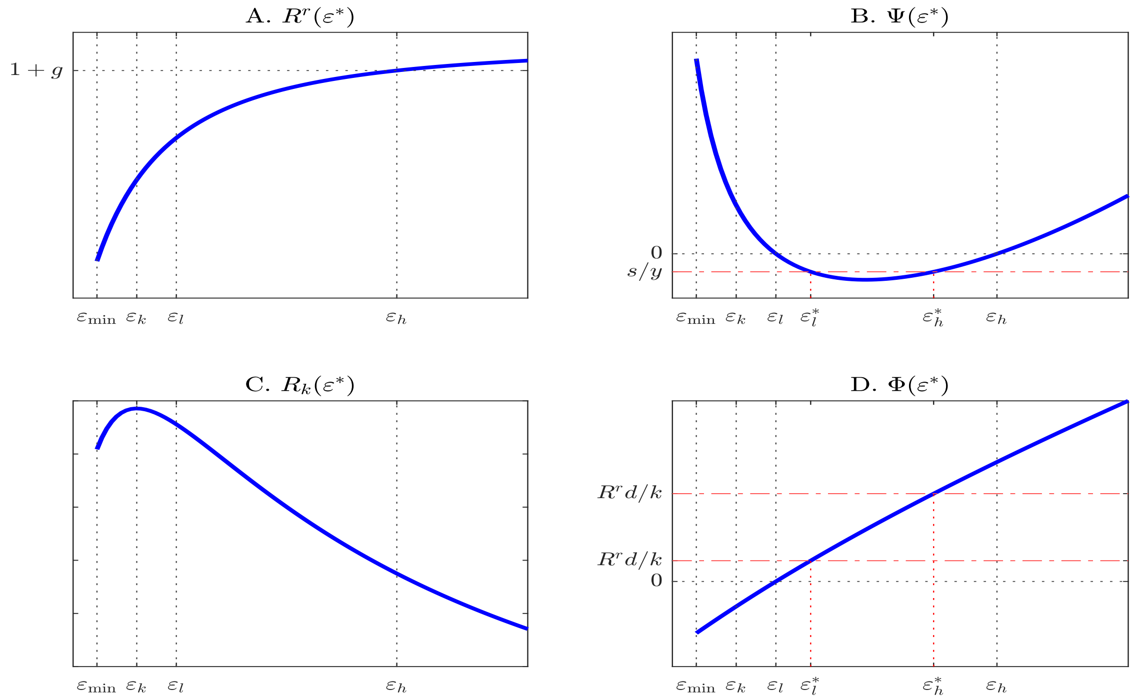

Figure 2: Functions $R^r(\varepsilon^*), R_k(\varepsilon^*), \Phi(\varepsilon^*), \Psi(\varepsilon^*)$, and determination of steady state.

steady state in which detrended debt has a finite positive value ($d > 0$) due to liquidity premium supported by self-fulfilling beliefs. The following proposition establishes the condition under which unbacked public debt can be rolled over indefinitely.

Proposition 2 *Suppose that $\mu \geq 0$ is sufficiently small and the steady-state primary-surplus-to-output ratio is fixed at $s/y = 0$. Then there always exists a steady state in which the investment cutoff is ε_l given in Lemma 2, $d = 0$, and $R^r = R^r(\varepsilon_l)$. This is the unique steady state if $R^r(\varepsilon_l) > 1+g$. If $R^r(\varepsilon_l) < 1+g$, then there also exists another steady state in which the investment cutoff $\varepsilon_h \in (\varepsilon_l, \varepsilon_{\max})$ is the unique solution to the equation $R^r(\varepsilon_h) = 1+g$, the real interest rate is $R^r = 1+g$, and the real value of government liabilities relative to capital is given by $R^r d/k = \Phi(\varepsilon_h)$.*

The condition in this proposition is similar to that in Tirole (1985), Miao and Wang (2018), and Dong, Miao and Wang (2020); that is, the real interest rate $R^r(\varepsilon_l)$ in the bubbleless steady state must be lower than economic growth. Because the steady-state real interest rate in the standard model without financial frictions is equal to $(1+g)/\beta > 1+g$, unbacked public debt cannot be valued or rolled over. Figure 2 presents the case of $R^r(\varepsilon_l) < 1+g$. A notable feature is that $\Psi(\varepsilon^*)$ is U-shaped and crosses the horizontal axis at two points ε_l and ε_h.

The critical assumption to generate a low interest rate is the presence of financial frictions. The following proposition establishes the impact of the parameter μ, which captures the tightness of borrowing constraints or the degree of financial frictions.

Proposition 3 *As μ decreases, both ε_l and $R^r = R^r(\varepsilon_l)$ decrease.*

This proposition shows that as the borrowing constraints are tighter, the real interest rate becomes lower and hence the unbacked public debt is more likely to be valued and rolled over. When the real interest rate is sufficiently low such that $R^r(\varepsilon_l) < 1 + g$, another steady state emerges in which the interest rate is equal to the economic growth rate, $R^r(\varepsilon_h) = 1 + g$. In this case, the steady-state version of equation (14) becomes

$$1 = \beta \left[1 + \int_{\varepsilon_h}^{\varepsilon_{\max}} \left(\frac{\varepsilon}{\varepsilon_h} - 1 \right) dF(\varepsilon) \right].$$

The integral term in the above equation represents liquidity premium due to financial frictions. Without this term, the above equation cannot hold for $\beta \in (0, 1)$. Since $R^r(\varepsilon_h) = 1 + g$ and public debt D_t grows at the rate $1 + g$ in the steady state, the transversality condition cannot rule out a bubble in the steady state. Formally, it follows from (14) that the bubble component in the deterministic steady state satisfies

$$\lim_{T\to\infty} \frac{\beta^T \Lambda_{t+T}}{\Lambda_t} D_{t+T} Q_{t+T}^l = A_0 d \left(1 + g\right)^t \lim_{T\to\infty} \left(\frac{1+g}{R^r} \right)^T > 0. \tag{47}$$

In our model the government bonds are net worth of entrepreneurs and help them overcome borrowing constraints. They are willing to trade government bonds to insure against their idiosyncratic investment shocks. Thus government bonds command liquidity premium and can be valued even though they are not backed by any fiscal surplus.

3.3 Government Debt Backed by Fiscal Surplus

In this subsection we study the case in which there is fiscal surplus in the steady state, i.e., $s/y > 0$. Then public debt is backed by fiscal surplus. In this case we have the standard result.

Proposition 4 *Suppose that $\mu \geq 0$ is sufficiently small and the steady-state primary-surplus-to-output ratio is fixed at $s/y > 0$. Then there exists a unique steady state in which $R^r = R^r(\varepsilon_p) > 1 + g$, where $\varepsilon_p \in (\varepsilon_{\min}, \varepsilon_{\max})$ is the unique solution to equation (46). The real value of government liabilities relative to capital is given by $R^r d/k = \Phi(\varepsilon_p) > 0$.*

Because $R^r > 1 + g$ and public debt grows at the economic growth rate $1 + g$ in the steady state, the bubble component in the steady state is equal to zero by (47). Thus the real value of public debt is entirely determined by its fundamental value.

To prove Proposition 4. We consider two cases. First, if $R^r(\varepsilon_l) > 1 + g$, then we can show that there $\Psi(\varepsilon^*)$ is positive and increases with ε^* on $(\varepsilon_l, \varepsilon_{\max}]$. Thus there is a unique steady state cutoff ε_p such that (46) holds for $s/y > 0$ and $R^r(\varepsilon_p) > R^r(\varepsilon_p) = 1 + g$.

Figure 2 shows the case of $R^r(\varepsilon_l) < 1+g$. The curve $\Psi(\varepsilon^*)$ crosses the horizontal line with $s/y > 0$. We ignore the crossing point in the region $[\varepsilon_{\min}, \varepsilon_l]$ as the implied $R^r d/y < 0$ shown in Panel D. The crossing point ε_p must be in the region $[\varepsilon_h, \varepsilon_{\max}]$. Then we also have $R^r(\varepsilon_p) > R^r(\varepsilon_h) = 1+g$.

3.4 Sustainability of Fiscal Deficits

Can a permanent fiscal deficit be sustained in the long run? What is the maximum sustainable primary deficit in the long run? The following proposition address these questions.

Proposition 5 *Suppose that μ is sufficiently small and that $R^r(\varepsilon_l) < 1+g$. For any given $s/y \in (-\underline{s}, 0)$, where*

$$-\underline{s} = \min_{\varepsilon^* \in [\varepsilon_l, \varepsilon_h]} \Psi(\varepsilon^*) < 0,$$

there exist (at least) two steady states with $R^r(\varepsilon_l) < R^r(\varepsilon_l^*) < R^r(\varepsilon_h^*) < R^r(\varepsilon_h) = 1+g$, where $\varepsilon_l < \varepsilon_l^* < \varepsilon_h^* < \varepsilon_h$ and both ε_l^* and ε_h^* solve equation (46). The real value of government liabilities relative to capital is given by $R^r d/k = \Phi(\varepsilon_l^*)$ and $R^r d/k = \Phi(\varepsilon_h^*)$, respectively. If $s/y < -\underline{s}$, then there does not exist a steady state.

The critical condition in this proposition is the same as that in Proposition 2; that is, the steady-state real interest rate on the unbacked public debt must be lower than the economic growth rate. This condition can support not only a steady state with a positive value of the unbacked public debt as in Proposition 2, but also at least two other steady states in which primary deficits last forever. Figure 2 illustrates the case with exactly two steady states.

The multiplicity is due to the non-monotonicity of $\Psi(\varepsilon^*)$, which is similar to a tax Laffer curve. Intuitively, for $s/y < 0$ and $R^r < 1+g$, the government effectively taxes households

$$-\Psi(\varepsilon^*) = \left(\frac{1}{R^r} - \frac{1}{1+g}\right)\frac{R^r d}{y}$$

to cover primary deficits $-s/y > 0$ by equations (44) and (46). An increase in the real interest rate R^r reduces the "tax rate" $1/R^r - 1/(1+g)$, but it may raise the "tax base" $R^r d$ due to the liquidity premium. In particular, an increase in R^r reduces the liquidity premium q^l by (14), thereby reducing the capital price q^k. Then aggregate capital demand rises. The credit-constrained entrepreneurs need more public debt value $R^r d$ to raise their net worth to finance investment. Thus total "taxes" $-\Psi(\varepsilon^*)$ may first increase with R^r and later decrease with R^r. This implies that there may exist multiple interest rates such that $\Psi(\varepsilon^*) = s/y$ holds. Equivalently, there may exist multiple cutoffs ε^* such that this equation holds because the interest rate R^r increases monotonically with ε^*.

We can rewrite the deterministic steady-state version of (38) as

$$d = \lim_{T\to\infty} \sum_{k=0}^{T} \left(\frac{1+g}{R^r}\right)^{k+1} s + \lim_{T\to\infty} \left(\frac{1+g}{R^r}\right)^{T+1} d. \tag{48}$$

The first term represents the fundamental value, or the present value of deficits for $s < 0$, and the second term represents the bubble component. If $R^r < 1 + g$, then the fundamental value explodes to negative infinity. However, the bubble component explodes to positive infinity. Proposition 5 shows that the sum of these two components can be a finite positive value, which is the real value of public debt. The government can keep rolling over debt to finance fiscal deficits and repay debt at an interest rate lower than economic growth. As discussed in Section 3.2, entrepreneurs are willing to hold government bonds because these bonds provide self-insurance against idiosyncratic investment shocks. Thus a positive value of government bonds can be supported in equilibrium.

If the deficit-to-output ratio $|s|/y$ is too high, then the government may issue too much bonds that exceed the demand capacity of entrepreneurs. As a result an equilibrium does not exist and primary deficits cannot be sustained.

4 Quantitative Implications

In this section we study quantitative implications of our model by calibrating our model to the US data at quarterly frequency. We are interested in the following questions: What is the maximum sustainable level of the primary deficit? What is the implied value of public debt? What is the stability of the steady states and the local determinacy of equilibria around a steady state? What are the dynamic responses of the economy to a monetary or fiscal policy shock?

4.1 Calibration

Our model can generate multiple steady states depending on parameter values. To study the possibility of sustaining permanent primary deficits, we calibrate our model such that the conditions in Proposition 5 hold. Moreover, we calibrate our model such that the steady state with the lowest interest rate match some long-run moments in the US data over 1950-2019. We choose this steady state because there has been a secular decline in interest rates across almost all advanced economies.[7]

The calibrated parameters are listed in Table 1. We first set $\alpha = 0.33$ and $\beta = 0.99$ as in the standard macroeconomics literature. Set $g = 0.0315/4$ so that the annual real GDP growth rate is 3.15% as in the data. Set $\Pi = 1 + 0.0309/4$ to be consistent with the average annual inflation rate of 3.09% during the period 1950-2019. For simplicity we assume that G_t/Y_t is constant for all

[7] Kaas (2016) also calibrates his model such that the low-interest-rate steady state matches the data.

Table 1: Calibrated Parameters at Quarterly Frequency

Parameter	Values	Description	Target
α	0.33	Capital elasticity	Capital income share
g	0.79%	Labor efficiency growth	GDP growth
$\Pi - 1$	0.77%	Inflation target	Inflation rate
G/Y	11.1%	Government spending share	Government spending to GDP ratio
s/y	-0.28%	Primary surplus to output ratio	Primary surplus to GDP ratio
ξ	0.75	Price adjustment probability	Duration of price adjustments
σ	11	Goods elasticity of substitution	Price markup
β	0.99	Discount factor	
δ	1.334%	Depreciation rate	Equity return
μ	0.188	Capital pledgeability	Real interest rate
η	0.42	Pareto shape	Investment-to-GDP ratio
$\epsilon_{\min}$	0.58	Pareto scale	Normalization $\mathbb{E}[\varepsilon] = 1$
ψ	3.75	Labor disutility	Number of hours worked

t and calibrated to match the long-run average 11.1% in the data. Set the steady-state primary-surplus-to-GDP ratio to -0.28%, which matches the long-run average in the data. Set $\xi = 0.75$ and $\sigma = 11$ so that the duration of price adjustments is four quarters and the steady-state markup is $\sigma/(\sigma-1) = 1.1$, consistent with the DNK literature. We choose ψ such that the steady-state labor is equal to 0.25 as in the business cycles literature.

It remains to calibrate three parameters δ, η, and μ. We adopt the Pareto distribution for the idiosyncratic investment shock $F(\varepsilon) = 1-(\varepsilon/\varepsilon_{\min})^{-\frac{1}{\eta}}$ and set $\varepsilon_{\min} = 1-\eta$ so that the unconditional mean is 1. We set $\delta = 1.334\%$ so that the steady-state equity (or investment) return $R_k/q^k + (1-\delta)$ is equal to 4% per annum. As is well known the equity premium is about 6% per annum in the data. Our steady-state target of 4% appears reasonable given that risk premium is absent in the deterministic steady state. We set $\eta = 0.420$ and $\mu = 0.188$ so that the real interest rate and the investment-to-GDP ratio in the steady state with low interest rate are equal to 1.9% per annum and 17.4%, respectively, as in the data.[8] The calibrated $\mu = 0.188$ is in line with those reported in Miao, Wang and Xu (2015) and Dong, Miao and Wang (2020).

4.2 Maximum Sustainable Deficit

Using the calibrated parameters given in Table 1, we can calculate the maximum sustainable deficit-to-output ratio $|s|/y$ as in Proposition 5. As shown in Figure 3, there are two steady states with primary surplus $s/y < 0$. The maximum sustainable deficit-to-output ratio is 0.30%, which is smaller than the number of 0.834% in Kaas (2016). When the economy is at its maximum sustainable deficit to output ratio, the annual net real interest rate is $4 * (R^r - 1) = 2.16\%$ and the

[8] We use the data of the nominal interest rates for the entire portfolio of the U.S. government bonds from Hall et al. (2018) and the GDP deflator to obtain real interest rates.

public-debt-to-annual-GDP ratio is $d/(4y) = 29.91\%$.

As s/y increases from -0.30% to 0, the smaller steady-state interest rate declines until the investment cutoff decreases to ε_l. In the meantime, the larger steady-state interest rate increases to $1+g$ until the investment cutoff rises to ε_h. When s/y further increases from zero to a positive number, real interest rate R^r increases from $1+g$. Moreover, capital, output, and the public debt to GDP ratio all increase. As $s/y \to +\infty$, $R^r \to (1+g)/\beta$ and $d/y \to +\infty$.

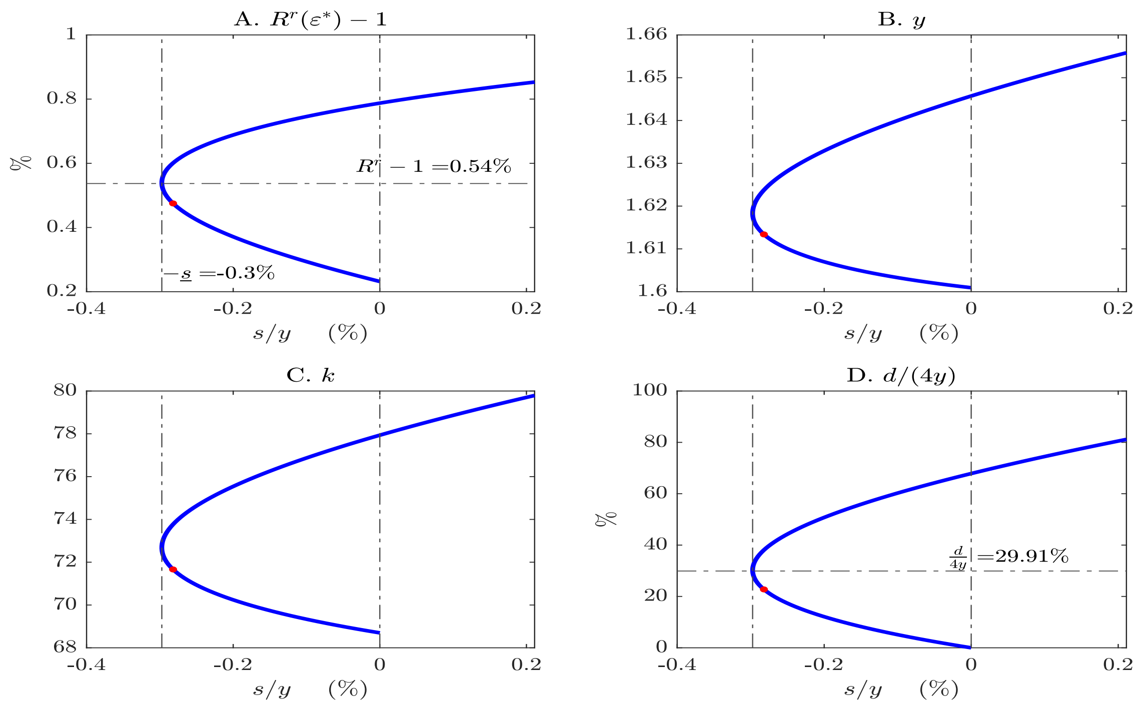

Figure 3: Steady state values for various primary-surplus-to-output ratios s/y. The dot point in each curve shows the steady state of the model under the calibration in Table 1.

Figure 3 shows an interesting property. Suppose that the economy always stays in the low-interest-rate steady state as the primary deficit declines. Then the real interest rate, capital, output, and public-debt-to-output ratio all decline. Intuitively, as deficit $|s|/y$ declines, the investment cutoff ε^* declines, the liquidity premium increases, and hence the real interest rate declines. Then the government reduces debt issuance. But this hurts entrepreneurs because government bonds are their net worth used to finance investment spending. As a result, entrepreneurs accumulates less capital and the steady-state output declines. The opposite results obtain if the economy always stay in the high-interest-rate steady state as the primary deficit declines.

Notice that our model generated maximum deficit-to-output ratio is lower than those estimated in the literature. Using OLG models, Chalk (2000) finds that primary deficits up to 5.2% are sustainable, while Bullard and Russell (1999) calibrate a similar model with a primary deficit of 1.9%. Using an infinitely-lived agent model with financial frictions, Kaas (2016) finds that the maximum deficit-to-output ratio is 0.837%.

Chalk (2000) calibrates the real interest rate to a lower value of 1.2% per annum. If we follow the same calibration strategy as in Section 4.1 but target his value of real interest rate with $R^r = 1 + 1.2\%/4$, we obtain $\delta = 1.28\%$, $\mu = 0.175$ and $\eta = 0.418$. The new calibration generated maximum sustainable deficit-to-output ratio is -0.39%. Given this ratio, the net real interest rate is $4 * (R^r - 1) = 1.96\%$ per annum and the public-debt-to-annual-output ratio is $d/(4y) = 33.31\%$. Thus the maximum sustainable deficit-to-output ratio and public-debt-to-output ratio do not change much.

The difference in estimates is likely due to the structural differences between OLG and infinitely-lived agents models. One possible explanation is that OLG models with hump-shaped earnings profiles permit the government to roll over larger stocks of debt. For infinitely-lived agents models with credit constraints, such large deficits are not sustainable for plausible parameter values.

4.3 Local Determinacy

Given our calibrated parameter values in Table 1, there are two steady states for the detrended system as shown in Figure 3. In this subsection we study the stability of these steady states and local determinacy of equilibria around each of these steady states. Due to the complexity of our model, we are unable to derive analytical results. We thus use numerical methods. We first linearize the detrended system around a steady state and then study the determinacy and stability of the linearized system using Klein's (2000) method. The linearized system is presented in Appendix D.

We first consider a real version of our model by removing the pricing and monetary policy block and setting $\Pi_t = 1$ for all t. Following Kaas (2016), we set $\phi_s = 0$ in (29) or assume that S_t/Y_t is exogenously given. We summarize the detrended system by a system of 13 equations for 13 variables $\{R_t, R_{kt}, \lambda_t, \varepsilon_t^*, q_t^k, q_t^l, w_t, d_t, k_t, N_t, y_t, c_t, i_t\}$ given in Appendix B. The predetermined variables are $\{R_t, d_t, k_t\}$. The steady-state allocation remains the same as in the monetary model. Using the same calibration as in Table 1, we find that the low-interest-rate steady state is a saddle and the high-interest-rate steady state is a source, a result similar to Kaas (2016). This implies that the equilibrium around the low-interest-rate is locally unique, but there is no local equilibrium around the high-interest-rate steady state.

Now we consider our monetary model, which generates very different results. In this case we need to stabilize both inflation and public debt. The determinacy depends on the policy rules in (29) and (30). The critical parameters are the policy response coefficients (ϕ_s, ϕ_π). Figure 4 presents the determinacy region for the calibrated model under different policy mix (ϕ_s, ϕ_π). For ease of numerical computations, we consider only values of ϕ_s in $(-1, 1)$ and $\phi_\pi > 0$.

As shown in Figure 4, the vertical line $\phi_s^* = 1/\beta - 1$ and the horizontal line $\phi_\pi^* = 1$ partition the policy parameter space into four regions for the flexible-price model of Leeper (1991): (i) unique

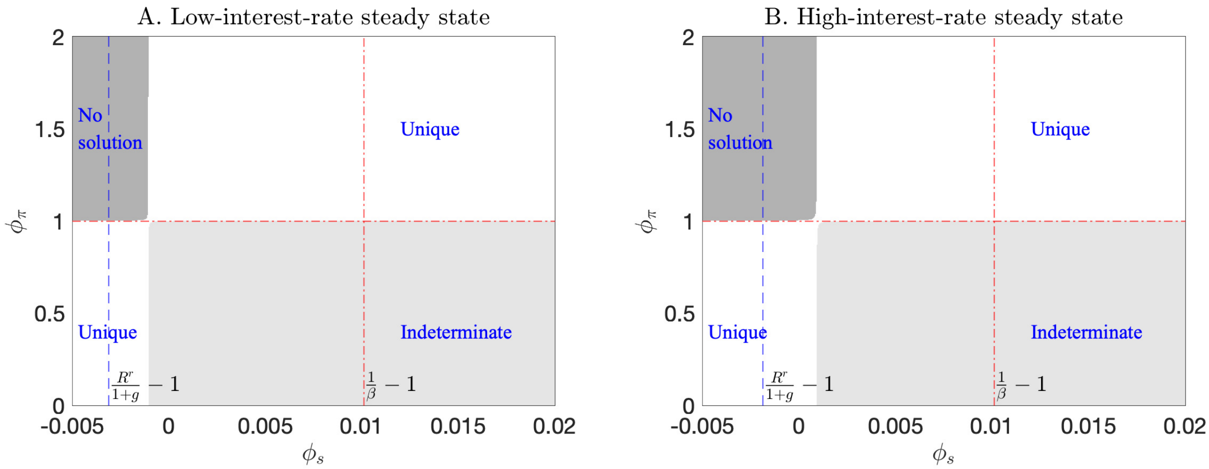

Figure 4: Local determinacy regions for the policy mix parameters $\{\phi_s, \phi_\pi\}$ for $s/y = -0.28\%$.

equilibrium for $\phi_s > 1/\beta - 1$ and $\phi_\pi > 1$ (active monetary policy and passive fiscal policy, regime M); (ii) unique equilibrium for $\phi_s < 1/\beta - 1$ and $\phi_\pi < 1$ (passive monetary policy and active fiscal, regime F); (iii) no bounded equilibrium for $\phi_s < 1/\beta - 1$ and $\phi_\pi > 1$ (active monetary policy and active fiscal policy); and (iv) indeterminate equilibria for $\phi_s > 1/\beta - 1$ and $\phi_\pi < 1$ (passive monetary policy and passive fiscal policy). This result still holds for the standard DNK model without capital (e.g., Woodford (2003)).[9] It also holds for the standard DNK model with capital if the distortions in the economy are not too large (e.g., Lubik (2003)).

By contrast, our model with financial frictions delivers different results. We focus our analysis on the determinacy around the low-interest-rate steady state illustrated in Panel A of Figure 4, as the analysis for the high-interest-rate steady state is similar. We find that there are three disjoint regions of the policy parameter space. The two regions that ensure a unique equilibrium in Leeper (1991) become one region in our model.[10] Unlike in Leeper (1991), the boundaries of this region are nonlinear. The Taylor principle threshold $\phi_\pi^* = 1$ is no longer the critical value to stabilize inflation and the steady-state net interest payment $R^r/(1+g) - 1$ is no longer the critical value to stabilize debt dynamics.

To understand the intuition, we use the linearized fiscal policy rule equation and the government budget constraint to derive

$$\widetilde{d}_t = \left(\frac{R^r}{1+g} - \phi_s\right)\widetilde{d}_{t-1} + \frac{R^r}{1+g}\frac{d}{y}\left(\widehat{R}_{t-1} - \widehat{\Pi}_t\right) - z_{st}, \tag{49}$$

where a variable with a tilde denotes derivation from its steady state relative to steady-state output (e.g., $(\widetilde{d}_t - d)/y$) and a variable with a hat denotes log deviation from its steady state (e.g., $\widehat{\Pi}_t = (\widehat{\Pi}_t - \Pi)/\Pi$). In the standard model without financial frictions (e.g., Leeper (1991)),

[9] Woodford (2003, Proposition 4.11) considers a fiscal policy rule that reacts to the maturity value of real public debt $R_t d_t$, instead of d_t in (29). But the result is essentially the same.

[10] Canzoneri et al. (2011) and Cui (2016) obtain similar results in different models.

the steady-state real interest expense is given by $R^r/(1+g)-1=1/\beta-1$ and the short-run real interest rate $\widehat{R}_{t-1}-\mathbb{E}_{t-1}\widehat{\Pi}_t$ is independent of public debt $\widehat{d}_{t-1}$. Thus we obtain the standard critical value $1/\beta-1$. By contrast, the long-run real interest expense in our model with financial frictions is $R^r/(1+g)-1$, which is less than zero and lower than $1/\beta-1$. Importantly, the real interest rate $\widehat{R}_{t-1}-\mathbb{E}_{t-1}\widehat{\Pi}_t$ responds to public debt $\widetilde{d}_{t-1}$ due to the liquidity premium.[11] Thus the stability of debt cannot be determined by the coefficient of $\widetilde{d}_{t-1}$ in (49) alone.

Let us still apply Leeper's (1991) definition of passive/active policy thresholds $\phi_\pi^*=1$ and $\phi_s^*=R^r/(1+g)-1$.[12] We also keep his terminology of regime M and regime F. Then Figure 4 shows that both active and passive monetary policy can be combined with a passive fiscal policy to ensure price determinacy. Policy parameters in regime M may not ensure determinacy. We need fiscal policy to be more passive to ensure determinacy. Formally, for $\phi_\pi>1$, the fiscal policy parameter ϕ_s must be sufficiently larger than $R^r/(1+g)-1$ to ensure determinacy because of the positive relation between the real interest rate and public debt. For $\phi_\pi<1$, the fiscal policy parameter ϕ_s must be small to ensure determinacy. But we do not need $\phi_s<R^r/(1+g)-1$. There are two nonlinear boundaries for the determinacy region. Both boundaries are increasing curves.

An important feature of our model is that there may exist two steady states given the same long-run primary-deficit-to-output ratio (see Figures 2 and 3). While the determinacy property around the high-interest-rate steady state is similar to that for the other steady state as illustrated in Panel B of Figure 4, the multiplicity of the steady states generates some interesting implications absent from the literature.

First, a fiscal and monetary policy mix is important not only for local determinacy of equilibria around a particular steady state, but also for selecting a particular steady state. For example, suppose that there exist two steady states for the same $s/y<0$ as shown in Figure 2. Let d_l (d_h) be the detrended public debt value in the steady state with low (high) interest rate. Either one of them can be used as a debt target in the fiscal policy rule (29). Then this rule combined with the monetary policy rule (30) can select a local equilibrium around a particular steady state.

Second, the determinacy region for the two steady states are different as illustrated in Figure 4. This means that, given the same policy response coefficients ϕ_s and ϕ_π, the local equilibrium is determinate around one steady state, but may be indeterminate around the other steady state. One particular example is that for an economy around the low-interest-rate steady state, a policy mix with $\phi_s=0$ and $\phi_\pi=1.5$ implies that the model has a unique bounded solution and the economy is in regime M. However, for the same policy mix, if the economy is around the high-interest-rate steady state, the model does not have any bounded solutions. In this case, the strong reaction of monetary policy to inflation ($\phi_\pi>1$) will increase the interest expense and lead to an explosive

[11] See Dominguez and Gomis-Porqueras (2019) for a similar discussion.

[12] More formally, monetary policy is active if $|\phi_\pi|>1$, and passive if $|\phi_\pi|<1$. Fiscal policy is active if $|R^r/(1+g)-\phi_s|>1$, and passive if $|R^r/(1+g)-\phi_s|<1$.

path for public debt. Another example is that around the high-interest-rate steady state, a policy mix with $\phi_s = 0$ and $\phi_\pi = 0.8$ can guarantee equilibrium determinacy. However, the same policy mix leads to indeterminacy around the low-interest-rate steady state.

Third, Woodford (2001) discusses the interest-rate-peg policy which corresponds to $\phi_s = \phi_\pi = 0$ in our model. As shown in Panel B of Figure 4, these policy parameters determine a unique equilibrium around the high-interest-rate steady state. However, they fall in the indeterminacy region for the low-interest-rate steady state as shown in Panel A of Figure 4.[13] The intuition is as follows.

The fiscal rule $\phi_s = 0$ is an active policy in the standard DNK model, in which the steady-state interest rate R^* for log utility satisfies $R^* = (1+g)/\beta > (1+g)$. With positive interest expenses ($R^* > 1+g$) and without raising taxes for $\phi_s = 0$, the government must raise an explosive amount of debt eventually to cover accumulated interest expenses starting from any initially given government liabilities. To obtain a unique bounded equilibrium, debt must be derived as a forward-looking solution and the initial inflation is adjusted to ensure the government budget constraint is satisfied (see equations (28) and (38)). This also suggests that the stabilization of public debt constantly relies on the debt revaluation through surprise inflation, with the latter being possible only if monetary policy is passive enough (e.g., $\phi_\pi = 0$). This is regime F discussed in Leeper (1991) and Woodford (2001).

By contrast, the fiscal rule $\phi_s = 0$ is a passive policy in our calibrated model because $R^r < (1+g)$. Importantly, the real interest rate $\widehat{R}_{t-1} - \mathbb{E}_t \widehat{\Pi}_t$ responds to debt value $\widetilde{d}_{t-1}$ due to the liquidity premium. We are unable to derive a closed-form solution to check determinacy. Our numerical results show that the fiscal rule $\phi_s = 0$ is passive enough to stabilize public debt by the fiscal authority itself for the local equilibrium around the low-interest-rate steady state. When monetary policy is passive with $\phi_\pi = 0$, inflation is also stabilized given any initial level of inflation. Thus the low-interest-rate steady state is a sink and equilibria around this steady state is indeterminate of degree one. We will illustrate this point further in the next subsection.

Our numerical results also show that the interest rate response to debt is so strong that debt dynamics cannot be stabilized by the fiscal authority itself around the high-interest-rate steady state. We then obtain a unique saddle-path equilibrium around that steady state as in regime F of the standard DNK model discussed above, even though both fiscal policy $\phi_s = 0$ and monetary policy $\phi_\pi = 0$ are passive in our model.

[13] Bassetto and Cui (2018) find a similar result in different models.

4.4 Impulse Responses

In this subsection we study the impulses responses of the calibrated model economy to a one percent shock to monetary and fiscal policy rules in (29) and (30).[14] We focus on the local equilibrium dynamics around the low-interest-rate steady state. The local dynamics around the high-interest-rate steady state are qualitatively similar. For regime M, we set $\phi_s = 1/\beta - 1$, $\phi_\pi = 1.5$, and for regime F, we set $\phi_s = R^r / (1+g) - 1 = -0.0031$, $\phi_\pi = 0.8$. These parameters lead to equilibrium determinacy as shown in Figure 4. To introduce persistence, we also set $\rho_s = \rho_\pi = 0.5$.

We first consider the impact of a one percentage point increase in the interest rate (a contractionary monetary policy shock) as shown in Figure 5. For regime M, we obtain the conventional impulse responses. In particular, the real interest rate rises, but consumption, investment, labor, and output all decline on impact. Thus inflation declines. The evolution of the real value of public debt is backward-looking. The initial decline in inflation leads to a higher real value of debt liabilities. Without immediate adjustments in the primary surplus, the government has to issue more public debt and therefore public debt rises initially. The rising interest expense further raises future debt value. But with the endogenous response of primary surplus, the value of public debt declines later on and goes back to its steady state level. Figure 5 also shows the relationship among real interest $(\widehat{R}_t^r)$, real debt $(\widetilde{d}_t)$, liquidity premium $\left(\widehat{q}_t^l\right)$, and Tobin's Q $\left(\widehat{q}_t^k\right)$. The last panel shows the dynamics of markup $(-\widehat{p}_{wt})$, which is important in the standard DNK model.

For regime F, a positive interest-rate shock also generates a contractionary impact on quantities initially. But there are two major differences from regime M. First, inflation rises on impact, but the real value of public debt declines. The interpretation by the standard FTPL is that the real value of public is equal to the present value of future surpluses discounted by the stochastic discount factor (fundamental value), which is related to real interest rates. If the price level does not change, then the real value of public debt would decline. Because nominal debt is predetermined, the government budget constraint (28) would be violated. Thus the initial price level (or inflation) must rise. This argument could fail because the present value may not be well defined when the interest rate is less than economic growth and when the government runs persistent primary deficits.

By contrast, we demonstrate that the real value of public debt is equal to fundamental value plus a bubble component in regime F of our calibrated model with long-run fiscal deficits. As equation (48) shows, the present value of future surpluses approaches negative infinity as the real interest rate is too low. But the bubble component approaches positive infinity to ensure the sum is a positive finite value. An increase in the real interest rate reduces the bubble component of the public debt and hence its real value. Thus the initial price or inflation must rise to satisfy the government budget constraint as nominal debt is predetermined (see (28) and (38)). In regime F

[14] See Kim (2003) for impulse responses to various shocks in a standard DNK model without capital. We have conducted a similar analysis for a standard DNK model with capital. Such results are available upon request.

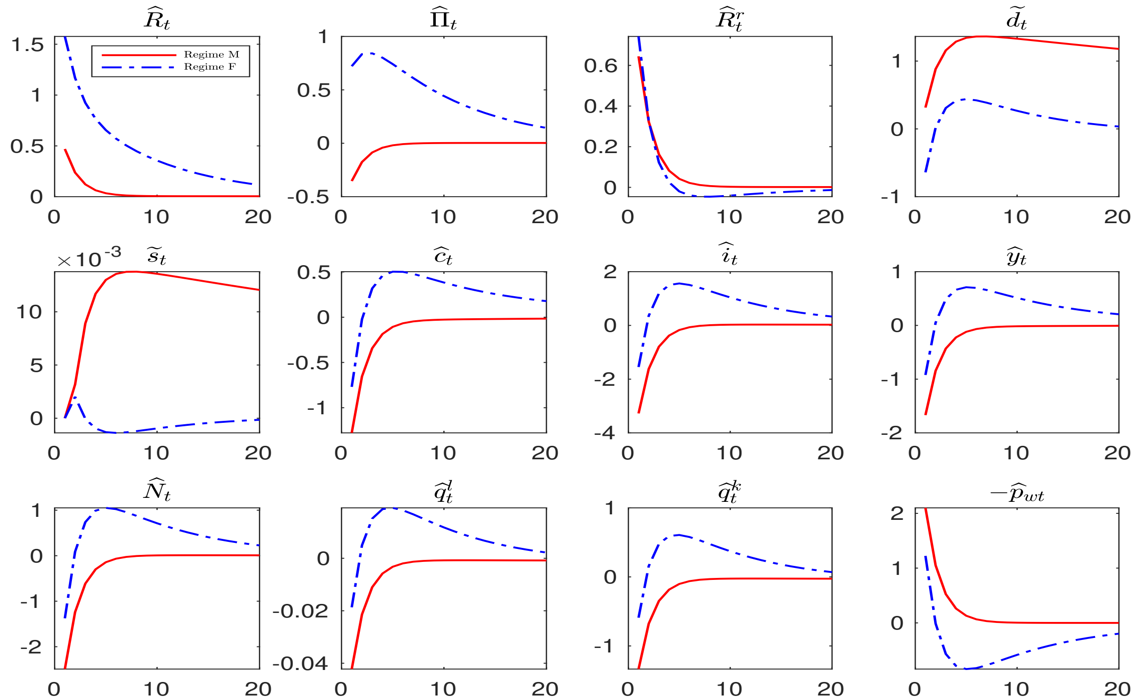

Figure 5: Impulse response functions to a positive 1% innovation shock to the nominal interest rate starting from the low-interest-rate steady state. All vertical axes represent percentage points.

(28) is an equilibrium condition.

Second, consumption, investment, and output rise above the steady state for some periods before they revert. The intuition is that government liabilities rise given a positive shock to the nominal interest rate. In the next period, entrepreneurs then make more investment and hire more workers due to the wealth effect. Households raise consumption as their wages rise. The initial drop of consumption, investment, labor, and output is due to the initial rise of the real interest rate as prices are sticky.

Next we consider the impact of a positive fiscal policy shock as shown in Figure 6. In the standard DNK model without financial frictions, this shock does not affect the real economy and inflation in regime M, because Ricardian equivalence holds and the determination of inflation is independent of the fiscal authority's behavior. For our model with financial frictions, this shock has an impact on the real economy, but the quantitative effect is very small in regime M. Unlike in the standard DNK model, the real value of debt in our model does not decline one-to-one as the primary surplus rises on impact because inflation responds to a surplus shock. Moreover, as households do not hold any bonds, they reduce consumption when primary surpluses/taxes rise. Households then increase labor supply causing output to rise on impact.

By contrast, a positive surplus shock has a significant contractionary impact on the economy in regime F. The interpretation by the FTPL is that the real value of public debt rises in response

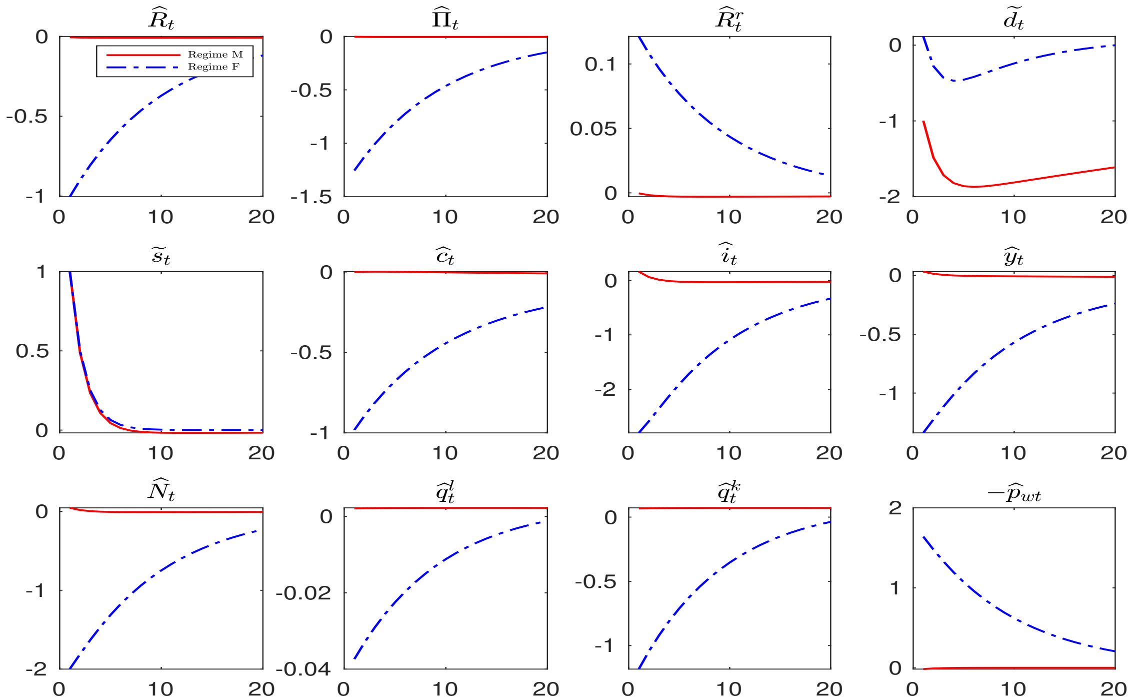

Figure 6: Impulse response functions to a positive 1% surplus shock starting from the low-interest-rate steady state. All vertical axes represent percentage points.

to the shock because its fundamental value (present value of future surpluses) rises, holding the discount rate (real interest rate) constant. Then the initial price or inflation must decline to balance the government budget. Given the passive interest-rate rule $\phi_\pi = 0.8$, the nominal interest rate declines less than the decline of inflation so that the real interest rate rises on impact, leading to an economic contraction. As discussed earlier, this argument could fail when the real interest rate is less than economic growth and when the government runs persistent deficits as the fundamental value may not be well defined. In our model public debt contains a bubble component. In response to a positive surplus shock, the real value of public debt rises because entrepreneurs believe the bubble component will rise. This belief is self-fulfilling in equilibrium.

Finally, consider the impact of the interest-rate-peg policy discussed by Woodford (2001), which corresponds to $\phi_s = \phi_\pi = 0$ in our model. This policy mix generates a unique determinate equilibrium in regime F in the standard DNK model and also in our model around the high-interest-rate steady state. As discussed in the previous subsection, this policy mix represents a passive monetary policy and passive fiscal policy mix. It generates indeterminate equilibria around the low-interest-rate steady state as shown in Figure 4. We now introduce a sunspot shock to the inflation expectation and study its impact in Figure 7. Formally, let $\widehat{\Pi}_t = \mathbb{E}_{t-1}\widehat{\Pi}_t + \eta_t$, where η_t is an exogenous independently and identically distributed sunspot shock.

Figure 7 shows two important properties: (i) a sunspot shock to inflation can be self-fulfilling;

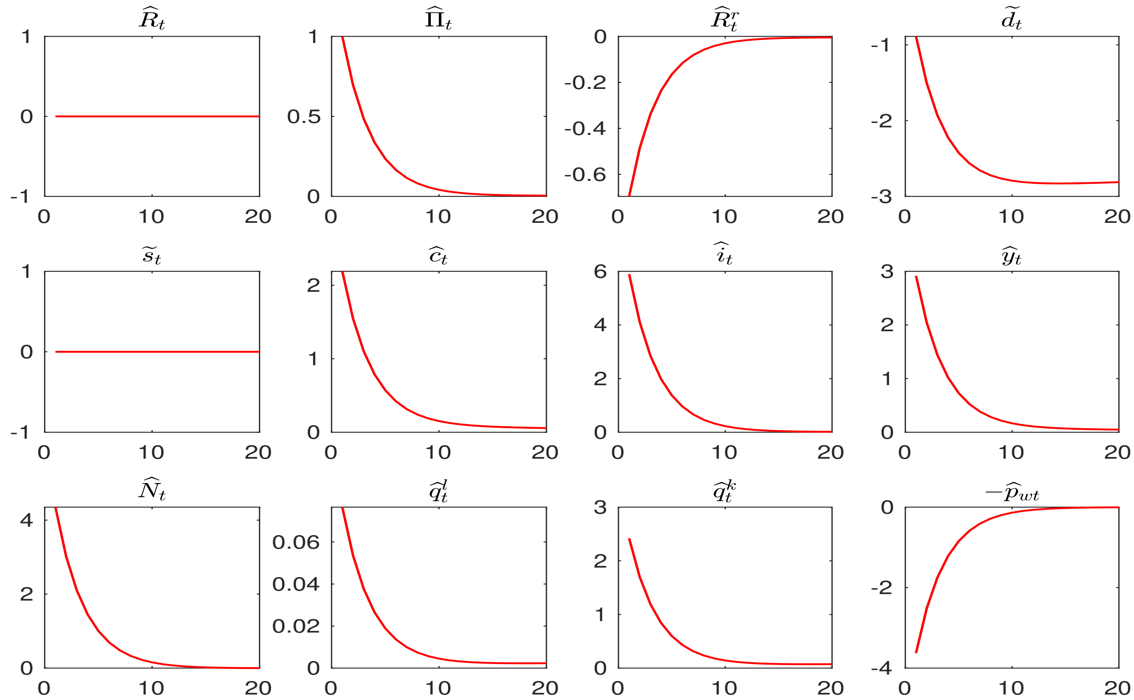

Figure 7: Impulse response functions to a 1% sunspot increase in inflation starting from the low-interest-rate steady state. All vertical axes represent percentage points.

and (ii) a sunspot increase in inflation is expansionary. Intuitively, given $\phi_\pi = 0$, a surprised 1% increase of inflation does not change the nominal interest rate. As a result, the real interest rate declines. It follows that Tobin's Q rises and firms increase their investment and labor demand. The increased labor demand raises the real wage and hence households raise consumption due to the wealth effect. With higher real wages, firms face higher marginal costs and thus set higher prices, justifying initial subjective beliefs about an increase in inflation.

As discussed in the previous subsection the fiscal rule $\phi_s = 0$ constitutes a passive policy around the low-interest-rate steady state in our model, instead of an active policy. The primary surplus does not respond to changes in the real value of public debt and remains at its negative steady-state value over time given our calibration. But public debt is backward-looking and can be stabilized by the fiscal policy. As shown in Figure 7, a sunspot increase in inflation reduces the real value of public debt on impact. With declined negative (net) real interest rates, the government issues less new debt to repay old debt over time due to decreased interest expenses. Formally, as $R^r/(1+g) < 1$ in equation (49), debt $\widetilde{d}_t$ can be stabilized even for $\phi_s = 0$. By contrast, in the standard DNK model, $R^r/(1+g) = 1/\beta > 1$. Thus a fiscal policy without adjusting taxes for $\phi_s = 0$ cannot stabilize debt unless monetary policy is passive by allowing inflation to revalue debt. In this case the fiscal policy with $\phi_s = 0$ in the standard DNK model is active.

5 Conclusion

We have provided a DNK model with financial frictions to study the interactions of monetary and fiscal polices in a world with low interest rates and high public debt. Our key assumption is that entrepreneurs face uninsurable idiosyncratic investment risk and credit constraints. Government bonds provide liquidity service and raise entrepreneurs' net worth. Multiple steady states with positive values of public debt can be supported for a given permanent deficit-to-output ratio. The steady-state interest rates are less than economic growth and public debt contains a bubble component. We analyze the determinacy regions of policy parameter space and find that a large set of monetary and fiscal policy parameters in either regime M or regime F can achieve debt and inflation stability given persistent fiscal deficits.

Our paper has focused on positive policy questions. It would be interesting to study some normative questions: What are optimal monetary and fiscal policies in a world with low interest rates? What is the welfare cost of a monetary and fiscal policy given persistent primary deficits and low interest rates? We leave these questions for future research.

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Online Appendix

A Proofs

Proof of Proposition 1: The entrepreneur's objective is to solve the following dynamic programming problem:

$$V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) = \max_{\{I_{jt}, D_{jt}, B_{jt}\}} C_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}\left(K_{jt}, B_{jt}, D_{jt}, \varepsilon_{jt+1}\right), \tag{A.1}$$

subject to

$$K_{jt} = (1-\delta) K_{jt-1} + \varepsilon_{jt} I_{jt}, \tag{A.2}$$

$$B_{jt} \geq -\mu K_{jt-1}, \tag{A.3}$$

$$C_{jt} + I_{jt} + B_{jt} + D_{jt} = R_{kt} K_{jt-1} + \frac{R_{t-1}}{\Pi_t} B_{jt-1} + \frac{R_{t-1}}{\Pi_t} D_{jt-1}, \tag{A.4}$$

$$C_{jt} \geq 0. \tag{A.5}$$

We conjecture that the value function takes the following form

$$V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) = \phi_t^k\left(\varepsilon_{jt}\right) K_{jt-1} + \phi_t^b\left(\varepsilon_{jt}\right) B_{jt-1} + \phi_t^d\left(\varepsilon_{jt}\right) D_{jt-1}, \tag{A.6}$$

where $\phi_t^i(\varepsilon_{jt})$, $i \in \{k, b, d\}$, satisfy

$$q_t^k = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^k(\varepsilon) dF(\varepsilon), \tag{A.7}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^b(\varepsilon) dF(\varepsilon), \tag{A.8}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^d(\varepsilon) dF(\varepsilon). \tag{A.9}$$

Substituting (A.2), (A.4), and the above conjecture into the Bellman equation (A.1), we have

$$\begin{aligned}
&V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) \\
= &\max_{\{I_{jt}, D_{jt}, B_{jt}\}} \left(R_{kt} + (1-\delta) \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^k(\varepsilon) dF(\varepsilon)\right) K_{jt-1} \\
&+ \frac{R_{t-1}}{\Pi_t} B_{jt-1} + \frac{R_{t-1}}{\Pi_t} D_{jt-1} + \left[\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^k(\varepsilon) dF(\varepsilon) \varepsilon_{jt} - 1\right] I_{jt} \\
&+ \left[\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^b(\varepsilon) dF(\varepsilon) - 1\right] B_{jt} + \left[\beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int \phi_{t+1}^d(\varepsilon) dF(\varepsilon) - 1\right] D_{jt}.
\end{aligned} \tag{A.10}$$

Optimal choices of B_{jt} and D_{jt} imply that (A.8) and (A.9) must hold in equilibrium. Otherwise, all entrepreneurs will either save or borrow at the same time, contradicting the market-clearing conditions for bonds.

Since $I_{jt} \geq 0$ and $C_{jt} \geq 0$, it follows that $I_{jt} = 0$ if $\varepsilon_{jt} < 1/q_t^k \equiv \varepsilon_t^*$; but the firm makes as much investment as possible so that $C_{jt} = 0$ if $\varepsilon_{jt} > \varepsilon_t^*$. It follows from (A.4) that when $\varepsilon_{jt} > \varepsilon_t^*$, we have

$$I_{jt} = -B_{jt} - D_{jt} + R_{kt}K_{jt-1} + \frac{R_{t-1}}{\Pi_t}B_{jt-1} + \frac{R_{t-1}}{\Pi_t}D_{jt-1}, \tag{A.11}$$

$$D_{jt} = 0, \quad B_{jt} = -\mu K_{jt-1}. \tag{A.12}$$

Consider first the case where $\varepsilon_{jt} < \varepsilon_t^*$ and $I_{jt} = 0$. The entrepreneurs are indifferent between borrowing and saving. Substituting the decision rules into (A.10) and reorganizing yield

$$\begin{aligned} &V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) \\ &= \max_{\{I_{jt}, D_{jt}, B_{jt}\}} \left(R_{kt} + (1-\delta)q_t^k\right) K_{jt-1} + \frac{R_{t-1}}{\Pi_t}B_{jt-1} + \frac{R_{t-1}}{\Pi_t}D_{jt-1}. \end{aligned}$$

Notice that (A.8) and (A.9) ensure that the indeterminacy of B_{jt} and D_{jt} does not affect the value function.

Matching the coefficients, we have

$$\begin{aligned} \phi_t^k(\varepsilon_{jt}) &= R_{kt} + (1-\delta)q_t^k, \\ \phi_t^b(\varepsilon_{jt}) &= \phi_t^d(\varepsilon_{jt}) = \frac{R_{t-1}}{\Pi_t}. \end{aligned}$$

Next we consider the case where $\varepsilon_{jt} > \varepsilon_t^*$. Substituting (A.11) and (A.12) into (A.10) and reorganizing yield

$$\begin{aligned} &V_t\left(K_{jt-1}, B_{jt-1}, D_{jt-1}, \varepsilon_{jt}\right) \\ &= \max_{\{I_{jt}, D_{jt}, B_{jt}\}} \left(R_{kt} + (1-\delta)q_t^k + R_{kt}\left(q_t^k\varepsilon_{jt} - 1\right) - \mu\left(1 - q_t^k\varepsilon_{jt}\right)\right) K_{jt-1} \\ &\quad + \frac{R_{t-1}}{\Pi_t}\left(q_t^k\varepsilon_{jt}\right) B_{jt-1} + \frac{R_{t-1}}{\Pi_t}\left(q_t^k\varepsilon_{jt}\right) D_{jt-1}. \end{aligned}$$

Matching the coefficients yields

$$\begin{aligned} \phi_t^k(\varepsilon_{jt}) &= R_{kt}\left(1 + \left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1\right)\right) + (1-\delta)q_t^k + \mu\left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1\right), \\ \phi_t^b(\varepsilon_{jt}) &= \phi_t^d(\varepsilon_{jt}) = \frac{R_{t-1}}{\Pi_t}\left(q_t^k\varepsilon_{jt}\right) = \frac{R_{t-1}}{\Pi_t}\left(1 + \left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1\right)\right). \end{aligned}$$

Combining the two cases above, we have

$$\begin{aligned} \phi_t^k(\varepsilon_{jt}) &= R_{kt}\left(1 + \max\left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1, 0\right)\right) + (1-\delta)q_t^k + \mu \max\left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1, 0\right), \\ \phi_t^b(\varepsilon_{jt}) &= \phi_t^d(\varepsilon_{jt}) = \frac{R_{t-1}}{\Pi_t}\left(1 + \max\left(\frac{\varepsilon_{jt}}{\varepsilon_t^*} - 1, 0\right)\right), \end{aligned}$$

for $\varepsilon_{jt} \in [\varepsilon_{\min}, \varepsilon_{\max}]$. Substituting these two equations into (A.7), (A.8) and (A.9), we obtain (13) and (14).

Finally, for the entrepreneur's objective to be finite, the value function must satisfy the following condition by the Bellman equation (A.1):

$$\lim_{i\to\infty} \mathbb{E}_t \frac{\beta^i \Lambda_{t+i}}{\Lambda_t} V_{t+i}\left(K_{jt+i-1}, B_{jt+i-1}, D_{jt+i-1}, \varepsilon_{jt+i}\right) = 0.$$

Using equations (A.6)-(A.9) we can derive transversality conditions (16). Q.E.D.

Proof of Lemma 1: Taking derivative of $R_k(\varepsilon^*)$ in (42) and reorganizing yields

$$\frac{\partial R_k(\varepsilon^*)}{\partial \varepsilon^*} = \frac{\mu \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) - (\beta^{-1}(1+g) - 1 + \delta) F(\varepsilon^*)}{\left[\int_{\varepsilon_{\min}}^{\varepsilon_{\max}} \max\left(\varepsilon, \varepsilon^*\right) dF(\varepsilon)\right]^2}. \tag{A.13}$$

The numerator in (A.13) is strictly decreasing in ε^*, with the maximum and the minimum being $\mu\mathbb{E}\left[\varepsilon\right] \geq 0$ and $-(\beta^{-1}(1+g) - 1 + \delta) < 0$, respectively. Hence, by the intermediate value theorem, there exists a unique threshold $\varepsilon_k \in [\varepsilon_{\min}, \varepsilon_{\max}]$ such that

$$\mu \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) - (\beta^{-1}(1+g) - 1 + \delta) F(\varepsilon_k) = 0.$$

And it follows that $\partial R_k(\varepsilon^*)/\partial\varepsilon^* > 0$ if $\varepsilon^* < \varepsilon_k$; $\partial R_k(\varepsilon^*)/\partial\varepsilon^* \leq 0$ if $\varepsilon^* \geq \varepsilon_k$. Moreover, we have $\varepsilon_k = \varepsilon_k(\mu)$ strictly increasing and $\lim_{\mu\to 0} \varepsilon_k = \varepsilon_{\min}$. Q.E.D.

Proof of Lemma 2: By Lemma 1, on $[\varepsilon_k, \varepsilon_{\max}]$, $R_k(\varepsilon^*)$ is decreasing and thus $\Phi(\varepsilon^*)$ is increasing. By (42), we compute

$$R_k(\varepsilon_k) = \frac{(1+g)/\beta - 1 + \delta - \mu \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) + \mu\varepsilon_k(1 - F(\varepsilon_k))}{\varepsilon_k F(\varepsilon_k) + \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)}. \tag{A.14}$$

By Lemma 1, we have

$$\frac{\partial R_k(\varepsilon^*)}{\partial \varepsilon^*}|_{\varepsilon_k} = 0.$$

Thus,

$$\mu \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) - (\beta^{-1}(1+g) - 1 + \delta) F(\varepsilon_k) = 0. \tag{A.15}$$

Using this equation, we can eliminate $F(\varepsilon_k)$ in (A.14) to obtain

$$R_k(\varepsilon_k) = \frac{(1+g)/\beta - 1 + \delta - \mu \int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)}{\int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)}.$$

Substituting this expression into (43) yields

$$\Phi(\varepsilon_k) = -\frac{(\beta^{-1} - 1)(1+g)}{\int_{\varepsilon_k}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)} < 0.$$

Since $\Phi(\varepsilon_{\max}) = +\infty$ and $\Phi(\varepsilon_k) < 0$ and Φ is increasing on $[\varepsilon_k, \varepsilon_{\max}]$, it follows from the intermediate value theorem that there exists a unique value $\varepsilon_l \in (\varepsilon_k, \varepsilon_{\max})$ such that $\Phi(\varepsilon_l) = 0$.

By (42), we have

$$R_k(\varepsilon_{\min}) = \frac{(1+g)/\beta - 1 + \delta - \mu(\mathbb{E}[\epsilon] - \varepsilon_{\min})}{\mathbb{E}[\varepsilon]}.$$

Substituting this expression into (43) yields

$$\Phi(\varepsilon_{\min}) = -\frac{(\beta^{-1} - 1)(1+g) + \mu\varepsilon_{\min}}{\mathbb{E}\varepsilon} < 0.$$

When $\mu = 0$, we have

$$\Phi(\varepsilon_{\min}) = -\frac{(\beta^{-1} - 1)(1+g)}{\mathbb{E}\varepsilon} < 0.$$

By (A.15), ε_k is an implicit continuous function of μ and $\varepsilon_k \to \varepsilon_{\min}$ as $\mu \to 0$. By continuity, for sufficiently small $\mu \geq 0$, we have $\Phi(\varepsilon^*) < 0$ for $\varepsilon^* \in [\varepsilon_{\min}, \varepsilon_k]$. Q.E.D.

Proof of Proposition 2: By the assumption and Lemma 2, the investment cutoff ε^* in any steady state must satisfy $\varepsilon^* \geq \varepsilon_k$. Since $\Phi(\varepsilon_l) = 0$, by (43) and setting $\varepsilon^* = \varepsilon_l$, we have $d = 0$. Thus (44) or (46) is satisfied for $s = 0$. We deduce that $\varepsilon^* = \varepsilon_l$ is the steady-state cutoff for $s = 0$. This is the only steady state with $d = 0$ because $\Phi(\varepsilon^*)$ increases with $\varepsilon^* \in [\varepsilon_k, \varepsilon_{\max}]$ by Lemma 1.

Suppose that there is another steady state with $d > 0$ if $R^r(\varepsilon_l) > 1 + g$. Then (44) implies that $R^r(\varepsilon^*) = 1 + g$ for $s = 0$. Since $R^r(\varepsilon^*)$ increases with ε^* and since $R^r(\varepsilon_l) > 1 + g$, we must have the steady state cutoff $\varepsilon^* < \varepsilon_l$. Since $R_k(\varepsilon^*)$ decreases with ε^* on $(\varepsilon_k, \varepsilon_l)$, it follows (43) that Φ increases with ε^* on $(\varepsilon_k, \varepsilon_l)$. Thus we have $\Phi(\varepsilon^*) < \Phi(\varepsilon_l) = 0$ for $\varepsilon^* \in (\varepsilon_k, \varepsilon_l)$, contradicting equation (43) as $d > 0$ and $R^r > 0$.

If $R^r(\varepsilon_l) < 1 + g$, we show that there is another steady state with $d > 0$. It follows from (44) we must have $R^r = 1 + g$. Since $R^r(\varepsilon^*)$ is a continuous and increasing function and since $R^r(\varepsilon_l) < 1 + g$ and $R^r(\varepsilon_{\max}) = (1+g)/\beta > 1 + g$, by the intermediate value theorem there is a unique solution $\varepsilon^* = \varepsilon_h \in (\varepsilon_l, \varepsilon_{\max})$ such that $R^r(\varepsilon^*) = 1 + g$. We then have $R^r = R^r(\varepsilon_h) = 1 + g$ in the steady state. It follows from (43) that $R^r d/k = \Phi(\varepsilon_h)$. Q.E.D.

Proof of Proposition 3: Recall that ε_l satisfies $\Phi(\varepsilon_l^*) = 0$. Total differentiating this equation and reorganizing yield

$$\frac{\mathrm{d}\varepsilon_l}{\mathrm{d}\mu} = -\frac{\left(1 + \frac{\partial R_k(\varepsilon_l)}{\partial \mu}\right)\int_{\varepsilon_l}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)}{\frac{\partial R_k(\varepsilon_l)}{\partial \varepsilon_l}\int_{\varepsilon_l}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon) - (\mu + R_k(\varepsilon_l))\varepsilon_l F'(\varepsilon_l)}.$$

By (42), we have $1 + \partial R_k(\varepsilon_l)/\partial\mu > 0$ and that $\partial R_k(\varepsilon_l)/\partial\varepsilon_l < 0$. Thus we have $\mathrm{d}\varepsilon_l/\mathrm{d}\mu > 0$. By (41), $R^r(\varepsilon^*)$ increases with ε^*. It follows that both ε_l and $R^r(\varepsilon_l)$ increase with μ. Q.E.D.

Proof of Proposition 4: By Lemma 2, for a sufficiently small $\mu \geq 0$, we only need to consider steady-state the investment cutoffs in $[\varepsilon_k, \varepsilon_{\max}]$. It follows from Lemma 1 that $R_k(\varepsilon^*)$ is a decreasing function of $\varepsilon^* \in [\varepsilon_k, \varepsilon_{\max}]$. Thus $\Phi(\varepsilon^*)$ increases with $\varepsilon^* \in [\varepsilon_k, \varepsilon_{\max}]$ by (43). We also know that $R^r(\varepsilon^*)$ increases with $\varepsilon^* \in [\varepsilon_{\min}, \varepsilon_{\max}]$. By (46) we have

$$\Psi(\varepsilon^*) = \left[1 - \frac{1+g}{R^r(\varepsilon^*)}\right] \frac{\alpha}{R_k(\varepsilon^*)} \Phi(\varepsilon^*). \tag{A.16}$$

Thus $\Psi(\varepsilon^*)$ is a product of three increasing functions on $[\varepsilon_k, \varepsilon_{\max}]$. Since $\Phi(\varepsilon_l) = 0$ and $\Phi(\varepsilon^*) < \Phi(\varepsilon_l) = 0$ for $\varepsilon^* \in [\varepsilon_k, \varepsilon_l]$, we will focus on the region $[\varepsilon_l, \varepsilon_{\max}]$ as equation (43) must hold with $R^r d \geq 0$. On this region $\Phi(\varepsilon^*) \geq 0$.

Suppose that $R^r(\varepsilon_l) > 1 + g$. Then we have

$$1 - \frac{1+g}{R^r(\varepsilon^*)} > 1 - \frac{1+g}{R^r(\varepsilon_l)} > 0$$

for $\varepsilon^* > \varepsilon_l > \varepsilon_k$. Since $\Phi(\varepsilon_l) = 0$, we have $\Phi(\varepsilon^*) > 0$ for $\varepsilon^* > \varepsilon_l$. Thus, as a product of three positive increasing functions on $[\varepsilon_l, \varepsilon_{\max}]$, $\Psi(\varepsilon^*)$ increases with $\varepsilon^* \in [\varepsilon_l, \varepsilon_{\max}]$. Since $\Psi(\varepsilon_l) = 0$ and $\Psi(\varepsilon_{\max}) = +\infty$, it follows from the intermediate value theorem that there exists a unique solution $\varepsilon_p \in (\varepsilon_l, \varepsilon_{\max})$ to equation (46). Then $R^r(\varepsilon_p) > R^r(\varepsilon_l) > 1 + g$.

Suppose that $R^r(\varepsilon_l) < 1 + g$. Then Proposition 2 shows that there exists $\varepsilon_h \in (\varepsilon_l, \varepsilon_{\max})$ such that $R^r(\varepsilon_h) = 1 + g$ and $\Psi(\varepsilon_h) = 0$. Thus $R^r(\varepsilon^*) > 1 + g$ for $\varepsilon^* \in [\varepsilon_h, \varepsilon_{\max}]$ by the monotonicity of $R^r(\varepsilon^*)$. It follows that $\Psi(\varepsilon^*)$ increases with $\varepsilon^* \in [\varepsilon_h, \varepsilon_{\max}]$ because $\Psi(\varepsilon^*)$ is a product of three positive increasing functions on $[\varepsilon_h, \varepsilon_{\max}]$. The intermediate value theorem implies that there exists a unique cutoff $\varepsilon_p \in (\varepsilon_h, \varepsilon_{\max})$ such that $\Psi(\varepsilon_p) = s/y > 0$. Then we have $R^r(\varepsilon_p) > R^r(\varepsilon_h) = 1 + g$.

For $\varepsilon^* \in (\varepsilon_l, \varepsilon_h)$, we have $R^r(\varepsilon^*) < R^r(\varepsilon_h) = 1 + g$ and thus $\Psi(\varepsilon^*) < 0$. There cannot exist a steady state with $s/y > 0$ by (46). Q.E.D.

Proof of Proposition 5: As in the proof of Proposition 4, for a sufficiently small $\mu \geq 0$, we only need to consider the region $[\varepsilon_l, \varepsilon_{\max}]$ for the steady state investment cutoff. By assumption, $R^r(\varepsilon_l) < 1 + g$. By the proof of Proposition 4, $\Psi(\varepsilon^*)$ is positive and increases with $\varepsilon^* \in (\varepsilon_h, \varepsilon_{\max}]$. But $\Psi(\varepsilon^*)$ is negative for $\varepsilon^* \in (\varepsilon_l, \varepsilon_h)$. Moreover, $\Psi(\varepsilon_h) = \Psi(\varepsilon_l) = 0$. Let $\underline{s}$ be defined as in the proposition. By the intermediate value theorem, for any $s/y \in (-\underline{s}, 0)$, there exist at least two steady-state cutoffs ε_l^* and ε_h^* with $\varepsilon_l < \varepsilon_l^* < \varepsilon_h^* < \varepsilon_h$ such that (46) holds. Q.E.D.

B Detrended Equilibrium System

The model exhibits long-run growth. To find a steady state and to study the dynamics around a steady state, we need to detrend the model around a long-run growth path. We consider transformations of $x_t = X_t/A_t$ for any variable $X_t \in \{K_t, D_t, S_t, Y_t, W_t, C_t, I_t\}$. Let $G_t = G_y Y_t$ where

$G_y \in (0,1)$ is an exogenous constant. For the marginal utility, we denote $\lambda_t = A_t \Lambda_t$. Then the detrended system can be summarized by the following 20 equations in 20 variables $\{R_{kt}, k_t,\ R_t,$ $q_t^k,\ q_t^l,\ \varepsilon_t^*,\ d_t,\ s_t,\ \Pi_t,\ p_t^*,\ \Gamma_t^a,\ \Gamma_t^b,\ \Delta_t,\ w_t,\ \lambda_t,\ p_{wt},\ N_t,\ y_t,\ c_t,\ i_t\}$, where $\{R_{-1},\ \Delta_{-1},\ d_{-1},\ k_{-1}\}$ and $\{z_{mt}, z_{st}\}$ are given exogenously:

1. The capital return,
$$R_{kt} = \alpha\left(1+g\right)^{1-\alpha} p_{wt} k_{t-1}^{\alpha-1} N_t^{1-\alpha}. \tag{B.1}$$

2. Evolution of capital,
$$(1+g)k_t = (1-\delta)k_{t-1} + \left(\left(\mu + R_{kt}\right)k_{t-1} + \frac{R_{t-1}}{\Pi_t} d_{t-1}\right)\int_{\epsilon_t^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \tag{B.2}$$

3. The nominal interest rate,
$$1 = \frac{\beta}{1+g}\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t}\frac{R_t}{\Pi_{t+1}}\left(1 + q_{t+1}^l\right). \tag{B.3}$$

4. Tobin's Q,
$$q_t^k = \frac{\beta}{1+g}\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} R_{kt+1} q_{t+1}^l + \frac{\beta}{1+g}\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1}^k (1-\delta) + \frac{\beta\mu}{1+g}\mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1}^l. \tag{B.4}$$

5. Liquidity premium,
$$q_t^l = \int_{\varepsilon_t^*}^{\varepsilon_{\max}} \left(q_t^k \varepsilon - 1\right) dF(\varepsilon). \tag{B.5}$$

6. Investment cutoff,
$$\varepsilon_t^* = 1/q_t^k. \tag{B.6}$$

7. Government budget constraint,
$$\frac{R_{t-1}}{\Pi_t}\frac{d_{t-1}}{1+g} = s_t + d_t. \tag{B.7}$$

8. Fiscal policy rule,
$$\left(s_t - s\right)/y = \phi_s (d_{t-1} - d)/y + z_{st}. \tag{B.8}$$

9. Monetary policy rule,
$$R_t = R\left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \exp(z_{mt}). \tag{B.9}$$

10. Pricing rule,
$$p_t^* = \frac{\Gamma_t^a}{\Gamma_t^b}. \tag{B.10}$$

11. Numerator in the pricing rule,
$$\Gamma_t^a = \lambda_t p_{wt} y_t + \beta\xi\mathbb{E}_t\left(\frac{\Pi_{t+1}}{\Pi}\right)^{\sigma}\Gamma_{t+1}^a. \tag{B.11}$$

12. Denominator in the pricing rule,

$$\Gamma_t^b = \lambda_t y_t + \beta \xi \mathbb{E}_t \left(\frac{\Pi_{t+1}}{\Pi} \right)^{\sigma-1} \Gamma_{t+1}^b. \tag{B.12}$$

13. Evolution of inflation,

$$1 = \left[\xi \left(\frac{\Pi}{\Pi_t} \right)^{1-\sigma} + (1-\xi)\, p_t^{*1-\sigma} \right]^{\frac{1}{1-\sigma}}. \tag{B.13}$$

14. Price dispersion,

$$\Delta_t = (1-\xi) p_t^{*-\sigma} + \xi \left(\frac{\Pi}{\Pi_t} \right)^{-\sigma} \Delta_{t-1}. \tag{B.14}$$

15. Labor demand,

$$w_t = (1-\alpha)(1+g)^{-\alpha} p_{wt} k_{t-1}^{\alpha} (N_t)^{-\alpha}. \tag{B.15}$$

16. Labor supply,

$$w_t = \frac{\psi}{\lambda_t}. \tag{B.16}$$

17. Marginal utility,

$$\lambda_t = 1/c_t. \tag{B.17}$$

18. Aggregate output,

$$y_t = \Delta_t^{-1} (1+g)^{-\alpha} k_{t-1}^{\alpha} (N_t)^{1-\alpha}. \tag{B.18}$$

19. Aggregate investment,

$$(1+g) i_t = \left((\mu + R_{kt}) k_{t-1} + \frac{R_{t-1}}{\Pi_t} d_{t-1} \right) (1 - F(\varepsilon_t^*)). \tag{B.19}$$

20. Resource constraint,

$$c_t + i_t = (1 - G_y) y_t. \tag{B.20}$$

For the real version of our model, we set $p_{wt} = \Pi_t = \Delta_t = 1$ and $R_t = R_t^r$ in the above system and the detrended equilibrium system consists of 13 equations (B.1)-(B.7), and (B.15)-(B.20) in 13 variables $\{R_t,\, R_{kt},\, \lambda_t,\, \varepsilon_t^*,\, q_t^k,\, q_t^l,\, w_t,\, d_t,\, k_t,\, N_t,\, y_t,\, c_t,\, i_t\}$.

C Steady-State System

We study the nonstochastic steady state of the detrended system with s/y and Π being exogenously given. Define real interest rate as $R^r = R/\Pi$. Let variables without time subscripts denote their steady state values. By the steady-state version of (B.13), we obtain $p^* = 1$. It then follows from (B.14) that $\Delta = 1$. Combining (B.10), (B.11), and (B.12), we have $p_w = 1$, $\Gamma^a = \Gamma^b = \lambda y/(1-\beta\xi)$. With w and λ being eliminated by using (B.16) and (B.17), and noting that $z_s = z_m = 1$, we obtain a steady-state system of 11 equations in 11 variables $\{R^r,\, R_k,\, \varepsilon^*,\, q^k,\, d,\, k,\, N,\, y,\, c,\, i, q^l\}$:

1. The capital return,

$$R_k = \alpha (1+g)^{1-\alpha} k^{\alpha-1} N^{1-\alpha}. \tag{C.1}$$

2. Evolution of capital,

$$(g+\delta)k = ((\mu + R_k) k + R^r d) \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon). \tag{C.2}$$

3. Nominal interest rate,

$$1 = \frac{\beta}{1+g} R^r \left(1 + q^l\right). \tag{C.3}$$

4. Tobin's Q,

$$q^k = \frac{\beta}{1+g} R_k \left(1+q^l\right) + \frac{\beta}{1+g} q^k (1-\delta) + \frac{\beta}{1+g} \mu q^l. \tag{C.4}$$

5. Liquidity premium,

$$q^l = \int_{\varepsilon^*}^{\varepsilon_{\max}} \left(q^k \varepsilon - 1\right) dF(\varepsilon). \tag{C.5}$$

6. Investment cutoff,

$$\varepsilon^* = 1/q^k. \tag{C.6}$$

7. Government budget constraint,

$$\left(\frac{R^r}{1+g} - 1\right) \frac{d}{y} = \frac{s}{y}. \tag{C.7}$$

8. Labor demand,

$$\psi c = (1-\alpha)(1+g)^{-\alpha} k^\alpha N^{-\alpha}. \tag{C.8}$$

9. Aggregate output,

$$y = (1+g)^{-\alpha} k^\alpha N^{1-\alpha}. \tag{C.9}$$

10. Aggregate investment,

$$(1+g)i = [(\mu + R_k) k + R^r d] (1 - F(\varepsilon^*)). \tag{C.10}$$

11. Resource constraint,

$$c + i = (1 - G_y) y. \tag{C.11}$$

As discussed in Section 3, the investment cutoff ε^* can be solved for by combining (C.3), (C.4), (C.5), (C.6), and (C.7). Given the inflation target Π, we obtain the nominal interest rate $R = R^r(\varepsilon^*)\Pi$. By (C.6), $q^k = 1/\varepsilon^*$. By (C.5), we derive q^l. With $R^r = R^r(\varepsilon^*)$, $R_k = R_k(\varepsilon^*)$ and $R^r d/k = \Phi(\varepsilon^*)$, we can determine y/k from $R_k = (1+g)\alpha y/k$ and $d/k = \Phi(\varepsilon^*)/R^r(\varepsilon^*)$. Noticing that equation (C.10) pins down the value of i/k, we can derive $i/y = (i/k)/(y/k)$. Using the resource constraint and the exogenously given G_y, we obtain $c/y = 1 - G_y - i/y$. Dividing (C.8) over (C.9) and reorganizing yield the steady-state value of labor:

$$N = \frac{1-\alpha}{\psi}/(\frac{c}{y}).$$

Then by noting that $R_k = R_k(\varepsilon^*) = \alpha(1+g)^{1-\alpha}k^{\alpha-1}N^{1-\alpha}$, we can solve for k. Combining with the ratios given above, we can then determine y, d, i, c, w, and s. Finally, we have $\Gamma_a = \Gamma_b = (y/c)/(1-\beta\xi)$.

D Linearized System

Let $\widehat{x}_t = (x_t - x)/x$ denote the log deviation from steady state for any variable x_t except for the surplus s_t and public debt d_t. For these two variables we consider level deviation relative to output, $\widetilde{d}_t = (d_t - d)/y$ and $\widetilde{s}_t = (s_t - s)/y$, instead of log deviation, because d may be zero and s may be negative.

By standard linearizations of New Keynesian model, we know the deviation of price dispersion $\widehat{\Delta}_t$ is of second-order. Thus we ignore the law of motion for the price dispersion. Moreover, the New Keynesian block can be summarized by the New-Keynesian Phillips curve. Hence, we can further eliminate $\widehat{p}^*_t$, $\widehat{\Gamma}^a_t$, and $\widehat{\Gamma}^b_t$. Then the linearized model can be summarized by a system of 16 equations in 16 variables, $\widehat{R}_{kt}$, $\widehat{k}_t$, $\widehat{R}_t$, $\widehat{q}^k_t$, q^l_t, $\widehat{\varepsilon}^*_t$, $\widetilde{d}_t$, $\widetilde{s}_t$, $\widehat{\Pi}_t$, $\widehat{p}_{wt}$, $\widehat{w}_t$, $\widehat{\lambda}_t$, $\widehat{N}_t$, $\widehat{y}_t$, $\widehat{c}_t$, and $\widehat{i}_t$, where $\widehat{R}_{-1}$, $\widetilde{d}_{-1}$, and $\widehat{k}_{-1}$ are predetermined, and z_{st} and z_{mt} are exogenous AR(1) processes:

1. The capital return,

$$\widehat{R}_{kt} = \widehat{p}_{wt} + (\alpha-1)\widehat{k}_{t-1} + (1-\alpha)\widehat{N}_t. \tag{D.1}$$

2. Evolution of capital,

$$\begin{aligned}(1+g)\widehat{k}_t =& (1-\delta)\widehat{k}_{t-1} - \left(\mu + R_k + \frac{R^r d}{k}\right)\varepsilon^{*2} f(\varepsilon^*)\widehat{\varepsilon}^*_t \\ &+ \int_{\varepsilon^*}^{\varepsilon_{\max}} \varepsilon dF(\varepsilon)\left((\mu+R_k)\widehat{k}_{t-1} + R_k\widehat{R}_{kt} + \frac{R^r d}{k}\left(\widehat{R}_{t-1} - \widehat{\Pi}_t\right) + \frac{R^r y}{k}\widetilde{d}_{t-1}\right).\end{aligned} \tag{D.2}$$

3. Nominal interest rate,

$$\widehat{R}_t - \mathbb{E}_t\widehat{\Pi}_{t+1} = \mathbb{E}_t\left(\widehat{\lambda}_t - \widehat{\lambda}_{t+1}\right) - \frac{q^l}{1+q^l}\mathbb{E}_t\widehat{q}^l_{t+1}. \tag{D.3}$$

4. Tobin's Q,

$$\begin{aligned}\widehat{q}^k_t =& \mathbb{E}_t\left(\widehat{\lambda}_{t+1} - \widehat{\lambda}_t\right) + \frac{\beta}{1+g}\frac{R_k(1+q^l)}{q^k}\mathbb{E}_t\widehat{R}_{kt+1} \\ &+ \frac{\beta}{1+g}\frac{(\mu+R_k)\,q^l}{q^k}\mathbb{E}_t\widehat{q}^l_{t+1} + \frac{\beta}{1+g}(1-\delta)\mathbb{E}_t\widehat{q}^k_{t+1}.\end{aligned} \tag{D.4}$$

5. Liquidity premium,

$$\widehat{q}_t^l = -\frac{1-F(\varepsilon^*)}{q^l \varepsilon^*}\widehat{\varepsilon}_t^*. \tag{D.5}$$

6. Investment cutoff,

$$\widehat{\varepsilon}_t^* = -\widehat{q}_t^k. \tag{D.6}$$

7. Government budget constraint,

$$\widetilde{s}_t + \widetilde{d}_t = \frac{R^r}{1+g}\widetilde{d}_{t-1} + \frac{R^r}{1+g}\frac{d}{y}\left(\widehat{R}_{t-1} - \widehat{\Pi}_t\right). \tag{D.7}$$

8. Fiscal policy rule,

$$\widetilde{s}_t = \phi_s \widetilde{d}_{t-1} + z_{st}. \tag{D.8}$$

9. Monetary policy rule,

$$\widehat{R}_t = \phi_\pi \widehat{\Pi}_t + z_{mt}. \tag{D.9}$$

10. New-Keynesian Phillips curve,

$$\widehat{\Pi}_t = \beta \mathbb{E}_t \widehat{\Pi}_{t+1} + \kappa \widehat{p}_{wt}, \tag{D.10}$$

where $\kappa = (1-\xi)(1-\beta\xi)/\xi$.

11. Labor demand,

$$\widehat{w}_t = \widehat{p}_{wt} + \alpha \widehat{k}_{t-1} - \alpha \widehat{N}_t. \tag{D.11}$$

12. Labor supply,

$$\widehat{w}_t = -\widehat{\lambda}_t. \tag{D.12}$$

13. Marginal utility,

$$\widehat{\lambda}_t = -\widehat{c}_t. \tag{D.13}$$

14. Aggregate output,

$$\widehat{y}_t = \alpha \widehat{k}_{t-1} + (1-\alpha)\widehat{N}_t. \tag{D.14}$$

15. Aggregate investment,

$$\begin{aligned}(1+g)\frac{i}{k}\widehat{i}_t =& [1-F(\varepsilon^*)]\left[(\mu + R_k)\widehat{k}_{t-1} + R_k \widehat{R}_{kt} + \frac{R^r d}{k}\widehat{R}_{t-1} - \frac{R^r d}{k}\widehat{\Pi}_t + \frac{R^r y}{k}\widetilde{d}_{t-1}\right] \\ &- \left(\mu + R_k + \frac{R^r d}{k}\right) f(\varepsilon^*)\varepsilon^* \widehat{\varepsilon}_t^*.\end{aligned} \tag{D.15}$$

16. Resource constraint,

$$\frac{c}{y}\widehat{c}_t + \frac{i}{y}\widehat{i}_t = (1-G_y)\widehat{y}_t. \tag{D.16}$$