Central Bank Digital Currency: A Corporate Finance Perspective*

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Abstract

We build models with an interest-bearing central bank digital currency (CBDC) to investigate the impacts of issuing CBDC on banking and the macroeconomy. In an economy where CBDC is the only medium of exchange, a higher interest rate on CBDC promotes deposits and firm investment because CBDC and bank deposits are complements. The interest rate on reserves can affect bank lending and investment when the reserve constraint binds. In our extensions, cash and interest-bearing CBDC can coexist, where the coexistence requires the central bank to adjust either the CBDC interest rate or the interest rate on reserves in some equilibrium. Our results suggest that the design of CBDC and banking matters for understanding how CBDC affects the macroeconomy.

Key words: CBDC, Corporate Finance, Banking, Interest Rate on Reserves

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1 Introduction

The current paper money system has been challenged by private cryptocurrencies since Bitcoin was created in 2009. This has pushed policy makers to explore and researchers to study the possibility of introducing a central bank digital currency (CBDC) in recent years. Indeed, the blockchain technology may bring a revolution to the current financial system and central banking, since it can support decentralized payment without the need of a third-party that controls the currency or payment network. Despite their huge price volatilities, Bitcoin-like cryptocurrencies have gained great popularity among individual and institutional investors. By early 2021, there are more than 4000 types of cryptocurrencies in total (still growing), and the market value of Bitcoin (top 1 cryptocurrency) even reached US$ 1 trillion and that of Ethereum (top 2) is around US$ 172 billion.

In contrast to Bitcoin-like cryptocurrencies, stablecoins like "Libra" (now "Diem")[1] and USDT, try to minimize the price volatility of cryptocurrencies, and also build more connections between cryptocurrencies and major currencies like US dollars and others. These private cryptocurrencies have generated great concerns among central banks and policy makers. In addition, the COVID-19 Pandemic has made cash payments even less appealing in major advanced and emerging economies. By Oct. 2020, a survey conducted by BIS found that "*80% of central banks are engaged in investigating CBDC and half have progressed past conceptual research to experimenting and running pilots*" (BIS, 2020). China is one of the pioneer countries in experimenting CBDC (it is called DC/EP, "digital currency and electronic payment", in China), and has run several pilot projects in multiple cities, since 2020.

Given the forthcoming CBDC, our paper addresses the following research questions. Firstly, how would CBDC interact with financial intermediation and monetary policy? One

[1] On June 18, 2019, Facebook and its partners issued the white paper of "Libra" (now change the name to "Diem") , which is a new cryptocurrency with the mission "to enable a simple global currency and financial infrastructure that empowers" over 2.7 billion Facebook users. It will still use the blockchain technology, but the design is to be a "stablecoin" which aims to minimize the price volatility, with the full backup of reserves from a basket of multiple fiat currencies and credible government securities. Compared to Bitcoin-like cryptocurrencies, these features make Libra more possible to serve as a "currency", i.e., serving as a medium of exchange, a unit of account and a store of value. Therefore, the news of Libra has caused huge shocks and concerns among central banks and financial regulators all over the world. On June 30, 2019, Agustin Carstens, the general manager of BIS, urged that "central banks may have to issue their own digital currencies *sooner than expected*" (FT, 2019).

advantage of CBDC is that it can be bear an interest rate. Can the CBDC interest rate become a new monetary policy tool? What are the implications from conducting the negative-interest-rate (NIR) policy, as what have happened in Japan, Euro Zone and some European countries in recent years? Given that cash has no interests, how can cash and CBDC coexist? How does monetary policy affect financial intermediation, investment and the macroeconomy when both cash and CBDC are available in the economy?

Before answering these questions, we need clearly define CBDC. First, CBDC is *fiat digital money*, *not* private cryptocurrency per se. Fundamentally, CBDC is "centralized", since it is central bank high power money, directly issued and controlled by a central bank. This makes it very different from private cryptocurrencies which are "decentralized", and can support peer-to-peer settlements, either issued through some algorithm (like Bitcoin), or by some private enterprises (like Diem).

Second, CBDC is different from cash. Although they are both fiat monies, they are in different forms: CBDC is digital while cash is physical paper money. Cash is costly to produce (the cost of printing, counterfeiting technology development, etc.), costly to carry, and hard to track the transactions (used in anonymous transactions, so cash can circulate in underground economy). Given its digital form, CBDC can potentially keep track histories of transactions. Furthermore, it can be interest-bearing. This is another stark difference between CBDC and cash, since it is impossible to pay interests to cash. From a policy perspective, CBDC interest rate can become a new policy tool, and central bank can set the interest rate as positive or negative, when necessary.

Third, CBDC is different from bank deposits despite both of them are in the form of "digital accounts", and seem very close to each other. Bank deposits are "inside money" while CBDC belongs to "outside money". The key difference is that there is no insolvency issues for CBDC, as bank deposits may have these issues from financial institutions even there is deposit insurance.[2]

Having defined CBDC, we now explain what we do in this paper to tackle the above

[2]Here we suppose, in general, central bank of a country is trustworthy, and can function well as the last resort for the whole financial system, while commercial banks may have insolvency issues, particularly in the time of crisis. Obviously, we exclude the extreme cases of not-so-trustworthy central banks as in Latin American and other countries.

research questions. We start from a benchmark model with only CBDC, where we explicitly model a frictional deposit market and a frictional loan market. Entrepreneurs hold CBDC, and may or may not have investment opportunities. If they do not have investment opportunities (labeled as type-0 entrepreneurs), they deposit the idle CBDC at banks in a frictional deposit market. If they do ((labeled as type-1 entrepreneurs), they use CBDC as down payment, then apply for bank loans in a frictional loan market, to acquire capital and produce final output.

We consider several monetary policy tools. One is a traditional tool of changing money growth rate (equivalent of changing inflation rate at steady states). The second is a new tool of changing CBDC interest rate. Banks in our model are subject to a reserve requirement, but the central bank pays interests to the reserves. Hence, we can also consider two additional policy tools: changing reserve ratio and changing the interest rate on reserves, which can play non-trivial roles in some general equilibrium.

There are *two main results* from the benchmark model. The *first* result is that a higher CBDC interest rate tends to have a positive on deposits and investment, but its effect on bank lending varies, depending on the type of general equilibrium. This result is in sharp contrast with findings in existing models of CBDC. For example, in Andolfatto (2018) and Keister and Sanches (2018), CBDC and bank deposits are substitutes so that a higher CBDC interest rate tends to crowd out deposits and reduce investment. An exception is in Chiu et al. (2019) where CBDC and bank deposits are still substitute. Owing to the imperfect competition in the deposit market, a higher CBDC interest rate may help limit bank's market power and force banks to offer a higher deposit rate to prevent people from switching bank deposits to CBDC. Therefore, deposits and loans increase in response to the higher CBDC interest rate. In their model, the CBDC interest rate serves as a floor for the deposit rate. The critical difference between our model and these existing models is that CBDC and banks are complements in the spirit of Berentsen et al. (2007). Banks help channel liquidity from type-0 entrepreneurs to type-1 entrepreneurs. The complementarity between CBDC and bank deposits makes a higher CBDC interest rate more favorable to deposits and investment. This important message from the benchmark model is that the relationship between CBDC and banking matters when it comes to assessing the macroeconomic effects

of CBDC.

The *second* result is that the interest rate on reserves and the reserve requirement ratio can be independent monetary policy tools in the equilibrium where the reserve constraint binds. When the reserve constraint does not bind, the interest rate on reserves and the reserve ratio do not affect general equilibrium allocation. The binding reserve constraint implies that both the interest rate on reserves and the reserve ratio can directly affect the amount of loans being issued. Therefore, investment and bank lending are also affected. In particular, a higher interest rate on reserves or a lower reserve ratio makes entrepreneurs hold less CBDC, but allows banks to issue more loans. It has a positive effect on bank lending but overall negative effect on investment. The interest rate on reserves is usually used by central banks as a lower bound in the channel system or floor system. We find that it has a potential role in affecting bank lending and investment.

To understand how cash and CBDC interact, we extend the benchmark model by adding cash to the portfolio of entrepreneurs[3], and consider two extreme scenarios where only cash or only CBDC can be accepted by banks as deposits. In the first scenario, banks can only accept cash as deposits and turn it into reserves. Banks can help entrepreneurs to store CBDC, but it cannot be used as reserves, which resembles the assumptions in Andolfatto (2018). In the second scenario, banks can only accept CBDC as deposits and turn it into reserves. This is to capture the features of fast growing new types of banks, i.e., Internet banks, or the online banking business operated by traditional banks, where banks mainly deal with digital or electronic money, and do not accept cash. Surprisingly, for these two scenarios, cash and CBDC can coexist in all general equilibrium, with a non-zero CBDC interest rate. We can clearly derive the coexisting conditions, for all general equilibrium. However, in some general equilibrium, the central bank needs to adjust the CBDC interest rate and the interest on reserves simultaneously to ensure the coexistence of cash and CBDC. Only in the reserve

[3]In the benchmark model or the extended one, we model entrepreneurs hold CBDC, or the portfolio of cash and CBDC. Someone may not feel it as intuitive as modelling individuals holding CBDC or the portfolio for daily transactions. However, corporate cash holding has been an important issue for firms in the U.S. and other advanced economies since 1980s (see Bates et al. 2009, Azar et al. 2015, Graham and Leary 2018 and many other corporate finance papers). Graham and Leary (2018) documented the current level of average cash holdings is around 25% of assets, for US firms. Furthermore, this issue is highly related to the financing decision of firms, as we explicitly show in the paper, i.e., the internal and external finance issues of firms.

constrained equilibrium, cash and CBDC can coexist, and both the CBDC interest rate and the interest rate on reserves can be independent policy tools.

Adding cash to the benchmark is mainly to capture the initial stage of issuing CBDC, where it is more realistic cash and CBDC coexist. Although no country has issued CBDC yet (China or Sweden may be the first country to do so), a good reference is to see what happened in the banknote demonetisation of India. [4] During the initial stage of demonetisation, the "old" money and "new" money coexist for a limited period. Our extension shows that the central bank can use monetary policy tools to have cash and (non-zero interest rate) CBDC coexist. This again highlights the role of CBDC interest rate as a new policy tool, even when cash and CBDC coexist.

Literature Review Our paper is related to three lines of literature. The first line is literature related to CBDC, including Keister and Sanches (2018), Andolfatto (2018), and Chiu et al. (2019). There are also a few policy reports on CBDC, such as Bordo and Levin (2017) and Berentsen and Schar (2018). This literature has not had many papers since CBDC belongs to very new and frontier research.

Keister and Sanches (2018) build a model where both central bank money and private bank deposits are used in exchange, to study the effects of introducing CBDC on interest rates, economic activity, and welfare. They have competitive banking, and CBDC and bank deposits as substitutes in the model. Their results show that introducing CBDC tends to promote efficiency in exchange and raises welfare, but also crowds out bank deposits and decreases investment. In contrast, with the setting of non-competitive banking, Andolfatto (2018) and Chiu et al. (2019) both study the impacts of issuing CBDC on banking. Their difference is that Andolfatto (2018) uses an OLG model with monopolistic banking, while Chiu et al. (2019) use the framework of New Monetarism model with a competitive loan market, but a cournot-oligopolistic deposit market.

In all of these three papers, CBDC and bank deposits are modelled as substitutes in exchange, which is very different from our complementary setting of CBDC and bank deposits.

[4] On 8 November, 2016, the Government of India announced the demonetisation of all NIR 500 and 1000 banknotes of the Mahatma Gandhi Series, over a period of fifty days until 30 December, 2016 (Wikipedia). It also announced the issuance of new NIR 500 and 2000 banknotes in exchange for the demonetized ones. Hence, the demonetisation can be regarded as a "natural experiment" of changing fiat money system.

In addition, the focus of our paper is different from these papers. They focus on the impacts of CBDC on banking, investment and welfare, while our paper not only gets involved with these aspects, but also focuses on how effective CBDC interest rate can be a new monetary policy tool, and discusses the policy of negative interest rate.[5] Our extended models with both CBDC and cash also address important CBDC design issues, while this coexisting issue is either ignored or not fully addressed in those papers. Furthermore, we clearly focus on a corporate finance perspective, which is very different from those papers as well.

The second line is banking literature. There are many papers on banking since the canonical paper of Diamond & Dyvbig (1983). Here we just list a few that are highly related to our paper. Banks in our models accept idle liquidity as bank deposits from those who do not need liquidity, and then make loans to those who need. This is also the key mechanism to make CBDC and bank deposits become complements in the models. The role of banks is similar to Berentsen et al. (2007). However, they focus on a consumer finance perspective in the model, where there is no capital, agents are consumers, and banking is perfectly competitive, both in the deposit market and the loan market. In contrast, we focus on a corporate finance perspective, where agents are entrepreneurs, and we model both the deposit market and loan market as frictional ones. Our paper is also related to Rocheteau et al. (2018b) in the corporate finance perspective. The frictional loan market is similar to theirs, but we also explicitly model a frictional deposit market, which is absent in their paper. Not withstanding the totally different focuses: we focus on CBDC and the effects of introducing CBDC on banking and macroeconomy, while they focus on the pass-though and transmission mechanism of monetary policy from a corporate finance perspective. There are also a lot of other papers studying banking, such as Williamson (2012), Gu et al. (2013), Brunnermeier and Sannikov (2016), Dong et al. (2017), etc.

The third line of literature is about cryptocurrency and blockchain, including Chiu and Koeppl (2017), Hendry and Zhu (2017), Huberman et al. (2017), Abadi and Brunnermeier

[5] There are some papers related to negative interest rates, including He et al. (2008), Rocheteau et al. (2018a), Dong and Wen (2017), and Groot and Haas (2018). He et al.(2008) and Rocheteau et al. (2018a) use New Monetarism models and can generate negative interest rate for assets. Dong and Wen (2017) and Groot and Haas (2018) study the negative interest rate policy which has happened in some advanced economies (such as Japan, Euro Zone, and some Europrean countries), but neither of them is related to CBDC.

(2018), Schilling and Uhlig (2018), Dong et al. (2019), etc. These papers help understand cryptocurrency and blockchain technology, particularly how cryptocurrencies are different from fiat money. Our paper differs from these papers since CBDC is not cryptocurrency per se.

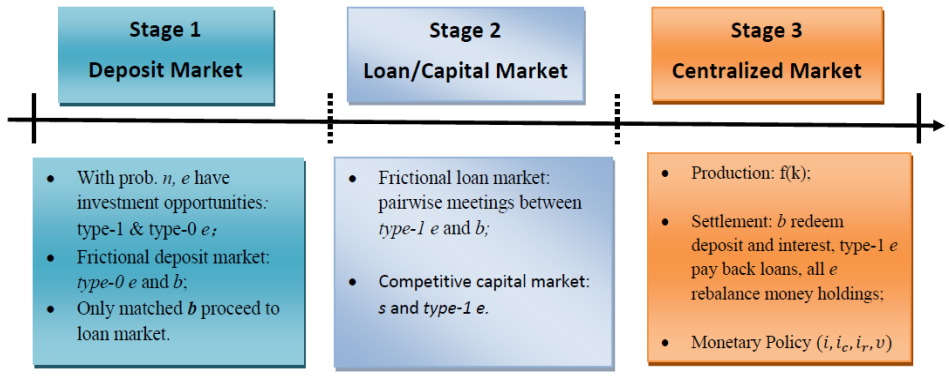
The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 introduces the benchmark model, where CBDC is the only medium of exchange. Section 4 is the policy analysis for the benchmark model. Section 5 extends the benchmark model by adding cash, and considers two scenarios: cash only deposits and CBDC only deposits. Section 6 discusses more design issues on CBDC, and concludes the paper.

2 Environment

Time is discrete and continues forever. Each period consists of three stages: Stage 1 is a decentralized deposit market; Stage 2 has a decentralized loan market, and a competitive capital market operating in parallel; and Stage 3 is a centralized market (CM). There are three types of agents: entrepreneurs (e), suppliers (s) and banks (b). There is a measure one of entrepreneurs, who are subject to an investment shock. With a probability n, $n > 1/2$, an entrepreneur has an investment opportunity and needs to acquire capital for production. With the rest probability $1 - n$, the entrepreneur does not have an investment opportunity. We label them as type-1 and type-0 entrepreneurs, respectively. The investment shock is realized at the beginning of each period. Suppliers can provide capital in the capital market. As in Rocheteau et al. (2018b), the measure of suppliers is irrelevant due to constant returns. There is a measure one of banks that need to first take deposits in the deposit market, satisfying a reserve requirement, and then issue loans in the loan market. Banks are owned by all entrepreneurs equally.

In the benchmark model, we assume that a central bank issues only CBDC m_c. This resembles the scenario when CBDC completely phases out paper money. We consider and discuss the coexistence of cash and CBDC in Section 5. CBDC is a fiat digital money with the price ρ, measured by CM numeriare goods x. It is interest-bearing, with a nominal interest rate i_c paid every period. The timeline of a representative period is shown in Figure

2, and the details of each stage are as follows.



At Stage 1, all banks go to the deposit market to take deposits, in order to make loans in the subsequent loan market. After the investment shock is realized, type-0 e go to the deposit market to deposit their idle balances. We assume a simple matching technology in the deposit market: short-side being served. Given the measure of type-0 e is $1-n$, the matching probability for type-0 e is 1 and that for banks is $1-n$. Those banks who do not get deposits will not proceed to the loan market, due to a reserve requirement which requires bankers to hold a fraction υ $(0 < \upsilon < 1)$ of total assets in the form of reserves.[6] Banks and entrepreneurs bargain over the terms of the deposit contract. For simplicity, we suppose banks make take-it-or-leave-it offers, to model banks (almost) have all of the bargaining power in the deposit market in the real world. [Do we have evidence that banks have all the bargaining power in the deposit market?]

Banks obtaining deposits and type-1 e participate in Stage 2, i.e., the loan market. For simplicity, we again assume a simple matching technology: short-side being served. Given that the measure of type-1 e is n and that of banks is $1-n$, the matching probability for entrepreneurs is $(1-n)/n$ and that for banks is 1 since $n > 1/2$. Banks and entrepreneurs bargain over the terms of the loan contract, including a down payment p (in the form of CBDC), a loan service fee ϕ and a loan size ℓ. Then the banked type-1 e can use the down payment plus the loan to acquire capital from suppliers. To ensure the repayment of loans, we follow Rocheteau et al. (2018b) to assume that a fraction χ of entrepreneur's output is

[6] Notice that, to model the reserve requirement, Rocheteau et al. (2018b) introduce an interbank market where bankers can borrow at some policy rate, while we introduce a deposit market to do so.

pledgeable. It implies that banks can uncover χ fraction of output in the case of default. As for unbanked type-1 e, i.e., those who do not get loans from banks, the entrepreneurs use internal finance to purchase capital in the competitive capital market, where suppliers provide capital at the market price q_k.

At Stage 3, all agents participate in the competitive centralized market. Entrepreneurs who deposit at Stage 1 redeem their deposits, and entrepreneurs who borrow in the loan market repay the loans and banking service fees. Banks distribute all profits to entrepreneurs. Entrepreneurs use capital k to produce, with the technology $f(k)$, $f'(k) > 0$, $f''(k) < 0$ and satisfying the Inada conditions. All agents can consumer numeriare goods x in CM. If they work, x is negative.

The government is only active at Stage 3. Suppose it is a consolidated monetary and fiscal authority, and the fiscal policy passively accommodates monetary policy, so that we could focus on monetary policy. The budget constraint of the government is,

$$G + T = (\pi - i_c)M_c + (\pi - i_r)\, M_r, \tag{1}$$

where G is government spending, T is lump-sum transfers, (i_c, i_r) are interests paid on CBDC and reserves. The LHS in (1) refers to the total government expenditure, while the RHS is the seigniorage revenues net of CBDC interest payment, where (M_c, M_r) denote the supply of CBDC and reserves by the central bank in the beginning of CM. There are *four types of monetary policy tools. The first one* is to change the growth rate of CBDC, μ, with

$$\frac{M_c}{M_{c-}} = 1 + \mu,$$

where $1 + \mu \equiv \rho/\hat{\rho}$, and $\mu = \pi$ at steady states (π is inflation rate). The Fisher equation implies that $1 + i = (1 + \pi)/\beta$. Here i can be interpreted as the nominal interest rate of illiquid bonds, which measure the opportunity cost of holding fiat money with zero interest. *The second one* is to set the interest rate of CBDC, i_c, with $i_c < i$ because of the no-arbitrage condition, but it is possible to have $i_c \geq 0$, or $i_c < 0$. When $i_c < 0$, it resembles the scenario that the central bank conducts NIR policy. *The third one* is to change the interest rate of

bank reserves, i_r, $i_r \geq 0$. When $i_r = 0$, the central bank pays zero interest to bank reserves, as in a typical floor system in normal times; when $i_r > 0$, i.e., reserves become interest-bearing, then i_r can be another independent policy tool, no matter in a floor system or a channel system. *The fourth monetary policy tool* is to change the required reserve ratio υ, $0 < \upsilon < 1$.

3 Model

In the model, we use U^j, V^j and W^j to denote the value functions for type-j agent at Stage 1, 2, 3, where $j = \{e, b, s\}$, to represent entrepreneurs, banks, and suppliers. For $j = e$, we have U_i^e, V_i^e and W_i^e, $i = \{0, 1\}$, to differentiate type-0 and type-1 entrepreneurs once the investment shock is realized in the beginning of Stage 1.

We start from Stage 3 in the current period, followed by Stage 1 and 2 in the next period. In the beginning of Stage 3, there are two types of entrepreneurs, type-1 e and type-0 e, since the investment shock is already realized at Stage 1. For type-1 e,

$$W_1^e(z_c, \ell, k) = \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\}$$
$$\text{st. } x + (1+\mu)\hat{z}_c = (1+i_c)z_c - \ell + f(k) + T + \Pi,$$

where $z_c \equiv \rho m_c$ is real balances of CBDC, $\hat{z}_c = \hat{\rho}\hat{m}_c$ is the real balances of CBDC carried to next period, ℓ denotes the amount of loans if borrowing in the previous loan market, $f(k)$ is the final output with capital input k, and (T, Π) represent transfers from the government and profits distributed by banks. The LHS of the budget constraint is consumption expenditure x plus real balances of CBDC carried to next period, while the RHS is net revenues out of loan repayment. Substituting x from the budget constraint, we have,

$$W_1^e(z_c, \ell, k) = z_c(1+i_c) - \ell + f(k) + T + \Pi + \max_{\hat{z}_c}\{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c)\}.$$

For type-0 e,

$$\begin{aligned} W_0^e(z_c, d) &= \max_{x, \hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z}_c &= z_c(1+i_c) + d(1+i_d) + T + \Pi, \end{aligned}$$

where d is the real balances of deposits, i_d is the nominal deposit rate. The major difference from type-1 e is that type-0 e redeems deposit interests, but do not hold any capital or need to repay bank loans, at Stage 3. Similarly, we have,

$$W_0^e(z_c, d) = z_c(1+i_c) + d(1+i_d) + T + \Pi + \max_{\hat{z}_c}\{-(1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}_c).$$

It is clear that entrepreneurs will choose the same $\hat{z}_c$ independent of their previous types,

$$1 + \mu = \beta \frac{\partial \mathbb{E} U^e(\hat{z}_c)}{\partial \hat{z}_c}. \tag{2}$$

Banks distribute their profits to entrepreneurs, where $\Pi = \sum \Pi_b$ aggregates all profits from active banks in this period. For each bank,

$$W^b(z_c, z_r, \ell, d) = z_c(1+i_c) + z_r(1+i_r) + \ell - d(1+i_d) + \beta U^b,$$

where z_r denoting the real balances of required reserves. We can rewrite the above value function as,

$$W^b(\Pi_b) = \Pi_b + \beta U^b$$

where $\Pi_b \equiv z_c(1+i_c) + z_r(1+i_r) + \ell - d(1+i_d)$. In practice, the interest rate paid on reserves forms the lower bound of the interest rate channel in a typical channel system. Since the 2008 Great Recession, the U.S. Federal Reserve who used to adopt a typical floor system, have also started to pay interests to bank reserves. Therefore, we include a policy instrument i_r to model this important monetary policy tool and consider how i_r shall be used in an economy where CBDC could also bear interests.

For a supplier,

$$W^s = \omega + \beta V^s,$$

where ω is the wealth upon entering CM, and V^s is the value function in the capital market at Stage 2 of next period (since suppliers are only active at Stage 2 and 3).

Moving to Stage 1 in the next period, after the investment shock is realized, type-1 e will directly proceed to the loan/capital market at Stage 2 and type-0 e enter the deposit market to deposit their idle balances. For entrepreneurs,

$$\mathbb{E}U^e\left(\hat{z}_c\right) = nU_1^e\left(\hat{z}_c\right) + (1-n)\,U_0^e\left(\hat{z}_c\right).$$

where $U_1^e(\hat{z}_c) = V_1^e\left(\hat{z}_c\right)$ and $U_0^e\left(\hat{z}_c\right) = W_0^e\left(\hat{z}_c - d, d\right)$ for $d \leq \hat{z}_c$. For banks,

$$U^b = (1-n)\,V^b\left(z_r, d\right) + nV^b\left(0,0\right),$$

since the matching technology in the deposit market is short side (type-0 e) being served, and banks who do not get deposits will exit from the market. That is, $V^b\left(0,0\right) = W^b\left(0\right) = 0$, with $\Pi_b = 0$. Here, we let $z_r = d$, representing banks use all of the deposits as reserves, so as to make loans later.

In the loan market at Stage 3, type-1 e and banks with deposits meet. For type-1 e,

$$V_1^e\left(\hat{z}_c\right) = \frac{1-n}{n} W_1^e\left(\hat{z}_c - \frac{p_b}{1+i_c}, \ell, k_b\right) + \frac{2n-1}{n} W_1^e\left(\hat{z}_c - \frac{p_m}{1+i_c}, 0, k_m\right),$$

where we again use a simple matching function $\min\{n, 1-n\} = 1-n$ (given $n > 1/2$). Therefore, with probability $(1-n)/n$, a type-1 e successfully matches with a bank and uses down payment p_b, $p_b \leq \hat{z}_c(1+i_c)$ to get a loan ℓ to acquire capital k_b. With the rest probability, type-1 e does not obtain loans and can only resort to internal finance p_m, $p_m \leq \hat{z}_c(1+i_c)$, to acquire capital k_m. Notice the subscripts (b, m) denote terms related to banked and unbanked type-1 e. For a bank,

$$V^b\left(z_r, d\right) = -\left(k_b - p_b\right) + W^b\left(p_b, z_r, \ell, d\right),$$

where $\ell = k_b - p_b + \phi$ is the total amount the type-1 e repays the bank. More specifically, $k_b - p_b$ represents the loan size to support the purchase of capital and ϕ is the banking service fee.

For suppliers in the capital market,

$$V^s = \max_k \{-k + W^s(q_k k)\},$$

which leads to $q_k = 1$.

3.1 Bargaining

After defining the value functions, we consider how the deposit contract and loan contract are determined. In the deposit market, the surplus of banks and type-0 e are

$$\begin{aligned} V^b(z_r, d) - V^b(0,0) &= \phi + d(i_r - i_d) \\ W_0^e(\hat{z}_c - d, d) - W_0^e(\hat{z}_c, 0) &= d(i_d - i_c). \end{aligned}$$

Let γ be the bargaining share of banks, $0 < \gamma < 1$. The Nash bargaining in the deposit market is

$$\max_{d, i_d} [\phi + d(i_r - i_d)]^\gamma [(i_d - i_c) d]^{1-\gamma}$$
$$\text{st. } d \leq \hat{z}_c.$$

For simplicity, suppose banks make take-it-or-leave-it offer. Then $\gamma = 1$ and $(i_d - i_c) d = 0$. It implies that $i_d = i_c$. We let $d = \hat{z}_c$ since type-0 e should be indifferent to deposit or not.

In the loan market, the surplus of type-1 e is,

$$\begin{aligned} & W_1^e\left(\hat{z}_c - \frac{p_b}{1+i_c}, \ell, k_b\right) - W_1^e\left(\hat{z}_c - \frac{p_m}{1+i_c}, 0, k_m\right) \\ & = f(k_b) - k_b - \phi - \Delta_m, \end{aligned}$$

where $\Delta_m \equiv f(k_m) - p_m$. For unbanked type-1 e, we have $p_m = k_m = \hat{z}_c(1+i_c)$. The bank's surplus is ϕ. Let the bank's bargaining power be θ. Then taking d and $\hat{z}_c$ as given, the Kalai bargaining problem is

$$\max_{p_b,\phi,k_b} \phi$$

$$\text{st. } \phi = \theta\left[f(k_b) - k_b - \Delta_m\right], \tag{3}$$

$$k_b - p_b + \phi \leq \chi f(k_b), \tag{4}$$

$$k_b - p_b \leq \delta d(1+i_r), \tag{5}$$

$$p_b \leq \hat{z}_c(1+i_c), \tag{6}$$

where $\delta \equiv 1/\upsilon - 1$, υ is the reserve ratio. The first constraint (4) indicates the collateral constraint for type-1 e: he uses χ fraction of final output $f(k)$ as collateral, to get bank loans. The second constraint (5) indicates the reserve constraint for a bank: the amount of lending is constrained by the amount of reserves (plus interests) held by the bank. Banks need to satisfy the reserve requirement, and the maximum amount of loans issued by the bank is constrained by the rest (excess) reserves plus interests. The third constraint (6) indicates the down payment p_b cannot exceed the real balances of CBDC plus interests by type-1 e. We let $p_b = \hat{z}_c(1+i_c)$ because there is no benefit for type-1 e to keep some extra CBDC given that $i_c \leq i$.

We set up the Lagrangian function,

$$\begin{aligned}\mathcal{L}(k_b,\lambda_1,\lambda_2) = &\max_{k_b,\lambda_1,\lambda_2} \theta\left[f(k_b) - k_b - \Delta_m\right] \\ &- \lambda_1\left\{k_b - \hat{z}_c(1+i_c) + \theta\left[f(k_b) - k_b - \Delta_m\right] - \chi f(k_b)\right\} \\ &- \lambda_2\left[k_b - \hat{z}_c(1+i_c) - \delta(1+i_r)d\right].\end{aligned}$$

The FOCs with respect to $(k_b, \lambda_1, \lambda_2)$ are,

$$k_b : \theta[f'(k_b) - 1] = \lambda_1[(\theta - \chi)f'(k_b) + 1 - \theta] + \lambda_2$$

$$\lambda_1 : k_b - \hat{z}_c(1 + i_c) + \theta[f(k_b) - k_b - \Delta_m] - \chi f(k_b) = 0$$

$$\lambda_2 : k_b - \hat{z}_c(1 + i_c) - \delta(1 + i_r)d = 0,$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$. Hence, there are three cases to consider as follows.

Case 1: $\lambda_1 = 0$ and $\lambda_2 = 0$. Neither the collateral constraint nor the reserve constraint binds. Then we have $k_b = k^*$, with $f'(k^*) = 1$, and

$$\phi = \theta[f(k^*) - k^* - \Delta_m].$$

Case 2: $\lambda_1 > 0$ and $\lambda_2 = 0$. The collateral constraint binds but the reserve constraint does not. We have

$$(\theta - \chi)f(k_b) + (1 - \theta)k_b = \hat{z}_c(1 + i_c) + \theta\Delta_m \tag{7}$$

$$\lambda_1 = \frac{\theta[f'(k_b) - 1]}{(\theta - \chi)f'(k_b) + 1 - \theta} > 0 \tag{8}$$

to solve for (k_b, λ_1) and ϕ can be solved from (3).

Case 3: $\lambda_1 = 0$ and $\lambda_2 > 0$. The reserve constraint binds but the collateral constraint does not. We solve for (k_b, λ_2) from

$$k_b = \delta d(1 + i_r) + \hat{z}_c(1 + i_c) \tag{9}$$

$$\lambda_2 = \theta[f'(k_b) - 1] > 0,$$

and ϕ is solved from (3). Notice that a fourth case with $\lambda_1 > 0$ and $\lambda_2 > 0$ is not generically possible because both the collateral and reserve constraints can be used to solve for k_b.

3.2 General Equilibrium

After solving for the deposit contract and the loan contract, we can use these conditions to find the choice of $\hat{z}_c$ at Stage 3. The term that is important for the determination of $\hat{z}_c$ is $\beta \mathbb{E} U^e(\hat{z}_c)$

$$\begin{aligned}\mathbb{E}U^e(\hat{z}_c) &= nU_1^e(\hat{z}_c) + (1-n)U_0^e(\hat{z}_c) \\ &= (1-n)[f(k_b) - k_b - \phi - \Delta_m] + n[\hat{z}_c(1+i_c) - p_m + f(k_m)] \\ &+ (1-n)[d(1+i_d) + (\hat{z}_c - d)] + nW_1^e(0,0) + (1-n)W_0^e(0,0).\end{aligned}$$

With $p_m = \hat{z}_c(1+i_c) = k_m$, $d = \hat{z}_c$, and $i_d = i_c$, we have

$$\begin{aligned}\frac{\partial \mathbb{E}U^e(\hat{z}_c)}{\partial \hat{z}_c} &= (1-n)(1-\theta)\{[f'(k_b)-1]\frac{\partial k_b}{\partial \hat{z}_c} - [f'(k_m)-1](1+i_c)\} \\ &\quad + n[f'(k_m)-1](1+i_c) + (1+i_c).\end{aligned} \tag{10}$$

To get $\partial k_b / \partial \hat{z}_c$, we have three cases to consider for the general equilibrium analysis.

In case 1, $k_b = k^*$ and $\partial k_b / \partial \hat{z}_c = 0$. Using (2) and (10), we solve for k_m from

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1]. \tag{11}$$

where we define $A \equiv n - (1-n)(1-\theta)$. In (11), the LHS and RHS represent the cost and benefit of holding an additional unit of CBDC, respectively. Knowing (k_b, k_m), (3) gives the equilibrium value of ϕ. In this case, neither the collateral constraint nor the reserve constraint binds. We label this case 1 equilibrium as unconstrained equilibrium.

In case 2, k_b is determined in (7). Only the collateral constraint binds. We label this type of equilibrium collateral constrained equilibrium. It follows that

$$\frac{\partial k_b}{\partial \hat{z}_c} = \frac{(1+i_c)[\theta f'(k_m) + (1-\theta)]}{(\theta - \chi) f'(k_b) + 1 - \theta} > 0.$$

From (2) and (10), k_m solves

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + \frac{(n - A)\left[\theta f'(k_m) + (1 - \theta)\right]}{(\theta - \chi) f'(k_b) + 1 - \theta}[f'(k_b) - 1]. \tag{12}$$

When the collateral constraint binds, having an additional unit of CBDC helps relax the collateral constraint and increase the loan size, which further leads to a higher k_b. The second term in the RHS of (12) shows this additional benefit of CBDC. We then solve for (k_b, ϕ) from (7) and (3).

In case 3, (9) includes the bank's reserve d and CBDC $\hat{z}_c$ held by the type-1 e. Notice that d comes from deposits by another type-0 e. Despite that $d = \hat{z}_c$ in equilibrium, an entrepreneur's choice of $\hat{z}_c$ should not affect k_b through d. It implies that $\partial k_b / \partial \hat{z}_c = 1 + i_c$. Again, (2) and (10) help determine k_m from

$$\frac{i - i_c}{1 + i_c} = A[f'(k_m) - 1] + (n - A)[f'(k_b) - 1]. \tag{13}$$

The binding reserve constraint means that an additional unit of CBDC helps relax the reserve constraint and raise the value of k_b. This additional benefit is reflected by the second term in (13). Finally, (9) leads to

$$k_b = \left[\delta \frac{1 + i_r}{1 + i_c} + 1\right] k_m \tag{14}$$

and ϕ is given by (3). With the binding reserve constraint, we label this type of equilibrium as the reserve constrained equilibrium.

4 Policy Analysis

The equilibrium conditions in the three cases that we have outlined, include four monetary policy parameters (i, i_c, i_r, υ). In this section, we use the baseline model to analyze how monetary policy affect investment, output and banking activities. The interest-bearing aspect of CBDC has raised concerns that CBDC could lead to financial disintermediation. In particular, the interest-bearing CBDC can crowd out bank deposits so that banks' lending

activities can be adversely affected. This concern of CBDC views CBDC and bank deposits as substitutes. In our model, banks can take CBDC as deposits and issue loans. The reserve requirement induces banks to take deposits to make loans. CBDC and deposits become complements. We highlight this new perspective of CBDC because how CBDC and banking activities interact depend on the design of CBDC. In Andolfatto (2018), banks can only store CBDC but cannot accept CBDC as deposits to issue loans. We consider a design of CBDC and banking such that banks can take CBDC as deposits. As long as banks satisfy the reserve requirement, they can make loans to facilitate investment. In such an environment, we can examine how the central bank can set the interest rate on CBDC i_c, the interest rate on reserves i_r and the growth rate of money supply μ which is equivalent to i. In addition, the reserve requirement υ serves as a parameter that is related to banking policy.

Given the three types of equilibrium, we proceed to investigate the effects of monetary policy in each type of equilibrium. Notice that i_r and υ enter into the equilibrium conditions only in the reserve constrained equilibrium. In the other two types of equilibrium, they do not affect the equilibrium allocation as the reserve constraint does not bind. Both i and i_c affect the equilibrium allocation in all types of equilibrium. To be more specific, we focus on k_b, k_m, aggregate investment $K = (1-n)\,k_b + (2n-1)k_m$, aggregate lending $L = (1-n)\,(k_b - k_m)$, the real deposit rate $r_d = (1+i_d)\,/\,(1+\pi) - 1$ and the real loan rate $r_\ell = \phi/\,(k_b - k_m)$.

In the unconstrained equilibrium, type-1 e can invest the optimal amount of capital k^*, which is independent of the policy parameters. We summarize in Proposition 1 the effects of changing i and i_c. Type-0 e can afford more k_m when i_c increases. Notice that for a given amount of CBDC, the higher interest rate allows entrepreneurs to purchase more capital. It does not necessarily imply that entrepreneurs would hold more CBDC. Since type-1 e borrow $k^* - k_m$ from banks, a higher i_c reduces the amount of bank lending. The simple take-it-or-leave-it offer makes the nominal deposit rate equal to i_c. Therefore, the real deposit rate increases. The real loan rate r_ℓ depends on the banking fee ϕ and the amount of lending $k^* - k_m$.

Proposition 1 *In an unconstrained equilibrium, a higher i_c leads to a higher k_m, a higher*

K, a lower L, a higher r_d and a lower r_ℓ.

Proof 1 *For the effects of changing i_c, we have*

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{1+i}{Af''(k_m)(1+i_c)^2} > 0 \\
\frac{\partial \phi}{\partial i_c} &= -\theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial L}{\partial i_c} &= -(1-n)\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{(k^*-k_m)\frac{\partial \phi}{\partial i_c} + \phi\frac{\partial k_m}{\partial i_c}}{(k^*-k_m)^2}.
\end{aligned}$$

Using the expression of $\partial\phi/\partial i_c$ and $\partial k_m/\partial i_c$, we derive

$$\begin{aligned}
\frac{\partial r_\ell}{\partial i_c} &= \frac{-(k^*-k_m)\,\theta[f'(k_m)-1]+\phi}{(k^*-k_m)^2}\frac{\partial k_m}{\partial i_c} \\
&\simeq \phi-(k^*-k_m)\,\theta\,[f'(k_m)-1] \\
&= f(k^*)-f(k_m)-f'(k_m)(k^*-k_m) < 0
\end{aligned}$$

due to the concavity of the production function.

The results for the collateral constrained equilibrium are given in Proposition 2. As in the last case, a higher i_c allows type-0 e to purchase more k_m and type-1 e to have more down payment. Holding more down payment also enables type-1 e to borrow more from banks, so a higher interest rate of CBDC encourages lending and investment. This result is in sharp contrast to the result that CBDC can lead to disintermediation. That is, CBDC competes with deposits and crowds out lending. See Andolfatto (2018) and Keister and Sanches (2020) for examples. The key difference between our model and those models is that CBDC does not substitute bank deposits. Rather, banking is designed such that banks can accept CBDC as deposits. In some senses, CBDC and deposits are substitutes in Andolfatto and Keister and Sanches (2020), but they are complements in our model.

Proposition 2 *In a collateral constrained equilibrium, a higher i_c leads to a higher k_m, a higher k_b, a higher K, a higher L and a higher r_d, but the effect on r_ℓ is ambiguous.*

Proof 2 *For the effects of changing i_c, we use (7) and (12) to derive*

$$\begin{aligned}\frac{\partial k_m}{\partial i_c} &= -\frac{(1+i)[(\theta-\chi)f'(k_b)+1-\theta]}{D(1+i_c)^2} > 0 \\ \frac{\partial k_b}{\partial i_c} &= -\frac{(1+i)[1+\theta(f'(k_m)-1)]}{D(1+i_c)^2} > 0,\end{aligned}$$

where $D \equiv B_2[1+\theta(f'(k_m)-1)]f''(k_b) + B_1[(\theta-\chi)f'(k_b)+1-\theta]f''(k_m) < 0$ *and*

$$\begin{aligned}B_1 &= \frac{\theta n[f'(k_b)-1]+A[1-\chi f'(k_b)]}{\theta[f'(k_b)-1]+1-\chi f'(k_b)} > 0 \\ B_2 &= \frac{(n-A)(1-\chi)[1+\theta(f'(k_m)-1)]}{[(\theta-\chi)f'(k_b)+1-\theta]^2} > 0.\end{aligned}$$

For the aggregate lending L,

$$\begin{aligned}\frac{\partial L}{\partial i_c} &\simeq \frac{\partial k_b}{\partial i_c} - \frac{\partial k_m}{\partial i_c} \\ &= -\frac{(1+i)\{\chi f'(k_b)+\theta[f'(k_m)-f'(k_b)]}{D(1+i_c)^2} > 0.\end{aligned}$$

However, the effects on ϕ and r_ℓ are ambiguous, where

$$\begin{aligned}\frac{\partial \phi}{\partial i_c} &= -\frac{\theta(1+i)\{(1-\chi)f'(k_b)-[1-\chi f'(k_b)]f'(k_m)\}}{D(1+i_c)^2} \lesseqgtr 0 \\ \frac{\partial r_\ell}{\partial i_c} &= \frac{\frac{\partial \phi}{\partial i_c}(k_b-k_m)-\phi\frac{\partial(k_b-k_m)}{\partial i_c}}{(k_b-k_m)^2} \lesseqgtr 0.\end{aligned}$$

In the case that $\partial\phi/\partial i_c < 0$, we will have $\partial r_\ell/\partial i_c < 0$.

In the reserve constrained equilibrium, we can examine how the interest rate on reserves i_r and the reserve requirement υ affect investment and banking in addition to i_c.

Proposition 3 *In a reserve constrained equilibrium, a higher i_r leads to a lower k_m, a higher k_b, a lower K assuming $f'''(k) > 0$, a higher L. A change in i_r does not affect r_d and has an ambiguous effect on r_ℓ. The effects of υ are the opposite as the effects of i_r. In terms of i_c, a higher i_c leads to a higher k_m and a higher r_d, but its effect on k_b is ambiguous in general.*

Proof 3 *From (13) and (14),*

$$\begin{aligned}
\frac{\partial k_m}{\partial i_r} &= -\frac{\delta k_m}{1+\frac{\delta(1+i_r)}{1+i_c}+\frac{Af''(k_m)}{(n-A)f''(k_b)}} < 0 \\
\frac{\partial k_b}{\partial i_r} &= -\frac{Af''(k_m)}{(n-A)f''(k_b)}\frac{\partial k_m}{\partial i_r} > 0 \\
\frac{\partial K}{\partial i_r} &= (1-n)\frac{\partial k_b}{\partial i_r} + n\frac{\partial k_m}{\partial i_r} \\
&= \left[n-(1-n)\frac{Af''(k_m)}{(n-A)f''(k_b)}\right]\frac{\partial k_m}{\partial i_r} \\
&\simeq \frac{A-n^2 f''(k_b)+nA\left[f''(k_b)-f''(k_m)\right]}{(n-A)f''(k_b)}
\end{aligned}$$

If the production function $f(k)$ satisfies $f'''(k) > 0$, then $\partial K/\partial i_r < 0$. Since L depends on $k_b - k_m$, we have $\partial L/\partial i_r > 0$. The real deposit rate does not change as the nominal deposit rate equals i_c. The real loan rate depends on ϕ and $k_b - k_m$, but both ϕ and $k_b - k_m$ increase when i_r increase. Furthermore,

$$\begin{aligned}
\frac{\partial r_\ell}{\partial i_r} &= \frac{\theta\left[f'(k_b)\frac{\partial k_b}{\partial i_r} - f'(k_m)\frac{\partial k_m}{\partial i_r}\right](k_b-k_m) - \theta\left[f(k_b) - f(k_m)\left(\frac{\partial k_b}{\partial i_r} - \frac{\partial k_m}{\partial i_r}\right)\right]}{(k_b-k_m)^2} \\
&\simeq \frac{(n-A)f''(k_b)\left\{(k_b-k_m)f'(k_m) - \left[f(k_b)-f(k_m)\right]\right\} - Af''(k_m)\left\{(k_b-k_m)f'(k_b) - \left[f(k_b)-f\right.\right.}{(n-A)f''(k_b)}
\end{aligned}$$

has an ambiguous sign. Similar to i_r, the reserve requirement υ enters only into (14). It has the opposite effects as i_r.

For the effects of i_c,

$$\frac{\partial k_m}{\partial i_c} = \frac{(n-A)(k_b-k_m)f''(k_b) - (1+i)/(1+i_c)}{(n-A)\left[\delta(1+i_r)+(1+i_c)\right]f''(k_b) + A(1+i_c)f''(k_m)} > 0.$$

Given that $i_d = i_c$, $\partial r_d/\partial i_c > 0$. The effects of i_c on (k_b, K, L, r_ℓ) are generally ambiguous.

As in the other two cases, a higher i_c leads to a higher k_m because it directly benefits entrepreneurs that rely on internal finance. In the reserve constrained equilibrium, k_b is constrained by the amount of reserves. Reserves held by banks earn interests on reserves, but forego the interests paid on CBDC. A higher i_c indirectly tightens the reserve constraint.

The overall effect of i_c on k_b becomes ambiguous. As a result, its effects on investment and lending are ambiguous.

The reserve constrained equilibrium is an interesting case as the interest rate on reserve plays a non-trivial role. In practice, the interest rate on excess reserves forms a lower bound for the channel system and the floor system operated by modern central banks. Our model suggests that the interest rate on reserves can have an additional role in affecting bank lending and investment in the economy.

We also consider to special cases of i_r. The first special case is $i_r = 0$ and the second special case is $i_r = i_c$. In the first case, reserves do not bear any interests. In the second case, reserves earn the same interest rate as CBDC. This special case makes reserves less costly. The reserve constraint implies that $k_b = (1 + \delta) k_m$. Type-1 e's investment is proportional to type-0 e's investment. It follows that $\partial k_b / \partial i_c > 0$, $\partial K / \partial i_c > 0$, $\partial L / \partial i_c > 0$ and $\partial r_\ell / \partial i_c < 0$.

5 Cash and CBDC

Our benchmark model considers an extreme scenario where CBDC is the only asset in the economy. It shows that interest-bearing CBDC does not necessarily lead to financial disintermediation. The complementary role of CBDC and banking makes CBDC essential for banking business. In practice, countries that consider the adoption of CBDC will feature the coexistence of traditional cash and CBDC. Therefore, we use this section to consider when and how cash and CBDC can coexist in a model where money and banking serve complementary roles.

The environment remains very similar to the benchmark model, except that entrepreneurs can hold a portfolio of cash and CBDC. We consider the natural case where cash and CBDC have the same value and the same growth rate μ.[7] While entrepreneurs can hold both assets, we consider two extreme scenarios where either cash or CBDC can be accepted by banks as deposits at Stage 1. Specifically, the first scenario in terms of banking and money is that banks can take only cash as deposits and cash can be turned into reserves. Banks can help

[7] It might be interesting to consider an endogenous exchange rate between cash and CBDC. We want to focus on the simple and more natural scenario where they have the same value.

entrepreneurs to store CBDC, but CBDC cannot be used as reserves. This way of modeling CBDC can be found in Andolfatto (2018), where banks can help individuals to store CBDC, but cannot use CBDC to issue loans.

In the second scenario, we consider the opposite banking arrangement compared with the first scenario. That is, banks accept only CBDC as deposits and CBDC can be turned into reserves. Banks do not accept cash deposits and cash cannot be used by banks as reserves. This type of banking arrangement shares feature of online banking where banks deal with electronic assets and physical cash is generally not accepted as deposits.

In the following, we briefly outline how adding cash into the model and adjusting the banking arrangement change the value functions and decisions. We generalize the bargaining power to $0 < \gamma < 1$ in this section. The results from these two models can help us understand the potential economic tradeoff between cash and CBDC, and how banking arrangements or regulations can affect these tradeoffs. We begin with the first scenario where cash is accepted as deposits.

5.1 Cash Only Deposits

5.1.1 Value Functions

At Stage 3, for type-1 e,

$$\begin{aligned} W_1^e(z, z_c, \ell, k) &= = \max_{x,\hat{z},\hat{z}_c} \{x + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c)\} \\ \text{st. } x + (1+\mu)\hat{z} + (1+\mu)\hat{z}_c &= z + (1+i_c)z_c - \ell + f(k) + T + \Pi. \end{aligned}$$

Then,

$$W_1^e(\omega, \ell, k) = \omega - \ell + f(k) + T + \Pi +$$

$$\max_{\hat{z},\hat{z}_c} \{-(1+\mu)\hat{z} - (1+\mu)\hat{z}_c + \beta \mathbb{E} U^e(\hat{z}, \hat{z}_c)\},$$

where $\omega \equiv z + (1+i_c)z_c$, representing financial wealth of cash and CBDC (including CBDC interests) of type-1 e. The FOCs are,

$$\begin{aligned} 1+\mu &= \beta \frac{\partial \mathbb{E}U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}} \\ 1+\mu &= \beta \frac{\partial \mathbb{E}U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c}. \end{aligned}$$

Similarly, for type-0 e, the unconstrained maximization problem is

$$W_0^e(\omega, d) = \omega + (1+i_d)d + T + \Pi + \max_{\hat{m}} \{-(1+\mu)\hat{z} - (1+\mu)\hat{z}_c + \beta \mathbb{E}U^e(z, \hat{z})\}.$$

The FOCs wrt $\hat{z}$ and $\hat{z}$ is the same as typ-1 e.

For banks,

$$W^b(\Pi_b) = W^b(\omega, z_r, d, \ell) = \Pi_b + \beta U^b$$

where

$$\Pi_b = \omega + (1+i_r)z_r - (1+i_d)d + \ell.$$

For suppliers,

$$W^s = \omega + \beta V^s.$$

At Stage 1 next period, banks and type-0 e are active in the deposit market,

$$\mathbb{E}U^e(\hat{z}, \hat{z}_c) = nU_1^e(\hat{z}, \hat{z}_c) + (1-n)U_0^e(\hat{z}, \hat{z}_c).$$

For type-0 and type-1 e,

$$U_0^e(\hat{z}, \hat{z}_c) = W_0^e(\hat{z} - d, \hat{z}_c, d) \text{ and } U_1^e(\hat{z}, \hat{z}_c) = V_1^e(\hat{z}, \hat{z}_c).$$

For banks,

$$U^b = (1-n)V^b(z_r, d) + nV^b(0,0).$$

All deposits will be used as reserves $z_r = d$. If a bank does not take deposits, the bank

cannot make loans. Again, $V^b(0,0) = W^b(0)$.

At stage 2 of the loan market, banks with deposits and type 1 entrepreneurs meet. For a type-1 entrepreneur,

$$V_1^e(z,\hat{z}) = \frac{1-n}{n} W_1^e(\omega - p_b, \ell, k_b) + \frac{2n-1}{n} W_1^e(\omega - p_m, 0, k_m).$$

For a bank,

$$V^b(z_r, d) = -(k_b - p_b) + W^b(p_b, z_r, d, \ell),$$

where again $\ell = k_b - p_b + \phi$.

5.1.2 Bargaining

In the deposit market, a bank's surplus is given by

$$V^b(z_r, d) - V^b(0) = \phi + (i_r - i_d)\, d,$$

taking into consideration that banks use the deposits from type-0 entrepreneurs as reserves, i.e., $z_r = d$. For a type 0 entrepreneur, the surplus is

$$W_0^e(\hat{z} - d, \hat{z}_c, d) - W_0^e(\hat{z}, \hat{z}_c, 0) = i_d d.$$

Let γ be the bank's bargaining share in the deposit market where $0 < \gamma \leq 1$. The Nash bargaining is

$$\max_{d, i_d} [\phi + (i_r - i_d)\, d]^{\gamma} (i_d d)^{1-\gamma} \text{ st. } d \leq \hat{z}.$$

In the extreme case that banks make take-it-or-leave-it offer, $\gamma = 1$, we have $i_d = 0$ as in the benchmark model. For $0 < \gamma < 1$, the FOC wrt i_d gives

$$i_d d = (1-\gamma)(\phi + i_r d).$$

It is natural to consider $d = \hat{z}$ because type 0 entrepreneurs should prefer to deposit all cash given that $\gamma < 1$.

In the loan market, type-1 entrepreneur's surplus is

$$\begin{aligned} & W_1^e(\omega - p_b, \ell, k_b) - W_1^e(\omega - p_m, 0, k_m) \\ & = f(k_b) - k_b - \phi - [f(k_m) - p_m]. \end{aligned}$$

If the entrepreneur does not take loans from the bank, the entrepreneur uses all $\hat{z}$ and $\hat{z}_c$ to purchase k_m. It follows $p_m = \hat{z} + (1 + i_c)\hat{z}_c = k_m$. Define $\Delta_m \equiv f(k_m) - \hat{z} - (1 + i_c)\hat{z}_c$. The bank's surplus in the loan market is ϕ. Let the bank's bargaining power be θ. The Kalai bargaining problem is

$$\begin{aligned} & \max_{k_b, p_b, \phi} \phi \\ & \text{st. } \phi = \theta[f(k_b) - k_b - \Delta_m], \\ & k_b - p_b + \phi \leq \chi f(k_b), \\ & k_b - p_b \leq \delta(1 + i_r) d, \\ & p_b \leq \hat{z} + (1 + i_c)\hat{z}_c. \end{aligned}$$

We focus on $p_b = \hat{z} + (1 + i_c)\hat{z}_c$ because there is no benefit for type 1 entrepreneurs to keep some extra assets given that $i_c \leq i$.

We can set up the Lagrangian

$$\begin{aligned} \mathcal{L} = & \max_{k_b, \lambda_1, \lambda_2} \theta[f(k_b) - k_b - \Delta_m] \\ & - \lambda_1 \{k_b - \hat{z} - (1 + i_c)\hat{z}_c + \theta[f(k_b) - k_b - \Delta_m] - \chi f(k_b)\} \\ & - \lambda_2 [k_b - \delta(1 + i_r) d - \hat{z} - (1 + i_c)\hat{z}_c]. \end{aligned}$$

The FOCs wrt $(k_b, \lambda_1, \lambda_2)$ are

$$k_b : \theta \left[f'(k_b) - 1 \right] = \lambda_1 \left[(\theta - \chi) f'(k_b) + 1 - \theta \right] + \lambda_2,$$

$$\lambda_1 : k_b - \hat{z} - (1 + i_c) \hat{z}_c + \theta \left[f(k_b) - k_b - \Delta_m \right] - \chi f(k_b) = 0,$$

$$\lambda_2 : k_b - \delta (1 + i_r) d - \hat{z} - (1 + i_c) \hat{z}_c = 0.$$

Case 1: $\lambda_1 = 0$ and $\lambda_2 = 0$. Neither constraint is binding, and we have $f'(k_b) = 1$.

$$\phi = \theta \left[f(k^*) - k^* - \Delta_m \right].$$

Case 2: $\lambda_1 > 0$ and $\lambda_2 = 0$. The collateral constraint binds but the reserve constraint does not bind. We have

$$\theta \left[f'(k_b) - 1 \right] = \lambda_1 \left[(\theta - \chi) f'(k_b) + 1 - \theta \right]$$

and

$$\begin{aligned} \phi &= \theta \left[f(k_b) - k_b - \Delta_m \right] \\ &= \chi f(k_b) - k_b + \hat{z} + (1 + i_c) \hat{z}_c \end{aligned}$$

to solve for (k_b, ϕ, λ_1).

Case 3: $\lambda_1 = 0$ and $\lambda_2 > 0$. The reserve constraint binds but the collateral constraint does not bind. We solve for (k_b, ϕ, λ_2) from

$$\begin{aligned} k_b &= \delta (1 + i_r) d + \hat{z} + (1 + i_c) \hat{z}_c \\ \phi &= \theta \left[f(k_b) - k_b - \Delta_m \right] \\ \lambda_2 &= \theta \left[f'(k_b) - 1 \right]. \end{aligned}$$

Notice that a fourth case with $\lambda_1 > 0$ and $\lambda_2 > 0$ is not generically possible because both

the collateral constraint and the reserve constraint can be used to solve k_b.

5.1.3 General Equilibrium

To determine an entrepreneur's asset choice, we first find the term $\mathbb{E}U^e(\hat{z}, \hat{z}_c)$, where

$$\begin{aligned}\mathbb{E}U^e(\hat{z}, \hat{z}_c) &= nU_1^e(\hat{z}, \hat{z}_c) + (1-n)U_0^e(\hat{z}, \hat{z}_c) \\ &= (1-n)\left[f(k_b) - k_b - \phi\right] + + (2n-1)\left[f(k_m) - k_m\right] + \hat{z} + (1+i_c)\hat{z}_c \\ &+ (1-n)\left[(1-\gamma)\left(\phi^d + i_r d\right)\right] + nW_1^e(0,0,0) + (1-n)W_0^e(0,0).\end{aligned}$$

Notice that we introduce a new notation ϕ^d to represent the banking fee that a depositor's bank earns in the loan market. For type-0 entrepreneurs, it is ϕ^d that matters for their choices of assets. Given that $k_m = \hat{z} + (1+i_c)\hat{z}_c$, we have

$$\frac{\partial k_m}{\partial \hat{z}} = 1 \text{ and } \frac{\partial k_b}{\partial \hat{z}} = 1 + i_c.$$

The expressions of $\partial k_b/\partial \hat{z}$ and $\partial k_b/\partial \hat{z}_c$ depend on the specific type of banking equilibrium.

In Case 1, $f'(k_b) = 1$ so that k_b does not depend on $\hat{z}$ and $\hat{z}_c$.

$$\begin{aligned}\frac{\partial \mathbb{E}U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}} &= \left[(2n-1) + \theta(1-n)\right]\left[f'(k_m) - 1\right]\frac{\partial k_m}{\partial \hat{z}} \\ &+ (1-n)(1-\theta)\frac{\partial k_b}{\partial \hat{z}} + (1-n)(1-\gamma)\left(\frac{\partial \phi^d}{\partial \hat{z}} + i_r\right) + 1.\end{aligned}$$

Notice that in case 1, banking fee does not depend on the depositor's cash balance. It follows that the FOC wrt $\hat{z}$ is

$$i = \left[(2n-1) + \theta(1-n)\right]\left[f'(k_m) - 1\right] + (1-n)(1-\gamma)i_r.$$

where $1 + i = \rho/(\beta\hat{\rho})$. Similarly, the FOC wrt $\hat{z}_c$ yields

$$\frac{i - i_c}{1 + i_c} = \left[(2n-1) + \theta(1-n)\right]\left[f'(k_m) - 1\right].$$

For cash and CBDC to coexist, it requires

$$\frac{i - i_c}{1 + i_c} = i - (1 - n)(1 - \gamma) i_r. \tag{15}$$

In Case 2, k_b is solved from

$$\theta \left[f(k_b) - k_b - f(k_m) + k_m \right] = \chi f(k_b) - k_b + k_m.$$

We have

$$\begin{aligned} \frac{\partial k_b}{\partial \hat{z}} &= \frac{\theta f'(k_m) + (1 - \theta)}{(\theta - \chi) f'(k_b) + 1 - \theta}, \\ \frac{\partial k_b}{\partial \hat{z}_b} &= \frac{\left[\theta f'(k_m) + (1 - \theta)\right](1 + i_c)}{(\theta - \chi) f'(k_b) + 1 - \theta}. \end{aligned}$$

Then the FOCs wrt to $\hat{z}$ and $\hat{z}_c$ become

$$\begin{aligned} i &= (1 - n)(1 - \theta)\left[f'(k_b) - 1\right] \frac{\theta f'(k_m) + 1 - \theta}{(\theta - \chi) f'(k_b) + 1 - \theta} \\ &\quad + \left[(2n - 1) + \theta(1 - n)\right]\left[f'(k_m) - 1\right] + (1 - n)(1 - \gamma) i_r \\ \frac{i - i_c}{1 + i_c} &= (1 - n)(1 - \theta)\left[f'(k_b) - 1\right] \frac{\theta f'(k_m) + 1 - \theta}{(\theta - \chi) f'(k_b) + 1 - \theta} \\ &\quad + \left[(2n - 1) + \theta(1 - n)\right]\left[f'(k_m) - 1\right]. \end{aligned}$$

Again, in case 2, the reserve constraint does not bind. The banking fee does not depend on the depositor's cash balance. For cash and CBDC to coexist, we require the same condition as in case 1.

In case 3, $k_b = \delta(1 + i_r) d + \hat{z} + (1 + i_c) \hat{z}_c$. Notice that the bank's reserve d comes from some other type 0 entrepreneur's deposits. Therefore, the deposit's cash balance can affect its bank's banking fee and

$$\frac{\partial \phi_d}{\partial d} = \theta \left[f'(k_b) - 1\right] \delta (1 + i_r).$$

Despite that $d = \hat{m}$ in equilibrium, an individual's choice of $\hat{m}$ should not affect k_b through the deposit channel. Therefore,

$$\frac{\partial k_b}{\partial \hat{z}} = 1 \text{ and } \frac{\partial k_b}{\partial \hat{z}_c} = 1 + i_c.$$

It follows that

$$\begin{aligned} i &= (1-\theta)(1-n)\left[f'(k_b) - 1\right] + \left[(2n-1) + \theta(1-n)\right]\left[f'(k_m) - 1\right] \\ &\quad + (1-n)(1-\gamma)\, i_r + (1-n)(1-\gamma)\,\theta\left[f'(k_b) - 1\right]\delta(1+i_r) \\ \frac{i - i_c}{1+i_c} &= (1-\theta)(1-n)\left[f'(k_b) - 1\right] + \left[(2n-1) + \theta(1-n)\right]\left[f'(k_m) - 1\right]. \end{aligned} \tag{16}$$

The coexistence of cash and CBDC implies

$$\frac{i - i_c}{1+i_c} = i - (1-n)(1-\gamma)\, i_r - (1-n)(1-\gamma)\,\theta\left[f'(k_b) - 1\right]\delta(1+i_r), \tag{17}$$

which is used to solve for k_b and either one of the FOCs can be used to solve for k_m. Knowing (k_m, k_b), the reserve constraint is used to solve for $\hat{z}$,

$$k_b = \delta(1+i_r)\,\hat{z} + k_m. \tag{18}$$

5.1.4 Policy Analysis

In Case 1 and Case 2, (15) ensures that cash and CBDC can coexist. Suppose (15) is satisfied. Cash does not have any nominal return, but when banks use cash as reserves, cash earns an interest rate i_r. This additional value of cash generates a tradeoff between cash and CBDC. Condition (15) requires a specific relationship between i_c and i_r such that entrepreneurs are indifferent between holding cash and CBDC. The comparative statics results remain the same as in the benchmark model. As long as $\hat{z} + (1+i_c)\,\hat{z}_c = k_m$, the exact portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate.

When (15) is not satisfied, either cash or CBDC should be driven out of existence.

Suppose i_r is too low to satisfy (15), cash is not attractive enough for entrepreneurs to use. Entrepreneurs optimally choose to hold CBDC. However, CBDC cannot be accepted as deposits. If banks do not take deposits, they cannot issue loans. The economy will function as if banks do not exist. All type-1 entrepreneurs will rely on internal finance to purchase capital. Suppose i_r is too high to satisfy (15), cash will dominate CBDC and becomes the only asset chosen by entrepreneurs. In this case, the economy effectively functions as the benchmark economy with $i_c = 0$. Given that the central bank can adjust both i_r and i_c, it can give up one policy tool (either i_r or i_c) to ensure the condition in (15) is satisfied. Therefore, achieving coexistence of cash and CBDC can be achieved through monetary policy.

Case 3 is a more interesting case. Cash and CBDC can coexist, and the portfolio choice of $(\hat{z}, \hat{z}_c)$ is determinate. There is no need for the central bank to sacrifice any monetary policy tool to maintain the coexistence of cash and CBDC. The binding reserve constraint implies that the interests earned on deposits depend on the banking fee, which in turn depends on the size of deposits. Compared to case 1 and case 2, cash has the additional value through affecting the return on deposits. Therefore, the tradeoff between cash and CBDC depends on i_r and the extra return on cash through the banking fee. Entrepreneurs hold a determinate portfolio of cash and CBDC. The coexistence of cash and CBDC allows us to investigate how cash and CBDC interact.

When i_c increases, the LHS of (17) decreases, which implies that k_b should decrease. From (16), a lower k_b leads to a higher k_m. Notice that $k_b - k_m = \delta (1 + i_r) \hat{z}$. We know that $\hat{z}$ should decrease and $\hat{z}_c$ would increase as a result of a higher i_c. To summarize, we have

$$\frac{\partial \hat{z}}{\partial i_c} < 0, \ \frac{\partial \hat{z}_c}{\partial i_c} > 0, \ \frac{\partial k_m}{\partial i_c} > 0, \ \frac{\partial k_b}{\partial i_c} < 0 \text{ and } \frac{\partial L}{\partial i_c} < 0$$

The higher interest rate on CBDC induces entrepreneurs to hold more CBDC, which reduces their need for cash. Since only cash serves as reserves, type-0 entrepreneurs deposit less cash and banks issue less loans to banked type-1 entrepreneurs. In this sense, the higher return CBDC crowds out deposits, which reduces the amount of lending in the economy. For type-1 entrepreneurs, unbanked entrepreneurs purchase capital using their own portfolios consisting of cash and CBDC. It turns out that the increase in the holding of CBDC

dominates the decrease in the holding of cash. Unbanked entrepreneurs are able to raise k_m in response to a higher i_c. In contrast, despite that banked entrepreneurs' own portfolios allow them to purchase more capital, the reduction in bank lending leads to a lower k_b in response to a higher i_c.

5.2 CBDC Only Deposits

5.2.1 Value Functions

At Stage 3, the value functions for type-1 e, type-0 e, banks and suppliers, are the same as Section 5.1.1, so we skip that part algebra here.

Now we move to Stage 1 next period. For e,

$$\mathbb{E}U^e\left(\hat{z},\hat{z}_c\right) = nU_1^e\left(\hat{z},\hat{z}_c\right) + (1-n)\,U_0^e\left(\hat{z},\hat{z}_c\right),$$

where $U_1^e(\hat{z},\hat{z}_c) = V_1^e\left(\hat{z},\hat{z}_c\right)$ and $U_0^e\left(\hat{z},\hat{z}_c\right) = W_0^e\left(\omega-d,d\right)$ for $d \leq \hat{z}_c$. For banks,

$$U^b = (1-n)\,V^b\left(z_r,d\right) + nV^b\left(0,0\right).$$

Notice banks only accept CBDC as deposits, so d only comes from CBDC holdings of type-0 e, not cash. Hence, the deposit contract and bargaining problem is the same as the BM model.

At Stage 2, first type-1 e and banks with deposits meet in the loan market, then type-1 e turns to suppliers to acquire capital. For type-1 e,

$$\begin{aligned} V_1^e\left(\hat{z},\hat{z}_c\right) &= \frac{1-n}{n}[W_1^e(\omega-p_b,\ell,k_b) - W_1^e(\omega-p_m,0,k_m)] + W_1^e(\omega-p_m,0,k_m) \\ &= \frac{1-n}{n}[f(k_b)-k_b-\phi-(f(k_m)-p_m)] + W_1^e(\omega-p_m,0,k_m), \end{aligned}$$

where p_b, $p_m \leq \hat{z}+(1+i_c)\hat{z}_c$. That is, for banked e, down payment p_b cannot exceed the total amount of cash and CBDC (including CBDC interests); for unbanked e, internal finance p_m

cannot exceed the total amount as well. For banks,

$$V^b(z_r, d) = -(k_b - p_b) + W^b(p_b, z_r, d, \ell),$$

where again $\ell = k_b - p_b + \phi$.

For suppliers in the capital market, it is the same as BM model.

5.2.2 Bargaining

For the loan contract, the bargaining problem is almost the same as in the BM model, except $p_b \leq \hat{z} + (1 + i_c)\hat{z}_c$. We again need to consider three cases, and the bargaining solutions for $\{p_b, \phi, k_b\}$ are very similar to Case 1-3 in the BM model, except now $p_b = \hat{z} + (1 + i_c)\hat{z}_c$. Hence, we skip the algebra details here.

As for the deposit contract, the bargaining problem is almost the same as the BM model, since banks only accept CBDC as deposits. However, now we consider a more general case, $0 < \gamma < 1$, instead of T-I-O-L-T offer. Hence,

$$\max_{d, i_d} [\phi + d(i_r - i_d)]^{\gamma} [(i_d - i_c) d]^{1-\gamma}$$

$$\text{st. } d \leq \hat{z}_c.$$

WLOG, suppose $d \leq \hat{z}_c$ binds, then the FOC for i_d is,

$$(1 - \gamma)\phi = [\gamma(i_d - i_c) + (1 - \gamma)(i_d - i_r)]d.$$

Hence,

$$i_d \cdot d = (1 - \gamma)\phi + [\gamma i_c + (1 - \gamma)i_r]d, \tag{19}$$

which shows i_d is related to ϕ, from the solutions of the loan contract. Notice only CBDC can be accepted as deposits in both the BM model and this extended model. However, with a generalized Nash bargaining, we obtain a different result on i_d, compared to $i_d = i_c$, in the BM model.

5.2.3 General Equilibrium

Similarly, after solving the deposit and loan contracts, we can use those solutions to sort out the asset choice for $(\hat{z}, \hat{z}_c)$ at Stage 3. We have,

$$\begin{aligned}
\mathbb{E}U^e(\hat{z}, \hat{z}_c) &= nU_1^e(\hat{z}, \hat{z}_c) + (1-n)U_0^e(\hat{z}, \hat{z}_c) \\
&= n\{\frac{1-n}{n}(1-\theta)[f(k_b) - k_b - \Delta_m] + \hat{z} + (1+i_c)\hat{z}_c - p_m + f(k_m)\} \\
&\quad +(1-n)[\hat{z} + (1+i_c)(\hat{z}_c - d) + (1+i_d)d] + nW_1^e(0,0,0) + (1-n)W_0^e(0,0) \\
&= (1-n)(1-\theta)[f(k_b) - k_b] + [n - (1-n)(1-\theta)][f(k_m) - k_m] + n[\hat{z} + (1+i_c)\hat{z}_c] \\
&\quad +(1-n)[\hat{z} + (1+i_c)(\hat{z}_c - d) + (1+i_d)d] + nW_1^e(0,0,0) + (1-n)W_0^e(0,0).
\end{aligned}$$

Since cash can not be accepted as deposits, d, i_d are irrelevant to $\hat{z}$. And $p_m = \hat{z} + (1+i_c)\hat{z}_c = k_m$, hence,

$$\frac{\partial \mathbb{E}U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}} = (n-A)[f'(k_b) - 1]\frac{\partial k_b}{\partial \hat{z}} + A[f'(k_m) - 1].$$

With $d = \hat{z}_c$, and i_d from (19), first we rewrite $\mathbb{E}U^e(\hat{z}, \hat{z}_c)$ as,

$$\begin{aligned}
\mathbb{E}U^e(\hat{z}, \hat{z}_c) &= (1-n)(1-\theta)[f(k_b) - k_b] + [n - (1-n)(1-\theta)][f(k_m) - k_m] + n[\hat{z} + (1+i_c)\hat{z}_c] \\
&\quad +(1-n)\{\hat{z} + (1+i_c)(\hat{z}_c - d) + d + (1-\gamma)\phi + [\gamma i_c + (1-\gamma)i_r]d\} \\
&\quad +nW_1^e(0,0,0) + (1-n)W_0^e(0,0).
\end{aligned}$$

Then,

$$\begin{aligned}
\frac{\partial \mathbb{E}U^e(\hat{z}, \hat{z}_c)}{\partial \hat{z}_c} &= (n-A)[f'(k_b) - 1]\frac{\partial k_b}{\partial \hat{z}_c} + A[f'(k_m) - 1](1+i_c) \\
&\quad +(1+i_c) + (1-n)(1-\gamma)[\frac{\partial \phi}{\partial \hat{z}_c} + i_r - i_c],
\end{aligned}$$

To get $\partial k_b/\partial \hat{z}$, $\partial k_b/\partial \hat{z}_c$, $\partial \phi/\partial \hat{z}_c$, again we have three cases to consider for the general equilibrium analysis.

Case 1: $\lambda_1 = 0$**,** $\lambda_2 = 0$

With $k_b = k^*$, $\phi = \theta[f(k^*) - k^* - \Delta_m]$, we have $\partial \phi/\partial \hat{z}_c = 0$ since the reserve constraint

does not bind, and ϕ is not relevant to $\hat{z}_c$. Then we can rewrite the FOCs for $\hat{z}$ and $\hat{z}_c$ as,

$$
\begin{aligned}
i &= A[f'(k_m) - 1] && (20)\\
\frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + (1 - n)(1 - \gamma)\frac{i_r - i_c}{1 + i_c} && (21)
\end{aligned}
$$

Therefore, to have cash and CBDC coexist, it requires,

$$
i_r = -\frac{[1 + i - (1 - n)(1 - \gamma)]i_c}{(1 - n)(1 - \gamma)}. \tag{22}
$$

As in the BM, we label Case 1 as unconstrained equilibrium.

Case 2: $\lambda_1 > 0$**,** $\lambda_2 = 0$

Similarly, we have $\partial\phi/\partial\hat{z}_c = 0$. With the collateral constraint binding, we have

$$
(\theta - \chi)f(k_b) + (1 - \theta)k_b = \hat{z} + (1 + i_c)\hat{z}_c + \theta\Delta_m. \tag{23}
$$

Then we can derive,

$$
\begin{aligned}
\frac{\partial k_b}{\partial \hat{z}} &= \frac{1 + \theta[f'(k_m) - 1]}{(\theta - \chi)f'(k_b) + 1 - \theta}\\
\frac{\partial k_b}{\partial \hat{z}_c} &= \frac{1 + \theta[f'(k_m) - 1]}{(\theta - \chi)f'(k_b) + 1 - \theta}(1 + i_c).
\end{aligned}
$$

Hence, we can rewrite the FOCs for $\hat{z}$ and $\hat{z}_c$ as,

$$
\begin{aligned}
i &= A[f'(k_m) - 1] + \frac{(n - A)[f'(k_b) - 1]\{1 + \theta[f'(k_m) - 1]\}}{(\theta - \chi)f'(k_b) + 1 - \theta} && (24)\\
\frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + \frac{(n - A)[f'(k_b) - 1]\{1 + \theta[f'(k_m) - 1]\}}{(\theta - \chi)f'(k_b) + 1 - \theta} + \frac{(1 - n)(1 - \gamma)(i_r - i_c)}{1 + i_c} && (25)
\end{aligned}
$$

And we can derive the same condition as (22), for coexisting of cash and CBDC. As in the BM, we label Case 2 as collateral constrained equilibrium.

Case 3: $\lambda_1 = 0$**,** $\lambda_2 > 0$

When the reserve constraint is binding,

$$k_b = \hat{z} + (1+i_c)\hat{z}_c + \delta(1+i_r)d.$$

Then we have

$$\frac{\partial k_b}{\partial \hat{z}} = 1, \frac{\partial k_b}{\partial \hat{z}_c} = 1 + i_c.$$

Given $k_m = \hat{z} + (1+i_c)\hat{z}_c$, we have,

$$d = \hat{z}_c = \frac{k_b - k_m}{\delta(1+i_r)} \tag{26}$$

Hence, we can rewrite the FOCs for $\hat{z}$ and $\hat{z}_c$ as,

$$\begin{aligned} i &= A[f'(k_m) - 1] + (n - A)[f'(k_b) - 1] && (27) \\ \frac{i - i_c}{1 + i_c} &= A[f'(k_m) - 1] + [n - A + \frac{\theta\delta(1-n)(1-\gamma)(1+i_r)}{1+i_c}][f'(k_b) - 1] && (28) \\ &\quad + \frac{(1-n)(1-\gamma)(i_r - i_c)}{1+i_c}. \end{aligned}$$

The coexisting of cash and CBDC implies,

$$f'(k_b) - 1 = -\frac{(1+i)i_c + (1-n)(1-\gamma)(i_r - i_c)}{\theta\delta(1-n)(1-\gamma)(1+i_r)}. \tag{29}$$

Using (29), we can derive k_b, then derive k_m, $\hat{z}_c$ by (27) and (26). As in the BM model, we label Case 3 as reserve constrained equilibrium.

5.2.4 Policy Analysis

In the unconstrained equilibrium, to have cash and interest-bearing CBDC coexist, (22) need to be satisfied. It means, we either have $i_r > 0$ then $i_c < 0$, or $i_r < 0$ then $i_c > 0$. Since cash is zero interest, the central bank either imposes a negative CBDC interest rate, then compensate banks with positive interests on reserves, or impose a positive CBDC interest rate, then charge negative interests on reserves, so that cash and CBDC can coexist in the

economy. Then entrepreneurs are indifferent in holding either of them. It also means i_c cannot be an independent policy tool, in this coexisting scenario. As mentioned before, as long as $\hat{z} + (1 + i_c)\hat{z}_c = k_m$, the exact portfolio of $(\hat{z}, \hat{z}_c)$ is indeterminate.

However, when (22) is not satisfied, particularly under the condition CBDC dominates, the model economy collapses to a CBDC-only economy as the BM model (except with the generalization of bargaining, $0 < \gamma < 1$). Then again i_c becomes an independent and effective policy tool, and the effects and mechanism of changing i_c are quite similar as the BM model as well. The effects of changing i_c on i_d (then on r_d) become ambiguous, since Nash bargaining cannot guarantee i_d to have an one-to-one change with i_c any more. It is also possible for the economy to collapse as a cash-only one, if the marginal cost of holding cash is lower than that of holding CBDC. But banks cannot survive in the economy since they only accept CBDC as deposits. Proposition 4 summarizes all of these as follows.

Proposition 4 *In an unconstrained equilibrium, cash and CBDC can coexist when (22) is satisfied, but i_c cannot be an independent policy tool. When $-\{[(1+i)-(1-n)(1-\gamma)]i_c\}/(1-n)(1-\gamma) < i_r < \{i - [1-(1-n)(1-\gamma)]i_c\}/(1-n)(1-\gamma)$, CBDC dominates, and a higher i_c leads to a higher k_m, a higher K, a lower L, a lower ϕ and a lower r_ℓ, but the effect on r_d is ambiguous. When $i_r < -\{[(1+i)-(1-n)(1-\gamma)]i_c\}/[(1-n)(1-\gamma)]$, cash dominates, and the economy collapses to a cash economy without banks.*

For the collateral constrained equilibrium, we get the same condition (22) for the coexisting of cash and interest-bearing CBDC. But again the central bank need to adjust i_c and i_r simultaneously, so that i_c cannot be an independent policy tool. When (22) is not satisfied, depending on the marginal costs of holding cash and holding CBDC, the economy with two fiat monies may collapse to a CBDC-only or cash-only economy. Proposition 5 summarizes all of these as follows.

Proposition 5 *In a collateral constrained equilibrium, cash and CBDC can coexist when (22) is satisfied, but i_c cannot be an independent policy tool. When $i_r > -\{[(1+i)-(1-n)(1-\gamma)]i_c\}/[(1-n)(1-\gamma)]$, CBDC dominates, and a higher i_c leads to a higher k_m, a higher k_b, a higher K, a higher L, but the effect on r_d, r_ℓ, are ambiguous. When*

$i_r < -\{[(1+i)-(1-n)(1-\gamma)]i_c\}/[(1-n)(1-\gamma)]$, *cash dominates, and the economy collapses to a cash economy without banks.*

The reserve constrained equilibrium is a more interesting case, since cash and CBDC can coexist, and the portfolio of $(\hat{z}, \hat{z}_c)$ is determinate. The central bank does not need to sacrifice any monetary policy tool to maintain the coexistence of cash and CBDC. The mechanism is similar to Case 3 of Section 5.1, except now it is CBDC, instead of cash, that has the additional value in affecting the return on deposits. Hence, when i_c increases, entrepreneurs are willing to hold more CBDC, less cash, in general. A higher i_c also leads to a higher deposit rate, then type-0 entrepreneurs are willing to deposit more CBDC, which further leads to more loans issuing by banks, and thus higher investment k_b from banked type-1 e. But it turns out the effect of decreasing cash holding dominates, so that the total internal finance for unbanked type-1 e decreases, then the investment k_m decreases subsequently. In general, however, a higher i_c leads to higher aggregate investment K.

We can also easily prove the effects of changing i_r and the effects of changing υ. Proposition 6 summarizes all of the results as follows.

Proposition 6 *In a reserve constrained equilibrium, cash and CBDC can coexist when* $i_r > -\{[(1+i)-(1-n)(1-\gamma)]i_c\}/(1-n)(1-\gamma)$, *and the effects of changing* i_c *leads to a higher* $\hat{z}_c$ *(then a higher* d*), a lower* $\hat{z}$*, a lower* k_m*, a higher* k_b*, a higher* K *(if* $f'''(k) > 0$*), a higher* L*, a higher* d *or* $\hat{z}_c$*, a higher* r_d*, but the effect on* r_ℓ *is ambiguous. As for the effects of changing* i_r*, it has opposite effects on* k_m*,* k_b*,* K*,* L*,* d *or* $\hat{z}_c$*, compared to changing* i_c*, and the effects on* r_d *and* r_ℓ *are ambiguous. And the effects of changing* υ *are exactly the opposite as changing* i_r*.*

See the Appendix for Proof of Prop. 4 – 6.

6 Discussion and Conclusion

To study the effects of introducing CBDC, we firstly build a benchmark model where CBDC is the only medium of exchange in the economy. An important feature of CBDC is that

the central bank can pay interests for CBDC through digital accounts. The CBDC interest rate can serve as a new monetary policy tool, in additional to the traditional policy tools where the central bank change the growth rate of CBDC, or the required reserve ratio for banks, or the interest rate on reserves (a more recent policy tool, and has been popular among major advanced economies since the 2008 Global Financial Crisis). In our benchmark model, we assess how this new policy tool affects interest rates, banking, investment and the macroeconomy.

The main findings in the benchmark model is as follows. When the central bank raises the interest rate paid on CBDC, it makes CBDC a more attractive asset to hold. In all of the three cases, we show that an increase in the CBDC interest rate leads to higher investment by unbanked entrepreneurs (whose CBDC holdings serve as internal finance). And in two out of three cases, aggregate investment (from both banked and unbanked entrepreneurs) increases in response to a higher CBDC interest rate. Hence, a higher CBDC interest rate tends to have positive impacts on firm investment. As for the impacts on banking, the complementarity between CBDC and bank deposits makes a higher CBDC interest rate favorable to deposits, but may decrease or increase bank loans, across various general equilibrium regimes. Therefore, interest-bearing CBDC will not necessarily lead to financial disintermediation, or decrease bank lending.

Furthermore, the benchmark model also shows the interest rate on reserves and the reserve requirement ratio can be independent policy tools in the reserve constrained equilibrium. When the reserve constrain binds, both the interest rate on reserves and the reserve ratio can directly affect the amount of loans being issued, and indirectly affect firm investment. And it turns out that a higher interest rate on reserves or a lower reserve ratio has a positive effect on bank lending but overall negative effect on investment. Although the CBDC interest rate and the interest rate on reserves serve as independent policy tools in the model economy, it may be interesting to further investigate how these two interact in the economy, since the central bank can adjust the former to directly change the cost of holding liquidity for entrepreneurs, and adjust the latter to directly change the cost of holding liquidity for banks.

To consider more CBDC design choices, we extend the benchmark model by adding paper

money to the portfolio holdings of entrepreneurs. This is to make us better understand how CBDC interacts with the existing fiat money system. We consider two scenarios for the extension: cash only deposits, and CBDC only deposits. Due to interesting interactions between CBDC interest rate and the interest rate on reserves, cash and interest-bearing CBDC can always coexist in all general equilibrium cases. When the reserve constraint is not binding, the central bank can adjust these two interest rates simultaneously, for coexisting of two fiat monies; when it does bind, the central bank does not need to sacrifice any monetary polity tool to maintain the coexistence of cash and CBDC. That is, both CBDC interest rate and the interest rate on reserves can serve as independent policy tools.

CBDC is a frontier research topic, and there are urgent needs to examine its design issues among central banks and policy makers. One issue is, once issuing CBDC, how should it be issued, through an independent CBDC account system provided by the central bank, or through the current banking infrastructure? What are the key factors to justify the choices? This may depend on the size of the economy, and also the banking structure. For example, in China, the central bank has confirmed it will introduce CBDC through a two-tier system, with the central bank as the first tier, and commercial banks and digital payment platforms as the second tier. It quite makes sense since commercial banks actually dominate the financial system, and digital payment platforms are growing fast, in China. But it will be interesting to study how the two-tier system affects the current financial system, and how commercial banks compete with digital payment platforms in the wholesale and retail CBDC.

Another issue is the privacy designs of CBDC: should it be anonymous or not? at which level of anonymity should it be designed? If every one hold a digital account of CBDC, the central bank can directly access to the transaction and financial history of individuals. In contrast, cash transaction is anonymous, which can be good (to protect our privacy), or bad (for the risks of getting stolen and counterfeiting, or using in underground economy). This is also related to data sharing issues in the current era of digital economy (Jones and Tonetti, 2018, Easley et al. 2018), but becomes even more important in the context of CBDC designs.

Our paper is among the initial attempts to shed light on the issues of CBDC. There are many more to be explored, and we leave them for future research.

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A Proof for Proposition 4 − 6

Proof for Proposition 4:

In Section 5.2.3, we already prove cash and CBDC can coexist when (22) is satisfied. Easily, we can see, if $i_r > -\frac{[1+i-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)}$, the marginal cost of holding one more unit of cash is higher than that of holding one more unit of CBDC, i.e., $i > \frac{i-i_c-(1-n)(1-\gamma)(i_r-i_c)}{1+i_c}$, based on (20) and (21). Hence, CBDC will dominate the model economy.

For the effects of changing i_c, using (21), we can derive

$$\frac{\partial k_m}{\partial i_c} = \frac{(1-n)(1-\gamma)(1+i_r)-(1+i)}{Af''(k_m)(1+i_c)^2} \cong 1+i-(1-n)(1-\gamma)(1+i_r).$$

Since $k_m < k^*$, with $f'(k^*) = 1$, we have

$$\begin{aligned} f'(k_m)-1 &= \frac{i-i_c-(1-n)(1-\gamma)(i_r-i_c)}{A(1+i_c)} > 0 \\ &\rightarrow i_r < \frac{i-[1-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)} \\ &\rightarrow 1+i-(1-n)(1-\gamma)(1+i_r) > (1+i_c)[1-(1-n)(1-\gamma)] > 0. \end{aligned}$$

Hence, we can prove $\partial k_m/\partial i_c > 0$.

To summarize, when $-\frac{[1+i-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)} < i_r < \frac{i-[1-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)}$, CBDC dominates the economy (we can also easily prove $\frac{i-[1-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)} - \frac{(1-n)(1-\gamma)-(1+i)}{(1-n)(1-\gamma)} = \frac{i(1+i_c)}{(1-n)(1-\gamma)} \geq 0$). Also notice now $k_m = (1+i_c)\hat{z}_c$, not including $\hat{z}$ any more.

Furthermore, we have,

$$
\begin{aligned}
\frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k^*}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} > 0 \\
\frac{\partial L}{\partial i_c} &= -(1-n)(\frac{\partial k^*}{\partial i_c} - \frac{\partial k_m}{\partial i_c}) < 0 \\
\frac{\partial \phi}{\partial i_c} &= -\theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_c} < 0 \\
\frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\partial k_m/\partial i_c - \hat{z}_c}{1+i_c} \lesseqgtr 0 \\
\frac{\partial i_d}{\partial i_c} &= \frac{(1-\gamma)}{\hat{z}_c}\frac{\partial \phi}{\partial i_c} + \frac{\gamma i_c + (1-\gamma)i_r - i_d}{\hat{z}_c}\frac{\partial \hat{z}_c}{\partial i_c} + \gamma \lesseqgtr 0 \\
\frac{\partial r_d}{\partial i_c} &= \frac{1}{1+\pi}\frac{\partial i_d}{\partial i_c} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{\phi - \theta[f'(k_m)-1](k^*-k_m)}{(k^*-k_m)^2}\frac{\partial k_m}{\partial i_c} \\
&\cong \theta[f(k^*) - k^* - (f(k_m)-k_m)] - \theta(k^*-k_m)[f'(k_m)-1] \\
&= \theta[f(k^*) - f(k_m) - f'(k_m)(k^*-k_m)] < 0.
\end{aligned}
$$

Lastly, when $i_r < -\frac{[1+i-(1-n)(1-\gamma)]i_c}{(1-n)(1-\gamma)}$, cash dominates, but banks cannot survive since they can only accept CBDC as deposits. Entrepreneurs can still use cash as internal finance to acquire capital, but cannot get any bank loans. Hence, it will collapse as a cash economy without banks.

Proof for Proposition 5:

When $i_r > \frac{(1-n)(1-\gamma)-(1+i)}{(1-n)(1-\gamma)}$, again $i > \frac{i-i_c-(1-n)(1-\gamma)(i_r-i_c)}{1+i_c}$, based on (24) and (25), therefore, CBDC will dominate the model economy, i.e., $\hat{z}=0$, $\hat{z}_c > 0$. For the effects of changing i_c, using (23) and (25), we have,

$$
\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= -\frac{[1+i-(1-n)(1-\gamma)][1+\theta(f'(k_m-1)]}{(1+i_c)D} > 0 \\
\frac{\partial k_b}{\partial i_c} &= -\frac{[1+i-(1-n)(1-\gamma)][(\theta-\chi)f'(k_b)+1-\theta]}{(1+i_c)D} > 0,
\end{aligned}
$$

where $D < 0$, with the same expression as the BM. Furthermore, we can derive,

$$
\begin{aligned}
\frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k_b}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} > 0 \\
\frac{\partial L}{\partial i_c} &= \frac{(1-n)[1+i-(1-n)(1-\gamma)]}{(1+i_c)D}\{\theta[f'(k_b)-f'(k_m)] - \chi f'(k_b)\} > 0 \\
\frac{\partial \phi}{\partial i_c} &= -\frac{\theta[1+i-(1-n)(1-\gamma)]}{(1+i_c)D}\{f'(k_b)-f'(k_m)+\chi f'(k_b)[f'(k_m)-1]\} \lesseqgtr 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{(k_b-k_m)\partial\phi/\partial i_c - \phi(\partial k_b/\partial i_c - \partial k_m/\partial i_c)}{(k_b-k_m)^2} \lesseqgtr 0 \\
&= -\frac{\theta[1+i-(1-n)(1-\gamma)]}{(1+i_c)D}\{[f'(k_b)-f'(k_m)][\theta(f(k_b)-f(k_m))+(1-\theta)(k_b-k_m)] + \chi f'(k_b)[(k_b - \\
\frac{\partial d}{\partial i_c} &= \frac{\partial \hat{z}_c}{\partial i_c} = \frac{\partial k_m/\partial i_c - \hat{z}_c}{1+i_c} \lesseqgtr 0 \\
\frac{\partial i_d}{\partial i_c} &= \frac{(1-\gamma)}{\hat{z}_c}\frac{\partial \phi}{\partial i_c} + \frac{\gamma i_c + (1-\gamma)i_r - i_d}{\hat{z}_c}\frac{\partial \hat{z}_c}{\partial i_c} + \gamma \lesseqgtr 0 \\
\frac{\partial r_d}{\partial i_c} &= \frac{1}{1+\pi}\frac{\partial i_d}{\partial i_c} \lesseqgtr 0.
\end{aligned}
$$

Notice when $\partial\phi/\partial i_c < 0$, we have $\partial r_\ell/\partial i_c > 0$, although the effects of changing i_c on ϕ and r_ℓ are ambiguous in general.

Lastly, when $i_r < -\frac{1+i-(1-n)(1-\gamma)}{(1-n)(1-\gamma)}$, similarly, cash dominates, but banks cannot survive since they can only accept CBDC as deposits. Entrepreneurs can still use cash to acquire capital, but cannot get any bank loans. Hence, it will collapse as a cash economy without banks.

Proof for Proposition 6:

For the reserve constrained equilibrium, since $k_b < k^*$, then $f'(k_b) > f(k^*) = 1$. Hence, using (29), we can derive $i_r > -[1+i-(1-n)(1-\gamma)]i_c/[(1-n)(1-\gamma)]$. In addition, for the effects of changing i_c, we have,

$$\frac{\partial k_b}{\partial i_c} = -\frac{1+i-(1-n)(1-\gamma)}{\theta\delta(1-n)(1-\gamma)(1+i_r)f''(k_b)} > 0.$$

Then, using (27) and $\partial k_b/\partial i_c$, we can derive

$$\begin{aligned}
\frac{\partial k_m}{\partial i_c} &= \frac{(n-A)[1+i-(1-n)(1-\gamma)]}{A\theta\delta(1-n)(1-\gamma)(1+i_r)f''(k_m)} < 0 \\
\frac{\partial L}{\partial i_c} &= (1-n)(\frac{\partial k_b}{\partial i_c} - \frac{\partial k_m}{\partial i_c}) \\
&= -(1-n)\frac{[Af''(k_m)+(n-A)f''(k_b)][1+i-(1-n)(1-\gamma)]}{Af''(k_m)f''(k_b)\theta\delta(1-n)(1-\gamma)(1+i_r)} > 0
\end{aligned}$$

Furthermore, we have,

$$\begin{aligned}
\frac{\partial K}{\partial i_c} &= (1-n)\frac{\partial k_b}{\partial i_c} + (2n-1)\frac{\partial k_m}{\partial i_c} \\
&= \frac{[1+i-(1-n)(1-\gamma)][(2n-1)(n-A)f''(k_b) - A(1-n)f''(k_m)]}{A\theta\delta(1-n)(1-\gamma)(1+i_r)f''(k_m)f''(k_b)} \\
&\cong (2n-1)(n-A)f''(k_b) - A(1-n)f''(k_m) \\
\frac{\partial \phi}{\partial i_c} &= \theta[f'(k_b)-1]\frac{\partial k_b}{\partial i_c} - \theta[f'(k_m)-1]\frac{\partial k_m}{\partial i_c} > 0 \\
\frac{\partial r_\ell}{\partial i_c} &= \frac{(k_b-k_m)\partial\phi/\partial i_c - \phi(\partial k_b/\partial i_c - \partial k_m/\partial i_c)}{(k_b-k_m)^2} \lesseqgtr 0 \\
&\cong \theta\{(k_b-k_m)f'(k_b) - [f(k_b)-f(k_m)]\}\frac{\partial k_b}{\partial i_c} + \theta[f(k_b)-f(k_m)-(k_b-k_m)f'(k_m)]\frac{\partial k_m}{\partial i_c} \\
\frac{\partial \hat{z}_c}{\partial i_c} &= \frac{\partial d}{\partial i_c} = \frac{1}{\delta(1+i_r)}(\frac{\partial k_b}{\partial i_c} - \frac{\partial k_m}{\partial i_c}) > 0 \\
\frac{\partial \hat{z}}{\partial i_c} &= \frac{\partial k_m}{\partial i_c} - \hat{z}_c - (1+i_c)\frac{\partial \hat{z}_c}{\partial i_c} < 0 \\
\frac{\partial i_d}{\partial i_c} &= \frac{(1-\gamma)}{\hat{z}_c}\frac{\partial \phi}{\partial i_c} + \frac{\gamma i_c + (1-\gamma)i_r - i_d}{\hat{z}_c}\frac{\partial \hat{z}_c}{\partial i_c} + \gamma > 0 \\
&= \frac{\gamma i_c + (1-\gamma)i_r - i_d - \theta(1-\gamma)(1+i_c)[f'(k_m)-1]}{(1+i_c)\hat{z}_c}\frac{\partial k_m}{\partial i_c} + \frac{\theta(1-\gamma)[f'(k_b)-1]}{\hat{z}_c}\frac{\partial k_b}{\partial i_c} + \frac{i_d-(1-\gamma)i_r+}{1+i_c} \\
\frac{\partial r_d}{\partial i_c} &= \frac{1}{1+\pi}\frac{\partial i_d}{\partial i_c} > 0.
\end{aligned}$$

Notice $\partial K/\partial i_c \cong (2n-1)(n-A)f''(k_b) - A(1-n)f''(k_m)$. With $A \equiv n-(1-n)(1-\theta)$, we can easily prove $(2n-1)(n-A) - A(1-n) = -\theta n(1-n) < 0$, so $0 < (2n-1)(n-A) < A(1-n)$. Suppose $f'''(k) > 0$, then we can prove $\partial K/\partial i_c > 0$. Also for the result for $\partial i_d/\partial i_c$, $\gamma i_c + (1-\gamma)i_r - i_d = -(1-\gamma)\phi/\hat{z}_c < 0$, and $i_d - (1-\gamma)i_r + \gamma = (1-\gamma)\phi/\hat{z}_c + \gamma(1+i_c) > 0$. Hence, with $\partial k_m/\partial i_c < 0$, $\partial k_b/\partial i_c > 0$, we can derive $\partial i_d/\partial i_c > 0$.

Similarly, for the effects of changing i_r, we have,

$$\begin{aligned}
\frac{\partial k_b}{\partial i_r} &< 0, \frac{\partial k_m}{\partial i_r} > 0, \frac{\partial L}{\partial i_r} < 0, \frac{\partial d}{\partial i_r} < 0, \\
\frac{\partial \phi}{\partial i_r} &< 0, \frac{\partial K}{\partial i_r} \gtrless 0, \frac{\partial r_d}{\partial i_r} \gtrless 0, \frac{\partial r_\ell}{\partial i_r} \gtrless 0.
\end{aligned}$$

Notice we can also prove $\partial K/\partial i_r < 0$, assuming $f'''(k) > 0$.

As for the effects of changing the required reserve ratio υ, it only appears in (29), with $\delta = 1/\upsilon - 1$. It has the opposite effects as i_r.