Rational Inattention, Financial Heterogeneity and Effectiveness of Monetary Policy*

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Abstract

How do financial constraints affect firms' information acquisition and, thus, the effectiveness of monetary policy over the business cycle? This paper addresses these questions in a rational inattention model in which heterogeneous firms face both (aggregate) monetary shocks and (idiosyncratic) productivity shocks. The model predicts that, due to strategic complementarity in pricing decisions, firms with a binding financial constraint pay more attention to monetary shocks than unconstrained ones. At the aggregate level, heterogeneity in attention allocation implies that prices become more responsive to shocks as the fraction of constrained firms increases. As a result, in the calibrated version of the model, monetary policy is less powerful in recessions, when financial frictions are more severe. The model is consistent with firms' heterogeneous attention to macroeconomic conditions and with the state-dependent effectiveness of monetary policy, two features supported both by the empirical evidence provided in the paper and by the existing literature.

Keywords: Rational Inattention, Financial Heterogeneity, Monetary non-neutrality

JEL Codes: E3, E44, D8

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1 Introduction

Firms collect and analyze information about various uncertainties to guide their decisions, among which activeness and willingness can be significantly affected by firms' characteristics. The rapidly growing literature on rational inattention demonstrates thorough insights, how one can address empirical puzzles by assuming that agents cannot attend to all available information due to cognitive limitations. Most of those models, however, assume firms with homogeneous characteristics, for example, same financial conditions. The prominent literature on financial frictions has been showing how financial constraints might affect firms' behaviour, and hence the business fluctuations. For example, Bernanke et al. (1999) show that firms' access to credit market can amplify and propagate shocks to the macroeconomy. Nonetheless, the feature of financial constraints in shaping firms' information acquisition behaviour remains an open question. This paper shows that a model embedding financial friction within the rational inattention framework can generate divergence in firms' both attentiveness and responsiveness to macroeconomic conditions, which could be supported both by the empirical evidence provided in the paper and by the existing literature. Additionally, this model provides rich implications regarding the state-dependent effectiveness of monetary policy, which a rational inattention model with representative firms can hardly deliver.

In this paper, I study a stochastic dynamic general equilibrium model that builds on the seminal rational inattention framework in Mackowiak and Wiederholt (2009) where firms facing aggregate monetary shock and idiosyncratic productivity shock. Further, I introduce a new element: financial constraint, which affects firms' borrowing capacity and hence their pricing behaviour. The road-map of theoretical analysis is divided into two parts. One is a simple general equilibrium model with a simplified information set, which produces analytical results regarding firms' attention allocation, the other is a calibrated full dynamic model, which provides quantitative analysis about effectiveness of monetary policy. In the simple model, I assume that firms have perfect information about previously realized shocks, which facilitates analyzing firms' attention allocation; this assumption is relaxed in the full dynamic model.

Throughout the paper, I assume that firms with relatively low productivity face ex-ante binding constraints that limit the amount of capital they can rent, while other unconstrained firms can freely adjust their capital input.[1] The main results from the simple model are: financially constrained firms strategically choose to learn weakly more about aggregate monetary shock compared to unconstrained firms. Additionally, due to strategic complementarity, this greater incentive of constrained firms tends to increase while unconstrained firms shift their attention more towards monetary conditions.

The mechanism that drives the main results is firms' pricing sensitivity to different shocks. With a constant return to scale production function, unconstrained firms' pricing decisions are independent of the aggregate price once they are conditional on nominal money supply, i.e. there is no strategic complementarity. Nonetheless, as constrained firms cannot achieve an optimal input combination, the only input they can adjust to produce a committed amount of goods is labour. For constrained firms, real rigidity mimics a decreasing return to scale production function. As a consequence, their pricing decisions become strategic complement, i.e. their firm-specific prices are positively affected by the aggregate price.[2] This effect delivers a stronger incentive for constrained firms to shift their attention towards monetary policy shock, in order to monitor aggregate demand. However, since this incentive stems from complementarity, constrained firms will not pay more attention to aggregate conditions unless unconstrained firms start to do so when the uncertainty of aggregate shock is relatively low.[3][4] This feature is also consistent with what Hellwig and Veldkamp (2009) describes as "knowing what others know" since constrained firms will pay no attention to aggregate conditions if unconstrained firms remain careless about it. Concerning actual price responsiveness, since constrained firms pay more attention to monetary policy shocks, which offsets their sluggish response due to real rigidity, and unconstrained firms' profit-maximizing prices depend on the capital rental rate,

[1]Under this assumption, constrained firms are relatively small firms in terms of revenue and employment.

[2]This result is similar to the setting proposed by Woodford (2001) and Paciello (2012); however, this paper contributes by using financial constraint to rationalize the strategic complementary price of constrained firms.

[3]As in this paper, monetary policy shock is the only aggregate shock, I will use 'aggregate shock' and 'monetary policy shock' interchangeably.

[4]Mackowiak and Wiederholt (2009) also discuss this strategic complementarity. However, as firms in their model are homogeneous, once the equilibrium price response is derived and plugged into firms' individual pricing decision, strategic complementarity disappears.

which is affected by previously realized shocks. The combined effect of these forces is that, after monetary policy shock occurs, an unconstrained firms prices are less responsive compared to those of a constrained firm. Therefore, monetary non-neutrality generally decreases with the fraction of constrained firms in the economy.[5]

A prominent feature of this model is its ability to match empirical findings in both firms' attention allocation behaviour and firms' price responsiveness to monetary policy shock. Regarding attentiveness heterogeneity,e I use firm size as a proxy for firms' financial condition, i.e. smaller firms are more likely to be financially constrained.[6] To support the validity of this proxy, I take qualitative microdata from the German manufacturing subset of the IFO Business Expectation Panel and verify the strong correlation between firm size and financial condition. Using survey data collected by Coibion, Gorodnichenko, and Kumar (2018), I can test the theoretical prediction that smaller firms make systemically smaller errors when asked to recall the previous aggregate variables like unemployment rate and the output gap. These results are in line with Coibion et al. (2018), who demonstrate that smaller firms also pay relatively more attention to the inflation level. One can link the empirical facts by the well-known relationship in the Phillips curve and Okuns Law. Concerning price responsiveness heterogeneity, the theoretical prediction of this model is consistent with the empirical facts documented in Balleer, Hristov, and Menno (2017), i.e. that constrained firms respond faster to monetary policy shocks, both upwards and downwards.

The general weaker sensitivity of unconstrained firms is instructive for studying the state-dependent effectiveness of monetary policy. Some recent empirical findings have documented that monetary policy is less powerful in stimulating real growth during a recession. Whereas, a traditional rational inattention model with a representative firm can hardly explain this fact. As Bloom et al. (2018) calibrate, the volatility of idiosyncratic shock escalates more than that of aggregate shock during a recession. Considering the representative firm rational inattention

[5]Nonetheless, once the economy enters into a particularly volatile situation when aggregate uncertainty is sufficiently high, and constrained firms have allocated all their attention to analyzing Macroeconomic uncertainties, their price response to aggregate shock will be lower than unconstrained firm in the short run. The reason behind is that constrained firms do not have enough attention to offset the slow responsiveness caused by financial friction.

[6]This approach is widely adopted in the literature, for example Kashyap et al. (1994) and Gertler and Gilchrist (1994).

model, firms should pay less attention to aggregate shock and hence more effective monetary policy, which contradicts the empirical literature. The model proposed in this paper can reconcile this effect perfectly. Given that more firms are likely to be constrained during a recession, this composition effect can not only offset the relative volatility effect but also makes monetary policy weaker during a recession. Calibration using aggregate data from the US shows that increasing the fraction of constrained firms from 13.4% (calibrated value) to 50% can induce about a 25% loss as the real effect of monetary policy.

Related Literature This paper connects to several strands of literature. Firstly, it is closely related to the burgeoning literature on rational intention, which has proliferated since Sims (2003). The approach with limited information processing capacity has been applied to different aspects of the economy, among which the application to pricing decisions is the main focus of this paper. Specifically, Mackowiak and Wiederholt (2009) show how rational inattentive price setters can generate large and enduring monetary non-neutrality, conditional on assuming one order of magnitude more volatile idiosyncratic shock compared to nominal aggregate shock. Paciello (2012) show analytically how monetary policy feedback rules affect firms' allocating attention characterization in a general equilibrium model. Afrouzi (2019) solves the dynamic general equilibrium model with inattentive price-setters and strategic complementarities, which shows how a firm's attention to aggregate variables varies with the number of their competitors. Pasten and Schoenle (2016) extend Mackowiak and Wiederholt (2009)'s framework to a multi-product setting and demonstrate that monetary non-neutrality quickly vanishes as the number of goods per firm rises. Turén et al. (2018) studies a dynamic rational inattention model with price-setting firms having heterogeneous information acquisition cost. This model builds on Mackowiak and Wiederholt (2009) and contributes to the literature by allowing the co-existence of two types firms, and it shows how financial heterogeneity and information friction can jointly generate heterogeneous information acquisition behaviour, along with heterogeneous responsiveness. In addition, this paper is capable of rationalizing contemporary empirical findings for the state-dependent effectiveness of monetary policy, which the representative firm rational inattention model can scarcely address.

Secondly, this paper relates to a vast literature which studies how monetary policy shock affects firms differently through financial friction. The feature of financial friction has long been discussed by seminal papers such as Bernanke, Gertler, and Gilchrist (1999). Many empirical papers, including Kashyap, Lamont, and Stein (1994) and Gertler and Gilchrist (1994), have argued that smaller and presumably more credit constrained firms are more responsive to monetary policy, along with several other dimensions. Balleer et al. (2017) contribute to the literature by showing that financially constrained firms respond faster to monetary policy shocks, both upwards and downwards. This paper theoretically rationalizes the effect of financial friction in shaping firms' responsiveness to monetary policy shock through firms' information acquisition behaviour.

Thirdly, this paper contributes to explaining and extending recent micro empirical facts regarding firms' beliefs about macroeconomic variables. The growth of new data has accelerated in recent years, and this has verified several theoretical features of rational inattention. I enumerate the most relevant ones for the purposes of this research. Coibion et al. (2018) indicate that firms in New Zealand, where inflation has been stable over the last 20 years, appear to be inattentive to macroeconomic variables such as inflation, output gap and even GDP. In contrast, Borraz and Orlik (2019) document that firms in Uruguay exhibit a very high degree of attention to current inflation conditions which they link to the countrys historically high volatile inflationary experience. In the literature, some new cross-sectional evidence about firms' heterogeneous attentiveness to aggregate conditions is also in the spotlight. For example, Coibion et al. (2018) document the presence of significant heterogeneity in attentiveness across firms. Notably, larger and older firms systematically report larger errors in the values of realized aggregate variables and, more significantly, larger forecasting errors. I add to these empirical facts by testing other macroeconomic variables, output gap and unemployment rate, with the same New Zealand survey data.

Lastly, I contribute to the literature on the state-dependent effectiveness of monetary policy. A number of recent papers, including Vavra (2014), Tenreyro and Thwaites (2016), and

Alpanda et al. (2019) argue that monetary policy is less effective in stimulating real growth during a recession than expansion through a business investment channel, an uncertainty channel and a household borrowing channel, respectively. Instead, I use an empirical justified rational inattention model to show how financial heterogeneity can rationalize the state-dependent effectiveness of monetary policy as well as accounting for counter-cyclical aggregate and idiosyncratic volatility in the business cycle, as documented in Bloom (2014), Vavra (2014), Bloom et al. (2018) and Baker et al. (2016).

The paper is organized as follows. Section 2 illustrates the primary mechanism of financial friction shaping firms' information acquisition in a simplified general equilibrium model. Section 3 derives and illustrates a set of testable predictions regarding firms' attention heterogeneity and price responsiveness heterogeneity. Section 4 presents empirical regularities to test theoretical predictions. Section 5 discusses the full dynamic model numerical solution. Section 6 studies the implications for the state-dependent effectiveness of monetary policy. In Section 7, I discuss my extended model with heterogeneous information processing capacity. Section 8 concludes the paper. Moreover, all the technical derivations, as well as proofs of all the propositions and corollaries, are included in the Appendices.

2 Model

In this section, I build a general equilibrium model to illustrate the role of financial friction in shaping firms' behaviours and monetary neutrality. The model presented here is a particular case, in terms of information structure, of the full dynamic general equilibrium model that is specified in Section 4. While the general dynamic model has to be solved using computational methods, the solution to this model is in closed-form, which provides insights for interpreting the results from the full model.

2.1 Setup

Time is discrete and infinite, periods are indexed by $t \in T \equiv \{0, 1, 2, ..., \}$. The economy is populated by a representative household deriving utility from consuming a final good C_t and disutility from providing labour L_t, a continuum of firms $i \in [0,1]$ setting prices and hiring labour to produce, and a government controlling money supply M_t in accordance with specific money supply rule.

2.2 Household

Problem The representative household consists a consumer and large enough number of workers who takes the nominal prices of goods and wages as given and forms demand over products from different firms. Household's preferences in period $t = 0$ are given by

$$U_0 = E \sum_{t=0}^{\infty} \beta^t \left(\log \left(C_{\mathrm{t}} \right) - \phi_L L_t \right), \tag{1}$$

where $\beta \in (0,1)$ is the discount factor. The assumption of logarithmic utility makes analytical characterization easier but can be generalized to constant relative risk aversion (CRRA) utility at the expense of some extra notation. The linear disutility of labour indicates infinite Frisch elasticity.[7] The final consumption good C_t is a composite index of all firms' products in period t, which is modelled by the Dixit-Stiglitz aggregator with elasticity of substitution $\nu > 1$

$$C_t = \left[\int_0^1 \left(C_{i,t} \right)^{\frac{\nu-1}{\nu}} di \right]^{\frac{\nu}{\nu-1}}. \tag{2}$$

The representative household's objective is to maximize (2) with respect to a sequence of variables $\{C_t, L_t, C_{i,t}, B_{t+1}, M_{t+1}^d\}_{t=0}^{\infty}$ subject to the following sequence of budget constraint flow,

[7] The log-utility assumption implies that the level of nominal interest rate is proportional to the growth rate of aggregate demand, which is an approach that has been widely used in rational inattention literature (for instance, see Afrouzi (2019) and Paciello (2012)). The linear disutility in labour is a common assumption in the models addressing monetary non-neutrality which eliminates the source of across industry strategic complementarity from the household side (for instance, see Afrouzi (2019) and Golosov and Lucas Jr (2007)).

for $t = 0, 1, ...,$

$$s.t. \qquad M_t + B_t \leq W_t L_t + R_t B_{t-1} + (M_{t-1} - P_{t-1}C_{t-1}) + \int_0^1 \Pi_{i,t} di \quad \forall t \geq 0, \tag{3}$$

where P_t is the price of the final consumption good, B_{t-1} are the households demand for nominal government bonds between periods $t-1$ and t, R_t is the nominal gross interest rate between $t-1$ and t on those bond holdings, W_t is the nominal wage rate in period t, $\Pi_{i,t}$ is the nominal profits of firm i in period t. The representative household can freely transform his pre-consumption wealth in period t into money balances, M_t, and bond holdings, B_t. Given the main purpose of this paper, I assume that households are fully informed about prices and wages.[8] Accordingly, $E[\cdot]$ represents the perfect information expectation operator.

The purpose of holding money is to purchase consumption goods. I assume that the representative household faces the following cash-in-advance constraint

$$\int_0^1 P_{i,t} C_{i,t} di = M_t. \tag{4}$$

where $P_{i,t}$ is the price of differentiated good i. The representative household also faces a no-Ponzi-scheme condition. I assume for simplicity that the gross nominal interest rate is larger than 1 for all t to guarantee that (4) is always binding. I introduce the cash-in-advance constraint to obtain a mapping from the monetary policy instrument, i.e., the control of nominal aggregate supply, to the monetary policy target, i.e., the nominal interest rate.

We show in Appendix that household's optimal behaviour implies the demand function of variety i:

$$C_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\nu} C_t. \tag{5}$$

[8] As this paper is mainly studying the implication of rational inattention for firms, I abstract from the information friction for households. This approach is same as in Afrouzi (2019), Paciello (2012) and Paciello and Wiederholt (2014), etc.

and the aggregate price index

$$P_t = \left[\int_0^1 \left(P_{i,t} \right)^{1-\nu} di \right]^{\frac{1}{1-\nu}}. \tag{6}$$

When I have binding cash-in-advance constraint, the household's intertemporal Euler equation and optimal labour supply equation are given by

$$\frac{1}{R_{t+1}} = \beta E \left[\frac{M_t}{M_{t+1}} \right]$$
$$W_t = \phi_L M_t$$

The log-utility of household leads to a duality between constructing monetary policy either in terms of controlling nominal interest rates or aggregate demand. Furthermore, the linear disutility in labour, which corresponds to an infinite Frisch elasticity of labour supply, implies that the nominal wage is proportional to the aggregate nominal demand.

2.3 Monetary Policy

For simplicity, I assume that the monetary authority sets its policy, in terms of the aggregate money supply, following an log-AR(1) process with persistence rate $\rho_M \in [0, 1)$

$$\ln M_t^s = (1 - \rho_M) \ln \bar{M} + \rho_M \ln M_{t-1}^S + \varepsilon_{M,t} \tag{7}$$

where $\varepsilon_{M,t}$ is the only aggregate uncertainty, which is i.i.d and normally distributed monetary policy disturbance, $\varepsilon_{M,t} \sim N(0, \sigma_M^2)$, $\bar{M}$ is the non-stochastic steady state value of aggregate demand. We assume away the feedback in the money supply rule.[9]

[9] Some literature assume a log-AR(1) process of the aggregate demand growth rate, which guarantees a stationary inflation process instead of price level. See, Mankiw and Reis (2002), Woodford (2001), and Afrouzi (2019). To have a more realistic case, the equation (7) can be generalised to $\ln M_t = \phi_P \ln P_t + \phi_C \ln \frac{C_t}{C_t^*} + \rho_M \ln M_{t-1} + \varepsilon_{M,t}$ which accounts for the feedback of price and consumption level, as in Paciello (2012).

2.4 Firms

The economy consists of a continuum of monopolistic competitive firms indexed by $i \in I = [0, 1]$, that are producing differentiated varieties of outputs $C_{i,t}$. Firms take aggregate price index and wage as given while committing to produce the realized level of demand that their price induces. After setting prices, firms then hire labour and rent capital to produce in accordance with the following production function

$$Y_{i,t} = A_{i,t} L_{i,t}^{\alpha} K_{i,t}^{1-\alpha}, \tag{8}$$

where $Y_{i,t}$ is output, $L_{i,t}$ is labour input and $K_{i,t}$ is capital input of firm i in period t. The parameter $\alpha \in (0, 1)$ is the elasticity of output with respect to labour input. $A_{i,t}$ is idiosyncratic productivity in period t, which follows a stationary log-AR(1) process

$$\log A_{i,t} = (1 - \rho_A) \log A_i + \rho_A \log A_{i,t-1} + \varepsilon_{i,t}^{A},$$

where the parameter $\rho_A \in [0, 1)$ and the innovation $\varepsilon_{i,t}^{A}$ is the only idiosyncratic uncertainty which is i.i.d and normally distributed $\varepsilon_{i,t}^{A} \sim N(0, \sigma_A^2)$.[10] Firm heterogeneity originates from A_i, which is the non-stochastic steady state value of firm i's productivity. Additionally, the support of A_i is assumed as $A_i \in \{A_L,\ A_H\}$, and the fraction of firms having low productivity A_L is ϕ i.e., $Prob(A_i = A_L) = \phi$, which will be elaborated in the following subsection. [11]

Firm i's nominal profit in period t is given by

$$\Pi_{i,t} = P_{i,t} Y_{i,t} - W_t L_{i,t} - R_t^K K_{i,t} \tag{9}$$

[10] This is equivalent to assuming that the variance of productivity innovation is identical across all firms.

[11] The distribution of A_i is not critical for the fundamental results given the following reasons: as will be elaborated later, the heterogeneity of A_i is introduced to rationalize the fact that smaller, i.e. low productive, firms will be financially constrained, which turns the productivity heterogeneity into financial heterogeneity; additionally, I study the model in the form of log-deviation form steady state, hence the non-stochastic distribution of A_i is not crucial for the final results.

where R_t^K is the capital rental price which is equal to

$$R_t^K = \frac{P_{t-1}}{\beta} \frac{M_t}{M_{t-1}}. \tag{10}$$

In this model, firms utilize capital for production and pay their rental cost both in period t. Analogy with the typical capital investment setting, one could view this framework as firm managers choosing a state-contingent capital investment plan after all shocks are realized in the end of period $t-1$ for period t production through external financing and pay interest rate in period t. In this way, capital rental price could be pinned down as previous formula if assuming capital full depreciation. In Appendix A.2, I explicitly explain how this capital rental price is derived and rationalized.

2.4.1 Financial Friction

Financial friction is introduced by limiting the amount of capital that firms can rent.[12] Intuitively, the status of financial constraint being binding or not should be endogenously determined by firms optimal choice of capital and total collateral. Nonetheless, since the primary purpose of this ingredient is to illustrate how constrained, and unconstrained firms differ in their behaviours of allocating attention to Macroeconomic condition, I assume that there are two types of firms populated in this economy, *constrained* firms and *unconstrained* firms with non-stochastic steady state productivity A_L and A_H, respectively. The fraction of constrained firms is assumed to be ϕ. In addition, constrained firms are facing a binding constraint in the sense that they can borrow only a certain amount of capital each period, K_i, which is lower than the non-stochastic optimal amount of capital constrained firms would like to choose, K_i^*,[13] i.e.,

$$\text{If } A_i = A_L, \text{ then } K_i \leq K_i^* = \frac{P_i^{-\nu} P^{\nu} C}{A_i} \left(\frac{1-\alpha}{\alpha} \frac{W}{R^K}\right)^{\alpha}. \tag{11}$$

[12] The approach to model the financial constraint is widely adopted in the literature, for instance, Evans and Jovanovic (1989), Buera and Shin (2011), Moll (2014) and Mehrotra and Sergeyev (2020).

[13] This is equivalent to assume that firms have full information about their financial constraint status.

Under this assumption, firms are categorized into two groups, and they are identical within each group.[14] In a dynamic setting with endogenously determined financial constraint, e.g., Moll (2014), the maximum capital that firms are allowed to rent is determined by firms' last period wealth, which plays as a state variable for constrained firms. Hence, constrained firms cannot internalize this value to infer the actual state of $K^*_{i,t}$. To ease notation, I use K_i, which is not time-dependent, to denote the capital that constrained firm i can rent, which will be internalized by firm i as a constant value.[15]

2.4.2 Price setting behaviour

After knowing the binding status of financial constraint, firms need to decide their desired price level. Given (5), (8) and (9), the profit-maximizing price under perfect information are given by

$$P^*_{i,t} = \begin{cases} [\frac{\nu}{\alpha(\nu-1)}]^{\frac{\alpha}{\psi}} W_t^{\frac{\alpha}{\psi}} C_t^{\frac{1-\alpha}{\psi}} P_t^{\frac{(1-\alpha)\nu}{\psi}} K_i^{\frac{\alpha-1}{\psi}} A_{i,t}^{-\frac{1}{\psi}} & \text{if constrained} \\ \frac{\nu}{(\nu-1)\alpha^\alpha(1-\alpha)^{1-\alpha}} W_t^\alpha (R_t^K)^{1-\alpha} A_{i,t}^{-1} & \text{if unconstrained} \end{cases} \tag{12}$$

where $\psi \equiv \alpha + (1-\alpha)\nu$ denotes the degree of real rigidity. After rearrangement, the log-deviation of firms' desired price from the non-stochastic steady state could be expressed as a function of aggregate and idiosyncratic shocks, together with other endogenous variables.

Lemma 1 *The log-deviation of different types of firm's desired price under perfect information can be categorized as*

$$p^*_{i,t} = \begin{cases} \frac{1}{\psi} m_t + \frac{\psi-1}{\psi} p_t - \frac{1}{\psi} a_{i,t} & \textit{if constrained} \\ m_t - a_{i,t} & \textit{if unconstrained} \end{cases} \tag{13}$$

where small letters $x_t \equiv \log X_t - \log \bar{X}$ denotes the value of X_t in log-deviations from the non-stochastic steady state.[16]

[14] The form of financial constraint could also be represented by limiting firms' capital rental ability with their initial wealth, and the intuition is similar: firms with less wealth cannot sufficiently support their optimal capital choice which leads to financial constrained status, see Appendix A.1.

[15] This is equivalent as assuming in the neighbourhood of the non-stochastic steady state, and the collateral constraint is always satisfied with equality for constrained firms, and slack for unconstrained firms. This assumption allows me to employ standard approximation methods when analyzing attention allocation behaviours. In turn, this will require a bound on the amplitude of stochastic driving forces in the model. See, for example, Monacelli (2009).

[16] Note that equation (13) illustrates firms' price in the first period right after shock occurs. The impulse response

Here $\frac{\psi-1}{\psi}$ measures the degree of strategic complementarity in constrained firms' desired price.[17] A higher ψ results in a smaller price adjustment of constrained firms in response to shocks but higher strategic complementarity. Whereas, the price of unconstrained firms (depicted in the second line) shows no complementarity to the current aggregate price. Apart from the money supply and aggregate price level of the last period, unconstrained firms' desired price is merely responding to two exogenous shocks. The key reason why the difference emerges is that constrained firms cannot freely adjust their capital input to match the optimal input combination, which leads to an increasing marginal cost function of constrained firms. Therefore, the desired price of constrained firms is dependent on its production scale, which is then dependent on the aggregate demand level. On the other hand, due to the constant return to scale production function, unconstrained firms have constant marginal cost, which does not change with the production scale.

2.5 Information Structure

Following the seminal literature of rational inattention (for instance, Sims (2003); Mackowiak and Wiederholt (2009)), I assume that firms are *rationally inattentive*, and in each period t the information set of the decision maker is

$$\begin{aligned}\mathcal{I}_{i,t} &= \mathcal{I}_{i,t-1} \cup \{s_{i,t}, A_{i,t-1}, M_{t-1}\} \\ &= \mathcal{I}_{i,-1} \cup \left\{ \{s_{i,\tau}\}_{\tau=0}^{t}, \{A_{i,\tau}\}_{\tau=0}^{t-1}, \{M_\tau\}_{\tau=0}^{t-1} \right\},\end{aligned}$$

where $\mathcal{I}_{i,-1}$ is the initial information set of decision maker from firm i and $s_{i,t}$ is the signal that she receives in period t. By assuming that firms have perfect information about previous realized shocks helps to abstract intertemporal information acquisition decisions and allow us to get closed-form solutions for this simple model. In the dynamic model this assumption will be relaxed. Firms choose their optimal signal $s_{j,t}$ from a set of available signals, S_t. Specifically,

of unconstrained firms' price after the first period will be dependent on previous aggregate variables due to the fact that their price is dependent on aggregate price and monetary shock of last period.

[17] Through out this paper, I assume that constrained firms' price change in the same direction with aggregate price, but less sensitive than unconstrained firms. This assumption is valid as long as $\nu > 1$. This assumption is in line with common sense in the literature.

I assume that the signals about fundamentals are of the form:[18]

Assumption 1 *The signal available to firm j in period t is a two-dimensional vector:*

$$s_{i,t} = (s^1_{i,t}\ ,\ s^2_{i,t})'$$

where

$$s^1_{i,t} = m_t + \eta^M_{i,t} \tag{14}$$

$$s^2_{i,t} = a_{i,t} + \eta^A_{i,t} \tag{15}$$

where $s^1_{i,t}$ is the noisy signal about aggregate fundamental with $\eta^M_{i,t} \sim \mathcal{N}(0, \tau^2_M)$, while $s^2_{i,t}$ is idiosyncratic noisy signals about firm specific productivity with noise $\eta^A_{i,t} \sim \mathcal{N}(0, \tau^2_A)$.

Additionally, I assume that (i) the noisy term in the signal is due to limited attention of firm's manager, (ii) the process $\{a_{i,t}\},\ i \in [0,1]$ are pairwise independent and independent of $\{q_t\}$, (iii) the process $\{\eta^M_{i,t}\}$ and $\{\eta^A_{i,t}\},\ i \in [0,1]$ are mutually and pairwise independent. The benefits of these assumptions are as follows. First, the assumption formalizes the initial idea that being attentive to aggregate fundamentals and being attentive to idiosyncratic conditions are different activities. Secondly, these assumptions are simplification of reality that has the important advantage of introducing an endogenous information choice into an otherwise standard general equilibrium framework, while keeping the model tractable enough to allow for a closed-form solution. This solution provides valuable information on the interaction between the different components of the model. Assumption 1 will be relaxed in the dynamic benchmark model.

The key assumption of rational inattention models is that price setters are constrained in the flow of information that they can process at every period t:

$$\mathcal{I}\bigg(\{M_t, A_{i,t}\}\ ;\ \{s_{i,t}\}\bigg) \leq \kappa$$

[18]This assumption is not essentially necessary for the result. Using the theoretical results in Mackowiak and Wiederholt (2009) and Maćkowiak, Matějka, and Wiederholt (2018), one can easily prove that: signals with the format shown in Assumption 1 are optimal for firms.

where the information flow $\mathcal{I}(x, y)$ is a measure of mutual information between random variables x and y in bits,[19] the parameter κ denotes firm's total attention, measuring the information flow per time unit. This constraint states that the average per period amount of information a firm can process about all the uncertainty in economy is upper bounded by κ. In the extension section, I consider the case when firms having heterogeneous information processing capacity.

2.6 Firm's problem

Firms maximize the expected discounted stream of profits by choosing how precisely to observe the respective signals and hence their optimal prices. Firms take as given the stochastic processes and choose prices of every period without any adjustment costs. Additionally, by assuming that firms have perfect information about realized shocks, the problem of each firm is essentially static. For the ease of exposition, I decompose the decision-making behaviour as a three-stage process within each period:

Stage 1: In the end of period $t-1$, firm managers with information set $\mathcal{I}_{i,t-1}$ allocate their attention and choose the optimal signals $s_{i,t} \in \mathcal{S}_t$.

Stage 2: At the beginning of period t, firm managers receive signal and realized shock of last period, their information set is updated to $\mathcal{I}_{i,t}$.

Stage 3: With the new information set, firm managers make optimal pricing strategy $P_{i,t} : \mathcal{I}_{i,t} \to \mathbb{R}_+,\ \forall i \in I$

Note that agents make a purely static decision every period, and the link across different periods is only through the information set. Hence, firm i's problem can be represented as

$$\max_{\{s_{i,t}\} \in S_t} E\left[\sum_{t=1}^{\infty} \beta^t \Pi(P_{i,t}, P_t, Y_t, M_t, A_{i,t})\right] \tag{16}$$

[19] $\mathcal{I}(x, y)$ is Shannon's mutual information function. In this paper, I focus on Gaussian random variables, in which case $\mathcal{I}(x, y) = H(M_t, A_{i,t}) - H(M_t, A_{i,t}|s_{i,t}) = \frac{1}{2}\log_2(\sigma_x^2) - \frac{1}{2}\log_2(\sigma_{x|y}^2)$

with

$$P_{i,t} = \arg\max_{P_{i,t}} E\big[\Pi(P_{i,t}, P_t, Y_t, M_t, A_{i,t}) \mid \mathcal{I}_{i,t}\big] \tag{17}$$

and subject to

$$\mathcal{I}\bigg(\{M_t, A_{i,t}\}\ ;\ \{s_{i,t}\}\bigg) \leq \kappa \tag{18}$$

In order to have an analytical solution of the attention allocation problem, in this section, I consider a second-order Taylor approximation of the discounted sum of future profits around the non-stochastic steady state, in deviation from the discounted value of profits under full information profit-maximizing behaviour. Additionally, under **Assumption** 1 and the independence assumption of shocks, choosing signal $s_{i,t}$ is equivalent to choosing how much attention allocated to each shock. Let $\kappa_{M,i} = \frac{1}{2}\log_2(\frac{\sigma_M^2}{\tau_M^2} + 1)$ denote the attention allocated to aggregate conditions and let $\kappa_{A,i} = \frac{1}{2}\log_2(\frac{\sigma_A^2}{\tau_A^2}+1)$ denote the attention allocated to idiosyncratic conditions, where $i \in [0, 1]$. Hence, I can rewrite firm's problem as minimizing profit loss by choosing the optimal composition of attention, where π_{11}^i denotes the derivative of profits twice with respect to the good price.[20]

$$\min_{\kappa_{M,i},\kappa_{A,i}} \sum_{t=1}^{\infty} \beta^t \frac{|\pi_{11}^i|}{2} E\big[(p_{i,t} - p_{i,t}^*)^2\big] \tag{19}$$

subject to

$$p_{i,t} = E[p_{i,t}^* | s_{i,t}] \tag{20}$$

and the information flow constraint

$$\kappa_{M,i} + \kappa_{A,i} \leq \kappa. \tag{21}$$

The information flow constraint reflects a trade-off for firm managers: increasing the pre-

[20]See Appendix for the derivation of such approximation.

cision of signals about aggregate shock (i.e., pay more attention to aggregate demand shock), force them to decrease the precision of signals about idiosyncratic productivity shock.

Since capital rental rate is dependent on previous shock and price, I conjecture that the equilibrium price level is a log-linear function of all nominal aggregate demand shock

$$p_t = \sum_{\tau=0}^{\infty} h_{t-\tau}^{t} m_{t-\tau} \tag{22}$$

where $h_{t-\tau}^{t}$ denotes the response of current price p_t to aggregate nominal demand of period $t-\tau$. This conjecture will be verified.

To analyse firms' attention allocation in period t, I assume temporarily that the economy was previously in non-stochastic steady state which implies that $m_{t-\tau} = p_{t-\tau} = 0,\ \forall \tau \in (1, \infty)$.[21] Therefore, the price response of current period is $p_t = h_t^t m_t$, and I can rewrite the profit-maximizing price (13) as

$$p_{i,t}^{*} = \begin{cases} \dfrac{1}{\psi}\big([1+(\psi-1)h_t^t]m_t - a_{i,t}\big) & \text{if constrained} \\ \\ \quad m_t - a_{i,t} & \text{if unconstrained} \end{cases} \tag{23}$$

and within each category, firms' price deviation is independent of their size. This equation implies that under perfect information, unconstrained firm's price deviation is a function of two shocks only, where the coefficients are exogenous. Regarding constrained firm's price deviation, its response to productivity shock is exogenous, however, the response to aggregate monetary shock is endogenous since the equilibrium price response to monetary shock h_t^t will translate into constrained firm's price response due to strategic complementarity.

Given this, we can represent the actual price set by firm i given information set $\mathcal{I}_{i,t}$ as

$$p_{i,t} = E[p_{i,t}^{*}|\mathcal{I}_{i,t}] = \xi_{M,i}\left[1-(\frac{1}{4})^{\kappa_M}\right](m_t+\eta_{i,t}^{M}) - \xi_{A,i}\left[1-(\frac{1}{4})^{\kappa_A}\right](a_{i,t}+\eta_{i,t}^{A}), \tag{24}$$

[21] To fit into a generalized economy that was not in steady state previously, the price response derived here can be viewed as the price response to a transitory aggregate shock, or the instantaneous price response to current period's aggregate shock. In section 3.2.2, I will explicitly derive the impulse response of price to current and all previous aggregate shock later this section. Additionally I will show that this simplification has no effect on firm's attention allocation behaviour other than simplifying notation.

where

$$
\xi_{M,i} = \begin{cases} \dfrac{1+(\psi-1)h_t^t}{\psi} \equiv \xi_{M,C}, & \text{if constrained} \\ \qquad 1 \quad \equiv \xi_{M,U}, & \text{if unconstrained} \end{cases} \qquad \xi_{A,i} = \begin{cases} \dfrac{1}{\psi} \equiv \xi_{A,C}, & \text{if constrained} \\ 1 \equiv \xi_{A,U}, & \text{if unconstrained} \end{cases} \tag{25}
$$

denotes the sensitivity of firms' price with respect to each shock. It is clear that unconstrained firm's price is equally sensitive to monetary shock and productivity shock. Whereas constrained firm are weakly more sensitive to monetary shock since $\frac{1+(\psi-1)h_t^t}{\psi} \geq \frac{1}{\psi}$ when $h_t^t \geq 0$, which is a guaranteed and intuitive condition in this model. The key mechanism here is that constrained firms become decreasing return to scale once their capital choice is fixed, then their price decision will be affected then their optimal price will respond to the equilibrium price, which responds to aggregate monetary policy shock, i.e., h_t^t.

2.7 Competitive Equilibrium

A competitive equilibrium for this economy is an allocation for household $\{C_{i,t}, M_t^d, L_t, B_t\}_{(i,t)\in I\times T}$, a signal sequence $\{s_{i,t}\}_{(i,t)\in I\times T}$, firm prices $\{P_{i,t}\}_{(i,t)\in I\times T}$ for firms given initial information set $\{\mathcal{I}_{i,0}\}_{i\in I}$, realised production and labour demand of firms $\{Y_{i,t}, L_{i,t}^d\}_{(i,t)\in I\times T}$, and a set of prices including equilibrium price, interest rates and wages $\{P_t, R_t, W_t\}_{t\in T}$, such that the following are true:

1. Household: maximize (1) subject to (2) and (3),
2. Firms: solve the problem described from (16) to (18),
3. Equilibrium price P_t satisfies (6),
4. Monetary Policy: $\{M_t\}_{t\in T}$ satisfies the monetary policy rule described in (7),
5. Both goods market and labour market clears in every time period $t \in T$:

   $C_{i,t} = Y_{i,t},\ L_t = \int_0^1 L_{i,t}^d di.$

3 Theoretical Results

In this section, I study the optimal attention allocation of firms under different financial status as well as the equilibrium price response to money supply shock. By assuming the particular information set described in the previous section, I can obtain an analytical solution regarding firms' attention allocation and price responsiveness.

3.1 Attention Allocation

3.1.1 Constrained Firms

We first illustrate the optimal attention allocation problem of constrained firms. Recall firm's problem described in (19)-(21), which depends on the equilibrium dynamics of the desired price. The unique solution for the attention allocated to money supply shock is given by

$$\kappa^*_{M,C} = \begin{cases} 0 & \text{if } \xi_C \sigma_r \in (0, 2^{-\kappa}] \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2(\xi_C^2 \sigma_r^2) & \text{if } \xi_C \sigma_r \in (2^{-\kappa}, 2^{\kappa}] \\ \kappa & \text{if } \xi_C \sigma_r \in (2^{\kappa}, \infty) \end{cases} \tag{26}$$

where $\xi_C \equiv 1 + (\psi - 1)h_t^t$ denotes the relative importance of aggregate shock to idiosyncratic shock for financially constrained firms, and $\sigma_r^2 \equiv \frac{\sigma_M^2}{\sigma_A^2}$ denotes the relative uncertainty. I present details of the solution in Appendix A.2. The attention of constrained firms allocated to aggregate shock is weakly increasing both in its importance and its volatility relative to idiosyncratic shocks. The intuition behind is straightforward, when aggregate condition is more volatile than idiosyncratic condition, firm's decision maker will shift more attention to aggregate condition since the marginal benefit is higher. Likewise, if firm's desired optimal decision (price setting) is more sensitive to changes in aggregate condition, firm manager will pay more attention to it. Additionally, recall (22), $h_t^t \geq 0$ denotes the equilibrium response of aggregate price to nominal demand shock. Hence, constrained firms' attention to aggregate uncertainty is weakly increasing with equilibrium price's response to aggregate uncertainty, on account of strategic complementarity.

3.1.2 Unconstrained Firms

The optimal attention allocation problem of unconstrained firms is relatively simpler than that of constrained firms due to the constant return to scale (CRS) production function. The unique solution for the attention allocated to money supply shock is given by

$$\kappa^*_{M,U} = \begin{cases} 0 & \text{if } \sigma_r \in (0, 2^{-\kappa}] \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2(\sigma_r^2) & \text{if } \sigma_r \in (2^{-\kappa}, 2^{\kappa}] \\ \kappa & \text{if } \sigma_r \in (2^{\kappa}, \infty) \end{cases} \tag{27}$$

The interpretation of unconstrained firms' optimal attention allocation is analogous to those of constrained firms apart from their identical sensitivity to aggregate demand shock and idiosyncratic productivity shock, which is due to the non-strategic complementarity generated by CRS production function. Consequently, unconstrained firms will not increase their attention to aggregated condition, even observing equilibrium price moves with aggregate shock.

Proposition 1 *Financially constrained firms allocate weakly more attention to aggregate conditions than what financially unconstrained firms do:* $\kappa_{M,C} \geq \kappa_{M,U}, \forall \sigma_r \in (0, \infty)$

Proof. See Appendix A.A.1 ■

This proposition directly follows from the optimal solution (26) and (27) of constrained and unconstrained firms, respectively. The intuition behind this proposition is that once becoming financially constrained, firms cannot adopt their optimal inputs combination. If aggregate shock is stable enough, no firm will be attentive to aggregate condition. Nonetheless, once the relative volatility reaches a certain threshold, due to strategic complementarity, constrained firms encounter higher nominal marginal cost which makes them more sensitive to aggregate conditions relative to idiosyncratic condition, compared to unconstrained firms. Therefore, as the relative uncertainty grows, constrained firms will shift their attention towards aggregate condition at a higher speed than unconstrained firms until they reach their limit of information processing capacity. This theoretical result is consistent with Coibion et al. (2018), which find that smaller firms (a proxy of constrained firms) pay more attention to inflation rate and output gap.

3.2 Equilibrium Price Response and Firms' Behaviour

Characterizing the equilibrium and optimal action of households and firms delivers the necessary tools to study one of my primary purposes which is how the equilibrium price response to aggregate nominal demand shock. Keep in mind that through this section, the economy was assumed to be in steady state in previous periods. Hence, the price response I study in this subsection is the contemporaneous response in the period when shock take place, i.e. on-impact price response. Before deriving the equilibrium price, I make a further assumption which has nothing to do with the key results but simplify the calibration process.

According to the aggregate price index, integrate price (24) over all i yields

$$p_t = \left[\omega_C \phi \xi_{M,C}(1 - 2^{-2\kappa^*_{M,C}}) + \omega_U (1-\phi)(1 - 2^{-2\kappa^*_{M,U}})\right] m_t, \tag{28}$$

where $\omega_C = \left(\frac{P^C}{P}\right)^{1-\nu}$, $\omega_U = \left(\frac{P^U}{P}\right)^{1-\nu}$, denote the steady state price ratio of constrained and unconstrained firms to aggregate price index, respectively. In the following I will refer these ratios as each group of firms' 'driving force' in shifting equilibrium price deviation. [22]

Lemma 2 $\phi\omega_C + (1-\phi)\omega_U = 1$, *and* $\omega_C < \omega_U$.

Proof. See Appendix A. ■

The aggregation of $p_{i,t}$ requires an additional condition, which is the number of constrained and unconstrained firms are both sufficiently large so that the idiosyncratic shocks average

[22] The introduction of this assumption is just to simplify the functional form of the equilibrium price representation from $p_t = \int_0^1 \omega_i \xi_{M,i}(1 - 2^{-2\kappa^*_{M,i}}) m_t$ to the aforementioned form, which does not affect the analytical results. To link two-level productivity case to continuous-level productivity case, one can view ω_C as the average 'driving force' of all firms with $A_i \leq \bar{A}$.

out within each group when prices are aggregated, i.e., $\int_0^\phi \eta_{i,t}^{M,C} di = \int_0^\phi \eta_{i,t}^{A,C} di = \int_\phi^1 \eta_{i,t}^{M,U} di = \int_\phi^1 \eta_{i,t}^{A,U} di = 0$[23]. Therefore, the equilibrium price level under rational inattention can be solved from a fixed point problem of the mapping between (28) and conjecture (22).

Proposition 2 *There exists a stationary equilibrium where the equilibrium price p_t response to current aggregate demand shock m_t in the following way*

$$h := h_t^t = \begin{cases} 0 & \text{if } \sigma_r \in \left(0,\ 2^{-\kappa}\right], \\ 1 - 2^{-\kappa}\sigma_r^{-1} & \text{if } \sigma_r \in \left(2^{-\kappa},\ \Lambda_\sigma\right], \\ \dfrac{\omega_C\phi(1-2^{-2\kappa}) + \omega_U(1-\phi)\psi(1-2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})} & \text{if } \sigma_r \in (\Lambda_\sigma,\ 2^\kappa], \\ \dfrac{[\omega_C\phi + \omega_U(1-\phi)\psi](1-2^{-2\kappa})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})} & \text{if } \sigma_r \in (2^\kappa, \infty). \end{cases}$$

where

$$\Lambda_\sigma = \frac{2^{-\kappa}(\psi-1) + 2^\kappa}{\psi}.$$

Proof. See Appendix A. ■

Note that the four stages of equilibrium price response are essentially characterized using the combination of firms' attentive behaviour. If $\sigma_r \in (0,\ 2^{-\kappa}]$, neither constrained firms nor unconstrained firms care about aggregate shock. If $\sigma_r \in (2^{-\kappa},\ \Lambda_\sigma]$, both groups of firms are at their interior optimal attention allocation which implies non-zero attention to both aggregate shock and idiosyncratic shock. If $\sigma_r \in (\Lambda_\sigma,\ 2^\kappa]$, constrained firms have allocated all their attention to nominal shock, whereas unconstrained firms are still in their optimal interior allocation. As constrained firms pay weakly more attention to aggregate nominal shock, they will reach their processing limit earlier than unconstrained firms as σ_r increase. That is to say, Λ_σ denotes the threshold of relative uncertainty when constrained firms shift all their attention to monetary

[23] See Appendix for more details

policy shock. If $\sigma_r \in (2^\kappa, \infty)$, both groups of firms allocate all their resources into processing information about aggregate nominal shock.

3.2.1 Firms' Attention Allocation in Equilibrium

The implications of the response of equilibrium price level to monetary shock are abundant after substituting the results of **Proposition 2** into Equation (26) to determine the equilibrium attention allocation of constrained firms. First, there exists feedback effects for constrained firms, as their optimal prices are strategic and complement the aggregate price level.[24] If the volatility of nominal shock is sufficiently large when unconstrained firms start to shift their attention towards aggregate shock, then constrained firms will pay even more attention to aggregate condition.

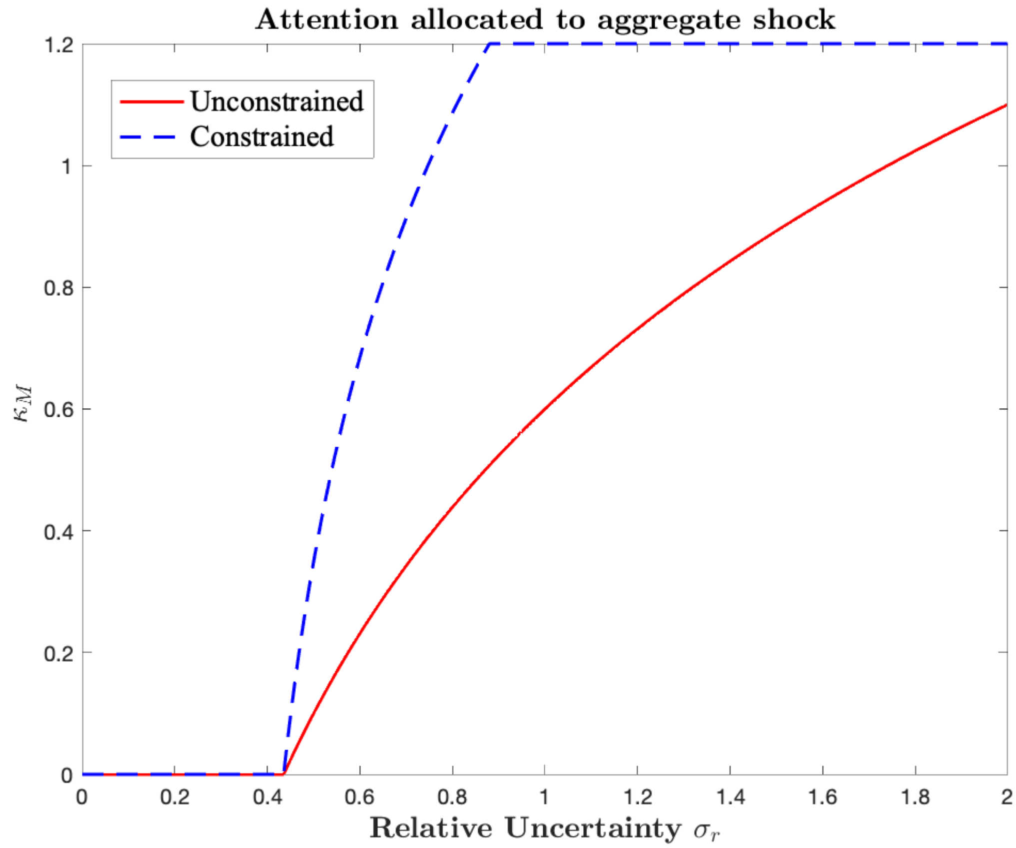
Lemma 3 *In equilibrium, constrained firms will pay positive attention to aggregate shock if and only if unconstrained firms pay positive attention to aggregate shock,* $\kappa_{M,C} > 0 \iff \kappa_{M,U} > 0, \forall \sigma_r \in (0, \infty)$.

Proof. See Appendix A. ■

Second, as shown in **Lemma 3**, in equilibrium, there exist no circumstances whereby only constrained firms pay attention to aggregate shock. Recall the optimal pricing decision (23), and unconstrained firms are equally sensitive to current monetary policy and idiosyncratic productivity shocks.However, regarding constrained firms, the only motivation to devote relatively more attention to tracking monetary shock other than idiosyncratic shock comes from the strategic complementarity in their pricing decisions. If the relative volatility σ_r is low enough so that unconstrained firms are entirely careless about m_t, then constrained firms will find no incentive to track it since aggregate price does not responding to m_t. Figure 1 illustrates these results by showing how each group of firms' attention allocated to aggregate shock changes with relative uncertainty σ_r. If σ_r reaches $2^{-\kappa}$, both constrained and unconstrained firms start

[24] See, Woodford (2002), Mackowiak and Wiederholt (2009) and Acharya (2017) for more discussion about the strategic complementarity of pricing decisions.

Figure 1: Firms' attention allocation



NOTE: This figure illustrates how firm's attention allocated to aggregate shock changes with the relative standard deviation between aggregate shock and idiosyncratic shock. The red solid line is for unconstrained firms, the blue dashed line is for constrained firms.

to devote a positive amount of attention to monetary shock simultaneously. Once both types of firms start tracking, as relative uncertainty increases, constrained firms continue to shift their attention to m_t faster than unconstrained firms, due to strategic complementarity. Nonetheless, if I assume identical information processing capacity across all firms, constrained firms reach their limit κ with a lower σ_r than unconstrained ones.

3.2.2 Price Response in Equilibrium

Recall that the derived aggregate price response h_t^t essentially refers to the current price response to current aggregate shock. If the economy was not in its steady state previously, since the nominal aggregate demand is log-AR(1) process, then aggregate price p_t should also respond to previous aggregate conditions $m_{t-\tau}, \ \forall\tau \in (1, \infty)$, as conjectured in equation (22). After solving a fixed point problem, the dynamic equilibrium price response at any arbitrary period to the correlated aggregate demand is illustrated in the following proposition

Proposition 3 *Suppose that the first period after steady state when aggregate shock occur is denoted as $t = 1$, then the response of equilibrium price to nominal aggregate demand at any following period t is*

$$p_t = \sum_{\tau=0}^{t-1} h^t_{t-\tau} m_{t-\tau} \tag{29}$$

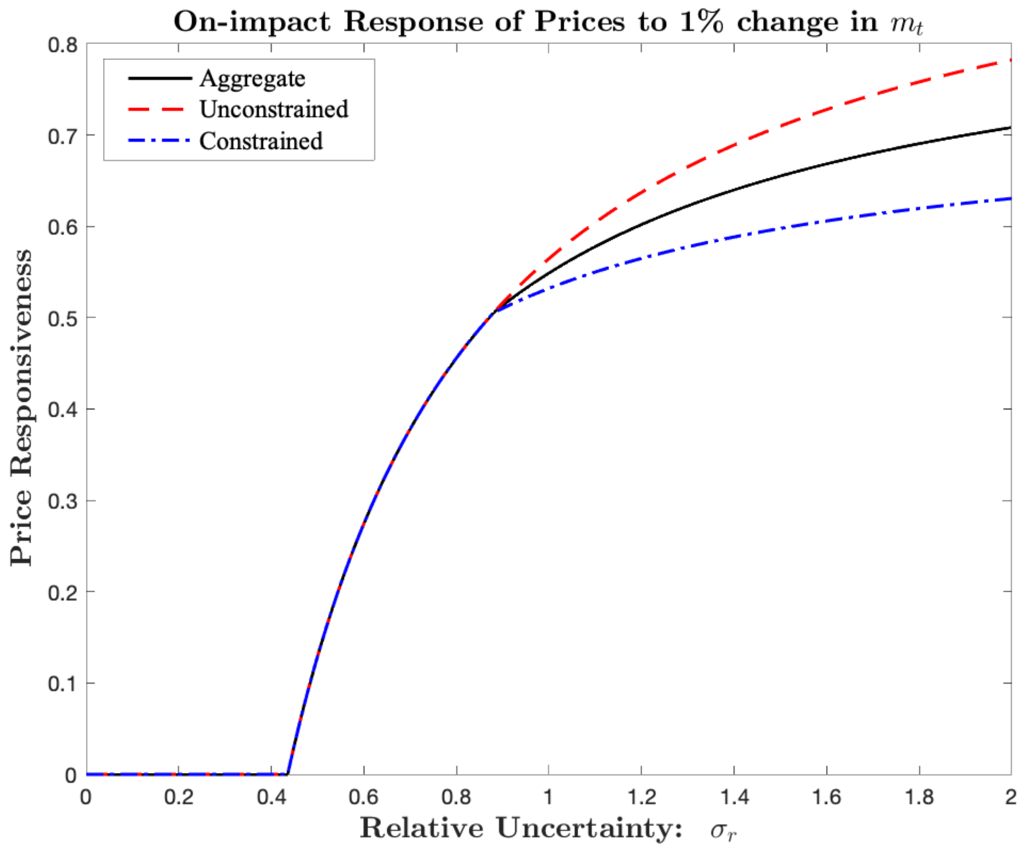
where $h^t_{t-\tau}$ is the period t equilibrium price response to shock $m_{t-\tau}$ and equal to

$$h^t_{t-\tau} = \begin{cases} h & \text{if } \tau = 0 \\ \dfrac{\psi(1-\omega_C\phi)(\alpha - 1 + (1-\alpha)h)}{\psi - \omega_C\phi(\psi - 1)} & \text{if } \tau = 1 \\ \dfrac{\psi(1-\omega_C)(1-\alpha)h^{t-1}_{t-\tau}}{\psi - \omega_C\phi(\psi-1)} & \text{if } \tau > 1 \end{cases} \tag{30}$$

From the previous proposition, it is clear that rational inattention is directly affects the on-impact response of the aggregate price to the current aggregate shock, i.e., h, and indirectly affects the price response to realised shocks, $h^t_{t-\tau}$, through h. In fact, price response mimics an infinite order moving average process by responding to both current and previous aggregate conditions. Whereas, firms' attention allocation behaviour is static, no matter whether the economy was previously in a steady state or not.

To analyse the dynamic price response, it is crucial to understand the instantaneous price response. I can decompose the instantaneous response to aggregate shock, h, into the response of two groups of firms. Unlike the distinction of firms' attention allocated to monetary shock, firms show partially ambiguous distinction in their actual instantaneous price response to monetary shock. As shown in Figure 2, the responsiveness of equilibrium price and each kind of firms' price are all increasing in the relative uncertainty. Even though constrained firms are paying weakly more attention to m_t, their price response to current m_t could be lower than that of unconstrained firms if σ_r is sufficiently high. The mechanism stems from the real rigidity gen-

Figure 2: Response of Prices to Current Aggregate Demand Shock



NOTE: This figure illustrates the response of prices to a 1% monetary shock when $\omega_C = 0.8$. The black line is for aggregate price, the red dashed line is for price of unconstrained firms and the green dashed line is for price of constrained firms.

erated by financial friction. Given this real rigidity, constrained firms cannot freely adjust their capital to produce, and hence they cannot reach their optimal input choice as unconstrained firms can. Consequently, for any $\sigma_r \in (2^{-\kappa}, \Lambda_\sigma)$, constrained firms' advantage from their more precise signals about current aggregate shock is used fully to offset the sluggish responsiveness caused by financial friction. As long as both categories of firms are within their optimal interior allocation, their prices respond to current aggregate shock identically. Once σ_r exceeds the threshold where constrained firms consume all their capacity in tracking aggregate conditions, then constrained firms' prices become less responsive to aggregate shock compared to unconstrained firms, because they no longer have superiority in information to offset the effect of real rigidity. It is worth noting that this difference in responsiveness shrinks when ω_C decrease.

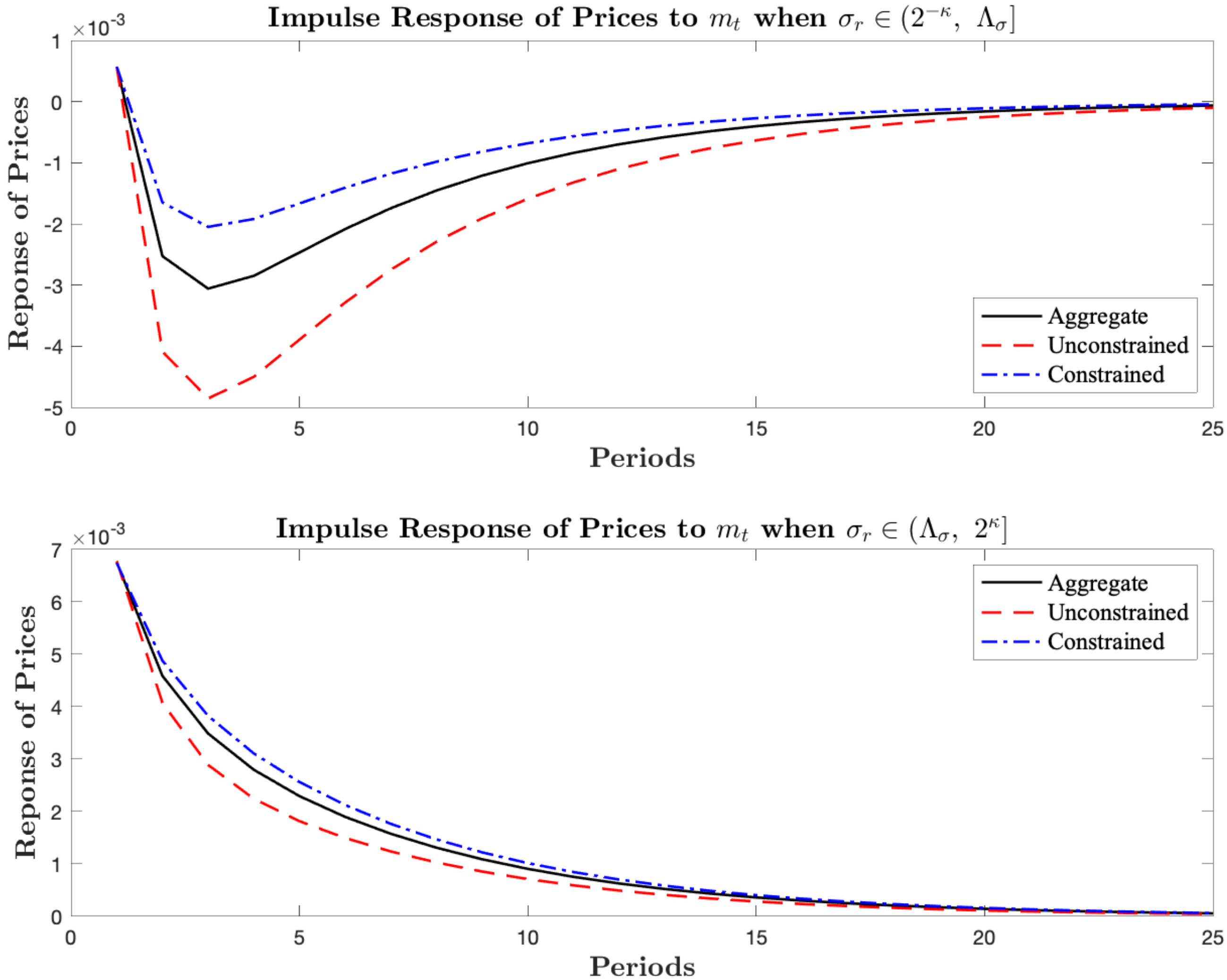
Additionally, the distance of responsiveness keeps increasing until unconstrained firms reach their information processing limit.

The dynamic price response also takes into account the response to previous aggregate nominal demand as well, which facilitates analysing the implications for monetary non-neutrality. Figure 3 illustrates the impulse response of equilibrium price to aggregate shocks of one standard deviation in period $t = 1$ with $\sigma_r \in (2^{-\kappa}, \Lambda_\sigma]$ and $\sigma_r \in (\Lambda_\sigma, 2^\kappa]$, respectively. The two impulse response differ dramatically. When $\sigma_r \in (2^{-\kappa}, \Lambda_\sigma]$, constrained firms and unconstrained firms allocate only part of their total capacity to aggregate shock; consequently, neither category of firm can fully adjust their price to accommodate aggregate nominal demand change which leads to a sluggish equilibrium price. In the following period, when the previous shock is revealed, unconstrained firms have incentive to decrease their prices because M_{t-1} will reduce the capital rental rate R_t^K. However, since P_{t-1} does not compensate for such an effect, unconstrained firms find it optimal to reduce current prices, and the aggregate price goes down. When $\sigma_r \in (\Lambda_\sigma, 2^\kappa]$, constrained firms put all their capacity into m_t and unconstrained firms allocate only partial capacity to m_t. The equilibrium price p_{t-1} is responsive enough to accommodate the change in R_t^K caused by M_{t-1}. As a result, prices will not drop after a positive money supply shock.

Regarding the distinction in price response between constrained firms and unconstrained firms, it is noteworthy that constrained firms respond to previous shocks only through a response to equilibrium price. In comparison, aggregate price responds to a previous shock simply because unconstrained firms directly respond to it. Consequently, the response of constrained firms to shocks in previous periods is not as strong as that of unconstrained firms, and the damping effect of a previous effect on optimal price is larger for unconstrained firms. There is an exception when $\sigma_r > \Lambda_\sigma$ as constrained firms have consumed all their capacity in tracking aggregate capacity and can no longer compensate for the sluggish price responsiveness caused by financial friction. Therefore, unconstrained firms will respond more to aggregate shock in the early periods after shocks occur, see lower panel of 3.

In Section 5 with the generalized information set with which firms do not observe realized

Figure 3: Response of Equilibrium Price to Aggregate Demand Shock



NOTE: This figure illustrates the impulse response of aggregate price after a 1% monetary shock when $\sigma_r \in (2^{-\kappa}, \Lambda_\sigma]$ (upper panel) and $\sigma_r \in (\Lambda_\sigma, 2^{\kappa}]$ (lower panel). The red dash line is for the price response of unconstrained firm, the black solid line is for the aggregate price response, the blue point-dashed line is for the price response of constrained firm.

shocks since constrained firms find it optimal to allocate more attention to aggregate shock, which will help to overcome a sluggish response due to real rigidity, they will be even more responsive to aggregate shock. Hence, even the instantaneous response is more extensive than that of unconstrained firms.

3.2.3 Monetary Non-neutrality Analysis

The price response to aggregate demand shock depicted in **Proposition 2** and **Proposition 3** allow us to analyse how monetary non-neutrality will be affected by different variables.

Proposition 4 *For any interior solution of either category of firms, monetary non-neutrality is strictly decreasing in the relative uncertainty:* $\frac{\partial h^t_{t-\tau}}{\partial \sigma_r} > 0,\ \forall \sigma_r \in (2^{-\kappa}, 2^{\kappa}),\ \tau \in (0, \infty)$

Proof. See Appendix A. ■

This proposition demonstrates the fundamental spirit of rational inattention theory. The marginal benefit of being attentive to aggregate shock is larger when its volatility increases. Hence, irrespective of their financial status, firms will allocate more resources in tracking aggregate shock, and equilibrium price responsiveness will increase if relative volatility σ_r rise.[25]

Proposition 5 *If either category of firms are paying non-zero attention to aggregate shock, monetary non-neutrality is strictly increasing in the degree of real rigidity* ψ*:* $\frac{\partial h^t_{t-\tau}}{\partial \psi} < 0,\ \forall \sigma_r \in (2^{-\kappa}, \infty),\ \tau \in [0, \infty)$.

Proof. See Appendix A. ■

It is worth noting that although the instantaneous price response h is independent of ψ for $\sigma_r \in (2^{-\kappa}, \Lambda_\sigma)$, smaller real rigidity can postpone the starting point where all firms start to pay positive attention to aggregate shock, i.e., $\frac{\partial \Lambda_\sigma}{\partial \psi} < 0$. Hence, higher real rigidity implies higher non-neutrality of money for interior solutions of firm attention.

Proposition 6 *For any interior solution of both categories of firms, monetary non-neutrality is weakly decreasing in the fraction of constrained firms,* $\frac{\partial h^t_{t-\tau}}{\partial \phi} \leq 0, \forall \sigma_r \in (0, \Lambda_\sigma],\ \tau \in [0, \infty)$,

- *If* $\tau = 0$*, price response is independent of* ϕ*,* $\frac{\partial h}{\partial \phi} = 0$
- *If* $\tau > 0$*, price response is strictly increasing in* ϕ*,* $\frac{\partial h^t_{t-\tau}}{\partial \phi} > 0$

[25] See Sims (2003) and Mackowiak and Wiederholt (2009) for more details.

Proof. See Appendix A. ■

Knowing firms are showing heterogeneous responsiveness to aggregate shock, the fraction of constrained firms ϕ should have a substantial effect on monetary non-neutrality. This proposition illustrates the core implication of this model: for an economy populated by both constrained firms and unconstrained firms when $\sigma_r \in (0, \Lambda_\sigma]$, the instantaneous equilibrium price response when aggregate occurs is not affected by the fraction of constrained firms, which resembles the equilibrium response of an economy populated only with inattentive homogeneous firms that produce differentiated goods. Though constrained firms are relatively less responsive due to real rigidity, their advantageous attention allocated to aggregate shock help them in responding to aggregate shock. As a result, constrained firms can maintain the same responsiveness as unconstrained firms in the first period of shock. Afterwards, in the following periods, the aggregate price response will increase with the fraction of constrained firms since the previous shock only partially dampens constrained firms' price. The key point is that when $\phi = 1$, aggregate price is always positive since constrained firms' price response is not subject to previous aggregate nominal demand. If the volatility of monetary shock is sufficiently large that $\sigma_r \in (\Lambda_\sigma, \infty)$, as σ_r increases, constrained firms can no longer allocate additional attention to aggregate shock so as to offset the effect of real rigidity. Therefore, the higher fraction of constrained firms, the less responsive is the instantaneous equilibrium price to aggregate shock.

3.2.4 Comparison with Perfect Information Model

In an economy with only financial friction but all firms receive perfect information about both aggregate and idiosyncratic shocks, unconstrained firms' price deviation will be a one to one mapping to the deviation of nominal aggregate deviation, i.e., money is neutral for unconstrained firms. Recall that price response of constrained firms to nominal shock are partially determined by the aggregate response (strategic complementarity), and partially determined by the same responsiveness as it of unconstrained firms. Therefore, the equilibrium price responsiveness is equal to 1 given the production function (8), and not vary no matter how the fraction of constrained firms changes. Money is entirely neutral for an economy with only financial friction but absent from information friction, and the fraction of constrained firms do not affect the aggregate price response.

Whereas, under rational inattention setting when the relative uncertainty σ_r is sufficiently large, the fraction of constrained firms have an ambiguous impact on monetary non-neutrality through the combined effect of strategic complementarity, real rigidity and capital rental rate. If σ_r is so low that constrained firms allocate non-zero attention to both aggregate and idiosyncratic shock, then monetary non-neutrality is always decreasing with ϕ. Whereas if σ_r is high enough that constrained firm put all capacity in analysing aggregate condition, then the instantaneous price response is decreasing in ϕ, but long-run price response will be increasing in ϕ. These results are illustrated in Figure 3.

3.2.5 Comparison with Financial Frictionless Model

Now consider an economy without financial friction, i.e., only populated with unconstrained firms. The instantaneous equilibrium price response h will be as follows,

$$h = \begin{cases} 0 & \text{if } \sigma_r \in \left(0,\ 2^{-\kappa}\right], \\ 1 - 2^{-\kappa}\sigma_r^{-1} & \text{if } \sigma_r \in \left(2^{-\kappa},\ 2^{\kappa}\right], \\ 1 - 2^{-2\kappa} & \text{if } \sigma_r \in (2^{\kappa}, \infty). \end{cases}$$

Compared with what Proposition 2 illustrates, the aggregate price in a frictionless financial economy is categorized into three stages by a different value of σ_r instead of four stages. For $\sigma_r \in (0, \Lambda_\sigma]$, the two economy are identical in terms of aggregate price response. Whereas, the third stage, i.e., $\sigma_r \in (\Lambda_\sigma, 2^{\kappa})$ depict the key difference, which is that the real rigidity amplifies monetary non-neutrality as a consequence of financial friction. Specifically, in the third stage, unconstrained firms' price response is still identical to the frictionless financial economy. However, since constrained firms have already devoted all their information processing capacity in tracking aggregate shock, they cannot offset the sluggish response due to real rigidity by shifting more attention to aggregate shock.

4 Relation Between Model Predictions and Empirical Regularities

This section provides recent empirical evidence to test the crucial theoretical mechanism and predictions of the model. To do so, I use both qualitative microdata from the German manufacturing subset of the IFO Business Expectation Panel (BEP) as well as quantitative data from a survey of firms from in New Zealand, which is designed and conducted by Coibion et al. (2018). This paper contributes empirically to the literature that has used the same datasets in the following ways: (1) it documents the strong negative correlation between firms' financing difficulties and their size, (2) it documents that smaller firms are more influenced by the economic policy in general, (3) it further verified the stylized fact in Coibion et al. (2018) that larger firms are less attentive to inflation and its substantial correlated variables: unemployment rate and output gap.

4.1 Firm Size and Financial Condition

Since New Zealand survey data collected by Coibion et al. (2018) is the lack of firms' financial information, I need to find a good proxy for the financial condition of firms. In the model studied previously, firm size and its financial constraint status is a one-to-one mapping, which is in line with Gertler and Gilchrist (1994) who consider firm size as a reasonable proxy for capital market access. I test this assumption again with the BEP dataset, which contains questions to measure this assumption directly. Besides, the quality and validity of BEP dataset have been widely exploited by the literature, see, Bachmann and Elstner (2015) and Ehrmann (2005). The first informative question asks firms whether their domestic production is currently constrained. Conditional on this question, firms are then asked if their production are constrained by difficulties of financing. Table 1 reports the results of regressing the dummy variable firms employment using OLS and Logit regression. The effect of firm size in reducing the difficulties of financing is significant and robust after including other controls and fixed effects.

Table 1: Firm Size, Firm age and Financing Difficulty

	(1)	(2)	(3)	(4)	(5)
log(employment)	-0.00815***	-0.00329**	-0.00837***	-0.00286	-0.424***
	(0.00124)	(0.00146)	(0.00164)	(0.00194)	(0.0454)
Constrain	-0.0798***	-0.0760***	-0.0759***	-0.0727***	-7.415***
	(0.00312)	(0.00121)	(0.00364)	(0.00140)	(0.414)
Firm age			-0.0000943**	0.000173	-0.00371***
			(0.0000417)	(0.000157)	(0.00128)
Industry FE	Yes	No	Yes	No	Yes
Firm FE	No	Yes	No	Yes	No
Constant	0.229***	0.181***	0.231***	0.159***	6.067***
	(0.0127)	(0.00776)	(0.0151)	(0.0151)	(0.572)
Observations	123341	123360	86913	86915	86913
R^2	0.0693	0.0683	0.0647	0.0548	

Notes: The dependent variable is a binary variable that equal to 1 if a firm reports its domestic production activities are currently constrained by difficulties in financing, and 0 otherwise. Variable Constrain is a binary variable that equals to 1 if firm reports its domestic production activities are currently constrained, and 2 otherwise. Column (1) to column (4) report the estimates using OLS, and column (5) reports the estimates using Logit regression. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

This stylized fact is not unique to Germany. Using a cross-country survey, Beck, Demirgüç-Kunt, and Maksimovic (2005) document that smaller firms report to face significant more financing obstacles.[26] Though, lack of financial data of firms in New Zealand survey, we can conclude that firm size could be considered as a reasonable proxy variable for firms' financial situation.

[26]Rich literature in corporate finance has also been studying the correlation between firm size and financial condition. For example, Ratti et al. (2008) document that large firms are less credit constrained than small firms using data of non-financial firms in 14 European countries. Hadlock and Pierce (2010) use qualitative information from financial filings to propose a new measure of financial constraints and argue that firm size and age are particularly useful predictors of financial constraint level.

4.2 Firm Size and Knowledge about Aggregate Information

The main predictions of the model are that under the rational inattention setting, smaller firms pay weakly more attention to nominal aggregate demand shock than to idiosyncratic productivity shock, see Proposition 1. The fundamental reason behind this is that smaller firms are more affected by aggregate conditions rather than idiosyncratic shock. Using BEP data, I find that smaller firms report being more influenced by economic policy, which rationalizes the fact why those firms pay more attention to macroeconomic conditions, see Table 2.

Table 2: Firm Size, Firm Age and Economic Policy Influence

	(1)	(2)	(3)	(4)
log(employment)	-0.0275***	-0.0204***	-0.0276***	-0.0204***
	(0.00725)	(0.00671)	(0.00744)	(0.00788)
log(investment)	-0.0129***	-0.0135***	-0.0129***	-0.0136***
	(0.00472)	(0.00396)	(0.00477)	(0.00492)
Firm age			-0.000215*	-0.000202*
			(0.000121)	(0.000122)
West/East			-0.0156	-0.0108
			(0.0211)	(0.0216)
Industry FE	No	Yes	No	Yes
Constant	0.789***	0.880***	0.820***	0.898***
	(0.0270)	(0.0405)	(0.0419)	(0.0545)
Observations	29763	29763	29086	29086
R^2	0.0114	0.0187	0.0108	0.0179

Notes: The dependent variable is the adjusted response to the following question: "our investment activity is influenced positively/negatively by economic policy in general". The survey answer is 1 for "strong inducement", 2 for "slight inducement", 3 for "no influence" , 4 for "slight negative influence" and 5 for "strong negative influence". So as to capture the magnitude of influence, the variable value equal to 3 for "strong negative influence" and "strong inducement", 2 for "slight negative influence" and "slight inducement", and 0 for no influence. Variable "West/East" is a dummy variable that equals to 1 if firm is from west Germany and 0 otherwise. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Regarding direct measurement of inattention, I use the New Zealand survey, whose details have been comprehensively discussed in Kumar et al. (2015) and Coibion et al. (2018). The survey was conducted among a random sample of firms in New Zealand with broad sectoral coverage. So far, six waves of the survey have been completed over the time frame of September 2013 to July 2016. With this survey, Coibion et al. (2018), find that smaller firms make significant smaller errors when asked to recall the inflation level in the preceding twelve months.

To further verify the empirical facts, I use unemployment and output gap as dependent variables and conduct the same empirical estimation regarding firms' inattention and size. The survey asked firms for their beliefs about what *"the unemployment rate currently is in New Zealand"* and *"By how much higher or lower than normal do you think the current level of overall economic activity is"*. In light of Coibion et al. (2018)'s empirical strategies, I construct the "errors" made by firms concerning the two macroeconomic variables by subtracting their reported beliefs from the actual level and estimate the following regressions (using unemployment for example):

$$|unemp_t - B_t^i(unemp_t)| = \beta_0 + \beta_1 L_i + \mu X_i + \epsilon_i$$

where $unemp_t$ denotes the actual current unemployment rate and $B_t^i(unemp_t)$ denotes firm i's belief about the current unemployment rate. L_i denotes employment, X_i consists of the same set of firms and manager characteristics as in Table 4 of Coibion et al. (2018). In Table 3, column (2) and (3) reports the estimates using unemployment error and output gap error as dependent variables and indicates that larger firms make larger errors about the two variables. The similar patterns of the unemployment rate and inflation can be rationalized by the Phillips curve, which explains the stylized fact between unemployment rate and inflation in historical data. See Figure 10 for the correlation between inflation and unemployment in New Zealand after adopting an inflation targeting rule. Since the New Keynesian Phillips Curve considers output gap instead of unemployment, I find a similar positive effect of firm size on their inattentive level concerning the output gap. Additionally, this feature persists when measures are changed to forecast errors in next year's inflation and unemployment level.

Table 3: Firm Size, Firm age and Financing Difficulty

Variables	(1) Inflation	(2) Unemployment	(3) Output Gap
log(age)	0.11***	0.04*	1.18***
	(0.03)	(0.02)	(0.22)
log(employment)	0.384***	0.06**	3.83***
	(0.06)	(0.03)	(0.24)
Labor share of costs	-0.01	0.01***	0.03*
	(0.00)	(0.00)	(0.02)
Foreign trade share	0.01***	-0.00	0.015**
	(0.00)	(0.00)	(0.01)
Number of competitors	-0.01***	0.01***	-0.04***
	(0.00)	(0.00)	(0.01)
Average margin	0.00	0.03***	0.06***
	(0.00)	(0.00)	(0.02)
Price relative to competitors	0.01	-0.00	0.04***
	(0.00)	(0.01)	(0.02)
Firm's past price changes	-1.17***	0.13	-0.04***
	(0.26)	(0.11)	(0.02)
Industry PPI inflation	-0.01	-0.00	-0.07**
	(0.01)	(0.00)	(0.00)
Expected size of price change	-0.00	0.00	-0.04
	(0.0)	(0.02)	(0.03)
Duration until price change	0.03***	0.00	0.38***
	(0.01)	(0.00)	(0.04)
Absolute slope of profit function	-0.20***	0.00	-1.61***
	(0.04)	(0.02)	(0.21)
Industry FE	Yes	Yes	Yes
Observations	2,912	1,164	3146
R^2	0.799	0.089	0.45

Notes: Column (1) of the table replicates the results of Coibion et al. (2018). Column (2) of the table reports estimates of firms' nowcast absolute error about current unemployment rate. Column (4) reports estimates of firms' nowcast absolute error about output gap. The table reports Huber-robust estimates. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

5 Full Model and Solution

This section presents the quantative results obtained for a dynamic full model. From this section onwards, I relax two assumptions to generalize the model. First, **Assumption** 1 is relaxed to allow signal formation other than "true state plus white noise error". Second, firms no longer have perfect information about realized shocks, hence, their information set become

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,t-1} \cup \{s_{i,t}\} = \mathcal{I}_{i,-1} \cup \{s_{i,\tau}\}_{\tau=0}^{t}.$$

This form of information set is widely adopted in rational inattention literature, which is consistent with the empirical findings that firms are inattentive to both current and previous shocks. According to the empirical facts discussed in section 4, this information set is more realistic since firms appear to be remarkably uninformed about previous variables like inflation, output growth, etc. Additionally, this full model allows firms acquiring information about past shocks which would be beneficial for future usage. Recall the optimal pricing decision of unconstrained firms

$$p_{i,t}^{*} = (\alpha - 1)\ln m_{t-1} + (1-\alpha)\ln p_{t-1} + \ln m_t - \ln a_{i,t},$$

which indicates that if the value of realized shock M_{t-1} is not included in firm's information set $\mathcal{I}_{i,t}$, then firm might find it optimal to acquire information about previous states which is useful for their pricing decision in the future given that nominal demand shock is serially correlated.

Since any stationary AR(p) process can be represented as MA(∞) process, let the moving average representations of m_t and $a_{i,t}$ be given by $m_t = \sum_{l=0}^{\infty} a_l \varepsilon_{t-l}^{M}$ and $a_{i,t} = \sum_{l=0}^{\infty} b_l \varepsilon_{i,t-l}^{A}$. Therefore, firm's profit loss minimization problem becomes

$$\min_{c,d,f,g} E\left[(p_{i,t} - p_{i,t}^{*})^2\right]$$

subject to the equations for $\hat{m}_{i,t}$ and $\hat{a}_{i,t}$ and information flow constraint

$$\hat{m}_t = \sum_{l=0}^{\infty} c_l \varepsilon_{t-l}^{M} + \sum_{l=0}^{\infty} d_l \eta_{i,t-l}^{A}, \quad \hat{a}_{i,t} = \sum_{l=0}^{\infty} f_l \varepsilon_{i,t-l}^{M} + \sum_{l=0}^{\infty} g_l \eta_{i,t-l}^{A}$$

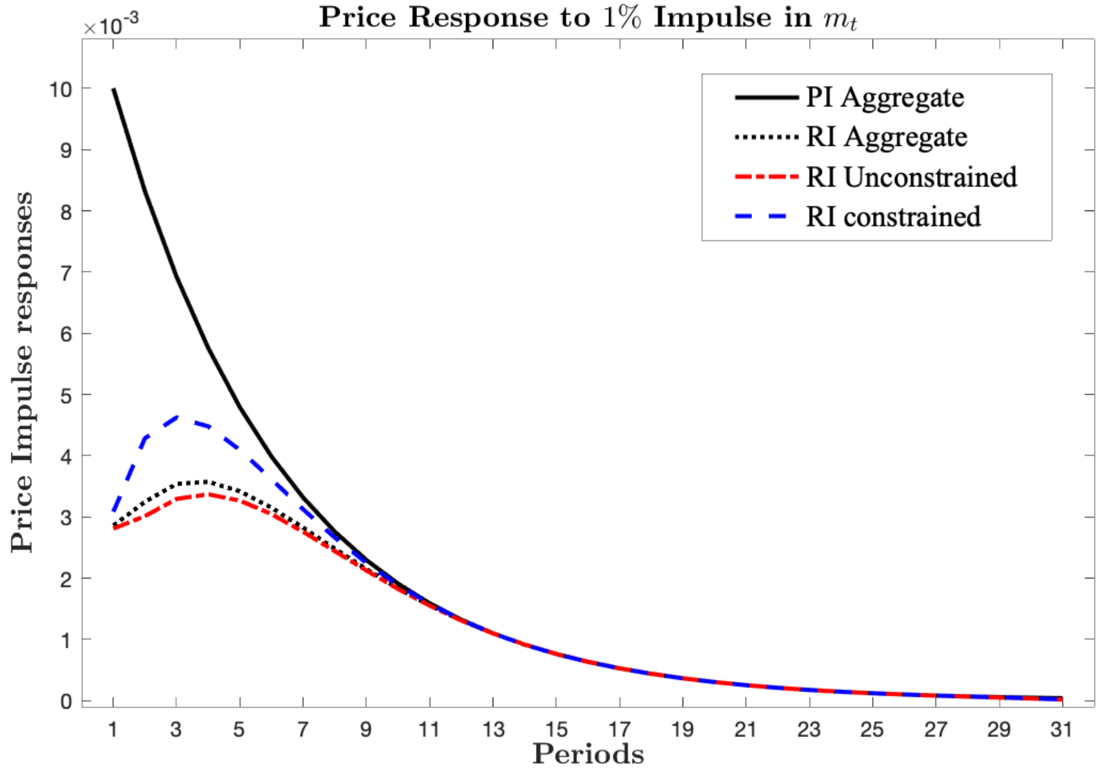
$$\mathcal{I}\big(\{m_t\}, \{\hat{m}_{i,t}\}\big) + \mathcal{I}\big(\{a_{i,t}\}, \{\hat{a}_{i,t}\}\big) \leq \kappa.$$

The optimal attention allocation in this full model could be different from the simple model, given the fact that unconstrained firms might find it optimal to acquire information about previous monetary supply to facilitate their pricing decision. Since solving this full model analytically is more challenging than the simple case due to the fact that information acquisition is now dynamic, I follow the numerical solution in Section 7 of Mackowiak and Wiederholt (2009) to firstly guess the equilibrium price and then solve firms' attention problem in previous equations which also gives firms' best price response. Eventually, I compute the equilibrium price and update the previous guess until a fixed point is reached.

Given certain parameters calibrated to match the U.S. economy (which will be explained in the next section), the solution shows that constrained firms allocated 43% of their attention analysing aggregate monetary shock. In contrast, unconstrained firms allocated only 18% of their total capacity to such shock. This result is in line with the theoretical results stressed in **Proposition 1** when σ_r are set to the calibrated value. In this full model, when firms cannot observe the realised value of previous shocks, they would find the current signal being helpful in the future pricing decision. The mechanism is not only through the serial correlation of shocks as in Mackowiak and Wiederholt (2009) but also because of that unconstrained firms' price depend on previous shock and previous price, which induce constrained firms' pricing decision being dependent on previous shock and price.

Regarding price response, Figure 11 shows the impulse response of constrained firms' price to an 1% innovation in nominal aggregate demand and productivity shock and Figure 12 shows the response of unconstrained firms. Price response for both categories of firms are hump-shaped because unconstrained firms' price change depends on previous aggregate states, and since constrained firms' price is strategic complement, hence also show the hump-shaped pattern. The yellow lines illustrate the firms' response to the noise term, which is decaying exponentially. Figure 4 compares the price response of different groups of firms after 1% innovation in m_t, where constrained firms respond more in their price than what unconstrained firms do until the 12th quarter. The effect of ϕ in shaping monetary non-neutrality is illustrated in Figure 5. As the fraction of constrained firms grow, monetary non-neutrality is decreasing due to the fact that aggregate price becomes more responsive to monetary shock. As previously discussed

Figure 4: Price Response to Aggregate shock



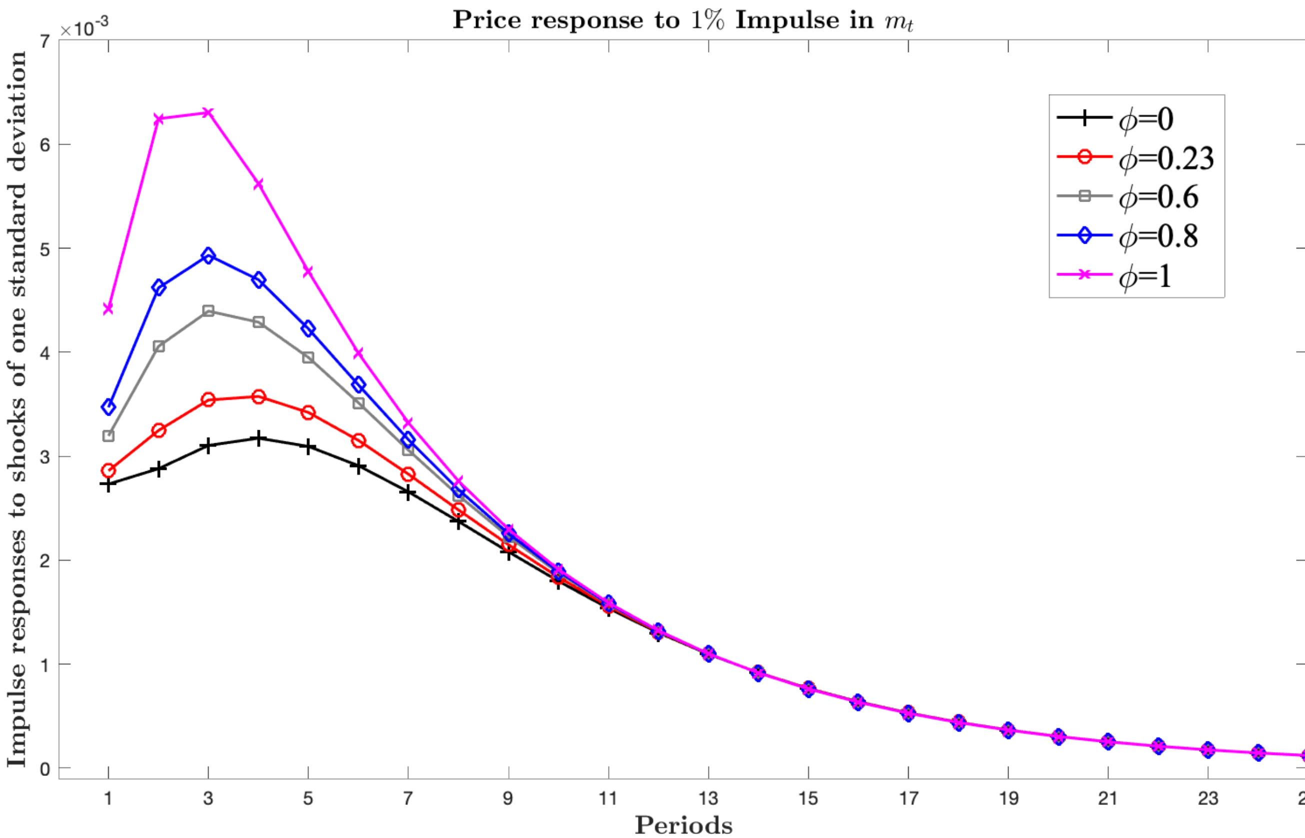
NOTE: This figure illustrates the impulse response of price after a 1% monetary shock with calibrated parameters. The red dash line is for the price response of unconstrained firm, the black dot line is for the aggregate price response, the blue point-dashed line is for the price response of constrained firmthe black solid line is for the price response of aggregate price under perfect information.

in the comparison between different categories of firms, constrained firms adjust faster from period 1 to period 12. Therefore, with the severity of financial friction, aggregate price response is amplified, which induces lower monetary non-neutrality.

6 Effectiveness of Monetary Policy Implications

This rational inattention model, integrated with financial heterogeneity, offers the possibility to analyse monetary non-neutrality with different levels of financial friction. Using calibrated parameters, I further show that this model has important implications for the state-dependent effectiveness of monetary policy and more explaining power for empirical facts compared to the rational inattention model with representative firms. The calibrated parameters are presented

Figure 5: Response of Aggregate Price to Aggregate shock



NOTE: This figure illustrates how the impulse response of price after a 1% monetary shock varies with the fraction of constrained firms. The black line is for the price response when $\phi = 0$, the red line is for the price response when $\phi = 0.23$, the grey line is for the price response when $\phi = 0.6$, the blue line is for the price response when $\phi = 0.8$, the purple line is for the price response when $\phi = 1$

in Table 4.

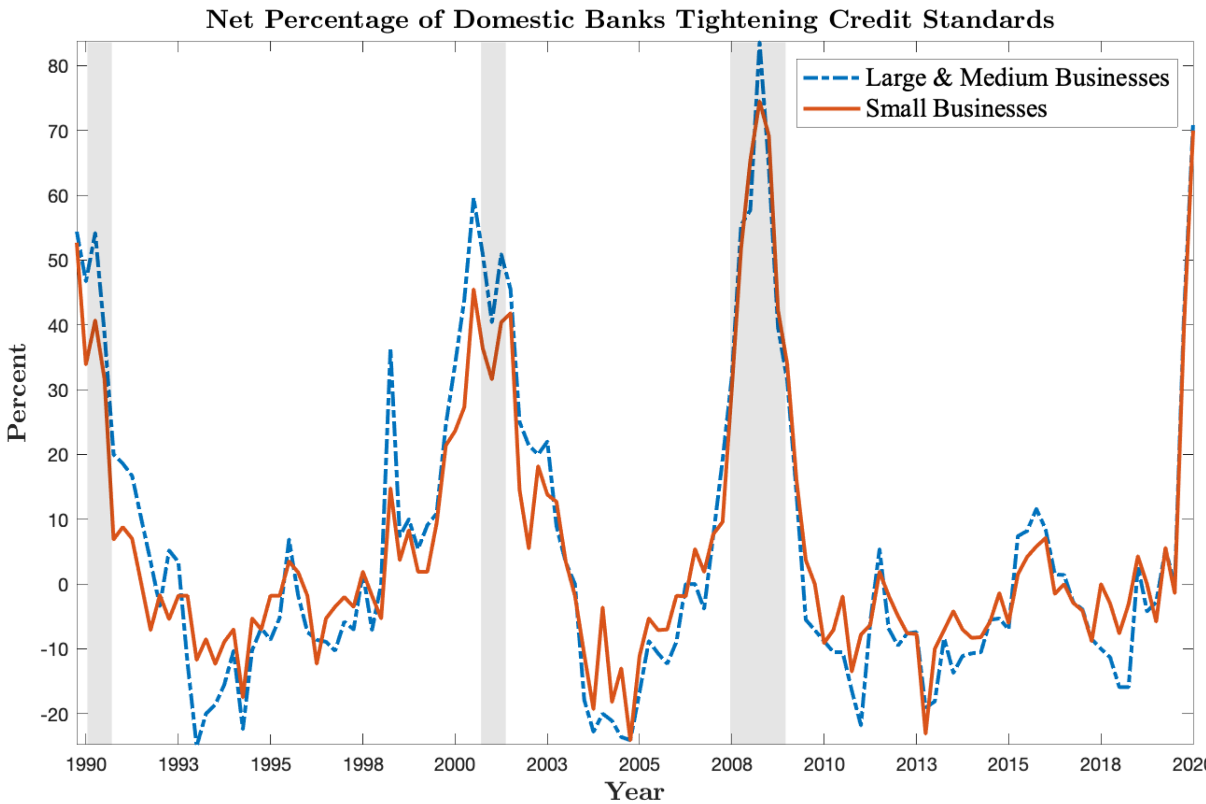
Recent empirical literature, which studies the time-varying effectiveness of monetary policy, has documented that a nominal stimulus is less powerful during a recession than expansion. For example, Tenreyro and Thwaites (2016) investigate how the response of the US economy to monetary policy shocks depends on the state of the business cycle using a local projection method and conclude that shocks to the federal funds rate are more potent in expansions than in recessions. Similarly,Alpanda et al. (2019) find that the impact of monetary policy shocks on output and most other macroeconomic and financial variables is smaller during periods of economic downturns, using data from 18 advanced economies. Vavra (2014) investigates this question in the time-varying volatility channel and documents that monetary policy is less effective in increasing real output during periods of high volatility than during regular times.

Nonetheless, the fundamental rational inattention model presented in Sims (2003) and Mackowiak and Wiederholt (2009) cannot fully capture the state-dependent effectiveness of monetary policy after allowing for time-varying volatility. As documented in Bloom (2014), Vavra (2014), Bloom et al. (2018) and Baker et al. (2016), uncertainties, either aggregate or idiosyncratic, are mostly counter-cyclical. Regarding aggregate uncertainty, Baker er al (2016) point out that the VIX index rises by 58% on average during recessions. This trend does not deviate greatly once changed to economic policy uncertainty, which shows a 51% increase during recessions. Notwithstanding, the idiosyncratic uncertainty shows more dramatic differences during the business cycle. Bloom et al. (2018) find that the variance of plants sales growth rates rose by a massive 152 percent during the Great Recession. Besides, the calibrated value of idiosyncratic productivity in Bloom et al. (2018) rises by 3.33 times in recession, whereas the aggregate productivity is only 1.92 times more volatile in recession state. Given that idiosyncratic productivity shock is more volatile compared to aggregate monetary policy, the rational inattention model with representative firms should predict that firms allocate less attention to monetary policy shock which will consequently lead to a greater real effect, i.e., more effective monetary policy during recessions. Obviously, this theoretical implication, due to relative uncertainty change, contradicts with the empirical findings illustrated in the last paragraph.

This rational inattention model integrated with financial heterogeneity is capable of resolving this empirical puzzle as the composition effect is also important for equilibrium price response. During recessions, two channels will drive the effectiveness of monetary policy in different directions. On the one hand, as discussed in **Proposition** 4, the decrease in relative uncertainty $\sigma_r = \frac{\sigma_M}{\sigma_A}$ will enhance the real effect of monetary policy shock.

On the other hand, as discussed in **Proposition** 4, monetary non-neutrality will decrease with the fraction of constrained firms ϕ during recessions. It is widely accepted that more firms become financially constrained during an economic downturn, for example, during the most recent recession initiated by COVID-19, even a monopolistic firm like Boeing was constrained

Figure 6: Percentage of Banks Tightening Credit Standards



NOTE: This figure illustrates how much tighter credit standards have become on commercial and industrial lines of credit. Tighter credit standards are a proxy for reductions in the supply of credit. The blue point-dashed line is the net percentage of domestic banks that report to have tightened their credit standards for large and medium businesses. The red solid line is the net percentage of domestic banks that report to have tightened their credit standards for small businesses. Shading indicates U.S. recession periods. Sources: Board of Governors of the Federal Reserve System

by liquidity. To further verify this conjecture, I present the credit supply change along the business cycle in the US using the Senior Loan Officer Opinion Survey on Bank Lending Practices conducted by the Federal Reserve. Figure 6 illustrates the fact that during recessions, on the credit supply side, commercial banks dramatically raised their standards when providing loan to firms regardless of their size. Hence, the fraction of firms facing difficulties in accessing financing should be countercyclical.

The overall effect of these two forces remains ambiguous, which will be determined by calibrated parameters. For this rational inattention model with financial friction, the main parameters are the volatility of aggregate shock and idiosyncratic shock under recession and expansion; the information processing capacity, κ; the degree of real rigidity, ψ, the fraction of firms with binding capital constraint, ϕ; and the 'driving force', ω_U and ω_C.

The volatility of idiosyncratic productivity is taken from the estimation of Bloom et al. (2018) as $\sigma_A^{Recession} = 0.13, \ \ \sigma_A^{Expansion} = 0.039$, which implies 3.33 times higher idiosyncratic uncertainty during recession than expansion. The stochastic process for nominal aggregate demand is calibrated using the nominal GDP data of the U.S. from 1972 to 2010 so as to be consistent with Bloom et al. (2018). I assume that the aggregate monetary policy process follows a Markov-Switching log-AR(1) model

$$m_t = \rho_M^{s_t} m_{t-1} + \varepsilon_{M,t}, \quad \varepsilon_{M,t} \sim N(0, \sigma_M^{s_t}), \quad s_t \in \{Recession, Expansion\},$$

and then estimate the time-varying volatility and persistent rate of aggregate nominal shock using Expectation-Maximization algorithm. The estimated results are $\rho_M^{Expansion} = 0.96, \ \ \rho_M^{Recession} = 0.88, \ \ \sigma_M^{Recession} = 0.0114, \ \ \sigma_M^{Expansion} = 0.0048$, which implies a 2.38 times increase in volatility during recession. In line with Bloom (2014), the uncertainty of idiosyncratic shock increases more dramatically during recession compared to that of aggregate uncertainty estimated using Markov-Switching log-AR(1) process. Overall, the idiosyncratic uncertainty is around ten times as large as aggregate uncertainty in either economic state which is consistent with what Mackowiak and Wiederholt (2009) calibrate.

The information processing capacity is one of the key parameters for rational inattention models. Coibion and Gorodnichenko (2015) and Afrouzi (2019) propose a new approach to measure the degree of information rigidity in forecasts of aggregate inflation from the data by regressing firms' ex-post mean forecast errors on their ex-ante mean forecast revisions. Following this approach, the capacity that firms allocate to inflation is calibrated to 0.75 from the New Zealand survey. However, the capacity estimated from this approach is simply the capacity firms allocated to one uncertainty variable, i.e., inflation, which accounts for only part of a firm's total capacity. Due to the limitations in calibrating κ, I choose three different levels to calibrate the model: 0.75, as in Afrouzi (2019) (0.75); 1.5; and 3, as in Mackowiak and Wiederholt

(2009).[27]

I choose the elasticity of substitution in final goods production $\nu = 5$, which yields average markup of 25%, while the labour coefficient α is calibrated as $\alpha = 0.54$ so as to match the manufacturing industry of the US jointly with ν. Autor et al. (2020) revaluate the labour share drop in the US and document that the aggregate labour share in Manufacturing in 2012 is around 32.5% which is equal to $\frac{\nu-1}{\nu}\alpha$.

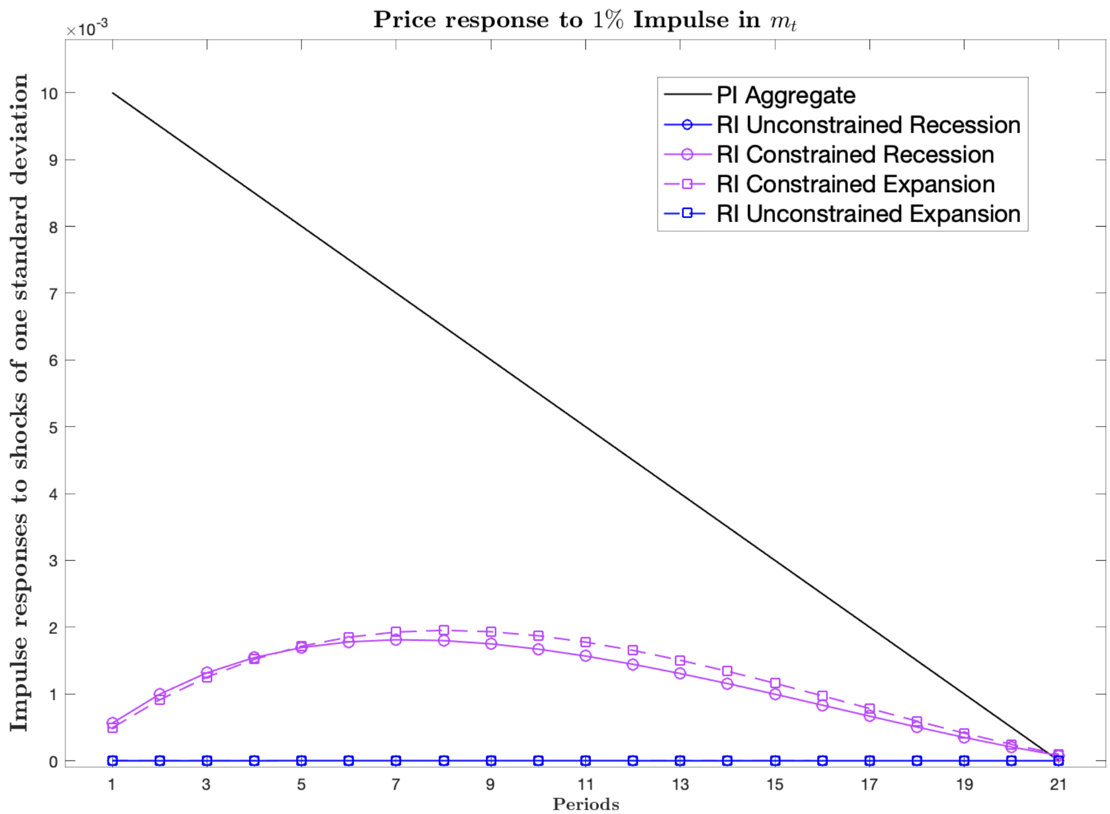
The fraction of constrained firms is a crucial parameter which determines the level of monetary non-neutrality when constrained firms have reached their limit in processing aggregate information. To calibrate this parameter, I use the World Bank's Enterprise Surveys dataset, which consists of periodical surveys of firms in countries around the world to analyse their characteristics across different sectors of an economy. The sampling of firms is designed to be representative of the structure of each economy and captures a variety of firms from different sizes. Among the high-income OECD economies, the share of firms that report being fully credit constrained and partially credit constrained is 3.6% and 9.8%, respectively. I combine these two categories of firms as financially constrained firms and this implies $\phi = 13.4\%$.[28] Since this survey was mainly conducted globally between 2010 and 2016, and we can view this fraction as the fraction of constrained firms during expansion. The 'driving force', ω_C and ω_U, i.e., steady state price ratios can be pinned down by firms' marginal cost differences. Given that in the model price is a marginal cost multiplied by a constant markup, I choose $\omega_C = 0.92$ and $\omega_U = \frac{1-\phi\omega_C}{1-\phi} = 1.02$, which implies a 5% higher marginal cost of constrained firms than unconstrained firms.

With these parameter adjustments, I can study how the composition effect can accommodate the relative uncertainty effect when shifting monetary policy effectiveness along the business cycle. Figure 7 illustrates how firms' prices change during recession and expansion. Since σ_r decreases in an economic downturn, firms will theoretically allocate more attention to idiosyn-

[27] The value 1.5, which is half of what Mackowiak and Wiederholt (2009) choose and twice of what Afrouzi (2019) choose, yields a ratio of posterior variance to prior variance of 0.25.

[28] Balleer et al. (2017) report that using the German BEP survey, an average of 5% of constrained firms according to the production measure and about 25% of constrained firms according to the banking measure regarding balance sheet. The fraction implied by the World Bank's Enterprise Surveys dataset of 13.4% is between these two values.

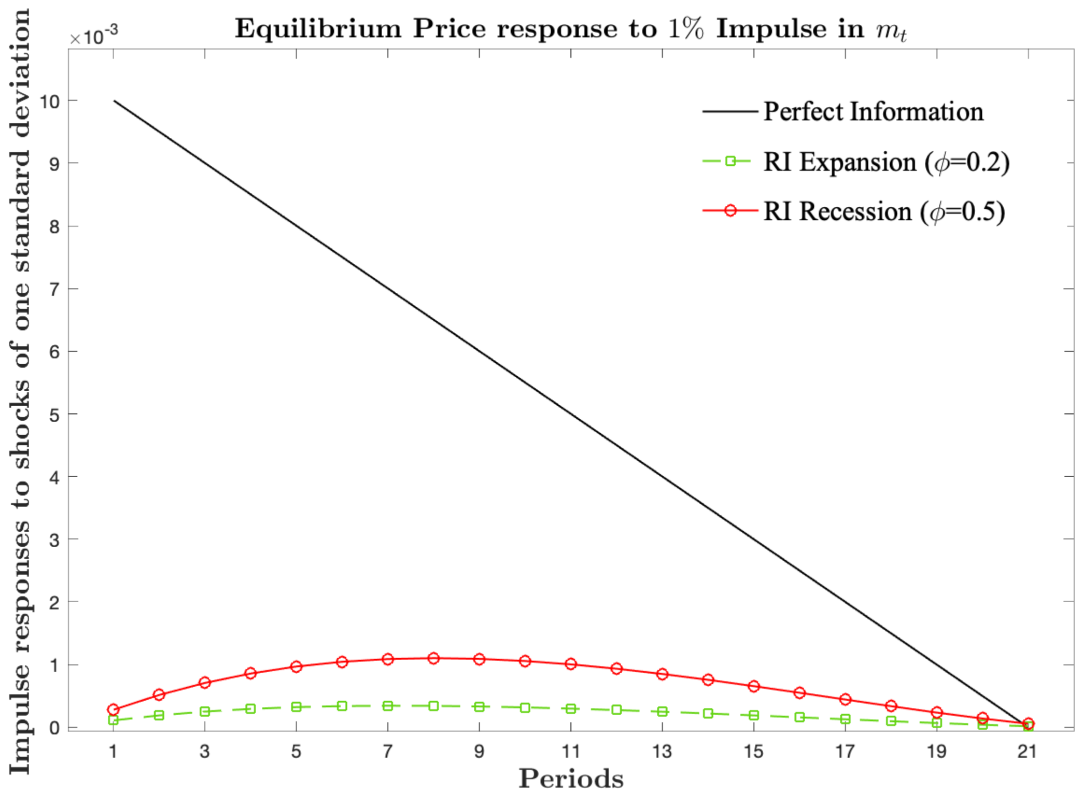
Figure 7: Response of Inflation and Output to Aggregate shock



cratic shock and hence be less responsive to monetary policy shock. In Figure 7, unconstrained firms pay almost no attention to aggregate shock, whereas constrained firms' responsiveness decreases in a recession, as the solid purple line illustrates. If it was not for the composition effect, the real effect of a 1% impulse in m_t should have grown in a recession, because the aggregate price will be less responsive if ϕ is constant. However, Figure 8 shows how the aggregate price responds in different states of the economy. Specifically, the red dashed line represents the aggregate price during expansion when $\phi = 13.4\%$, as calibrated. With of 15% more firms being constrained, this composition effect can fully accommodate the relative uncertainty effect and implies a similar cumulative (16 periods) impulse response to real consumption. Moreover, when ϕ is set to 50% to represent an economic downturn, as depicted by the solid red line, the cumulative impulse response of GDP could even drop by 25% compared with the dashed line.[29]

[29] I still need to calibrate how the fraction of constrained firms shift during recession to have a precise understanding about the policy implications.

Figure 8: Response of Inflation and Output to Aggregate shock



Overall, this model delivers a comprehensive explanation regarding the state-dependent effectiveness of monetary policy, which the previous rational inattention literature fails to capture.

7 Extensions

7.1 Heterogeneous Capacity

The model setting implies that larger and more productive firms are usually unconstrained firms. However, it is natural to expect that larger firms possess higher information processing capacity than smaller firms, i.e., $\kappa_H > \kappa_L$, given that larger firms usually have more monetary/human resources. Under this assumption, the equilibrium price response can be categorized with extra two more stages compared to homogeneous capacity case, which are: first, only unconstrained firms pay positive attention to aggregate shock, i.e., $\kappa_M^C = 0 < \kappa_M^U$, second, constrained firms allocate all attention to aggregate shock whereas unconstrained firms are at their interior solution, i.e., $\kappa_M^C = \kappa_L < \kappa_M^U = \frac{1}{2}\kappa_H + \frac{1}{4}\log_2(\sigma_r^2)$. The following proposition sums up the equilibrium price response.

Proposition 7 *The equilibrium aggregate price response to aggregate shock at different σ_r is as follows*

$$h = \begin{cases} 0 & \textit{if } \sigma_r \in \left(0,\ 2^{-\kappa_H}\right], \\ (1-\omega_C\phi)(1-2^{-\kappa_H}\sigma_r^{-1}) & \textit{if } \sigma_r \in \left(2^{-\kappa_H},\ \Lambda_1\right], \\ \dfrac{\omega_C\phi(1-2^{-\kappa_L}\sigma_r^{-1}) + (1-\omega_C\phi)\psi(1-2^{-\kappa_H}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)} & \textit{if } \sigma_r \in (\Lambda_1,\ \Lambda_2], \\ \dfrac{\omega_C\phi(1-2^{-2\kappa_L}) + (1-\omega_C\phi)\psi(1-2^{-\kappa_H}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa_L})} & \textit{if } \sigma_r \in (\Lambda_2,\ 2^{\kappa_H}], \\ \dfrac{\omega_C\phi(1-2^{-2\kappa_L}) + (1-\omega_C\phi)\psi(1-2^{-2\kappa_H})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa_L})} & \textit{if } \sigma_r \in (2^{\kappa_H}, \infty). \end{cases}$$

where

$$\Lambda_1 = \frac{2^{-\kappa_L} + (\psi-1)(1-\omega_C\phi)2^{-\kappa_H}}{1+(\psi-1)(1-\omega_C\phi)}$$

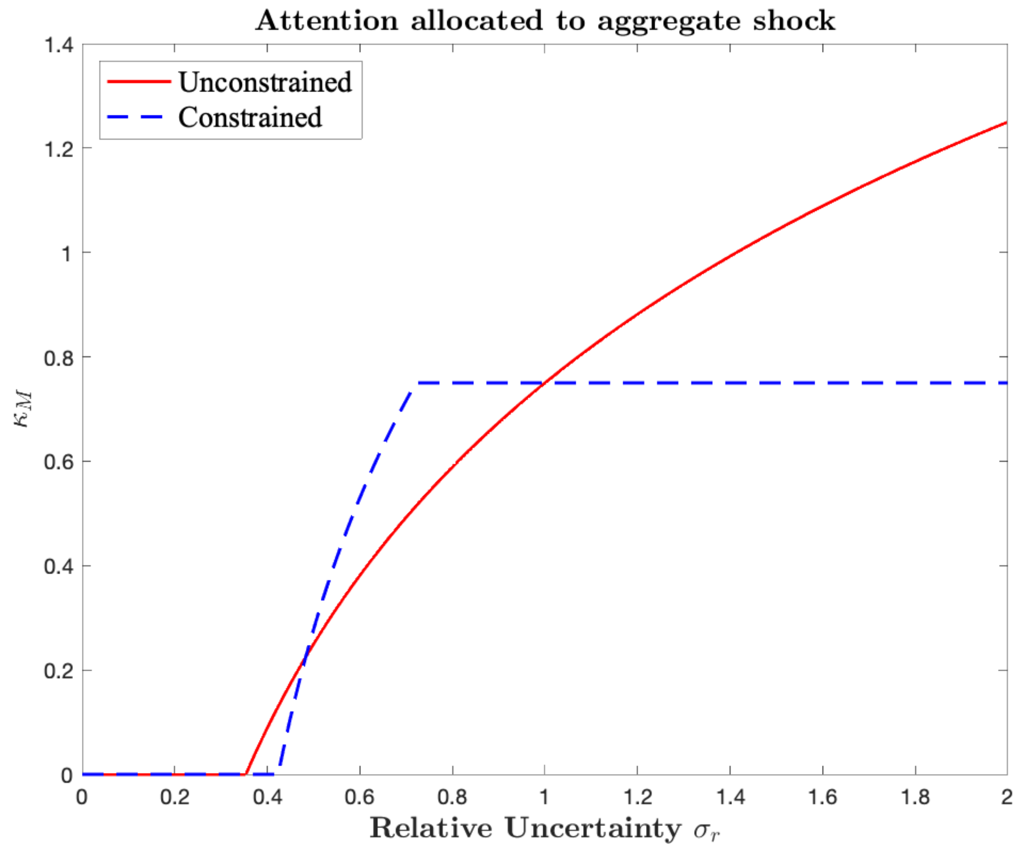
$$\Lambda_2 = \frac{2^{\kappa_L}[\psi - \omega_C\phi(\psi-1)] + (\psi-1)[\omega_C\phi 2^{-\kappa_L} + \psi(1-\omega_C\phi)2^{-\kappa_H}]}{\psi[\psi-\omega_C\phi(\psi-1)]}$$

Proof. See Proof of Proposition 2. ■

Here Λ_1 indicates the threshold when constrained firms start to allocate positive amount of attention to aggregate shock, Λ_2 indicates the threshold when constrained firms reach their information processing limit.

With heterogeneous information processing capacity, the distinction between attention allocation of two categories are slightly different. Unconstrained (large) firms, given their advantage in total capacity, will start being attentive to aggregate shock at a lower level of σ_r than constrained (small) firms. With σ_r increasing, constrained firms will speed up in shifting more attention to aggregate shock and overpass unconstrained firm at a certain level of σ_r, only if

Figure 9: Attention allocated to Aggregate shock under Heterogeneous Capacity



NOTE: This figure illustrates how firm's attention allocated to aggregate shock changes with the relative standard deviation between aggregate shock and idiosyncratic shock. The red solid line is for unconstrained firms, the blue dashed line is for constrained firms.

the gap between κ_L and κ_H is not sufficiently large. Figure 9 illustrates the difference: unconstrained firms are more attentive to aggregate shock if $\sigma_r \in (2^{-\kappa_H}, \Lambda_3) \cup (2^{2\kappa_L-\kappa_H}, \infty)$, and constrained firms pay more attention to aggregate shocks if $\sigma_r \in (\Lambda_3, 2^{2\kappa_L-\kappa_H})$, where

$$\Lambda_3 = \frac{2^{-\kappa_L}(\psi-1)\omega_C\phi + 2^{-\kappa_H}\psi(\psi-\omega_C\phi(\psi-1))}{(\psi-\omega_C\phi(\psi-1))(\psi-2^{\kappa_L-\kappa_H})}.$$

Regarding price responses, due to the fact that constrained firms are experiencing an disadvantage in total processing capacity, the effect of strategic complementarity on attention cannot fully offset their less responsiveness due to financial friction. Therefore, even for an interior solution of both categories of firms, constrained firms are still less responsive than unconstrained firms. As a consequence, monetary non-neutrality will be enlarged compared to the case with identical processing capacity.

8 Conclusion

Whether monetary policy can be effective or not partially depends on the attentiveness of economic agents; however, the attention allocated to macroeconomic conditions varies with a firms characteristics. In contrast to the common knowledge that larger firms should be more aware of how the economy is running thanks to their advantageous resources, empirical findings show that smaller firms might have a more accurate understanding of the big picture. In this paper, I develop a model with heterogeneous firms to make the link between financial friction and information acquisition and show that what matters for price-setters mainly depends on their pricing sensitivity. When firms are constrained in freely adjusting their production input, they will pay more attention to aggregate shock, which is relatively more important. Besides, due to strategic complementarity, constrained firms shift their attention to aggregate conditions faster as the variance in aggregate shock increases. As a result, constrained firms are more informed about macroeconomic conditions compared to unconstrained firms.

Concerning actual price responsiveness heterogeneity, constrained firms are generally more responsive to aggregate monetary shock due first to their higher attentive level, and Second to their immunity to capital rental rate change. The generally lower responsiveness of unconstrained firms is instructive for studying the state-dependent effectiveness of monetary policy. The latest empirical findings have documented the phenomenon that monetary policy is less potent in stimulating real growth during a recession, which the traditional rational inattention model with representative firms can hardly reconcile. during a recession, the volatility of idiosyncratic shock escalates more than that of aggregate shock. If applied to the representative firm rational inattention model, a firm will pay less attention to aggregate shock and hence have a more effective monetary policy, which contradicts the empirical literature. The model proposed in this paper can perfectly reconcile this effect, given that more firms are likely to be constrained during a recession, and this composition effect can not only offset the relative volatility effect but also deliver a less effective monetary policy during a recession. Calibration using aggregate data from the US shows that increasing the fraction of constrained firms from 13.4% to 50% can induce about a 25% loss in the real effect of monetary policy.

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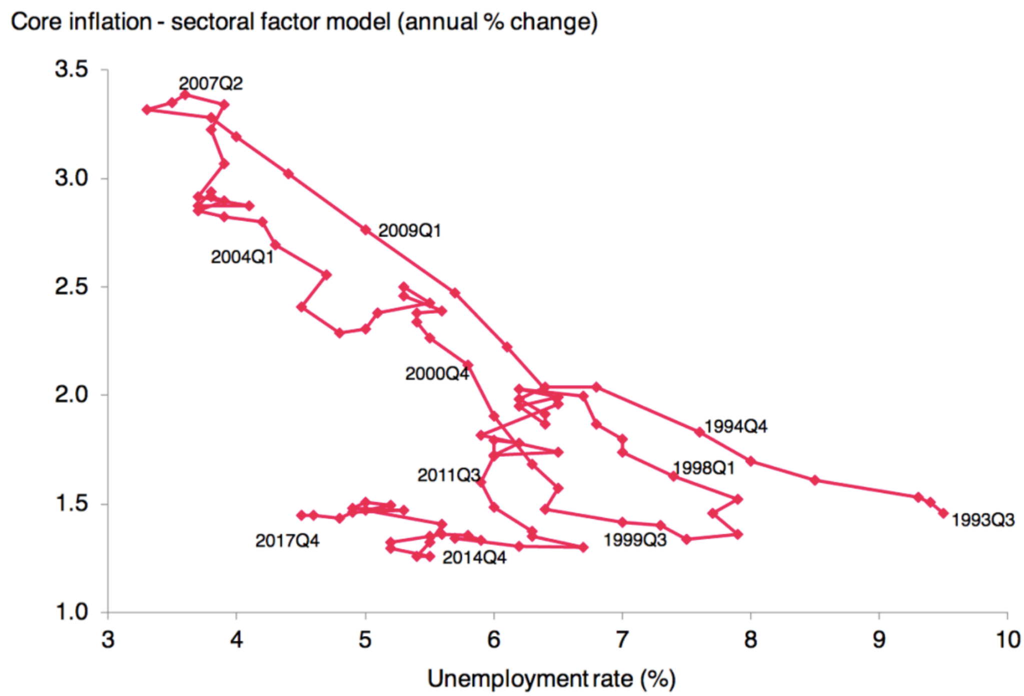
Figure 10: Inflation and Unemployment rate of New Zealand



Source: Reserve Bank of New Zealand, Statistics New Zealand

Figure 11: Response of Constrained Firm's Price to Shocks

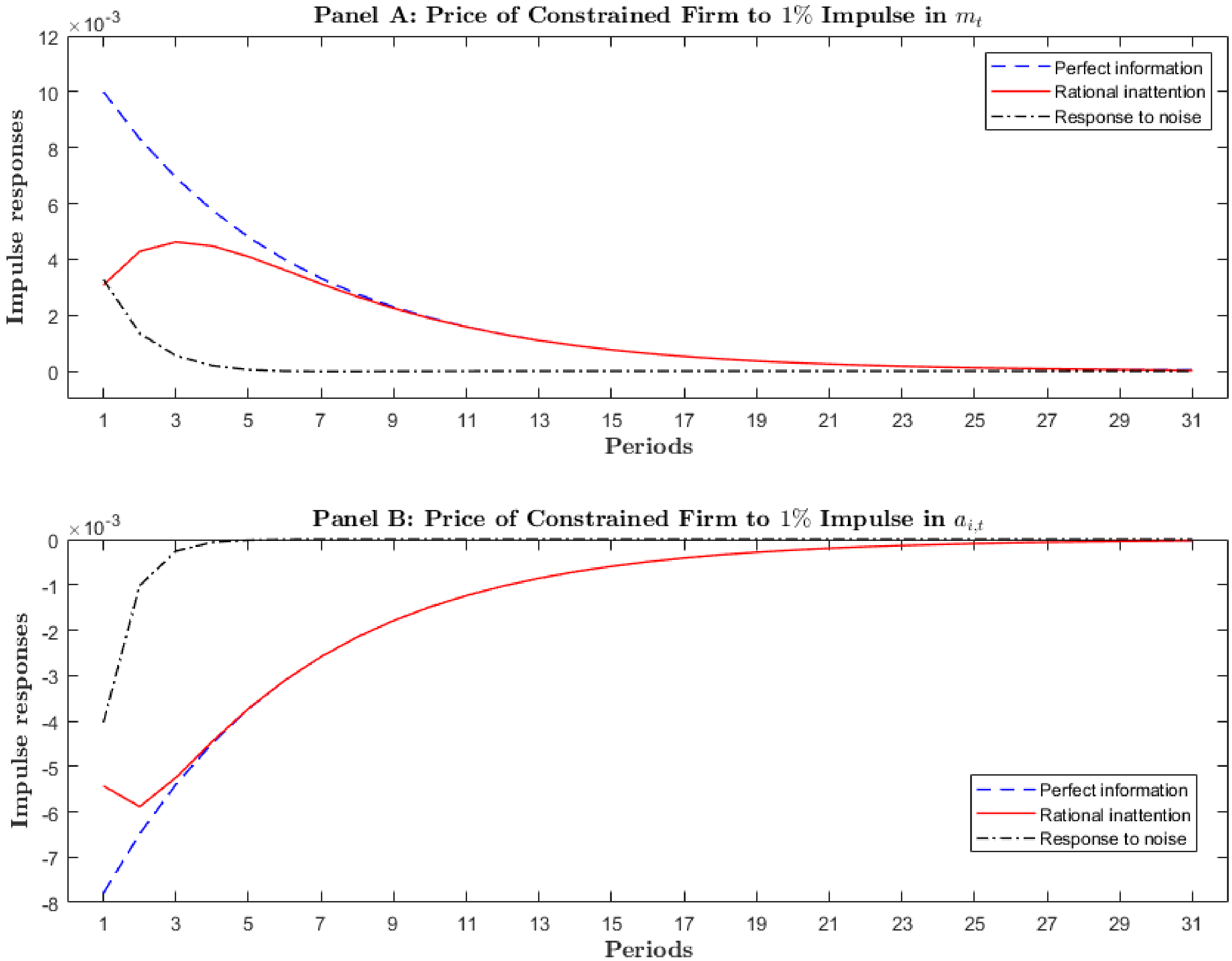


Figure 12: Response of Unconstrained Firm's Price to Shocks

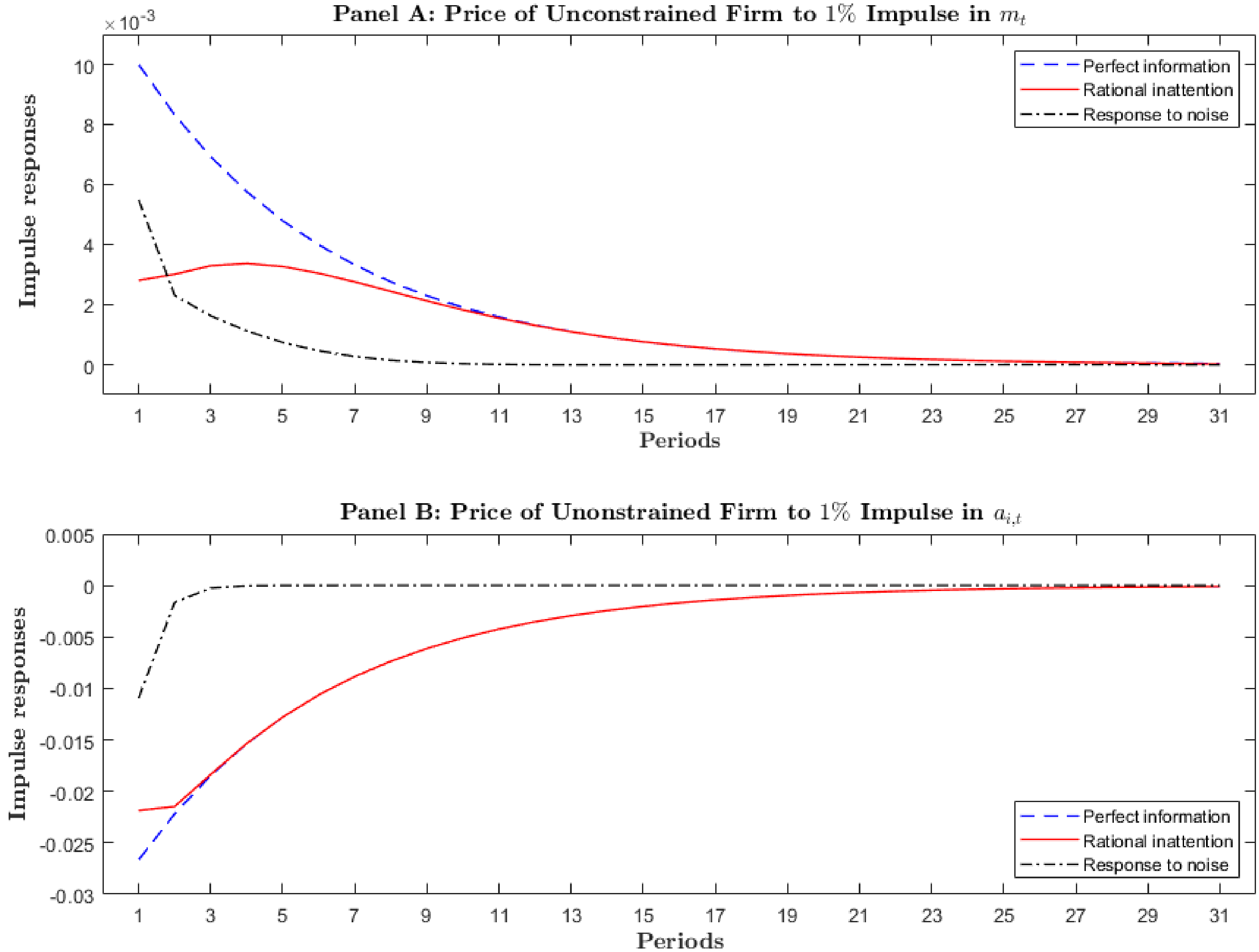


Figure 13: Response of Inflation and Output to Aggregate shock

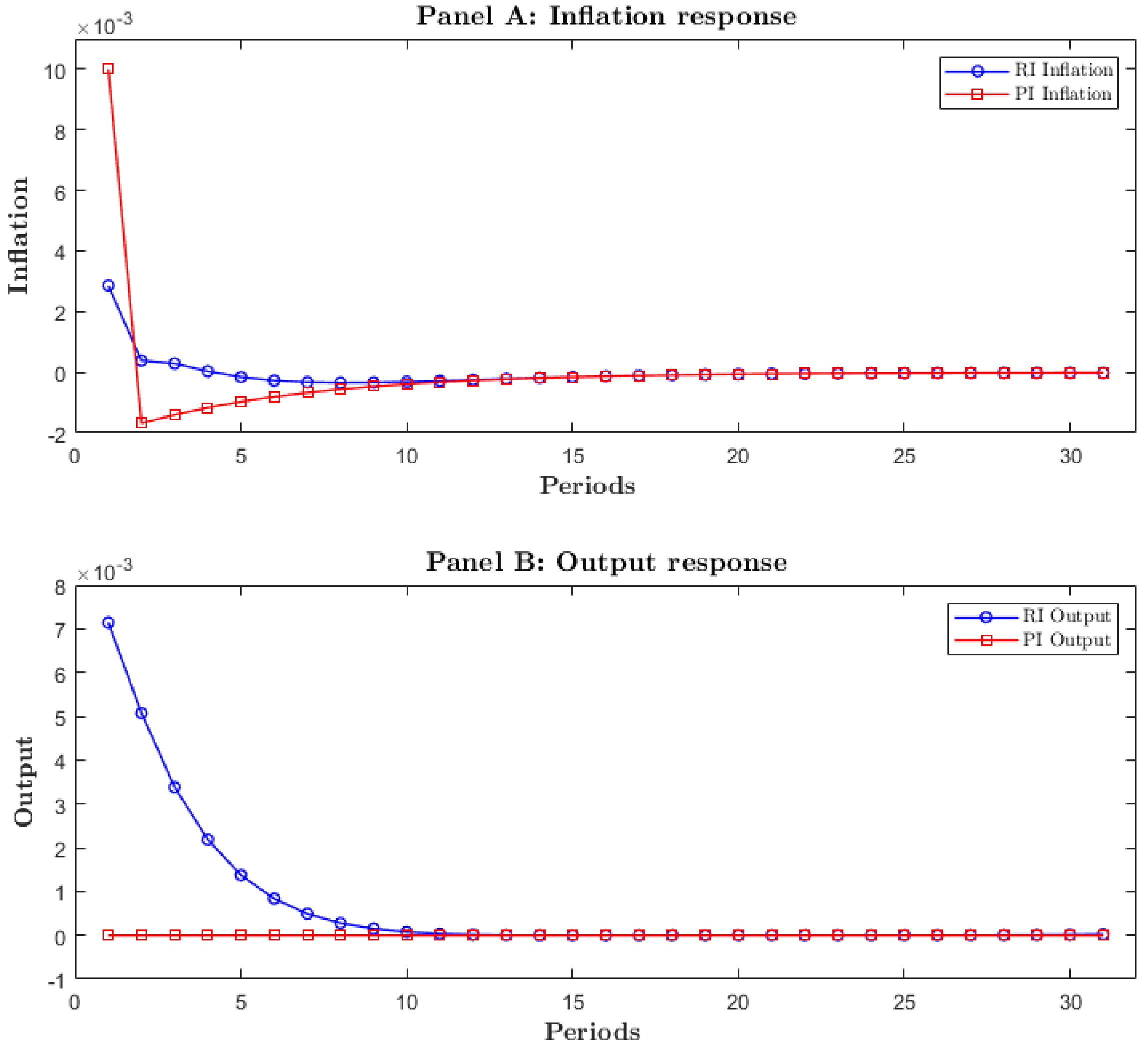


Figure 14: Response of Prices to Aggregate shock when $\phi = 0.61$

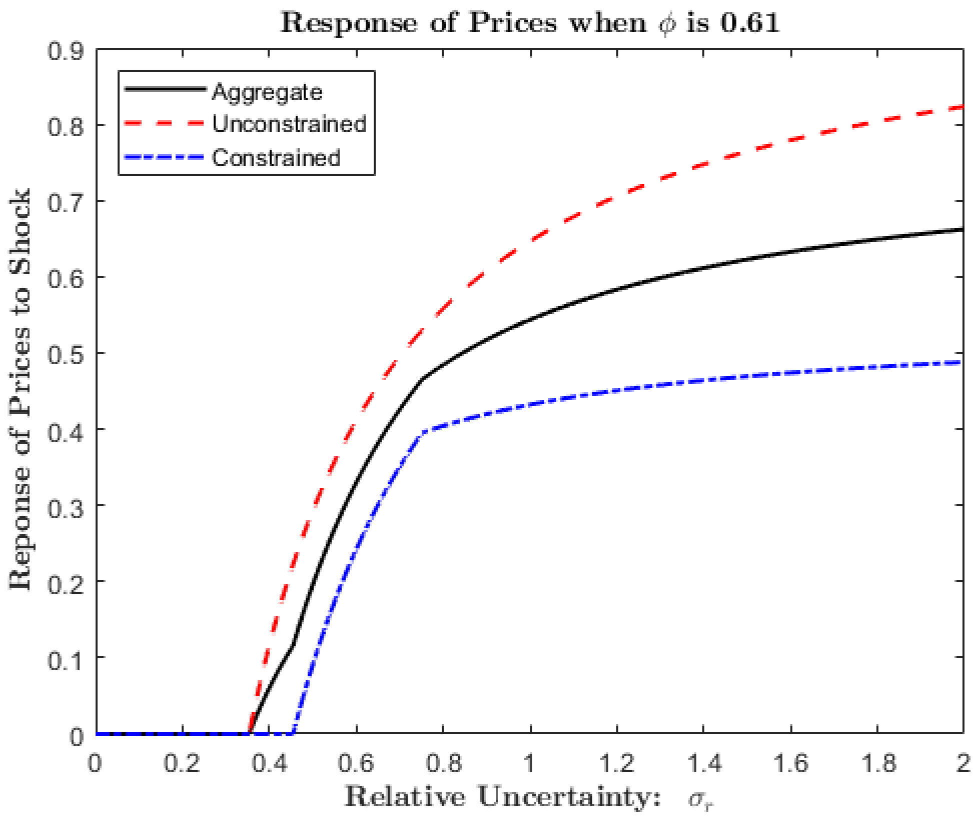


Table 4: Calibration Parameters

Parameters	Description	Value	Moment Matched
β	Discount factor	0.99	Quarterly discount factor
ν	Elasticity of substitution	5	25% Average Mark-up
α	Coefficient of Labour	0.54	Autor et al. (2020)
$\sigma_A^{Expansion}$	Standard deviation of idiosyncratic shock during expansion	0.0130	Bloom et al. (2018)
$\sigma_A^{Recession}$	Standard deviation of idiosyncratic shock during recession	0.039	Bloom et al. (2018)
$\sigma_M^{Expansion}$	Standard deviation of aggregate shock during expansion	0.0048	Nominal GDP of the U.S.
$\sigma_M^{Recession}$	Standard deviation of aggregate shock during recession	0.0114	Nominal GDP of the U.S.
ϕ	Fraction of constrained firms	0.134	World Bank Enterprise Survey
κ	Information processing capacity	1.5	Afrouzi (2019), Mackowiak and Wiederholt (2009)

Appendix

A Proofs for Lemmas and Propositions

A.1 Proof of Proposition 1

Proof. Note that $\xi_C \equiv 1 + (\psi - 1)h$ denotes the relative importance of aggregate shock to idiosyncratic shock for constrained firms and $\psi \equiv \alpha + (1-\alpha)\nu > 1$, $h \geq 0$. Hence, $\xi_C \geq 1$.

If $\sigma_r < \frac{2^{-\kappa}}{\xi_C} \leq 2^{-\kappa}$, then $\kappa^*_{M,C} = \kappa^*_{M,U} = 0$, i.e., both types of firm pay no attention to aggregate shock.

If $\frac{2^{-\kappa}}{\xi_C} < \sigma_r \leq 2^{-\kappa}$, then $\kappa^*_{M,C} = \frac{1}{2}\kappa + \frac{1}{4}\log_2(\xi_C^2\sigma_r^2) \geq \kappa^*_{M,U} = 0$, i.e., Constrained firms pay no less attention than unconstrained firms.

If $2^{-\kappa} < \sigma_r \leq \frac{2^{\kappa}}{\xi_c}$, then $\kappa^*_{M,C} = \frac{1}{2}\kappa + \frac{1}{4}\log_2(\xi_C^2\sigma_r^2) \geq \kappa^*_{M,U} = \frac{1}{2}\kappa + \frac{1}{4}\log_2(\sigma_r^2)$, i.e., constrained firms pay more attention than unconstrained firms.

If $\frac{2^{\kappa}}{\xi_c} < \sigma_r \leq 2^{\kappa}$, then $\kappa^*_{M,C} = \kappa \geq \kappa^*_{M,U} = \frac{1}{2}\kappa + \frac{1}{4}\log_2(\sigma_r^2)$, i.e., constrained firms pay no less attention than unconstrained firms.

If $2^{\kappa} < \sigma_r$, then $\kappa^*_{M,C} = \kappa = \kappa^*_{M,U} = \kappa$, i.e., both types of firms allocate all their attention to aggregate shock.

To sum up, $\kappa_{M,C} \geq \kappa_{M,U}$. ■

A.2 Proof of Lemma 2

Proof. First, given the property of CES aggregator, I have

$$(\phi\omega_C P^{1-\nu} + (1-\phi)\omega_U P^{1-\nu})^{\frac{1}{1-\nu}} = P$$
$$\Rightarrow \phi\omega_C + (1-\phi)\omega_U = 1$$

Second, from firm's optimal pricing decision, I have

$$P^C = \left(\frac{\nu}{\alpha(\nu-1)}\right)^{\frac{\alpha}{\psi}} \left(\frac{W^\alpha}{A}\right)^{\frac{1}{\psi}} (P^\nu C)^{\frac{1-\alpha}{\psi}} (\varrho_i \bar{K})^{\frac{\alpha-1}{\psi}}$$
$$P^U = \frac{\nu}{\nu-1} \frac{R_K^{1-\alpha} W^\alpha}{A\alpha^\alpha(1-\alpha)^{1-\alpha}}.$$

The capital rental constraint (11) implies

$$\varrho_i \bar{K} \leq \frac{P_i^{-\nu} P^\nu C}{\mu_A} (\frac{1-\alpha}{\alpha} \frac{W}{R^K})^\alpha,$$

Combine the previous three equations I have $P^C > P^U$, hence $\omega_C < \omega_U$. ■

A.3 Proof of Proposition 2

Proof. The derivation of Proposition 2 comes from the fixed point of solving the equilibrium price response. I now know that the optimal price that firms set under imperfect information are

$$p^{*C}_{i,t} = \xi_{M,C}(1-2^{-2\kappa^*_{M,C}})(m_t+\eta^{M,C}_{i,t}) - \xi_{A,C}(1-2^{-2(\kappa-\kappa^*_{M,C})})(a_{i,t}+\eta^{A,C}_{i,t})$$
$$p^{*U}_{i,t} = \xi_{M,U}(1-2^{-2\kappa^*_{M,U}})(m_t+\eta^{M,U}_{i,t}) + \xi_{A,U}(1-2^{-2(\kappa-\kappa^*_{M,U})})(a_{i,t}+\eta^{A,U}_{i,t})$$

where $\eta^{M,C}_{i,t}, \eta^{M,U}_{i,t}$, denotes the noise in nominal shocks of constrained firms and unconstrained firms, respectively. Similarly, the denotation applies to $\eta^{A,C}_{i,t}, \eta^{A,U}_{i,t}$.

The aggregate price response $p_t = hm_t$ can be solved from a fixed point problem. Recall that the utility-based price index is

$$P_t = \left(\int_0^1 P^{1-\nu}_{i,t} di\right)^{\frac{1}{1-\nu}} = \left(\int_0^{\phi} (P^C_{i,t})^{1-\nu} di + \int_{\phi}^1 (P^U_{i,t})^{1-\nu} di\right)^{\frac{1}{1-\nu}}$$

where the parameter ϕ denotes the fraction of constrained firms.

Thus, for the aggregate price of all firms $P_t = \left[\int_0^1 \left(P_{i,t}\right)^{1-\nu} dj\right]^{\frac{1}{1-\nu}}$, I have

$$\ln P_t = \frac{1}{1-\nu} \ln \int_0^1 \left(P_{i,t}\right)^{1-\nu} di$$

(first-order Taylor expansion)

$$= \frac{1}{1-\nu} \ln \underbrace{\left[\int_0^{\phi} \left(P^C_{i,t}\right)^{1-\nu} di + \int_{\phi}^1 \left(P^U_{i,t}\right)^{1-\nu} di\right]}_{P^{1-\nu}} + \frac{\int_0^1 P_j^{-\nu}(P_{i,t}-P_j)di}{P^{1-\nu}}$$

$$= \ln P + \int_0^{\phi} \frac{(P^C_{i,t}-P^C)(P^C)^{-\nu}}{P^{1-\nu}} di + \int_{\phi}^1 \frac{(P^U_{i,t}-P^U)(P^U)^{-\nu}}{P^{1-\nu}} di$$

$$= \ln P + \int_0^1 \frac{(P_{i,t}-P_j)(P_j)^{-\nu}}{P^{1-\nu}} di$$

Since $P^C \neq P^U$, I cannot interpret $\frac{(P^C_{i,t}-P^C)(P^C)^{-\nu}}{P^{1-\nu}}$ as the deviation from steady state.

$$\frac{(P^C_{i,t}-P^C)(P^C)^{-\nu}}{P^{1-\nu}} = \frac{P^C_{i,t}-P^C}{P^C}\left(\frac{P^C}{P}\right)^{1-\nu}$$

However, I can rewrite the aggregate price deviation as a weighted average function of two groups prices,

$$
\begin{aligned}
p_t &= \underbrace{(\frac{P^C}{P})^{1-\nu}}_{\equiv \omega_C} \int_0^\phi p_{i,t}^C dj + \underbrace{(\frac{P^U}{P})^{1-\nu}}_{\equiv \omega_U} \int_\phi^1 p_{i,t}^U dj \\
&= \omega_C \int_0^\phi \left[\xi_{M,C}(1-2^{-2\kappa^*_{M,C}})(m_t + \eta_{i,t}^{M,C}) - \xi_{A,C}(1-2^{-2(\kappa-\kappa^*_{M,C})})(a_{i,t}+\eta_{i,t}^{A,C}) \right] dj \\
&\quad + \omega_U \int_\phi^1 \left[\xi_{M,U}(1-2^{-2\kappa^*_{M,U}})(m_t + \eta_{i,t}^{M,U}) + \xi_{A,U}(1-2^{-2(\kappa-\kappa^*_{M,U})})(a_{i,t}+\eta_{i,t}^{A,U}) \right] dj \\
&= \left[\omega_C \phi \xi_{M,C}(1-2^{-2\kappa^*_{M,C}}) + \omega_U (1-\phi)\xi_{M,U}(1-2^{-2\kappa^*_{M,U}}) \right] m_t
\end{aligned}
$$

(If firms are totally heterogeneous, i.e., $P_i \neq P_j,\ if\ i \neq j$, then $p_t = \int_0^1 (\frac{P_i}{P})^{1-\nu} p_{i,t} di$)

The last equality comes from the assumption that $\int_0^\phi \eta_{j,t}^{M,C} dj = \int_0^\phi \eta_{j,t}^{A,C} dj = \int_\phi^1 \eta_{j,t}^{M,U} dj = \int_\phi^1 \eta_{j,t}^{A,U} dj = 0$. In words, **Assumption: The number of constrained firms is sufficiently large so that the idiosyncratic shocks average out.**

Due to the property of CES aggregator, I have

$$
\begin{aligned}
\phi \omega_C P^{1-\nu} + (1-\phi)\omega_U P^{1-\nu} &= P^\nu \\
\Rightarrow \omega_U &= \frac{1-\phi\omega_C}{1-\phi}
\end{aligned}
$$

Additionally, $\left(\frac{P^C}{P}\right)^{1-\nu}$ is decreasing with P_C, which means that, the larger the steady state price, the smaller effect its deviation has in affecting aggregate price deviation. The value of steady state price weight does not matter for the theoretical results. I will calibrate the value deliberately.

Given that two types of firms might respond to shocks differently at a certain level of shocks' volatility. The aggregate price response can be divided into five scenarios. Use σ_r^2 to denote the relative volatility $\frac{\sigma_M^2}{\sigma_A^2}$, I have

$$
p_t = \begin{cases}
0 & \text{if } \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{-2\kappa}, \\
\omega_C \phi \xi_{M,C} \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\xi_C^2\sigma_r^2)]}\right) m_t & \text{if } \sigma_r^2 < 2^{-2\kappa} < \xi_C^2\sigma_r^2 < 2^{2\kappa}, \\
\Big[\omega_C \phi \xi_{M,C} \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\xi_C^2\sigma_r^2)]}\right) & \\
+\omega_U (1-\phi) \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\sigma_r^2)]}\right)\Big] m_t & \text{if } 2^{-2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{2\kappa}, \\
\Big[\omega_C \phi \xi_{M,C} \left(1 - 2^{-2\kappa}\right) & \\
+\omega_U (1-\phi) \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\sigma_r^2)]}\right)\Big] m_t & \text{if } 2^{-2\kappa}\sigma_r^2 < \sigma_r^2 < 2^{2\kappa} < \xi_C^2\sigma_r^2, \\
\Big[\omega_C \phi \xi_{M,C} \left(1 - 2^{-2\kappa}\right) + \omega_U (1-\phi) \left(1 - 2^{-2\kappa}\right)\Big] m_t & \text{if } 2^{-2\kappa}\sigma_r^2 < 2^{2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2.
\end{cases}
$$

Since ξ_C contains the response of aggregate price to monetary shock, the equilibrium price level is the fixed point of the mapping between the conjectured law of motion $p_t = hm_t$ and actual law of motion (previous equations). Because the response of first scenario is 0, I start from the second one.

1. For the 2nd scenario, $\sigma_r^2 < 2^{-2\kappa} < \xi_C^2\sigma_r^2 < 2^{2\kappa}$

$$
\begin{aligned}
h &= \omega_C \phi \xi_{M,C} \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\xi_C^2\sigma_r^2)]}\right) \\
&= \omega_C \phi \frac{1 + (1-\alpha)(\sigma-1)h}{\alpha + (1-\alpha)\sigma} (1 - 2^{-\kappa} \frac{\sigma_A}{\sigma_M \left|(1 + (1-\alpha)(\sigma-1)h)\right|})
\end{aligned}
$$

There are two possibilities here:

(a) Assume that $\xi_C = 1 + (1-\alpha)(\sigma-1)h > 0$, I have

$$\Rightarrow h = \frac{\omega_C \phi(1 - 2^{-\kappa}\sigma_r^{-1})}{\alpha + (1-\alpha)\sigma - \omega_C\phi(1-\alpha)(\sigma-1)}$$

We take this solution back to the optimal attention allocation problem of constrained firm to check the validity,

$$\begin{aligned}
\xi_C^* &= 1 + \frac{(1-\alpha)(\sigma-1)\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)} \\
&= \frac{\psi - (1-\alpha)(\sigma-1)\omega_C\phi 2^{-\kappa}\sigma_r^{-1}}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)} \\
&= \frac{\psi - (\psi-1)\omega_C\phi 2^{-\kappa}\sigma_r^{-1}}{\psi - \omega_C\phi(\psi-1)}
\end{aligned}$$

Then both $\kappa_{M,C}^* \in (0,1)$ and $\kappa_{M,U}^* \in (0,1)$ are optimal choice at the fixed point if and only if

$$\left[\frac{\psi - (1-\alpha)(\sigma-1)\omega_C\phi 2^{-\kappa}\sigma_r^{-1}}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)}\right]^2 \sigma_r^2 \in (2^{-2\kappa}, 2^{2\kappa})$$

and

$$\sigma_r < 2^{-\kappa}$$

(b) Assume that $\xi_C = 1 + (1-\alpha)(\sigma-1)h < 0$ $(i.e.,\ \ h < 0)$, I have

$$h = \frac{\omega_C\phi(1+2^{-\kappa}\sigma_r^{-1})}{\alpha + (1-\alpha)\sigma - \omega_C\phi(1-\alpha)(\sigma-1)} < 0$$

$$\Rightarrow \quad \alpha + (1-\alpha)\sigma < \omega_C\phi(1-\alpha)(\sigma-1)$$

$$\Rightarrow \quad \alpha + (1-\alpha)\omega_C\phi + (1-\omega_C\phi)(1-\alpha)\sigma < 0$$

which is not possible since $\alpha \in [0,1],\ \omega_C \in [0,1],\ \phi \in [0,1]$. Hence, this contradiction rules out the possibility of decreasing equilibrium price given positive money supply shock.

2. For the 3rd scenario, $2^{-2\kappa} < \sigma_r^2 < \xi_C^2 \sigma_r^2 < 2^{2\kappa}$

$$h = \omega_C \phi \xi_{M,C} \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\xi_C^2 \sigma_r^2)]}\right) + \omega_U (1-\phi)\xi_U \left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\sigma_r^2)]}\right)$$
$$= \omega_C \phi \frac{1 + (1-\alpha)(\sigma-1)h}{\psi}\big(1 - 2^{-\kappa}\frac{\sigma_r^{-1}}{|1 + (1-\alpha)(\sigma-1)h|}\big) + \omega_U (1-\phi)\big(1 - 2^{-\kappa}\frac{\sigma_A}{\sigma_M}\big)$$

There are two possibilities here:

(a) $1 + (1-\alpha)(\sigma-1)h > 0$

$$\Rightarrow h = \frac{\omega_C \phi (1 - 2^{-\kappa}\sigma_r^{-1}) + \omega_U (1-\phi)(1 - 2^{-\kappa}\sigma_r^{-1})\psi}{\psi - \omega_C \phi (1-\alpha)(\sigma-1)}$$

We take this solution back to the optimal attention allocation problem of constrained firm to check the validity,

$$\xi_C^* = 1 + (1-\alpha)(\sigma-1)h$$
$$= \frac{\psi - (1-\alpha)(\sigma-1)\left[\omega_C \phi 2^{-\kappa}\sigma_r^{-1} - \omega_U (1-\phi)(1 - 2^{-\kappa}\sigma_r^{-1})\psi\right]}{\psi - \omega_C \phi (1-\alpha)(\sigma-1)}$$

Then both $\kappa_{M,C}^* \in (0,1)$ and $\kappa_{M,U}^* \in (0,1)$ are optimal choice at the fixed point if and only if

$$\xi_C^* \sigma_r \in (2^{-\kappa}, 2^{\kappa}) \quad \text{and} \quad \sigma_r \in (2^{-\kappa}, 2^{\kappa})$$

(b) $1 + (1-\alpha)(\sigma-1)h < 0$ and $2\alpha - 1 > 0$

$$\Rightarrow h = \frac{\omega_C \phi (1 + 2^{-\kappa}\sigma_r^{-1}) + \omega_U (1-\phi)(1 - 2^{-\kappa}\sigma_r^{-1})(\psi)}{\psi - \omega_C \phi (1-\alpha)(\sigma-1)}$$

As I have proved in previous scenario, the denominator cannot be non-positive, hence to have $h < 0$, I need to have,

$$\begin{aligned}
1 - 2^{-\kappa}\sigma_r^{-1} &< 0 \\
1 &< 2^{-\kappa}\sigma_r^{-1} \\
\sigma_r^2 &< 2^{-2\kappa}
\end{aligned}$$

which is contradicting with the optimal attention allocation condition $2^{-2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{2\kappa}$.

Therefore, for the third scenario, the only solution is

$$\Rightarrow h = \frac{\omega_C\phi(1 - 2^{-\kappa}\sigma_r^{-1}) + \omega_U(1-\phi)(1 - 2^{-\kappa}\sigma_r^{-1})\psi}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)}$$

3. For the 4th scenario, $2^{-2\kappa} < \xi_U^2\sigma_r^2 < 2^{2\kappa} < \xi_C^2\sigma_r^2$

$$\begin{aligned}
h &= \omega_C\phi\xi_{M,C}\left(1 - 2^{-2\kappa}\right) + \omega_U(1-\phi)\left(1 - 2^{-2[\frac{\kappa}{2} + \frac{1}{4}\log_2(\xi_U^2\sigma_r^2)]}\right) \\
&= \omega_C\phi\frac{1 + (1-\alpha)(\sigma-1)h}{\psi}\left(1 - 2^{-2\kappa}\right) + \omega_U(1-\phi)\big(1 - 2^{-\kappa}\frac{\sigma_A}{\sigma_M}\big)
\end{aligned}$$

$$\begin{aligned}
h &= \frac{\omega_C\phi(1 - 2^{-2\kappa})}{\psi} + \frac{\omega_C\phi(1 - 2^{-2\kappa})(1-\alpha)(\sigma-1)}{\psi}h + \omega_U(1-\phi) - 2^{-\kappa}\frac{\omega_U(1-\phi)\sigma_A}{\sigma_M} \\
\Rightarrow h &= \frac{\omega_C\phi(1 - 2^{-2\kappa}) + \omega_U(1-\phi)(1 - 2^{-\kappa}\sigma_r^{-1})\psi}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)(1 - 2^{-2\kappa})}
\end{aligned}$$

We take this solution back to the optimal attention allocation problem of constrained firm to check the validity,

$$\begin{aligned}\xi_C^* &= 1 + (1-\alpha)(\sigma-1)h \\ &= \frac{\psi\left[1 + (1-\alpha)(\sigma-1)\omega_U(1-\phi)(1-2^{-\kappa}\sigma_r^{-1})\right]}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)(1-2^{-2\kappa})}\end{aligned}$$

Then both $\kappa_{M,C}^* \in (0,1)$ and $\kappa_{M,U}^* \in (0,1)$ are optimal choice at the fixed point if and only if

$$\xi_C^*\sigma_r > 2^\kappa \cap \sigma_r \in (2^{-\kappa}, 2^\kappa)$$

4. For the 5th scenario, $2^{-2\kappa} < 2^{2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2$

$$\begin{aligned} h &= \omega_C\phi\xi_{M,C}\left(1-2^{-2\kappa}\right) + \omega_U(1-\phi)\xi_U\left(1-2^{-2\kappa}\right) \\ &= \left[\omega_C\phi\frac{1+(1-\alpha)(\sigma-1)h}{\alpha+(1-\alpha)\sigma} + \omega_U(1-\phi)\right]\left(1-2^{-2\kappa}\right) \\ \Rightarrow h &= \frac{\omega_C\phi + \omega_U(1-\phi)\psi}{\psi - \omega_C\phi(1-\alpha)(\sigma-1)(1-2^{-2\kappa})}(1-2^{-2\kappa})\end{aligned}$$

We take this solution back to the optimal attention allocation problem of constrained firm to check the validity,

$$\begin{aligned}\xi_C^* &= 1 + (1-\alpha)(\sigma-1)h \\ &= \frac{(\psi)\left[1 + (1-\alpha)(\sigma-1)\omega_U(1-\phi)(1-2^{-2\kappa})\right]}{\alpha + (1-\alpha)\sigma - \omega_C\phi(1-\alpha)(\sigma-1)(1-2^{-2\kappa})}\end{aligned}$$

Then both $\kappa_{M,C}^* \in (0,1)$ and $\kappa_{M,U}^* \in (0,1)$ are optimal choice at the fixed point if and only if

$$\xi_C^*\sigma_r > 2^\kappa \cap \sigma_r > 2^\kappa$$

To sum up, the equilibrium price level under rational inattention is the fixed point of the mapping between conjecture $p_t = hm_t$ and the actual law of motion in five different scenarios.

$$p_t = \begin{cases} 0 & \text{if } \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{-2\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})}{\psi-\omega_C\phi(1-\alpha)(\sigma-1)}m_t & \text{if } \sigma_r^2 < 2^{-2\kappa} < \xi_C^2\sigma_r^2 < 2^{2\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})+\omega_U(1-\phi)(1-2^{-\kappa}\sigma_r^{-1})\psi}{\psi-\omega_C\phi(1-\alpha)(\sigma-1)}m_t & \text{if } 2^{-2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{2\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-2\kappa})+\omega_U(1-\phi)(1-2^{-\kappa}\sigma_r^{-1})\psi}{\psi-\omega_C\phi(1-\alpha)(\sigma-1)(1-2^{-2\kappa})}m_t & \text{if } 2^{-2\kappa} < \sigma_r^2 < 2^{2\kappa} < \xi_C^2\sigma_r^2, \\ \dfrac{\omega_C\phi+\omega_U(1-\phi)\psi}{\psi-\omega_C\phi(1-\alpha)(\sigma-1)(1-2^{-2\kappa})}(1-2^{-2\kappa})m_t & \text{if } 2^{2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2. \end{cases} \tag{31}$$

$$p_t = \begin{cases} 0 & \text{if } 0 < \sigma_r < 2^{-\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})}{\psi-\omega_C\phi(\psi-1)}m_t & \text{if } \sigma_r < 2^{-\kappa} < \dfrac{\psi-(\psi-1)\omega_C\phi 2^{-\kappa}\sigma_r^{-1}}{\psi-\omega_C\phi(\psi-1)}\sigma_r < 2^{2\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})+\omega_U(1-\phi)\psi(1-2^{-\kappa}\sigma_r^{-1})}{\psi-\omega_C\phi(\psi-1)}m_t & \text{if } 2^{-2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2 < 2^{2\kappa}, \\ \dfrac{\omega_C\phi(1-2^{-2\kappa})+\omega_U(1-\phi)\psi(1-2^{-\kappa}\sigma_r^{-1})}{\psi-\omega_C\phi(\psi-1)(1-2^{-2\kappa})}m_t & \text{if } 2^{-2\kappa} < \sigma_r^2 < 2^{2\kappa} < \xi_C^2\sigma_r^2, \\ \dfrac{\omega_C\phi(1-2^{-2\kappa})+\omega_U(1-\phi)\psi(1-2^{-2\kappa})}{\psi-\omega_C\phi(\psi-1)(1-2^{-2\kappa})}m_t & \text{if } 2^{2\kappa} < \sigma_r^2 < \xi_C^2\sigma_r^2. \end{cases}$$

$$h = \begin{cases} 0 & \text{if } \sigma_r < 2^{-\kappa}, \\ h_1 = \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)} & \text{if } \sigma_r < 2^{-\kappa} < \big[1+(\psi-1)h_1\big]\sigma_r < 2^{\kappa}, \\ & \equiv \dfrac{1}{2^{\kappa}[1+(\psi-1)h_1]} < \sigma_r < \dfrac{1}{2^{\kappa}} \\ h_2 = \dfrac{\omega_C\phi(1-2^{-\kappa}\sigma_r^{-1}) + \omega_U(1-\phi)\psi(1-2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)} & \text{if } 2^{-\kappa} < \sigma_r < \big[1+(\psi-1)h_2\big]\sigma_r < 2^{\kappa}, \\ h_3 = \dfrac{\omega_C\phi(1-2^{-2\kappa}) + \omega_U(1-\phi)\psi(1-2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})} & \text{if } 2^{-\kappa} < \sigma_r < 2^{\kappa} < \big[1+(\psi-1)h\big]\sigma_r, \\ h_4 = \dfrac{\omega_C\phi(1-2^{-2\kappa}) + \omega_U(1-\phi)\psi(1-2^{-2\kappa})}{\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})} & \text{if } 2^{\kappa} < \sigma_r < \big[1+(\psi-1)h\big]\sigma_r. \end{cases}$$

We can prove that the second scenario does not exist since without any strategic complementarity, constrained firms will not pay attention to aggregate shock, see **Proof of Lemma 2.** Hence, there does not exist a situation where only constrained firms pay attention to aggregate shock, so $h_1 = 0$.

For the third scenario, I need to pin down the range of σ_r by substituting h_2 into the range

$$2^{-\kappa} \le \sigma_r < \frac{2^{-\kappa}(\psi-1)(\omega_C\phi + \omega_U(1-\phi)\psi) + 2^{\kappa}(\psi - \omega_C\phi(\psi-1))}{\psi + \omega_U(1-\phi)\psi(\psi-1)}$$

Similarly, for the fourth scenario, I get

$$\frac{2^{-\kappa}(\psi-1)(\omega_C\phi + \omega_U(1-\phi)\psi) + 2^{\kappa}(\psi - \omega_C\phi(\psi-1))}{\psi + \omega_U(1-\phi)\psi(\psi-1)} \le \sigma_r \le 2^{\kappa}$$

Therefore, I have the thresholds for five scenarios and the corresponding price responses. ■

A.4 Proof of Lemma 3

Proof. Recall the price response (31), in the second scenario I have

$$h = \frac{\omega_C \phi(1 - 2^{-\kappa}\sigma_r^{-1})}{\psi - \omega_C \phi(\psi - 1)} \quad \text{if } \sigma_r < 2^{-\kappa} < \big[1 + (\psi - 1)h_1\big]\sigma_r < 2^{\kappa},$$

Substitute h into the set of σ_r I get,

$$\begin{aligned} &\sigma_r < 2^{-\kappa} < \frac{\sigma_r \psi - (\psi - 1)\omega_C \phi 2^{-\kappa}}{\psi - \omega_C \phi(\psi - 1)} \\ &\Rightarrow 2^{-\kappa} < \sigma < 2^{-\kappa} \end{aligned}$$

which implies contradiction. Hence, the second scenario does not exist. ■

A.5 Proof of Proposition 6

Proof. The proof simply follows from taking first order derivative of the price response function with respect to σ_r, ψ and ϕ.

$$\frac{\partial h}{\partial \phi} = \begin{cases} 0 & \text{if } \sigma_r \in \left(0,\ 2^{-\kappa}\right], \\ 0 & \text{if } \sigma_r \in \left(2^{-\kappa}, \Lambda_\sigma\right], \\ \dfrac{\psi\big[(1-2^{-2\kappa})\omega_C[1-\omega_U(1-2^{-\kappa}\sigma_r^{-1})] - (1-2^{-\kappa}\sigma_r^{-1})\omega_U\psi(1-\omega_C(1-2^{-2\kappa}))\big]}{[\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})]^2} & \text{if } \sigma_r \in (\Lambda_\sigma,\ 2^{\kappa}], \\ \dfrac{(1-2^{-2\kappa})\psi\big[\omega_C[1-\omega_U(1-2^{-2\kappa})] - \omega_U\psi[1-\omega_C(1-2^{-2\kappa})]\big]}{[\psi - \omega_C\phi(\psi-1)(1-2^{-2\kappa})]^2} & \text{if } \sigma_r \in (2^{\kappa}, \infty). \end{cases}$$

■

B Derivations and Supportive Documents

B.1 Alternative way of introducing financial constraint

Assume that the initial firm wealth follows an arbitrary distribution with cumulative distribution function as $\mathcal{F}(\Delta)$. Borrowing from Moll (2014) and Mehrotra and Sergeyev (2018), I introduce the financial constraint in in the following way

$$K_{i,t} \leq \lambda \Delta_{i,t-1}$$

where $\Delta_{i,t-1}$ is the wealth of firm i from last period and λ denotes the leverage ratio. This form of financial constraint implies that firms with insufficient initial wealth can only rent a constrained level of capital which is below their optimal capital rental choice. Since this last period's wealth is a state variable which is independent of firm's attention allocation problem, the main results remain the same.

B.2 Capital Rental Price

The capital rental price R_t^K could be rationalized in the following way. Suppose that we have a dynamic economy where firm choose investment for next period's production by borrowing external fund, following Bernake et al (1999), the optimal decision should satisfy

$$E_t\left(\frac{P_{t+1}MPK_{t+1} + Q_{t+1}(1-\delta)}{Q_t}\right) = 1 + E_t(i_{t,t+1}).$$

where Q_t is capital price at period t, MPK_{t+1} is the marginal production of capital and $i_{t,t+1}$ is the nominal interest rate. We can derive simplified rental price of capital to be used in the simple model

1. Assume fully depreciating capital ($\delta = 1$), we have the capital rental price as

$$P_{t+1}MPK_{t+1} = R_{t,t+1}^K = (1 + i_{t,t+1})Q_t$$

2. Assume that capital price equal to final consumption good price, i.e., $P_t = Q_t$, and consumption good can be freely transformed to capital, we have

$$MPK_{t+1} + 1 - \delta = \frac{1 + i_{t,t+1}}{\Pi_{t+1}} = 1 + r_{t,t+1}$$
$$MPK_{t+1} = r_{t,t+1} + \delta$$

Combining previous two assumptions, we have the rental price of capital in the following form

$$P_{t+1}MPK_{t+1} = R^K_{t,t+1} = (1 + r_{t,t+1})P_{t+1} = \frac{1}{\beta}E_t(\frac{C_{t+1}}{C_t})P_{t+1} = \frac{1}{\beta}E_t\left[\frac{M_{t+1}}{M_t}\right]P_t$$

Note that this rental rate is the price of choosing capital used for production at period $t+1$. Back to our simple model, the capital rented K_t is utilized for production at period t, and the rental price be paid should be R^K_t. In a nominal economy, the rental price of capital is

$$R^K_t = \frac{1}{\beta}E_{t-1}(\frac{C_t}{C_{t-1}})P_t = \frac{1}{\beta}E_{t-1}\left[\frac{M_t}{M_{t-1}}\right]P_{t-1}$$

Recall firm manager's decision making behaviour in the end of period $t-1$ is lined up as:

Stage 1: Firm managers with information set $\mathcal{I}_{i,t-1}$ allocate their attention and choose the optimal signals $s_{i,t} \in \mathcal{S}_t$.

Stage 2: Firm managers receive signals and their information set is updated to $\mathcal{I}_{i,t}$.

Stage 3: With the new information set, firm managers make their optimal pricing strategy $P_{i,t}$ and choose the optimal input combination (minimized production cost) to produce the committed goods

Given the time line of firm's decision making process, I further assume that the capital supplying intermediary rent capital to firm in the end of period $t-1$. Additionally, the rental price is identical across all firms.

B.3 The Log-quadratic Approximation of Profit Function

Log-quadratic approximation of the profit function

$$\begin{aligned}\Pi_{i,t} &= P_{i,t}Y_{i,t} - W_t L_{i,t} - R_{i,t}K_{i,t} \\ &= \pi(p_{i,t}, p_t, m_t, a_{i,t})\end{aligned}$$

$$\hat{\pi}_{i,t} = \pi_1 p_{i,t} + \frac{\pi_{11}}{2} p_{i,t}^2 + \pi_{12} p_{i,t} p_t + \pi_{13} p_{i,t} m_t + \pi_{14} p_{i,t} a_{i,t}$$

- Desired price given full information and LQ profit function

$$\text{FOC}(p_{i,t}): \quad p_{i,t}^* = -\frac{\pi_{12}}{\pi_{11}} p_t - \frac{\pi_{13}}{\pi_{11}} m_t - \frac{\pi_{14}}{\pi_{11}} a_{i,t}$$

- Optimal price given information set $\mathcal{I}_{i,t}$ and LQ profit function

$$p_{i,t} = E[p_{i,t}^* | \mathcal{I}_{i,t}] = -E[\frac{\pi_{12}}{\pi_{11}} p_t + \frac{\pi_{13}}{\pi_{11}} m_t | s_{i,t}^M] - E[\frac{\pi_{14}}{\pi_{11}} a_{i,t} | s_{i,t}^A]$$

Therefore, the profit function can be written as

$$
\begin{aligned}
\Pi_{i,t} &= P_{i,t}Y_{i,t} - W_t L_{i,t} - R_{i,t}K_{i,t} \\
&= \Pi(P_{i,t}, P_t, M_t, A_{i,t}) \\
&= \Pi(\bar{P}e^{p_{i,t}}, \bar{P}e^{p_t}, \bar{M}e^{M_t}, \bar{A}e^{A_{i,t}}) \\
&= \pi(p_{i,t}, p_t, m_t, a_{i,t})
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_{i,t} &= \pi_1 p_{i,t} + \frac{\pi_{11}}{2}p_{i,t}^2 + \pi_{12}p_{i,t}p_t + \pi_{13}p_{i,t}m_t + \pi_{14}p_{i,t}a_{i,t} \\
\pi_{11} &= \lambda_j^{\sigma-1}\bar{P}^{\sigma-1}\bar{M}\left[(1-\sigma)^2\bar{P}_j^{1-\sigma} - \sigma^2\bar{P}_j^{\sigma}\bar{R}_j A^{-1}\right] \\
\pi_{12} &= (\sigma-1)\lambda_j^{\sigma-1}\bar{P}^{\sigma-1}\bar{M}\left[(1-\sigma)\bar{P}_j^{1-\sigma} + \sigma\bar{P}_j^{\sigma}\bar{R}_j A^{-1}\right] \\
\pi_{13} &= \lambda_j^{\sigma-1}\bar{P}^{\sigma-1}\bar{M}\left[(1-\sigma)\bar{P}_j^{1-\sigma} - \sigma\bar{P}_j^{\sigma}f_1(\bar{R},\lambda_j)\beta^{-1}A^{-1}\right] \\
\pi_{14} &= \lambda_j^{\sigma-1}\bar{P}^{\sigma-1}\bar{M}\left[(1-\sigma)\bar{P}_j^{1-\sigma} - \sigma\bar{P}_j^{\sigma}\bar{R}_j A^{-1}\right]
\end{aligned}
$$

B.4 The Optimal Attention Allocation

Firm's problem contains three decisions: the optimal input choice to minimize cost; the optimal prices given input choices and signals; the optimal signal to maximize profit. The cost minimization problem is solved in Appendix A.5. Here I show the optimal decisions for price and attention allocation.

Constrained Firms

For those firms who are financially constrained, the optimal pricing decision under perfect information can be derived as follows

$$
\begin{aligned}
&\max_{P_{i,t}}\ P_{i,t}Y_{i,t} - W_t L_{i,t} - R_t\bar{K} = P_{i,t}^{1-\nu}P_t^{\nu}C_t - W_t(\frac{P_{i,t}^{-\nu}P_t^{\nu}C_t}{\bar{K}^{1-\alpha}A_t})^{\frac{1}{\alpha}} - R_t\bar{K} \\
\Rightarrow\quad & P_{i,t}^{1+\frac{(1-\alpha)\nu}{\alpha}} = \frac{\nu}{\alpha(\nu-1)}\frac{W_t}{A_{i,t}^{\frac{1}{\alpha}}}(\lambda_j^{\nu-1}P_t^{\nu}C_t)^{\frac{1-\alpha}{\alpha}}\bar{K}^{\frac{\alpha-1}{\alpha}} \\
\Rightarrow\quad & P_{i,t}^{\alpha+(1-\alpha)\nu} = [\frac{\nu}{\alpha(\nu-1)}]^{\alpha}\frac{W_t^{\alpha}}{A_{i,t}}(\lambda_j^{\nu-1}P_t^{\nu}C_t)^{1-\alpha}\bar{K}^{\alpha-1} \\
& P_{i,t}^{\alpha+(1-\alpha)\nu} = [\frac{\nu}{\alpha(\nu-1)}]^{\alpha}\frac{W_t^{\alpha}}{A_{i,t}}(\lambda_j^{\nu-1}P_t^{\nu-1}M_t)^{1-\alpha}\bar{K}^{\alpha-1} \\
& P_{i,t}^{\alpha+(1-\alpha)\nu} = [\frac{\nu}{\alpha(\nu-1)}]^{\alpha}\frac{W_t^{\alpha}M_t^{1-\alpha}}{A_{i,t}}(\lambda_j^{\nu-1}P_t^{\nu-1})^{1-\alpha}\bar{K}^{\alpha-1} \\
\Rightarrow\quad & \ln P_{i,t} = \mathcal{C} + \frac{\alpha}{\alpha+(1-\alpha)\nu}\ln M_t + \frac{1-\alpha}{\alpha+(1-\alpha)\nu}\ln M_t + \frac{(1-\alpha)(\nu-1)}{\alpha+(1-\alpha)\nu}\ln P_t \\
&\qquad - \frac{1}{\alpha+(1-\alpha)\nu}\ln A_{i,t} \\
&\quad = \mathcal{C} + \frac{1}{\alpha+(1-\alpha)\nu}(\ln C_t - \ln A_{i,t}) + \ln P_t
\end{aligned}
$$

where $\mathcal{C} = \frac{\alpha}{\alpha+(1-\alpha)\nu}\ln\frac{\nu}{\alpha(\nu-1)} + \frac{(\nu-1)(1-\alpha)}{\alpha+(1-\alpha)\nu}\ln\lambda_i + \frac{\alpha-1}{\alpha+(1-\alpha)\nu}\ln\bar{K}$ and $\lambda_j = 1$ throughout the main body of this paper. Thus, the log-deviation of firm i's optimal price under perfect information is

$$
\begin{aligned}
p_{i,t}^{*} &= \ln P_{i,t} - \ln P_i \\
&= \frac{\alpha}{\alpha+(1-\alpha)\nu}m_t + \frac{1-\alpha}{\alpha+(1-\alpha)\nu}m_t + \frac{(1-\alpha)(\nu-1)}{\alpha+(1-\alpha)\nu}p_t - \frac{1}{\alpha+(1-\alpha)\nu}a_{i,t} \\
&= \frac{1}{\psi}m_t + \frac{\psi-1}{\psi}p_t - \frac{1}{\psi}a_{i,t}
\end{aligned}
$$

Define $\frac{1}{\psi} = \frac{1}{\alpha+(1-\alpha)\nu}$ as the degree of real rigidity. Small case notation generically denotes log-deviations from steady-state levels throughout.

Guess that the equilibrium price responds to aggregate shock as $p_t = hm_t$, then the perfect information pricing rule for all constrained firms is

$$
p_{i,t}^{*} = \underbrace{\frac{1+(1-\alpha)(\sigma-1)h}{\alpha+(1-\alpha)\sigma}}_{\xi_{M,C}}m_t - \underbrace{\frac{1}{\alpha+(1-\alpha)\sigma}}_{\xi_{A,C}}a_{i,t}
$$

The optimal price with imperfect information is (by law of total variance)

$$p_{i,t}^C = E[p_{i,t}^*|s_{i,t}] = \xi_{M,C}\frac{\sigma_M^2}{\sigma_M^2+\tau_M^2}(m_t+\eta_{i,t}^M) - \xi_{A,C}\frac{\sigma_A^2}{\sigma_A^2+\tau_A^2}(a_{i,t}+\eta_{i,t}^A)$$

After second-order Taylor approximation, the loss of profit due to price deviation is

$$\begin{aligned}
&\frac{\pi_{11}}{2}E(p_{i,t}^C - p_{i,t}^*)^2\\
=&\frac{\pi_{11}}{2}\left(\xi_{M,C}\frac{\sigma_M^2}{\sigma_M^2+\tau_M^2}(m_t+\eta_{i,t}^M) - \xi_{A,C}\frac{\sigma_A^2}{\sigma_A^2+\tau_A^2}(a_{i,t}+\eta_{i,t}^A) - (\xi_{M,C}m_t - \xi_{A,C}a_{i,t})\right)^2\\
=&\frac{\pi_{11}}{2}\left(-\xi_{M,C}\frac{\tau_M^2}{\sigma_M^2+\tau_M^2}m_t + \xi_{M,C}\frac{\sigma_M^2}{\sigma_M^2+\tau_M^2}\eta_{i,t}^M + \xi_{A,C}\frac{\tau_A^2}{\sigma_A^2+\tau_A^2}a_{i,t} - \xi_{A,C}\frac{\sigma_A^2}{\sigma_A^2+\tau_A^2}\eta_{i,t}^A\right)^2\\
=&\frac{\pi_{11}}{2}\left(\xi_{M,C}^2\frac{(\tau_M^2)^2\sigma_M^2+(\sigma_M^2)^2\tau_M^2}{(\sigma_M^2+\tau_M^2)^2} + \xi_{A,C}^2\frac{(\tau_A^2)^2\sigma_A^2+(\sigma_A^2)^2\tau_A^2}{(\sigma_A^2+\tau_A^2)^2}\right)\\
=&\frac{\pi_{11}}{2}\left(\xi_{M,C}^2\underbrace{\frac{\tau_M^2}{\sigma_M^2+\tau_M^2}\sigma_M^2}_{\sigma_{M|s}^2} + \underbrace{\xi_{A,C}^2\frac{\tau_A^2}{\sigma_A^2+\tau_A^2}\sigma_A^2}_{\sigma_{A|s}^2}\right)\\
=&\frac{\pi_{11}}{2}\left(\xi_{M,C}^2(\frac{1}{4})^{\kappa_M}\sigma_M^2 + \xi_{A,C}^2(\frac{1}{4})^{\kappa-\kappa_M}\sigma_A^2\right)
\end{aligned}$$

Then firm minimise the profit loss by choosing κ_M subject to information flow constraint

$$\underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_M^2}{\tau_M^2}+1\right)}_{\kappa_M} + \underbrace{\frac{1}{2}\log_2\left(\frac{\sigma_A^2}{\tau_A^2}+1\right)}_{\kappa_A} \le \kappa$$

By taking FOC with respect to κ_M

$$
\begin{aligned}
\xi_{M,C}^2\sigma_M^2\frac{1}{4}^{\kappa_M} &= \xi_{A,C}^2\frac{1}{4}^{\kappa-\kappa_M}\sigma_A^2 \\
2\kappa_M\log_2\frac{1}{4} &= \kappa\log_2\frac{1}{4}+\log_2\frac{\xi_{A,C}^2\sigma_A^2}{\xi_{M,C}^2\sigma_M^2} \\
\Rightarrow \kappa_{M,C}^* &= \frac{\kappa}{2}+\frac{1}{4}\log_2\frac{\xi_{M,C}^2\sigma_M^2}{\xi_{A,C}^2\sigma_A^2} \\
&= \frac{\kappa}{2}+\frac{1}{4}\log_2\big[(\underbrace{1+(1-\alpha)(\sigma-1)h}_{\xi_C})^2\frac{\sigma_M^2}{\sigma_A^2}\big]
\end{aligned}
$$

Since I will have corner solutions, the optimal attention allocated to monetary shocks is

$$
\kappa_{M,C}^* = \begin{cases} \kappa & \text{if } \xi_C^2\frac{\sigma_M^2}{\sigma_A^2}\geq 2^{2\kappa} \\ \frac{1}{2}\kappa+\frac{1}{4}\log_2(\xi_C^2\frac{\sigma_M^2}{\sigma_A^2}) & \text{if } \xi_C^2\frac{\sigma_M^2}{\sigma_A^2}\in[2^{-2\kappa},2^{2\kappa}], \text{i.e., } \xi_C\frac{\sigma_M}{\sigma_A}\in[2^{-\kappa},2^{\kappa}] \\ 0 & \text{if } \xi_C^2\frac{\sigma_M^2}{\sigma_A^2}\leq 2^{-2\kappa} \end{cases}
$$

Unconstrained Firms

For firms that are free from financial constraint, the process for deriving the optimal choices are similar with constrained firms. The perfect information profit maximizing price of unconstrained firm is

$$
\begin{aligned}
P_{i,t}^* =& \frac{\nu}{\nu-1}\frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{W_t^\alpha R_{K,t}^{1-\alpha}}{A_{i,t}} \\
& \text{substituting (10)} \\
=& \frac{\nu}{\nu-1}\frac{P_{t-1}^{1-\alpha}}{\beta^{1-\alpha}\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{W_t^\alpha\big(M_t/M_{t-1}\big)^{1-\alpha}}{A_{i,t}} \\
=& \frac{\nu}{\nu-1}\frac{M_{t-1}^{\alpha-1}P_{t-1}^{1-\alpha}}{\beta^{1-\alpha}\alpha^\alpha(1-\alpha)^{1-\alpha}}\frac{(\phi_L)^\alpha M_t}{A_{i,t}}
\end{aligned}
$$

The optimal price that rationally inattentive unconstrained firm i sets is given by

$$
\begin{aligned}
P_{i,t} =& E\left[P^*_{i,t}|\mathcal{I}_{i,t}\right] \\
=& \frac{\nu}{\nu-1}\frac{M_{t-1}^{\alpha-1}P_{t-1}^{1-\alpha}}{\beta^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\frac{(\phi_L)^{\alpha}E[M_t|\mathcal{I}_{i,t}]}{E[A_{i,t}|\mathcal{I}_{i,t}]}
\end{aligned}
$$

Though, the optimal price of unconstrained firm depends on previous state variable, I can argue that under Gaussian i.i.d shocks, there will be no intertemporal strategic decisions for firm to make. First of all, nominal demand is completely exogenous, which will not be affected by any kind of firm's decision. Secondly, given the fact that there are infinite number of firms, each firm's pricing decision has no impact on the aggregate price index. Hence, firm's pricing behaviour remain static under Gaussian i.i.d shocks. I will show, numerically, how it will become dynamic when shocks are serially correlated.

Assume that the economy is perturbed from the steady state (at time $t-1$), then at time t the optimal price under perfect information is

$$
p^*_{i,t} = m_t - a_{i,t}
$$

Similarly, I take second order Taylor-expansion to get the approximated profit loss

$$
\begin{aligned}
&\frac{\pi^U_{11}}{2}(p_{j,t}-p^*_{j,t})^2 \\
=&\frac{\pi^U_{11}}{2}\left((\frac{1}{4})^{\kappa_M}\sigma^2_M + (\frac{1}{4})^{\kappa-\kappa_M}\sigma^2_A\right)
\end{aligned}
$$

where π^U_{11} denotes the second order approximation parameter of unconstrained firms.

The optimal amount of attention allocated to aggregate shock is κ_M

$$
\begin{aligned}
\sigma^2_M \frac{1}{4}^{\kappa_M} &= \frac{1}{4}^{\kappa-\kappa_M}\sigma^2_A \\
2\kappa_M \log_2\frac{1}{4} &= \kappa\log_2\frac{1}{4} + \log_2\frac{\sigma^2_A}{\sigma^2_M} \\
\Rightarrow \kappa^*_{M,U} &= \frac{\kappa}{2} + \frac{1}{4}\log_2\frac{\sigma^2_M}{\sigma^2_A}
\end{aligned}
$$

Since I will have corner solutions, the optimal attention allocated to monetary shocks is

$$\kappa^*_{M,U} = \begin{cases} \kappa & \text{if } \frac{\sigma_M^2}{\sigma_A^2} \geq 2^{2\kappa} \\ \frac{1}{2}\kappa + \frac{1}{4}\log_2(\frac{\sigma_M^2}{\sigma_A^2}) & \text{if } \frac{\sigma_M^2}{\sigma_A^2} \in [2^{-2\kappa}, 2^{2\kappa}] \\ 0 & \text{if } \frac{\sigma_M^2}{\sigma_A^2} \leq 2^{-2\kappa} \end{cases}$$

B.5 Log-deviation of Aggregate Price Index

See Proof of Proposition 2 for the derivation of log-deviation of aggregate price index.

B.6 Solution for Household's Optimal Decisions

The household optimization solution of representative household's problem consists of demand functions for each firm-specific product, labour supply functions for each product line derived from the first order conditions. The resulting demand functions are give by

$$C_{i,t} = \lambda_{i,t}^{\nu-1} \left(\frac{P_{i,t}}{P_t} \right)^{-\nu} C_t. \tag{32}$$

The solution also delivers price indices for composite goods at two stages respectively

$$P_t = \left[\int_0^1 \left(\frac{P_{i,t}}{\lambda_{j,t}} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}},$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{M_t}{M_{t+1}} \right]$$

Labour Supply

$$W_t = \phi_L P_t C_t = \phi_L M_t$$

B.7 Firm's Production Input Choices

Firms need to choose the optimal combination between capital and labour so as to minimize cost and maximize profit function:

$$\Pi_{i,t} = P_{i,t}Y_{i,t} - W_{i,t}L_{i,t} - r^K_{i,t}K_{i,t}$$

The cost minimization problem is

$$\min_{K_{i,t},L_{i,t}} W_{i,t}L_{i,t} + r^K_{i,t}K_{i,t}$$

$$\text{s.t. } Y_{i,t} = A_{i,t}(L^{\alpha}_{i,t}K^{1-\alpha}_{i,t})^{\phi}$$

The solution for cost minimization is

$$K_{j.t} = \left(\frac{1-\alpha}{\alpha}\frac{W_{i,t}}{r^K_{i,t}}\right)^{\alpha} \quad (\frac{Y_{i,t}}{A_{i,t}})^{1/\phi}$$

$$L_{i,t} = \left(\frac{\alpha}{1-\alpha}\frac{r^K_{i,t}}{W_{i,t}}\right)^{1-\alpha} (\frac{Y_{i,t}}{A_{i,t}})^{1/\phi}$$

The ratio between two inputs is

$$\frac{K_{i,t}}{L_{i,t}} = \frac{1-\alpha}{\alpha}\frac{W_{i,t}}{r^K_{i,t}}$$

The marginal cost is

$$MC_{i,t} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\frac{W^{\alpha}_{i,t}r^{1-\alpha}_{i,t}}{A_{i,t}}$$

which is not changing with output for unconstrained firm. Price is

$$P_{i,t} = \frac{\nu}{\nu-1}\frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\frac{W^{\alpha}_{i,t}r^{1-\alpha}_{i,t}}{A_{i,t}}$$

Thus, the profit of firm j in period t is

$$\begin{aligned}
Y_{i,t}(P_{i,t} - MC_{i,t}) =& (\frac{\nu}{\nu-1}MC_{i,t} - MC_{i,t})Y_{i,t} = \frac{1}{\nu-1}MC_{i,t}Y_{i,t} \\
=& \lambda_{i,t}^{\nu-1}\frac{1}{\nu-1}MC_{i,t}(\frac{P_{i,t}}{P_t})^{-\nu}Y_t \\
=& \lambda_{i,t}^{\nu-1}\frac{1}{\nu-1}MC_{i,t}^{1-\nu}\frac{\nu}{\nu-1}^{-\nu}P_t^{\nu}Y_t
\end{aligned}$$

which is increasing in $\lambda_{i,t}$.