Superstar Firms and Frictional Labor Market

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Abstract

Large firms receive more applicants per vacancy given wages. I hypothesize that being large itself is one reason why they attract more applicants. Then, I build a directed search model with superstar and small firms to understand how such a difference affects the labor market. In the model, superstar firms attract more workers given wages because job searchers more easily recognize their vacancies. I show that if this recruiting advantage is large enough compared to the productivity gap between superstar and small firms, then small firms post inefficiently lower wages in equilibrium to avoid hiring competition. Thus, the equilibrium is inefficient, and wage distribution becomes more polarized. Superstar firms enjoy a higher profit than that of the efficient case because job search value is lower. As a result, wages and labor shares of both superstar and small firms are lower. The model suggests that the rise of superstar firms can contribute to the wage polarization and fall in the labor share through its labor market competition effects. Thus, helping small businesses in hiring can improve both welfare and distribution.

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1 Introduction

This paper studies a frictional labor market with two types of firms. One type of firm, which I denote 'superstar,' is more productive and more famous, so that vacancy information from these firms reaches many job searchers. The other 'small' firms are less efficient in producing goods and less renowned than superstar firms. This paper investigates how such a difference in renown affects the aggregate labor market in equilibrium, mainly focusing on wage distribution and efficiency.

In the past decades, we have seen the rise of superstar firms like Google, Apple, Amazon, and Facebook. Such a trend is not confined to the IT industry but rather widespread phenomena across sectors (Autor et al., 2020; De Loecker et al., 2020). What does this trend imply to the labor market? Many channels have been proposed, including distributional effects across different markups and labor market power, and this paper proposes another channel: heterogeneous recruiting efficiency. There are several reasons to expect that large and more productive firms are more efficient in recruiting, and one reason is their size itself. For instance, job searchers would recognize large firms' vacancy information more quickly because large firms are widely known. Alternatively, job searchers may value being an employee of large firms given that all compensations are equal.

I provide supporting empirical evidence using Employment Opportunity Pilot Projects(EOPP): the number of job applications for a vacancy is positively correlated with firm size.[1] The correlation survives after controlling for compensations, job characteristics, and proxy for recruiting intensity. The correlation is not because of applicants' expectation that large firms will hire more applicants since the offer-applicants ratio decreases in firm size. Furthermore, employers' subjective difficulty in finding unskilled workers decreases in firm size. These patterns imply that workers disproportionately recognize and apply to large firms' vacancy more.[2]

How does such heterogeneity affect the aggregate labor market? To answer the question, I build a directed search model with two types of vacancies: superstar and small.[3] Superstar vacancies have higher productivity when matched with a worker. Furthermore, they are more visible than small vacancies: each superstar vacancy is observed by a larger fraction of job searchers within a submarket. Therefore, the effective number of job searchers per vacancy is larger for superstar vacancies, so that they enjoy a higher vacancy-filling probability

[1] Rebien et al. (2020) finds the same relationship in Germany.

[2] Previous literature finds that a firm's reputation or image positively influences the quality and quantity of job applicants (Belt and Paolillo, 1982; Turban and Cable, 2003; Lange et al., 2011).

[3] In the model, there is no distinction between 'firm' and 'vacancy' because of the linear production and vacancy cost.

than small vacancies in the same submarket. Vacancies post wages understanding that this different renown leads to a different vacancy-filling probability given wage, in addition to the standard trade-off between wage and vacancy-filling probability. The other side of the market consists of homogeneous job searchers. Job searchers choose a wage to search, understanding that the vacancy type distribution affects the job-finding probability.

I first investigate the social planner's allocation subject to the same matching technology. In this environment, the social planner choose two numbers: the number of job searchers per each type of vacancy, which is called queue length. When choosing the queue lengths, the social planner equalizes the marginal social value of a job searcher across two types of vacancy. It requires that the vacancy-filling probability ratio is the inverse of the productivity ratio. Therefore, in the constrained efficient allocation, the queue length depends on firms' productivity, independent of how famous they are.

I then study the equilibrium allocation and show that the equilibrium is not constrained efficient if the difference in renown is large compared to the productivity gap. To understand the intuition, suppose two types' productivity is almost identical. Then, the social planner would choose almost the same queue length for each type. This allocation is supported in equilibrium if two types choose almost the same wage. However, if two wages are almost identical, superstar vacancies prefer to search in the small vacancy market even though the wage is suboptimal because they can enjoy a strictly higher vacancy-filling probability by competing with small vacancies in hiring. Such incentive disappears only if the wage difference is sufficiently large; thus the equilibrium features inefficiency and wage polarization.

What is the direction of distortion if the equilibrium is not constrained efficient? I show that it is small vacancies that choose an suboptimal wage. In the equilibrium, given the value of job search, superstars' choice is not distorted: wage and queue length are the same as if there are no small vacancies in the economy. On the other hand, the wage of small vacancies is constrained by the incentive of superstar vacancies, so that the wage is much lower than the counterfactual economy without superstar vacancies. There is also a general equilibrium effect through the fall in the value of job search, which pushes down wages paid by both types. As a result, superstar vacancies receive a larger profit than what they could have received in the constrained efficient allocation. Whether small vacancies are better off depends on the relative size of the direct incentive effect and general equilibrium effect.

This paper sheds light on an additional channel through which the rise of superstar firms contributes to the increase in inequality. It also points out that the wage polarization itself can indicate inefficient allocation if it is driven by the increased heterogeneity in firms' recruiting efficiency. The model also has an implication on labor share. In the equilibrium, the labor share of both types falls because small vacancies are constrained to pay a lower wage, and superstar vacancies do not need to compensate much due to the lower value of job search. Thus, the fall in labor share implies a completely different meaning for superstar and small firms even though the direction of changes coincides.

Despite the rise of superstar firms, small businesses still dominate the labor market. They consist of 99% of U.S employer firms and about half of private-sector employment.[4] These firms face various difficulties, including financial constraints, and hiring a qualified candidate is also one of big challenges (Williamson et al., 2002). While their lower compensation explains a significant part of why they suffer in finding a qualified candidate, this paper suggests that their lower compensation may result from the fact that they are small, instead of their low productivity. Thus, if a policymaker helps them in recruiting markets, as a policy agency does in financial markets, it can positively affect aggregate welfare and distribution.

The contribution of this paper is twofold. First, this paper sheds light on a new mechanism through which the product market concentration induces the wage polarization and fall in the labor share.[5] While the linkage has been already proposed (Autor et al., 2020; Barkai, 2020; De Loecker et al., 2020), most papers focus on the labor market effects through higher markups charged by large firms while assuming a perfectly competitive labor market. Instead, this paper points out a mechanism in the opposite direction: the polarization in the labor demand side can cause higher markups in product markets. Second, this paper also contributes to the directed search literature by allowing that some agents are more famous than the others (Moen, 1997; Burdett et al., 2001; Shi, 2009). In doing so, I made a clear distinction between the renown and search or recruiting efforts (Pissarides, 2000; Gavazza et al., 2018). While this paper focuses on the labor market context, the intuition can be applied to other frictional markets. For instance, this paper provides another explanation of why larger banks charge a higher mortgage rate (Allen et al., 2019).

2 Empirical motivation

2.1 Data

The ideal data for examining the recruiting efficiency across firms must contain the number of job applicants, number of vacancies, and characteristics of firms, jobs, and workers.

[4]Source: Small Business Administration, www.sba.gov/advocacy/7540/42371.

[5]Each of these trends are well established in previous literature. For instance, the rise in product markets concentration has been documented in Autor et al. (2020); Barkai (2020); De Loecker et al. (2020). The secular increase in wage inequality has been documented in many papers, including Autor et al. (2006); Autor and Dorn (2013). The fall in labor share has been documented in Elsby et al. (2013); Karabarbounis and Neiman (2014).

Unfortunately, few datasets cover such information altogether. In this regard, the Employment Opportunity Pilot Projects(EOPP) provides valuable information about firms' recruiting behaviors. The data were collected in 1980 and 1982 to evaluate policies that help firms hire and train workers. The data contains various aspects about recruiting and hiring, including the number of vacancies, job applications, interviews, and offers. Also, it asks the demographics of the last hire, which represents the position's requirement, and some information about firms such as the number of employees, industry, and sales. As it is a rare combination of information, recent papers still use the dataset (Faberman and Menzio, 2018; Wolthoff, 2018).

In the data, the survey unit is an 'employer,' represented by a single account number. It can be a single establishment or several establishments under a unique account number. Table 1 shows summary statistics. After excluding non-respondents, the total number of samples in the 1982 survey is 3,411. The average number of employees is about 30.5, but as the size distribution is highly skewed to the right so that 84% of employers are smaller than the average size. These employers have 2.9 vacancies on average and receive 3.8 job applications unconditionally, and 9.3 job applications conditional on at least one application is received. Among these, 5.8 candidates are interviewed, and 3.1 of them get an offer. Note that many employers interview and give offers to candidates even without formal applications. The average number of offers made among the employers who had not received any formal application is 1.9. It shows that firms rely on both formal and informal methods to hire workers.

Insert Table 1 here.

On top of the total number of vacancies, applications, interviews, and offers that employers have, the survey asks these numbers for a specific position that is lastly filled. For the specific position, employers open 1.3 vacancies on average; thus, a vacancy roughly represents a position in the data. Employers receive 10.2 applicants for the position, and then about 6 of them are interviewed, and 1.3 of them receive offers. The survey also asks demographics about the person who fills this position, such as gender and education.

In the empirical analysis, I mainly focus on the numbers for a position lastly filled for two reasons. First, the number of applications, interviews, offers made are more interpretable as no firm reports 0 application. Second, the demographics and some job characteristics are specific to the position lastly filled.

2.2 Firm size, the number of applicants, queue length, and offers

The baseline regression equation is the following:

$$y_i = \alpha + x_i'\beta_0 + z_i'\beta_1 + \gamma \log(w_i) + \delta \log(E_i) + \eta \log(S_i) + \epsilon_i \tag{1}$$

y_i is the variable of interests: the number of applicants, queue length, and offer rate. The queue length is the number of applicants divided by the number of vacancies for a position. The offer rate is the ratio between the number of offers to applicants. x_i is characteristics of workers, which include age, gender, and schooling. To control schooling, I classify educational attainment into five categories: less than high school, high school graduate, some colleges less than BA, Bachelor's degree, and higher than BA. z_i is characteristics of firms or jobs, including industry, region, and employment type. w_i is starting wage, and γ captures the effect of wage on y_i. I include the hours spent for recruiting E_i into controls. S_i is the number of employees, and η is the parameter that governs the effects of firm size.

Insert Table 2 here.

The first column in Table 2 is the regression result for the number of applicants. In all specifications, industry and location are controlled. The result shows a positive relationship between the firm size and the number of applicants. The magnitude is significant; when causally interpreted, 1% increase in firm size implies 1.06 more applicants for a position. The second column is the result for the queue length. The discrepancy is because some positions had multiple vacancies to be filled. Still, there is a significant and positive relationship between firm size and queue length. These results show that large firms receive more applicants per position and per vacancy controlling for workers' and jobs' characteristics.

One possibility is that workers apply to large firms more since large firms hire more workers per vacancy. The data tells that it is not likely the case since the number of offers to applications ratio decreases in the firm size, the third column in Table 2. Thus, workers applying to large firms experience a lower probability of getting an offer than those who apply to small firms given wages because of more competition.

Insert Figure 1 here.

To provide additional supports, in Figure 1, I draw the subjective easiness that employers feel when they find workers. To be specific, the survey asks the following question: "Generally speaking, how difficult or easy would you say it is to find reliable unskilled workers at "reasonable" wages in your location?" The answer takes integer values of 1(very difficult) - 4(easy). Considering that unskilled workers are relatively homogeneous in productivity and wage, the answer can roughly indicate the easiness of hiring controlling for the worker characteristics. Figure 1 shows that large firms are more easily finding workers.

Why do large firms recruit job applicants more easily? One potential reason is non-financial compensations. This non-financial compensation is not necessarily what firms directly provide, such as insurance benefits. Instead, workers may find a more lucrative job transition opportunity in the future when employed by a well-known company. Alternatively, workers may prefer to be a member of big corporations rather than a smaller one. Another possibility is that workers disproportionately recognize vacancy information from large firms more. For instance, a vacant job in Google will be more well-advertised than that of a small software company, and this will be true even if the two companies guarantee the same compensation and put the same recruiting efforts.

In reality, both reasons would coexist so that large firms receive more applicants than small firms given compensations provided and recruiting efforts. This seemingly small difference can lead to a large wage gap if, for instance, the difference makes the small software company give up competing with Google in the same labor market and recruit workers from different, and possibly low-wage markets. Indeed, large and small firms seem to search for workers in different markets in terms of wages and recruiting methods.[6] In the model part, I formally examine how such *ex ante* heterogeneity in recruiting efficiency affects the aggregate labor market.

3 Model

3.1 Environment

3.1.1 Agents

I consider a single period matching model with workers and vacancies. There is a unit measure of homogeneous workers who are currently unemployed and searching for a job. I normalize the value of home production to 0. Unlike workers, there are two types of vacancies $j \in \{1(\text{superstar}), 2(\text{small})\}$. I assume that each type's measure is exogenous and denoted the measure of vacancies by m_j. A match between a worker and a vacancy produces output

[6] For instance, small firms more heavily use informal methods, such as family referrals, while large firms rely more on formal methods (Rebien et al., 2020).

y_j.

Assumption 3.1. $y_1 > y_2 > y_1 \exp\left(-\frac{1}{m_1}\right)$

I assume that superstar vacancies are more productive following most previous literature. However, the qualitative implications of the model still hold even if small vacancies are more productive. The second condition guarantees that small vacancies are searching for workers in equilibrium.

One may concern that there is no notion of firm size nor its dynamics in the model. By assuming m_j is exogenous, this paper ignores the entry and exit as well as expansion and shrink of firms. I deliberately omit these features because incorporating these features requires modeling the cause of the rise of superstar firms. Instead of exploring the cause, this paper solely focuses on the labor market responses when there are already two types of vacancies in the market, which is another crucial question. To answer the question, assuming exogenous m_j is not a problem as long as there is no increasing or decreasing return to scale in the matching process, which is a typical assumption in papers dealing with firm size distribution (Elsby and Michaels, 2013; Kaas and Kircher, 2015; Schaal, 2017).

3.1.2 Search and matching

Search is directed by wages. There is a continuum of submarkets indexed by wages w, and vacancies and workers choose w in which they search for the counterpart. Denote the measure of workers in a submarket by u, and the measure of each vacancy type by v_j. Note that the labor market is not separated by vacancy type. It captures the idea that firms are competing for similar workers regardless of their size as long as they guarantee similar compensations. This competition and endogenous responses of vacancies are crucial in the model.

Departing from the standard search models, vacancies differ in their renown. Specifically, each superstar vacancy in a submarket w is observed by $z \leq 1$ fraction of workers in the submarket. On the other hand, each small vacancy in the same submarket is observed only by z/η of workers in the submarket where $\eta > 1$. Thus, η captures the degree of the difference in renown. The larger the η is, the more heterogeneous in how famous they are. Note that there are other interpretations of η. For instance, workers may recognize all vacancies equally, but disproportionately apply to superstar vacancies for various reasons. η can also capture such tendency as qualitative properties of the model do not depend on how and why η emerges.

The difference in renown η affects the matching probability through the effective queue length for each vacancy type. For superstar vacancies, the effective measure of workers in a submarket is zu because it is the measure of workers who recognize their vacancies. Each of

zu workers observe $zv_1 + (z/\eta)v_2$ measure of wage postings, thus the effective queue length for superstar vacancies is:

$$\theta_1 \equiv \frac{zu}{zv_1 + (z/\eta)v_2} = \frac{\eta u}{\eta v_1 + v_2} \tag{2}$$

On the other hand, the effective measure of workers for small vacancies is $(z/\eta)u$ while each of them observe the same $zv_1 + (z/\eta)v_2$ measure of wage postings. Thus, the effective queue length for small vacancies is:

$$\theta_2 \equiv \frac{(z/\eta)u}{zv_1 + (z/\eta)v_2} = \frac{u}{\eta v_1 + v_2} = \theta_1/\eta \tag{3}$$

Given the queue length, the vacancy-filling probability is governed by a Urn-ball matching function $q(\theta) = 1 - \exp(-\theta)$. Superstar vacancies have a higher vacancy-filling probability $q(\theta_1)$ than that for small vacancies $q(\theta_2)$. From the vacancy-filling probability, the aggregate matching function $\mathcal{M}(v_1, v_2, u)$ can be recovered.

$$\mathcal{M}(v_1, v_2, u) = v_1 q(\theta_1) + v_2 q(\theta_2) \tag{4}$$

The job-finding probability is given by $\mathcal{M}/u$ from the constant return to scale property. Note that the job-finding probability for a worker not only depends on θ_1 and θ_2, but also depends on the vacancy type distribution in a submarket. Denote the fraction of superstar vacancies by μ. Then, the job-finding probability of a worker $p(\theta_1, \theta_2, \mu) \equiv p(\theta, \mu)$ is expressed by the following:

$$p(\theta, \mu) = \tilde{\mu}\frac{1 - \exp(-\theta_1)}{\theta_1} + (1 - \tilde{\mu})\frac{1 - \exp(-\theta_2)}{\theta_2} \tag{5}$$

$$\tilde{\mu} = \frac{\mu}{\mu + (1 - \mu)\frac{1}{\eta}} \tag{6}$$

where $\tilde{\mu}$ is the fraction of superstar vacancies adjusted by firms' different renown. Intuitively, with probability $\tilde{\mu}$, a worker meets a superstar type. In that case, the job-finding probability is $q(\theta_1)/\theta_1$. Similarly, with probability $1-\tilde{\mu}$, a worker meets a small vacancy, and the resulting job-finding probability becomes $q(\theta_2)/\theta_2$. I provide a micro-foundation of this matching function in Section 3.1.3.

One feature of the model is worth emphasizing. If there is only one type of vacancy in a submarket, η does not play a role. It is apparent from the definition of θ_j when either one of v_j is 0. This is because being observed by more workers has two opposite effects. On the one hand, being observed by more workers increase the matching probability. On the other hand, it intensifies coordination failure because more workers observe the same vacancy. These two effects exactly cancel out each other if all vacancies are alike. However, superstar vacancies can benefit if others in the same submarket are less renowned than them.

It highlights how recruiting efforts and firms' renown are distinct. In previous literature on recruiting or search efforts, the parameter usually appears as total factor productivity of the matching function (Davis et al., 2013; Gavazza et al., 2018).[7] Therefore, if all firms put more recruiting efforts, it increases the total number of matches. On the other hand, all firms being more famous does not affect the aggregate number of matches. Still, being more famous alone benefits the firm that becomes more famous.

3.1.3 Micro-foundation of the matching function

In this subsection, I provide a micro-foundation of the matching function used in the previous section. Suppose that there are N workers and M vacancies. Among M vacancies, μM vacancies are superstar vacancies, and $(1-\mu)M$ vacancies are small. As I interest in the limit case $N, M \to \infty, N/M \to t$, I regard $M_1 \equiv \mu M, M_2 \equiv (1-\mu)M, zM$, and $(z/\eta)M$ as integers.

Consider the following matching environment: First, each vacancy sends its advertisement to multiple workers. In doing so, superstar vacancies send advertisements to zM-number of workers while small vacancies send advertisements to $(z/\eta)M$-number of workers. Then, workers choose one vacancy randomly from all the advertisements received, independent of vacancy type. Lastly, vacancies select one worker randomly among the pool of applicants, if any exist.

Assume that each vacancy chooses workers to advertise with an equal probability, and sending advertisements is independent across all vacancies. In such a case, the limit vacancy-filling probability for each type q_j and job-finding probability p are the followings:

$$q_1 = 1 - \exp(-\theta_1), \quad q_2 = 1 - \exp(-\theta_2), \quad p = \tilde{\mu}\frac{1-\exp(-\theta_1)}{\theta_1} + (1-\tilde{\mu})\frac{1-\exp(-\theta_2)}{\theta_2} \tag{7}$$

$$\text{where} \quad \theta_1 = \lim_{N\to\infty} \frac{\eta N}{\eta M_1 + M_2}, \quad \theta_2 = \lim_{N\to\infty} \frac{N}{\eta M_1 + M_2} \tag{8}$$

The proof is in the appendix. One essential assumption for this derivation is that workers randomly choose one advertisement from all advertisements received, independent of vacancy type. There are several ways of understanding this assumption. For instance, workers may

[7] Shimer (2004) shows that how to model search efforts is crucial for their aggregate implications and demonstrates a model that has a similar flavor to this paper. However, his paper considers homogeneous workers and firms, while this paper is mainly interested in heterogeneous cases.

not be able to distinguish vacancy type from reading an advertisement, or some platform may automatically select one and only one advertisement for workers. This platform can be a real platform or a modeling tool for introducing the concept that some firms are more famous. Alternatively, it is the assumption that captures workers' preference over different firm type.

The micro-foundation in this section is similar to the price advertisement model of Butters (1977) and Burdett et al. (2001), except that all vacancies post the same wage. One can interpret the matching process of this section as what happens in each submarket. The analysis can be extended by assuming that each vacancy writes its wage on the advertisement, and workers first choose a wage and then choose a vacancy given the wage. In such a case, workers and firms must have a rational expectation on both θ and μ. I will describe how this rational expectation is determined both on- and off-the-path equilibrium in Section 3.4.1.

3.2 Constrained efficient allocation

I will first describe the social planner's allocation of this environment before introducing the individual's problem. The social planner can choose the measure of workers and vacancies of each type in submarkets subject to the same matching frictions. For convenience, assume that the social planner can create submarkets onto an arbitrary set S. Then, the social planner's problem can be expressed by the following.

$$W = \max_{\theta_j(s), v_j(s)} \int_S q(\theta_j) v_j y_j ds, \qquad \text{s.t} \qquad \int_S v_j ds = m_j, \quad j = 1, 2 \tag{9}$$

$$\int_S \theta_2(\eta v_1 + v_2) ds = 1 \tag{10}$$

$$\theta_1/\eta = \theta_2 \tag{11}$$

The objective function of the social planner's problem is to maximize the expected output. The social planner faces the same matching frictions represented by $q(\cdot)$ and $\theta_1/\eta = \theta_2$. The social planner also takes the exogenous measure of workers and vacancies as given.

Intuitively, the social planner wants to allocate the appropriate number of workers to each vacancy type according to its productivity. This task cannot be achieved if both types coexist in the same submarket. This is because their queue lengths are tied through $\theta_1/\eta = \theta_2$, but there is no matching efficiency gain from mixing types. Therefore, the constrained efficient allocation should feature full-separation. Analytically, $v_j > 0$ should satisfy the following first order condition:

$$v_1 > 0 \Rightarrow q(\theta_1) y_1 - \lambda\theta_1 = \rho, \quad v_2 > 0 \Rightarrow q\left(\frac{\theta_1}{\eta}\right) y_2 - \lambda\frac{\theta_1}{\eta} = \rho \tag{12}$$

where ρ is the Lagrange multiplier for vacancies, and λ is the Lagrange multiplier for workers. Given q, y_1, λ, ρ, the queue length θ_1 cannot satisfy the both conditions simultaneously.

In the constrained efficient allocation, there are only two markets, one for each type. Denote the constrained efficient queue length by θ_j^E. Since the markets are fully separated, the queue length θ_j^E is independent of η. Because η does not appear anywhere else, the constrained efficient allocation is the same whether $\eta = 1$ or $\eta > 1$.

The first order condition requires that the marginal value of a worker to each type coincides with the social value of a worker.

$$\lambda^E = q'(\theta_1^E)y_1 = q'(\theta_2^E)y_2 \tag{13}$$

where λ^E is the Lagrange multiplier of worker in the social planner's problem. Equation (13) implies that the marginal vacancy-filling probability ratio is the inverse of the productivity ratio independent of η in the constrained efficient allocation.

3.3 Individual's problems

Each vacancy chooses a wage to maximize the expected profit Π_j, taking the equilibrium queue length $\theta_j(w) : \mathbb{R}_+ \to \mathbb{R}_+, j = 1, 2$ as given. Note that j-type vacancy only cares about θ_j, but not about θ_{-j} and $\mu(w) : \mathbb{R}_+ \to [0, 1]$.

$$\Pi_j = \max_w q(\theta_j)(y_j - w) \tag{14}$$

Each worker chooses a wage while taking the equilibrium relationship between θ and w, and μ and w as given.

$$U = \max_w p(\theta, \mu)w \tag{15}$$

U is the value of job search in this model.

3.4 Equilibrium

3.4.1 Definition

Definition 1. *An equilibrium consists of the value of job search $U \in \mathbb{R}_+$, the expected profits for a vacancy $\Pi_j \in \mathbb{R}_+$, the queue length function $\theta_j(w) : \mathbb{R}_+ \to \mathbb{R}_+$, the rational belief*

$\mu(w) : \mathbb{R}_+ \to [0, 1]$, the measure of vacancy $v_j(w) : \mathbb{R}_+ \to \mathbb{R}_+$, and the active markets $\mathcal{P} \subseteq \mathbb{R}_+$ such that

1. *(Definition of profits and value of search) Given θ and μ, Π_j and U satisfy Equation (14) and (15).*

2. *(Active markets) $w \in \mathcal{W}$ only if $v_1(w) > 0$ or $v_2(w) > 0$.*

3. *(Workers' optimality) For $w \in \mathcal{W}$, $p(\theta, w)w \geq U$.*

4. *(Vacancies' optimality) If $v_j(w) > 0$, then $q(\theta_j)(y_j - w) \geq \Pi_j$.*

5. *(Consistent belief) For $w \in \mathcal{W}$, $\mu(w) = \frac{v_1(w)}{v_1(w)+v_2(w)}$.*

6. *(Off-the-path belief) For all $w \in \mathbb{R}$, θ and μ satisfies the following:*

$$\mu(w) > 0 \Rightarrow q(\theta_1)(y_1 - w) \geq \Pi_1, \quad \mu(w) < 1 \Rightarrow q(\theta_2)(y_2 - w) \geq \Pi_2 \tag{16}$$

7. *(Consistent supply) For $u = \theta_2(\eta v_1 + v_2)$,*

$$\int_{\mathcal{W}} v_j(w)dw = m_j, j = 1, 2 \quad \int_{\mathcal{W}} u(w)dw = 1 \tag{17}$$

Active markets are submarkets where non-zero workers and vacancies exist in equilibrium. Workers' optimality requires that if there is a non-zero measure of job searchers in w, then w must maximize search value. Vacancies' optimality means that if there is a non-zero measure of vacancies in w, this submarket must be profit-maximizing. Consistent belief requires that the belief μ must coincide with the actual fraction of superstar vacancies in active markets. Consistent supply condition implies that all workers and vacancies are searching.

The off-the-path belief condition determines θ and μ for submarkets that are not in active markets. The condition requires that if the rational belief puts a strictly positive probability for superstar vacancies ($\mu > 0$), then it must be optimal for them to create a vacancy in that market ($q(\theta_1)(y_1 - w) \geq \Pi_1$). Similarly, the belief assigns a strictly positive probability for small vacancies ($\mu < 1$) only if it is optimal for them to enter the submarket ($q(\theta_2)(y_2 - w) \geq \Pi_2$). This condition pins down the belief and the queue length for all w. Note that the way this paper pins down the off-the-path belief is consistent with previous literature that studies a directed search model with adverse selection (Guerrieri and Shimer, 2014; Chang, 2017).

3.4.2 Equilibrium properties

In this subsection, I characterize the equilibrium by documenting a series of equilibrium properties. Then, the existence and uniqueness result naturally follows.

Lemma 1. *In an equilibrium, there exists* $\bar{w} \in \mathbb{R}$ *such that* $\mu(w) = 0$ *for all* $w < \bar{w}$ *and* $\mu(w) = 1$ *for all* $w > \bar{w}$.

Lemma 1 is the result of single-crossing property:[8] For $w_h > w_l$ and $\theta_h < \theta_l$, $(y_2 > w_h)$

$$q(\theta_h)(y_1 - w_h) = q(\theta_l)(y_1 - w_l) \Rightarrow q\left(\frac{\theta_h}{\eta}\right)(y_2 - w_h) < q\left(\frac{\theta_l}{\eta}\right)(y_2 - w_l) \tag{18}$$

In words, if superstar vacancies are indifferent between posting w_h and w_l, then small vacancies strictly prefer to post the lower wage w_l than the higher wage w_h. Small vacancies are willing to accept a bigger sacrifice in the queue length than superstar vacancies for a unit decrease in wages because their queue length is multiplied by a factor of $1/\eta$. As a result, if both types are optimal to search in $\bar{w}$, small vacancies are never optimal to enter $w > \bar{w}$ markets when superstar vacancies are optimal to do so. Similarly, superstar vacancies are never optimal to enter $w < \bar{w}$ when small vacancies are indifferent between $\bar{w}$ and w.

While Lemma 1 disciplines the equilibrium belief μ as a step-function, there is still a possibility of pooling at $\bar{w}$. However, active markets in equilibrium do not feature pooling because it is not optimal for workers to search for $\bar{w}$.

Proposition 1. *An equilibrium is full-separation, i.e.,* $\mu(w) \in \{0, 1\}$ *for all* $w \in \mathcal{W}$.

Proposition 1 is the result from the fact that the job-finding probability depends on μ. Since the vacancy-filling probability is independent of μ, θ_1 (so does θ_2) is a continuous function of w in equilibrium. On the other hand, $p(\theta, \mu)$ is decreasing in μ, thus $p(\theta, \mu)$ has a decreasing jump at $\bar{w}$ if $\mu(\bar{w}) \in (0, 1)$. It means that workers prefer searching for $\bar{w} - \epsilon$ to $\bar{w}$, thus $\bar{w}$ cannot be an active market in equilibrium.

Proposition 2. *An equilibrium* (U, Π_1, Π_2) *solves the following maximization problem:*

$$\Pi_1 = \max_{\theta, w} q(\theta)(y_1 - w) \qquad \textit{subject to} \qquad \frac{q(\theta)}{\theta} w = U \tag{19}$$

$$\Pi_2 = \max_{\theta, w} q(\theta)(y_2 - w) \qquad \textit{subject to} \qquad \frac{q(\theta)}{\theta} w = U, \quad q(\eta\theta)(y_1 - w) \leq \Pi_1 \tag{20}$$

[8] Since the vacancy-filling rate is different between two types because of η, the single-crossing property is not immediate from $y_1 > y_2$. The proof is in the appendix.

Proposition 2 fully characterizes the equilibrium given the value of job search U. Equation (19) determines the expected profit of superstar vacancies Π_1 along with equilibrium wage and queue length. Then, the profit of small vacancies Π_2 and associated wage and queue length are derived recursively from Equation (20). A general equilibrium of the model is then the value of job search U that clears the labor market, i.e., $\theta_1 m_1 + \theta_2 m_2 = 1$. Since θ_1 and θ_2 are continuously decreasing in U, an equilibrium exists and unique.

Proposition 3. *An equilibrium exists. The equilibrium is unique.*

Insert Figure 2 here.

Most results of this paper can be derived from Proposition 2. To better understand the intuition, consider a model without heterogeneity in firms' renown, $\eta = 1$. Figure 2 represents the equilibrium in this case. The black line is the indifference curve for workers on (w, θ)-plane, and the red (blue) line is the indifference curve for superstar (small) vacancies. In the equilibrium, workers' and vacancies' indifference curves are tangent to each other, representing the efficiency property of directed search equilibrium. Thus, the equilibrium represented in Figure 2 coincides with the social planner's allocation.

Insert Figure 3 here.

Insert Figure 4 here.

Then, consider that there is heterogeneity in renown, but the degree of the heterogeneity is small. Figure 3 illustrates this case. Note that the superstar vacancies face a more favorable queue length $\eta\theta$ if only small vacancies exist in the submarket. Therefore, any market with only small vacancies should not lie above the IC curve $q(\eta\theta)(y_1 - w) = \Pi_1$ (red-dashed); otherwise, superstar vacancies enter the market. While the IC curve is always below the equilibrium indifference curve (red) $q(\theta)(y_1 - w) = \Pi_1$, if η is not large enough, the IC curve is above the wage that small vacancies are willing to choose. In such a case, the equilibrium is the same as $\eta = 1$ case, and the equilibrium coincides with the constrained efficient allocation.

It is no longer the case if η is large. Figure 4 represents the case. As η gets bigger, the IC shifts downward more. When this curve passes below the wage originally chosen by small vacancies, small vacancies cannot choose that wage as it violates the incentive constraint of superstar vacancies. At the lower wage chosen, the small vacancies' indifference curve (blue) and workers' indifference curve are not tangent; thus, the allocation is inefficient. Also, as small vacancies are pushed downward, the wage distribution becomes more unequal.

Note that Figure 4 only illustrates partial equilibrium effects given U. In the general equilibrium, since small vacancies are inefficiently posting, the value of job search U also falls. It makes the workers' indifference curve (black) shift toward the left. It makes both types' wages to fall compared to the partial equilibrium when U is fixed. Therefore, both types' wages are lower than the equilibrium without heterogeneity in Figure 2.

Figure 3 and 4 show when the incentive constraint is more likely to bind. One can easily see that the constraint never binds if $\eta = 1$ because the IC curve and indifference curve coincide. The constraint is more likely to bind if η is large, or y_1 and y_2 are similar so that the constrained efficient wages are close. Thus, it concludes that the equilibrium features inefficiency if the difference in firms' renown is large enough compared to the productivity difference.

In the equilibrium, superstar vacancies' choices are not distorted given U. It is different from the model with adverse selection (Guerrieri and Shimer, 2014; Chang, 2017) where the lowest type's choice is not distorted. The discrepancy arises because, in this paper, superstar vacancies are the agent who has an incentive to enter the market for the other type. Therefore, small vacancies face an additional incentive constraint of superstar vacancies in Equation (19).

Proposition 4. *The equilibrium does not coincides with the social planner's allocation if and only if* $\eta > \bar{\eta}(y_1, y_2)$. $\bar{\eta}$ *is increasing in* y_1 *and decreasing in* y_2 *as long as* $y_1 > y_2$. *Also,* $\bar{\eta}(y_1, y_2) \to 1$ *when* $|y_1 - y_2| \to 0$.

Proposition 5. *In the equilibrium, the followings hold:*

1. *If* $\eta \leq \bar{\eta}$, *then* $\theta_j = \theta_j^E, w_j = w_j^E$ *for* $j = 1, 2$ *and* $U = \lambda^E$.
2. *If* $\eta > \bar{\eta}$, *then*
   - *(Inefficiency)* $U < \lambda^E$ *and* $U = q'(\theta_1)y_1 < q'(\theta_2)y_2$.
   - *(Wage distribution)* $w_1 < w_1^E, w_2 < w_2^E$.

Proposition 4 and 5 rigorously formulate the intuition explained. These summarize the main results of this paper. When η is small given productivity difference ($\eta \leq \bar{\eta}$), then the equilibrium is constrained efficient. However, if η is large, then the equilibrium features additional inefficiency in addition to search frictions. The value of job search U is lower than the social value of a worker λ^E because small vacancies post inefficient wage $w_2 < w_2^E$. At this wage level, the marginal value of a job searcher is higher for small vacancies $q'(\theta_1)y_1 < q'(\theta_2)y_2$. While small vacancies want to post higher wages to attract more workers, it is impossible because if small vacancies attract such many workers in $w > w_2$, then it is profitable for superstar vacancies to enter the market and use their hiring advantage. Since the value of job search U is lower than λ^E, superstar vacancies do not need to compensate workers as much as they do in the constrained efficient case, $w_1 < w_1^E$.

The wage gap likely increases because w_2 falls through the direct incentive effect and indirect equilibrium effect through U, while w_1 falls only through the indirect equilibrium effect. In the next section, I numerically confirm it and show that the equilibrium exhibits a larger wage dispersion. It means that the rise of superstar firms can intensify wage polarization through the competition for workers in the labor market. The wage polarization itself does not necessarily imply economic inefficiency, although it harms distribution if the polarization is solely driven by productivity. However, this paper points out that wage polarization can imply inefficient allocation if it arises from increased heterogeneity in firms' renown. Proposition 5 also shows that the rise of superstar firms can cause the fall in labor share in both superstar and small firms. It complements previous literature that finds a negative correlation between the market concentration and labor share across industries (De Loecker et al., 2020).

4 Numerical analysis

In the model, there are four parameters to choose: m_j and y_j for each $j = 1, 2$. For the rest of this section, I set $m_1 + m_2 = 1$ to match the average queue length to 1 roughly. It implies the job-finding probability 63% if all workers and vacancies are in the same submarket, close to the monthly U-E rate. I normalize the productivity of small vacancies y_2 to 1. I vary the other parameters m_1 and y_1 according to the context of exercises.

4.1 How big η does induce inefficiency?

This paper's main result relies on the fact that when η is large enough, then the equilibrium features inefficiency. Then, the natural question is how big is the 'large enough.' To answer this question, I draw the threshold $\bar{\eta}$ as a function of the productivity gap $(y_1 - y_2)/y_2$.

Insert Figure 5 here.

Figure 5 illustrate the result for two different m_1. The immediate takeaway from Figure 5 is that $\bar{\eta}$ is not too large. When superstar vacancies are 30% more productive than small vacancies, relatively small $\eta \approx 1.065$ is enough to create inefficiency. Even if superstar vacancies are twice more productive than small vacancies, the estimated TFP ratio between the 90th and 10th percentile in 4-digit industry (Syverson, 2004), $\eta \approx 1.367$ generates an inefficient equilibrium given $m_1 = 0.4$. It decreases to $\eta \approx 1.268$ when m_1 is set to 0.1. Although η is not directly measurable, it would not be an extreme assumption that the top 10% firms' hiring information reaches 27% more workers than the bottom 10% firms' hiring information. Of course, more detailed data is necessary to examine whether the current economy is in the efficient or inefficient region. I put this concern aside for future research.

The relationship between $\bar{\eta}$ and $(y_1 - y_2)/y_2$ exhibits a mild convexity, and shifts upward when m_1 becomes smaller. Thus, when there is few superstar vacancies, the equilibrium is less likely to suffer inefficiency given η and productivity gap.

4.2 How big are the effects of η on wages?

The next question is that, then how big are the effects of η on wages. While the previous section demonstrates that the threshold level $\bar{\eta}$ is moderate, it may not be a concern if the effects of η on real allocations are minor. To examine its magnitude, I draw each type's wage as a function of η. In doing so, I first assume a moderate difference in productivity, and relatively large fraction of superstar vacancies: $y_1 = 1.2, m_1 = 0.4$.

Insert Figure 6 here.

Figure 6 illustrates the result. Both w_1 and w_2 fall once η exceeds a threshold level, and the slope is steeper for w_2. Thus, it intensifies the wage inequality. In this example, in the region where $\eta > \bar{\eta}$, the elasticity of wage to η is about 0.7 for superstar vacancies, and about 1.5 for small vacancies. Small vacancies are much more elastic because there are both direct and indirect effects.

Insert Figure 7 here.

Figure 7 shows the effects on wage inequality and labor share. It clearly shows that the increase in η induces a higher inequality and lower labor share. The magnitude of the effects is significant and large: 1% increase in η induces about 0.9% increase in the wage ratio w_1/w_2, and 1.16% decrease in the labor share. The effect on labor share is especially huge: increase in η only from 1.04 to 1.14 generates the decline of labor share that is comparable to the actual decline over past several decades (Karabarbounis and Neiman, 2014).

The large effects of η might come from the assumption that the productivity difference is small. To explore this possibility, I then examine a case where superstar and small vacancies are substantially different: $y_1 = 2, m_1 = 0.1$.

Insert Figure 8 here.

Insert Figure 9 here.

Figure 8 and 9 represent the result. The overall shape of the graph is identical to previous case, Figure 6 and 7. The elasticity of wage to η is slightly higher in this case: the elasticity of w_1 is about 1.1 and the elasticity of w_2 is about 1.7. The elasticity of wage ratio is slightly smaller in this case: 1% increase in η induces 0.78% increase in w_1/w_2. The elasticity of labor share exhibits much bigger magnitude, 1.65 compared to 1.16 in previous case. It confirms that even if the productivity gap is large so that the threshold $\bar{\eta}$ is large, once η exceeds this threshold, its effects on wages are substantial.

4.3 Welfare

In Figure 10, I draw welfare as a function of η. In this economy, welfare is the sum of the job search value and firms' profits, which coincides with the total production. I set $y_1 = 1.4$ and calculate the relationship between welfare and η under two cases: i) there are few superstar vacancies ($m_1 = 0.2$), ii) majority of vacancies are superstar vacancies ($m_1 = 0.8$).

Insert Figure 10 here.

The increase in η reduces the welfare, as analytically shown.[9] Compared to its large effects on wages, the change in η has a relatively mild effect on aggregate welfare. The increase in

[9]Note that the increase in η can arise either because superstar vacancies become more famous or because small vacancies become less known. The analysis is regardless of the cause.

η from 1 to 1.5 reduces welfare by 1.6% when $m_1 = 0.2$, and by 0.4% when $m_1 = 0.8$. The welfare effect is larger when there are fewer superstar vacancies and more small vacancies because it is the small vacancy that chooses the suboptimal choice. While it is difficult to interpret this number directly as there is no empirical measure of η, the implied elasticity of $0.032 = (1.6/50)$ is much smaller than the responsiveness of wage.

Insert Figure 11 here.

Insert Figure 12 here.

The aggregate effects are moderate because the opposite changes in firms' profits and job search value cancel out each other's effect. Figure 11 and 12 decompose welfare into U, Π_1, Π_2. While the change in η induces only 1.4% (0.6%) fall in aggregate welfare when $m_1 = 0.2$ ($m_1 = 0.8$), the value of job search U decreases by 40% (5.6%) due to the same change in η. Most of this effect is masked by the increase in firms' profits, especially Π_1.

Interestingly, both superstar firms' profits and small firms' profits increase in Figure 11. While the increase in η reduces U and raises Π_1, the directional effect on Π_2 depends on other parameter values. This is because the direct effect and general equilibrium effect work in the opposite direction. The general equilibrium effect likely dominates when there are fewer superstar vacancies in the economy since the direct incentive effect does not depend on the measure of superstar vacancies, while the general equilibrium effect increases in the measure of small vacancies. Thus, in Figure 12 where there are fewer small vacancies, the increase in η does not induce the increase in small vacancies' profits.

5 Conclusion

This paper studies how the difference in recruiting efficiency arising from firm size can affect the aggregate labor market. I hypothesize that large firms are more efficient in hiring because they are large. To support this idea, I provide empirical evidence using EOPP data, which shows a positive relationship between firm size and easiness of hiring. Then, I build a directed search model with superstar and small firms to understand how difference in firms' renown can affect wages and inequality. Using the model, I show that if the difference in firms' renown is large compared to the productivity gap, then the equilibrium features inefficiency.

When the equilibrium is inefficient, small firms pay inefficiently lower wages so that wage distribution becomes more polarized. Superstar firms can pay less because the job searchers' value falls; therefore, the aggregate labor share falls.

This paper suggests a new labor market channel through which the rise of superstar firms induces wage polarization and a fall in labor share. While these links have already been suggested in previous literature, they have a different meaning and policy implications. For instance, the model proposes that if a policymaker can help small businesses in the hiring process, it can improve both efficiency and inequality.

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A Appendix

A.1 Derivation of the matching function

In this section, I will derive the Urn-Ball matching function under the environment in Section 3.1.3. Recall that there are N workers and M vacancies. μ-fraction of vacancies are superstar vacancies and $1-\mu$ fraction of vacancies are small vacancies. I will consider the case where $\lim N, M \to \infty$ while $\lim M/N \to t$.

Consider a superstar vacancy k that sends advertisements to zN-number of workers. For each worker $i = \{1, \cdots, zN\}$, let X_i^N be the number of advertisements that i received. Since the worker i received an advertisement from k, X_i^N can be expressed by the following.

$$X_i^N = B_1 + B_2 + 1, \quad \text{where} \quad B_1 \sim Bin(M_1 - 1, z), B_2 \sim Bin(M_2, z/\eta) \tag{21}$$

B_j is the number of advertisements from j-type vacancy. Given X_i^N, the worker i applies to the vacancy k with probability $\frac{1}{X_i^N}$. Thus, the vacancy k receives no applicant with probability $f^N(X_1^N, \cdots, X_{zN}^N)$:

$$f^N(X_1^N, \cdots, X_{zN}^N) = \prod_{i=1}^{zN} \left(1 - \frac{1}{X_i^N}\right) \tag{22}$$

The probability of interest is the limit of the expectation, i.e., $\lim_{N\to\infty} E\left[f^N(X_1^N, \cdots, X_{zN}^N)\right]$. Note that taking expectation directly to f^N is not straightforward because X_i^N is not independent to each other. Thus, I will derive the probability limit of f^N.

Proposition 6.

$$\log f^N(X_1^N, \cdots, X_{zN}^N) \xrightarrow{p} -\frac{1}{\phi} \tag{23}$$

where ϕ is the probability limit of $\frac{X_i^N}{zN}$.

$$\frac{X_i^N}{zN} \xrightarrow{p} \mu t + \frac{(1-\mu)t}{\eta} \equiv \phi \tag{24}$$

Proof. Define a sequence of function $g^N(x)$:

$$g^N(x) = zN \log\left(1 - \frac{1}{zNx}\right) \tag{25}$$

Note that $g^N(x) \to -1/x$ as $N \to \infty$. From the definition of f^N and g^N,

$$\log f^N(X_1^N, \cdots, X_{zN}^N) = \frac{1}{zN}\sum_{i=1}^{zN} g^N\left(\frac{X_i^N}{zN}\right) \tag{26}$$

Thus, it is enough to show $\frac{1}{zN}\sum_{i=1}^{zN} g^N\left(\frac{X_i^N}{zN}\right) \xrightarrow{p} -\frac{1}{\phi}$.

$$\begin{aligned} & Pr\left(\left|\frac{1}{zN}\sum_{i=1}^{zN} g^N\left(\frac{X_i^N}{zN}\right) - \left(-\frac{1}{\phi}\right)\right| > \delta\right) && (27)\\ \leq\ & Pr\left(\frac{1}{zN}\sum_{i=1}^{zN}\left\{\left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| + \left|g^N(\phi) - \left(-\frac{1}{\phi}\right)\right|\right\} > \delta\right) && (28)\end{aligned}$$

For sufficiently large N, $\left|g^N(\phi) - \left(-\frac{1}{\phi}\right)\right| < \frac{\delta}{2}$. Thus,

$$\begin{aligned} & Pr\left(\left|\frac{1}{zN}\sum_{i=1}^{zN} g^N\left(\frac{X_i^N}{zN}\right) - \left(-\frac{1}{\phi}\right)\right| > \delta\right) && (29)\\ \leq\ & Pr\left(\frac{1}{zN}\sum_{i=1}^{zN}\left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (30)\\ \leq\ & Pr\left(\bigcup_i \left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (31)\\ \leq\ & \sum_{i=1}^{zN} Pr\left(\left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (32)\\ =\ & zNPr\left(\left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (33)\end{aligned}$$

Note that $g^{N\prime}(x) = \frac{zNx}{(zNx-1)x^2}$. Therefore, if $x \geq C$ for some $C > 0$, $|g^{N\prime}(x)| \leq \frac{zNC}{(zNC-1)C^2} \leq C_2$ for sufficiently large N. Therefore, there exists $k > 0$ such that $|x - \phi| \leq k$ implies $|g^N(x) - g^N(\phi)| \leq \delta/2$ for large N. For such k and large enough N,

$$\begin{aligned} & zNPr\left(\left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (34)\\ =\ & zNPr\left(\left|\frac{X_i^N}{zN} - \phi\right| > k, \left|g^N\left(\frac{X_i^N}{zN}\right) - g^N(\phi)\right| > \frac{\delta}{2}\right) && (35)\\ \leq\ & zNPr\left(\left|\frac{X_i^N}{zN} - \phi\right| > k\right) && (36)\end{aligned}$$

From Hoeffding's inequality,

$$Pr\left(\left|\frac{X_i^N}{zN}-\phi\right|>k\right)\leq \exp\left(-2M(z/t)^2k^2\right) \tag{37}$$

therefore,

$$\begin{aligned} & Pr\left(\left|\frac{1}{zN}\sum_{i=1}^{zN} g^N\left(\frac{X_i^N}{zN}\right)-\left(-\frac{1}{\phi}\right)\right|>\delta\right) && (38)\\ \leq\ & zN\exp\left(-2M(z/t)^2k^2\right)<\epsilon && (39)\end{aligned}$$

for sufficiently large N. □

From the proposition, $\log f^N \xrightarrow{p} -1/\phi$, thus $f^N \xrightarrow{p} \exp(-1/\phi)$. Note that if $X_n \xrightarrow{p} \mu$ and $|X_n| \leq Y$ for some absolutely integrable Y, $E(X_n) \to \mu$. As $f^N \leq 1$ for all N, $E(f^N) \to \exp(-1/\phi)$. Note that t is the limit of N/M, which is the worker-vacancy ratio. Therefore,

$$\begin{aligned} 1/\phi &= \frac{1}{\mu t+(1-\mu)\frac{t}{\eta}} && (40)\\ &= \frac{\eta u}{\eta\mu v+(1-\mu)v}=\frac{\eta u}{\eta v_1+v_2}=\theta_1 && (41)\end{aligned}$$

It proves that the vacancy-filling probability for superstar vacancies is $q_1 = 1-\exp(-\theta_1)$. From the same derivation, the $q_2 = 1-\exp(-\theta_2)$.

To calculate the job-finding probability, denote the total number of matches as $\mathcal{M}(v_1, v_2, u)$. The above derivation implies the following:

$$\mathcal{M}(v_1,v_2,u)=v_1(1-\exp(-\theta_1))+v_2(1-\exp(-\theta_2)) \tag{42}$$

$\mathcal{M}$ is the aggregate matching function which exhibits the constant return to scale. The job-finding probability is $\mathcal{M}/u$, where

$$\begin{aligned} \frac{\mathcal{M}}{u} &= \frac{v_1(1-\exp(-\theta_1))}{u}+\frac{v_2(1-\exp(-\theta_2))}{u} && (43)\\ &= \frac{v_1\theta_1}{u}\frac{1-\exp(-\theta_1)}{\theta_1}+\frac{v_2\theta_2}{u}\frac{1-\exp(-\theta_2)}{\theta_2} && (44)\\ &= \tilde{\mu}\frac{1-\exp(-\theta_1)}{\theta_1}+(1-\tilde{\mu})\frac{1-\exp(-\theta_2)}{\theta_2} && (45)\end{aligned}$$

where $\tilde{\mu}=\frac{\mu}{\mu+(1-\mu)\frac{1}{\eta}}$.

A.2 Proof of single-crossing property 1

Let the equilibrium profits Π_1 and Π_2 be given. Consider the indifference curve for each type of vacancy on (θ, w)-plane, where θ is the queue length for superstar vacancies:

$$C_1 : (1 - \exp(-\theta))(y_1 - w) = \Pi_1 \tag{46}$$
$$C_2 : (1 - \exp(-\theta/\eta))(y_2 - w) = \Pi_2 \tag{47}$$

The single-crossing condition is satisfied if $dw/d\theta$ of C_1 is greater than $dw/d\theta$ of C_2 for all θ. Note that

$$\left.\frac{dw}{d\theta}\right|_{C_1} = \Pi_1 \frac{1 - \exp(-\theta)}{(1 - \exp(-\theta))^2} \tag{48}$$
$$\left.\frac{dw}{d\theta}\right|_{C_2} = \Pi_2 \frac{1}{\eta} \frac{1 - \exp(-\theta/\eta)}{(1 - \exp(-\theta/\eta))^2} \tag{49}$$
$$\tag{50}$$

Since Π_1 and Π_2 are equilibrium profits, the followings hold.

$$\Pi_1 > y_1 \left(1 - \frac{1 + m_1 + m_2}{m_1 + m_2} \exp\left(-\frac{1}{m_1 + m_2}\right)\right) \tag{51}$$
$$\Pi_2 < y_2 \left(1 - \frac{1 + m_1 + m_2}{m_1 + m_2} \exp\left(-\frac{1}{m_1 + m_2}\right)\right) \tag{52}$$

where the right-hand side of Equation (51) is the equilibrium profit for a superstar vacancy if all the other vacancies are also superstar. Similarly, the right-hand side of Equation (52) is the equilibrium profit for a small vacancy if all the other vacancies are also small. Therefore, the following sufficient condition holds:

$$\frac{y_1}{y_2} > \frac{\frac{1}{\eta}\exp(-\theta/\eta)}{(1 - \exp(-\theta/\eta))^2} \Big/ \frac{\exp(-\theta)}{(1 - \exp(-\theta))^2} \Rightarrow \left.\frac{dw}{d\theta}\right|_{C_1} > \left.\frac{dw}{d\theta}\right|_{C_2} \tag{53}$$

Note that $\frac{\frac{1}{\eta}\exp(-\theta/\eta)}{(1-\exp(-\theta/\eta))^2} \Big/ \frac{\exp(-\theta)}{(1-\exp(-\theta))^2}$ is monotonically decreasing in θ, and goes to 1 as $\theta \to 0$. Since $y_1 > y_2$, the single-crossing property holds in equilibrium.

	Unconditional		Cond. on (> 0)	
	Mean	S.d.	Mean	S.d.
The number of observation	3,411			
Firm size	30.496	3.470		
The number of				
vacancies	2.863	.319		
applications	3.797	.277	9.268	.531
interviews	5.780	.364		
offers	3.080	.339	3.083	.339
The number of (for each position)				
vacancies	1.276	.0313		
applications	10.171	.262		
interviews	6.024	.201		
offers	1.330	.030	1.331	.030
Hours spent for recruiting	11.607	.461		
Workers				
age	27.907	.194		
female	0.434			
years of education	12.732	.0373		
Jobs				
permanent jobs	0.834			
starting wage	5.144	0.061		

Table 1: Summary statistics

Table 2: Effect of firm size on the number of applications, queue length, and offer rate

	(1) # of applicants		(2) Queue length		(3) Offer rate	
log(Size)	1.0638***	(0.2778)	0.4682**	(0.2287)	-0.0214***	(0.0054)
log(Wage)	-0.4056	(0.4374)	-0.2839	(0.3646)	0.0168**	(0.0083)
log(Recruit hours)	1.0852***	(0.2267)	0.9209***	(0.1889)	0.0078*	(0.0044)
Less than HS	-0.9738	(1.5099)	-0.2642	(1.2535)	0.0250	(0.0287)
HS	1.8983	(1.6927)	1.6954	(1.4031)	0.0499	(0.0322)
Some college	0.1289	(2.1509)	0.2371	(1.7849)	-0.0070	(0.0411)
BA	-4.1721	(3.8209)	-4.1469	(3.1738)	0.1857**	(0.0729)
Age	-0.0323	(0.1381)	-0.0400	(0.1135)	-0.0001	(0.0026)
Age Sq.	0.0000	(0.0016)	0.0002	(0.0013)	0.0000	(0.0000)
Seasonal	4.4325	(2.7228)	-1.4611	(2.2657)	-0.1690***	(0.0533)
Permanent	3.3465**	(1.4656)	3.7362***	(1.2174)	-0.1898***	(0.0279)
Female	0.9032	(1.0693)	1.2080	(0.8869)	-0.1034***	(0.0203)
Constant	1.6444	(57.8552)	4.3202	(47.3335)	0.1044	(1.5159)
Observations	2058		2021		2021	
R^2	0.1190		0.1449		0.1459	
Adjusted R^2	0.072		0.098		0.100	

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

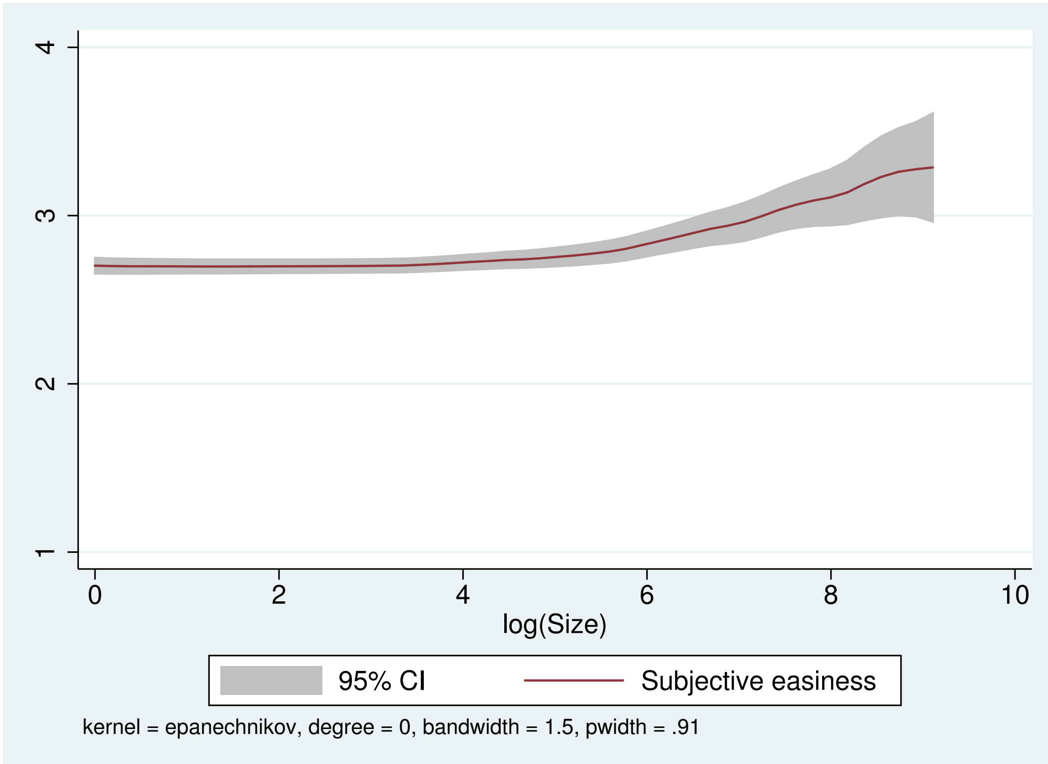

Figure 1: Subjective easiness

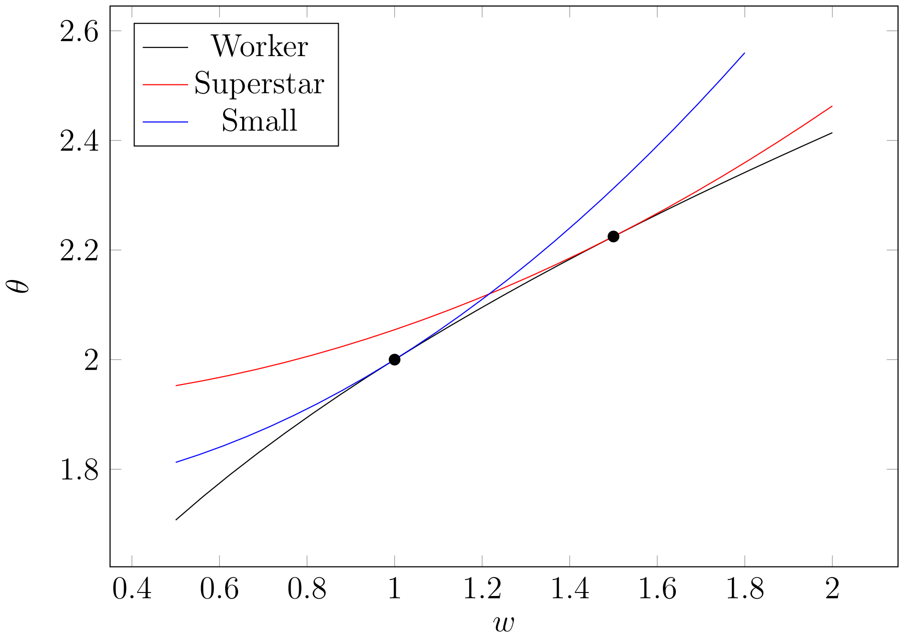

Figure 2: Directed search equilibrium

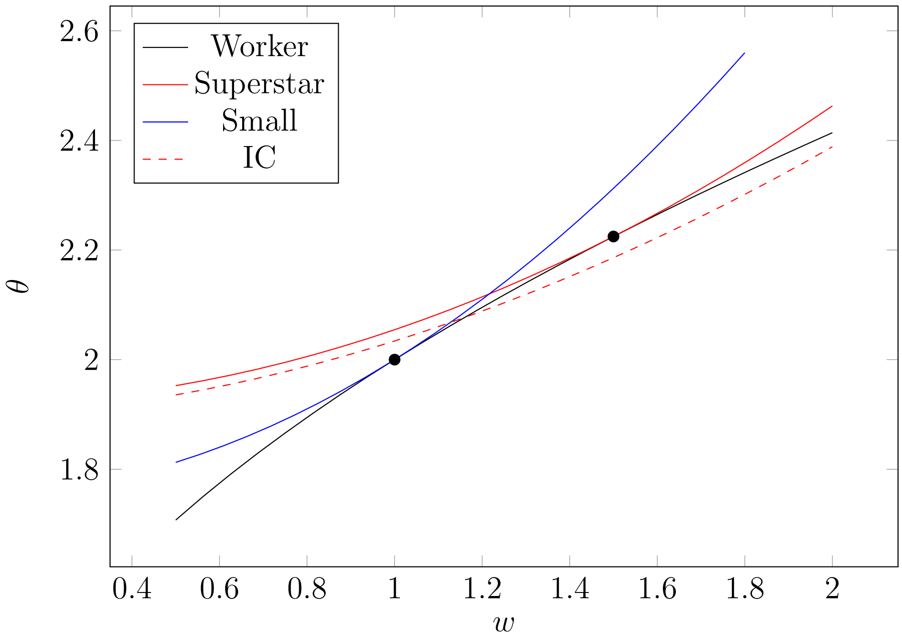

Figure 3: Non-binding case (when η is small)

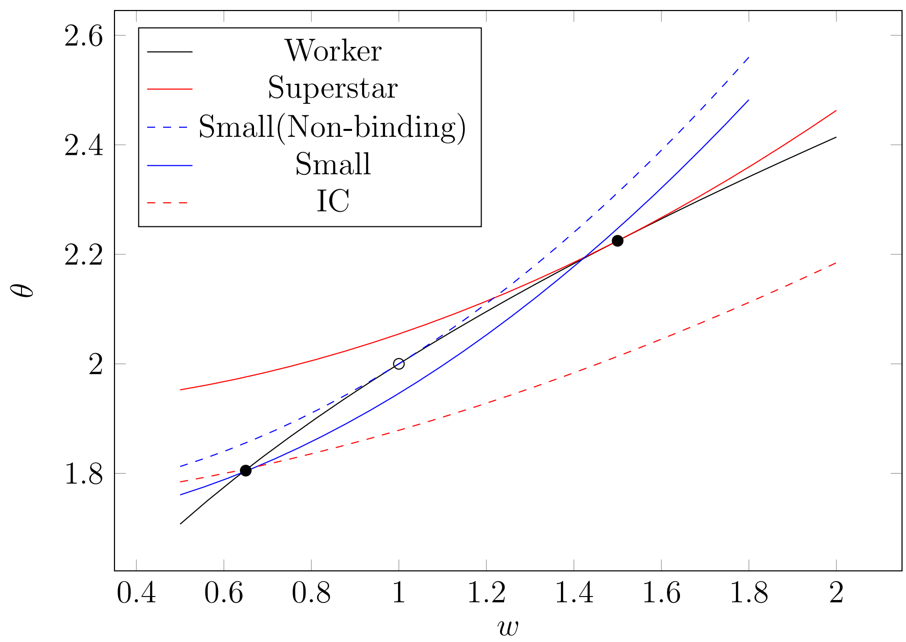

Figure 4: Binding case (when η is large)

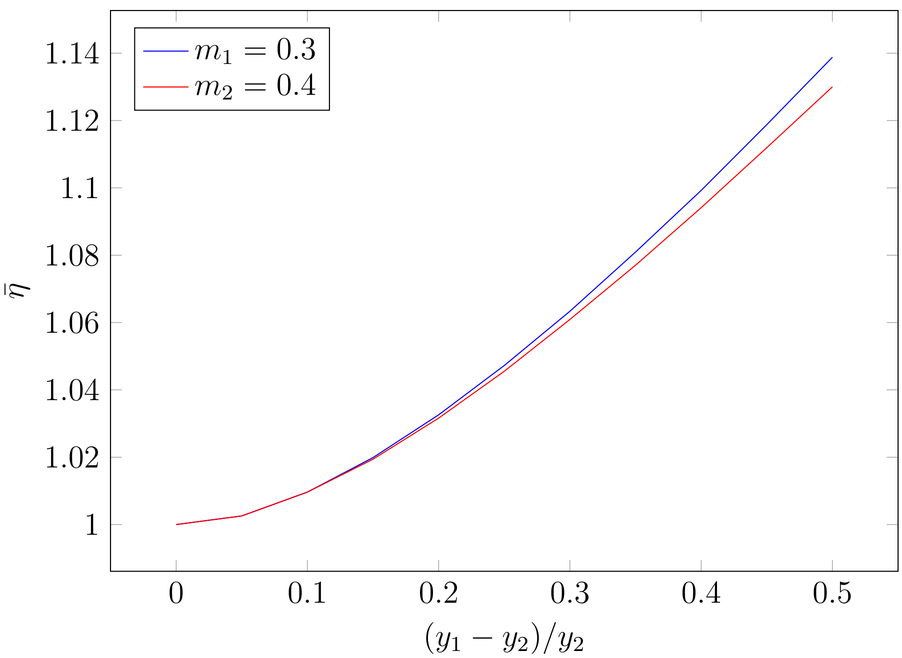

Figure 5: $\bar{\eta}$ as a function of productivity gap

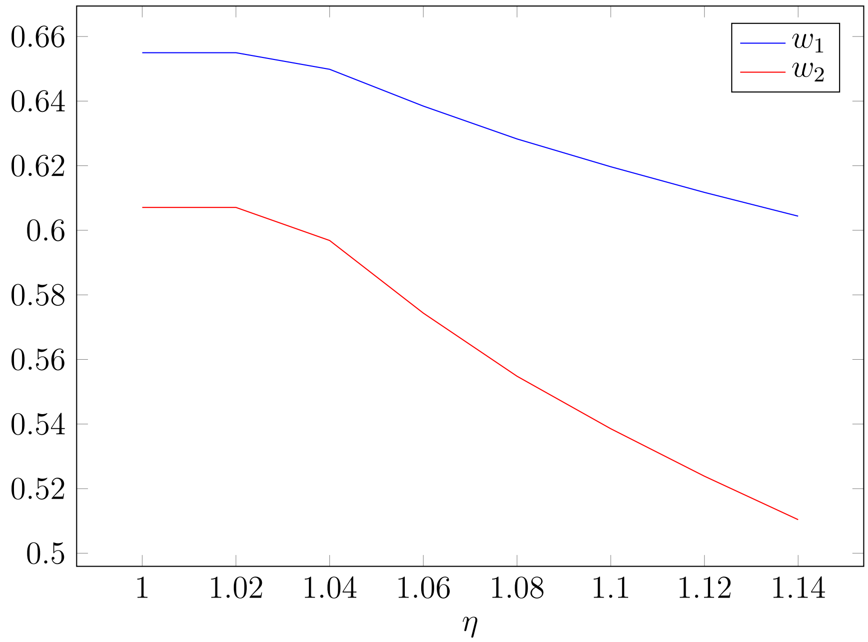

Figure 6: Wages as a function of η

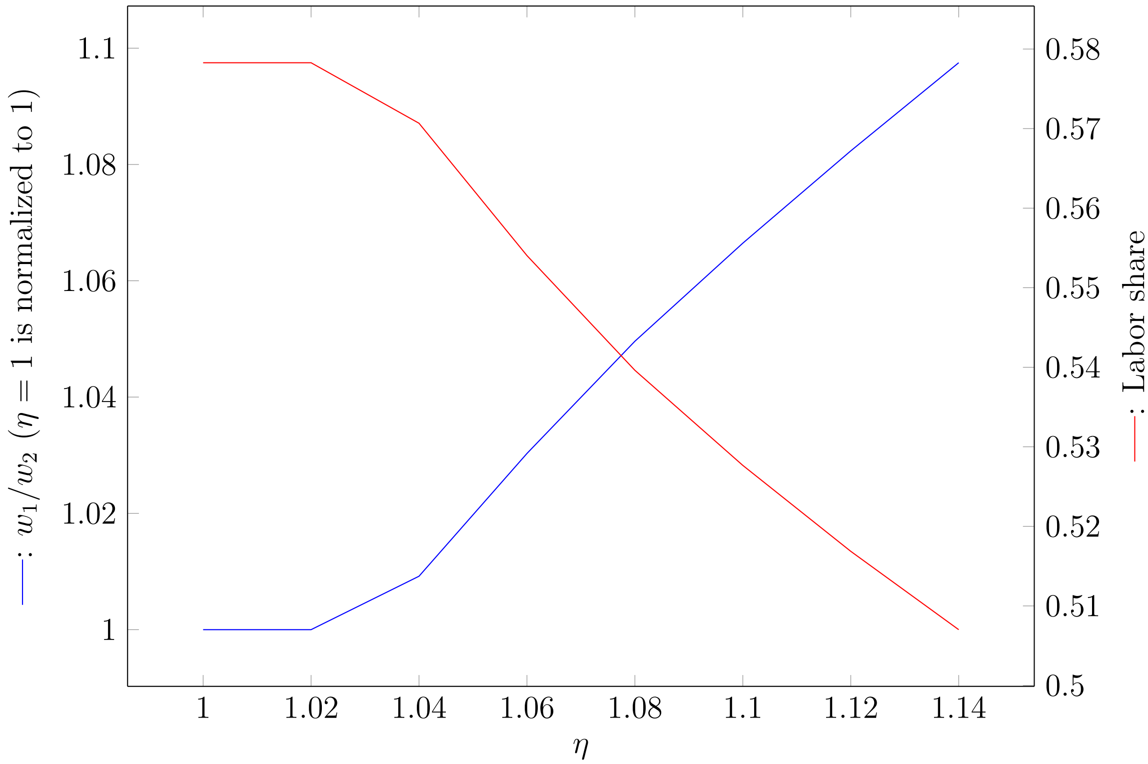

Figure 7: Inequality and labor share

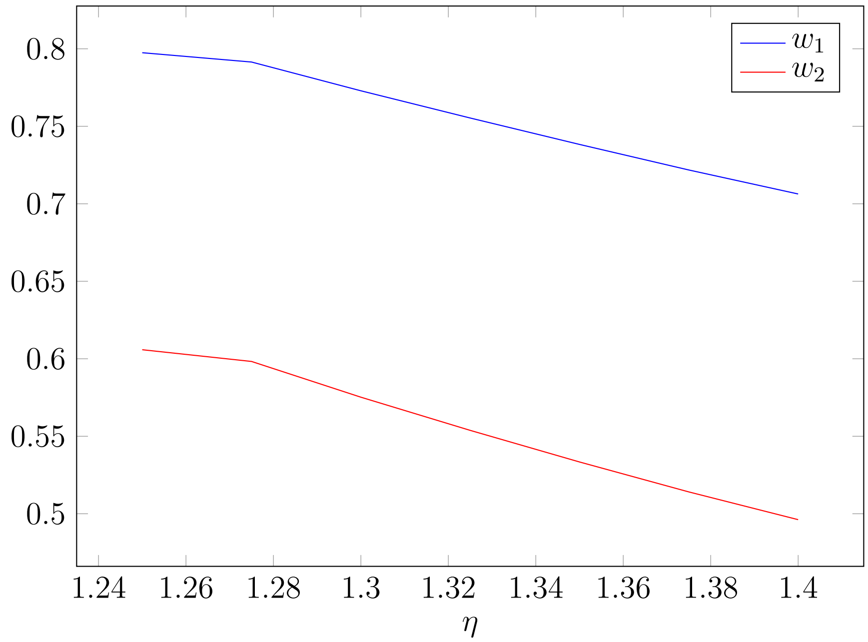

Figure 8: Wages as a function of η

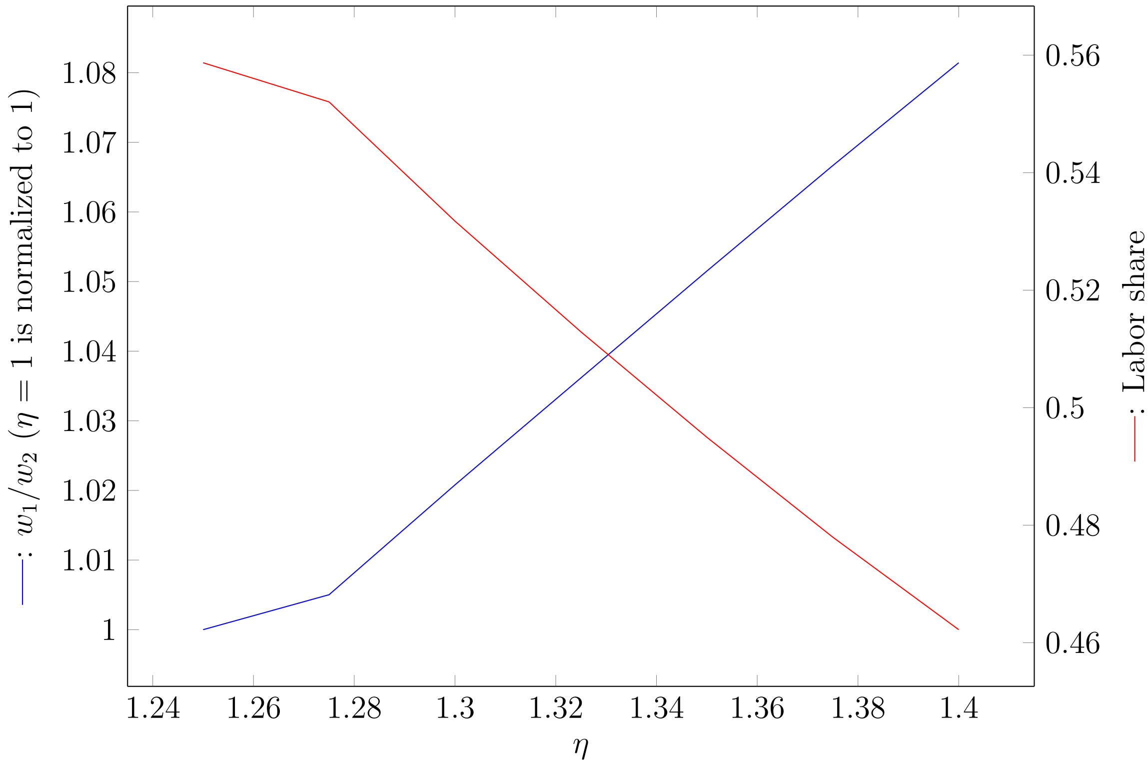

Figure 9: Inequality and labor share

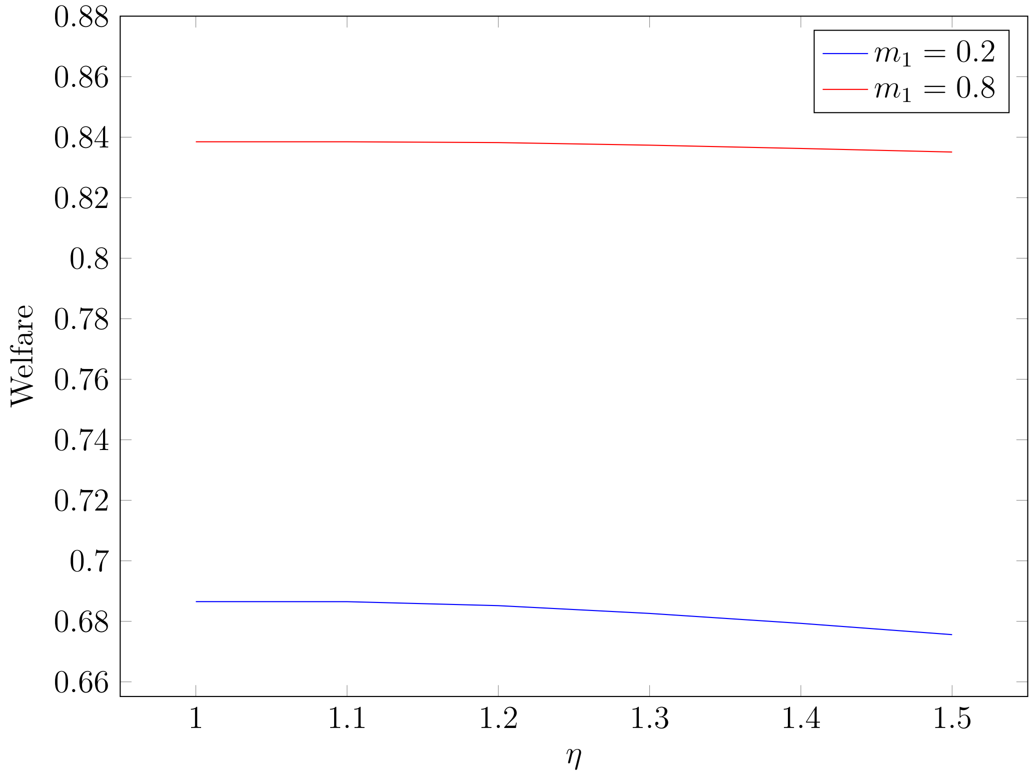

Figure 10: Welfare as a function of η

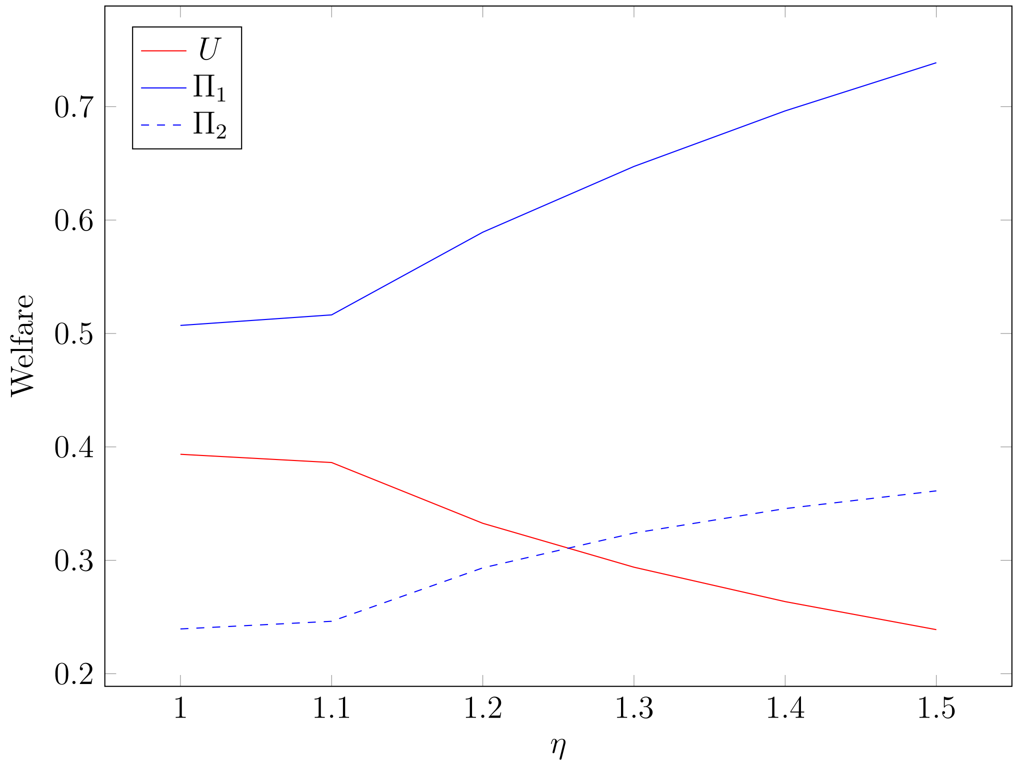

Figure 11: Profits and value of search ($m_1 = 0.2$)

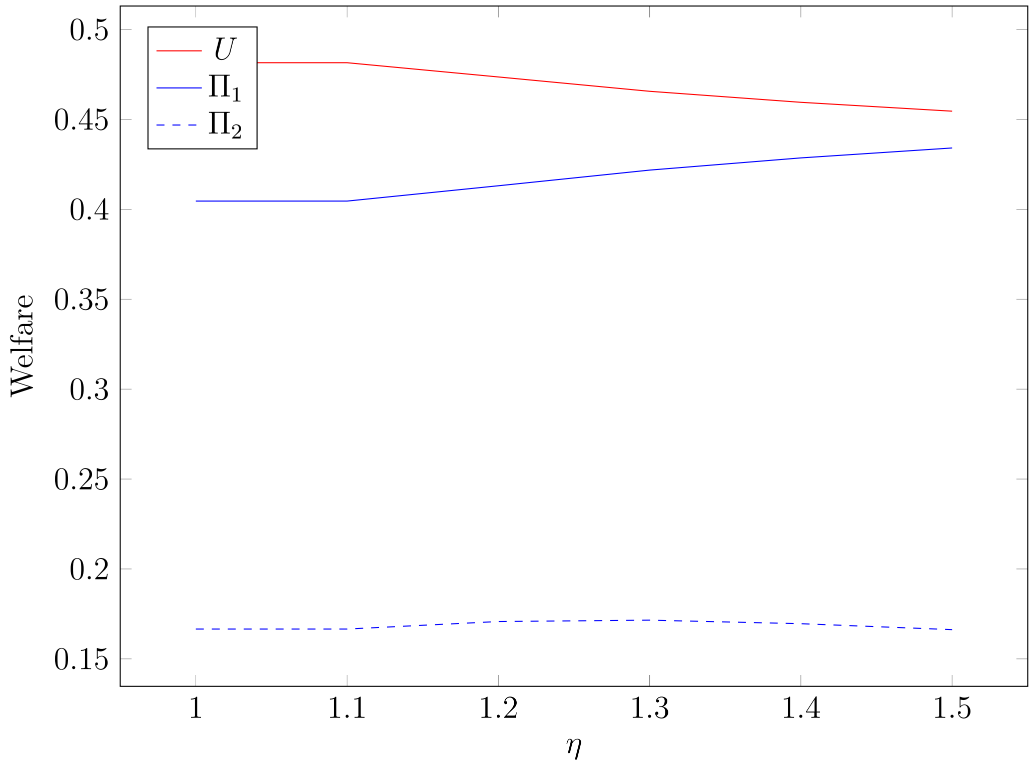

Figure 12: Profits and value of search ($m_1 = 0.8$)