Occupational Reallocations within and across Firms: Implications for the Labor-Market Polarization*

Toshihiko Mukoyama† Georgetown University

Naoki Takayama‡ Hitotsubashi University

Satoshi Tanaka§ University of Queensland

February 2021

Abstract

This paper analyzes how labor-market frictions interact with firms' decisions of reallocating workers across different occupations during the process of labor-market polarization. First, using two datasets in the US, we measure occupational reallocations within and across firms over the recent years. Second, we compare the patterns of occupational reallocations within and across firms between the US and Germany. We show that within-firm reallocations contribute significantly to the decline of employment in routine occupations in Germany, whereas the corresponding contribution is significantly small in the US. This outcome is consistent with the fact that firing restrictions are stronger in Germany. Third, we construct a general equilibrium model of firm dynamics and quantitatively analyze the impact of firing costs on the patterns of occupational reallocations and macroeconomic outcomes.

Keywords: occupational reallocations, firing costs, labor-market polarization

JEL Classifications: E24, J24, J62

*For comments and discussions, we thank Gianluca Violante and Matthias Doepke as well as other seminar participants at 2019 GRIPS-UT Macroeconomics and Policy Workshop in Tokyo, Virtual Australian Macroeconomics Seminar (VAMS), Virtual East Asia Macroeconomics Seminar (VEAMS), FRB New York, the Canon Institute for Global Studies, Waseda University, Osaka University, and Kyoto University. We acknowledge financial support from the Institute of Economic Research at Hitotsubashi University.

†Contact information: tm1309@georgetown.edu

‡Contact information: ntakayama@ier.hit-u.ac.jp

§Contact information: s.tanaka@uq.edu.au

1 Introduction

The recent US economy has experienced a significant decline in the employment in middle-skilled routine occupations. This phenomenon, often called the "labor-market polarization," has received large attention in the literature of macroeconomics and labor economics. The polarization is often attributed to the technological change that allows the firms to automate routine tasks by substituting workers by machines.

From the perspective of firms, the process of automation calls for reallocations of occupations: reducing the employment of occupations that are substituted by automation and increasing the employment of occupations that complement automation. Given the heterogeneity of technology adoption and other various (time-varying) factors across firms, the transformation of occupational mix likely accompanies reallocations of workers across firms.

How do firms reallocate workers across occupations under different labor-market environments? We ask this question with particular focus on different labor-market institutions. Several decades of literature has contrasted the US economy and the continental European economy as having starkly different labor-market institutions (see for example, Mortensen et al. (1999) and Ljungqvist and Sargent (2008)). One specific aspect that has caught attention of many researchers is the ease of firing. In our context, it is natural to infer that reallocating workers across occupations through firing and hiring in an environment where firing a worker imposes a substantial cost to the firm.

Using micro-level panel datasets from the US and Germany, we first show there are substantial occupational reallocations *within* a firm in both countries. Second, we compare the contributions of intra-firm occupational reallocations on the labor-market polarization in both countries. After developing a novel decomposition formula, we show that within-firm reallocations contribute more to the decline of routine occupation employment in Germany than in the US. Third, we construct a model of firm dynamics in general equilibrium to analyze the quantitative effect of firing costs to occupational reallocations within and across firms.

Our theoretical framework includes two types of firm-level (idiosyncratic) productivity shocks. The first is a Hicks-neutral productivity shock, commonly employed in the Hopenhayn (1992)-type framework. This shock affects all tasks in a symmetric manner. The second is an automation shock, which influences the marginal products of different tasks differently. We consider two different model experiments with different specifications of the automation shocks. In the first experiment, we highlight the role of intra- and inter-firm reallocation frictions in shaping the aggregate labor-market response to the automation shocks. In the second experiment, we analyze the effect of firing taxes on the transition dynamics and misallocation of labor across firms and across different tasks.

Our paper is motivated by the recent empirical literature in labor economics, which documents sizable within-firm occupational reallocations in several European countries during the labor-market polarization. Behaghel et al. (2012) is one of the earliest papers, which finds occurrence of within-firm occupational reallocations following firm's adoption of information and communication technologies (ICT) in French establishment data. Battisti et al. (2017) and Dauth et al. (2018) report similar evidence using the German establishment data when there are ICT shocks or industrial robot-exposure shocks. Our paper builds on these empirical findings, while we compare the differences in job polarization process between the U.S. and the European countries. We further evaluate the role of labor-market institutions on the intra- and inter-firm reallocations of workers.

In the macroeconomic literature, several recent papers build general equilibrium models and quantitatively analyze the process of labor-market polarization. For example, Eden and Gaggl (2018) and vom Lehn (2019) are two papers which analyze polarizing labor market using dynamic general equilibrium models. These papers focus on accounting for the changes in the occupational employment shares in the aggregate, using models with a representative-firm. In contrast, we study worker reallocations across occupations *and* across firms, explicitly taking the firm heterogeneity into account. We construct a novel theoretical framework, which is a natural extension of Hopenhayn's (1992) and Hopenhayn and Rogerson's (1993) heterogeneous-firm model. This framework enable us to analyze heterogeneous firm dynamics during the process of job polarization.

Finally, our paper also relates to the literature that studies the differences in labor-market institutions between the US and European countries. Among those studies, Mortensen et al. (1999) is the closest to our paper in spirit. They analyze how skill-biased shocks, interacting with different policy regimes, explain the rise in unemployment in Europe. While our paper's focus is on labor-market polarization and occupational reallocations, we share the similar motivation to the paper: different in labor-market institutions can result in different responses to technology shocks.

The paper is organized as follows. The next section conducts the empirical analysis. Section 3 constructs a general equilibrium model of firm dynamics. Section 4 analyzes the model quantitatively. Section 5 concludes.

2 Empirical findings

In this section, we document the patterns of occupational reallocations in the US and Germany. Both countries have experienced significant changes in the occupational composition in their labor markets in the past decades (see Acemoglu and Autor (2011) for the US and Böhm et al. (2016) for Germany). The patterns of reallocation, however, are markedly different across these two countries, as we will show below.

2.1 Data

Here, we outline the datasets used in our analysis. For the US, we use two datasets: the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS). For Germany, we use the Sample of Integrated Labor Market Biographies (SIAB).

2.1.1 United States

The Survey of Income and Program Participation (SIPP) is a dataset of household-based panel survey, administrated by the US Census Bureau. We use the following seven panels fo the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels which have the sample of 14,000 to 52,000 individual.[1] With these SIPP panels, we identify the workers' job and occupation switches on both yearly and monthly basis.

The other dataset for the US is the Current Population Survey (CPS), which is administered by the US Census Bureau.[2] The CPS is conducted with a sample of around 60,000 households and consists of the basic monthly questions focusing on labor force participation and supplemental questions such as the annual March income supplement. We utilize the longitudinal aspect of the Basic Monthly CPS from 1994-2019.

With each of the SIPP and the CPS datasets, we identify occupational switches when a worker change his/her occupations across three broadly occupational group, *cognitive*, *routine*, and *manual*, defined by Acemoglu and Autor (2011).[3]. Among those occupational switches, we further identify those which involve employer switches and those which don't by looking into changes in job IDs. We identify those switches either yearly or monthly basis.

2.1.2 Germany

The Sample of Integrated Labor Market Biographies (SIAB) is the dataset based on the administrative employment records in Germany. The data are provided by the Institute for Employment Research (IAB) for research. The data set has 2% sample of employment histories from the entire German employment for the period 1975–2017. It contains information on the starting and

[1]Appendix B.1 details the SIPP data.
[2]Appendix B.2 details the CPS data.
[3]The occupation groups are listed in Appendix B.4.

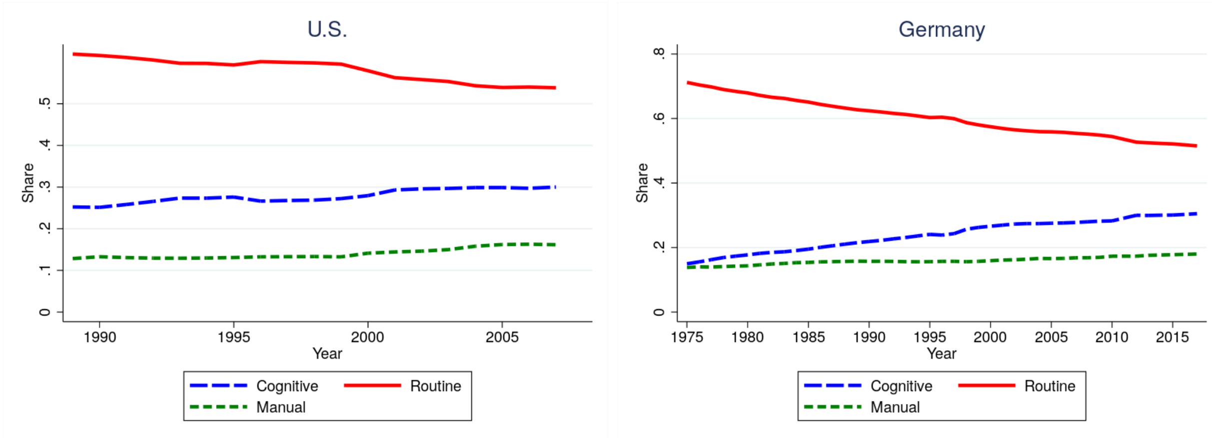


Figure 1: Occupational Employment Share in the US (SIPP, left) and Germany (SIAB, right)

ending dates of each employment spell with an employer identification number and an occupation classification code.

With the occupation information, we first create three broad occupation classifications (cognitive, routine, and manual) as similar to those for the US, following Böhm et al. (2016). We then identify job and occupation switches at annual frequency, and document within-firm (within-establishment) and across-firm (across-establishment) patterns of occupational reallocations, as similar to those in the U.S.

2.2 Time-series patterns of stocks

Figure 1 plots the share of employment across occupations for the US (SIPP) from 1989 to 2007, and for Germany (SIAB) from 1975 to 2017. As described earlier, the occupations are divided into *cognitive*, *routine*, and *manual* by the nature of tasks performed by each occupation. As is commonly observed in the literature (see Acemoglu and Autor (2011)), the share of the routine-occupation employment has declined both in the U.S. and Germany over the periods of time. In contrast, cognitive and manual occupations have gained employment shares. This phenomenon is often referred to as the labor-market polarization.[4]

2.3 Gross occupational switches within and across firms

How did the switch across occupations occur? The novel perspective of this paper is to divide the occupational switches into within-firm switches and across-firms switches. When the labor-market polarization is referred to as "the loss of middle-skilled jobs," a common perception is that these

[4]We confirm the pattern for the US from the SIPP with the CPS in Figure 5 in Appendix A.

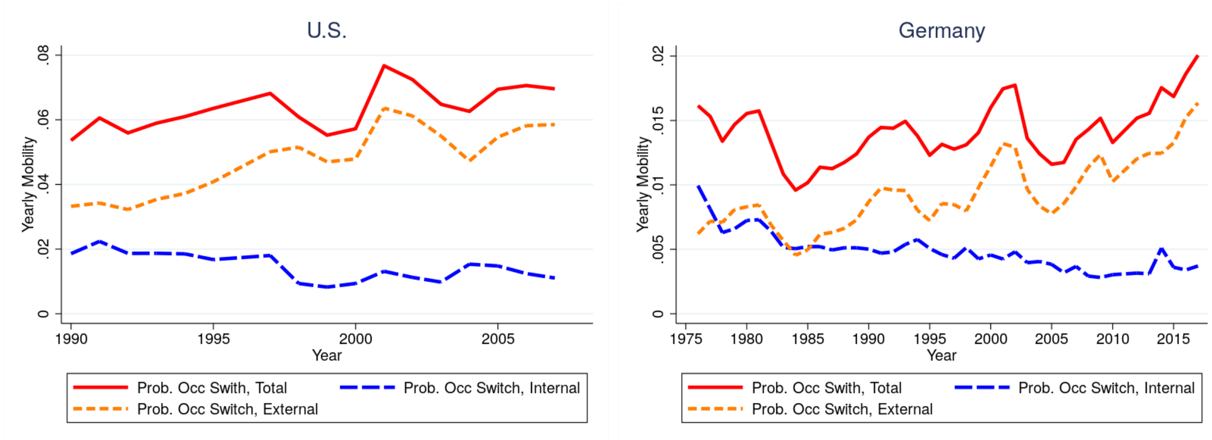


Figure 2: Yearly Internal/External Occupational Mobility in the US (SIPP, left) and Germany (SIAB, right)

job disappearances accompany worker separations. It turns out that, contrary to this perception, a significant fraction of occupational reallocation occur within firms.

Figure 2 plots the annual occupational switching rates for the US (SIPP) and for Germany (SIAB). The occupational switching rate at year t is defined as the number of workers who switch occupations across the above three categories between year $t-1$ and t divided by year t employment. For the *external* occupational switching rate, we use the workers who switched both their employers and the occupations as the denominator. For the *internal* occupational switching rate, we consider workers who changed the occupations but remained in the same employer. Note that whereas Figure 1 describes the change in shares and thus reflects the net flows of the workers across occupations, Figure 2 contains all gross worker flows.

As is clear from Figure 2, a significant portion of occupational reallocation happens within a firm both in the US and Germany, while the level of mobility is lower in Germany in general. These figures confirm that, for both countries, within-firm reallocations are potentially an important channel for job polarization. However, these numbers in Figure 2 are gross flow rates. Thus, we need to carefully connect them to the net employment changes in occupations. We discuss on the methodology and the results of within- and across-firm reallocations to job polarization in Section 2.4.

For the US, to see that this result is not specific to SIPP, we compare the statistics from SIPP and CPS in Figure 6 in Appendix A. To facilitate the comparison, both graphs are drawn in monthly frequency. While the levels of occupational switching rate are different across datasets, the fact that the internal occupational reallocation occupies an important fraction of the total occupational switches remains the same.

The mobility estimated from CPS reassures the non-negligible reallocation within a firm. From Figure 6, we can see that inter-firm reallocation always occupies a significant portion of the transition, although the estimates are different in level from the results estimated by SIPP. In addition, the series from CPS is more noise-ridden than from the SIPP.

2.4 Decomposition of the occupational employment share changes

To quantify the role of occupational switches within and across firms in the process of the labor-market polarization, we decompose the change in each occupational employment share into contributions of internal and external occupational changes.

Let ℓ_{it} be the stock of employment of occupation i at time t. Also let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the total employment. The employment share at time t for occupation i is ℓ_{it}/E_t. We decompose the change in the (log) employment share of occupation i from period t to period $t+T$:

$$\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right) \approx \Bigg[\underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1} - f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{Internal Net Flow}} + \underbrace{\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1} - f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{External EE Net Flow}} + \underbrace{\sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1} - f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}}_{\text{External Net Flow from/to Uemployment and OLF}} - \underbrace{\sum_{\tau=0}^{T-1}\Delta^{E}_{t+\tau,t+\tau+1}}_{\text{Total Employment Effect}}\Bigg]. \tag{1}$$

The derivation of the above equation (1) is in Appendix C. The equation shows that the cumulative change in employment share is decomposed into four components on the right-hand side. The first term is the contribution of within-firm occupational switches. The notation $f^{ji,s}_{t+\tau,t+\tau+1}$ is the gross worker flow from occupation j to occupation i between time $t+\tau$ and $t+\tau+1$, conditional on staying with the same employer. The term $f^{ij,s}_{t+\tau,t+\tau+1}$ is the worker flow in the opposite direction. Therefore, $\sum_{j\neq i} f^{ji,s}_{t+\tau,t+\tau+1} - \sum_{j\neq i} f^{ij,s}_{t+\tau,t+\tau+1}$ is the sum of inflow minus the sum of outflow from the viewpoint of occupation i. Thus, this is the net inflow due to the internal occupational switches. Similarly, the second term is the contribution of between-firm, employer-to-employer, occupational switches. The third term represents the net inflow from the unemployment and out of labor force. And, the fourth term is the change in the occupational employment share due to the change in the total employment.

Table 1: Decompositions of occupational employment share changes for the US (SIPP) and Germany (SIAB)

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
U.S.	1989	2007	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.25	0.30	0.17	0.01	0.17
Routine	0.62	0.54	−0.14	0.00	−0.14
Manual	0.13	0.16	0.23	−0.01	0.25
Germany	1975	2017	$\log(\Delta\text{share})$	Internal	External
Cognitive	0.15	0.30	0.71	0.17	0.54
Routine	0.71	0.52	−0.32	−0.05	−0.27
Manual	0.14	0.18	0.26	−0.04	0.30

Below, we call the first term in the right-hand side as *internal flow* and the sum of the second to fourth terms as *external flow.* We do not distinguish from the second to the fourth terms largely for the purpose comparability. In particular, it is difficult to make the distinction of the third and fourth terms comparable across the US and Germany. Given that our focus is on the internal flow, the most important task here is to distinguish the internal flow and other occupational switches.

Table 1 implements the decomposition formula (1) to SIPP for the US and SIAB for Germany from the periods 1989–2006 and 1975–2019, respectively. The frequency is annual. We have eliminated the internal movements into managerial occupations from the internal flow (therefore these are categorized into the external flow) because our focus is the labor-market polarization and the managerial promotions are unlikely to have been influenced by the polarization process.

There are striking differences between the US and Germany. As seen in Column (4) and (5), internal switches play almost no role to explain the rise of the cognitive employment and the decline in the routine employment for the US. In contrast, we observe that internal switches have non-negligible contributions for the changes in occupational employment for Germany. We graphically plot the cumulative contributions of net flows to occupational employment changes over time in Figure 7 and 8 in Appendix A. These patterns were evident even after several robustness checks.[5]

It is natural to infer the differences in the labor-market institutions between the US and Europe play some role in the different patterns of the occupational switches in Table 1. In the next two sections, we analyze a model of heterogeneous firms to examine the role of firing taxes during the

[5] The SIPP is not exempt from a sample attrition problem as in other survey data. A sample attrition problem could potentially generate biases in the decomposition results. Therefore, we run a robustness check by running the decomposition with the balanced panels of the SIPP in Appendix D.1.

process of labor-market polarization.

3 Model

Our model is a heterogeneous-firm general equilibrium model based on Hopenhayn (1992) and Hopenhayn and Rogerson (1993). Our model features a CES production structure with three broad types of occupations (cognitive, routine, and manual) and two firm-level shocks (an automation shock in addition to the standard Hicks-neutral total factor productivity (TFP) shock). When facing the TFP and automation shocks, firms try to adjust the numbers of workers in the occupations internally (that is, within-firm reallocation of workers) and externally (that is, hiring and/or firing). As will be detailed below, the degree of internal and external adjustments depends on firing taxes and reorganization costs firms face when reallocating workers within and across firms.

3.1 Consumers

Time is discrete. We assume that a infinitely-lived representative consumer exists. The consumer supplies labor and receive wage income. She also owns the firm, so that she receives the profit and also incurs the costs. The consumer is a price-taker and maximizes the utility

$$\sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$C_t = w_t N_t + \Pi_t + R_t - T_t.$$

Here, $U(\cdot,\cdot)$ is the period utility function, C_t is consumption at period t, and N_t is the labor supply. On the income side, w_t is the wage rate, Π_t is the profit from production in the firm. The firm pays firing tax, which is lump-sum rebated to the consumer as R_t. We assume firms have to pay reorganization cost (detailed below) when they reallocate workers across occupation and within the firm. The total reorganization cost is denoted as T_t.

3.2 Firms

Firms produce the consumption goods using labor. They act competitively in both product market and the labor market. The production process involves three different tasks, manual (m), cognitive (c), and and routine (r). A firm is subject to two kinds of idiosyncratic shocks. The first shock is the standard Hicks-neutral TFP shock s_h. The second shock is an automation shock s_a, representing the introduction of new technology (machines). After observing the shocks, a firm makes hiring decisions, where employment at task $i \in \{m, c, r\}$ is denoted as n_i. The production function is specified as

$$f(\mathbf{n}, \mathbf{s}) = s_h \mathbf{F}^{\alpha},$$

where $\alpha \in (0, 1)$ is the returns-to-scale parameter,

$$\mathbf{F}(n_m, \mathbf{G}) = \left(\mu_m n_m^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \mu_m) \mathbf{G}^{\frac{\sigma_m - 1}{\sigma_m}} \right)^{\frac{\sigma_m}{\sigma_m - 1}},$$

where $\sigma_m \geq 0$ is the elasticity of substitution parameter and n_m is the manual labor,

$$\mathbf{G}(n_c, \mathbf{M}) = \left(\mu_c n_c^{\frac{\sigma_c - 1}{\sigma_c}} + (1 - \mu_c) \mathbf{M}^{\frac{\sigma_c - 1}{\sigma_c}} \right)^{\frac{\sigma_c}{\sigma_c - 1}},$$

where $\sigma_c \geq 0$ is the elasticity of substitution parameter and n_m is the cognitive labor,

$$\mathbf{M}(n_r, s_a) = \left(\mu_r n_r^{\frac{\sigma_r - 1}{\sigma_r}} + (1 - \mu_r) s_a^{\frac{\sigma_r - 1}{\sigma_r}} \right)^{\frac{\sigma_r}{\sigma_r - 1}},$$

where $\sigma_r \geq 0$ is the elasticity of substitution parameter and n_r is the routine labor. One can interpret s_a as a capital stock that arrives exogenously. This specification of the production function is in line with the existing literature on labor-market polarization. For example, $\sigma_r = \infty$ corresponds to Cortes et al. (2017); $\sigma_c = 1$ case corresponds to Autor and Dorn (2013). Using the same specification as above, vom Lehn (2018) estimates the values of σ_i and μ_i.

Changing occupational employment from one period to next may require the firm to pay certain costs for adjustment. To describe these costs, first we introduce new notations. Let us denote the this period employment of occupation $i \in \{m, c, r\}$ as n'_i. The previous period employment of occupation i is denoted as n_i. Firms decide the vector $\mathbf{n}' \equiv \{n'_m, n'_c, n'_r\}$ given this period shocks $\mathbf{s} = \{s_h, s_a\}$ and $\mathbf{n} \equiv \{n'_m, n'_c, n'_r\}$. In other words, $\mathbf{s}$ and $\mathbf{n}$ are the state variables for the firm's employment decision $\mathbf{n}'$.

When increasing the number of occupational hires ($n'_i > n_i$), the firm has to bring in new workers into that occupation from inside the firm or from outside the firm. Define $\tilde{n}'_i \in [0, n'_i]$ as the internal workers (from any occupations but from the same firm) who now work in occupation i this period. Then $\tilde{n}'_i - n_i$ is the number of internally-moved workers and $n'_i - \tilde{n}'_i$ is the number of workers who are externally brought in. Furthermore, define $\hat{n}'_i \in [0, \min\{n_i, \tilde{n}'_i\}]$ be internal workers who stayed in the same occupation i (that is, the same firm and the same occupation) from the previous period. Then $\tilde{n}'_i - \hat{n}'_i$ is the number of workers who are internally brought into that occupation within the firm. Let $\tilde{\mathbf{n}}'$ be the vector of $\tilde{n}_i'$ and $\hat{\mathbf{n}}'$ be the vector of $\hat{n}_i'$.

In other words, the firm have three layers of decision: (i) how many to hire this period $\mathbf{n}'$; (ii) within $\mathbf{n}'$, how many come from the same firm ($\tilde{\mathbf{n}}'$), and (iii) how many in $\tilde{\mathbf{n}}'$ are the ones from the same occupation ($\hat{\mathbf{n}}$). Clearly $\hat{n}_i$ cannot exceed n_i, and the sum of $\tilde{n}_i'$ has to be less than the sum of n_i.

We assume there are two types of costs for adjustment. The first is the firing tax, imposed by the government. We denote it $g(\mathbf{n}, \tilde{\mathbf{n}}')$ and assume that it takes the form of

$$g(\mathbf{n}, \tilde{\mathbf{n}}') = \tau \left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}_i' \right),$$

where $\tau \geq 0$ is the tax rate. We also assume that a firm has to incur a reorganization cost, $h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}')$, taking the form of

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}_i' - \hat{n}_i'),$$

where $H_i(\cdot)$ is an increasing function. In the quantitative analysis below, we consider a quadratic form of H_i function:

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=0}^{k} \kappa_i (\max\{\tilde{n}_i' - \hat{n}_i', 0\})^2.$$

where $\kappa_i \geq 0$.

Formally, the firm's problem is

$$V(\mathbf{n}, \mathbf{s}) = \max_{\mathbf{n}', \tilde{\mathbf{n}}', \hat{\mathbf{n}}'} f(\mathbf{n}', \mathbf{s}) - w\mathbf{1} \cdot \mathbf{n} - g(\mathbf{n}, \tilde{\mathbf{n}}') - h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') - c_f + \frac{1}{1+r_t} \max\{E_{\mathbf{s}'}[V(\mathbf{n}', \mathbf{s}')], -g(\mathbf{n}, 0)\},$$

subject to

$$\sum_{i=m,c,r} \tilde{n}_i' \leq \sum_{i=m,c,r} n_i,$$

where

$$g(\mathbf{n}, \tilde{\mathbf{n}}') = \tau \left(\sum_{i=m,c,r} n_i - \sum_{i=m,c,r} \tilde{n}_i' \right),$$

and

$$h(\tilde{\mathbf{n}}', \hat{\mathbf{n}}') = \sum_{i=m,c,r} H_i(\tilde{n}_i' - \hat{n}_i').$$

Here, c_f is the fixed operation cost and r_t is the return to saving for the consumer (there are no aggregate shocks so that there is only one return). We assume that firm decides whether to exit at the beginning of the following period, before seeing the next period's shocks.

3.3 Equilibrium

We assume free entry of firms. Anyone can start a firm by paying the cost $c_e > 0$.

$$\int V(\mathbf{0}, \mathbf{s}) d\nu(\mathbf{s}) = c_e,$$

where $\mathbf{0}$ is the zero vector, $\nu(\mathbf{s})$ is the distribution of initial $\mathbf{s}$. In the competitive equilibrium, the firms' goods supply and the consumer's goods demand are equated. We assume that the labor is ex ante homogeneous, and the total labor supply of consumers has to be equal to the total labor demand by firms.

4 Quantitative Analysis

The empirical analysis in Section 2 highlights a significant difference in how firms react to the labor market polarization process. Motivated by this outcome, we quantitatively assess how labor market institutions affect the reallocation of workers across occupations and firms. In particular, we examine the effects of reallocation frictions within and across firms.

We conduct two experiments. First, by comparing two steady states with AR(1) shocks in s_h and s_a, we examine how reallocation frictions τ and κ_i affect the output, employment, and productivity of the macroeconomy. The exercise is directly comparable to Hopenhayn and Rogerson's (1993) experiment on the firing tax. The purpose of this section is to illustrate the different types of misallocations induced by two different shocks.

Second, we consider the transition process of automation. In particular, we analyze an experiment where we start from all firms' s_a is a low value $\underline{s_a}$ and randomly selected firms moves into a high value $\overline{s_a}$. We assume $\underline{s_a} < \overline{s_a}$. We interpret this transition of s_a from $\underline{s_a}$ to $\overline{s_a}$ as the process of automation. The macroeconomy gradually moves from one steady state, where no firm is automated, to the other steady state, where all firms are automated. As the automation continues, the composition of occupations also changes over time. This experiment is directly comparable to the empirical results in Section 2.

4.1 Misallocation due to reallocation frictions

The first experiment closely follows Hopenhayn and Rogerson (1993). We utilize the same specification for the consumer's utility as Hopenhayn and Rogerson (1993):

$$U(C_t, N_t) = \log(C_t) - \xi N_t.$$

Table 2: Output, Employment, and Labor Productivity in the Models with and without Frictions

$\tau = 0.1$						
	s_h **shock only**			s_a **shock only**		
	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.2$	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.2$
Y	91.1	91.0	91.0	91.9	91.4	89.9
L	92.3	92.0	91.8	94.4	94.3	93.1
Y/L	98.6	98.9	99.0	97.4	96.9	96.6

The TFP and automation shocks follow AR(1) processes

$$\log(s_h') = 0.95 \log(s_h) + \epsilon_h,$$

where

$$\epsilon_h \sim N(0, 0.1^2),$$

and

$$\log(s_a') = 0.95 \log(s_a) + \epsilon_a,$$

where

$$\epsilon_a \sim N(0, 5^2).$$

For the parameters, we set $\alpha = 2/3$, $c_f = 0.15$, $\mu_m = 0.1$, $\mu_c = 0.4$, $\mu_r = 0.6$, $\sigma_m = 0.5$, $\sigma_c = 0.5$, $\sigma_r = 3.0$, and $\beta = 1/1.04$. For this experiment, we will focus on the economy's steady state. Thus, from the representative consumer's Euler equation, the firm's discount factor $1/(1+r) = \beta$.

In Table 2, we repeat the Hopenhayn and Rogerson's (1993) exercise, comparing the output level, the employment level, and the labor productivity in the model with frictions with those in the model without frictions. We normalize the numbers for our baseline model without frictions ($\tau = 0$) equal to 100.

The results in Table 2 highlights two potential misallocations: within a firm and across firms. For the TFP shock (s_h), as seen in Column 1 to 3, the model behavior is similar to Hopenhayn and Rogerson (1993): lower output, lower employment, misallocation across firms (lower Y/L). The reorganization cost κ does not have large effects on these outcomes, as s_h affects all occupations in a firm in the same direction. In contrast, for the automation shock (s_a), the reorganization cost κ plays a role as shown in Columns 4 to 6, because for a low value of κ, the firms can reallocate labor internally (avoid within-firm misallocation).

Table 3 compute the gross inflow rates into occupations. There are two separate rows: internal

Table 3: Internal and External Employment Adjustments in the Models with and without Frictions

$\tau = 0.1$						
	s_h **shock only**			s_a **shock only**		
	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.2$	$\kappa = 0.0$	$\kappa = 0.1$	$\kappa = 0.2$
Internal	1.8	1.7	1.5	3.9	3.1	1.9
External	−2.8	−2.8	−2.7	−7.4	−8.1	−7.7

Note: Column *Internal* shows internal gross inflow from other occupations within the firm (outflow is a mirror image). Column *External* shows external gross inflow from outside the firm.

and external. The internal flows are the movement of workers across occupations within a firm. The external flows are movements from outside the firm.

The numbers in the table are the annual flow rate differences (in percentage points) from the $\tau = 0.0$ case. When $\tau = 0.0$, the firm can adjust all occupational employment through hiring and firing without costs for adjustments, regardless of the value of κ. Thus the internal flows in the table represents the flow rates when $\tau = 0.1$. When $\tau = 0.1$, internal flows contribute positively to the occupational adjustment. This behavior is qualitatively consistent with the US and German economy in Section 2, given the common perception that it is more difficult to fire workers in Germany.

In every column, the inflow increases and the outflow decreases with τ. Adding up the rows in each column reveals that the effect of τ on the total flow is negative. This outcome is consistent with the increase in misallocation in Table 2.

As in the case of the output and employment, the effect of the reorganization cost κ is very different depending on the nature of the productivity shock. The magnitude of κ has a strong effect in the case of s_a shocks. When an s_a shock hits, reallocating labor across occupations becomes beneficial. When κ is small, the reallocation is possible through internal movements, even when τ is large. The internal movement can work as a channel for the firm to avoid misallocation. When κ is large, there is less incentive to reallocate labor within a firm and the misallocation is more severe as a result.

4.2 Transition dynamics with one-time automation shock

Here, we consider an alternative setting for the automation shock. We formulate the s_a process to mimic one-time technology adoption (automation) by each firm. At the individual firm level, s_a starts from a value $\underline{s_a}$ and move to a larger value $\overline{s_a}$ randomly. We assume that all firms starts from $\underline{s_a}$ and $\overline{s_a}$ is the absorbing state. Thus, in the aggregate, we have to compute the transition path between the initial steady state where no firms are automated ($s_a = \underline{s_a}$ for all firms) and the final steady state where all firms are automated ($s_a = \overline{s_a}$ for all firms). We examine how the polarization invoked by the automation interacts with the firing tax. The computation of the model is substantially more challenging compared to the previous experiment, where the comparison is between two steady states. Computational details for this section are provided in the Appendix.

Below we assume a quasi-linear period utility

$$U(C_t, N_t) = C_t - \xi \frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},$$

where $\xi > 0$ and $\eta > 0$ are parameters. This specification implies that the firm's discount factor is also β, even outside the steady state.

In this section, we simplify the model and assume that the internal reallocation occurs only between cognitive and routine occupations. This assumption is motivated by the analysis of German data in Section 2, where the internal reallocation contributes to the polarization mainly between the cognitive and routine occupations.

Let $\mathbf{n} = (n_m, n_c, n_r)$ be the previous period's occupational employment, $\mathbf{n}' = (n'_m, n'_c, n'_r)$ be the current period's employment decision, x' be the workers who internally reallocated from routine to cognitive occupations, and $\mathbf{s} = (s_h, s_a)$ be the TFP shock and the automation shock. The firm's problem can then be written as

$$\begin{aligned} V(\mathbf{n}, \mathbf{s}) \quad = \quad & \max_{\mathbf{n}', x'}[-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'), 0\} + \max\{n_r - (n'_r + x'), 0\}) \\ & -\kappa x'^2 + f(\mathbf{n}', \mathbf{s}) - w\mathbf{1} \cdot \mathbf{n}' - c_f \\ & + \frac{1}{1+r} \max\{\mathbb{E}_{\mathbf{s}'}[V(\mathbf{n}', \mathbf{s}'|\mathbf{s})], -\tau \max\{\sum_i (n'_i - 0), 0\}\}], \end{aligned}$$

subject to

$$\begin{aligned} n'_m & \geq 0, \\ n'_c & \geq x', \\ n'_r & \geq 0, \\ 0 & \leq x' \leq n_r. \end{aligned}$$

We assume the economy is initially in a steady state where all firms have $s_a = \underline{s_a}$ and expect it to stay constant forever. Then, at a point in time (call time 0), the economy unexpectedly shifts to a new regime where a firm's s_a can move randomly. In particular, after time 0, at any point s_a can move from $\underline{s_a}$ to $\overline{s_a}$ with probability ϕ. The switch of the regime is permanent, and all economic agents understand the nature of the switch. At the firm level, the transition from $\underline{s_a}$ to $\overline{s_a}$ is one time and permanent: once s_a moves to $\overline{s_a}$, it stays at that value. Because this s_a shock is idiosyncratic, the macroeconomy experiences transition from the steady state where all firms have $s_a = \underline{s_a}$ to another steady state where all firms have $s_a = \overline{s_a}$. We interpret this transition dynamics as the process of labor-market polarization, driven by automation at each firm.

To analyze the macroeconomic dynamics of this transition, first we have to compute the initial and final steady state. In the initial steady state where no firms automate, a firm's dynamic programming problem is

$$
\begin{aligned}
& \underline{V}(\mathbf{n}, s_h; \underline{s_a}) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s_a}) - \underline{w}\mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s_a})|s_h], \\
& \qquad -\tau \max\{\sum_j (n'_j - 0), 0\}\}].
\end{aligned}
$$

At the final state where all firms have completed the automation, the Bellman equation is

$$
\begin{aligned}
& \overline{W}(\mathbf{n}, s_h; \overline{s_a}) \\
= \ & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n'_m, 0\} + \max\{n_c - (n'_c - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n'_r + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \qquad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s_a}) - \overline{w}\mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\overline{W}(\mathbf{n}', s'_h; \overline{s_a})|s_h], \\
& \qquad -\tau \max\{\sum_j (n'_j - 0), 0\}\}].
\end{aligned}
$$

We impose the free-entry condition in each steady state:

$$
\int \underline{V}(\mathbf{0}, s_h; \underline{s_a}) d\nu(s_h) = c_e
$$

and

$$
\int \overline{W}(\mathbf{0}, s_h; \overline{s_a}) d\nu(s_h) = c_e.
$$

The calibrated parameters are: $\xi = 2.17$, $\eta = 2$, $\alpha = 2/3$, $c_f = 0.1$, $c_e = 0.059$, $\mu_m = 0.1$, $\mu_c = 0.2$, $\mu_r = 0.9$, $\sigma_m = 0.5$, $\sigma_c = 0.5$, $\sigma_r = 2.0$, $\beta = \frac{1}{1.04}$, $\underline{s_a} = 1$, and $\overline{s_a} = \exp(0.4)$. We assume that $\log(s_h)$ follows an AR(1) process:

$$\log(s_h') = 0.95 \log(s_h) + \epsilon_h,$$

where

$$\epsilon_h \sim N(0, .1^2).$$

After computing the initial and final steady states, we compute the transition dynamics. We set the probability of automation, ϕ, to be 2.5 percent (per year). The value functions for the firms that are not yet automated are written as

$$\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s_a}) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \underline{s_a}) - w_t \mathbf{1} \cdot \mathbf{n}' - c_f \\
& \quad +\beta \max\{\mathbb{E}_{s_h'}[\phi W_{t+1}(\mathbf{n}', s_h'; \overline{s_a}) + (1-\phi) V_{t+1}(\mathbf{n}', s_h'; \underline{s_a}) | s_h], -\tau \max\{\sum_j (n_j' - 0), 0\}\}],
\end{aligned}$$

and the firms which are already hit by the automation shock solves the Bellman equation

$$\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s_a}) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} [-\tau(\max\{n_m - n_m', 0\} + \max\{n_c - (n_c' - x'(\mathbf{n}, \mathbf{n}')), 0\} + \max\{n_r - (n_r' + x'(\mathbf{n}, \mathbf{n}')), 0\}) \\
& \quad -\kappa x'(\mathbf{n}, \mathbf{n}')^2 + f(\mathbf{n}', s_h; \overline{s_a}) - w_t \mathbf{1} \cdot \mathbf{n}' - c_f \\
& \quad +\beta \max\{\mathbb{E}_{s_h'}[W_{t+1}(\mathbf{n}', s_h'; \overline{s_a}) | s_h], -\tau \max\{\sum_j (n_j' - 0), 0\}\}].
\end{aligned}$$

The value function for entrants is assumed to be

$$V_t^e(\mathbf{0}, s_h; \underline{s_a}) \quad = \quad \psi_t W_t(\mathbf{0}, s_h; \overline{s_a}) + (1-\psi_t) V_t(\mathbf{0}, s_h; \underline{s_a}) + c_f.$$

Here we assume that entrants automate immediately with a time-varying probability ψ_t. The probability ψ_t is assumed to be equal to the share of incumbent automated firms at the beginning of the period:

$$\psi_t \quad = \quad \frac{\int i_{t-1}^W(d\mathbf{n}, ds_h; \overline{s_a})}{\int i_{t-1}^V(d\mathbf{n}, ds_h; \underline{s_a}) + \int i_{t-1}^W(d\mathbf{n}, ds_h; \overline{s_a})},$$

where $i_t^V(\mathbf{n}, s_h; \underline{s_a})$ and $i_t^W(\mathbf{n}, s_h; \overline{s_a})$ are the measures of surviving non-automated and automated firms at the end of the period t (incumbents at the beginning of the next period). We impose the

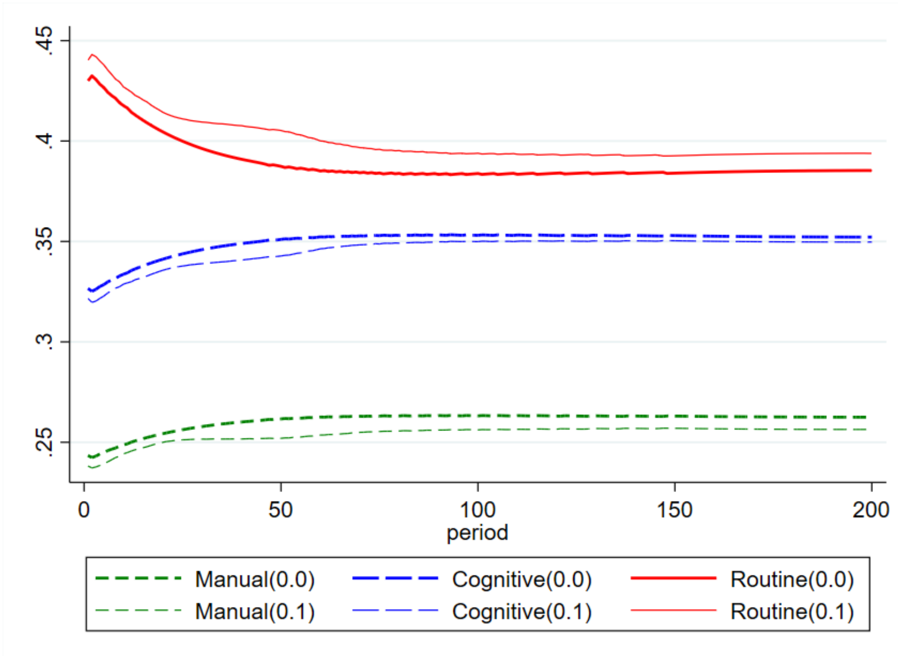


Figure 3: Occupation Share on Transition Path

free-entry condition at each period t:

$$\int V_t^e(\mathbf{0}, s_h; \underline{s_a}) d\nu(s_h) = c_e.$$

We solve the general equilibrium of this model and simulate the transition dynamics. Figure 3 draws the share of occupations from $t = 0$ to $t = 200$. The bold lines are from the simulation without the firing tax ($\tau = 0.0$) and the fine lines are with the firing tax ($\tau = 0.1$). We can note two results from the figure. First, the one-time automation shock generates a plausible pattern of polarization, as we see in the data. The share of routine employment declines and the share of cognitive and manual employment increases over time. Second, with the firing tax, the initial steady states final steady states show weaker degree of polarization and the transition to the new steady state is somewhat smaller.

In Figure 4, we examine the evolution of labor productivity Y/L. Again, the bold line corresponds to $\tau = 0.0$ and the fine line is with $\tau = 0.1$. Similarly to the previous section, the firing tax generates misallocation and the fine line is always below the bold line. Moreover, the misallocation worsens over the transition path, that is, in an automated economy the effect of misallocation is more severe.

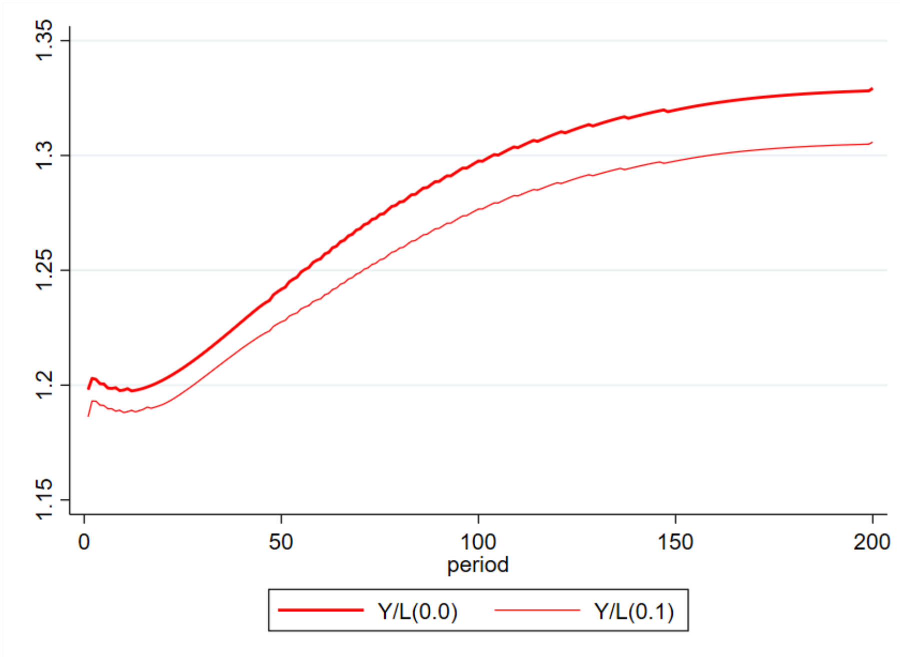


Figure 4: Y/L on Transition Path

5 Conclusion

In this paper, we analyzed how the labor-market friction interacts with the firms' decisions of reallocating workers in different occupations when the economy is facing a labor-market polarization. Using datasets from the US and Germany, we documented that within-firm reallocations contribute more to the decline of routine occupation employment in Germany, where firing restrictions are stronger than in the US.

We then built a model of firm dynamics with occupational mobility and labor-market frictions. Our model naturally extends Hopenhayn (1992) and Hopenhayn and Rogerson (1993) to multiple occupations and automation shocks. We analyzed how the labor market frictions interact with firm-level shocks in generating misallocations and how the transition dynamics during the labor-market polarization are affected by the presence of the reallocation frictions across firms and occupations.

After calibrating the model, we conducted two theoretical experiments. First, we considered two shocks, one is the standard Hicks-neutral shock and the other is a shock that represents automation. We assume that the automation shock takes an AR(1) form. We showed that the labor market friction affects aggregate employment and productivity, where the latter is induced by misallocation of resources. This result is similar to the standard firm-dynamics framework. A novel result is that the degree of misallocation is affected by the cost of internal reallocation only for the automation shock. The comparison between positive and zero firing tax qualitatively resembles the comparison between Germany and the US in the data.

For the second experiment, we still assume two shocks, but the automation shock is now a jump from zero to one, representing technology adoption. It was shown that the misallocation due to labor market frictions not only affects the eventual steady state, but also the transition dynamics.

References

Acemoglu, Daron and David Autor, "Skills, Tasks and Technologies: Implications for Employment and Earnings," *Handbook of Labor Economics*, 2011, *4*, 1043–1171.

Autor, David H and David Dorn, "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," *The American Economic Review*, 2013, pp. 1553–1597.

Battisti, Michele, Christian Dustmann, and Uta Schönberg, "Technological and Organizational Change and the Careers of Workers," *Working Paper*, 2017.

Behaghel, Luc, Eve Caroli, and Emmanuelle Walkowiak, "Information and Communication Technologies and Skill Upgrading: the Role of Internal vs External Labour Markets," *Oxford Economic Papers*, 2012, pp. 490–517.

Böhm, Michael, Hans-Martin von Gaudecker, and Felix Schran, "The Polarization of Task Prices in Germany, 1985–2010," *Working Paper*, 2016.

Dauth, Wolfgang, Sebastian Findeisen, Jens Suedekum, and Nicole Woessner, "Adjusting to Robots: Worker-Level Evidence," *Working Paper*, 2018.

Drew, Julia A Rivera, Sarah Flood, and John Robert Warren, "Making Full Use of the Longitudinal Design of the Current Population Survey: Methods for Linking Records across 16 Months," *Journal of Economic and Social Measurement*, 2014, *39* (3), 121–144.

Eden, Maya and Paul Gaggl, "On the Welfare Implications of Automation," *Review of Economic Dynamics*, 2018, *29*, 15–43.

Hopenhayn, Hugo A., "Entry, Exit, and Firm Dynamics in Long Run Equilibrium," *Econometrica*, 1992, *60* (5), 1127–1150.

Hopenhayn, Hugo and Richard Rogerson, "Job Turnover and Policy Evaluation: A General Equilibrium Analysis," *Journal of Political Economy*, 1993, *101* (5), 915–938.

Ljungqvist, Lars and Thomas J Sargent, "Two questions about European unemployment," *Econometrica*, 2008, *76* (1), 1–29.

Mortensen, Dale T, Christopher A Pissarides et al., "Unemployment Responses to'Skill-Biased'Technology Shocks: The Role of Labour Market Policy," *Economic Journal*, 1999, *109* (455), 242–265.

Moscarini, Giuseppe and Kaj Thomsson, "Occupational and Job Mobility in the US," *Scandinavian Journal of Economics*, 2007, *109* (4), 807–836.

Stinson, Martha H, "Technical Description of SIPP Job Identification Number Editing in the 1990-1993 SIPP Panels," *Working Paper*, 2003.

vom Lehn, Christian, "Labor Market Polarization, the Decline of Routine Work, and Technological Change: A Quantitative Analysis," *Journal of Monetary Economics*, 2019.

Appendix

A Data

A.1 Additional Tables and Figures

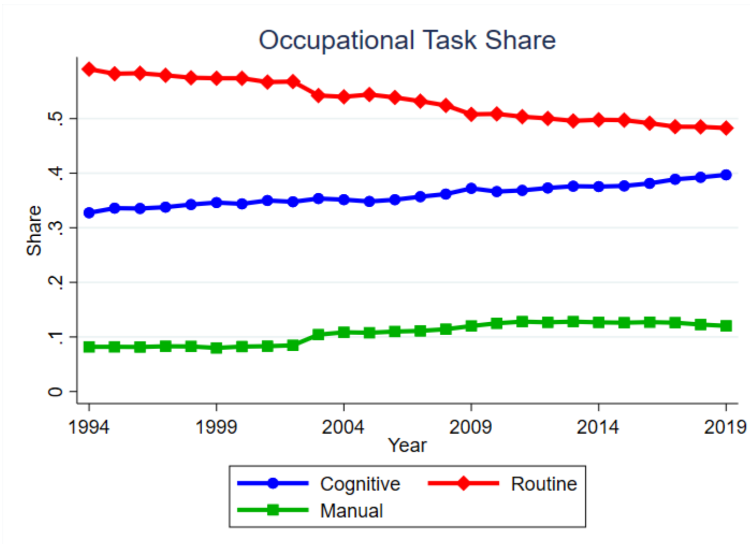
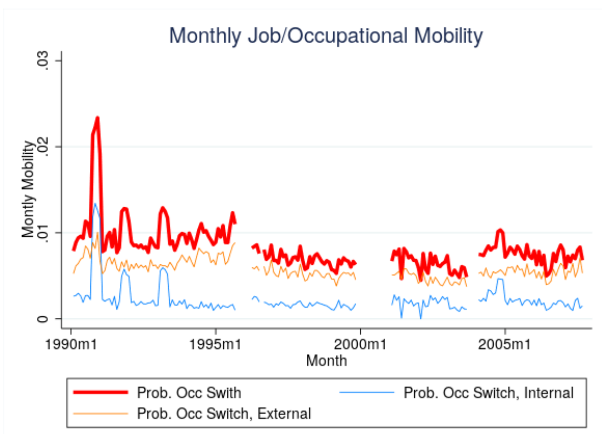


Figure 5: Occupational Employment Share in the U.S., CPS, 1994–2019



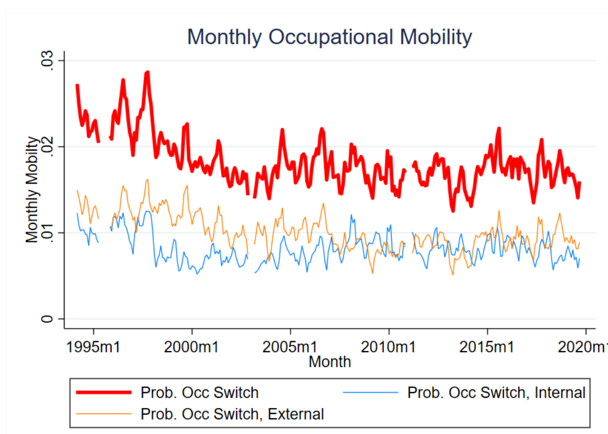


Figure 6: Monthly Internal/External Occupational Mobility in the U.S., SIPP and CPS

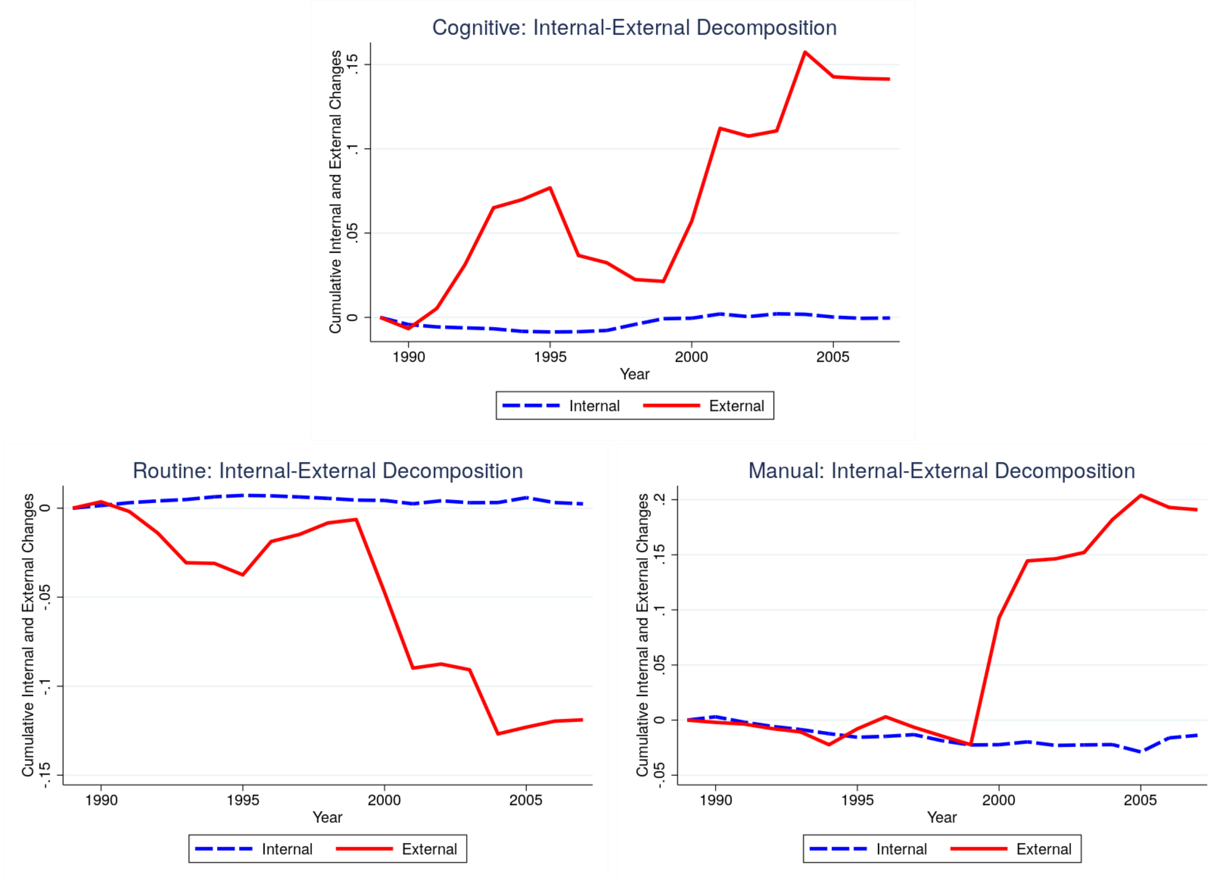


Figure 7: Cumulative Changes in Occupational Employment in the U.S., SIPP, 1993–2007

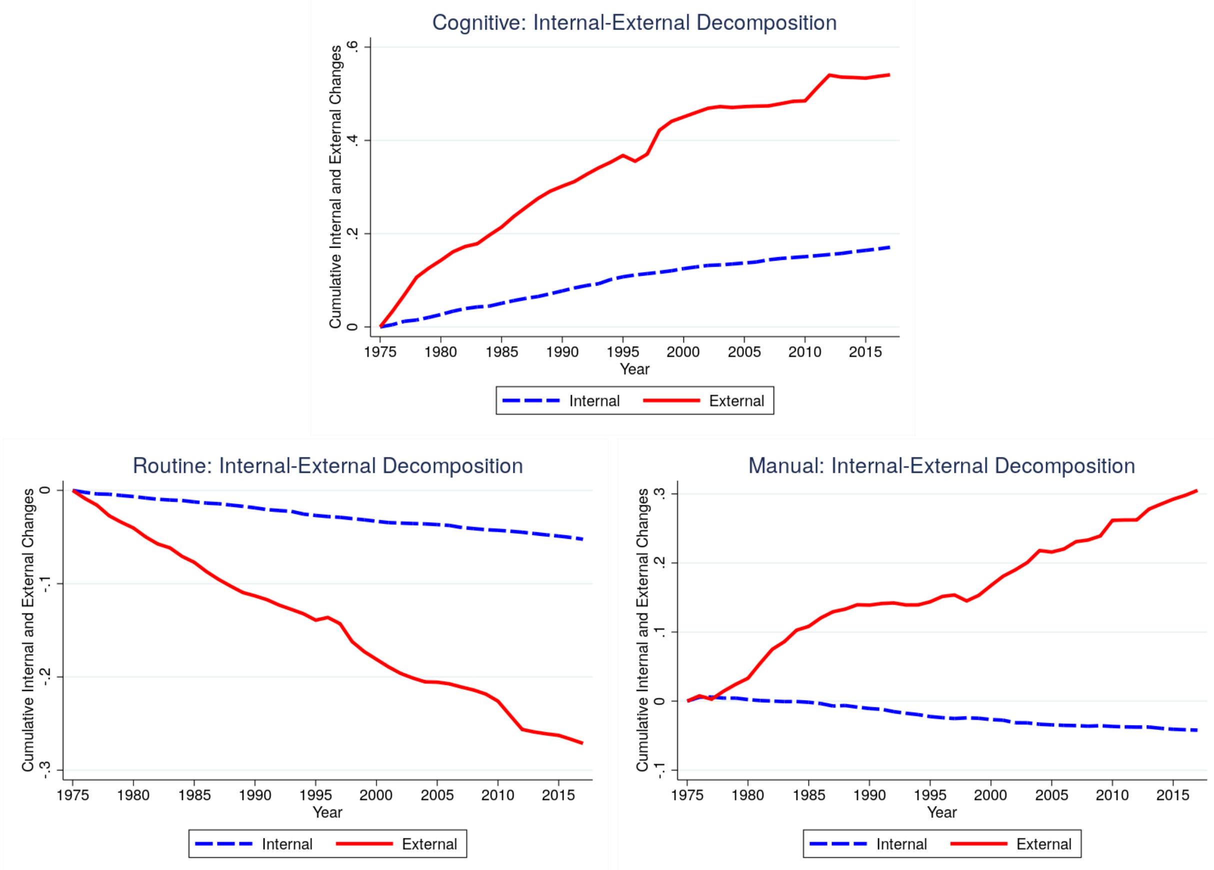


Figure 8: Cumulative Changes in Occupational Employment in Germany, SIAB, 1975–2017

B Data

B.1 Survey of Income and Program Participation (SIPP)

B.1.1 Data description

The Survey of Income and Program Participation (SIPP) is a dataset of household-based panel survey, administrated by the US Census Bureau. We use the following seven panels fo the SIPP for our analysis: 1990, 1991, 1992, 1993, 1996, 2001, and 2004 panels which have the sample of 14,000 to 52,000 individual. Each panel selects a nationally representative sample of households, interviewed every 4 months. Individuals are asked to provide their employment information as detailed as on weekly basis. With these SIPP panels, we identify the workers' job and occupation switches on both yearly and monthly basis. As noted in Stinson (2003), the 1990--1993 panels subject to substantial miscoding of their job IDs.

Thus, we use the revised job IDs provided by the U.S. Census Bureau. We do not use the panels earlier than 1990 as no revised job IDs are provided. We are not able to use the 2008 panel because the U.S. Census Bureau's data cleaning procedure has made occupational switches within firms unidentifiable for that panel.

B.1.2 Sample selection

Select observations where an individual is between age 23 and 55. We drop observations where an individual works in a public sector or as a self-employed. We also drop observations where no occupation information is available.

B.1.3 Data cleaning

In the SIPP, workers are asked to list 2 employers for each week. When a worker has two occupations at the same time, we select the occupation for their primary job by using the SIPP's primary job identifier. We drop the observations of managerial occupations to eliminate the flows due to promotions. We drop observations of the following occupations:

- Legislators,
- Chief executives and general administrators, public administration
- Administrators and officials, public administration
- Administrators, protective services

- Financial managers
- Personnel and labor relations managers
- Purchasing managers
- Managers, marketing, advertising, and public relations
- Administrators, education and related fields
- Managers, medicine and health
- Postmasters and mail superintendents
- Managers, food serving and lodging establishments
- Managers, properties and real estateFuneral directors
- Managers, service organizations, n.e.c.
- Managers and administrators, n.e.c.

B.1.4 Attrition

One of the major problems in longitudinal survey data is that individuals can drop from the sample over time. The SIPP is not exempt from this problem as well, which creates biases in the decomposition results. Therefore, we run a robustness check by running the decomposition with the balanced panels of the SIPP in Appendix D.1.

B.2 Current Population Survey (CPS)

B.2.1 Data description

The Current Population Survey (CPS), administered by the US Census Bureau, is conducted with a sample of around 60,000 households and consists of the basic monthly questions focusing on labor force participation and supplemental questions such as the annual March income supplement. We utilize the longitudinal aspect of the Basic Monthly CPS from 1994-2019. In the data, each individual shows up in the records at most eight times: respondents are contacted monthly for the first four consecutive months followed by eight months of gap and then the monthly interview resumes for the last four months. Following Moscarini and Thomsson (2007), we exploit the observations of transition between the second and the third months of the 4-8-4 sampling scheme. We use the Public Use Microdata File of the Basic Monthly CPS files from January 1994 to October 2019, which are obtained from the DataWeb FTP of the US Census Bureau. The respondents are matched based on Drew et al. (2014).

B.2.2 Sample selection

For comparability with the estimates by SIPP, we restrict our focus on males between 23 and 55 years of age. We drop observations where an individual works in a public sector or as a self-employed. We also drop observations where no occupation information is available.

B.3 Institute of Employment Research (IAB) Data

B.3.1 Data description

We use the Sample of Integrated Labour Market Biographies (SIAB) for the period 1975–2017 for our analysis of German labor markets, provided by the Institute for Employment Research (IAB) in Germany. The dataset is a 2 % sample of the population of the Integrated Employment Biographies (IEB), which comprises all individuals who showed one of the following statuses at least once during the observation period: Employment subject to social security (recorded from 1975 onwards), marginal part-time employment (recorded from 1999 onwards), receipt of benefits in accordance with Social Code Book III (recorded from 1975 onwards) or Social Code Book II (recorded from 2005 onwards), registered with the Federal Employment Agency or at an institution responsible for implementing SGB II as a jobseeker (recorded from 1997 onwards), or participation in an employment or training measure (recorded from 2000 onwards). As one caveat of this data set, civil servants and self-employed workers are not included in the data.

B.3.2 Sample selection

Select individuals who have a German citizenship. Select individuals who have never worked in East Germany. Select observations where the individual is between age 23 and 55. Drop observations where no occupation information is available.

B.3.3 Data cleaning

When a worker has multiple job spells in a year, to identify the main occupation, we calculate the number of days worked for each job spell, and select an occupation that is associated with the job spell with the longest number of days. If the number of days worked days are not available, we do the same for the earnings per day. We drop the observations of managerial occupations to eliminate the flows due to promotions. We drop observations of the following occupations:

- Foremen, master mechanics
- Entrepreneurs, managing directors
- Members of Parliament, ministers
- Senior government officials
- Association leaders, officials

B.3.4 Attrition

Workers may disappear from the social security records for various reasons (leave the labor force, migrate abroad, become a public servant or a self-employed, or pass away). The IAB is probably adding new individuals to the sample every year to keep it as 2% of the entire population in Germany.

B.4 Occupational Groups

In this paper, occupations are classified into three broad groups defined by Acemoglu and Autor (2011). These groups groups are as follows:

1. Nonroutine cognitive: Professional, technical, management, business and financial occupations.
2. Routine: Clerical, administrative support, sales workers, craftsmen, foremen, operatives, installation, maintenance and repair occupations, production and transportation occupations, laborers.
3. Nonroutine manual: Service workers.

For the SIPP and CPS, we aggregate the US Census' Occupational Classification codes into these broader categories. For the SIAB, we follow Böhm et al. (2016) to group three-digit occupations into nine categories, and define the three groups, which correspond to those in Acemoglu and Autor (2011), as follows:

1. Nonroutine cognitive: Managers, professional, and technicians.

2. Routine: Craftspeople, sales personnel, office workers, production workers, operations and laborers.

3. Nonroutine manual: Service personnel.

C Decomposition method

Let ℓ_{it} be the stock of employment of occupation i at time t. Also let

$$E_t \equiv \sum_{i=c,r,m} \ell_{it}$$

be the employment. The employment share at time t for occupation i is

$$\frac{\ell_{it}}{E_t}.$$

We would like to decompose

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right) - \log\left(\frac{\ell_{it}}{E_t}\right)$$

into net flows.

$$\log(\ell_{it}) = \log\left(\sum_{j=c,r,m,k=s,d} f_{t-1,t}^{ji,k} + f_{t-1,t}^{Ui}\right) = \log\left(\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}\right).$$

Here U includes unemployment, out of labor force, and dropped/added sample. s is for the same firm, and d is for the different firm. Thus

$$\begin{aligned}
\log(\ell_{i,t+1}) - \log(\ell_{it}) &= \log\left(\frac{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ji,k} + f_{t,t+1}^{Ui}}{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}}\right) \\
&= \log\left(1 + \frac{\sum_{j\neq i,k=s,d}(f_{t,t+1}^{ji,k} - f_{t,t+1}^{ij,k}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU})}{\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}}\right) \\
&\approx \frac{\sum_{j\neq i,k=s,d}(f_{t,t+1}^{ji,k} - f_{t,t+1}^{ij,k}) + (f_{t,t+1}^{Ui} - f_{t,t+1}^{iU})}{\ell_{it}}
\end{aligned}$$

Note also that

$$\begin{aligned}
\log(E_{t+1}) - \log(E_t) &\approx \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \ell_{it} \frac{E_{t+1} - E_t}{E_t} \\
&= \frac{1}{\ell_{it}} \left(\sum_{j=c,r,m,k=s,d} f_{t,t+1}^{ij,k} + f_{t,t+1}^{iU}\right) \frac{E_{t+1} - E_t}{E_t}
\end{aligned}$$

Let

$$\Delta_{t,t+1}^E \equiv \frac{E_{t+1} - E_t}{E_t}.$$

Combining above, we have

$$\log\left(\frac{\ell_{i,t+1}}{E_{t+1}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)=\frac{1}{\ell_{it}}\left[\sum_{j\neq i}(f^{ji,s}_{t,t+1}-f^{ij,s}_{t,t+1})+\sum_{j\neq i}(f^{ji,d}_{t,t+1}-f^{ij,d}_{t,t+1})+(f^{Ui}_{t,t+1}-f^{iU}_{t,t+1})-\ell_{it}\Delta^E_{t,t+1}\right].$$

To calculate the cumulative changes from period t to period $t+T$, note that

$$\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)=\sum_{\tau=0}^{T-1}\left[\log\left(\frac{\ell_{i,t+\tau+1}}{E_{t+\tau+1}}\right)-\log\left(\frac{\ell_{i,t+\tau}}{E_{t+\tau}}\right)\right].$$

Then, we can apply the decomposition formula to have

$$\begin{aligned}&\log\left(\frac{\ell_{i,t+T}}{E_{t+T}}\right)-\log\left(\frac{\ell_{it}}{E_t}\right)\\&=\left[\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,s}_{t+\tau,t+\tau+1}-f^{ij,s}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}+\sum_{\tau=0}^{T-1}\sum_{j\neq i}\frac{f^{ji,d}_{t+\tau,t+\tau+1}-f^{ij,d}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}\right.\\&\qquad\left.+\sum_{\tau=0}^{T-1}\frac{f^{Ui}_{t+\tau,t+\tau+1}-f^{iU}_{t+\tau,t+\tau+1}}{\ell_{i,t+\tau}}-\sum_{\tau=0}^{T-1}\Delta^E_{t+\tau,t+\tau+1}\right].\end{aligned}$$

D Robustness

D.1 Balanced Panel for SIPP

To check the robustness of our results in Table 1, and Figure 7 and 8 for the sample attrition issue of the SIPP sample, we create balanced panel for the SIPP and run the decomposition again. That is, we select the individuals who report their labor market status without any missing observations over the sample period of each SIPP panel, and use created, the balanced panel data for our analysis. Our internal-external decomposition results don't change the patterns even for the balanced panel case as seen in Table 4 and Figure 9.

Table 4: Decompositions of occupational employment share changes for the US (SIPP), balanced panel

	Occupational employment share			Decomposed contributions	
	(1)	(2)	(3)	(4)	(5)
U.S.	1989	2007	log (Δshare)	Internal	External
Cognitive	0.29	0.33	0.14	0.03	0.11
Routine	0.60	0.53	−0.12	0.01	−0.13
Manual	0.11	0.13	0.18	−0.10	0.29

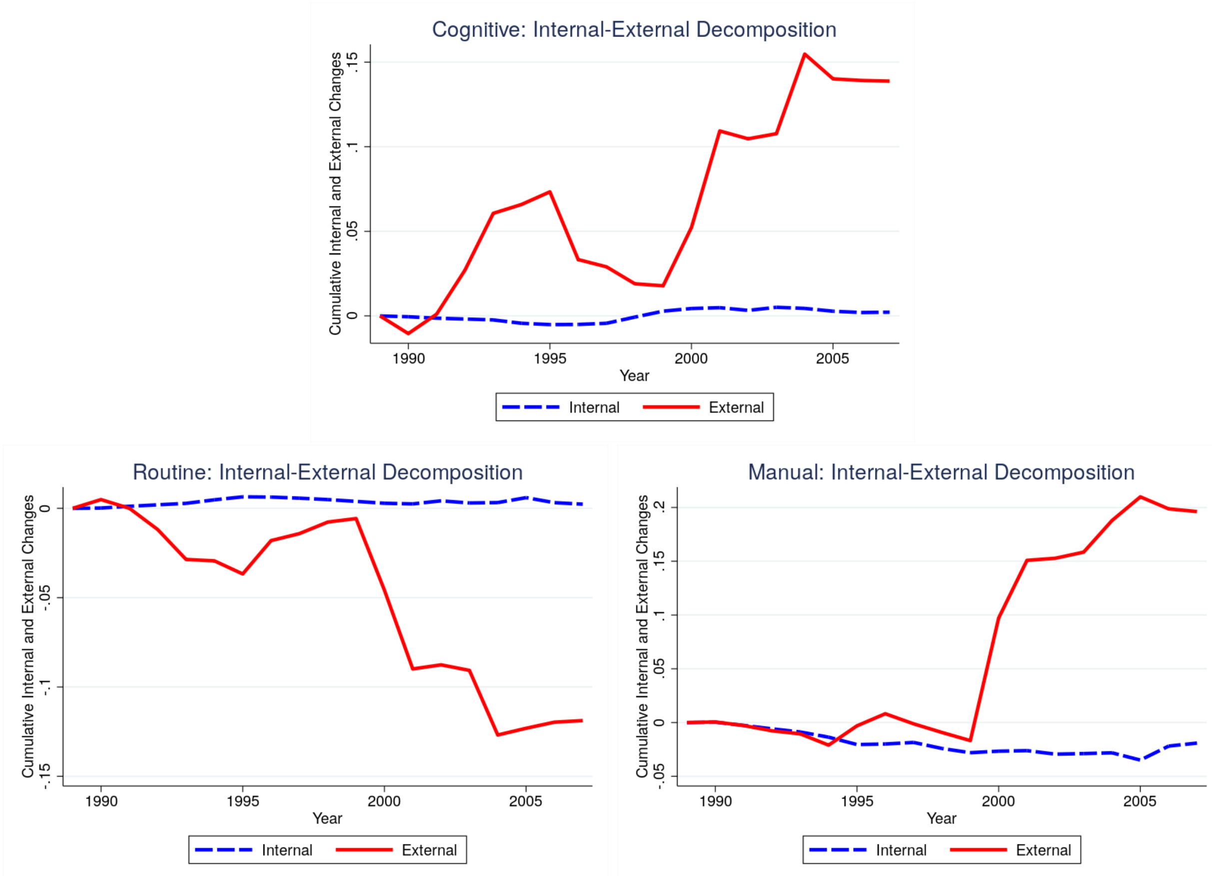


Figure 9: Cumulative Changes in Occupational Employment in the U.S., SIPP, 1993–2007, Balanced Panel

E Computing Transition

We want to compute the transition path between the initial steady state and the final steady state. Let us overview the objects for computation. The notation is in the general form without simplifying.

First, the value functions are specified as follows. Assuming the linear utility with respect to consumption

$$U(C,N) \quad = \quad C - \xi \frac{N^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}},$$

we can set the discount factor for firms to be constant β as $U_1(C',N')/U_1(C,N)$ is always unity. Thus, the value function for incumbents at the initial state where all firms are non-automated is written as

$$\begin{aligned}
& \underline{V}(\mathbf{n}, s_h; \underline{s_a}) \\
= \quad & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s_a}) - \underline{w} \mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\underline{V}(\mathbf{n}', s'_h; \underline{s_a}) | s_h], \\
& -\tau \max\{\sum_j (n'_j - 0), 0\}\} \Bigg]
\end{aligned}$$

and at the final state where all firms are automated is written as

$$\begin{aligned}
& \overline{W}(\mathbf{n}, s_h; \overline{s_a}) \\
= \quad & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \overline{s_a}) - \overline{w} \mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\overline{W}(\mathbf{n}', s'_h; \overline{s_a}) | s_h], \\
& -\tau \max\{\sum_j (n'_j - 0), 0\}\} \Bigg].
\end{aligned}$$

As for entrants,

$$\begin{aligned}
\underline{V}^e(\mathbf{0}, s_h; \underline{s_a}) &= \underline{V}(\mathbf{0}, s_h; \underline{s_a}) + c_f - c_e, \\
\overline{W}^e(\mathbf{0}, s_h; \overline{s_a}) &= \overline{W}(\mathbf{0}, s_h; \overline{s_a}) + c_f - c_e,
\end{aligned}$$

where all entrants at the initial state are non-automated and all entrants at the final state are automated.

On the transition path, the value functions for non-automated and automated incumbent firms are written as

$$
\begin{aligned}
& V_t(\mathbf{n}, s_h; \underline{s_a}) \\
= & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s_a}) - w_t \mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\phi W_{t+1}(\mathbf{n}', s'_h; \overline{s_a}) + (1-\phi) V_{t+1}(\mathbf{n}', s'_h; \underline{s_a}) | s_h], \\
& -\tau \max\{\sum_j (n'_j - 0), 0\}\} \Bigg],
\end{aligned}
$$

and

$$
\begin{aligned}
& W_t(\mathbf{n}, s_h; \overline{s_a}) \\
= & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \overline{s_a}) - w_t \mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[W_{t+1}(\mathbf{n}', s'_h; \overline{s_a}) | s_h], \\
& -\tau \max\{\sum_j (n'_j - 0), 0\}\} \Bigg]
\end{aligned}
$$

respectively. The value function for entrants are

$$
V^e_t(\mathbf{0}, s_h; \underline{s_a}) \quad = \quad \psi_t W_t(\mathbf{0}, s_h; \overline{s_a}) + (1-\psi_t) V_t(\mathbf{0}, s_h; \underline{s_a}) + c_f - c_e,
$$

as the entrants become automated with probability ψ_t and become non-automated with probability $1 - \psi$.

Secondly, the distributions of firms are defined as below. Let $m^V_t(\mathbf{n}, s_h; \underline{s_a})$ and $m^W_t(\mathbf{n}, s_h; \overline{s_a})$ be the measures of non-automated and automated firms right after entrants enter in the period t, e_t be the mass of entrant firms, and $i^V_t(\mathbf{n}, s_h; \underline{s_a})$ and $i^W_t(\mathbf{n}, s_h; \overline{s_a})$ be the measures of surviving non-automated and automated firms at the end of the period t (incumbents at the beginning of the next period). The counterparts at the initial steady state are denoted by $\underline{m}^V(\mathbf{n}, s_h; \underline{s_a})$, $\underline{e}$ and $\underline{i}^V(\mathbf{n}, s_h; \underline{s_a})$. As for the final steady state, they are $\overline{m}^W(\mathbf{n}, s_h; \overline{s_a})$, $\overline{e}$ and $\overline{i}^W(\mathbf{n}, s_h; \overline{s_a})$.

Finally, the probability ψ_t is assumed to be equal to the share of incumbent automated firms at the beginning of the period as

$$
\psi_t \quad = \quad \frac{\int i^W_{t-1}(d\mathbf{n}, ds_h; \overline{s_a})}{\int i^V_{t-1}(d\mathbf{n}, ds_h; \underline{s_a}) + \int i^W_{t-1}(d\mathbf{n}, ds_h; \overline{s_a})}.
$$

We compute these objects by the following steps.

E.1 Preparation

We discretize the labor and the shock and the grid points are denoted by $(n_m^{g_m}, n_c^{g_c}, n_r^{g_r}) = \mathbf{n}^{\mathbf{g_n}}$ and $s_h^{g_h}$ where integer $g_. \in \{1, ..., g_.^{max}\}$. Later, we want to redistribute the weight of an off-grid point $\mathbf{n}$ to the neighboring grid points such as$\mathbf{n}^{\mathbf{g_n}}$ by the following discrete measure G such that

$$G(\mathbf{n}, \mathbf{n}^{\mathbf{g_n}}) \quad = \quad \begin{cases} \frac{\prod |n_j^{g_j'} - n_j|}{\prod_j |n_j^{g_j'} - n_j^{g_j}|} & \text{if } n_j \text{ is between } n_j^{g_j} \text{ and } n_j^{g_j'} \text{ including endpoint for all } j = m, c, r, \\ 0 & \text{otherwise,} \end{cases}$$

where g_j' is either $g_j - 1$ or $g_j + 1$. The transition probability from $s_h^{g_h}$ to $s_h^{g_h'}$ is denoted by $P(s_h^{g_h'} | s_h^{g_h})$.

While $(\beta, \eta, \phi, \tau, \kappa, c_f)$ are given from outside model, c_e and ξ are pinned down within model. First, assuming $\tau = 0$ and $\underline{w}$=1, we solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s_a})$ by value function iteration. Then, as $V(\mathbf{0}, s_h; \underline{s_a}) + c_f$ is pinned down, c_e can be set to satisfy the free entry condition

$$\sum_{g_h} \underline{V}^e(\mathbf{0}, s_h^{g_h}; \underline{s_a}) \nu(s_h^{g_h}) \quad = \quad 0,$$

where ν is probability for initial s_h. Next, assuming the entrant mass $\underline{e}$ is unity and simulating the above firms' decision rule repeatedly as

$$\begin{aligned} \underline{m}^{V,new}(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s_a}, \underline{e} = 1) \quad &= \quad \underline{i}^{V,new}(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}, \underline{e} = 1) + 1\{\mathbf{n}^{\mathbf{g_n}'} = \mathbf{0}\} \nu(s_h^{g_h'}), \\ \underline{i}^{V,new}(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g_h'}; \underline{s_a}, \underline{e} = 1) \quad &= \quad \sum_{\mathbf{g_n}} \sum_{g_h} \Big[G(\underline{\mathbf{n}}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g_h'} | s_h^{g_h}) \underline{m}^{V,old}(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}, \underline{e} = 1) \\ &\quad 1\{\mathbb{E}_{s_h^{g_h'}}[\underline{V}(\mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), s_h^{g_h'}; \underline{s_a}) | s_h^{g_h}] \geq -\tau \max\{\mathbf{1} \cdot \underline{\mathbf{n}}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), 0\}\} \Big], \end{aligned}$$

we can obtain an invariant distribution of firms $\underline{m}^V(\mathbf{n}, s_h; \underline{s_a}, \underline{e} = 1)$ and $\underline{i}^V(\mathbf{n}, s_h; \underline{s_a}, \underline{e} = 1)$ with the unit entry mass. Note $\underline{m}^V$ consists of the incumbents after exiting occurs and the entrants right after entering and $\underline{i}^V$ consists of the surviving incumbents and entrants after exiting occurs. Then, the labor demand with unit mass entry is $\underline{N} = \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) \underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}, \underline{e} = 1)$. Then, by the intra-temporal optimality,

$$\xi \quad = \quad \frac{\underline{w}}{\underline{N}^{\frac{1}{\eta}}}.$$

E.2 Computing initial and final steady states

Setting $\tau > 0$ and using c_e pinned down above, we guess the free entry wage $\underline{w}$, solve for $\underline{V}$ and the corresponding decision rule $\underline{\mathbf{n}}'(\mathbf{n}, s_h; \underline{s_a})$ by value function iteration, and check if $\underline{w}$

satisfies the free entry condition. If the entry value is positive, increase $\underline{w}$ and *vice versa,* and repeat it until $\underline{V}^e$ satisfies the free entry condition. Similarly for $\overline{w}$ and $\overline{W}$. Assuming $\underline{e}$ and $\overline{e}$ are unity, we compute the invariant distribution $\underline{m}^V(\mathbf{n}, s_h; \underline{s_a}, \underline{e} = 1)$ and $\overline{m}^W(\mathbf{n}, s_h; \overline{s_a}, \overline{e} = 1)$ by using the obtained decision rule similarly to 1.1. Pin down $\underline{e}$ by the supply and demand of total labor L

$$
\begin{aligned}
\underline{L} &= \left(\frac{\underline{w}}{\xi}\right)^{\eta} \\
&= \underline{e} \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) \underline{m}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}, \underline{e} = 1),
\end{aligned}
$$

and the measure consistent with the labor-market equilibrium is obtained by $\underline{m}^V(\mathbf{n}, s_h; \underline{s_a}) = \underline{e} \cdot \underline{m}^V(\mathbf{n}, s_h; \underline{s_a}, \underline{e} = 1)$. Similarly for $\overline{e}$ and $\overline{m}^W(\mathbf{n}, s_h; \overline{s_a})$.

E.3 Guessing ψ_t

Let the transition occur from $t = 1$ to $t = T$. We guess the path of ψ_t from $\psi_1 = 0$ to $\psi_T = 1$.

E.4 Backward induction

Based on the guess of ψ_t, we solve for w_t,V_t and W_t and corresponding decision rules $\mathbf{n}'_t(\mathbf{n}, s_h; \underline{s_a})$ and $\mathbf{n}'_t(\mathbf{n}, s_h; \overline{s_a})$ by backward induction from T to 1, while we set $W_{T+1} = \overline{W}$ and $V_{T+1} = \overline{V}$ which is a hypothetical non-automated value function at the final steady state and obtained by solving

$$
\begin{aligned}
& \overline{V}(\mathbf{n}, s_h; \underline{s_a}) \\
= \; & \max_{\mathbf{n}' \geq \mathbf{0}} \Bigg[-\tau \max\{\sum_j (n_j - \tilde{n}'_j(\mathbf{n}, \mathbf{n}')), 0\} - \sum_j \kappa_j (\max\{\tilde{n}'_j(\mathbf{n}, \mathbf{n}') - n_j, 0\})^2 \\
& + f(\mathbf{n}', s_h; \underline{s_a}) - \overline{w} \mathbf{1} \cdot \mathbf{n}' - c_f + \beta \max\{\mathbb{E}_{s'_h}[\phi \overline{W}_{t+1}(\mathbf{n}', s'_h; \overline{s_a}) + (1 - \phi)\overline{V}(\mathbf{n}', s'_h; \underline{s_a}) | s_h], \\
& -\tau \max\{\mathbf{1} \cdot \mathbf{n}', 0\}\} \Bigg].
\end{aligned}
$$

At each t, first we guess the free entry wage w_t, solve for V_t and W_t and the decision rules, update the guess of w_t and repeat this until V_t^e satisfies the free entry condition. Then proceed to $t - 1$.

E.5 Simulating forward

Using the decision rules obtained above for $t = 1, ..., T$ and the supply and the demand of labor are described as

$$
\begin{aligned}
L_t &= \left(\frac{w_t}{\xi}\right)^{\eta}, \\
&= \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) i_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) + \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}) i_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}) \\
&+ e_t[\psi_t \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \overline{s_a}) \nu(s_h^{g_h}) + (1 - \psi_t) \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \underline{s_a}) \nu(s_h^{g_h})],
\end{aligned}
$$

where the first term in the right hand side is the labor demand from non-automated incumbents from previous period, the second is the demand from automated incumbents and the third is the demand from entrants. It follows that the entrant mass e_t at period t is pinned down as

$$
\frac{\left(\frac{w_t}{\xi}\right)^{\eta} - \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) i_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) - \sum_{\mathbf{g_n}} \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}) i_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a})}{\psi_t \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \overline{s_a}) \nu(s_h^{g_h}) + (1 - \psi_t) \sum_{g_h} \mathbf{1} \cdot \mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \underline{s_a}) \nu(s_h^{g_h})}.
$$

Then, we can compute $m_t^V(\mathbf{n}, s_h; \underline{s_a})$, $i_t^V(\mathbf{n}, s_h; \underline{s_a})$, $m_t^W(\mathbf{n}, s_h; \overline{s_a})$ and $i_t^W(\mathbf{n}, s_h; \overline{s_a})$ as follows. First,

$$
m_t^V(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g'_h}; \underline{s_a}) = i_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) + 1\{\mathbf{n}^{\mathbf{g_n}'} = \mathbf{0}\} e_t (1 - \psi_t) \nu(s_h^{g'_h}),
$$

that is the sum of the non-automated incumbents from the last period and the entrants which enter as non-automated. Second,

$$
\begin{aligned}
i_t^V(\mathbf{n}^{\mathbf{g_n}'}, s_h^{g'_h}; \underline{s_a}) &= \sum_{\mathbf{g_n}} \sum_{g_h} \Big[G(\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g'_h} | s_h^{g_h}) (1 - \phi) i_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) \\
& 1\{\mathbb{E}_{s_h^{g'_h}} [\phi W_{t+1}(\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), s_h^{g'_h}; \overline{s_a}) + (1 - \phi) V_{t+1}(\mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), s_h^{g'_h}; \underline{s_a}) | s_h^{g_h}] \\
& \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}'_t(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), 0\}\}\Big] \\
&+ e_t \sum_{g_h} \Big[G(\mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \underline{s_a}), \mathbf{n}^{\mathbf{g_n}'}) P(s_h^{g'_h} | s_h^{g_h}) (1 - \phi)(1 - \psi_t) \nu(s_h^{g_h}) \\
& 1\{\mathbb{E}_{s_h^{g'_h}} [\phi W_{t+1}(\mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \overline{s_a}), s_h^{g'_h}; \overline{s_a}) + (1 - \phi) V_{t+1}(\mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \overline{s_a}), s_h^{g'_h}; \underline{s_a}) | s_h^{g_h}] \\
& \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}'_t(\mathbf{0}, s_h^{g_h}; \underline{s_a}), 0\}\}\Big],
\end{aligned}
$$

where the first term in the right hand side is the surviving non-automated incumbents which do not become automated at the end of period t and the second is the surviving entrants which enter as non-automated and do not become automated at the end of period t. Third,

$$m_t^W(\mathbf{n}^{\mathbf{g_n}\prime}, s_h^{g_h'}; \overline{s_a}) \quad = \quad i_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}) + 1\{\mathbf{n}^{\mathbf{g_n}\prime} = \mathbf{0}\} e_t \psi_t \nu(s_h^{g_h'}),$$

that is the sum of the automated incumbents from the last period and the entrants which enter as automated. Fourth,

$$\begin{aligned}
i_t^W(\mathbf{n}^{\mathbf{g_n}\prime}, s_h^{g_h'}; \overline{s_a}) \quad = \quad & \sum_{\mathbf{g_n}} \sum_{g_h} \Big[G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) \phi i_{t-1}^V(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}) \\
& 1\{\mathbb{E}_{s_h^{g_h'}}[\phi W_{t+1}(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \overline{s_a}) + (1-\phi) V_{t+1}(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \underline{s_a}) | s_h^{g_h}] \\
& \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \underline{s_a}), 0\}\} \Big] \\
+ \quad & e_t \sum_{g_h} \Big[G(\mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \underline{s_a}), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) \phi (1-\psi_t) \nu(s_h^{g_h}) \\
& 1\{\mathbb{E}_{s_h^{g_h'}}[\phi W_{t+1}(\mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \overline{s_a}) + (1-\phi) V_{t+1}(\mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \underline{s_a}) | s_h^{g_h}] \\
& \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \underline{s_a}), 0\}\} \Big] \\
+ \quad & \sum_{\mathbf{g_n}} \sum_{g_h} \Big[G(\mathbf{n}_t'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) i_{t-1}^W(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}) \\
& 1\{\mathbb{E}_{s_h^{g_h'}}[W_{t+1}(\mathbf{n}'(\mathbf{n}^{\mathbf{g_n}}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \overline{s_a}) | s_h^{g_h}] \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}, 0\}\} \Big] \\
+ \quad & e_t \sum_{g_h} \Big[G(\mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}), \mathbf{n}^{\mathbf{g_n}\prime}) P(s_h^{g_h'} | s_h^{g_h}) \psi_t \nu(s_h^{g_h}) \\
& 1\{\mathbb{E}_{s_h^{g_h'}}[W_{t+1}(\mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}), s_h^{g_h'}; \overline{s_a}) | s_h^{g_h}] \geq -\tau \max\{\mathbf{1} \cdot \mathbf{n}_t'(\mathbf{0}, s_h^{g_h}; \overline{s_a}), 0\}\} \Big],
\end{aligned}$$

where the first term in the right hand side is the surviving non-automated incumbents which become automated at the end of period t, the second is the surviving non-automated entrants which become automated at the end of period t, the third is the surviving automated incumbents and the fourth is the surviving entrants which enter as automated. As for the period 1 measure, we set $m_1^V(\mathbf{n}, s_h; \underline{s_a}) = \underline{m}^V(\mathbf{n}, s_h; \underline{s_a})$, $i_1^V(\mathbf{n}, s_h; \underline{s_a}) = \underline{i}^V(\mathbf{n}, s_h; \underline{s_a})$, $m_1^W(\mathbf{n}, s_h; \overline{s_a}) = \overline{m}^W(\mathbf{n}, s_h; \overline{s_a})$ and $i_1^W(\mathbf{n}, s_h; \overline{s_a}) = \overline{i}^W(\mathbf{n}, s_h; \overline{s_a})$. After computing the measures, we can update the guess ψ_t by the definition of ψ_t and repeat 1.4 and 1.5 until ψ_t converges.