Self-Fulfilling Risk, Land Price and Macroeconomic Fluctuations*

Xuewen Liu† Pengfei Wang‡ Sichuang Xu§

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Abstract

The collapse of real estate price during the 2008 financial crisis is accompanied by a sharp surge in measured uncertainty. In this paper, we propose a tractable macroeconomic framework, linking uncertainty, housing price and the real economy. In the model, fluctuations in real estate price originating from changing perceptions about uncertainty transmits and propagates to the macroeconomy, generating boom-bust cycles. Our framework features self-fulfilling risk spike in the housing market, and is able to generate large volatility in price-rent rate as well as strong co-movement between housing price and macroeconomic aggregates. Quantitative exercise suggests risk panic is a leading driver of business-cycle fluctuations despite the presence of various competing structural shocks.

Keywords: Risk Panic, Self-Fulfilling Equilibria, Land Price, Uncertainty, Business Cycles.

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†Department of Finance, HKU Business School, The University of Hong Kong. Email: xuewenl@hku.hk.

‡PHBS Business School, Peking University. Email: pfwang@phbs.pku.edu.cn.

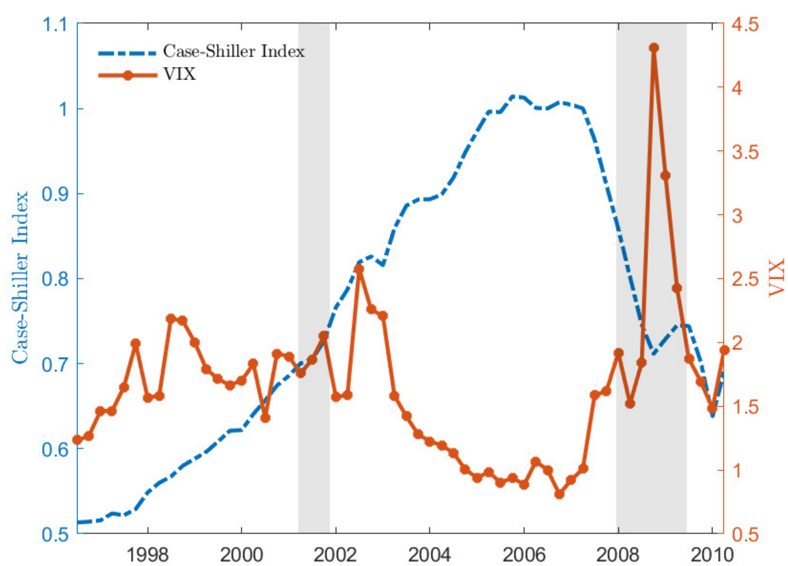
§School of Management and Economics, CUHK (Shenzhen). Email: xusichuang@cuhk.edu.cn.

1 Introduction

One striking feature of the 2008 financial crisis is that slumps in real estate price are accompanied by sharp surges in measured uncertainty. In Figure 1, Case-Shiller Home Price Index dropped for about 30% from the onset of recession to July-2009, and during the same period, macroeconomic uncertainty measured in VIX spiked by more than 200%. These dramatic shifts in uncertainties along with collapses in real estate market are followed by persistent declines in aggregate consumption and investment. Some economists hypothesize that this recession could have a self-fulfilling origin (Lucas and Stokey, 2011; Bacchetta et al., 2012a), and that heightened risk, or a perception of it, sets the economy into deep downturn (Bernanke, 2007). This paper formalize such an idea where self-fulfilling changes of risk originating from the housing market play an autonomous role in driving business cycle fluctuations.

We do so by constructing a tractable production economy with infinitely-lived agents, linking uncertainty, housing price and real macroeconomic variables. In the model, fluctuations in real estate price caused by changing perceptions of uncertainty ahead transmits and propagates to the macroeconomy and leads to boom-bust cycles. Our framework features self-fulfilling risk spike in the housing market that result in large drops in housing prices, and the theory can generate large volatility in housing price with modest fluctuations in rents. It also reproduce the strong co-movement pattern between housing price and macroeconomic aggregates, including investment, output and consumption.

Figure 1: Housing Price and Uncertainty



We consider two versions of the model, both under infinite horizon general equilibrium settings. The first one emphasis parsimony and tractability, and we use this model to deliver transparent illustration of the key impulse and propagation mechanism. It is then enriched to a medium-scale DSGE setting to access the quantitative property of the model's mechanisms.

The baseline model consists of a representative household, who optimally allocate funds between risky housing and riskless bond, and a credit constrained firm, who hold land for production and at the same time, use it as collateral. In such a model, land price is determine by households demand for housing, that is, the housing Euler equation. Fluctuations in land price leads to rise and fall in the value of collateralized land for firm, and through credit constraint, affects production. Inspired by the theoretical insight in Bacchetta et al. (2012a), we show when household is averse to price risk, its housing demand schedule features dynamic mapping of risk into itself, and nests sentiment driven equilibria characterized by collapsing land price and surging risk. To be clear, note housing price in our model is the combination of present value of future rents, discounted by an additional volatility term capturing the aversion generated from holding risky land,

$$q_t = \beta_h \mathbb{E}_t \left(1 + q_{t+1}\right) - \frac{\lambda}{\varphi} \mathrm{Var}_t \left(q_{t+1}\right).$$

In this equation, if households believe that certain sentiment variable, either related to economic fundamental or pure sunspots, matters for housing price, the perceived risk of future prices will increase. As a result, current housing price will indeed be affected, confirming household's belief and result in self-fulfilling fluctuations in land price. What is unique about this equilibrium is that sentiment shock moves the level of land price through affecting its (perceived) risk. In other words, waves of unfavorable sentiments lead to not only drops in land price, but also spikes in risk.

The collapse in housing price originating from unfavorable household sentiment brings two consequences for the real economy. For households, negative sentiment reduce their incentive to supply labor. When households believe holding housing becomes risky, they optimally re-balance their portfolio by reducing housing purchase and increase either bond holding or consumption expenditure. In equilibrium, they will do both. By wealth effect of labor supply, increasing consumption implies they prefer to work less at any given wage level. For entrepreneurs, as their land holding is pre-determined, labor demand schedule does not change. Equilibrium hours drop, leading to drops in output and entrepreneurial profit. Declining profit implies entrepreneur's net worth declines, so that they accumulate less land next period, and land reallocates from entrepreneurs to households. Land reallocation further reduce labor demand and result in larger drops of output. In the baseline model, declines in labor and land reallocation form a joint force that drives macroeconomic fluctuations.

For quantitative exercise, we enrich the baseline model into a medium-scale DSGE setting along three dimensions. First, we incorporate capital investment by assuming entrepreneurs produce using a combination of labor, land and capital. As investment expenditure is financed by collateralized debt, declining land price induced by unfavorable sentiment tightens entrepreneurs' borrowing constraint and result in drops in investment, generating co-movement between land price and macroeconomic aggregates. Second, we introduce nominal rigidity, which help amplifying the effect of sentiment through through a time-varying markup channel arising from fluctuations in aggregate demand. Finally, we allow for a series of common structural shocks in the DSGE literature including shocks to labor supply, collateral constraint, capital goods price, technology, monetary policy, and we also build in the standard modelling bells and whistles including habit formation and quadratic investment adjustment cost.

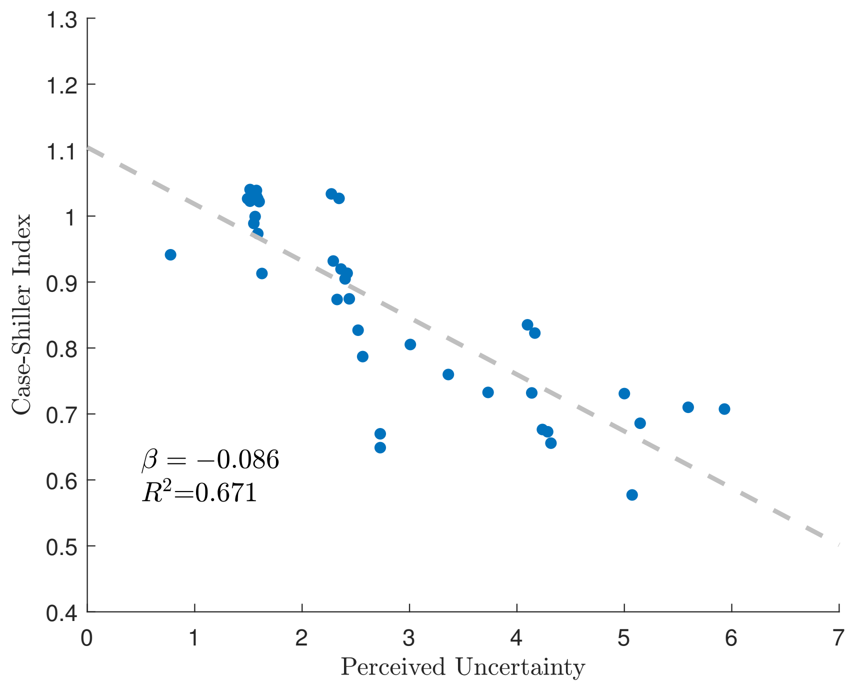
Fitting the model against aggregate U.S. time series, we find that, despite a wide array of competing shocks, sentiment shock emerges as a quantitatively important driver of business cycle. Variance decomposition exercise indicates sentiment accounts for about 87% in land price fluctuations, 43% of investment fluctuations, and 23% output fluctuations. Based on our estimation result, we conduct counterfactual experiment to quantify the model's ability in explaining the Great Financial Crisis, and we calculate that fluctuations in sentiment alone can explain almost all the drop in housing price around the crisis period as well a sharp spike in uncertainty. The model's internal propagation mechanism also help to generate 17.5% drop in investment, 2.3% in consumption, 4.3% in hours, and 6.2% in output, which is broadly consistent with aggregate data.

Empirical Supports on Housing Risk Channel. Housing wealth accounts on average a 27% of U.S. households' net worth (Poterba and Samwick, 1997), and due to price risk, it is also one of the most volatile items on homeowners' balance-sheet (Campbell and Cocco, 2007; Piazzesi and Schneider, 2016). As home purchasing is the largest financial decision for typical households, the risk associated with it is often an important consideration. Rosen et al. (1984) estimates a housing tenure model with uncertainty and finds a financial risk effect whereby housing price risk reduces housing demand. More recently, Han (2010) uses the Panel Study of Income Dynamics (PSID) data and obtains similar results. The risk-based housing demand function in our model is consistent with these micro-level evidence. To add empirical support at macro-level, we use the Michigan Survey of Consumer Sentiment to documents in Figure 2 a negative relationship between U.S. households' perception of uncertainty[1] and the housing price

[1] For measures of uncertainty perception, we use the Table 42 of Home Buying and Selling Conditions from Michigan Survey of Consumer Sentiment. The survey ask respondents opinions for home purchase, and their respond can fall into two categories: Good Time and Bad Time. There are several sub-categories if the respondent think it is a bad time: high price, high interest rate, cannot afford, bad investment, and uncertain future. We compute the perceived uncertainty measure as the percentage fraction of respondents with a "uncertain future" answer. The coverage is 2001–Q1 to 2010–Q2. For home price, we use the Case-Shiller Index.

from 2001 to 2010. The negative relationship is tight, with a (adjusted) R^2 of 0.67 and slope coefficient of -0.086, implying national-wide home price would drop by 8.6% when households' perception of uncertainty ahead doubles.

Figure 2: Housing Price and Uncertainty Perception



Literature. Our paper is related to three strands of literature. First, it connects to the recent literature linking housing market with macroeconomic fluctuations. Inspired by a series of work by Mian and Sufi (Mian and Sufi, 2011), this literature argues that the housing market was at the heart of the Great Recession, and build models where shocks that lead to rise and fall in housing price leads to macroeconomic fluctuations. While most research has been focused on housing price and its impact on aggregate consumption dynamics (Piazzesi and Schneider, 2016), a relatively small body of literature seek to explain the co-movement between housing prices and investment or employment fluctuations. Liu et al. (2013) develops and estimates a DSGE model where land is a collateral asset in firms' credit constraints, and identify housing demand shock as an important source of fluctuations in aggregate investment. Liu et al. (2016) shows that shocks move land price drives unemployment fluctuations. These papers do not model rental market explicitly, and predict that real estate price and rent move in comparable magnitude so that there is little variation in price-rent ratio, which is inconsistent with the data. In a recent paper, Miao et al. (2020) the liquidity premium channel and build model to jointly explain housing price-rent rate's high volatility and its co-movement with the business cycle. We contribute this line of research by developing a DSGE model with a novel risk-based channel where housing price fluctuations is driven by self-fulfilling risk-panics. At the same time, the model account for the volatility and co-movement pattern of housing price-rent rate and features transparent linking from the real estate market to the real economy.

Second, our paper is also related to the literature emphasizing fluctuations in uncertainty have an autonomous role in driving the business cycle. The pioneering work by Nick Bloom argues that uncertainty shock is an independent driving force of boom-bust cycles (Bloom, 2009; Bloom et al., 2018). An emerging literature propose that time-varying risk is a response of, instead of a source for business cycle fluctuations. Bachmann and Bayer (2013, 2014) calibrate heterogeneous-firm DSGE models to show time-varying firm-level risk through "wait-and-see" dynamics is unlikely a major source of business cycle fluctuations. Others build models to show that uncertainty can be an endogenous response due to either self-fulfilling risk-panic (Bacchetta et al., 2012b), learning from the action of others (Fajgelbaum et al., 2017), or information interdependence between financial markets and the real economy (Benhabib et al., 2019). We contribute this literature by presenting a DSGE model with a micro-founded endogenous uncertainty mechanism emphasizing the panic in housing market, and study how it transmitted and propagated into the real economy.

Finally, our paper belongs to the literature studying multiple equilibria, sunspot and the business cycle. In most of the literature, the role of sunspots is to randomize over multiple fundamental equilibria, and the self-fulfilling shifts in beliefs is about the *level* of a variable (for example, asset price, output, etc) (Lorenzoni, 2009; Angeletos and La'O, 2010; Barsky and Sims, 2012; Angeletos and La'O, 2013; Benhabib et al., 2015, 2016). There is also a literature focusing on self-fulfilling shifts in beliefs about *risk*, building either on static market participation (Pagano, 1989; Allen and Gale, 1994; Jeanne and Rose, 2002), or dynamic relation between the state variable and its future distribution (Bacchetta et al., 2012b; Bacchetta and van Wincoop, 2013, 2016). The fundamental insight of our model is based on Bacchetta et al. (2012b), but their model is too simple to calibrate to actual data of financial panics. Our contribution is to construct a infinite horizon production economy, linking risk-panic to real macroeconomic activity and perform quantitative investigation.

The rest of this article is organized as follows: In Section 2, we set up a parsimonious dynamic general equilibrium model, derive the theoretical results, and use these results to illustrate the key impulse and propagation mechanism. In Section 3, we enrich the model to a medium-scale DSGE setting, and we estimate the model using several U.S. time series, present the estimated results, the impulse response function, the variance decomposition, and based on the estimated results, we conduct a counterfactual crisis experiment. Section 4 concludes the article. Detailed derivations, proofs, and estimation procedures are provided in appendices.

2 Basic Model without Capital

The model is in infinite horizon and consists of two types of agents: a representative household and a representative entrepreneur. The household values consumption, housing service, and leisure. It supplies labor, purchase land (housing service), and saves in one period non-contingent bond. Risky land price generate dis-utility from holding land. The representative entrepreneur only values consumption, and uses land, labor as intermediate inputs to produce homogeneous consumption goods. The entrepreneur borrow from households, but due to credit market friction, her borrowing is constrained. We assume entrepreneur are less patient than household so that borrowing constrained binds in steady state.

Households. Household choose consumption, housing service, land holding, savings, and labor supply to maximize lifetime utility,

$$\max_{C_{ht},x_t,L_{ht},N_{ht},S_t,} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_{ht}^t \left[\log C_{ht} + \varphi x_t - \lambda \mathrm{Var}_t \left(\frac{Q_{lt+1}}{R_{t+1}} \right) L_t - \psi \frac{N_{ht}^{1+\upsilon}}{1+\upsilon} \right] \right\}, \tag{1}$$

where C_{ht} denotes consumption, x_t denotes housing (land) service, and φ measures marginal utility for housing rental; L_{ht} is household land holding; $\lambda \mathrm{Var}_t \left(\frac{Q_{lt+1}}{R_{t+1}} \right) L_t$ measures the dis-utility induced by (conditional) volatility for land price (normalized by rent), where λ represents the degree of risk aversion; household also have convex dis-utility in supplying labor N_{ht}, where υ measures the inverse of labor supply elasticity.

The flow of funds constraint is given by,

$$C_{ht} + Q_{lt}\left(L_{ht} - L_{ht-1}\right) + \frac{S_t}{R_{ft}} = w_t N_{ht} - R_t \left(x_t - L_{t-1}\right) + S_{t-1}. \tag{2}$$

where households use labor income $w_t N_{ht}$ and their debt repayment from last period, S_{t-1} to finance consumption, house purchasing, saving, and rental expenditure. The associated optimality conditions are,

$$\varphi C_{ht} = R_t, \tag{3}$$

$$\psi N_{ht}^{\upsilon} C_{ht} = w_t, \tag{4}$$

$$\frac{Q_{lt}}{C_{ht}} = \beta_h \mathbb{E}_t \left(\varphi + \frac{Q_{lt+1}}{C_{ht+1}} \right) - \lambda \mathrm{Var}_t \left(\frac{Q_{lt+1}}{R_{t+1}} \right), \tag{5}$$

$$1 = \beta_h R_{ft} \mathbb{E}_t \left(\frac{C_{ht}}{C_{ht+1}} \right), \tag{6}$$

where equation (3) equates marginal benefit from renting one unit of house to the rental rate; (4) is the labor supply equation; Equation (5) is the land pricing equation, which says the marginal cost of buying a house (in marginal utility terms) Q_{lt}/C_{ht}, equals to the marginal benefit of it, and we discuss the implication of this equation in the following subsection; Equation (6) is the bond Euler equation.

Sentiment Driven Equilibrium. As in Bacchetta et al. (2012a), risk aversion to asset price volatility opened the possibility for the existence of sentiment driven equilibria featuring self-fullfilling panics. To explain, note that in equation (3) housing rental rate and consumption is proportional. Plugging this relationship into (3) we have,

$$q_t = \beta_h \mathbb{E}_t (1 + q_{t+1}) - \frac{\lambda}{\varphi} \mathrm{Var}_t (q_{t+1}), \tag{7}$$

where

$$q_t := \frac{Q_{lt}}{R_t}$$

is the housing price-rental rate. From this equation, the equilibrium housing price-rental rate depends negatively on its perceived risk, $\mathrm{Var}_t (q_{t+1})$. Suppose there is a sentiment variable s_t and households believe risk depend on this variable, then by equation (7), q_t also depend on s_t. Hence, q_{t+1} depend on s_{t+1}. If the distribution of s_{t+1} depend on s_t, then $\mathrm{Var}_t (q_{t+1})$ will indeed depend on s_t, giving rise to sentiment equilibrium. Intuitively, if households believe sentiment matters for housing price, the perceived risk of future prices will increase. By risk-aversion, current housing price will indeed be affected, confirming their belief.

To formalize the analysis, suppose sentiment s_t follows an AR(1) process, $s_t = \rho_s s_{t-1} + \varepsilon_{st}$, where $\varepsilon_{st} \sim \mathcal{U}[-\bar{\varepsilon}, +\bar{\varepsilon}]$, uniform distribution from $-\bar{\varepsilon}$ to $+\bar{\varepsilon}$. The following proposition characterize the sentiment driven dynamics of housing price-rental rate.

Proposition 1. *The sentiment driven price-rental rate q_t is given by,*

$$q_t = \bar{q} - \phi s_t^2, \tag{8}$$

where s_t follows,

$$s_t = \rho_s s_{t-1} + \varepsilon_{st},$$

where ε_{st} follows uniform distribution, $\varepsilon_{st} \sim \mathcal{U}[-\bar{\varepsilon}, +\bar{\varepsilon}]$, and $\bar{q}$, ϕ are given by,

$$\phi = \frac{\varphi (1 - \beta_h \rho_s^2)}{4\lambda \sigma_\varepsilon^2 \rho_s^2}, \tag{9}$$

$$\bar{q} = \frac{1}{1 - \beta_h} \left\{ \beta_h - \phi \left[\beta \sigma_s^2 + \frac{\lambda}{\varphi} \phi \left(\omega_s^2 - \sigma_s^4 \right) \right] \right\}, \tag{10}$$

where $\omega_\varepsilon^2 := \mathbb{E}(\varepsilon_{st}^4)$, and $\sigma_\varepsilon^2 := \mathbb{E}(\varepsilon_{st}^2)$.

Proof. In Appendix (B). ■

Entrepreneurs. The representative entrepreneur produce homogeneous consumption goods by using land and labor as intermediate inputs, which is financed by borrowing from households. Entrepreneurs choose consumption C_{et}, land holding L_{et}, labor input N_{et}, and new debt issuance B_t, to maximize lifetime utility,

$$\max_{C_{et},B_t,L_{et}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_e^t \log C_{et} \right\}, \tag{11}$$

subject to the following flow of funds constraint,

$$C_{et} + Q_{lt} \left(L_{et} - L_{et-1} \right) + B_{t-1} = \max_{N_{et}} \{ Y_t - w_t N_{et} \} + \frac{B_t}{R_{ft}}, \tag{12}$$

in which entrepreneur finance consumption, new land purchase and wage bill, by using production revenue plus debt issuance. The production function is assumed to be Cobb-Douglas,

$$Y_t = A_t L_{et-1}^{\alpha} N_{et}^{1-\alpha},$$

where the decision of N_{et} is static,

$$w_t = (1-\alpha) \frac{Y_t}{N_{et}},$$

implying the flow of funds constraint (12) can be written as,

$$C_{et} + Q_{lt} \left(L_{et} - L_{et-1} \right) + B_{t-1} = z_t L_{et-1} + \frac{B_t}{R_{ft}}$$

where $z_t := \alpha \left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}} w_t^{\frac{\alpha-1}{\alpha}} A_t^{\frac{1}{\alpha}}$. Finally, entrepreneur's face the following collateral constriant,

$$B_t \leq \theta_t \mathbb{E}_t \left(Q_{lt+1} \right) L_{et}.$$

which says the amount that entrepreneurs can borrow is limitted by a faction of the value of the land holding. In similar spirit with Kiyotaki and Moore (1997) and Liu et al. (2013) , we interpret this type of credit constraint as reflecting the problem of costly contract enforcement: if the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital; since it is costly to liquidate the seized land and capital stock, the creditor can recoup up to a fraction θ_t of the total value of the collaterized land.

Assume this borrowing constraint binds, the flow of funds constraint becomes

$$C_{et} + \left(Q_{lt} - \frac{\theta_t \mathbb{E}_t \left(Q_{lt+1} \right)}{R_{ft}} \right) L_{et} = \left(z_t + Q_{lt} \right) L_{et-1} - B_{t-1}, \tag{13}$$

where the right hand side $(z_t + Q_{lt}) L_{et-1} - B_{t-1}$ is entrepreneurs beginning-of-period net worth. Equation (13) implies each one dollar of entrepreneurs saving will yield return in the amount of

$$\frac{z_{t+1} + Q_{lt+1} - \theta_t \mathbb{E}_t (Q_{lt+1})}{Q_{lt} - \frac{\theta_t \mathbb{E}_t(Q_{lt+1})}{R_{ft}}},$$

to explain, note for each dollar of saving, entrepreneur is buying land at price Q_{lt}, among which $\frac{\theta_t \mathbb{E}_t(Q_{lt+1})}{R_{ft}}$ is borrowed. The purchased land then yields return containing z_{t+1}, the marginal productivity, Q_{lt+1}, the capital gain, net the face value of debt $\theta_t \mathbb{E}_t (Q_{lt+1})$ that the entrepreneur needs to pay. Importantly, Cobb-Douglas production function implies this return depend only on aggregate variables. Combining with log utility assumption, it implies entrepreneur saves β_e fraction of its beginning-of-period net worth, and consumes the rest $1-\beta_e$ fraction. Formally, we have the following proposition characterizing entrepreneurs consumption and saving decision.

Proposition 2. *The representative entrepreneur's decisions are given by,*

$$C_{et} = (1-\beta_e) \left[(z_t + Q_{lt}) L_{et-1} - B_{t-1} \right], \tag{14}$$

$$L_t = \beta_e \frac{(z_t + Q_{lt}) L_{et-1} - B_{t-1}}{Q_{lt} - \frac{\theta_t \mathbb{E}_t(Q_{lt+1})}{R_{ft}}}, \tag{15}$$

$$B_t = \beta_e \frac{(z_t + Q_{lt}) L_{et-1} - B_{t-1}}{Q_{lt} - \frac{\theta_t \mathbb{E}_t(Q_{lt+1})}{R_{ft}}} \theta_t \mathbb{E}_t (Q_{lt+1}). \tag{16}$$

where $z_t := \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_t^{\frac{\alpha-1}{\alpha}} A_t^{\frac{1}{\alpha}}$ *measures the marginal productivity of land.*

General Equilibrium. In this economy, there are four state variables driving the model's dynamics: entrepreneurs and households land holding, technology and sentiments. Conditional on their initial values $\{L_{e,-1}, L_{h,-1}, A_{-1}, s_{-1}\}$, the dynamic general equilibrium can be defined as allocations $\{C_{et}, C_{ht}, B_t, S_t, L_{et}, L_{ht}, N_t\}_{t=0}^{\infty}$, and prices $\{w_t, Q_{lt}, R_{ft}, R_t\}_{t=0}^{\infty}$, satisfying housholds optimizations (3) to (6), entrepreneur optimization (14) to (16), budgets equations (2), (12), and market clearing conditions for output, labor, land, and bond:

$$Y_t = C_{ht} + C_{et}, \tag{17}$$

$$N_{et} = N_{ht}, \tag{18}$$

$$\bar{L} = L_{et} + L_{ht}, \tag{19}$$

$$B_t = S_t. \tag{20}$$

Inspecting the Mechanism. How is the impact of sentiment on macroeconomy? We now illustrate the propagation mechanism using three key equations linking households consumption C_{ht}, land price Q_{lt}, and hours worked N_t. These equations are the national accounts identity, the land pricing equation, and the labor market clearing equation,

$$A_t L_{et-1}^{\alpha} N_t^{1-\alpha} = C_{ht} + (1-\beta_e)\left[\left(\alpha A_t L_{et-1}^{\alpha-1} N_t^{1-\alpha} + Q_{lt}\right) L_{et-1} - B_{t-1}\right], \tag{21}$$

$$\frac{Q_{lt}}{C_{ht}} = \frac{1}{\varphi}\left(\bar{q} - \phi s_t^2\right), \tag{22}$$

$$\psi N_t^{\upsilon} C_{ht} = (1-\alpha) A_t L_{et-1}^{\alpha} N_t^{-\alpha}, \tag{23}$$

combining the three equations gives the following equilibrium relationship for C_{ht},

$$\begin{aligned} & [1-\alpha(1-\beta_e)]\left(\frac{1-\alpha}{\psi}\right)^{\frac{1-\alpha}{\alpha+\upsilon}}\left(A_t L_{et-1}^{\alpha}\right)^{\frac{1+\upsilon}{\alpha+\upsilon}} C_{ht}^{\frac{\alpha-1}{\alpha+\upsilon}} \\ = \; & \left[1+\varphi(1-\beta_e)\left(\bar{q}-\phi s_t^2\right) L_{et-1}\right] C_{ht} - (1-\beta_e) B_{t-1}, \end{aligned}$$

note the left hand side is a decreasing function of C_{ht}, and the right hand side is an increasing function of C_{ht}. A positive sentiment shock, i.e. drop in s_t^2, shift the right hand side inward and therefore reduce equilibrium household consumption C_{ht}, as in Figure 3. The intuition here is that good sentiment reduces housing price uncertainty, which in turn increase housing demand and decrease consumption. Turning to labor market, this drop in household consumption shift the labor supply curve inward and result in an higher equilibrium labor supply (the right panel of Figure 3). On impact of a favorable sentiment shock, higher labor supply increase equilibrium output, as labor and land holding are the only inputs in entrepreneurs' production function, and land holding is pre-determined. To see the response of housing price, we log-linearize the model (detailed derivation is in Appendix C) and obtain the following characterization, which shows that good sentiment shocks also lead land price to increase. When we introduce capital into entrepreneurs' production in the next section, increase in housing price will lead to increase in capital investment, because land is collateralized and increasing land price relaxes entrepreneurs' borrowing constraint.

Proposition 3. *Let $\hat{Z}_t$ denote the log deviation around variable Z_t's stochastic steady state, then the (log-linearized) dynamics of housing price and output can be shown to follow the following equations,*

$$\hat{Q}_t = \psi_l \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} + \psi_x \hat{x}_t, \tag{24}$$

$$\hat{Y}_t = \varrho_l \hat{L}_{et-1} + \varrho_b \hat{B}_{t-1} + \varrho_x \hat{x}_t, \tag{25}$$

where $x_t := s_t^2$, and ψ_l, ψ_b, ψ_x and $\varrho_l, \varrho_b, \varrho_x$ are constants given in Appendix C. It can be shown *that*

$$\psi_x < 0, \quad (26)$$

$$\varrho_x < 0, \quad (27)$$

that is, positive sentiment shock cause Q_t and Y_t to increase.

To visualize how shock to sentiment propagates to the real macroeconomy, we plot in Figure 5 the impulse and response function of housing price, output, labor and entrepreneur land holding to a one-time unit standard deviation unfavorable sentiment shock. On impact, households reduce labor supply, which is consistent with our previous qualitative analysis. As entrepreneur's land holding is pre-determined, labor demand schedule does not change. Therefore, equilibrium hours drop, leading to drops in output and entrepreneurial profit. Declining profit implies entrepreneur's net worth declines, so that they accumulate less land next period. Reallocation of land reallocates from entrepreneurs to households further reduce labor demand and result in larger drops of output. The declines in labor and land reallocation form a joint force that drives macroeconomic fluctuations.

3 A Medium-scale DSGE Model

In this section, we extend the basic model to a medium-scale DSGE model and access the model's quantitative ability in explaining business cycle fluctuations. In particular, we extend our illustrative model in previous section by introducing capital investment and nominal rigidity. As in the illustrative model, there are households and entrepreneurs. Entrepreneurs produce by employing labor, collateralizable land and physical capital to produce differentiated intermediate goods. Households consume, work, and purchase housing/land. In addition, there are a continuum of retailers, who combines the intermediated goods from entrepreneurs to produce final consumption goods. The retailers face cost in adjusting their output prices, which is the source of nominal rigidity. Central bank adjusts nominal interest rate using Taylor rule. To capture growth in macro variables, we introduce stochastic trends on aggregate TFP and on investment-specific technology as in Justiniano et al. (2009) and Liu et al. (2013). In addition, we also include several additional shocks that are standard in the DSGE literature to allow for other business-cycle drivers to compete with sentiment mechanism: transitory TFP shock, transitory investment-specific shock, labor disutility shock, collateral shock, and monetary policy shock. Details of the model is described as follows.

3.1 Model Description

Households. As in our previous illustrative model, households' maximizes their expected life-time utility function by choosing consumption C_{ht}, housing rental x_t, labor supply N_{ht}, and savings S_t,

$$\max_{C_{ht},x_t,L_{ht},N_{ht},S_t,} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_{ht}^t \left[\log\left(C_{ht} - \eta_h C_{ht-1}\right) + \varphi x_t - \lambda \text{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right) L_t - \psi \frac{N_{ht}^{1+\upsilon}}{1+\upsilon} \right] \right\}$$

subject to the following flow-of-funds constraint,

$$C_{ht} + Q_{lt}\left(L_{ht} - L_{ht-1}\right) + \frac{\tilde{S}_t}{\tilde{R}_{ft}} = w_t N_{ht} - R_t\left(x_t - L_{t-1}\right) + \frac{\tilde{S}_{t-1}}{\pi_t} + \Pi_t, \tag{28}$$

where C_{ht}, N_{ht}, x_t, are consumption, labor supply and housing rental, respectively. η_h measures internal habit formation, Π_t denotes lump-sum profits received from retailers, whose problem we shall describe below. Note that the above constraint is in real terms, and we define $\tilde{S}_t := S_t/P_t$ as the real bond holding, where S_t is bond in nominal terms and P_t denotes the price of consumption goods. The first order conditions associated with decision variables are given by (derivation in Appendix A),

$$C_{ht} \quad : \quad \Lambda_{ht} = \frac{1}{C_{ht} - \eta_h C_{ht-1}} - \beta E_t\left(\frac{\eta_h}{C_{ht+1} - \eta_h C_{ht}}\right) \tag{29}$$

$$x_t \quad : \quad \varphi = \Lambda_{ht} R_t \tag{30}$$

$$N_{ht} \quad : \quad \psi N_{ht}^{\upsilon} = \Lambda_{ht} w_t \tag{31}$$

$$L_{ht} \quad : \quad \frac{Q_{lt}}{C_{ht}} = \beta_h \mathbb{E}_t\left(\varphi + \frac{Q_{lt+1}}{C_{ht+1}}\right) - \lambda \text{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right) \tag{32}$$

$$\tilde{S}_t \quad : \quad 1 = \beta_h \tilde{R}_{ft} \mathbb{E}_t\left(\frac{C_{ht}}{C_{ht+1}} \frac{1}{\pi_{t+1}}\right) \tag{33}$$

where equations (29) to (33) are first order conditions on C_{ht}, x_t, N_{ht}, L_{ht} and $\tilde{S}_t$, respectively.

Final Goods and the Retail Sector. The final goods sector combines a basket of differentiated intermediated goods and turn them into consumption,

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where σ is the elasticity of substitution across these differentiated products. The above CES setting give rise to the following demand schedule,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\sigma} Y_t,$$

where $P_t = \left(\int_0^1 P_t(j)^{\frac{1}{1-\sigma}}\right)^{1-\sigma}$ is the aggregate price index. In the economy, intermediated goods is distributed by continuum of retailers, each producing differentiated products using the homogeneous intermediate goods from entrepreneurs according to the following production function,

$$Y_t(j) = X_t(j),$$

where $X_t(j)$ is intermediate input for retailer indexed by j, whose problem is to choose the price level to maximize its discounted profits. Following Rotemberg (1983), we assume price adjustment are subject to a quadratic cost, $\frac{\gamma}{2}\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)^2 Y_t$, where γ measures the cost of price adjustments and π is the steady state inflation rate. Given this, the problem of retailers is given by,

$$\max_{P_t(j)} \mathbb{E}_0 \left\{ \sum_{i=0}^{\infty} \beta^i \frac{\Lambda_{t+i}}{\Lambda_t} \left[\left(\frac{P_{t+i}(j)}{P_{t+i}} - p_{t+i} \right) Y_{t+i}(j) - \frac{\gamma}{2} \left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1 \right)^2 Y_t \right] \right\},$$

where p_t denotes the relative price of intermediate goods produced by entrepreneurs. Taking first order conditions with respect to $P_t(j)$, and impose symmetric equilibrium yields,

$$p_t = \frac{\sigma - 1}{\sigma} + \frac{\gamma}{\sigma} \left[\frac{\pi_t}{\pi} \left(\frac{\pi_t}{\pi} - 1 \right) - \mathbb{E}_t \left(\beta_h \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\pi_{t+1}}{\pi} \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{Y_{t+1}}{Y_t} \right) \right], \tag{34}$$

where π is steady state inflation rate. Note that in the case when cost of price adjustment is zero, $p_t = \frac{\sigma-1}{\sigma}$, which is the inverse of steady-state markup.

Entrepreneurs. Entrepreneurs produce intermediate goods and sell them to retailers. They have the following utility function,

$$\max_{C_{et}, B_t, L_{et}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_e^t \log\left(C_{et} - \eta_e C_{et-1}\right) \right\}, \tag{35}$$

subject to the following flow-of-funds constraint,

$$C_{et} + Q_{lt}\left(L_{et} - L_{et-1}\right) + \frac{\tilde{B}_{t-1}}{\pi_t} = p_t Y_t - w_t N_{et} - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{\tilde{R}_{ft}}, \tag{36}$$

where C_{et} and I_t are consumption and investment. L_{et}, Y_t, N_t, $\tilde{B}_{t-1}$ denote land holding, intermediate output, and real debt. Q_{lt}, p_t, $\tilde{R}_{ft}$, and w_t are the corresponding prices. The production function is a Cobb-Douglas combination of labor, as well as land and capital that is determined from last period,

$$Y_t = A_t \left(L_{et-1}^{\phi} K_{t-1}^{1-\phi} \right)^{\alpha} (N_{et})^{1-\alpha}. \tag{37}$$

The productivity shock A_t is a combination of permanent component transitory component, $A_t := A_t^p A_t^\tau$. The permanent component A_t^p have stochastic growth rate,

$$\begin{aligned} \log A_t^p &= \log A_{t-1}^p + \log \mu_t^A, \\ \log \mu_t^A &= (1 - \rho_{A^p}) \log \bar{\mu}^{A^p} + \rho_{A^p} \log \mu_{t-1}^A + \sigma_{A^p} \varepsilon_t^{A^p}, \end{aligned}$$

where $\bar{\mu}^{A^p}$ measures the average growth rate. The transitory component A_t^τ follows a standard log-AR(1) process,

$$\log A_t^\tau = \rho_{A^\tau} \log A_{t-1}^\tau + \sigma_{A^\tau} \varepsilon_t^{A^\tau}.$$

Q_{it} is the price of investment goods that also is a combination of permanet and transitory component $Q_{it} = Q_{it}^p Q_{it}^\tau$, where

$$\begin{aligned} \log Q_{it}^p &= \log Q_{it-1}^p + \log \mu_t^{Q_i}, \\ \log \mu_t^{Q_i} &= \left(1 - \rho_{Q_i^p}\right) \log \bar{\mu}^{Q_i} + \rho_{Q_i^p} \log \mu_{t-1}^{Q_i} + \sigma_{Q_i^p} \varepsilon_t^{Q_i}, \\ \log Q_{it}^\tau &= \rho_{Q_i^\tau} \log \log Q_{it-1}^\tau + \sigma_{Q_i^\tau} \varepsilon_t^{Q_i}. \end{aligned}$$

Maximizing over N_{et} simplifies equation (36) into,

$$C_{et} + Q_{lt} (L_{et} - L_{et-1}) + \frac{\tilde{B}_{t-1}}{\pi_t} = z_t L_{et-1}^{\gamma} K_{t-1}^{1-\gamma} - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{R_{ft}} \tag{38}$$

where $z_t := \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_t^{\frac{\alpha-1}{\alpha}} (p_t A_t)^{\frac{1}{\alpha}}$. Capital accumulation is subject to a quadratic adjustment cost,

$$K_t = (1 - \delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - g_I \right)^2 \right] I_t, \tag{39}$$

where g_I denotes the steady state growth rate of entrepreneurial investment. Following Kiyotaki and Moore (1997) and Iacoviello (2005), entrepreneur's borrowing is constrained by,

$$\tilde{B}_t \leq \theta_t \mathbb{E}_t \left[(1 + \pi_{t+1}) (Q_{lt+1} L_{et} + Q_{kt+1} K_t) \right]. \tag{40}$$

The associated first order conditions on $C_{et}, B_t, I_t, K_t, L_{et}$ are given by,

$$C_{et} \;:\; \Lambda_{et} = \frac{1}{C_{et} - \eta_e C_{et-1}} - \beta_t \left(\frac{\eta_e}{C_{et+1} - \eta_e C_{et}} \right) \quad (41)$$

$$N_t \;:\; w_t = p_t (1-\alpha) \frac{Y_t}{N_t} \quad (42)$$

$$B_t \;:\; 1 = \beta_e \tilde{R}_{ft} \mathbb{E}_t \left\{ \frac{\Lambda_{et+1}}{\Lambda_{et}} \frac{1}{\pi_{t+1}} \right\} + \frac{\xi_t}{\Lambda_{et}} \quad (43)$$

$$I_t \;:\; 1 = Q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] \quad (44)$$

$$+\beta \Omega \mathbb{E}_t \left[\frac{\Lambda_{et+1}}{\Lambda_{et}} Q_{kt+1} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \quad (45)$$

$$K_t \;:\; Q_{kt} = \frac{\xi_t}{\Lambda_{et}} \theta_t \mathbb{E}_t \left((1+\pi_{t+1}) Q_{kt+1} \right) + \beta_e \mathbb{E}_t \left\{ \frac{\Lambda_{et+1}}{\Lambda_{et}} \left[\alpha (1-\phi) p_t \frac{Y_{t+1}}{K_t} + (1-\delta) Q_{kt+1} \right] \right\} \quad (46)$$

$$L_{et} \;:\; Q_{lt} = \frac{\xi_t}{\Lambda_{et}} \theta_t \mathbb{E}_t \left((1+\pi_{t+1}) Q_{lt+1} \right) + \beta_e \mathbb{E}_t \left\{ \frac{\Lambda_{et+1}}{\Lambda_{et}} \left[\alpha \phi p_t \frac{Y_{t+1}}{L_{et}} + Q_{lt+1} \right] \right\} \quad (47)$$

where ξ_t is the Lagrange multiplier associated with the collateral constraint (40).

Monetary Policy. The central bank choose nominal interest following Taylor rule as in Christiano et al. (2011),

$$r_t = (1-\rho_r) \bar{r} + \rho_r r_{t-1} + (1-\rho_r) \left[\rho_\pi (\pi_t - \pi) + \rho_y (y_t - y) \right] + \eta_t^r,$$

where interest rates responds to deviations of inflation and output from their steady states. In the above equation, $r_t := \log \tilde{R}_{ft}$ is the logarithm of nominal interest rate; π_t is the inflation and π is its steady state; y_t is the detrended output level and y is its steady state; ρ_r captures the persistence of monetary policy, and η_t^r denote the monetary policy shock that evolves according to a log-AR(1) process,

$$\log \eta_t^r = \rho_m \log \eta_{t-1}^r + \sigma_m \varepsilon_t^m.$$

Equilibrium System. The equilibrium system is defined as follows. Given initial values,

$$\left\{ K_{-1}, L_{e,-1}, L_{h,-1}, A_{-1}, \tilde{B}_{-1}, \tilde{S}_{-1}, C_{h,-1}, C_{e,-1}, s_{-1} \right\},$$

the equilibrium is a set of allocations,

$$\left\{ C_{et}, C_{ht}, \Lambda_{et}, \Lambda_{ht}, K_t, \xi_t, \tilde{B}_t, \tilde{S}_t, L_{et}, L_{ht}, N_t, s_t, Q_{lt}, Q_{kt}, p_t \right\}_{t=0}^{\infty},$$

and prices

$$\left\{ w_t, Q_{lt}, Q_{kt}, \tilde{R}_{ft}, \pi_t \right\}_{t=0}^{\infty}$$

satisfying flow-of-funds constraint (28), (36), households optimization (29) to (33), entrepreneur optimization (41) to (47), taylor rule, market clearing conditions for bond, labor, intermediate and final goods, as well as land. Note the goods market clearing condition is now given by

$$Y_t = C_{ht} + C_{et} + \frac{K_t - (1-\delta) K_{t-1}}{Q_{it}} + \frac{\gamma}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 Y_t, \tag{48}$$

in which the output is either consumed, invested or spent on price adjustment by retailers.

3.2 Bayesian Estimation

We estimate our model using Bayesian method, and the detailed estimation procedure is described in Appendix I. In what follows, we discuss briefly the estimation method, the data used, the priors and the posteriors.

Data and Estimation Method. We use Bayesian method to fit the log-linearized model to 8 quarterly U.S. time series: land price, the inverse of quality-adjusted relative price of investment, real per capita consumption, real per capita investment, real per capita nonfarm nonfinancial business debt, the (utilization adjusted) total productivity, federal fund rate, and inflation. The sample period is 1975:Q1 to 2010:Q4 and in Appendix I, we show the observation equation linking model and data, and how these data is constructed.

Priors and Posteriors. Broadly speaking, these parameters can be sorted into two categories. Structural parameters–including Frisch elasticity, land share, investment adjustment cost, habits, and coefficients on Taylor rule–determines the model's internal propagation mechanism. Shock parameters, i.e. persistences and standard deviation of innovations, governs the dynamics of shock processes. In Table 2, we list one by one the prior distributions of estimated parameters as well as their posterior. The estimation results is broadly consistent with those used in the DSGE literature. For consistency, we set the prior for sentiment shock in line with the other shocks.

The following parameters are calibrated. We set risk aversion parameter λ/φ to 0.13 to match the average housing price to rental rate of 86.4. We set the discount rate of households to 0.9943. This, together with an quarterly inflation target of 0.5%, implies steady state nominal interest rate of 2% annually. We set discount of entrepreneurs to 0.9855, implies a steady state corporate bond spread of around 90 basis point, a number consistent with the yield spread of AAA-rated corporate bond. We set the average growth rate of technology g_A to be 1.0023 to match the quarterly growth rate of aggregate productivity in Fernald (2012), and similarly, we set the average growth rate of investment price to 1.0122. We set $\bar{\theta}$ to 0.80, so that steady state loan-to-value ratio is 4.00. We choose elasticity of substitution parameter $\sigma = 11$, and the cost of price adjustment $\gamma = 112$, so that the average markup of 10%, and that the slope of the

Phillips curve in the model corresponds to that implied by a Calvo model with a duration of price contracts of four quarters (Leduc and Liu, 2016). We set capital share $\alpha = 0.33$, capital depreciation $\delta = 0.036$, and land share $\alpha\phi = 0.026$, a value that is consistent that in Iacoviello (2005) and Liu et al. (2013). Finally, we normalize the average labor aversion parameter $\bar{\psi}$ and the marginal rental rate φ to 1. To the extent that we focus on first order approximation, these two parameters do not play a role in affecting model dynamics. Table 1 summarizes model parameterization. In Appendix I, we establish the mapping from the aforementioned parameters to model's steady states.

The estimation is conducted by log-linearizing the dynamic system around its stochastic steady state where entreprenuers' credit constraint (40) binds. Estimation is done by using the Matlab package Dynare, and we compute the posterior mode by Chris Sims's "csminwel" routine ("compute_mode = 4" in Dynare). Posterior distributions were obtained with the Markov Chain Monte Carlo (MCMC) algorithm, with an acceptance rate of 34%. We generated two parallel chains, each having 100,000 observations, and truncate the first 20% for both chains as burn-in. The posteriors for all the parameters are reported in the last four columns of Table 2. These estimations for parmameters for households, entrepreneurs, retailers, monetary authorities are broadly consistent with other estimates in the literature.

Remark on the Identification of Sentiment Parameters $\{\sigma_s, \rho_s, \lambda/\varphi\}$. Note equation (8) establishes a mapping from sentiment fluctuations to housing price-rent dynamics. It enables us to see transparently to what extent the sentiment process can lead to housing price-rental fluctuations, which is the key mechanism for our model. To this end, we provide a detailed explanation here by deriving three simple structural relationships from our model, which maps these three parameters for sentiment process to three distributional moments on the dynamics of U.S. house price-rent series.

First, the persistence of sentiment also governs the persistence of price to rental rate q_t,

$$\text{Corr}\left(q_t, q_{t-1}\right) = \rho_s^2, \tag{49}$$

Second, the risk aversion parameter λ determines how volatile price rental ratio is,

$$\text{Std}\left(q_t\right) = \frac{\varphi}{2\lambda\rho_s^2}\frac{1-\beta_h\rho_s^2}{1-\rho_s^2}\sqrt{\frac{4\rho_s^2+1}{5\left(1+\rho_s^2\right)}}, \tag{50}$$

Finally, given ρ_s and λ, the average of price-rental rate is determined by,

$$\text{Ave}\left(q_t\right) = \frac{\beta_h}{1-\beta_h} - \frac{\varphi\left(1-\beta_h\rho_s^2\right)}{4\lambda\rho_s^2}\left\{\frac{\beta_h}{1-\beta_h} + \frac{1}{5}\frac{1-\beta_h\rho_s^2}{\rho_s^2}\frac{1}{1-\beta_h} + \frac{1}{1-\rho_s^2}\right\} \tag{51}$$

The first equation is intuitive, as sentiment become more persistent, so will be price-rent ratio.

The second equation says price-rent will becomes more volatile as λ become smaller; The third equations says larger risk-aversion λ will increase average housing price-rental rate. Note σ_s does not show up in the above equations, neither does it affect model dynamics. Therefore, one can restrict attention to the identification of persistence and risk-aversion parameters only, and these sufficient-statistics-like formulas suggest tight identification of the two parameters utilizing information of the housing price/price-rental rate data.

3.3 Propagation Mechanisms

We have argued that fluctuations in sentiment drive changes in housing price rate, and through collateral constraint, cause macroeconomic variables to fluctuates. In this subsection, we first show the propagation mechanism in sticky price setting, and then discuss the estimation results. **The Mechanism**. When price flexible, low perceived house price volatility induce households to increase housing demand, and therefore reduces consumption. The reduction in consumption lead to an outward shift in labor supply. With labor demand curve does not change, equilibrium hours then increases, leading to a boom. In this section, we show how this analysis is enriched in the presence of nominal rigidity.

To fix idea, we restrict our attention to a limitting case where $\gamma = 0$, i.e. capital is not used for production. Note in this case, entrepreneurs consumption policy admits explicit solution,

$$C_{ht} = (1 - \beta_e) \left(\alpha p_t Y_t + Q_{lt} L_{et-1} - \frac{\tilde{B}_{t-1}}{\pi_t} \right), \tag{52}$$

combining it with equations (42), (48) we have,

$$p_t^{\frac{1-\alpha}{\upsilon+\alpha}} \left(A_t L_{et-1}^{\alpha} \right)^{\frac{1+\upsilon}{\upsilon+\alpha}} \left(\frac{1-\alpha}{\psi C_{ht}} \right)^{\frac{1-\alpha}{\upsilon+\alpha}} \tag{53}$$

$$= \frac{1}{\chi_t} \left\{ \left[1 + \varphi (1 - \beta_e) \left(\bar{q} - \phi s_t^2 \right) L_{et-1} \right] C_{ht} - (1 - \beta_e) \frac{\tilde{B}_{t-1}}{\pi_t} \right\}, \tag{54}$$

where $\chi_t := 1 - \frac{\gamma}{2} \left(\frac{\pi_t}{\pi} - 1 \right)^2 - (1 - \beta_e) \alpha p_t$. To assist illustration, we plot in Figure (4) how the left and right hand side of equation (53), as a function of C_{ht}, moves when the economy is hit by a favorable sentiment shock. First, for any given C_{ht}, drop in s_t^2 will lead to increase in the right hand side. This is because, by $\frac{Q_{lt}}{C_{ht}} = \bar{q} - \phi s_t^2$, when C_{ht} is given, drop in s_t^2 imply Q_{lt} must increase. According to equation (52), as Q_{lt} increases, entrepreneurs consumes more because their borrowing constrained is relaxed. With nominal price rigidity, increase in aggregate demand will leads to increase in p_t, leading to an outward shift of the left hand side. When this force is powerful enough, the outward shifting of the left hand side will eventually cause the equilibrium household consumption C_{ht} to increase. Turning to labor market. Increasing in C_{ht} will shift labor supply curve inward. But labor demand expand by more because of the increase in p_t (decrease in markup). This results in higher equilibrium labor. To summarize, in our model with nominal price rigidity, drop in perceived housing price volatility have the potential to generate a boom, with consumption, hours, and output all increase after positive sentiment shock. The following section investigates the model's quantitative potential in explaning business cycle dynamics.

3.4 Quantitative Results

Impulse Response Functions. The Figure 6 shows the impulse response function following unit standard deviation of sentiment shocks. Negative sentiment reduces housing price to rental rate, propagate through a sharp decline in housing price, which in turn tightens entrepreneur's borrowing constraint. Importantly, sentiment shock also lead to risk panics. Followed by a negative sentiment shocks, there is a spike of (conditional) volatility. Besides investment and labor, we also get a negative response on consumption after a unfavorable sentiment shocks. The reason, as we analyzed in section 3.3, is that markup is counter-cyclical.

For completeness, we also report in Figure 7 the impulse and response function of all other shocks in the model, and our finding here is consistent with that of the DSGE literature. For instance, for transitory technology shock, drop in productivity reduces equilibrium labor, consumption, investment and output, and the mechanism mostly work through a direct reduction in labor demand.

Shock Decomposition. By considering shock decomposition, we can gauge the relative importance of the shocks in driving business cycle fluctuations in land price and other key macroeconomic variables. In Table 3, we report the decomposition results of eight types of structural shocks at forecasting horizons from the impact period and six years after the initial shock. The following findings are worth noting.

First, sentiment shock drives most (around 90%) of housing price fluctuations. Through entrepreneurs credit constraints, housing price fluctuations causes a substantial fraction of fluctuations in investment (about 30% to 40%), output (about 15% to 35%), and labor hours (about 15% to 40%). Note that as sentiment shock is the only shock leading to risk panics, we can see that it explains all the fluctuations in the volatility of housing price to rental rate.

Second, aggregate productivity shocks, permanent or transitory, contributes little housing price fluctuations. Because productivity shock does not moves housing price, its impact is not amplified through credit constraints. It do explain, however, a substantial fractions in the fluctuations of consumption. Similarly, a labor supply shock or a patience shock explains little fluctuations in output, investment, and labor hours. This is also because this shock does not drives housing price, and therefore is not amplified through the credit-constraint channel.

Third, the combination of permanent and transitory investment shocks also emerge as the main driver of the business cycle. This is consistent with existing findings in the DSGE literature (e.g., (Justiniano et al., 2009)) and confirms that, apart from the inclusion of the sentiment shock, our exercises are quite typical.

Sticky Prices and Co-movement of Macroeconomic Variables. One issue in the DSGE literature is that demand shocks typically can generate co-movement across macroeconomic variables under sticky prices (Basu and Bundick, 2012). To show this, we conduct counterfactual experiment by shutting down the cost of price adjustment and re-estimate the flexible price version of our baseline model using the same time series. Figure 8 compares the two types of model. Consistent with our illustration in Section 3.3, the presence of nominal rigidity can leads to co-movement in consumption and output, as markup is counter-cyclical.

3.5 Crisis Experiment

To see our model's overall performance in explaining the Great Recession period from 2007:Q3 to 2009:Q2, we conduct a crisis experiment by using the estimated path of sentiment, so that we can access to what extent sentiment shock can generate the declines in macroeconomic variables observed in the Great Recession. To do this, we first estimate the time series paths of sentiments using the estimated model parameters. We then conduct an purification exercise on the estimated sentiments, and we finally construct simulated macroeconomic variables using sentiment shocks.

Estimation of Sentiment Shocks. Given our estimation strategy, the challenges in identifying sentiment shocks are as follows. In reality, sentiment shock could be correlated with a wide range of other shocks, namely, productivity, collateral, etc, could drive macroeconomic fluctuations through sentiment process s_t. In our model, the consequence of this is that sentiment innovations ε_{st} backed out from housing price-rent data may pick up economic fundamentals other than pure sentiments or technology. To address this issue, we extend the process of sentiments to the following in our estimation procedure,

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \sigma_x \varepsilon_t^x + \zeta_t,$$

where ζ_t captures the combined effect of all other fundamental shocks in affecting $\hat{x}_t$, and is defined by,

$$\zeta_t = \sum_{\upsilon} \rho_{\upsilon x} \sigma_\upsilon \varepsilon_t^\upsilon,$$

where $\upsilon \in \{A^\tau, A^p, Q_i^\tau, Q_i^p, \psi, \theta, m\}$, and $\rho_{\upsilon x}$ captures how correlated $\hat{x}_t$ and ε_t^υ is. This specification includes arbitrary linear structure whereby fundamental shocks originating from technology, investment price, labor supply, credit market, and monetary policy would affect housing price through sentiment. In Figure 9, we plot the estimated sentiment shock. Note that in this figure, episodes where unfavorable sentiment jumps most dramatically, i.e. the 1981–1982 recessions and the Great Recession of 2007–2009, also coincides with the periods where macroeconomic volatility is mostly heighted (Jurado et al., 2015). To further purify this series, we conduct an regression exercise by projecting ε_t^x on a wide range of important macroeconomic variables, where we include the consumption, labor supply, investment, consumer CPI, unemployment rate, and export (Milani, 2017; Angeletos et al., 2018). All variables are log-differenced, and we also include their lagged values up to 3 quarters.

Sentiment and Perceived Uncertainty. Do bad sentiments leads to higher perceived unceratinty? Ideally, one answer this this question by obtaining exogeneous proxy of sentiments, and establish a causal relationship between the two. But this is hard, if possible at all. Here, we show that in data, there exists a tight positive correlation between the two. In Figure 11, we plot the perceived uncertainty, proxied by the Michigan Consumer Survey on Consumer Uncertainty, against the lagged measure of sentiment innovations, recovered from our baseline model. These two variable have a correlation of 0.36. The regression coefficient suggests one standard deviation increase is asssociated with 0.5% more increase in the perceived uncertainty, and the relationship is statistically significant. Our choice of this uncertainty measure is that it is more related with consumers' perception, and therefore directly speak to the model's mechanism. In addition, the time span of the survey is consistent with our estimated sentiemnt, whereas for other common uncertainty measures such as VIX/VOX, the coverage is half the length of that of the Michigan Survey.

Crisis Experiment. With estimated sentiment shocks, we can see the extent to which sentiment shock alone can explain the Great Recession episode. In Figure (10), we shows the model generated path on land price and five other macroeconomic variables and compare them with data. To isolate the effect of sentiment, we set all other shocks to zero. There are four messages in this figure that would like to emphasis.

First, sentiment shocks play a crucial role in driving the decline of land price. The impact on land price are propagated through credit constraints to generate the declines of macroeconomic variables, leading to declines in output and business investment. The size of these predicted drops is roughly consistent with that of the data during the crisis period. Second, the model also imply uncertainty to increase during recessions that is largely in line with data. Third, sentiment can also generate modest drops in consumption (2%) and hours (4%). In data, the drops in the two variables are significantly larger (3.5% for consumption and 12% for hours). Forth, although sentiment can generate large drops in key macroeconomic variables (output and investment in particular), the effect of it not as persistent as we have observed in the data. For instance, the simulated path of investment and hours increases after 2008:Q4 while the data continues to decline after that. The reason for this is that other mechanisms beyond sentiments may connects the collapse in housing prices to the sharp contraction in macroeconomic activity in the Great Recession.

4 Conclusion

In this paper, we develop and estimate a dynamic general equilibrium model linking endogenous risk-panics, land price collapse, and the real economy. Our framework features self-fulfilling risk spike in the housing market that result in large drops in housing prices. The theory can generate large volatility in price-rent ratio, as well as the strong co-movement pattern between housing price and macroeconomic aggregates.

Because the Great Financial Crisis was an episode of spikes in uncertainty and collapse in land price, this article take a step in presenting theory focusing on the joint fluctuations of the two variables, as well as their relation to the business cycle. We illustrate an economic mechanism where self-fulfilling risk-panics drives housing price fluctuations, and where these fluctuations transmit and propagate to the real economy. Estimation exercise suggests the mechanism is an quantitatively important one, despite the presence of multiple competing mechanisms.

The framework abstracts from other aspects that we leave for further study. One such dimension is to include allow for credit constraint on the households side. Another one is to extend the model to incorporate the stock market in the model. We hope that the framework we develop in article lays the foundations for extending the model along these and other important dimensions.

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5 Tables and Figures

Figure 3: Illustration of Model Mechanism

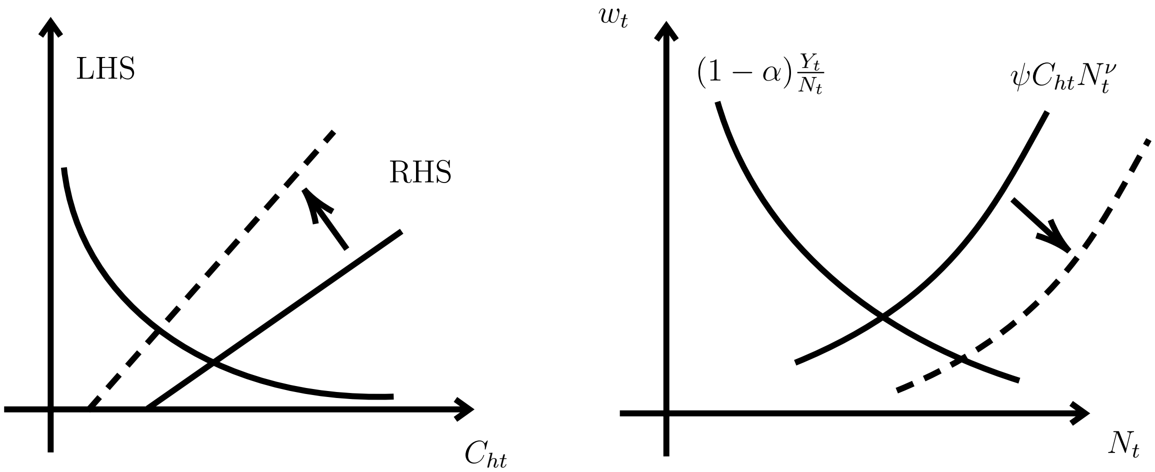


Figure 4: Illustration of Model Mechanism (Sticky Price)

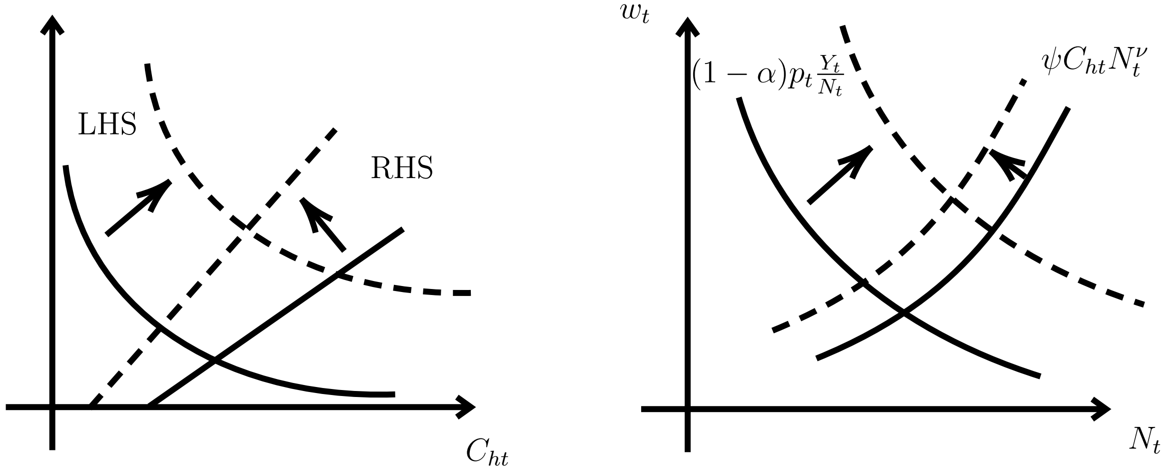
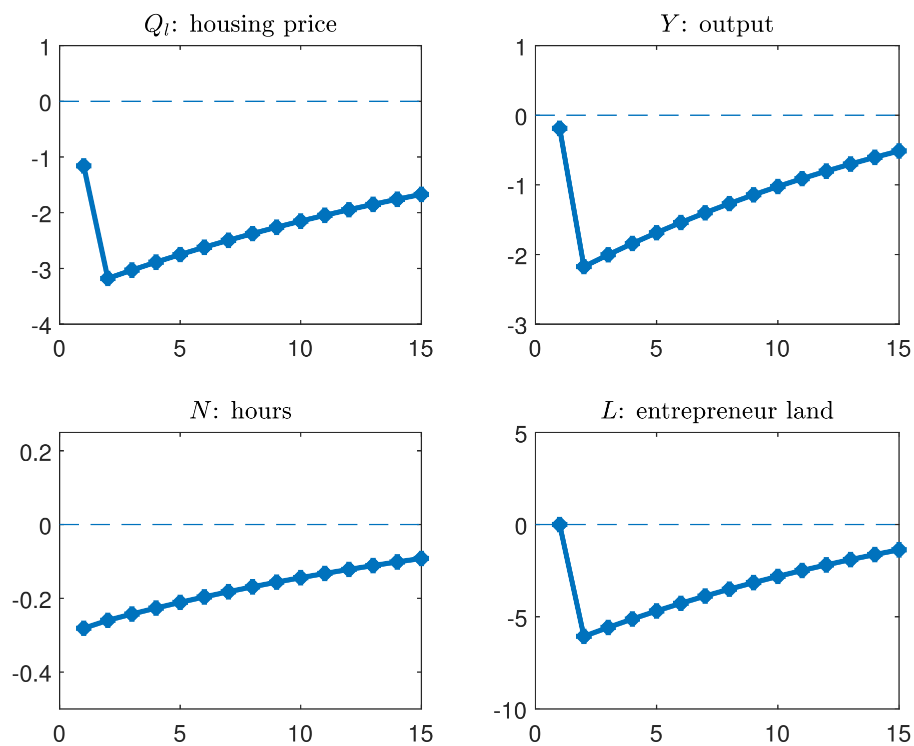


Figure 5: IRF for No-Capital Model (in %)



Note: This figure plots the impulse response function of housing price, output, hours and entrepreneur land. To do so, we pick parameters from Table 1 and 2.

Table 1: Calibrated Parameters

parameters	symbols	values	data/source
growth tech	g_A	1.0023	data
growth invp	g_I	1.0122	data
ave. price rent	$\bar{q}$	86.4450	data
disc household	β_h	0.9943	data
disc eentrepre	β_e	0.9855	data
ces aggregate	σ	11.0000	data
inf target	$\bar{\pi}$	1.0050	data
collat const	$\bar{\theta}$	0.8000	data
cost price adj	γ	112.0000	data
depreciation	δ	0.0360	literature
capital share	α	0.3300	literature
land share	ϕ	0.0800	literature

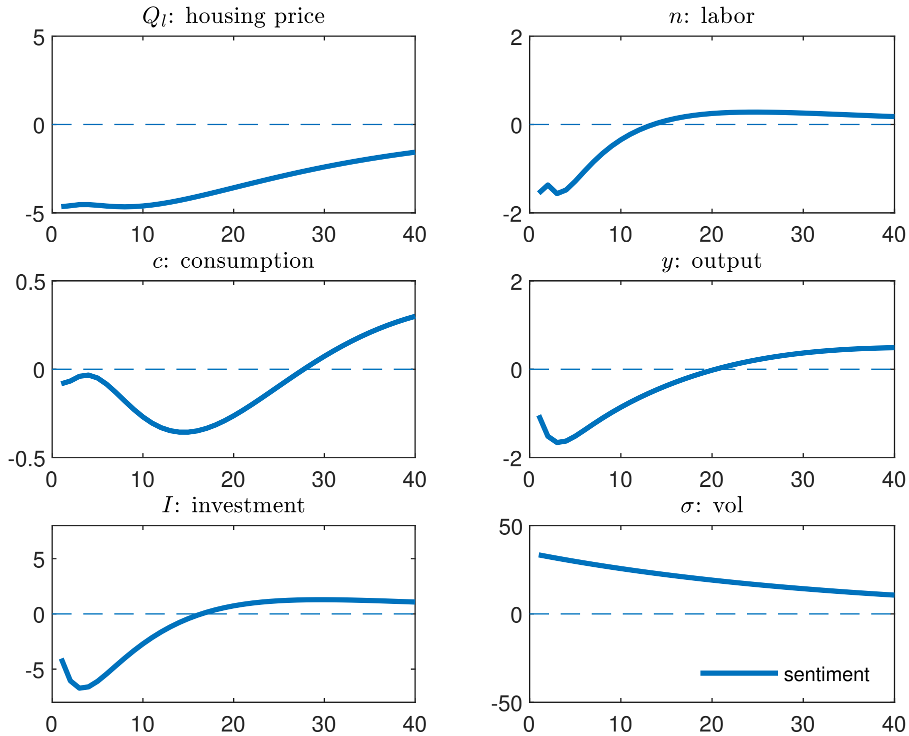
Note: This table lists the calibrated parameters (including the ones taken from literature) for the extended model with capital. Detailed description is given in Section 3.2.

Table 2: Estimated Parameters

		Prior					Posterior		
Parameter	Sym	Dist	a	b	Low	High	Mode	Low	High
structural para									
inv frisch elas	ν	IG(a, b)	0.5000	0.2000	0.4730	1.6360	0.5403	0.3976	0.7026
cost inv. adj	Ω	IG(a, b)	1.0000	0.5000	0.5914	1.5051	0.4481	0.3768	0.5277
habit entrep.	η_e	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.3874	0.2572	0.5225
habit hh	η_h	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.7663	0.7415	0.7903
taylor infl	ϕ_π	IG(a, b)	1.5000	0.2000	3.7579	4.2498	3.9862	3.7421	4.2425
taylor outp	ϕ_y	IG(a, b)	0.5000	0.2000	0.3163	0.7203	0.1765	0.1560	0.1984
shock para									
std collat	σ_θ	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0145	0.0133	0.0156
std inv (perm)	σ_{ip}	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0088	0.0080	0.0097
std inv (tran)	σ_i	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0028	0.0025	0.0032
std hour disu	σ_n	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0286	0.0254	0.0322
std tech (tran)	σ_a	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0129	0.0118	0.0140
std tech (perm)	σ_{ap}	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0069	0.0059	0.0080
std mp	σ_m	IG(a, b)	0.0100	2.0000	0.0037	0.0166	0.0044	0.0038	0.0051
pers nomial	ρ_r	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.5870	0.5202	0.6522
pers tech (trans)	ρ_a	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.7824	0.7406	0.8233
pers tech (perm)	ρ_{ap}	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.5315	0.4597	0.6047
pers senti	ρ_s	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.9854	0.9825	0.9883
pers collat	ρ_t	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.9796	0.9763	0.9831
pers disutility	ρ_n	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.9705	0.9647	0.9764
pers inv (trans)	ρ_i	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.9687	0.9543	0.9829
pers inv (perm)	ρ_{ip}	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.9764	0.9613	0.9899
pers mp	ρ_m	B(a, b)	0.5000	0.2000	0.2320	0.7680	0.5097	0.4409	0.5766
corr(a, s)	$\rho_{a,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	-0.0211	-0.2788	0.2330
corr(ap, s)	$\rho_{ap,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	0.0181	-0.2411	0.2812
corr(n, s)	$\rho_{n,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	-0.0607	-0.3089	0.1968
corr(t, s)	$\rho_{t,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	0.1099	-0.1417	0.3685
corr(i, s)	$\rho_{i,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	0.0096	-0.2440	0.2677
corr(ip, s)	$\rho_{ip,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	-0.0031	-0.2644	0.2630
corr(m, s)	$\rho_{m,s}$	B(a, b)	0.0000	0.5000	-0.2578	0.2578	-0.0057	-0.2584	0.2590

Note: "Low" and "High" denote the bounds of the 90% probability interval for the prior and the posterior distribution. IG denotes the inverse gamma distribution, B denote the beta distribution.

Figure 6: IRFs to one Std. Dev. Shocks



Note: This figure plots the impulse response (in percentage deviation from s.s.) to one standard deviation negative shock on sentiment. The model is solved using parameters in tables 1 and 2. For the estimated parameters, we use their posterior mean.

Table 3: Conditional Variance Decomposition

horizon	senti	trans tech	perm tech	collat	labor	tran inv	perm inv	money
				land price				
1Q	87.6735	2.7941	0.1524	0.0265	0.9507	1.7212	6.1905	0.4911
4Q	90.0861	1.5801	0.6474	0.0101	1.0427	0.8414	5.6274	0.1648
8Q	90.5577	0.8327	0.8448	0.0808	0.8131	1.0983	5.6563	0.1164
16Q	86.6168	0.6945	0.5189	0.2950	0.4741	4.8563	6.3960	0.1485
24Q	82.5797	0.6572	0.3797	0.3676	0.3478	8.5733	6.9365	0.1581
				uncertainty				
1Q	99.9950	0.0001	0.0000	0.0022	0.0000	0.0000	0.0027	0.0000
4Q	99.9950	0.0001	0.0000	0.0022	0.0000	0.0000	0.0027	0.0000
8Q	99.9950	0.0001	0.0000	0.0022	0.0000	0.0000	0.0027	0.0000
16Q	99.9950	0.0001	0.0000	0.0022	0.0000	0.0000	0.0027	0.0000
24Q	99.9950	0.0001	0.0000	0.0022	0.0000	0.0000	0.0027	0.0000
				consumption				
1Q	0.2075	79.1392	10.8278	0.0265	5.8677	0.0077	3.6318	0.2917
4Q	0.1432	45.9051	26.5198	0.1719	8.0652	1.0045	17.4672	0.7230
8Q	0.3936	24.6146	23.5854	0.6093	7.2309	9.2802	33.2047	1.0812
16Q	1.7924	11.2481	10.0432	1.2280	3.3931	33.3237	37.7952	1.1763
24Q	1.5642	7.2697	5.6196	1.1402	1.9190	47.1563	34.3393	0.9917
				hours				
1Q	40.3781	4.9738	0.1028	4.9295	0.2533	10.4715	29.1181	9.7729
4Q	29.3829	3.1956	0.4305	2.2018	0.2383	12.9275	45.8961	5.7272
8Q	24.1154	2.8831	1.1704	1.8013	0.2354	11.0158	55.0333	3.7454
16Q	18.5977	2.1899	1.3049	1.3892	0.2131	8.6517	64.8320	2.8216
24Q	17.0169	1.9660	1.1826	1.3606	0.1903	7.9450	67.8472	2.4913
				output				
1Q	28.3597	22.4151	1.6050	3.4623	5.6682	11.1745	20.4512	6.8640
4Q	32.8815	4.4312	0.6513	4.9752	2.0483	17.6515	31.8695	5.4916
8Q	27.2798	3.9041	0.3109	4.7494	1.2866	22.9059	35.7204	3.8430
16Q	18.5910	3.3580	0.2232	3.7867	0.8075	32.2221	38.2490	2.7624
24Q	14.2392	2.8764	0.2041	3.0463	0.6162	38.5601	38.1789	2.2787
				investment				
1Q	43.3085	3.9142	1.8729	5.2717	0.8035	18.6093	17.5396	8.6803
4Q	40.1412	9.4472	3.4613	5.3815	0.1012	18.5120	18.0204	4.9353
8Q	38.1981	8.1662	4.6652	5.3880	0.0962	20.5516	19.2827	3.6520
16Q	34.8443	7.0385	5.2463	5.1393	0.1296	22.6287	21.7373	3.2362
24Q	34.3537	6.8195	5.2360	4.9896	0.1374	22.5715	22.7469	3.1453

Figure 7: IRFs of All Shocks

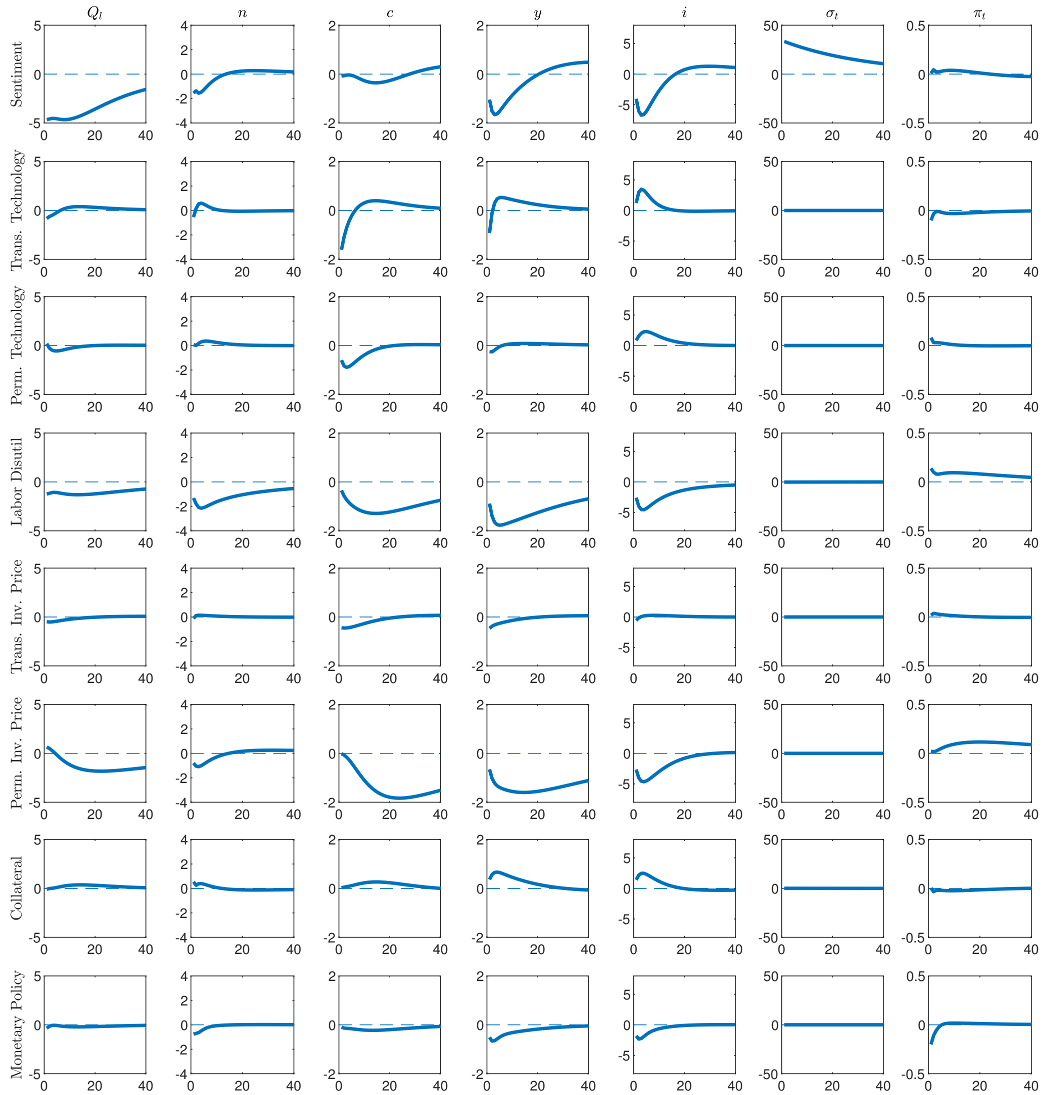
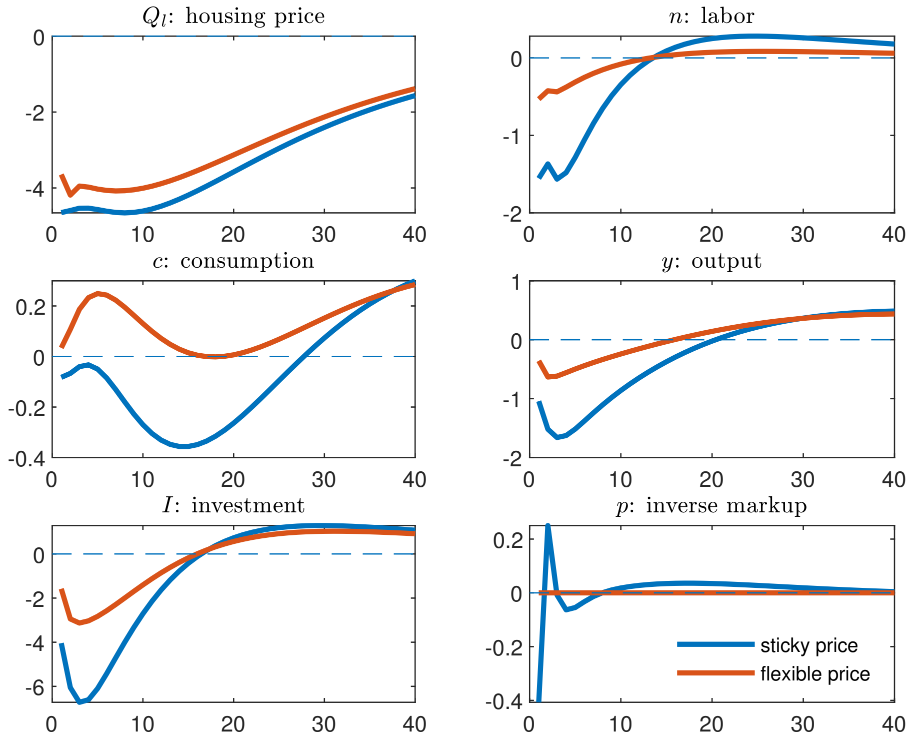
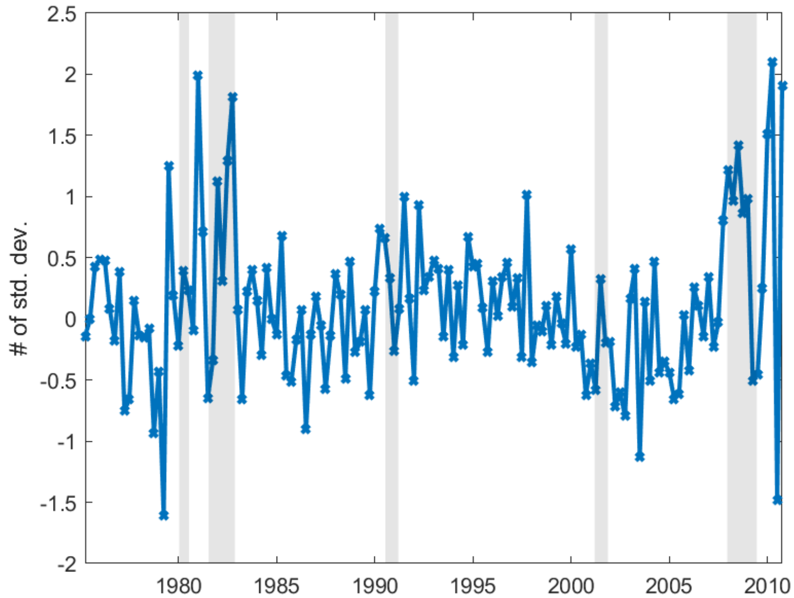


Figure 8: Sticky v.s. Flexible Prices



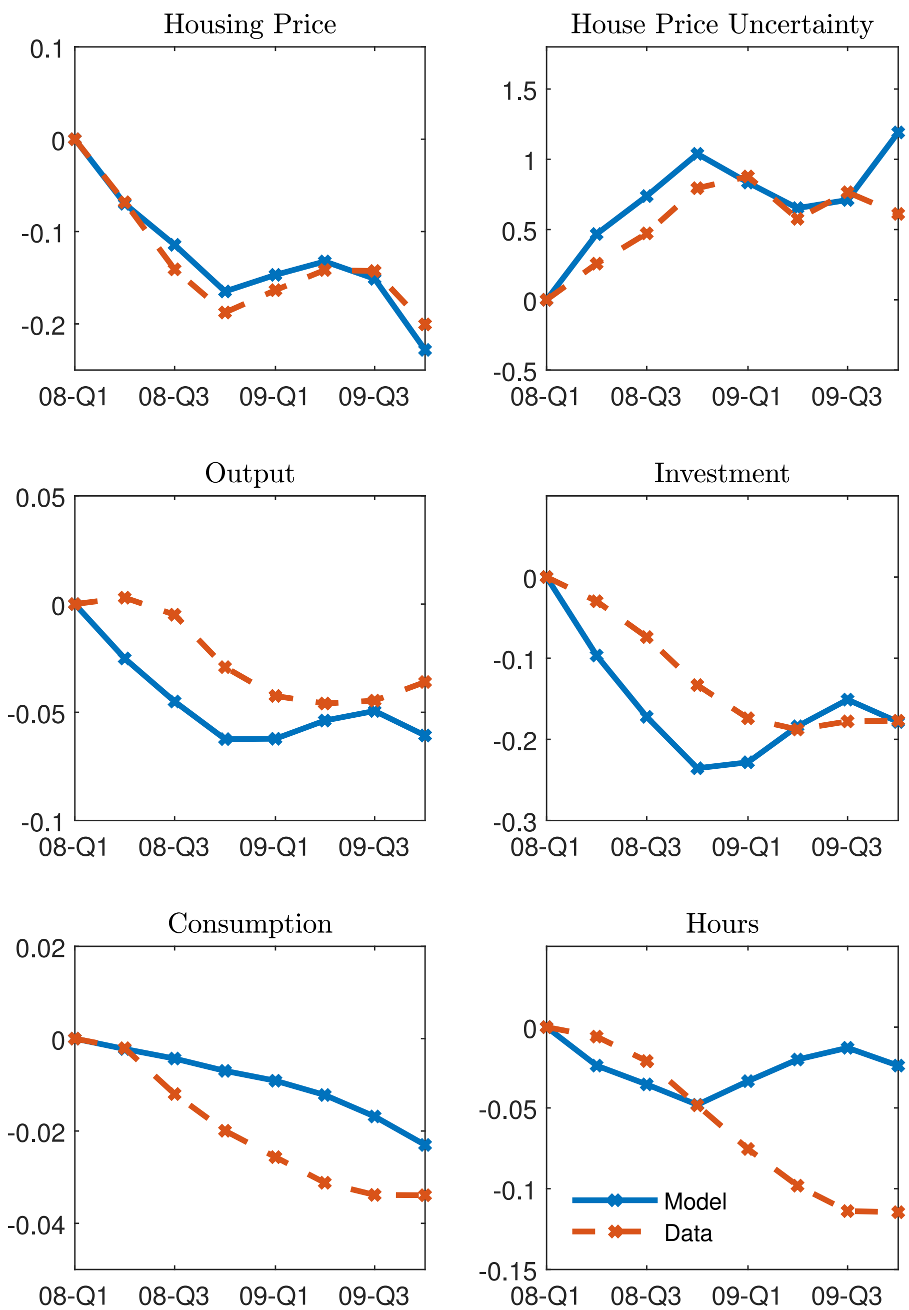
Note: This figure shows the impulse response for sticky price and flexible price models. For sticky price version, we use the estimated parameters in Table 1 and 2. Impulse response for the flexible price model is obtained by setting the cost of price adjustment parameter γ equal to zero.

Figure 9: Estimated Sentiment Innovations



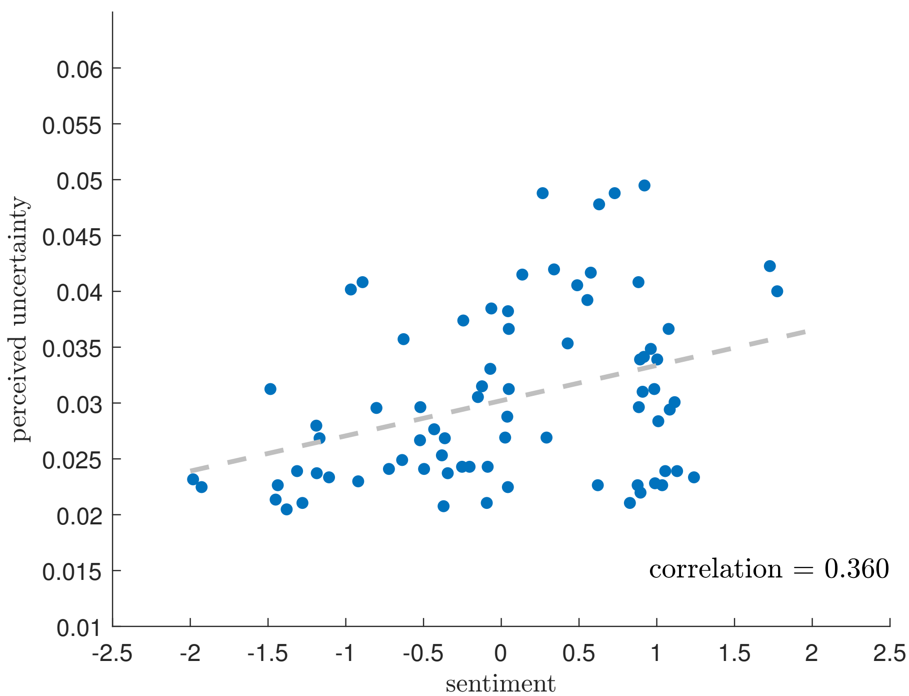
Note: This figure shows the estimated sentiment innovation ε_{xt}.

Figure 10: Crisis Experiment



Note: This figure shows the simulated dynamics of macroeconomic variables around crisis period. We first estimate the sentiment shock (in Figure 9), then we compute these paths by feeding the model with estimated parameters and shocks.

Figure 11: Sentiment and Uncertainty



Note: This figure plots the correlations for the perceived uncertainty against the estimated sentiment series (x_t in model). The perceived uncertainty is constructed from the Michigan Survey of Consumers as in Liu et al. (2017).

A Derivation of Equation (5)

The Lagrange is,

$$
\begin{aligned}
L \;=\; & \log\left(C_{ht}-\eta_h C_{ht-1}\right)+\varphi x_t-\lambda \mathrm{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right)L_t-\psi_t\frac{N_{ht}^{1+\upsilon}}{1+\upsilon} \\
& +\Lambda_{ht}\left\{w_t N_{ht}-R_t\left(x_t-L_{t-1}\right)+\frac{\tilde{S}_{t-1}}{\pi_t}+\Pi_t-C_{ht}-Q_{lt}\left(L_{ht}-L_{ht-1}\right)-\frac{\tilde{S}_t}{\tilde{R}_{ft}}\right\} \\
& +\beta_h \log\left(C_{ht+1}-\eta_h C_{ht}\right)+\varphi x_{t+1}-\lambda \mathrm{Var}_{t+1}\left(\frac{Q_{lt+2}}{R_{t+2}}\right)L_{t+1}-\psi_{t+1}\frac{N_{ht+1}^{1+\upsilon}}{1+\upsilon} \\
& +\beta_h\Lambda_{ht+2}\left\{w_{t+1}N_{ht+1}-R_{t+1}\left(x_{t+1}-L_t\right)+\frac{\tilde{S}_t}{\pi_{t+1}}+\Pi_{t+1}-C_{ht+1}-Q_{lt+1}\left(L_{ht+1}-L_{ht}\right)-\frac{\tilde{S}_{t+1}}{\tilde{R}_{ft+1}}\right\} \\
& +...
\end{aligned}
$$

First order condition on L_{ht} is given by,

$$
Q_{lt}\Lambda_{ht}=\beta E_t\left\{\Lambda_{ht+1}\left[R_{t+1}+Q_{lt+1}\right]\right\}-\lambda \mathrm{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right),
$$

where we have,

$$
Q_{lt}\frac{\Lambda_{ht}}{\varphi}=\beta E_t\left\{1+\frac{\Lambda_{ht+1}}{\varphi}Q_{lt+1}\right\}-\frac{\lambda}{\varphi}\mathrm{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right)
$$

and by using $\varphi=\Lambda_{ht+1}R_{t+1}$ we have,

$$
\frac{Q_{lt}}{R_t}=\beta_h E_t\left\{1+\frac{Q_{lt+1}}{R_{t+1}}\right\}-\frac{\lambda}{\varphi}\mathrm{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right)
$$

B Derivation of Equation (8)

Conjecture that,

$$
q_t=\bar{q}-\phi s_t^2, \tag{55}
$$

where

$$
s_t=\rho_s s_{t-1}+\varepsilon_t, \tag{56}
$$

with $\varepsilon_t \in [-\bar{\varepsilon}, +\bar{\varepsilon}]$. To verify, note,

$$\begin{aligned}
\mathbb{E}_t\left[1+q_{t+1}\right] &= \mathbb{E}_t\left[1+\left(\bar{q}-\phi s_{t+1}^2\right)\right] \\
&= 1+\bar{q}-\phi\mathbb{E}_t\left[s_{t+1}^2\right] \quad (57) \\
&= 1+\bar{q}-\phi\mathbb{E}_t\left[\rho^2 s_t^2+\varepsilon_{t+1}^2+2\rho s_t\varepsilon_{t+1}\right] \\
&= 1+\bar{q}-\phi\left(\rho_s^2 s_t^2+\sigma_s^2\right), \quad (58)
\end{aligned}$$

and note,

$$\begin{aligned}
\mathrm{Var}_t\left(q_{t+1}\right) &= \phi^2\mathrm{Var}_t\left(s_{t+1}^2\right) \\
&= \phi^2\mathrm{Var}_t\left(\varepsilon_{t+1}^2+2\rho_s s_t\varepsilon_{t+1}+\rho^2 s_t^2\right) \\
&= \phi^2\mathrm{Var}_t\left(\varepsilon_{t+1}^2+2\rho_s s_t\varepsilon_{t+1}\right) \\
&= \phi^2\left[\mathbb{E}_t\left(\varepsilon_{t+1}^2+2\rho_s s_t\varepsilon_{t+1}\right)^2-\left(\mathbb{E}_t\left(\varepsilon_{t+1}^2+2\rho_s s_t\varepsilon_{t+1}\right)\right)^2\right] \\
&= \phi^2\left[\mathbb{E}_t\left(\varepsilon_{t+1}^4+4\rho_s^2 s_t^2\varepsilon_{t+1}^2+4\rho_s s_t\varepsilon_{t+1}^3\right)-\sigma_s^4\right] \\
&= \phi^2\left(\omega_s^2+4\sigma_s^2\rho_s^2 s_t^2-\sigma_s^4\right),
\end{aligned}$$

where $\sigma_\varepsilon^2 := \mathbb{E}_t\left(\varepsilon_{t+1}^2\right)$, and $\omega_\varepsilon^2 := \mathbb{E}_t\left(\varepsilon_{t+1}^4\right)$. Therefore,

$$\begin{aligned}
q_t &= \beta_h\mathbb{E}_t\left(1+q_{t+1}\right)-\frac{\lambda}{\varphi}\mathrm{Var}_t\left(q_{t+1}\right) \\
\bar{q}-\phi s_t^2 &= \beta_h\left[1+\bar{q}-\phi\left(\rho_s^2 s_t^2+\sigma_\varepsilon^2\right)\right]-\frac{\lambda}{\varphi}\phi^2\left(\omega_\varepsilon^2+4\sigma_\varepsilon^2\rho_s^2 s_t^2-\sigma_\varepsilon^4\right),
\end{aligned}$$

matching coefficients yields,

$$\begin{aligned}
\bar{q} &= \beta_h\left(1+\bar{q}-\phi\sigma_\varepsilon^2\right)-\frac{\lambda}{\varphi}\phi^2\left(\omega_\varepsilon^2-\sigma_\varepsilon^4\right) \\
-\phi s_t^2 &= \beta_h\left[-\phi\rho_s^2 s_t^2\right]-\frac{\lambda}{\varphi}\phi^2\left(4\sigma_\varepsilon^2\rho_s^2 s_t^2\right)
\end{aligned}$$

or

$$\begin{aligned}
\bar{q} &= \frac{1}{1-\beta_h}\left\{\beta_h-\phi\left[\beta_h\sigma_\varepsilon^2+\frac{\lambda}{\varphi}\phi\left(\omega_\varepsilon^2-\sigma_\varepsilon^4\right)\right]\right\} \quad (59) \\
\phi &= \frac{\varphi\left(1-\beta_h\rho_s^2\right)}{4\lambda\sigma_\varepsilon^2\rho_s^2} \quad (60)
\end{aligned}$$

C Log-Linearized Model without Capital

We first list all the equations for the dynamic system and the log-linearize the system. (This section has been numerically verified by Dynare).

C.1 Households

The Household optimality conditions are given by

$$C_{ht} = \frac{R_t}{\varphi} \tag{61}$$

$$\psi N_{ht}^{\gamma} = \frac{w_t}{C_{ht}} \tag{62}$$

$$\frac{Q_{lt}}{R_t} = \bar{q} - \phi x_t \tag{63}$$

$$1 = \beta R_{ft} \mathbb{E}_t \left\{ \frac{C_{ht}}{C_{ht+1}} \right\} \tag{64}$$

C.2 Entrepreneurs

The budget is,

$$C_{et} + Q_{lt} \left(L_{et} - L_{et-1} \right) - \frac{B_t}{R_{ft}} = z_t L_{et-1} + B_{t-1} \tag{65}$$

and the optimality condition is

$$C_{et} = (1 - \beta_e) \left[(z_t + Q_{lt}) L_{e,t-1} - B_{t-1} \right] \tag{66}$$

$$L_{et} = \frac{1}{Q_{lt} - \theta_t \frac{\mathbb{E}_t[Q_{lt+1}]}{R_{ft}}} \beta_e \left[(z_t + Q_{lt}) L_{e,t-1} - B_{t-1} \right] \tag{67}$$

$$B_t = \theta_t \mathbb{E}_t \left[Q_{lt+1} \right] L_{et}. \tag{68}$$

C.3 Equilibrium Conditions

We have the market clearing conditions,

$$C_{ht} + C_{et} = A_t L_{et-1}^{\alpha} N_t^{1-\alpha}, \tag{69}$$

$$(1 - \alpha) \frac{A_t L_{et-1}^{\alpha} N_{et}^{-\alpha}}{C_{ht}} = \psi N_{ht}^{\upsilon}, \tag{70}$$

$$S_t = B_t, \tag{71}$$

$$L_{ht} + L_{et} = \bar{L}, \tag{72}$$

where $z_t = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} w_t^{\frac{\alpha-1}{\alpha}} A_t^{\frac{1}{\alpha}}$, and the shock processes,

$$\hat{x}_t = \rho_x \hat{x}_{t-1} + \sigma_x \varepsilon_{xt}, \tag{73}$$

where

$$\hat{x}_t := \log x_t - \log \frac{\sigma_\varepsilon^2}{1-\rho_s^2}, \tag{74}$$

is the log-deviation of sentiment shock $x_t := s_t^2$ from its stochastic average. To focus on the impact of sentiment, we omit technology fluctuations by setting $A = 1$ permanently.

C.4 (Stochastic) Steady State

Before log-linearizing the model, we first solve the stochastic steady state where the entrepreneurs' borrowing constraint binds. The stochastic steady state is then given by,

$$\begin{aligned}
\bar{x} &= \frac{\sigma_s^2}{1-\rho_s^2}, \\
\bar{N} &= \left(\psi \frac{1-\alpha\omega}{1-\alpha}\right)^{-\frac{1}{1+\upsilon}}, \\
\bar{Q} &= \left\{ \frac{\varphi(\bar{q}-\phi\bar{x})}{1-\alpha} \left[\frac{(1-\theta\beta_h)-(1-\theta)\beta_e}{\alpha\beta_e}\right]^{\frac{\alpha}{1-\alpha}} \psi \bar{N}^{\upsilon} \right\}^{\alpha-1},
\end{aligned}$$

where,

$$0 < \omega := \frac{(1-\beta_e)(1-\theta\beta_h)}{(1-\theta\beta_h)-(1-\theta)\beta_e} < 1$$

note other variables can be recoverd by the following relationships,

$$\begin{aligned}
\bar{L}_e &= \left[\frac{(1-\theta\beta_h)-(1-\theta)\beta_e}{\alpha\beta_e}\bar{Q}\right]^{\frac{1}{\alpha-1}} \bar{N}, \\
\bar{C}_h &= \frac{1-\alpha}{\psi} \bar{h}_e^{\alpha} \bar{N}^{-(\alpha+\upsilon)}, \\
\bar{R} &= \varphi \bar{C}_h, \\
\bar{C}^e &= \bar{L}_e^{\alpha} \bar{N}^{1-\alpha} - \bar{C}_h, \\
\bar{w} &= (1-\alpha) \bar{L}_e^{\alpha} \bar{N}^{-\alpha}, \\
\bar{z} &= \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} \bar{w}^{\frac{\alpha-1}{\alpha}}, \\
\bar{B} &= \frac{\theta\beta_e}{1-\theta\beta_h} \left[\bar{z} + (1-\theta)\bar{Q}\right] \bar{L}_e.
\end{aligned}$$

To derive the first equation, note from the budget of entrepreneurs, $C_e + (1-\beta_h) B = z L_e$. Plugging C_e, z, and B inside gives the first equation.

C.5 Log-Linearization

Let

$$u_t := Q_{lt} - \theta_t \frac{\mathbb{E}_t\left[Q_{lt+1}\right]}{R_{ft}}.$$

Step 1. We first establish the relationship between L_{et} and N_t. Note from entrepreneurs optimal decision,

$$\frac{C_{et}}{L_{et}} = \frac{1-\beta_e}{\beta_e}\left(Q_{lt} - \theta_t \frac{\mathbb{E}_t\left[Q_{lt+1}\right]}{R_{ft}}\right),$$

where

$$C_{et} = Y_t - C_{ht} = A_t L_{et-1}^{\alpha} N^{1-\alpha} - \frac{1-\alpha}{\psi} A_t L_{et-1}^{\alpha} N_{et}^{-(\alpha+\upsilon)},$$

therefore,

$$L_{et} = L_{et-1}^{\alpha} \frac{N_t^{1-\alpha} - \frac{1-\alpha}{\psi} N_t^{-(\alpha+\upsilon)}}{\frac{1-\beta_e}{\beta_e}\chi_t},$$

so that,

$$\begin{aligned}
\hat{L}_{et} &= \alpha \hat{L}_{et-1} - \hat{\chi}_t + \frac{\bar{N}^{1-\alpha}}{\bar{N}^{1-\alpha} - \frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}\left(1-\alpha\right)\hat{N}_t + \frac{\frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}{\bar{N}^{1-\alpha} - \frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}\left(\alpha+\gamma\right)\hat{N}_t \\
&= \alpha \hat{L}_{et-1} - \hat{\chi}_t + \underbrace{\frac{\left(1-\alpha\right)\bar{N}^{1-\alpha} + \left(\alpha+\upsilon\right)\frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}{\bar{N}^{1-\alpha} - \frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}}_{:=\eta}\hat{N}_t \\
&= \alpha \hat{L}_{et-1} + \eta \hat{N}_t - \hat{\chi}_t \\
&= \left(\alpha+\upsilon\right)\hat{N}_t + \hat{C}_{ht} + \eta \hat{N}_t - \hat{\chi}_t \\
&= \left(\alpha+\upsilon\right)\hat{N}_t + \hat{C}_t^h + \eta \hat{N}_t - \left\{\frac{1}{1-\theta\beta_h}\hat{Q}_{lt} - \frac{\theta\beta_h}{1-\theta\beta_h}\left(\mathbb{E}_t\left[\hat{Q}_{lt+1}\right] - \hat{R}_{ft}\right)\right\} \\
&= \left(\alpha+\upsilon+\eta\right)\hat{N}_t + \hat{C}_t^h - \hat{C}_t^h + \frac{1}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\hat{x}_t - \frac{\theta\beta_h}{1-\theta\beta_h}\mathbb{E}_t\left[\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\hat{x}_{t+1}\right] \\
&= \left(\alpha+\upsilon+\eta\right)\hat{N}_t + \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\hat{x}_t, \qquad (75)
\end{aligned}$$

where by steady state values, we can show,

$$\eta := \frac{\left(1-\alpha\right)\bar{N}^{1-\alpha} + \left(\alpha+\upsilon\right)\frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}}{\bar{N}^{1-\alpha} - \frac{1-\alpha}{\psi}\bar{N}^{-(\alpha+\upsilon)}} = \frac{1+\upsilon}{\alpha\omega} - \left(\alpha+\upsilon\right),$$

where $\omega := \frac{(1-\beta_e)(1-\theta\beta_h)}{(1-\theta\beta_h)-(1-\theta)\beta_e}$.

Step 2. Conjecture,

$$\hat{L}_{et} = \varrho_h \hat{L}_{et-1} + \varrho_b \hat{B}_{t-1} + \varrho_x \hat{x}_t, \qquad (76)$$

where ϱ_h, ϱ_b and ϱ_x are undetermined coefficients.

Step 3. Next, we log-linearize $\hat{Q}_{lt}$. By $\frac{Q_{lt}}{C_{ht}} \propto \bar{q} - \phi x_t$,

$$\begin{aligned}
\hat{Q}_t &= \hat{C}_{ht} + \widehat{\bar{q} - \phi x_t} \\
&= \hat{C}_{ht} - \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \hat{x}_t \\
&= \alpha \hat{L}_{et-1} - (\alpha + \upsilon) \hat{N}_t - \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \hat{x}_t \\
&= \alpha \hat{L}_{et-1} - \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \left(\hat{L}_{et} - \frac{1 - \theta \beta_h \rho_s^2}{1 - \theta \beta_h} \frac{\phi \bar{x}}{\bar{q} - \bar{x}} \hat{x}_t \right) - \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \hat{x}_t \\
&= \underbrace{\alpha - \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \varrho_h}_{\psi_h} \hat{L}_{et-1} \underbrace{- \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \varrho_b}_{\psi_b} \hat{B}_{t-1} \\
&\quad \underbrace{+ \frac{1 - \theta \beta_h \rho_s^2}{1 - \theta \beta_h} \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} - \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \varrho_x - \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}}}_{\psi_x} \hat{x}_t \\
&: = \psi_h \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} + \psi_x \hat{x}_t
\end{aligned} \tag{77}$$

Step 4. We log-linearize $\hat{R}_{ft}$. Note that by $\frac{Q_{lt}}{C_{ht}} \propto \bar{q} - \phi x_t$, we have,

$$\begin{aligned}
\hat{C}_{ht} &= \hat{Q}_{lt} + \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \hat{x}_t \\
&= \psi_h \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} - \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \left(\varrho_x - \frac{1 - \theta \beta_h \rho_s^2}{1 - \theta \beta_h} \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \right) \hat{x}_t,
\end{aligned}$$

so that

$$\begin{aligned}
\hat{R}_{ft} = &\left[\begin{array}{c} \left(\varrho_h \psi_h \hat{L}_{et-1} + \varrho_b \psi_h \hat{B}_{t-1} + \varrho_x \psi_h \hat{x}_t + \varrho_{1x} \psi_h \hat{x}_{t-1} \right) \\ - \frac{\alpha+\upsilon}{\alpha+\upsilon+\eta} \left(\varrho_x - \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h} \frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}} \right) \rho_s^2 \hat{x}_t - \frac{\alpha+\upsilon}{\alpha+\upsilon+\eta} \varrho_{1x} \hat{x}_t \end{array} \right] \\
&- \left\{ \psi_h \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} - \frac{\alpha + \upsilon}{\alpha + \upsilon + \eta} \left(\varrho_x - \frac{1 - \theta \beta_h \rho_s^2}{1 - \theta \beta_h} \frac{\phi \bar{x}}{\bar{q} - \phi \bar{x}} \right) \hat{x}_t \right\} + \psi_b \hat{B}_t
\end{aligned} \tag{78}$$

$$\begin{aligned}
= \; &\left[(\varrho_h - 1) \psi_h + \psi_b \varrho_h \frac{1 + \psi_h}{1 - \psi_b} \right] \hat{L}_{et-1} + \left[(\psi_h \varrho_b - \psi_b) + \psi_b \varrho_b \frac{1 + \psi_h}{1 - \psi_b} \right] \hat{B}_{t-1} \\
&+ \left\{ \begin{array}{c} \psi_b \left(\varrho_x \frac{1+\psi_h}{1-\psi_b} + \frac{\psi_x \rho_s^2 + \psi_{1x}}{1-\psi_b} \right) \\ - \left[(\rho_s^2 - 1) \frac{\alpha+\upsilon}{\alpha+\upsilon+\eta} \left(\varrho_x - \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h} \frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}} \right) + \frac{\alpha+\upsilon}{\alpha+\upsilon+\eta} \varrho_{1x} - \varrho_x \psi_h \right] \end{array} \right\} \hat{x}_t
\end{aligned} \tag{79}$$

Step 5. Then, we derive the log-deviation of $u_t = Q_t - \theta_t \frac{\mathbb{E}_t[Q_{t+1}]}{R_{ft}}$, we have,

$$\begin{aligned}
\hat{u}_t &= \frac{1}{1-\theta\beta_h}\left(\psi_h \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} + \psi_x \hat{x}_t\right) \\
&\quad - \frac{\theta\beta_h}{1-\theta\beta_h}\left\{\underbrace{\psi_h \hat{L}_{et} + \psi_b \hat{B}_t + \left(\psi_x \rho_s^2 + \psi_{1x}\right)\hat{x}_t}_{\mathbb{E}_t\left[\hat{Q}_{t+1}\right]} - \hat{R}_{ft}\right\} \\
&= \frac{\psi_h - \theta\beta_h\left(\varrho_h \psi_h + \psi_b \varrho_h \frac{1+\psi_h}{1-\psi_b}\right)}{1-\theta\beta_h}\hat{L}_{et-1} + \frac{\psi_b - \theta\beta_h\left(\varrho_b \psi_h + \psi_b \varrho_b \frac{1+\psi_h}{1-\psi_b}\right)}{1-\theta\beta_h}\hat{B}_{t-1} \\
&\quad + \frac{\psi_x - \theta\beta_h\left[\varrho_x \psi_h + \psi_b\left(\varrho_x \frac{1+\psi_h}{1-\psi_b} + \frac{\psi_x \rho_s^2 + \psi_{1x}}{1-\psi_b}\right) + \left(\psi_x \rho_s^2 + \psi_{1x}\right)\right]}{1-\theta\beta_h}\hat{x}_t + \frac{\theta\beta_h}{1-\theta\beta_h}\hat{R}_{ft} \\
&= \psi_h \hat{L}_{et-1} + \psi_b \hat{B}_{t-1} + \frac{1}{1-\theta\beta_h} \times \\
&\quad \left\{\begin{array}{c} \left(1-\theta\beta_h \rho_s^2\right)\psi_x - \theta\beta_h\left(\rho_s^2 - 1\right)\frac{\alpha+\upsilon}{\alpha+\upsilon+\eta}\left(\varrho_x - \frac{1-\theta\beta_h \rho_s^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\right) \\ -\theta\beta_h\left(\frac{\alpha+\upsilon}{\alpha+\upsilon+\eta}\varrho_{1x} + \psi_{1x}\right) \end{array}\right\}\hat{x}_t
\end{aligned}$$

Step 6. Finally, we have $L_{et} = \beta_e \frac{\alpha L_{et-1}^{\alpha} N_t^{1-\alpha} + Q_{lt} L_{et-1} - B_{t-1}}{Q_{lt} - \theta_t \frac{\mathbb{E}_t[Q_{lt+1}]}{R_{ft}}}$, log-linearize

$$\begin{aligned}
\hat{L}_{et} &= \alpha L_{et-1}^{\alpha} N_t^{1-\alpha} \widehat{+ Q_{lt}} L_{et-1} - B_{t-1} - \hat{u}_t \\
&= \frac{\alpha\beta_e \bar{L}_e^{\alpha} \bar{N}^{1-\alpha}}{\bar{N}\bar{Q}_l\left(1-\beta_h\theta\right)}\left[\alpha\hat{L}_{et-1} + (1-\alpha)\hat{N}_t\right] + \frac{\beta_e \bar{Q}_l \bar{L}_e}{\bar{L}_e \bar{Q}_l\left(1-\beta_h\theta\right)}\left(\hat{Q}_{lt} + \hat{L}_{et-1}\right) \\
&\quad - \frac{\beta_e \bar{B}}{\bar{L}_e \bar{Q}_l\left(1-\beta_h\theta\right)}\hat{B}_{t-1} - \hat{u}_t \\
&= \left[\frac{\left(1-\beta_h\theta\right) + \beta_e(\theta-1)}{1-\beta_h\theta}\left(\alpha + \frac{1-\alpha}{\alpha+\upsilon+\eta}\varrho_h\right) + \frac{\beta_e}{1-\beta_h\theta}\left(\psi_h + 1\right)\right]\hat{L}_{et-1} \\
&\quad + \left[\frac{\left(1-\beta_h\theta\right) + \beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\alpha+\upsilon+\eta}\varrho_b + \frac{\beta_e}{1-\beta_h\theta}\psi_b - \frac{\theta\beta_e}{1-\beta_h\theta}\right]\hat{B}_{t-1} \\
&\quad + \left\{\frac{\left(1-\beta_h\theta\right) + \beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\alpha+\upsilon+\eta}\left(\varrho_x - \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\right) + \frac{\beta_e}{1-\beta_h\theta}\psi_x\right\}\hat{x}_t - \hat{u}_t.
\end{aligned}$$

where we have used the relationship that $\bar{B} = \theta\bar{Q}_l\bar{L}_e$.

Step 7. Finally, we matching coefficients on ϱ_h, ϱ_b, and ϱ_x,

1. on $\hat{L}_{et-1}$

$$\frac{\left(1-\beta_h\theta\right) + \beta_e(\theta-1)}{1-\beta_h\theta}\left(\alpha + \frac{1-\alpha}{\alpha+\upsilon+\eta}\varrho_h\right) + \frac{\beta_e}{1-\beta_h\theta}\left(\psi_h + 1\right) - \psi_h = \varrho_h$$

rearranging terms

$$\varrho_h = \frac{\frac{\beta_e}{1-\beta_h\theta}(\alpha\theta+1)}{1-\frac{\frac{\beta_e}{1-\beta_h\theta}[\theta(1-\alpha)-(1+\upsilon)]+(1+\upsilon)}{\eta+\alpha+\upsilon}} \tag{80}$$

2. on $\hat{B}_{t-1}$

$$\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon}\varrho_b + \frac{\beta_e}{1-\beta_h\theta}\psi_b - \frac{\theta\beta_e}{1-\beta_h\theta} - \psi_b = \varrho_b,$$

or

$$\varrho_b = -\frac{\frac{\theta\beta_e}{1-\beta_h\theta}}{1-\frac{[(1-\beta_h\theta)+\beta_e(\theta-1)](1-\alpha)-[\beta_e-1+\beta_h\theta](\alpha+\upsilon)}{(1-\beta_h\theta)(\eta+\alpha+\upsilon)}} \tag{81}$$

3. on $\hat{x}_t$

$$\begin{aligned}\varrho_x \quad = \quad & \frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon}\left(\varrho_x - \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\right) \\ & + \frac{\beta_e}{1-\beta_h\theta}\psi_x \\ & - \frac{1}{1-\theta\beta_h}\left\{\begin{array}{c}(1-\theta\beta_h\rho_s^2)\psi_x \\ -\theta\beta_h(\rho_s^2-1)\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\left(\varrho_x - \frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\right)\end{array}\right\}\end{aligned}$$

cancelling terms and plugging ψ_x

$$\varrho_x = \frac{\left[\frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon} - \frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon}+1\right]\frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h} - \frac{\beta_e}{1-\theta\beta_h}}{1-\left[\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon} - \frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\right]}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}} \tag{82}$$

Step 8. Finally, to prove $\psi_x < 0$, we need,

$$\frac{1-\theta\beta_h\rho_x^2}{1-\theta\beta_h}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon} - \frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\varrho_x - \frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}} < 0,$$

plugging in ϱ_x, this is equivalent to,

$$\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\left[\begin{array}{c}\frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h} - \\ \frac{\left[\frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon} - \frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon}+1\right]\frac{1-\theta\beta_h\rho_s^2}{1-\theta\beta_h} - \frac{\beta_e}{1-\beta_h\theta}}{1-\left[\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon} - \frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\right]}\end{array}\right] < 1$$

for this to be true, we need $\frac{\eta+\alpha+\upsilon}{\alpha+\upsilon} > \frac{\frac{\beta_e}{1-\beta_h\theta}}{1-\left[\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\alpha+\gamma+\upsilon}-\frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\gamma}{\upsilon+\alpha+\gamma}\right]}$, or

$$1 < \frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\left[\frac{(1+\upsilon)}{\alpha(1-\beta_e)(\alpha+\upsilon)}-\frac{1-\alpha}{\alpha+\upsilon}\right].$$

note $\frac{(1+\upsilon)}{\alpha(1-\beta_e)(\alpha+\upsilon)}-\frac{1-\alpha}{\alpha+\upsilon}$ is a decreasing function in υ, becuase the derivative w.r.t. υ is,

$$\frac{1-\alpha}{(\alpha+\upsilon)^2}\left[1-\frac{1}{\alpha(1-\beta_e)}\right] < 0 \tag{83}$$

Therefore it reaching minimum as $\upsilon \to \infty$, so that

$$\begin{aligned}
&\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\left[\frac{(1+\upsilon)}{\alpha(1-\beta_e)(\alpha+\upsilon)}-\frac{1-\alpha}{\alpha+\upsilon}\right] \\
> \ &\frac{1}{\alpha}\frac{(1-\theta\beta_h)-(1-\theta)\beta_e}{(1-\theta\beta_h)(1-\beta_e)} \\
> \ &\frac{1}{\alpha} \\
> \ &1.
\end{aligned}$$

By equation (75), the response of $\hat{N}_t$ is thus given by

$$\begin{aligned}
\hat{N}_t = \ &\frac{\varrho_h}{\upsilon+\alpha+\gamma}\hat{L}_{et-1}+\frac{\varrho_b}{\upsilon+\alpha+\gamma}\hat{B}_{t-1} \\
&-\frac{1}{\upsilon+\alpha+\gamma}\frac{\frac{\beta_e}{1-\theta\beta_h}}{1-\left[\frac{(1-\beta_h\theta)+\beta_e(\theta-1)}{1-\beta_h\theta}\frac{1-\alpha}{\eta+\alpha+\upsilon}-\frac{\beta_e-(1-\beta_h\theta)}{1-\beta_h\theta}\frac{\alpha+\upsilon}{\eta+\alpha+\upsilon}\right]}\frac{\phi\bar{x}}{\bar{q}-\phi\bar{x}}\hat{x}_t,
\end{aligned}$$

so that

$$\hat{Y}_t = \alpha\hat{L}_{et-1}+(1-\alpha)\hat{N}_t.$$

D Derivation of Equations (49), (50), and (51)

Note that we have,

- By construction, sentiment innovation ε_{st} follows uniform distribution from $-\bar{\varepsilon}$ to $+\bar{\varepsilon}$, thus the density function is given by,

$$f(\varepsilon) = \begin{cases} \frac{1}{2\bar{\varepsilon}} & \text{if } \varepsilon \in [-\bar{\varepsilon}, +\bar{\varepsilon}] \\ 0 & \text{otherwise} \end{cases} \tag{84}$$

so that we have

$$\sigma_{\varepsilon}^{2} \ : \ = \mathbb{E}\left(\varepsilon_{t}^{2}\right) = \int_{-\bar{\varepsilon}}^{+\bar{\varepsilon}} \frac{1}{2\bar{\varepsilon}}\varepsilon^{2} d\varepsilon = \frac{1}{2\bar{\varepsilon}}\frac{\varepsilon^{3}}{3}|_{-\bar{\varepsilon}}^{+\bar{\varepsilon}} = \frac{1}{2\bar{\varepsilon}}\frac{2}{3}\bar{\varepsilon}^{3} = \frac{\bar{\varepsilon}^{2}}{3}$$

$$\omega_{\varepsilon}^{2} \ : \ = \mathbb{E}\left(\varepsilon_{t}^{4}\right) = \int_{-\bar{\varepsilon}}^{+\bar{\varepsilon}} \frac{1}{2\bar{\varepsilon}}\varepsilon^{4} d\varepsilon = \frac{1}{2\bar{\varepsilon}}\frac{\varepsilon^{5}}{5}|_{-\bar{\varepsilon}}^{+\bar{\varepsilon}} = \frac{1}{2\bar{\varepsilon}}\frac{2}{5}\bar{\varepsilon}^{5} = \frac{\bar{\varepsilon}^{4}}{5}$$

- Note that the process of s_t is given by,

$$s_t = \rho_s s_{t-1} + \varepsilon_t$$

$$s_t^2 = \rho_s^2 s_{t-1}^2 + \varepsilon_t^2 + 2\rho_s s_{t-1}\varepsilon_t$$

so that the process of $x_t := s_t^2$ is given by

$$x_t = \rho_s^2 x_{t-1} + 2\rho_s\sqrt{x_{t-1}}\varepsilon_t + \varepsilon_t^2 \tag{85}$$

and

$$\mathbb{E}\left[s_t^2\right] = \rho_s^2\mathbb{E}\left[s_{t-1}^2\right] + \mathbb{E}\left(\varepsilon_t^2\right)$$

$$\mathbb{E}\left[x_t\right] = \frac{\mathbb{E}\left(\varepsilon_t^2\right)}{1-\rho_s^2} = \frac{\bar{\varepsilon}^2}{3\left(1-\rho_s^2\right)} \tag{86}$$

- Note that,

$$
\begin{aligned}
\mathbb{E}\left[x_t^2\right] &= \mathbb{E}\left[(\rho_s s_{t-1}+\varepsilon_t)^4\right] \\
&= \mathbb{E}\left[\left(\rho_s^2 s_{t-1}^2+2\rho_s\varepsilon_t s_{t-1}+\varepsilon_t^2\right)\left(\rho_s^2 s_{t-1}^2+2\rho_s\varepsilon_t s_{t-1}+\varepsilon_t^2\right)\right] \\
&= \mathbb{E}\left[\begin{array}{c} \rho_s^4 s_{t-1}^4+\rho_s^2 s_{t-1}^2 2\rho_s\varepsilon_t s_{t-1}+\rho_s^2 s_{t-1}^2\varepsilon_t^2 \\ +2\rho_s\varepsilon_t s_{t-1}\rho_s^2 s_{t-1}^2+(2\rho_s\varepsilon_t s_{t-1})^2+2\rho_s\varepsilon_t s_{t-1}\varepsilon_t^2 \\ +\varepsilon_t^2\rho_s^2 s_{t-1}^2+\varepsilon_t^2 2\rho_s\varepsilon_t s_{t-1}+\varepsilon_t^4 \end{array}\right] && (87) \\
&= \mathbb{E}\left[\begin{array}{c} \rho_s^4 s_{t-1}^4+\rho_s^2 s_{t-1}^2\varepsilon_t^2 \\ +(2\rho_s\varepsilon_t s_{t-1})^2 \\ +\varepsilon_t^2\rho_s^2 s_{t-1}^2+\varepsilon_t^4 \end{array}\right] && (88) \\
&= \mathbb{E}\left[\rho_s^4 s_{t-1}^4+\rho_s^2 s_{t-1}^2\varepsilon_t^2+\varepsilon_t^2\rho_s^2 s_{t-1}^2+\varepsilon_t^2\varepsilon_t^2+4\rho_s^2\varepsilon_t^2 s_{t-1}^2\right] \\
&= \rho_s^4\mathbb{E}\left[x_t^2\right]+6\rho_s^2\mathbb{E}\left[\varepsilon_t^2 s_{t-1}^2\right]+\mathbb{E}\left(\varepsilon_t^4\right) && (89) \\
&\Longrightarrow && (90) \\
\mathbb{E}\left[x_t^2\right] &= \frac{6\rho_s^2\mathbb{E}\left[\varepsilon_t^2 s_{t-1}^2\right]+\mathbb{E}\left(\varepsilon_t^4\right)}{1-\rho_s^4} = \frac{6\rho_s^2\frac{\bar{\varepsilon}^2}{3(1-\rho_s^2)}\frac{\bar{\varepsilon}^2}{3}+\frac{\bar{\varepsilon}^4}{5}}{1-\rho_s^4} && (91) \\
&= \frac{\frac{2}{3}\frac{\rho_s^2}{1-\rho_s^2}+\frac{1}{5}}{1-\rho_s^4}\bar{\varepsilon}^4 && (92) \\
&\Longrightarrow && (93) \\
\mathrm{Var}\left(q_t\right) &= \phi^2\mathrm{Var}\left(x_t\right)=\phi^2\left\{\mathbb{E}\left[x_t^2\right]-\left(\mathbb{E}\left[x_t\right]\right)^2\right\} && (94) \\
&= \phi^2\bar{\varepsilon}^4\left\{\frac{\frac{2}{3}\frac{\rho_s^2}{1-\rho_s^2}+\frac{1}{5}}{1-\rho_s^4}-\frac{1}{9\left(1-\rho_s^2\right)^2}\right\} && (95) \\
&= \phi^2\frac{\bar{\varepsilon}^4}{9}\left\{\frac{6\frac{\rho_s^2}{1-\rho_s^2}+\frac{9}{5}}{1-\rho_s^4}-\frac{1}{\left(1-\rho_s^2\right)^2}\right\} && (96) \\
&= \phi^2\frac{\bar{\varepsilon}^4}{9}\left\{\frac{4}{5}\frac{4\rho_s^2+1}{\left(\rho_s^2-1\right)^2\left(\rho_s^2+1\right)}\right\} && (97) \\
\sqrt{\mathrm{Var}\left(q_t\right)} &= \phi\frac{2\bar{\varepsilon}^2}{3\sqrt{5}\left(1-\rho_s^2\right)}\sqrt{\frac{4\rho_s^2+1}{\rho_s^2+1}} && (98) \\
&= \frac{\varphi\left(1-\beta_h\rho_s^2\right)}{4\lambda\mathbb{E}_t\left(\varepsilon_{t+1}^2\right)\rho_s^2}\frac{2\bar{\varepsilon}^2}{3\sqrt{5}\left(1-\rho_s^2\right)}\sqrt{\frac{4\rho_s^2+1}{\rho_s^2+1}} && (99) \\
&= \frac{\left(1-\beta_h\rho_s^2\right)}{2\sqrt{5}\rho_s^2\left(1-\rho_s^2\right)}\frac{\varphi}{\lambda}\sqrt{\frac{4\rho_s^2+1}{\rho_s^2+1}} && (100)
\end{aligned}
$$

so that

$$\frac{\lambda}{\varphi} = \frac{1-\beta_h\rho_s^2}{2\sqrt{5}\rho_s^2\left(1-\rho_s^2\right)\sqrt{\operatorname{Var}\left(q_t\right)}}\sqrt{\frac{4\rho_s^2+1}{\rho_s^2+1}}$$

- Note that

$$\begin{aligned}
\operatorname{Ave}\left(q_t\right) &= \frac{1}{1-\beta_h}\left\{\beta_h - \phi\left[\beta_h\sigma_\varepsilon^2 + \frac{\lambda}{\varphi}\phi\left(\omega_\varepsilon^2 - \sigma_\varepsilon^4\right)\right]\right\} - \phi\mathbb{E}\left[x_t\right] \\
&= \frac{1}{1-\beta_h}\left\{\beta_h - \phi\left[\beta_h\frac{\bar{\varepsilon}^2}{3} + \frac{\lambda}{\varphi}\phi\left(\frac{\bar{\varepsilon}^4}{5} - \frac{\bar{\varepsilon}^4}{9}\right)\right]\right\} - \phi\frac{\bar{\varepsilon}^2}{3\left(1-\rho_s^2\right)} \\
&= \frac{\beta_h}{1-\beta_h} - \frac{\varphi\left(1-\beta_h\rho_s^2\right)}{4\lambda\rho_s^2}\left\{\frac{\beta_h}{1-\beta_h} + \frac{1}{5}\frac{1-\beta_h\rho_s^2}{\rho_s^2}\frac{1}{1-\beta_h} + \frac{1}{1-\rho_s^2}\right\}
\end{aligned}$$

so that

$$\frac{\lambda}{\varphi} = \frac{1-\beta_h\rho_s^2}{4\rho_s^2}\frac{\frac{\beta_h}{1-\beta_h} + \frac{1}{5}\frac{1-\beta_h\rho_s^2}{\rho_s^2}\frac{1}{1-\beta_h} + \frac{1}{1-\rho_s^2}}{\frac{\beta_h}{1-\beta_h} - \operatorname{Ave}\left(q_t\right)}$$

E Linearizing Sentiments

Define the log-deviation of sentiment around its stochastic steady state,

$$\hat{x}_t = \log x_t - \log\frac{\sigma_\varepsilon^2}{1-\rho_s^2},$$

where $x_t = s_t^2$, then

$$\frac{Q_{lt}}{R_t} = \bar{q} - \phi e^{\hat{x}_t + \log\frac{\sigma_\varepsilon^2}{1-\rho_s^2}},$$

note that by the process of x_t in equation (85),

$$\begin{aligned}
x_t &= \rho_s^2 x_{t-1} + 2\rho_s\sqrt{x_{t-1}}\varepsilon_t + \varepsilon_t^2 \\
e^{\hat{x}_t + \log\frac{\sigma_\varepsilon^2}{1-\rho_s^2}} &= \rho_s^2 e^{\hat{x}_{t-1} + \log\frac{\sigma_\varepsilon^2}{1-\rho_s^2}} + 2\rho_s\varepsilon_t e^{\frac{1}{2}\hat{x}_{t-1} + \frac{1}{2}\log\frac{\sigma_\varepsilon^2}{1-\rho_s^2}} + \varepsilon_t^2
\end{aligned}$$

or

$$e^{\hat{x}_t} = \rho_s^2 e^{\hat{x}_{t-1}} + 2\rho_s\sqrt{1-\rho_s^2}\frac{\varepsilon_t}{\sigma_s}e^{\frac{1}{2}\hat{x}_{t-1}} + \frac{\varepsilon_t^2}{\sigma_s^2}\left(1-\rho_s^2\right)$$

subtract the above equation by $1 = \rho_s^2 + (1-\rho_s^2)$,

$$e^{\hat{x}_t} - 1 = \rho_s^2\left(e^{\hat{x}_{t-1}} - 1\right) + 2\rho_s\sqrt{1-\rho_s^2}\frac{\varepsilon_t}{\sigma_s}e^{\frac{1}{2}\hat{x}_{t-1}} + \frac{\varepsilon_t^2 - \sigma_s^2}{\sigma_s^2}\left(1-\rho_s^2\right)$$

to a first order approximation (note we use $e^{\frac{1}{2}\hat{x}_{t-1}} = 1 + \frac{1}{2}\hat{x}_{t-1}$ and drop $\varepsilon_t \hat{x}_{t-1}$ term),

$$\hat{x}_t = \underbrace{\rho_s^2}_{:=\rho_x} \hat{x}_{t-1} + \underbrace{2\rho_s\sqrt{1-\rho_s^2}\frac{\varepsilon_t}{\sigma_\varepsilon} + \frac{\varepsilon_t^2 - \sigma_\varepsilon^2}{\sigma_\varepsilon^2}\left(1-\rho_s^2\right)}_{:=\tilde{\varepsilon}_t,}$$

then,

$$\mathrm{E}\left[\tilde{\varepsilon}_t\right] = 0,$$

and

$$\begin{aligned}
\mathrm{Var}\left[\tilde{\varepsilon}_t\right] &= \mathrm{Var}\left[2\rho_s\sqrt{1-\rho_s^2}\frac{\varepsilon_t}{\sigma_\varepsilon}\right] + \mathrm{Var}\left[\frac{\varepsilon_t^2 - \sigma_\varepsilon^2}{\sigma_\varepsilon^2}\left(1-\rho_s^2\right)\right] \\
&\quad + \mathrm{Cov}\left[2\rho_s\sqrt{1-\rho_s^2}\frac{\varepsilon_t}{\sigma_\varepsilon}, \frac{\varepsilon_t^2 - \sigma_\varepsilon^2}{\sigma_\varepsilon^2}\left(1-\rho_s^2\right)\right] \\
&= 4\rho_s^2\left(1-\rho_s^2\right) + \left(1-\rho_s^2\right)^2 \mathrm{Var}\left(\frac{\varepsilon_t^2}{\sigma_\varepsilon^2}\right) + 0 \\
&= 4\rho_s^2\left(1-\rho_s^2\right) + \left(1-\rho_s^2\right)^2 \frac{\mathbb{E}\left(\varepsilon_t^4\right) - \mathbb{E}\left(\varepsilon_t^2\right)^2}{\sigma_\varepsilon^4} \\
&= 4\rho_s^2\left(1-\rho_s^2\right) + \left(1-\rho_s^2\right)^2 \left(\frac{\omega_\varepsilon^2}{\sigma_\varepsilon^4} - 1\right) \\
&= 4\rho_s^2\left(1-\rho_s^2\right) + \frac{4}{5}\left(1-\rho_s^2\right)^2
\end{aligned}$$

where we use $\frac{\omega_\varepsilon^2}{\sigma_\varepsilon^4}$. In the case where $\rho_s^2 = 0.99$, we have $\mathrm{Std}[\varepsilon_{xt}] \approx \sqrt{0.04} = 0.2$. It is thus convenient to write down

$$\log x_t = \rho_x \log x_{t-1} + \sigma_x \varepsilon_{xt},$$

where

$$\rho_x = \rho_s^2 \tag{101}$$

$$\sigma_x = \sqrt{4\rho_s^2\left(1-\rho_s^2\right) + \frac{4}{5}\left(1-\rho_s^2\right)^2} \tag{102}$$

F Parameter Transformation for Estimation

- We have,

$$\phi = \frac{\varphi\left(1-\beta_h\rho_s^2\right)}{4\lambda\sigma_\varepsilon^2\rho_s^2}, \tag{103}$$

$$\bar{q} = \frac{1}{1-\beta_h}\left\{\beta_h - \phi\left[\beta_h\sigma_\varepsilon^2 + \frac{\lambda}{\varphi}\phi\left(\omega_\varepsilon^2 - \sigma_\varepsilon^4\right)\right]\right\}, \tag{104}$$

- Let $\tilde{q} = \bar{q} - \phi \frac{\sigma_\varepsilon^2}{1-\rho_\varepsilon^2}$, then guess,

$$\Theta := \{\sigma_\varepsilon, \rho_s, \tilde{q}, \beta_h\}, \tag{105}$$

so that,

$$\left\{\phi, \bar{q}, \frac{\lambda}{\varphi}, \omega_\varepsilon\right\} \tag{106}$$

are given by,

$$\phi(\Theta) = \frac{\frac{\beta_h}{1-\beta_h} - \tilde{q}}{\sigma_\varepsilon^2 \left[\frac{1}{1-\beta_h}\left(\beta_h + \frac{1-\beta_h\rho_s^2}{5\rho_s^2}\right) + \frac{1}{1-\rho_s^2}\right]} \tag{107}$$

$$\bar{q}(\Theta) = \frac{\beta_h}{1-\beta_h} - \phi(\Theta)\frac{\sigma_\varepsilon^2}{1-\beta_h}\left(\beta_h + \frac{1-\beta_h\rho_s^2}{5\rho_s^2}\right) \tag{108}$$

$$\omega_s(\Theta) = \frac{3}{\sqrt{5}}\sigma_s^2 \tag{109}$$

$$\frac{\lambda}{\varphi}(\Theta) = \frac{1-\beta_h\rho_s^2}{4\phi(\Theta)\sigma_\varepsilon^2\rho_s^2}$$

- Then the price-rental ratio is given by,

$$\frac{Q_{lt}}{R_t} = \bar{q} - \phi\frac{\sigma_s^2}{1-\rho_s^2}x_t = \bar{q}(\Theta) - \phi(\Theta)\frac{\sigma_\varepsilon^2}{1-\rho_s^2}x_t \tag{110}$$

where

$$\log x_t = \rho_x \log x_{t-1} + \sigma_x \varepsilon_{xt}. \tag{111}$$

G Dynamic System With Stochastic Trends

The model has two stochastic trends, permanent productivity shock and investment price shock,

$$\begin{aligned}
A_t &= A_t^p A_t^\tau \\
\log A_t^p &= \log A_{t-1}^p + \log \mu_t^A \\
\log A_t^\tau &= \rho_{A^\tau} \log A_{t-1}^\tau + \sigma_{A^\tau}\varepsilon_{A^\tau t} \\
\log \mu_t^A &= (1-\rho_{A^p})\log \bar{\mu}^A + \rho_{A^p}\log \mu_{t-1}^A + \sigma_{A^p}\varepsilon_{A^p t}
\end{aligned}$$

and,

$$
\begin{aligned}
Q_{it} &= Q^p_{it} Q^\tau_{it} \\
\log Q^p_{it} &= \log Q^p_{it-1} + \log \mu^{Q_i}_t \\
\log Q^\tau_{it} &= \rho_{Q^\tau_i} \log Q^\tau_{it-1} + \sigma_{Q^\tau_i} \varepsilon_{Q^\tau_i t} \\
\log \mu^{Q_i}_t &= \left(1 - \rho_{Q^p_i}\right) \log \bar{\mu}^{Q_i} + \rho_{Q^p_i} \log \mu^{Q_i}_{t-1} + \sigma_{Q^p_i} \varepsilon_{Q^p_i t}
\end{aligned}
$$

define,

$$\Gamma_t = \left[A_t Q^{(1-\phi)\alpha}_{it}\right]^{\frac{1}{1-\alpha(1-\phi)}}$$

so that,

$$
\begin{aligned}
\log g_{\gamma t} &= \frac{1}{1-\alpha(1-\phi)} \left[\Delta \log A_t + \alpha(1-\phi) \Delta \log Q_{it}\right] \\
&= \frac{1}{1-\alpha(1-\phi)} \left\{ \begin{array}{c} \Delta \left[\log A^p_t + \log A^\tau_t\right] \\ +\alpha(1-\phi) \Delta \left[\log Q^p_{it} + \log Q^\tau_{it}\right] \end{array} \right\} \\
&= \frac{1}{1-\alpha(1-\phi)} \left\{ \begin{array}{c} \log \mu^A_t + \alpha(1-\phi) \log \mu^{Q_i}_t \\ + \left[\log A^\tau_t - \log A^\tau_{t-1}\right] + \alpha(1-\phi) \left[\log Q^\tau_{it} - \log Q^\tau_{it-1}\right] \end{array} \right\} \quad (112)
\end{aligned}
$$

and,

$$\log g_{Q_{it}} = \log \mu^{Q_i}_t + \alpha(1-\phi) \left[\log Q^\tau_{it} - \log Q^\tau_{it-1}\right] \quad (113)$$

so that,

$$\log g_\gamma = \frac{\log \bar{\mu}^A + \alpha(1-\phi) \log \bar{\mu}^{Q_i}}{1-\alpha(1-\phi)}$$

so that,

$$g_\gamma = \left[\bar{\mu}^A \left(\bar{\mu}^{Q_i}\right)^{\alpha(1-\phi)}\right]^{\frac{1}{1-\alpha(1-\phi)}} \quad (114)$$

and,

$$g_{Q^i} = \bar{\mu}^{Q_i}.$$

G.1 Households

We have households problem,

$$\max_{\{C_{ht}, x_t, L_{ht}, N_{ht}, S_t\}} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t_{ht} \left[\log C_{ht} + \varphi x_t - \lambda \text{Var}_t \left(\frac{Q_{lt+1}}{R_{t+1}}\right) L_t - \psi \frac{N^{1+\upsilon}_{ht}}{1+\upsilon}\right] \right\},$$

flow of funds,

$$C_{ht}+Q_{lt}\left(L_{ht}-L_{ht-1}\right)+\frac{\tilde{S}_t}{\tilde{R}_{ft}}=w_tN_{ht}-R_t\left(x_t-L_{t-1}\right)+\frac{\tilde{S}_{t-1}}{\pi_t}+\Pi_t,$$

subject to,

$$\begin{aligned}L &= \log C_{ht}+\varphi x_t-\lambda \mathrm{Var}_t\left(\frac{Q_{lt+1}}{R_{t+1}}\right)L_t-\psi\frac{N_{ht}^{1+\upsilon}}{1+\upsilon}\\ &+\Lambda_{ht}\left\{W_tN_{ht}-R_t\left(x_t-L_{t-1}\right)+\frac{\tilde{S}_{t-1}}{\pi_t}+\Pi_t-C_{ht}-Q_{lt}\left(L_{ht}-L_{ht-1}\right)-\frac{\tilde{S}_t}{\tilde{R}_{ft}}\right\}\end{aligned}$$

First order condition gives,

$$\Lambda_{ht} = \frac{1}{C_{ht}-\eta_hC_{ht-1}}-\beta_h\mathbb{E}_t\left(\frac{\eta_h}{C_{ht+1}-\eta_hC_{ht}}\right) \tag{115}$$

$$\varphi = \Lambda_{ht}R_t \tag{116}$$

$$\psi_tN_{ht}^{\upsilon} = \Lambda_{ht}w_t \tag{117}$$

$$\frac{Q_{lt}}{R_t} = \bar{q}-\phi x_t \tag{118}$$

$$1 = \beta_h\tilde{R}_{ft}\mathbb{E}_t\left(\frac{\Lambda_{ht+1}}{\Lambda_{ht}}\frac{1}{\pi_{t+1}}\right) \tag{119}$$

and we can transform equation (115) into,

$$\begin{aligned}\Lambda_{ht}\Gamma_t &= \frac{\Gamma_t}{C_{ht}-\eta_hC_{ht-1}}-\beta_h\mathbb{E}_t\left(\frac{\eta_h\Gamma_t}{C_{ht+1}-\eta_hC_{ht}}\right)\\ \lambda_{ht} &= \frac{1}{\frac{C_{ht}}{\Gamma_t}-\eta_h\frac{C_{ht-1}}{\Gamma_{t-1}}\frac{\Gamma_{t-1}}{\Gamma_t}}-\beta_h\mathbb{E}_t\left(\frac{\eta_h}{\frac{C_{ht+1}}{\Gamma_{t+1}}\frac{\Gamma_{t+1}}{\Gamma_t}-\eta_h\frac{C_{ht}}{\Gamma_t}}\right)\end{aligned}$$

$$\lambda_{ht} = \frac{1}{c_{ht}-\frac{\eta_h}{g_{\gamma t}}c_{ht-1}}-\beta_h\mathbb{E}_t\left(\frac{\eta_h}{c_{ht+1}g_{\gamma t+1}-\eta_hc_{ht}}\right) \tag{120}$$

and equation (116) into,

$$\varphi=\lambda_{ht}r_t \tag{121}$$

and equation (117) into,

$$\psi_tn_t^{\upsilon}=\lambda_{ht}w_t \tag{122}$$

and equation (118) into,

$$\frac{q_{lt}}{r_t}=\bar{q}-\phi x_t \tag{123}$$

and equation (119) into,

$$
\begin{aligned}
1 &= \beta_h \tilde{R}_{ft} \mathbb{E}_t \left(\frac{\Lambda_{ht+1}}{\Lambda_{ht}} \frac{1}{\pi_{t+1}} \right) \\
1 &= \beta_h \tilde{R}_{ft} \mathbb{E}_t \left(\frac{\Lambda_{ht+1}\Gamma_{t+1}}{\Lambda_{ht}\Gamma_t} \frac{\Gamma_t}{\Gamma_{t+1}} \frac{1}{\pi_{t+1}} \right) && (124) \\
1 &= \beta_h \tilde{R}_{ft} \mathbb{E}_t \left(\frac{\lambda_{ht+1}}{\lambda_{ht} g_{\gamma t+1}} \frac{1}{\pi_{t+1}} \right) && (125)
\end{aligned}
$$

G.2 Intermediate Firms

We have,

$$
\max_{\{C_{et}, B_t, L_{et}\}} E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^e \log\left(C_{et} - \eta_e C_{et-1}\right) \right\},
$$

subject to the constraint,

$$
\begin{aligned}
C_{et} + Q_{lt}\left(L_{et} - L_{et-1}\right) + \frac{\tilde{B}_{t-1}}{\pi_t} &= p_t Y_t - w_t N_t - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{\tilde{R}_{ft}} && (126) \\
\tilde{B}_t &= \theta_t \mathbb{E}_t \left\{ \left(1 + \pi_{t+1}\right)\left(Q_{lt+1} L_{et} + Q_{kt+1} K_t\right) \right\} && (127) \\
K_t &= (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - g_\gamma g_{Q^i} \right)^2 \right] I_t && (128)
\end{aligned}
$$

where,

$$
Y_t = A_t \left(L_{et-1}^{\phi} K_{t-1}^{1-\phi} \right)^{\alpha} N_{et}^{1-\alpha} \qquad (129)
$$

the Lagrange problem is

$$
\begin{aligned}
L &= \log\left(C_{et} - \eta_e C_{et-1}\right) \\
&\quad + \Lambda_{et} \left\{ p_t Y_t - w_t N_t - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{\tilde{R}_{ft}} - C_{et} - Q_{lt}\left(L_{et} - L_{et-1}\right) - \frac{\tilde{B}_{t-1}}{\pi_t} \right\} \\
&\quad + \xi_t \left\{ \theta_t \mathbb{E}_t \left\{ \left(1 + \pi_{t+1}\right)\left(Q_{lt+1} L_{et} + Q_{kt+1} K_t\right) \right\} - \tilde{B}_t \right\} \\
&\quad + \chi_t \left\{ (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - g_\gamma \right)^2 \right] I_t - K_t \right\}
\end{aligned}
$$

this gives,

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_\gamma\right)^2\right] I_t \tag{130}$$

$$\tilde{B}_t = \theta_t \mathbb{E}_t \left[\pi_{t+1} \left(Q_{lt+1} L_{et} + Q_{kt+1} K_t\right)\right] \tag{131}$$

$$\Lambda_{et} = \frac{1}{C_{et} - \eta_e C_{et-1}} - \beta \mathbb{E}_t \left(\frac{\eta_e}{C_{et+1} - \eta_e C_{et}}\right) \tag{132}$$

$$Y_t = A_t \left(L_{et-1}^{\phi} K_{t-1}^{1-\phi}\right)^{\alpha} (N_{et})^{1-\alpha} \tag{133}$$

$$W_t = p_t (1-\alpha) \frac{Y_t}{N_t} \tag{134}$$

$$\frac{1}{\tilde{R}_{ft}} = \beta_e \mathbb{E}_t \left\{\frac{\Lambda_{et+1}}{\Lambda_{et}} \frac{1}{\pi_{t+1}}\right\} + \frac{\xi_t}{\Lambda_{et}} \tag{135}$$

$$\frac{1}{Q_{it}} = Q_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_\gamma\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - g_\gamma\right)\frac{I_t}{I_{t-1}}\right] \tag{136}$$

$$+\beta \Omega E_t \left[\frac{\Lambda_{et+1}}{\Lambda_{et}} Q_{kt+1} \left(\frac{I_{t+1}}{I_t} - g_\gamma\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \tag{137}$$

$$Q_{kt} = \frac{\xi_t}{\Lambda_{et}} \theta_t \mathbb{E}_t \left(\pi_{t+1} Q_{kt+1}\right) + \beta_e \mathbb{E}_t \left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha(1-\phi) p_t \frac{Y_{t+1}}{K_t} + (1-\delta) Q_{kt+1}\right]\right\} \tag{138}$$

$$Q_{lt} = \frac{\xi_t}{\Lambda_{et}} \theta_t \mathbb{E}_t \left(\pi_{t+1} Q_{lt+1}\right) + \beta_e \mathbb{E}_t \left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha \phi p_t \frac{Y_{t+1}}{L_{et}} + Q_{lt+1}\right]\right\} \tag{139}$$

$$C_{et} = z_t L_{et-1}^{\gamma} K_{t-1}^{1-\gamma} - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{R_{ft}} - Q_{lt}\left(L_{et} - L_{et-1}\right) - \frac{\tilde{B}_{t-1}}{\pi_t} \tag{140}$$

transform equation (130) into

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - g_\gamma g_{Q^i}\right)^2\right] I_t$$

$$\frac{K_t}{Q_{it}\Gamma_t} = (1-\delta) \frac{K_{t-1}}{Q_{it}\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\frac{I_t}{Q_{it}\Gamma_t}}{\frac{I_{t-1}}{Q_{it}\Gamma_t}} - g_\gamma g_{Q^i}\right)^2\right] \frac{I_t}{Q_{it}\Gamma_t}$$

$$k_t = (1-\delta) \frac{K_{t-1}}{Q_{it-1}\Gamma_{t-1}} \frac{Q_{it-1}\Gamma_{t-1}}{Q_{it}\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\frac{I_t}{Q_{it}\Gamma_t}}{\frac{I_{t-1}}{Q_{it-1}\Gamma_{t-1}} \frac{Q_{it-1}\Gamma_{t-1}}{Q_{it}\Gamma_t}} - g_\gamma g_{Q^i}\right)^2\right] \frac{I_t}{Q_{it}\Gamma_t}$$

$$k_t = \frac{1-\delta}{g_{\gamma t} g_{Q_t^i}} k_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{i_t}{i_{t-1}} g_{\gamma t} g_{Q_t^i} - g_\gamma g_{Q^i}\right)^2\right] i_t \tag{141}$$

equation (131) into

$$\begin{aligned}\tilde{B}_t &= \theta_t \mathbb{E}_t \left[\pi_{t+1} \left(Q_{lt+1} L_{et} + Q_{kt+1} K_t\right)\right] \\ \frac{\tilde{B}_t}{\Gamma_t} &= \theta_t \mathbb{E}_t \left[\pi_{t+1} \left(\frac{Q_{lt+1}}{\Gamma_{t+1}} \frac{\Gamma_{t+1}}{\Gamma_t} L_{et} + Q_{it} \frac{Q_{it+1}}{Q_{it+1}} Q_{kt+1} \frac{K_t}{\Gamma_t Q_{it}}\right)\right] \qquad (142) \\ b_t &= \theta_t \mathbb{E}_t \left[\pi_{t+1} \left(q_{lt+1} g_{\gamma t+1} l_{et} + \frac{q_{kt+1}}{g_{Q^i_{t+1}}} k_t\right)\right] \qquad (143)\end{aligned}$$

equation (132) into

$$\begin{aligned}\Lambda_{et} &= \frac{1}{C_{et} - \eta_e C_{et-1}} - \beta_e E_t \left(\frac{\eta_e}{C_{et+1} - \eta_e C_{et}}\right) \\ \lambda_{et} &= \frac{1}{c_{et} - \frac{\eta_e}{g_{\gamma t}} c_{et-1}} - \beta_e \mathbb{E}_t \left(\frac{\eta_e}{c_{et+1} g_{\gamma t+1} - \eta_e c_{et}}\right) \qquad (144)\end{aligned}$$

equation (133) into

$$\begin{aligned}Y_t &= A_t \left(L^{\phi}_{et-1} K^{1-\phi}_{t-1}\right)^{\alpha} \left(N_{et}\right)^{1-\alpha} \\ \frac{Y_t}{\Gamma_t} &= \frac{A_t \left(L^{\phi}_{et-1} K^{1-\phi}_{t-1}\right)^{\alpha} \left(N_{et}\right)^{1-\alpha}}{\left[A_t Q_{it}^{(1-\phi)\alpha}\right]^{\frac{1}{1-\alpha(1-\phi)}}} \\ y_t &= \frac{K_{t-1}^{\alpha(1-\phi)}}{\left[A_t^{\alpha(1-\phi)} Q_{it}^{(1-\phi)\alpha}\right]^{\frac{1}{1-\alpha(1-\phi)}}} n_t^{1-\alpha} l_{et-1}^{\alpha\phi} = \left(\frac{K_{t-1}}{\left[A_t Q_{it}\right]^{\frac{1}{1-\alpha(1-\phi)}}}\right)^{\alpha(1-\phi)} n_t^{1-\alpha} l_{et-1}^{\alpha\phi} \\ &= \left(\frac{K_{t-1}}{\Gamma_{t-1} Q_{it-1}} \frac{\Gamma_{t-1} Q_{it-1}}{\Gamma_t Q_{it}}\right)^{\alpha(1-\phi)} n_t^{1-\alpha} l_{et-1}^{\alpha\phi} \\ &= \left(g_{\gamma t} g_{Q^i_t}\right)^{\alpha(\phi-1)} \left(l^{\phi}_{et-1} k^{1-\phi}_{t-1}\right)^{\alpha} n_t^{1-\alpha} \qquad (145)\end{aligned}$$

equation (134) to

$$w_t = p_t \left(1-\alpha\right) \frac{y_t}{n_t} \qquad (146)$$

equation (135) to

$$\frac{1}{\tilde{r}_{ft}} = \beta_e \mathbb{E}_t \left\{\frac{\lambda_{et+1}}{g_{\gamma t+1} \lambda_{et}} \frac{1}{\pi_{t+1}}\right\} + \frac{\xi_t}{\lambda_{et}}$$

equation (136) to

$$\begin{aligned}
\frac{1}{Q_{it}} &= Q_{kt}\left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-g_\gamma g_{Q^i}\right)^2-\Omega\left(\frac{I_t}{I_{t-1}}-g_\gamma g_{Q^i}\right)\frac{I_t}{I_{t-1}}\right] \\
&\quad +\beta\Omega\mathbb{E}_t\left[\frac{\Lambda_{et+1}}{\Lambda_{et}}Q_{kt+1}\left(\frac{I_{t+1}}{I_t}-g_\gamma g_{Q^i}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \qquad (147) \\
\frac{1}{Q_{it}Q_{kt}} &= \left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-g_\gamma g_{Q^i}\right)^2-\Omega\left(\frac{I_t}{I_{t-1}}-g_\gamma g_{Q^i}\right)\frac{I_t}{I_{t-1}}\right] \\
&\quad +\beta\Omega\mathbb{E}_t\left[\frac{\Lambda_{et+1}}{\Lambda_{et}}\frac{Q_{kt+1}}{Q_{kt}}\left(\frac{I_{t+1}}{I_t}-g_\gamma g_{Q^i}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right] \qquad (148) \\
\frac{1}{q_{kt}} &= \left[1-\frac{\Omega}{2}\left(\frac{i_t}{i_{t-1}}g_{\gamma t}g_{Q^i_t}-g_\gamma g_{Q^i}\right)^2-\Omega\left(\frac{i_t}{i_{t-1}}g_{\gamma t}g_{Q^i_t}-g_\gamma g_{Q^i}\right)\frac{i_t}{i_{t-1}}g_{\gamma t}g_{Q^i_t}\right] \\
&\quad +\beta\Omega\mathbb{E}_t\left[\frac{\lambda_{et+1}}{g_{\gamma t+1}\lambda_{et}}\frac{q_{kt+1}}{q_{kt}}\frac{1}{g_{Q^i_{t+1}}}\left(\frac{i_{t+1}}{i_t}g_{\gamma t+1}g_{Q^i_{t+1}}-g_\gamma g_{Q^i}\right)\left(\frac{i_{t+1}}{i_t}g_{\gamma t+1}g_{Q^i_{t+1}}\right)^2\right] \qquad (149)
\end{aligned}$$

equation (138) to

$$\begin{aligned}
Q_{kt} &= \frac{\xi_t}{\Lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}Q_{kt+1}\right)+\beta_e\mathbb{E}_t\left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha\left(1-\phi\right)p_t\frac{Y_{t+1}}{K_t}+\left(1-\delta\right)Q_{kt+1}\right]\right\} \\
1 &= \frac{\xi_t}{\Lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}\frac{Q_{it+1}Q_{kt+1}}{Q_{it}Q_{kt}}\frac{Q_{it}}{Q_{it+1}}\right)+\beta_e\mathbb{E}_t\left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha\left(1-\phi\right)p_t\frac{Y_{t+1}}{Q_{kt}K_t}+\left(1-\delta\right)\frac{Q_{kt+1}}{Q_{kt}}\right]\right\} \\
1 &= \frac{\xi_t}{\lambda_{et}}\theta_t\mathbb{E}_t\left(\frac{q_{kt+1}}{q_{kt}}\frac{\pi_{t+1}}{g_{Q^i_{t+1}}}\right)+\beta_e\mathbb{E}_t\left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha\left(1-\phi\right)p_t\frac{\frac{Y_{t+1}}{\Gamma_t}}{Q_{it}Q_{kt}\frac{K_t}{\Gamma_t Q_{it}}}+\left(1-\delta\right)\frac{q_{kt+1}}{q_{kt}}\frac{1}{g_{Q^i_{t+1}}}\right]\right\} \\
1 &= \frac{\xi_t}{\lambda_{et}}\theta_t\mathbb{E}_t\left(\frac{q_{kt+1}}{q_{kt}}\frac{\pi_{t+1}}{g_{Q^i_{t+1}}}\right)+\beta_e\mathbb{E}_t\left\{\frac{\lambda_{et+1}}{g_{\gamma t+1}\lambda_{et}}\left[\alpha\left(1-\phi\right)p_t\frac{y_{t+1}g_{\gamma t+1}}{q_{kt}k_t}+\left(1-\delta\right)\frac{q_{kt+1}}{q_{kt}}\frac{1}{g_{Q^i_{t+1}}}\right]\right\} \\
q_{kt} &= \frac{\xi_t}{\lambda_{et}}\theta_t\mathbb{E}_t\left(\frac{\pi_{t+1}}{g_{Q^i_{t+1}}}q_{kt+1}\right)+\beta_e\mathbb{E}_t\left\{\frac{\lambda_{et+1}}{g_{\gamma t+1}\lambda_{et}}\left[\alpha\left(1-\phi\right)p_t\frac{y_{t+1}g_{\gamma t+1}}{k_t}+\left(1-\delta\right)\frac{q_{kt+1}}{g_{Q^i_{t+1}}}\right]\right\} \qquad (150)
\end{aligned}$$

equation (139) to

$$\begin{aligned}
Q_{lt} &= \frac{\xi_t}{\Lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}Q_{lt+1}\right)+\beta_e\mathbb{E}_t\left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha\phi p_t\frac{Y_{t+1}}{L_{et}}+Q_{lt+1}\right]\right\} \\
\frac{Q_{lt}}{\Gamma_t} &= \frac{\xi_t}{\Lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}\frac{Q_{lt+1}}{\Gamma_t}\right)+\beta_e\mathbb{E}_t\left\{\frac{\Lambda_{et+1}}{\Lambda_{et}}\left[\alpha\phi p_t\frac{Y_{t+1}}{\Gamma_t L_{et}}+\frac{Q_{lt+1}}{\Gamma_t}\right]\right\} \\
q_{lt} &= \frac{\xi_t}{\lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}q_{lt+1}g_{\gamma t+1}\right)+\beta_e\mathbb{E}_t\left\{\frac{\lambda_{et+1}}{g_{\gamma t+1}\lambda_{et}}\left[\alpha\phi p_t\frac{y_{t+1}}{l_{et}}g_{\gamma t+1}+q_{lt+1}g_{\gamma t+1}\right]\right\} \\
q_{lt} &= \frac{\xi_t}{\lambda_{et}}\theta_t\mathbb{E}_t\left(\pi_{t+1}q_{lt+1}g_{\gamma t+1}\right)+\beta_e\mathbb{E}_t\left\{\frac{\lambda_{et+1}}{\lambda_{et}}\left[\alpha\phi p_t\frac{y_{t+1}}{l_{et}}+q_{lt+1}\right]\right\}
\end{aligned}$$

equation (140) to

$$\begin{aligned}
C_{et} &= \alpha Y_t - \frac{I_t}{Q_{it}} + \frac{\tilde{B}_t}{R_{ft}} - Q_{lt}\left(L_{et} - L_{et-1}\right) - \frac{\tilde{B}_{t-1}}{\pi_t} \\
\frac{C_{et}}{\Gamma_t} &= \alpha \frac{Y_t}{\Gamma_t} - \frac{I_t}{\Gamma_t Q_{it}} + \frac{\tilde{B}_t}{\Gamma_t R_{ft}} - \frac{Q_{lt}}{\Gamma_t}\left(L_{et} - L_{et-1}\right) - \frac{\tilde{B}_{t-1}}{\Gamma_t \pi_t} \\
c_{et} &= \left(g_{\gamma t} g_{Q_t^i}\right)^{\alpha(\phi-1)} \left(l_{et-1}^{\phi} k_{t-1}^{1-\phi}\right)^{\alpha} n_t^{1-\alpha} - i_t + \frac{b_t}{\tilde{r}_{ft}} - q_{lt}\left(L_{et} - L_{et-1}\right) - \frac{b_{t-1}}{\pi_t g_{\gamma t}} \qquad (151)
\end{aligned}$$

G.3 Final Goods Firms

Final goods firms,

$$\begin{aligned}
p_t &= \frac{\sigma - 1}{\sigma} + \frac{\gamma}{\sigma}\left[\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) - \beta_h \mathbb{E}_t\left(\frac{\Lambda_{ht+1}}{\Lambda_{ht}} \frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{Y_{t+1}}{Y_t}\right)\right] \\
p_t &= \frac{\sigma - 1}{\sigma} + \frac{\gamma}{\sigma}\left[\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) - \beta_h \mathbb{E}_t\left(\frac{\lambda_{ht+1}}{g_{\gamma t+1}\lambda_{ht}} \frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{y_{t+1}}{y_t} g_{\gamma t+1}\right)\right] \\
p_t &= \frac{\sigma - 1}{\sigma} + \frac{\gamma}{\sigma}\left\{\frac{\pi_t}{\pi}\left(\frac{\pi_t}{\pi} - 1\right) - \beta_h \mathbb{E}_t\left[\frac{\lambda_{ht+1}}{\lambda_{ht}} \frac{\pi_{t+1}}{\pi}\left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{y_{t+1}}{y_t}\right]\right\}
\end{aligned}$$

G.4 Fed

We have,

$$\begin{aligned}
\log \tilde{R}_{ft} &= \log \tilde{R}_f + \rho_\pi\left(\log \pi_t - \log \pi\right) + \rho_y\left(\log y_t - \log y\right) \\
\log \tilde{r}_{ft} &= \log \tilde{r}_f + \rho_\pi\left(\log \pi_t - \log \pi\right) + \rho_y\left(\log y_{t+1} - \log y\right) \qquad (152)
\end{aligned}$$

G.5 Market Clearing

We have

$$y_t = c_{ht} + c_{et} + i_t + \frac{\gamma}{2}\left(\frac{\pi_t}{\pi} - 1\right)^2 y_t \qquad (153)$$

H Steady State

From household side optimization,

$$\lambda_h = \frac{g_\gamma - \beta_h \eta_h}{g_\gamma - \eta_h} \frac{1}{c_h} \tag{154}$$

$$\varphi = \lambda_h r \tag{155}$$

$$\psi n^\upsilon = \lambda_h w \tag{156}$$

$$\frac{q_l}{r} = \bar{q} - \phi x \tag{157}$$

$$\pi g_\gamma = \beta_h \tilde{r}_f \tag{158}$$

from entrepreneur optimization,

$$\frac{i}{k} = 1 - \frac{1-\delta}{g_\gamma g_{Q^i}} \tag{159}$$

$$b = \theta \pi \left(g_\gamma q_l l_e + \frac{k}{g_{Q^i}} \right) \tag{160}$$

$$\lambda_e = \frac{g_\gamma - \beta_e \eta_e}{g_\gamma - \eta_e} \frac{1}{c_e} \tag{161}$$

$$y = \left(g_\gamma g_{Q^i} \right)^{\alpha(\phi-1)} \left(l_e^\phi k^{1-\phi} \right)^\alpha n^{1-\alpha} \tag{162}$$

$$w = p\left(1-\alpha\right) \frac{y}{n} \tag{163}$$

$$\frac{1}{\tilde{r}_f} = \beta_e \frac{1}{g_\gamma \pi} + \frac{\xi}{\lambda_e} \tag{164}$$

$$1 = \theta \frac{\xi}{\lambda_e} \frac{\pi}{g_{Q^i}} + \frac{\beta_e}{g_\gamma} \left[\alpha \left(1-\phi\right) g_\gamma \frac{y}{k} + \frac{1-\delta}{g_{Q^i}} \right] \tag{165}$$

$$q_l = \frac{\xi}{\lambda_e} \theta \pi q_l g_\gamma + \beta_e \left(\alpha \phi \frac{y}{l_e} + q_l \right) \tag{166}$$

$$c_e = \alpha p y - i + \left(\frac{1}{\tilde{r}_f} - \frac{1}{\pi g_\gamma} \right) b \tag{167}$$

and fed,

$$p = \frac{\sigma - 1}{\sigma} \tag{168}$$

and market clearing,

$$y = c_h + c_e + i \tag{169}$$

H.1 Solving for Steady State

Step 1. : reduce system into $\{c_h, l_e, k, n, q_l\}$. Note that combining equation (154), (156), (163), and (168) gives,

$$c_h = \underbrace{\frac{1-\alpha}{\psi}\frac{g_\gamma - \beta_h\eta_h}{g_\gamma - \eta_h}\frac{\sigma-1}{\sigma}\left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)}}_{:=\chi_1} l_e^{\alpha\phi}k^{\alpha(1-\phi)}n^{-(v+\alpha)}, \tag{170}$$

and note equation (165) and (166) implies,

$$l_e^{\alpha\phi}k^{\alpha(1-\phi)-1}n^{1-\alpha} = \underbrace{\frac{1-\frac{\theta(\beta_h-\beta_e)}{g_\gamma g_{Q^i}}-\frac{\beta_e(1-\delta)}{g_\gamma g_{Q^i}}}{\frac{\sigma-1}{\sigma}\beta_e\alpha\left(1-\phi\right)\left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)}}}_{:=\chi_2} \tag{171}$$

$$l_e^{\alpha\phi-1}k^{\alpha(1-\phi)}n^{1-\alpha} = \underbrace{\frac{(1-\beta_e)-\theta\left(\beta_h-\beta_e\right)}{\frac{\sigma-1}{\sigma}\beta_e\alpha\phi\left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)}}}_{:=\chi_3} q_l \tag{172}$$

where we have used the relationship that,

$$\frac{\xi}{\lambda_e} = \frac{1}{\tilde{r}_f} - \beta_e\frac{1}{g_\gamma\pi} = \frac{1}{\tilde{r}_f} - \beta_e\frac{1}{\beta_h\tilde{r}_f} = \frac{1}{\tilde{r}_f}\left(1-\frac{\beta_e}{\beta_h}\right) \tag{173}$$

Also note from equation (169), $c_e = y - c_h - i$, plugging this equation inside equation (167), gives

$$\left(1-\alpha\frac{\sigma-1}{\sigma}\right)\left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)}\left(l_e^{\phi}k^{1-\phi}\right)^{\alpha}n^{1-\alpha} = c_h - \frac{1-\beta_h}{g_\gamma}\theta\left(g_\gamma q_l l_e + \frac{k}{g_{Q^i}}\right) \tag{174}$$

Finally, we combine equations (154), (155) and (157),

$$q_l = \underbrace{\varphi\frac{g_\gamma - \eta_h}{g_\gamma - \beta_h\eta_h}(\bar{q}-\phi x)}_{:=\chi_0} c_h \tag{175}$$

Step 2. we plug in the following equation into,

$$c_h = \chi_1 l_e^{\alpha\phi}k^{\alpha(1-\phi)}n^{-(v+\alpha)} \tag{176}$$

$$\chi_2 = l_e^{\alpha\phi}k^{\alpha(1-\phi)-1}n^{1-\alpha} \tag{177}$$

$$\chi_3 q_l = l_e^{\alpha\phi-1}k^{\alpha(1-\phi)}n^{1-\alpha} \tag{178}$$

$$q_l = \chi_0 c_h \tag{179}$$

combine equations (179), (176), and (178) solves $n(l_e)$,

$$n(l_e) = (\chi_3 \chi_0 \chi_1 l_e)^{\frac{1}{1+v}}, \tag{180}$$

plugging this equation inside equation (177) gives $k(l_e)$,

$$k(l_e) = \left(\frac{l_e^{\alpha\phi + \frac{1-\alpha}{1+v}}}{\chi_2 (\chi_3 \chi_0 \chi_1)^{\frac{\alpha-1}{1+v}}} \right)^{\frac{1}{1-\alpha(1-\phi)}}, \tag{181}$$

so that by equation (176),

$$c_h(l_e) = \chi_1 \left(l_e^{\phi} k^{1-\phi} \right)^{\alpha} n^{-(\alpha+v)}. \tag{182}$$

Step 3. Plugging equations (180) and (181) into (174) gives a non-linear function with l_e being the unknown,

$$\left(1 - \alpha \frac{\sigma - 1}{\sigma}\right) \left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)} \left(l_e^{\phi} k^{1-\phi} \right)^{\alpha} n^{1-\alpha} = c_h - \frac{1-\beta_h}{g_\gamma} \theta \left(g_\gamma q_l l_e + \frac{k}{g_{Q^i}} \right) \tag{183}$$

Step 4. We can recover other related variables. We have,

$$
\begin{aligned}
n &= \left(\chi_3\chi_0\chi_1 l_e\right)^{\frac{1}{1+\upsilon}}, & (184)\\
k\left(l_e\right) &= \left(\frac{l_e^{\alpha\phi+\frac{1-\alpha}{1+\upsilon}}}{\chi_2\left(\chi_3\chi_0\chi_1\right)^{\frac{\alpha-1}{1+\upsilon}}}\right)^{\frac{1}{1-\alpha(1-\phi)}}, & (185)\\
c_h &= \chi_1\left(l_e^{\phi}k^{1-\phi}\right)^{\alpha} l^{-(\alpha+\upsilon)}, & (186)\\
q_l &= \chi_0 c_h, & (187)\\
i &= \left(1-\frac{1-\delta}{g_\gamma g_{Q^i}}\right)k, & (188)\\
r_f &= \frac{\pi g_\gamma}{\beta_h}, & (189)\\
p &= \frac{\sigma-1}{\sigma}, & (190)\\
y &= \left(g_\gamma g_{Q^i}\right)^{\alpha(\phi-1)}\left(l_e^{\phi}k^{1-\phi}\right)^{\alpha} l^{1-\alpha}, & (191)\\
w &= (1-\alpha)\frac{\sigma-1}{\sigma}\frac{y}{n}, & (192)\\
c_e &= \frac{\sigma-1}{\sigma}y - wn - i + b\left(\frac{1}{r_f}-\frac{1}{\pi g_\gamma}\right), & (193)\\
\lambda_e &= \frac{g_\gamma-\beta_e\eta_e}{g_\gamma-\eta_e}\frac{1}{c_e}, & (194)\\
\lambda_h &= \frac{g_\gamma-\beta_h\eta_h}{g_\gamma-\eta_h}\frac{1}{c_h}, & (195)\\
r &= \frac{\varphi}{\lambda_e}, & (196)\\
\xi &= \frac{1}{r_f}\left(1-\frac{\beta_e}{\beta_h}\right)\lambda_e. & (197)
\end{aligned}
$$

I Detailed Estimation Procedure

I.1 Observation Equations

We have the following observation equations.

- For variables X scaled down by Γ_t, including $\{Y, B, A, Q\}$. The model-implied growth rate is given by,

$$
\begin{aligned}
\underbrace{\log\frac{X_t}{X_{t-1}}}_{\text{from data}} &= \log X_t - \log X_{t-1} \\
&= \log x_t + \log\Gamma_t - (\log x_{t-1} + \log\Gamma_{t-1}) \\
&= \log x_t - \log x_{t-1} + \frac{1}{1-\alpha(1-\phi)}\left[\log\left(A_t Q_{it}^{(1-\phi)\alpha}\right) - \log\left(A_{t-1}Q_{it-1}^{(1-\phi)\alpha}\right)\right] \\
&= \log x_t - \log x_{t-1} \\
&\quad + \frac{1}{1-\alpha(1-\phi)}\left[\log A_t - \log A_{t-1} + \alpha(1-\phi)(\log Q_{it} - \log Q_{it-1})\right] \qquad (198) \\
&= \log x_t - \log x_{t-1} \\
&\quad + \frac{1}{1-\alpha(1-\phi)}\left[\begin{array}{c} \log A_t^p - \log A_{t-1}^p + \log A_t^\tau - \log A_t^\tau \\ +\alpha(1-\phi)\left(\log Q_{it}^p - \log Q_{it-1}^p + \log Q_{it}^\tau - \log Q_{it-1}^\tau\right)\end{array}\right] \qquad (199) \\
&= \log x_t - \log x_{t-1} \qquad (200) \\
&\quad + \frac{1}{1-\alpha(1-\phi)}\left[\begin{array}{c} \log\mu_t^A + \left(\log A_t^\tau - \log A_{t-1}^\tau\right) \\ +\alpha(1-\phi)\left(\log\mu_t^{Q_i} + \log Q_{it}^\tau - \log Q_{it-1}^\tau\right)\end{array}\right] \qquad (201)
\end{aligned}
$$

to show how the growth rate is related with parameters. Note that

$$
\begin{aligned}
\log g_X &= \overline{\log\left(\frac{X_t}{X_{t-1}}\right)} = \frac{1}{1-\alpha(1-\phi)}\left[\log\mu^A + \alpha(1-\phi)\log\mu^{Q_i}\right] \\
g_X &= \left[\left(\mu^A\right)\left(\mu^{Q_i}\right)^{\alpha(1-\phi)}\right]^{\frac{1}{1-\alpha(1-\phi)}}
\end{aligned}
$$

- For variables scaled down by $\Gamma_t Q_{it} = (A_t Q_{it})^{\frac{1}{1-\alpha(1-\phi)}}$, including $\{K_t, I_t\}$. The model implied growth rate is given by,

$$
\underbrace{\log\frac{X_t}{X_{t-1}}}_{\text{from data}} = \log x_t - \log x_{t-1} + \frac{1}{1-\alpha(1-\phi)}\left[\begin{array}{c} \log\mu_t^A + \left(\log A_t^\tau - \log A_{t-1}^\tau\right) \\ + \left(\log\mu_t^{Q_i} + \log Q_{it}^\tau - \log Q_{it-1}^\tau\right)\end{array}\right]
$$

Note that

$$
g_X = \left(\mu^A \mu^{Q_i}\right)^{\frac{1}{1-\alpha(1-\phi)}}
$$

- To link data and model, we have

$$
\begin{aligned}
&\log\left(\frac{Y_t}{Y_{t-1}}\right)-\overline{\log\left(\frac{Y_t}{Y_{t-1}}\right)} \\
=\;&\log y_t-\log y_{t-1}+\frac{1}{1-\alpha(1-\phi)}\left[\begin{array}{c}\underbrace{\log\tilde{\mu}_t^A}_{:=\log\mu_t^A-\overline{\log\mu_t^A}}+\left(\log A_t^\tau-\log A_{t-1}^\tau\right) \\ +\alpha(1-\phi)\left(\underbrace{\log\tilde{\mu}_t^{Q_i}}_{:=\log\mu_t^{Q_i}-\overline{\log\mu_t^{Q_i}}}+\log Q_{it}^\tau-\log Q_{it-1}^\tau\right)\end{array}\right]
\end{aligned}
$$

- Therefore, in the model, the observation equations are,

$$
\begin{pmatrix}\Delta\log HPRICE_t\\ \Delta\log CONS_t\\ \Delta\log INV_t\\ \Delta\log TFP_t\\ \Delta\log INVPRC_t\\ \Delta\log DEBT_t\\ INFLATION_t\\ FEDFUND_t\end{pmatrix}=\begin{pmatrix}\Delta\log\Gamma_t\\ \Delta\log\Gamma_t\\ \Delta\log\Gamma_t\\ \Delta\log\Gamma_t\\ \Delta\log\Gamma_t\\ \Delta\log\Gamma_t\\ \pi\\ r\end{pmatrix}+\begin{pmatrix}\Delta q_{lt}\\ \Delta c_t\\ \Delta i_t\\ \Delta a_t\\ \Delta q_{it}\\ \Delta b_t\\ \pi_t\\ r_t\end{pmatrix}
$$

note in data, investment is in consumption units, $INV_t := \frac{I_t}{Q_{it}} = \frac{I_t}{Q_{it}\Gamma_t}\Gamma_t$. So that the observation equation is $\Delta\log INV_t = \Delta\log\Gamma_t + \Delta i_t$, where $i_t := \frac{I_t}{Q_{it}\Gamma_t}$.

I.2 Construction of Data

We draw data of land price, consumption, output, investment, corporate bond holding, and hours from Liu, Wang and Zha (2014). Inflation and federal fund rate is from FRED dataset.

J Shocks

The model have the following shocks,

$$
\begin{aligned}
\log A_t^\tau &= \rho_{A^\tau} \log A_{t-1}^\tau + \sigma_{A^\tau} \varepsilon_{A^\tau t} \\
\log \mu_t^A &= (1-\rho_{A^p}) \log \bar{\mu}^A + \rho_{A^p} \log \mu_{t-1}^A + \sigma_{A^p} \varepsilon_{A^p t} \\
\log Q_{it}^\tau &= \rho_{Q_i^\tau} \log Q_{it-1}^\tau + \sigma_{Q_i^\tau} \varepsilon_{Q_i^\tau t} \\
\log \mu_t^{Q_i} &= \left(1-\rho_{Q_i^p}\right) \log \bar{\mu}^{Q_i} + \rho_{Q_i^p} \log \mu_{t-1}^{Q_i} + \sigma_{Q_i^p} \varepsilon_{Q_i^p t} \\
\log x_t &= \rho_x^2 \log x_{t-1} + \sigma_x \varepsilon_{xt} \\
\log \psi_t &= \left(1-\rho_\psi\right) \log \bar{\psi} + \rho_\psi \log \psi_{t-1} + \sigma_\psi \varepsilon_{\psi t} \\
\log \theta_t &= (1-\rho_\theta) \log \bar{\theta} + \rho_\theta \log \theta_{t-1} + \sigma_\theta \varepsilon_{\theta t} \\
\log m_t &= \rho_m \log m_{t-1} + \sigma_m \varepsilon_{mt}
\end{aligned}
$$

K Calibrated Parameters

- Growth rate parameters: μ^A, and μ^{Q_i}: directly computed from the growth rate of technology shocks
  - let
  $$\mu^{Q_i} = \overline{\left(\frac{Q_{it}}{Q_{it-1}}\right)} = 1.0122$$
  where Q_{it} is the relative price of investment constructed in Liu, Wang and Zha (2014)
  - let
  $$\mu^A = \overline{\frac{A_t}{A_{t-1}}} = 1.0023$$
  where A_t is Fernald (capital-utilization adjusted) TFP series.
- Steady state parameters: $\bar{\psi}$, $\bar{\theta}$, $\bar{q} - \phi\frac{\sigma_s^2}{1-\rho_s^2}, \beta_h$, π, r_f
  - we have
  $$\bar{q}(\Theta) - \phi(\Theta)\frac{\sigma_s^2}{1-\rho_s^2} = \overline{\frac{Q_{lt}}{R_t}} = 86.4450$$
  note by equation (), $\bar{q}$ will be a combination of parameters.
  - we have π as the average inflation rate
  $$\pi = 1.005$$
  so that 2% per year (fed's target)

- by equation

$$\beta_h = \frac{\pi g_\gamma}{\tilde{r}_f} = \frac{\pi \left[\left(\mu^A\right)\left(\mu^{Q_i}\right)^{\alpha(1-\phi)}\right]^{\frac{1}{1-\alpha(1-\phi)}}}{\tilde{r}_f}$$

- average loan to value ratio $\bar{\theta} = 0.80$

$$\frac{b}{v-b} = \frac{\bar{\theta}}{1-\bar{\theta}} = 4.00$$

- ces aggregate $\sigma = 11$

$$\text{markup} = \frac{1}{\sigma - 1} = 0.10$$

- $\beta_h = 0.9855$ to match (real rate) of
- $\beta_e = 0.9855 \times 1.0089$ to match bond excessive return
- set the price adjustment cost parameter to $\Omega = 112$, so that, to a first-order approximation, the slope of the Phillips curve in our model corresponds to that implied by a Calvo model with a duration of price contracts of four quarters
- $\bar{\psi}$ normalized to, to a first order approximation, $\bar{\psi}$ does not affect dynamics, it only affect s.s.

- Feed in parameters
  - $\alpha = 0.33$
  - $\delta = 0.036$

- Real estate to output (Iacoviello, 2005). Note from the land Euler equation,

$$\begin{aligned}\frac{q_l l_{et}}{y} &= \frac{\beta_e \alpha \phi p}{(1-\beta_e) - \frac{\xi}{\lambda_e}\theta \pi g_\gamma} \\ &= \frac{\beta_e \alpha \phi \frac{\sigma-1}{\sigma}}{(1-\beta_e) - (\beta_h - \beta_e)\theta}\end{aligned}$$

using

$$\frac{\xi}{\lambda_e} = \frac{\beta_h}{\pi g_\gamma}\left(1 - \frac{\beta_e}{\beta_h}\right)$$

so that

$$\phi = \frac{q_l l_{et}}{y}\left[\frac{(1-\beta_e) - (\beta_h - \beta_e)\theta}{\beta_e \alpha \frac{\sigma-1}{\sigma}}\right]$$

so that the land value over output is given by

$$\frac{q_l l_e}{y} = \frac{\beta_e \alpha \phi \frac{\sigma-1}{\sigma}}{(1-\beta_e) - (\beta_h - \beta_e)\theta}$$

- All other parameters are estimated by using structural and shock parameters

L Flexible Price Model

The household side,

$$\begin{aligned}
\lambda_{ht} &= \frac{1}{c_{ht} - \frac{\eta_h}{g_{\gamma t}} c_{ht-1}} - \beta_h \mathbb{E}_t \left(\frac{\eta_h}{c_{ht+1} g_{\gamma t+1} - \eta_h c_{ht}} \right) \\
\varphi &= \lambda_{ht} r_t \\
\psi_t n_t^{\upsilon} &= \lambda_{ht} w_t \\
\frac{q_{lt}}{r_t} &= \bar{q} - \phi x_t \\
1 &= \beta_h R_{ft} \mathbb{E}_t \left(\frac{\lambda_{ht+1}}{\lambda_{ht} g_{\gamma t+1}} \right)
\end{aligned}$$

Transform equation (130) into

$$\begin{aligned}
k_t &= \frac{1-\delta}{g_{\gamma t} g_{Q_t^i}} k_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{i_t}{i_{t-1}} g_{\gamma t} g_{Q_t^i} - g_{\gamma} g_{Q^i} \right)^2 \right] i_t \\
b_t &= \theta_t \mathbb{E}_t \left(q_{lt+1} g_{\gamma t+1} l_{et} + \frac{q_{kt+1}}{g_{Q_{t+1}^i}} k_t \right) \\
\lambda_{et} &= \frac{1}{c_{et} - \frac{\eta_e}{g_{\gamma t}} c_{et-1}} - \beta_e \mathbb{E}_t \left(\frac{\eta_e}{c_{et+1} g_{\gamma t+1} - \eta_e c_{et}} \right) \\
y_t &= \left(g_{\gamma t} g_{Q_t^i} \right)^{\alpha(\phi-1)} \left(l_{et-1}^{\phi} k_{t-1}^{1-\phi} \right)^{\alpha} n_t^{1-\alpha} \\
w_t &= (1-\alpha) \frac{y_t}{n_t} \\
\frac{1}{R_{ft}} &= \beta_e \mathbb{E}_t \left(\frac{\lambda_{et+1}}{g_{\gamma t+1} \lambda_{et}} \right) + \frac{\xi_t}{\lambda_{et}} \\
\frac{1}{q_{kt}} &= \left[1 - \frac{\Omega}{2} \left(\frac{i_t}{i_{t-1}} g_{\gamma t} g_{Q_t^i} - g_{\gamma} g_{Q^i} \right)^2 - \Omega \left(\frac{i_t}{i_{t-1}} g_{\gamma t} g_{Q_t^i} - g_{\gamma} g_{Q^i} \right) \frac{i_t}{i_{t-1}} g_{\gamma t} g_{Q_t^i} \right] \\
&\quad + \beta \Omega \mathbb{E}_t \left[\frac{\lambda_{et+1}}{g_{\gamma t+1} \lambda_{et}} \frac{q_{kt+1}}{q_{kt}} \frac{1}{g_{Q_{t+1}^i}} \left(\frac{i_{t+1}}{i_t} g_{\gamma t+1} g_{Q_{t+1}^i} - g_{\gamma} g_{Q^i} \right) \left(\frac{i_{t+1}}{i_t} g_{\gamma t+1} g_{Q_{t+1}^i} \right)^2 \right] \\
q_{kt} &= \frac{\xi_t}{\lambda_{et}} \theta_t \mathbb{E}_t \left(\frac{1}{g_{Q_{t+1}^i}} q_{kt+1} \right) + \beta_e \mathbb{E}_t \left\{ \frac{\lambda_{et+1}}{g_{\gamma t+1} \lambda_{et}} \left[\alpha (1-\phi) \frac{y_{t+1} g_{\gamma t+1}}{k_t} + (1-\delta) \frac{q_{kt+1}}{g_{Q_{t+1}^i}} \right] \right\} \\
q_{lt} &= \frac{\xi_t}{\lambda_{et}} \theta_t \mathbb{E}_t \left(q_{lt+1} g_{\gamma t+1} \right) + \beta_e \mathbb{E}_t \left\{ \frac{\lambda_{et+1}}{\lambda_{et}} \left[\alpha \phi \frac{y_{t+1}}{l_{et}} + q_{lt+1} \right] \right\} \\
c_{et} &= \left(g_{\gamma t} g_{Q_t^i} \right)^{\alpha(\phi-1)} \left(l_{et-1}^{\phi} k_{t-1}^{1-\phi} \right)^{\alpha} n_t^{1-\alpha} - i_t + \frac{b_t}{\tilde{r}_{ft}} - q_{lt} \left(L_{et} - L_{et-1} \right) - \frac{b_{t-1}}{g_{\gamma t}}
\end{aligned}$$