Information Acquisition and the Pre-Announcement Drift

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Abstract: We present a dynamic Grossman-Stiglitz model with endogenous information acquisition to explain the pre-FOMC announcement drift. Because FOMC announcements reveal substantial information about the economy, investor's incentives to acquire information are particularly strong in days ahead of announcements. Information acquisition partially resolves the uncertainty for uninformed traders, and under generalized risk sensitive preferences (Ai and Bansal (2018)), lowers the discount rate and results in a stock market run-up. Because our theory does not rely on the leakage of information, it can simultaneously explain the low realized volatility during the pre-FOMC announcement period and the lack of correlation between pre- and post-announcement returns.

Keywords: Macroeconomic Announcement Premium, Pre-FOMC Announcement Drift, Asymmetric Information, Trading Volume.

JEL Code: D83, D84, G11, G12, G14

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1 Introduction

In this paper, we present a general equilibrium model with endogenous information acquisition to explain the pre-FOMC announcement drift documented by Lucca and Moench (2015). Information is publicly available but costly to acquire. Because FOMC announcements resolve substantial uncertainty of the aggregate economy and have a significant impact on the stock market, informed traders have particularly large information advantages in trading over uninformed traders before announcements are made. As a result, it is optimal for uninformed traders to start to acquire information days ahead of the announcements. Due to generalized risk sensitivity (Ai and Bansal (2018)) in preferences, as uncertainty resolves, equity market risk premium realizes shortly before announcements. More importantly, because the newly acquired information is from publicly available sources but not leakage of the content of the upcoming announcement, our theory can simultaneously explain the low realized volatility during the pre-FOMC announcement period, and the lack of correlation between pre- and post-announcement returns.

Stock market returns earned on FOMC announcement days account for almost 100% of the overall equity market risk premium since the mid-1990s. Ai and Bansal (2018) demonstrate that this phenomenon can be consistent with general equilibrium asset pricing models if investors have generalized risk sensitive preferences. However, the puzzling aspect of the FOMC announcement premium is that it is mostly realized in hours or a trading day before the actual announcements. If one is willing to assume that most of the time, the contents of FOMC announcements are leaked to the market in days before announcements, the example in Ai and Bansal (2018), illustrated in Figure 4 of their paper, provides a direct explanation for the pre-FOMC announcement drift. However, the existing evidence for information leakage is mostly anecdotal, and this extreme form of information leakage is implausible from an institutional point of view. More importantly, models with leakage of information typically have two counter-factual implications.

1. If the leaked information is slowly disclosed to the market, then the returns during the pre-announcement period will be highly positively correlated with returns upon announcements. However, as pointed out by Lucca and Moench (2015), the correlation between pre- and post-announcement returns has a negative point estimate and is not statistically different from zero.

2. The leakage of information will also imply that the realized volatility during the pre-announcement period must be significantly higher than that on non-announcement days. However, empirically, the realized volatility of market returns during the pre-announcement period is slightly lower than that on non-announcement days.

We propose a theory for the pre-FOMC announcement drift based on endogenous information acquisition on financial markets. The endogenous information acquisition in our model is consistent with the evidence documented by Fisher, Martineau, and Sheng (2020) that investor's attention peaks roughly three days before pre-scheduled FOMC announcements. Our theory does not rely on information leakage. Therefore it not only explains the existence of the pre-announcement drift but also the lack of correlation between pre- and post-announcement returns, as well as the low realized volatility during the pre-announcement period.

In our model, the long-run growth rate of the economy is governed by a latent state variable that is unobservable to all investors but periodically announced by the central bank. Information about the latent variable is available but costly to acquire. There are two groups of investors, informed and uninformed. Informed investors have zero cost of information acquisition and always observe the signals. Uninformed investors do not observe the signals unless they pay a cost to acquire them. We interpret informed investors as professional traders and uninformed investors as retail investors who normally pay less attention to stock market dynamics than professional traders but may choose to increase their attention if the benefit exceeds the cost of information acquisition.

In the above environment, uninformed investors have incentives to acquire information to avoid trading losses due to information asymmetry. This incentive is particularly strong and results in a sharp increase in information acquisition for uninformed investors in days ahead of FOMC announcements when the information advantage of informed traders peaks. As a result, our model provides a rational explanation for the pattern of investors' attentions ahead of macroeconomic announcements documented by Fisher, Martineau, and Sheng (2020).

As investors acquire more information, uncertainty resolves in days ahead of the FOMC announcements. Under generalized risk sensitive preferences, the discount rate drops, and the stock price rises. As in Ai and Bansal (2018), this mechanism produces a pre-FOMC announcement drift. Importantly, because newly acquired information has already been in the public domain and incorporated into stock prices, realized market volatility is low during this period, consistent with empirical evidence. As in the data, announcements reveal new information and result in a sudden increase in realized volatility and trading volume. In addition, the absence of leakage of information in our model implies that pre- and post- announcement returns are not positively correlated. These features of our model are broadly consistent with the empirical evidence of stock market dynamics presented in Lucca and Moench (2015).

Related Literature Our paper builds on the literature of macroeconomic announcement premium. Savor and Wilson (2013, 2014) are among the first to document the macroeconomic announcement premium. Ai and Bansal (2018) provide a reveal preference theory for the macroeconomic announcement premium. Wachter and Zhu (2020) develop a quantitative model of the macroeconomic announcement premium based on rare disasters. Ai, Bansal, Im, and Ying (2020) provide evidence for the impact of announcements on macroeconomic quantities as well as asset markets and develop a production-based asset pricing model to explain these facts. Ernst, Gilbert, and Hrdlicka (2019) present additional evidence for the macroeconomic announcement premium.

Within the above broader literature, our paper is more closely related to the FOMC announcement premium. Lucca and Moench (2015) document the pre-FOMC announcement drift and Cieslak, Morse, and Vissing-Jorgensen (2019) provide evidence for stock returns over the FOMC announcement cycles. Morse and Vissing-Jorgensen (2020) provide a study for the information transmission mechanism for Fed policies. Both Laarits (2020) and Ying (2020) provide models of pre-announcement drifts. Both papers rely on the arrival of new information during the pre-announcement period as in the example of Ai and Bansal (2018). Cocoma (2020) develops a general equilibrium with disagreement. However, her model predicts a low risk premium and low investor attention during the pre-announcement period.

Several recent empirical work document important facts related to investor attention and trading activities around FOMC announcement which provide a basis for the development of the theoretical model in this paper. Fisher, Martineau, and Sheng (2020) develop a macroeconomic attention index and provide a systematic study of the pattern of investor attention around macroeconomic announcements. Boguth, Grégoire, and Martineau (2018) emphasize the importance of press conferences in shaping market expectations. Hu, Pan, Wang, and Zhu (2020) document the dynamics of implied volatility around FOMC announcements. Ai, Bansal, Guo, and Yaron (2020) link the dynamics of implied volatility around announcements to investors' preference for early resolution of uncertainty. Bollerslev, Li, and Xue (2018) study the relationship between realized volatility and trading volume around FOMC announcements.

From the theoretical point of view, this paper builds on the noisy rational expectations literature pioneered by Grossman and Stiglitz (1980), Grossman (1981), and Hellwig (1980). The continuous-time and dynamic setup are directly related to Wang (1993, 1994), and the setup of the macroeconomic announcement is related to Han (2020).[1]

From the perspective of general equilibrium asset pricing, this paper belongs to the large literature that studies various aspects of equity market risk and risk compensation based on preferences with generalized risk sensitivity. To incorporate generalized risk sensitivity in a tractable way in the Grossman-Stiglitz setup, we use the recursive multiple prior setup of Chen and Epstein (2002). See also, Epstein and Schneider (2007). This preference is also related to the robust control preference of Hansen and Sargent (2007, 2008). We do not attempt to survey this large literature but refer the readers to Ai and Bansal (2018) for the references of preferences that satisfy generalized risk sensitivity and their applications in asset pricing.

The rest of the paper is organized as follows. In Section 2, we summarize stylized facts related to the FOMC announcement premium and the pre-FOMC announcement drift. We present our model in Section 4 and study its implications in Section 5. Section 6 concludes.

2 Stylized facts

We begin by summarizing the stylized facts about stock market dynamics around pre-scheduled FOMC announcements. All of the facts we list here are well established in the literature, and we simply use them as guidance for the development of the model.

1. The aggregate stock market exhibits high average returns starting from the previous trading

[1] An incomplete list of recent applications of the dynamic Grossman-Stiglitz models include Breon-Drish (2015), Bond and Goldstein (2015), Banerjee and Green (2015), Goldstein and Yang (2017), Albuquerque and Miao (2014), Andrei and Cujean (2017), Andrei, Cujean, and Wilson (2018), Sockin (2019), and Buffa, Vayanos, and Woolley (2019).

day until the release of the FOMC announcement. The 24-hour return including the pre-scheduled FOMC announcement is about 40 basis points on average (Lucca and Moench (2015)).

2. Investors' attention rises roughly three days before FOMC announcements and peaks right after FOMC announcements. Fisher, Martineau, and Sheng (2020) develop a macroeconomic attention index and show that investor attention rises roughly three days ahead of announcements.

3. The realized volatility before announcement hour is not significantly different between FOMC announcement days and non-FOMC announcement days. The market realized volatility peaks right after FOMC announcements.

4. The return realized during the pre-FOMC announcement period is slightly negatively correlated with returns after FOMC announcements.

5. The significant return of the pre-FOMC announcement drift is not associated with significant rises in trading volume. Trading volume only peaks after the FOMC announcement.

In the following section, we show that a dynamic noisy rational expectations (NREE) model with endogenous information acquisition, after incorporating generalized risk sensitive preferences, provides a unified explanation for the above facts.

3 An example of pre-FOMC announcement drift

In this section, we reproduce the simple example in Figure 4 of Ai and Bansal (2018) to illustrate how combining generalized risk sensitivity and information leakage can generate a pre-FOMC announcement drift, and why this version of the Ai and Bansal (2018) model cannot explain the volatility dynamics around FOMC announcements.

The Ai and Bansal (2018) model assumes a continuous-time setup where the aggregate consumption follows $\frac{dC_t}{C_t} = [x_t dt + \sigma dB_{C,t}]$, and the expected consumption growth, x_t is given by:

$$dx_t = b\left(\bar{x} - x_t\right) dt + \sigma_x dB_{x,t}, \tag{1}$$

where $\bar{x}$ is the long-run mean of x_t, b is the rate of mean reversion, σ_x is the volatility of the hidden state x_t, and $B_{x,t}$ is a standard Brownian motion independent of $B_{C,t}$. At time $t = T, 2T, 3T, \cdots$, pre-scheduled FOMC announcements reveal the true values of x_t. To model information leakage, we assume that starting at time $\tau < T$, all investors observe an additional signal s_t, which carries information about the upcoming announcement x_t:

$$s_t = x_t dt + \sigma_i dB_{i,t}. \tag{2}$$

where σ_i is the inverse of signal precision and $B_{i,t}$ is a mutually independent Brownian motion noise. is Ai and Bansal (2018) show that the posterior mean of x_t, denoted $\hat{x}_t$ can be written as:

$$d\hat{x}_t = b\left(\bar{x} - \hat{x}_t\right)dt + \frac{q_t}{\sigma}d\hat{B}_{C,t} + \frac{q_t}{\sigma_i}d\hat{B}_{i,t}, \tag{3}$$

where q_t is the posterior variance of x_t and $d\hat{B}_{C,t} = \frac{1}{\sigma}\left\{\frac{dC_t}{C_t} - \mathbb{E}_t\left[\frac{dC_t}{C_t}\right]\right\}$ and $d\hat{B}_{i,t} = \frac{1}{\sigma_i}\left\{ds_t - \mathbb{E}_t\left[ds_t\right]\right\}$ are innovations in the observation processes relative to investor belief.

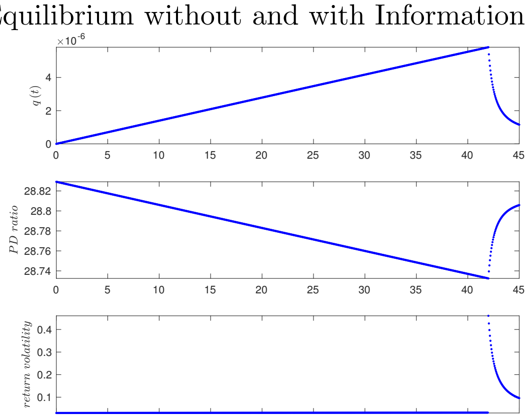
Assume that the investors have a multiplier robust control preference with a subjective discount rate of ρ, a unit IES, and a multiplier κ, the pricing kernel can be written as:

$$d\pi_t = -r_t dt - \sigma dB_{C,t} - \kappa\left[\left(1 + \frac{q_t}{(b+\rho)\,\sigma^2}\right)\sigma d\hat{B}_{C,t} + \frac{q_t}{(b+\rho)\,\sigma_i}d\hat{B}_{i,t}\right], \tag{4}$$

where the first term in the market price of risk, $\sigma dB_{C,t}$ comes from the standard log preference, and the term in the square bracket can be interpreted as probability distortions due to the preference for robustness.[2] As shown in Ai and Bansal (2018), the robust control preference satisfy generalized risk sensitivity and generates an announcement premium.

In the above example, between $t \in [\tau, T]$, investors observe an additional signal, s_t. In Figure 1 below, we plot the posterior variance (top panel), and average price-to-dividend ratio (middle panel) and the volatility of the market return (bottom panel) implied by the above model. To model leakage of information, we choose $\sigma_i = 0.01\%$ to be very small. Because the information is very precise, the posterior variance $\hat{q}_t$ drops sharply at $t = \tau$. At the same time, the average price-to-dividend ratio rises sharply. This is because leakage of information is associated with high volatility of the stochastic discount factor: the term $\frac{q_t}{(b+\rho)\sigma_i}$ in (4) is very large when σ_i is close to zero, generating a large risk premium in a short period ahead of announcements.

Figure 1: Equilibrium without and with Information Acquisition



This figure plots $\hat{q}_t$, the posterior variance for $\hat{x}_t$ (top panel), the average price-to-dividend ratio (middle panel), and the return volatility (bottom panel) over one announcement cycle. The agent starts to acquire information at time $\tau < T$.

[2]Different from Ai and Bansal (2018), here we assume robust control preference as Hansen and Sargent (2008) and continuous information arrival, because these assumptions allow us to link and compare to the asymmetric information model we develop in the rest of the paper.

The high volatility of the stochastic discount factor, however, is associated with high volatility of the posterior belief $\frac{q_t}{\sigma_i}$ in equation (3). In fact, the high volatility of $\hat{x}_t$ is the reason for the high volatility of the stochastic discount factor. As shown in Figure 1, the realized volatility rises sharply simultaneously as the prices-to-dividend ratio increases with leakage of information.

The above example illustrates a key difficulty for models that generate a pre-FOMC announcement drift based on the arrival of new information to the market, or leakage of information. In the data, the average excess during the pre-FOMC announcement period is roughly 40 bps per trading day, and that on non-announcement days is less than 2 bps. Holding the Sharpe ratio constant, to account for a 40 bps premium, the information leakage based story required a realized market volatility of twenty times higher during the pre-announcement period, whereas in the data, the realized market volatility in this period is slightly lower than that on non-announcement days. In the rest of the paper, we develop a noisy rational expectations model with asymmetric information to resolve the above puzzle.

4 Dynamic Model

This section develops a continuous-time NREE model with periodic macroeconomic announcements and with endogenous information acquisition. The model is a continuous-time version of the Grossman-Stiglitz model with generalized risk-sensitive preferences. The model setup is based on Han (2020).

4.1 Model Setup

The asset market Time is continuous and infinite. There is a unit measure of investors. An ω fraction of them are uninformed investors and $1-\omega$ fraction are informed. There are two assets available for trading, a stock and a risk-free bond. We assume that the risk-free return r is constant. The stock is the claim to the following dividend process:

$$dD_t = (x_t - D_t)\,dt + \sigma_D dB_{D,t}, \tag{5}$$

where D_t is the dividend flow, x_t is the long-run trend for the dividend flow, σ_D is the volatility of the dividend flow, and $B_{D,t}$ is an i.i.d. shock to the dividend payment modeled as a standard Brownian motion. We model the expected dividend flow as $x_t - D_t$, so that the dividend process is stationary. The assumption that the mean reversion rate equals to 1 is not important and can be relaxed without affecting most parts of the model. The long-run trend of the dividend flow, x_t, is itself mean reverting, modeled as an Ornstein-Uhlenbeck (OU) process as in (1). In addition, as is standard in the NREE literature, we assume that the total equity supply is a stochastic process and denote it as θ_t, where

$$d\theta_t = a\left(\bar{\theta} - \theta_t\right)dt + \sigma_\theta dB_{\theta,t}. \tag{6}$$

In the above equation, a is the rate of mean reversion, $\bar{\theta}$ is the long-run mean for θ_t, and σ_θ is the noisy supply volatility. We assume that Brownian motions $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$ are mutually independent.

Information and preference of informed investors We assume that the dividend process, D_t, is observable to all investors, but its long-run trend x_t and the total risky asset supply θ_t are not. At pre-scheduled times, $t = nT$, for $n = 1, 2, \cdots$, the monetary authority (central bank) makes periodic announcements that reveal the true value of x_t. Both the informed and the uninformed investors can observe D_t and the the pre-scheduled FOMC announcements and use them to update their beliefs about the latent variable that drives economic growth, x_t.

We assume that market research can produce a signal that is informative about x_t, denoted as s_t:

$$ds_t = x_t dt + \sigma_s dB_{s,t}, \tag{7}$$

where σ_s is the signal volatility and $B_{s,t}$ is a Brownian motion independent of $B_{D,t}$, $B_{x,t}$, and $B_{\theta,t}$. We think of s_t as information available in the pubic domain but costly to acquire. We interpret informed investors as professional investors who have a comparative advantage in acquiring information s_t. For simplicity, they have zero information acquisition cost and observe s_t at all times.

Informed investors maximize CARA utilities represented by $\left[\mathbb{E} \int_0^\infty -e^{-\rho t - \gamma C_t} dt\right]$, where C_t is the consumption at time t, ρ is the subjective time discount rate and γ is the absolute risk aversion.

GRS through recursive multiple prior preferences In order to generate an equilibrium announcement premium, we assume that the uninformed investors are concerned about model uncertainty, which is modeled by a robust control preference. The robust control model, as shown by Ai and Bansal (2018), satisfies generalized risk sensitivity. Formally, let $(\Omega, \mathcal{F}, P)$ be the probability space where all uncertainty in this economy is generated. The agent's preference is specified by a CARA utility function with the absolute risk aversion of γ and a set of probability models defined on $(\Omega, \mathcal{F}, P)$, denoted as $\mathcal{P}$. That is, the agent computes his time-t utility using:

$$V_t = \inf_{Q \in \mathcal{P}} \mathbb{E}_t^Q \left[\int_t^\infty -e^{-\rho s - \gamma C_s} ds \right]. \tag{8}$$

Here, $\mathcal{P}$ captures model uncertainty (Hansen and Sargent (2008)). That is, the agent is ambiguous about the true data generating process and entertains a set of probability models to compute the worse-case scenario over $\mathcal{P}$ when ranking stochastic consumption streams.

Information and beliefs The informed investors observe three sources of information about the latent variable x_t that drives the economic growth: the dividend process D_t, pre-scheduled FOMC announcements at $t = nT, n = 1, 2, \cdots$, and the signal process s_t obtained from market research. Denote $\hat{x}_t \equiv \hat{\mathbb{E}}_t [x_t]$ and $\hat{q}(t) \equiv \hat{\mathbb{E}}_t \left[(\hat{x}_t - x_t)^2 \right]$ as the posterior mean and variance of the informed investors for x_t. If the informed investors' prior for x_0 is a Gaussian distribution, then their posterior distribution for x_t is also Gaussian and can be characterized by the standard Kalman filter. Because FOMC announcements fully reveal the true value of x_t, we have $\hat{x}_t = x_t$ and $\hat{q}_t = 0$ at prescheduled announcements $t = nT$. After announcements, because $\hat{x}_t$ process evolves according to equation (9), $\hat{x}_t$ drifts away from the true value of x_t and $\hat{q}_t$ increases above zero, until the next announcement. Standard Kalman filter implies that the dynamics of $\hat{x}_t$ can be computed by:

$$d\hat{x}_t = b\left(\bar{x} - \hat{x}_t\right)dt + \frac{\hat{q}\left(t\right)}{\sigma_D} d\hat{B}_{D,t} + \frac{\hat{q}\left(t\right)}{\sigma_s} d\hat{B}_{s,t}, \tag{9}$$

where $d\hat{B}_{D,t} = dD_t - \hat{\mathbb{E}}_t\left[dD_t\right]$ and $d\hat{B}_{s,t} = ds_t - \hat{\mathbb{E}}_t\left[ds_t\right]$ are innovations in the observation processes relative to expectations.

In contrast, the uninformed investors do not observe s_t, until they pay a cost. To keep the structure simple, we assume that uninformed investors can choose to obtain information about s_t by paying a fixed cost K and a flow cost k per unit of time. Paying the cost allows uninformed investors to observe a noisy signal about the best forecast of x_t obtained by market research. That is, it allows uninformed investors to observe a signal of the form:

$$ds_t^i = \hat{x}_t dt + \sigma_i dB_t^i, \tag{10}$$

where B_t^i are i.i.d. across investors. We focus on symmetric equilibria where all uninformed investors start to acquire information at time τ. To save notation, assuming an uninformed investor choose to observe the signal during the period $[\tau, \tau']$, we write $ds_t^i = \hat{x}_t dt + \sigma_i\left(t\right) dB_t^i$ with $\sigma_i\left(t\right) = \sigma_i$ for $t \in [\tau, \tau']$ and $\sigma_i\left(t\right) = \infty$ otherwise.

It is convenient to denote the posterior mean of an uninformed investor as $\tilde{x}_t = \tilde{\mathbb{E}}_t\left[\hat{x}_t\right]$ and the posterior variance as $\tilde{q}\left(t\right) \equiv \tilde{\mathbb{E}}_t\left[\left(\tilde{x}_t - \hat{x}_t\right)^2\right]$, where $\tilde{\mathbb{E}}$ is the belief of an uninformed investor. We conjecture and later verify that the equilibrium price takes the following form

$$P_t = \phi\left(t\right) + \phi_D D_t - \phi_\theta\left(t\right)\theta_t + \phi_x\left(t\right)\hat{x}_t + \phi_\Delta\left(t\right)\tilde{x}_t, \tag{11}$$

where the sum of the two coefficients, $\phi_x\left(t\right) + \phi_\Delta\left(t\right) = \bar{\phi}_x$, is a constant. Note that equilibrium price contains information about the best prediction for x_t obtained by market research and uninformed investors should learn from it. Clearly, if we define $\Delta_t \equiv \hat{x}_t - \tilde{x}_t$ to be the difference between the beliefs of the informed and uninformed investors, price can be written as:

$$P_t = \phi\left(t\right) + \phi_D D_t - \phi_\theta\left(t\right)\theta_t + \bar{\phi}_x \hat{x}_t - \phi_\Delta\left(t\right)\Delta_t. \tag{12}$$

Learning from prices Here we describe the beliefs of uninformed investors in our model, which is the key for understanding the model's implications for the pre-FOMC announcement drift. It is convenient to define $\xi_t = \phi_x\left(t\right)\hat{x}_t - \phi_\theta\left(t\right)\theta_t - \frac{\hat{q}_t}{\sigma_D^2}\phi_x\left(t\right)D_t$ as the information content of prices, as observing ξ_t is the same as observing the equilibrium price. The uninformed traders observe three sources of information about $\hat{x}_t$, the dividend process, the equilibrium price, and the signal s_t^i after paying the information acquisition cost. Standard Kalman filter implies that the dynamics of $\tilde{x}_t$ can be written as:

$$d\tilde{x}_t = b\left(\bar{x} - \tilde{x}_t\right) dt + \frac{\hat{q}\left(t\right) + \tilde{q}\left(t\right)}{\sigma_D} d\tilde{B}_{D,t} + \nu\left(t\right) \sigma_\xi\left(t\right) d\tilde{B}_{\xi,t} + \frac{\tilde{q}\left(t\right)}{\sigma_i\left(t\right)} d\tilde{B}_{i,t}, \tag{13}$$

where $d\tilde{B}_{D,t} = dD_t - \tilde{\mathbb{E}}_t\left[dD_t\right]$, $d\tilde{B}_{\xi,t} = d\xi_t - \tilde{\mathbb{E}}_t\left[d\xi_t\right]$ and $d\tilde{B}_{i,t} = ds_{i,t} - \tilde{\mathbb{E}}_t\left[ds_{i,t}\right]$ are innovations in the observation processes relative to expectations. In the above expression, $\nu\left(t\right)$ is defined in equation (46) and the volatility of $d\xi_t$, $\sigma_\xi\left(t\right)$ is defined in (39) in Appendix 6.1.

Before τ, uninformed traders can only learn about $\hat{x}_t$ from the dividend process and the equilibrium price. After time information acquisition, they can also learn from the newly acquired information, $s_{i,t}$. Our notation in (13) incorporate both possibilities by using the convention $\sigma_i\left(t\right) = \sigma_i$ for $t \in \left[\tau, \tau'\right]$ and $\sigma_i\left(t\right) = \infty$ otherwise. It is important to note that in our setup, the endogenously acquired signals, s_t^i is about information that is already on the market, $\hat{x}_t$. s_t^i is not informative about the different between the true value of x_t and$\hat{x}_t$, which is only revealed upon announcements. In other words, the content of announcement in not revealed until after the announcement. This feature of our model is important in accounting for the volatility dynamics

To incorporate robustness, we assume that under the worst case probability, $d\tilde{B}_{D,t} = d\tilde{B}_{D,t}^{\kappa} - \kappa\left[\bar{\phi}_x \frac{\hat{q}(t)+\tilde{q}(t)}{\sigma_D} + \phi_D \sigma_D\right]$, $d\tilde{B}_{\xi,t} = d\tilde{B}_{\xi,t}^{\kappa} - \kappa\bar{\phi}_x \nu\left(t\right)\sigma_\xi\left(t\right)$, and $d\tilde{B}_{i,t} = d\tilde{B}_{i,t}^{\kappa} - \kappa\bar{\phi}_x \frac{\tilde{q}(t)}{\sigma_i}$ are Brownian motions with negative drifts, where $\tilde{B}_{D,t}^{\kappa}$, $\tilde{B}_{\xi,t}^{\kappa}$, and $\tilde{B}_{i,t}^{\kappa}$ are standard Brownian motions under the worst case probability. In Appendix, we show that the above probability distortions can be derived from a robust valuation problem, where κ is the robustness parameter, or the Lagrangian multiplier on the relative entropy constraint.

4.2 Equilibrium and Equilibrium Conditions

For simplicity, we will focus on stationary equilibriums in which equilibrium prices satisfy $P_t = P_{t \ mod \ T}$ and so do equilibrium quantities. That is, equilibriums are identical across announcement cycles. Without loss of generality, we can therefore focus on prices and quantities over the closed time interval $[0, T]$, because they repeat themselves within each announcement cycle. We use T^+ and T^- to denote the moment right after announcements and right before announcements, respectively. Whenever there is no confusion, time 0 should be understood as 0^+ and T should be understood as T^-.

Below we construct an equilibrium in which there exists a moment $\tau \in (0, T)$ such that for all $t \leq \tau$, all uninformed investors find it suboptimal to acquire any information, and after $t > \tau$, all uninformed investor acquire until the next announcement.

Equilibrium Definition A stationary equilibrium consists of a collection of pricing functions $\left\{\phi\left(t\right), \phi_D, \bar{\phi}_x, \phi_\theta\left(t\right), \phi_\Delta\left(t\right)\right\}$, demand functions of the informed, $\alpha\left(t, \theta_t, \Delta_t\right) = \alpha_0\left(t\right) + \alpha_\theta\left(t\right)\theta_t + \alpha_\Delta\left(t\right)\Delta_t$, demand functions for uninformed investors, $\beta\left(t, \tilde{\theta}_t\right) = \beta_0\left(t\right) + \beta_\theta\left(t\right)\tilde{\theta}_t$ such that:

1. Given the pricing function $\{\phi(t), \phi_D, \phi_\theta(t), \phi_x(t), \phi_\Delta(t)\}$, $\{\alpha_0(t), \alpha_\theta(t), \alpha_\Delta(t)\}$ represents the optimal portfolio demand for the informed investors.

2. Uninformed investors strictly prefer not to acquire information for all $t < \tau$. After time τ, uninformed investors prefer to acquire information.

3. Given their information set, $\{\beta_0(t), \beta_\theta(t)\}$ represents the optimal demand for the uninformed investors.

4. Markets clear, that is,

$$(1-\omega)\left[\alpha_0(t) + \alpha_\theta(t)\theta_t + \alpha_\Delta(t)\Delta_t\right] + \int \left[\beta_0(t) + \beta_\theta(t)\tilde{\theta}_t^i\right] di = \theta_t \tag{14}$$

   for all $t \in [0, T]$.

In the market clearing condition (14), because uninformed agents have different

Equilibrium beliefs Given the pricing equation (12), we define the excess return process as $dQ_t = (D_t - rP_t) + dP_t$. Informed investors can distinguish Δ_t from θ_t. We can combine equations (9) and (13) to derive the difference in belief as a diffusion process:

$$d\Delta_t = -a_\Delta(t)\Delta_t dt - \frac{\tilde{q}(t)}{\sigma_D} d\hat{B}_{D,t} + \frac{\hat{q}(t)}{\sigma_s}\left[1 - \phi_x(t)\nu(t)\right] d\hat{B}_{s,t} + \phi_\theta(t)\nu(t)\sigma_\theta dB_{\theta,t}, \tag{15}$$

where $a_\Delta(t)$ is defined in equation (54) in Appendix 6.1. Using the law of motion of the state variables, we can write the excess return as a diffusion process from the perspective of informed investors:

$$dQ_t = \left[e_0(t) + e_\theta(t)\theta_t + e_\Delta(t)\Delta_t\right] dt + \varrho_D(t) d\hat{B}_{D,t} + \left[1 + \phi_\Delta(t)\nu(t)\right]\sigma_\xi(t) d\hat{B}_{\xi,t}, \tag{16}$$

where the coefficients $e_0(t)$, $e_\theta(t)$, $e_\Delta(t)$, $\varrho_D(t)$ are given in Equation (68) in Appendix 6.1, and $\sigma_\xi(t) d\hat{B}_{\xi,t} = d\xi_t - \hat{\mathbb{E}}_t[d\xi_t]$ is the innovations of ξ_t relative to the informed investors' information.

Uninformed investors, however, cannot distinguish Δ_t from θ_t. Because they observe the prices, rational expectations imply $P_t = \tilde{\mathbb{E}}_t[P_t]$. This allows us to write the Equilibrium price (12) as:

$$P_t = \phi(t) + \phi_D D_t - \phi_\theta(t)\tilde{\theta}_t + \bar{\phi}_x \tilde{x}_t. \tag{17}$$

The law of motion of $\tilde{x}_t$ is given in equation (13). To derivate a law of motion for $\tilde{\theta}_t$, recall that observing prices is equivalent to observing $\xi_t = \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t$. Taking conditional expectation $\tilde{\mathbb{E}}_t$ on both sides, we have $\xi_t = \tilde{\mathbb{E}}_t[\xi_t]$. Therefore,

$$\xi_t = \phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t = \phi_x(t)\tilde{x}_t - \phi_\theta(t)\tilde{\theta}_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t) D_t. \tag{18}$$

We have: $\tilde{\theta}_t = \frac{\phi_x(t)}{\phi_\theta(t)}\tilde{x}_t - \frac{1}{\phi_\theta(t)}\xi_t - \frac{\hat{q}(t)}{\sigma_D^2}\frac{\phi_x(t)}{\phi_\theta(t)}D_t$. The law of motion of $\tilde{\theta}_t$ can therefore be written as:

$$d\tilde{\theta}_t = a\left(\bar{\theta} - \tilde{\theta}_t\right)dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t} + \left[\phi_x(t)\nu(t) - 1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}_{\xi,t}. \tag{19}$$

This allows us to write the excess return process dQ_t in terms of a diffusion process adapted to the information set of the uninformed investors:

$$dQ_t = \left[e_0(t) + e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\,d\tilde{B}_{D,t} + \left[1 + \phi_\Delta(t)\nu(t)\right]\sigma_\xi(t)\,d\tilde{B}_{\xi,t}. \tag{20}$$

Portfolio selection and information acquisition Informed investors in our model solve a simple portfolio selection problem. At time t,they maximize life-time utility, $\mathbb{E}_t\left[\int_0^\infty -e^{-\rho s - \gamma C_{t+s}}ds\right]$ by choosing consumption and portfolio holdings, $\{\alpha_{t+s}, C_{t+s}\}_{s=0}^\infty$, subject to the following law of motion of wealth:

$$dW_t = (W_t r - C_t)\,dt + \alpha_t dQ_t, \tag{21}$$

where the excess return process dQ_t is given in Equation (16). As a result, the value function for informed investors, denoted $\hat{V}(t, W, \theta, \Delta)$ satisfies the following HJB:

$$\hat{V}(t, W, \theta, \Delta) = max_{C,\alpha}\left\{u(C) + \hat{\mathcal{L}}^{C,\alpha}\hat{V}(t, W, \theta, \Delta)\right\},$$

where the operator $\mathcal{L}^{C,\alpha}$ is defined as:

$$\hat{\mathcal{L}}^{C,\alpha}\hat{V}(t, W_t, \theta_t, \Delta_t) = \lim_{h\to 0}\frac{1}{h}\hat{\mathbb{E}}_t^{C,\alpha}\left[\hat{V}(t_h, W_{t+h}, \theta_{t+h}, \Delta_{t+h}) - \hat{V}(t, W_t, \theta_t, \Delta_t)\right],$$

and the notation $\hat{\mathbb{E}}_t^{C,\alpha}$ emphasizes that the law of motion of wealth, (21) depends on the consumption and portfolio choice decisions.

Uninformed investors solve both an optimal consumption-investment problem and an optimal information acquisition problem. Let $\tilde{V}\left(t, \tilde{q}, W, \tilde{\theta}\right)$ be the value function of an uninformed investor who has not yet started to acquire information. Then $\tilde{V}\left(t, \tilde{q}, W, \tilde{\theta}\right)$ must satisfy:

$$\tilde{V}\left(t, \tilde{q}, W, \tilde{\theta}\right) = max_{C,\beta}\left\{u(C) + \mathcal{L}^{C,\beta}\tilde{V}\left(t, \tilde{q}, W, \tilde{\theta}\right), \tilde{V}^i\left(t, \tilde{q}, W - K, \tilde{\theta}\right)\right\}.$$

That is, at any time t an uninformed investor can choose to continue not to acquire information, in which case the law of motion of wealth will be given by $dW_t = (W_t r - C_t)\,dt + \beta_t dQ_t$, or to pay the fixed cost K and start to acquire information. $\tilde{V}^i\left(t, \tilde{q}, W, \tilde{\theta}\right)$ in the above equation is the value function for an uninformed investor who have paid the fixed cost and who have started to acquire information. The value function $\tilde{V}^i\left(t, \tilde{q}, W, \tilde{\theta}\right)$ satisfies the following HJB:

$$\tilde{V}^i\left(t, \tilde{q}, W, \tilde{\theta}\right) = max_{C,\beta}\left\{u(C - k) + \mathcal{L}^{C,\beta,i}\tilde{V}^i\left(t, \tilde{q}, W, \tilde{\theta}\right), \tilde{V}\left(t, \tilde{q}, W, \tilde{\theta}\right)\right\},$$

That is, if the investor choose to continue to acquire information, she has to pay a flow cost of k per unit of time. She also have an option to stopping acquiring information at any time.

Market clearing In our model, equilibrium price is pinned down by the market clearing condition (14). Using equation (18), $\phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t = \phi_x(t)\tilde{x}_t - \phi_\theta(t)\tilde{\theta}_t$, that is, $\tilde{\theta}_t = \theta_t - \frac{\phi_x(t)}{\phi_\theta(t)}\Delta_t$. Intuitively, because uninformed investors observe price, they can make mistakes about $\hat{x}_t$ and θ_t individually, but will not make a mistake about $\phi_x(t)\hat{x}_t - \phi_\theta(t)\theta_t$. This restriction implies that the only reason for the uninformed to be pessimistic about $\hat{x}_t$ is that they believe that the level of price is not justified by high fundamentals,$\hat{x}_t$, but by a lower supply θ_t. $\hat{x}_t - \tilde{x}_t$ and $\theta_t - \tilde{\theta}_t$ must have the same sign.

Using $\tilde{\theta}_t = \theta_t - \frac{\phi_x(t)}{\phi_\theta(t)}\Delta_t$ to replace $\tilde{\theta}_t$ in market clearing condition (14), we obtain the following restrictions on the portfolio decisions:

$$(1-\omega)\,\alpha_0(t) + \omega\beta_0(t) = 0, \tag{22}$$

$$(1-\omega)\,\alpha_\theta(t) + \omega\beta_\theta(t) = 1, \tag{23}$$

$$(1-\omega)\,\alpha_\Delta(t) - \omega\frac{\phi_x(t)}{\phi_\theta(t)}\beta_\theta(t) = 0. \tag{24}$$

In Appendix 6.1, we show that investor optimality and the above market clearing conditions jointly pin down the pricing functions $\{\phi(t), \phi_\theta(t), \phi_\Delta(t)\}$.

5 Model implications

We calibrate our model to match the overall market equity premium and evaluate its implications on the FOMC announcement premium, pre-FOMC announcement drift, and the pattern of realized volatility around FOMC announcements. We provide details of the parameter calibrations in the appendix and focus on its implication here.

The key mechanism of our model is that after each pre-scheduled announcements, the uncertainty of the economy builds up over time. Uninformed investors find it optimal to acquire information ahead of the next announcement. In this section, we focus on the following four implications of the endogenous information acquisition problem.

1. Investors' incentive to acquire information increases monotonically over time and peaks before announcements. Because information acquisition is costly, it is optimal to acquire information shortly before announcements.

2. As uninformed investors start to acquire information, stock returns and they future financial wealth become more correlated. Under generalized risk sensitivity, this higher correlation translates into a higher risk premium and leads to an increase in expected returns, or pre-FOMC announcement drift.

3. Because newly acquired information has already been incorporated in the market price through informed investors' belief, information acquisition does not lead to an increase in the realized volatility of the market.

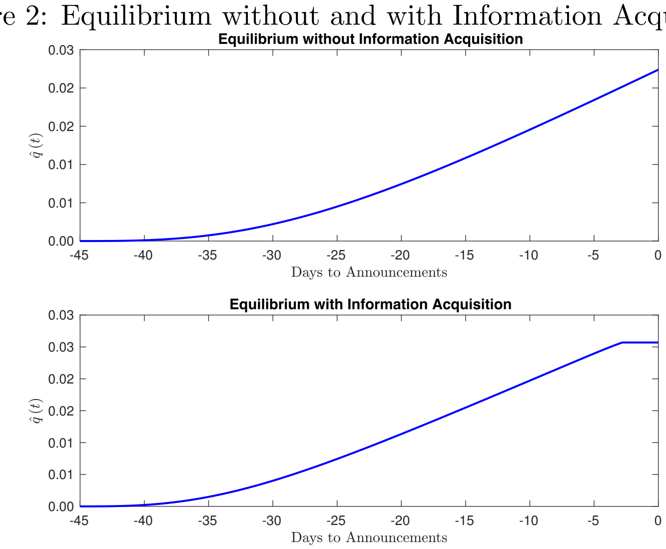
4. Upon announcement, the true value of x_t is revealed. As a result, realized volatility spike and so does trading volume.

We begin by analyzing the incentives for the endogenous information acquisition.

Timing of information acquisition In our model, periodical announcements are pre-scheduled. Uninformed investors do not find it optimal to acquire information until close to the upcoming announcements for two reasons. First, because announcements fully reveal the true value of x_t, initially after the previous announcement, both the informed and the uninformed investors have little uncertainty about x_t and there is no need to acquire additional information. As it gets closer to the next announcement, uncertainty slowly builds up, and the benefit of information acquisition rises.

Second, and more importantly, uncertainty about x_t resolves slowly prior to the announcement and the Brownian motions $B_{x,t}$ and $B_{s,t}$ evolve continuously. Therefore, the information advantage of informed traders over uninformed traders slowly increases over time. At the announcement, however, the true value of x_t is revealed and a large amount of information arrives at the market in a short period of time. Information acquisition prior to announcements is particularly important for uninformed investors because the information advantage that the informed investors accumulated over time will be fully realized at the announcements.

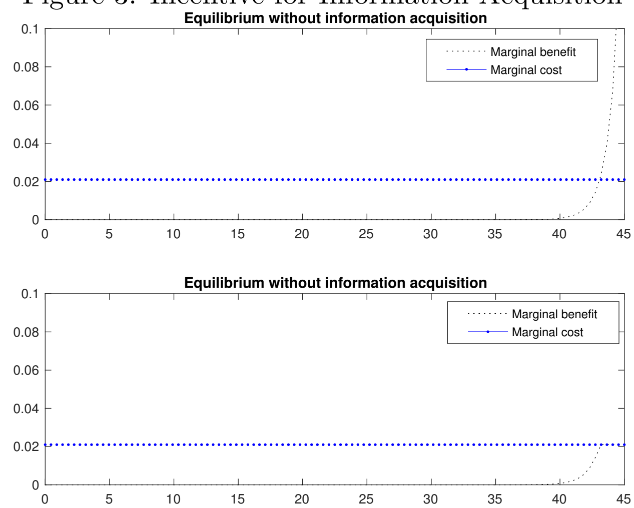
Figure 2: Equilibrium without and with Information Acquisition



This figure plots $\hat{q}_t$, the posterior variance of the uninformed investor's belief of $\hat{x}_t$ over one announcement cycle. The top panel is a model without information acquisition and the bottom penal is our benchmark economy with endogenous information acquisition. The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5, for example, stands for five days before announcements.

In Figure 2, we plot $\tilde{q}_t$, uninformed investors' posterior variance of $\hat{x}_t$. Our calibration features eight announcements per year and therefore each announcement cycle is 45 days. The top panel is the path of $\tilde{q}_t$ in an equilibrium without information acquisition, where $\tilde{q}_t$ increases monotonically from day 0 to day 45. The bottom panel of Figure 2 is $\tilde{q}_t$ in our benchmark model with endogenous information acquisition, where the decision to acquire information is made around 3 days before the announcement. As uninformed investors acquire information, the price becomes more informative, and $\tilde{q}_t$ drops sharply from day 42 to day 45.

Figure 3: Incentive for Information Acquisition

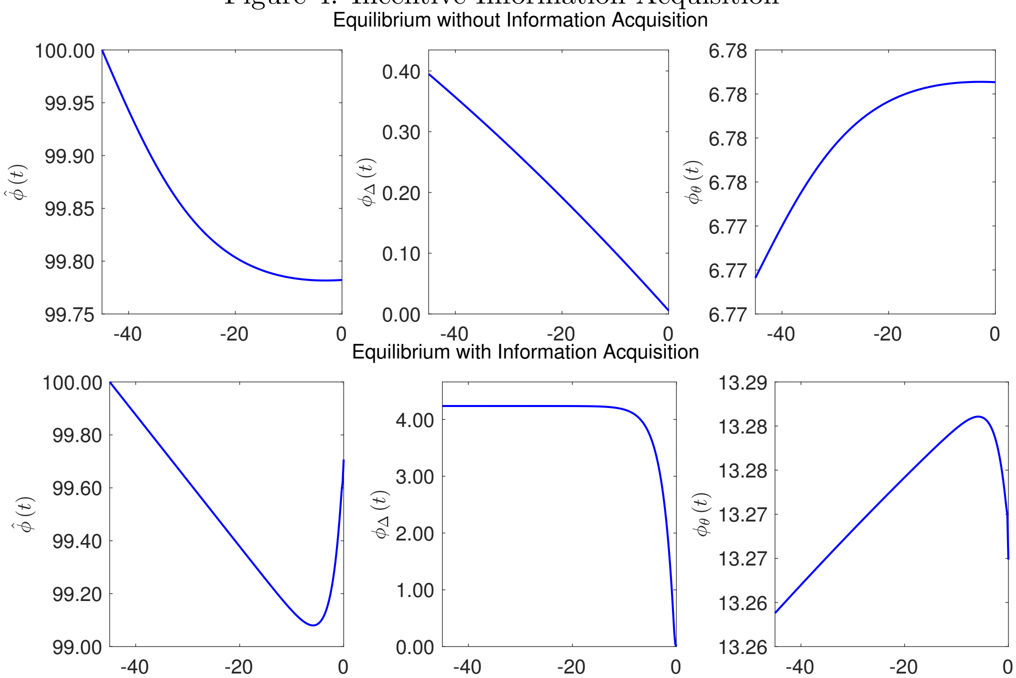


This figure plots the marginal benefit and marginal cost for the uninformed investors to acquire information over one announcement cycle. The top panel is a model without information acquisition and the bottom penal is our benchmark economy with endogenous information acquisition. The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5, for example, stands for five days before announcements.

In Figure 3, we plot the marginal cost and marginal benefit for the uninformed investors to acquire information in equilibrium without information acquisition (top panel) and those in the equilibrium with information acquisition. In both figures, the marginal benefit of information acquisition, as measured in consumption equivalent terms sharply increases in days ahead of the announcement. In the model without information acquisition, it keeps increasing until the announcement day. In the model with endogenous information acquisition, as more investors acquire information, the equilibrium price becomes more informative, and the marginal benefit of information acquisition equals to its marginal cost after day 42, when investors become indifferent towards information acquisition. The fact that investors start to acquire information endogenously in our model days ahead of the FOMC announcement provides a rational explanation for the increasing patterns of investors' attentions around macroeconomic announcements documented by Fisher, Martineau, and Sheng (2020).

Pre-FOMC announcement drift To understand the model's implications on pre-FOMC announcement drift, we plot the unconditional expectation of equilibrium price: $\hat{\phi}(t) = \mathbb{E}[P_t] = \phi(t) + \left[\bar{\phi}_x + \phi_D\right]\bar{x}$ as a function of time. Starting from time τ, as uninformed investors acquire information about the frontier research $\hat{x}_t$, the posterior variance drops, and the covariance between return and wealth increases. Under the robust control preference, this translate into a higher required return from the perspective of uninformed investors. The equilibrium price, $\hat{\phi}(t)$ must increase to compensate for uninformed investors. As the equilibrium price rises, the expected return from the perspective of informed traders increases and informed traders purchase more of the stock, while uninformed sell. The equilibrium price is determined by the market clearing condition that equalize supply to demand.

Figure 4: Incentive Information Acquisition

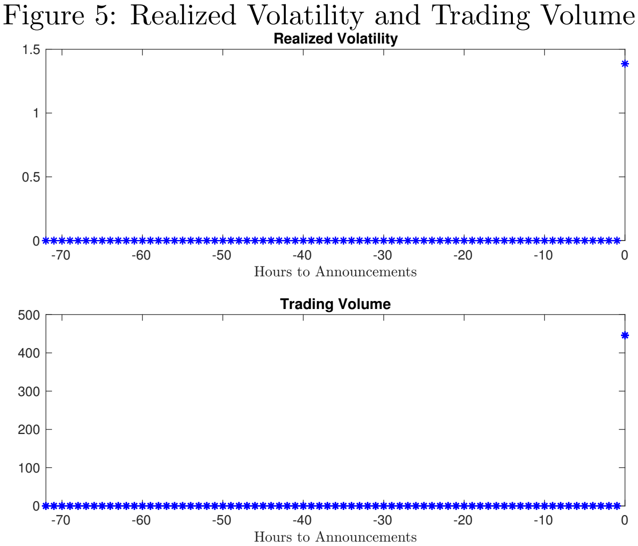


This figure plots the pricing functions $\hat{\phi}(t)$, $\phi_{\Delta}(t)$ and $\phi_{\theta}(t)$ of a model without information acquisition (the top panel) and those for our benchmark economy with endogenous information acquisition (the bottom panel). The horizon axis is the number of days before the upcoming announcement, which is normalized as 0. A -5 for example, stands for five days before announcements.

We plot pricing functions $\hat{\phi}(t)$, $\phi_{\Delta}(t)$ and $\phi_{\theta}(t)$ in Figure 4 for an economy without information acquisition in the top panel and those for our benchmark model with endogenous information acquisition in the bottom panel. In the model without information acquisition, $\hat{\phi}(t)$ monotonically decreases over time due to the ambiguity aversion of the uninformed traders. At time $t = 0$, a new announcement cycle starts and price level jumps to a higher level, which we normalize as 100. As in Ai and Bansal (2018) and Ai, Bansal, Guo, and Yaron (2020), ambiguity aversion generates an announcement premium which is realized upon announcements. In our model with endogenous information acquisition, however, equity price starts to climb up a couple of days before the announcement to generate a pre-announcement drift, as more and more uninformed traders start to acquire information.

Realized volatility and trading volumes The final implication of our model is the patterns of realized volatility and trading volume. As we emphasize earlier, the key feature of the data, as shown by Lucca and Moench (2015), is that realized volatility is in fact lower during the hours of pre-FOMC announcement drift compared to non-FOMC announcement days. Our model captures this features of the data quite well. Note that before announcements, price takes the form of (12). After information acquisition at time τ, as we show earlier, $\phi_\Delta(t) = 0$. That is, difference in opinion no longer impact price dynamics. As a result, the realized volatility during this period of information acquisition is in fact lower than normal days. In addition, trading volume in the data exhibit a similar pattern as realized volatility. As documented by Bollerslev, Li, and Xue (2018), realized volatility and trading volume are highly correlated around FOMC announcements.

Figure 5: Realized Volatility and Trading Volume



This figure plots the realized volatility (in the top panel) and trading volume (in the bottom panel) 72 hours (3 days) before announcements in our benchmark model with endogenous information acquisition.

We plot the model implied realized volatility (top panel) and trading volume (bottom panel) three days leading up to the announcement in Figure 5. Both realized volatility and trading volume in our model remains low during the period of information acquisition. As shown in Figure 5, realized volatility and trading volume increase sharply only at the announcement day, consistent with the patterns documented in Lucca and Moench (2015) and that by Bollerslev, Li, and Xue (2018).

Correlation between pre- and post- announcement return Another key evidence against a information leakage based story is the lack of correlation between pre- and post- announcement returns. Given the functional form of price in (12), the announcement return can be written as:

$$P_T^+ - P_T^- = \left[\phi_0\left(T^+\right) - \phi_0\left(T^-\right)\right] + \bar{\phi}_x\left(x_T - \hat{x}_T^-\right) + \left[\phi_\theta\left(T^-\right) - \phi_\theta\left(T^+\right)\right]\theta_T. \tag{25}$$

The term $\phi_0(T^+) - \phi_0(T^-)$ is announcement premium and is deterministic. The term $x_T - \tilde{x}_T$ is innovation of the true value of x_T relative to its expectation, and therefore cannot be predictable by publicly available information. Announcement return will not be predictable unless the term $\tilde{\theta}_T$

is. The return realized during the pre-announcement period can be written as:

$$P_T - P_\tau = [\phi_0(T) - \phi_0(\tau)] + \bar{\phi}_x(\hat{x}_T - \hat{x}_\tau) + [\phi_\theta(\tau)\theta_\tau - \phi_\theta(T)\theta_T].$$

In the above expression, the term $\phi_0(T) - \phi_0(\tau)$ is the pre-announcement drift, and the term $\hat{x}_T - \hat{x}_\tau$ is innovations in the rational expectation about x_T. The last term is the noisy supply in prices. Because the process θ_t is mean reverting, the pre- and post- announcement return in the above expressions are actually slightly negatively correlated. In our calibrated example, this correlation is -0.003. This feature of our model also matches the empirical evidence well.

6 Conclusion

In this paper, we develop a noisy rational expectations model with endogenous information acquisition and periodic announcements to account for the pre-FOMC announcement drift puzzle. We show that the endogenous information acquisition together with the generalized risk sensitive preference not only allows us to provide an equilibrium interpretation of the pre-FOMC announcement drift but also the stylized facts of the volatility dynamics and the trading volume around the FOMC announcements. We argue that models with information leakage have counter-factual implications on the volatility dynamics around announcements, and on the correlation between pre- and post-announcement returns. Our model does not assume information leakage and matches the empirical patterns of the FOMC announcement returns and volatility dynamics in the data quite well.

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Appendix

6.1 Equilibrium beliefs

The learning problem The optimal learning for the informed investor is a standard Kalman filter problem with the unobserved state variable given in (1) and the observed processes (5), (6), and (7). Applying Theorem 10.3 from Liptser and Shiryaev (2001), it is straightforward to show that the law of motion of the posterior mean satisfies (9) where the innovation processes for (5) and (7) are given by

$$d\hat{B}_{D,t} = \frac{1}{\sigma_D}\left[dD_t - (\hat{x}_t - D_t)\,dt\right], \text{and } d\hat{B}_{s,t} = \frac{1}{\sigma_s}\left(ds_t - \hat{x}_t dt\right). \tag{26}$$

The law of motion of the conditional variance $\hat{q}_t$ must satisfy the Riccati equation

$$d\hat{q}\,(t) = \left[\sigma_x^2 - 2b\hat{q}\,(t) - \left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}^2\,(t)\right]dt. \tag{27}$$

We can solve $\hat{q}\,(t) = \frac{\sigma_x^2\left(1-e^{-2\hat{b}(t+t^*)}\right)}{\left(\hat{b}-b\right)e^{-2\hat{b}(t+t^*)}+b+\hat{b}}$, where $\hat{b} = \sqrt{b^2 + \sigma_x^2\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)}$ and $t^* = \frac{1}{2\hat{b}}\ln\frac{\sigma_x^2+\left(\hat{b}-b\right)\hat{q}(0)}{\sigma_x^2-\left(\hat{b}+b\right)\hat{q}(0)}$.

The uninformed and the informed agree to disagree. Under the uninformed investor's distorted belief, the rational forecast of the informed should have been: $d\hat{x}_t^\kappa = b\left(\bar{x}^\kappa - \hat{x}_t^\kappa\right)dt + \frac{\hat{q}(t)}{\sigma_D}d\hat{B}_{D,t}^\kappa + \frac{\hat{q}(t)}{\sigma_s}d\hat{B}_{s,t}^\kappa$. The solution for $\hat{x}_t^\kappa$ can be written as (Note that $\hat{x}_0^\kappa = \hat{x}_0 = x_0$):

$$\hat{x}_\tau^\kappa = e^{-y(\tau)}\left\{x_0 + \int_0^\tau e^{y(t)}\left[b\bar{x}^\kappa dt + \frac{\hat{q}\,(t)}{\sigma_D}\frac{1}{\sigma_D}\left(dD_t + D_t dt\right) + \frac{\hat{q}\,(t)}{\sigma_s}\frac{1}{\sigma_s}ds_t\right]\right\}, \tag{28}$$

where $y\,(\tau) = \int_0^\tau \left(b + \hat{q}\,(s)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\right)ds$, for $\tau \in (0,T)$. Note that the solution for $\hat{x}_\tau$ takes a similar form except that we need to replace $\bar{x}^\kappa$ by $\bar{x}$. This implies that $\hat{x}_\tau^\kappa$ and $\hat{x}_\tau$ must be related by

$$\begin{aligned} K\,(\tau) \equiv \hat{x}_\tau - \hat{x}_\tau^\kappa &= e^{-y(\tau)}\int_0^\tau e^{y(t)}b\left(\bar{x} - \bar{x}^\kappa\right)dt = \kappa\sigma_x e^{-y(\tau)}\int_0^\tau e^{y(t)}dt \\ &= \kappa\sigma_x e^{-\int_0^\tau\left(b+\hat{q}(s)\left(\frac{1}{\sigma_D^2}+\frac{1}{\sigma_s^2}\right)\right)ds}\int_0^\tau e^{\int_0^t\left(b+\hat{q}(s)\left(\frac{1}{\sigma_D^2}+\frac{1}{\sigma_s^2}\right)\right)ds}dt. \end{aligned} \tag{29}$$

Now we can find out the distribution of $\hat{x}_t$ from the eyes of the uninformed. That is, we can represent the law of motion of $\hat{x}_t$ in terms of $d\hat{B}_{D,t}^\kappa$ and $d\hat{B}_{s,t}^\kappa$, what the uninformed trader think should be BM. Therefore, we can rewrite

$$d\hat{B}_{D,t} = d\hat{B}_{D,t}^\kappa - \frac{1}{\sigma_D}K\,(t)\,dt, \tag{30}$$

$$d\hat{B}_{s,t} = d\hat{B}_{s,t}^\kappa - \frac{1}{\sigma_s}K\,(t)\,dt. \tag{31}$$

The law of motion of $\hat{x}_t$ from the perspective of the uninformed is therefore

$$d\hat{x}_t = \left[b\left(\bar{x} - \hat{x}_t\right) - \hat{q}\,(t)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)K\,(t)\right]dt + \frac{\hat{q}\,(t)}{\sigma_D}d\hat{B}_{D,t}^\kappa + \frac{\hat{q}\,(t)}{\sigma_s}d\hat{B}_{s,t}^\kappa. \tag{32}$$

The uniformed observes two sources of information for $\hat{x}_t$. One is the dividend process:

$$
\begin{aligned}
dD_t &= (\hat{x}_t - D_t)\,dt + \sigma_D d\hat{B}_{D,t}. && (33)\\
&= (\hat{x}_t - K_t - D_t)\,dt + \sigma_D d\hat{B}^{\kappa}_{D,t} && (34)
\end{aligned}
$$

Information content of price In addition to observing the dividend, the uninformed trader also observes the prices process. We have assumed that the price process takes the form of equation (11).

Now, we think about the information content of price as before. Note that observing the price is equivalent to observing $\zeta_t \equiv \phi_x(t)\,\hat{x}_t - \phi_\theta(t)\,\theta_t$, because all other variables in (11) are known to the uninformed investors. Applying Ito's lemma, ζ_t can be represented as a Markov process given the state variables $\hat{x}_t$ and ζ_t itself:

$$
\begin{aligned}
d\zeta_t \quad = \quad & \left[b\bar{x}\phi_x(t) + \left(\left(a - b - \frac{\phi'_\theta(t)}{\phi_\theta(t)}\right)\phi_x(t) + \phi'_x(t)\right)\hat{x}_t + \left(\frac{\phi'_\theta(t)}{\phi_\theta(t)} - a\right)\zeta_t\right]dt\\
& + \frac{\hat{q}(t)}{\sigma_D}\phi_x(t)\,d\hat{B}_{D,t} + \frac{\hat{q}(t)}{\sigma_s}\phi_x(t)\,d\hat{B}_{s,t} - \sigma_\theta\phi_\theta(t)\,dB_{\theta,t}. && (35)
\end{aligned}
$$

It is convenient to define $\xi_t = \zeta_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t)\,D_t$ so that $(\hat{x}_t, D_t, \xi_t)$ has a state space representation and the innovations of dD_t and $d\xi_t$ are mutually independent. The dynamics of ξ_t is

$$
d\xi_t = \left[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + m_x(t)\,\hat{x}_t + \left(\frac{\phi'_\theta(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)\,D_t\right]dt + \sigma_\xi(t)\,d\hat{B}_{\xi,t}, \qquad (36)
$$

where the coefficients $m_x(t)$ and $m_D(t)$ are defined as

$$
\begin{aligned}
m_x(t) &= \left(a - b - \frac{\phi'_\theta(t)}{\phi_\theta(t)} - \frac{\hat{q}(t)}{\sigma_D^2}\right)\phi_x(t) + \phi'_x(t), && (37)\\
m_D(t) &= \frac{1}{\sigma_D^2}\left[\hat{q}(t)\,\phi_x(t)\left(1 - a + \frac{\phi'_\theta(t)}{\phi_\theta(t)}\right) - \hat{q}'(t)\,\phi_x(t) - \hat{q}(t)\,\phi'_x(t)\right]. && (38)
\end{aligned}
$$

It is convenient to define the volatility of ξ_t as

$$
\sigma_\xi(t) = \sqrt{\frac{\hat{q}^2(t)}{\sigma_s^2}\phi_x^2(t) + \sigma_\theta^2\phi_\theta^2(t)}, \qquad (39)
$$

and $\hat{B}_{\xi,t}$ is a standard Brownian motion that is independent of $\hat{B}_{D,t}$:

$$
d\hat{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)}\left[\frac{\hat{q}(t)}{\sigma_s}\phi_x(t)\,d\hat{B}_{s,t} - \sigma_\theta\phi_\theta(t)\,dB_{\theta,t}\right]. \qquad (40)
$$

Under the informed trader's information set, $\hat{B}_{\xi,t}$ is a standard Brownian motion that is independent of $\hat{B}_{D,t}$. From the uninformed perspective,

$$
d\hat{B}_{\xi,t} = d\hat{B}^{\kappa}_{\xi,t} - \frac{1}{\sigma_\xi(t)}\frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\,K(t)\,dt. \qquad (41)
$$

To summarize, the uninformed trader's learning problem can be written as follows. The state variable is (32), and the observation processes are:

$$dD_t = (\hat{x}_t - K_t - D_t)\,dt + \sigma_D d\hat{B}^{\kappa}_{D,t}, \tag{42}$$
$$d\xi_t = \left[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + m_x(t)\,\hat{x}_t + \left(\frac{\phi'_\theta(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)\,D_t - \frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\,K(t)\right]dt + \sigma_\xi(t)\,d\hat{B}^{\kappa}_{\xi,t} \tag{43}$$

Optimal filtering for the uninformed In order to solve the uninformed trader's optimal portfolio demand problem, we have to solve uninformed trader's learning problem about $\hat{x}_t$. Note that from the uninformed trader's perspective, $\hat{x}_t$ is not a rational belief, and the uninformed is not trying to learn about the rational belief $\hat{x}^{\kappa}_t$. He is trying to learn about $\hat{x}_t$, and the law of motion of $\hat{x}_t$ is (32). So as matter of notation, we use $\tilde{x}_t = \tilde{\mathbb{E}}[\hat{x}_t]$. Just as a matter of notation, $\tilde{x}_t$ is what uninformed think $\hat{x}_t$ should be. It does not satisfy the law of iterated expectation. In fact, $\tilde{x}_t = \tilde{\mathbb{E}}[\hat{x}_t] = \tilde{\mathbb{E}}[\hat{x}^{\kappa}_t + K(t)] = \tilde{\mathbb{E}}[x_t] + K(t)$. The uninformed think that $\hat{x}^{\kappa}_t$ should be the rational expectation, and $\hat{x}_t$ is "irrational exuberance".

To apply the Kalman filter, we will treat (32) as the unobservable state variable and (42) and (43) as the observation process. The Kalman filter can therefore be written as equation (13), where

$$d\tilde{B}_{D,t} = \frac{1}{\sigma_D}\left[dD_t - (\tilde{x}_t - K_t - D_t)\,dt\right] \tag{44}$$
$$d\tilde{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)}\left\{d\xi_t - \left[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + m_x(t)\,\tilde{x}_t + \left(\frac{\phi'_\theta(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)\,D_t - \frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\,K(t)\right]dt\right\} \tag{45}$$

and we denote

$$\nu(t) = \frac{1}{\sigma_\xi^2(t)}\left[\frac{\phi_x(t)}{\sigma_s^2}\hat{q}^2(t) + m_x(t)\,\tilde{q}(t)\right]. \tag{46}$$

The posterior variance for $\hat{x}_t$ from the uninformed investor's belief, $\tilde{q}(t)$ satisfies the following Riccati equation

$$d\tilde{q}_t = \left[\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}_t^2 - 2b\tilde{q}_t - \frac{(\hat{q}_t + \tilde{q}_t)^2}{\sigma_D^2} - \left(\frac{m_x(t)\,\tilde{q}_t + \frac{\phi_x(t)\hat{q}_t^2}{\sigma_s^2}}{\sigma_\xi(t)}\right)^2\right]dt \tag{47}$$

Using Equations (33) and (36), we can relate $\tilde{B}_{D,t}$ to $\hat{B}_{D,t}$ and $\tilde{B}_{\xi,t}$ to $\hat{B}_{\xi,t}$:

$$d\tilde{B}_{D,t} = \frac{1}{\sigma_D}\left[(\hat{x}_t - \tilde{x}_t + K_t)\,dt\right] + d\hat{B}_{D,t} \tag{48}$$
$$d\tilde{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)}\left[m_x(t)\,(\hat{x}_t - \tilde{x}_t) + \frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\,K(t)\right]dt + d\hat{B}_{\xi,t}. \tag{49}$$

Once $\tilde{q}_t$ is determined, the following variances and covariance can be compute from the law of total covariance. First, $\tilde{\mathbb{E}}(x_t) = \tilde{\mathbb{E}}(\hat{x}_t^\kappa) = \tilde{\mathbb{E}}(\hat{x}_t - K(t)) = \tilde{x}_t - K(t)$. In addition,

$$\begin{aligned} \tilde{Var}(x_t) &= \tilde{\mathbb{E}}\left[\hat{Var}^\kappa(x_t)\right] + \tilde{Var}\left(\hat{\mathbb{E}}^\kappa[x_t]\right) \\ &= \tilde{\mathbb{E}}[\hat{q}_t] + \tilde{Var}(\hat{x}_t^\kappa) = \hat{q}_t + \tilde{q}_t. \end{aligned} \tag{50}$$

Next, we compute $\tilde{Cov}(x_t, \hat{x}_t) = \tilde{\mathbb{E}}\left[\hat{Cov}^\kappa(x_t, \hat{x}_t)\right] + \tilde{Cov}\left(\hat{\mathbb{E}}^\kappa[x_t], \hat{\mathbb{E}}^\kappa[\hat{x}_t]\right) = \tilde{q}_t$. Therefore,

$$\tilde{Cov}(x_t, \theta_t) = \tilde{Cov}\left[x_t, \frac{1}{\phi_{\theta,t}}(\phi_{x,t}\hat{x}_t - \zeta_t)\right] = \frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t, \tag{51}$$

and $\tilde{Cov}(\hat{x}_t, \theta_t) = \frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t$. Finally,

$$\tilde{Var}(\theta_t) = \tilde{Cov}\left[\frac{1}{\phi_{\theta,t}}(\phi_{x,t}\hat{x}_t - \zeta_t), \frac{1}{\phi_{\theta,t}}(\phi_{x,t}\hat{x}_t - \zeta_t)\right] = \frac{\phi_{x,t}^2}{\phi_{\theta,t}^2}\tilde{q}_t. \tag{52}$$

Difference in Beliefs Because price is a function of both $\hat{x}_t$ and $\tilde{x}_t$, in order to solve investors' optimal portfolio choice problem, we need to figure out the belief about uninformed about $\hat{x}_t$ and the belief about the informed about $\tilde{x}_t$. The uninformed investors' do not know $\hat{x}_t$ exactly, but they can calculate its distribution, $\mathcal{N}(\tilde{x}_t, \tilde{q}_t)$ using (13) and (47).

The informed trader, of course observes everything that the uniformed observe and can calculate $\tilde{x}_t$ exactly. In particular, they can calculate where the "confused BMs", $\tilde{B}_{D,t}$ and $\tilde{B}_{\xi,t}$ using (48) and (49). This allows them to compute $\tilde{x}_t$ as:

$$\begin{aligned} d\tilde{x}_t = &\left\{b(\bar{x} - \tilde{x}_t) + \left[\frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D^2} + \nu(t)\, m_x(t)\right][\hat{x}_t - \tilde{x}_t] + \left[\frac{\tilde{q}(t)}{\sigma_D^2} + \frac{\hat{q}(t)}{\sigma_s^2}(\phi_x(t)\,\nu(t) - 1)\right]K(t)\right\}dt \\ &+ \frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D}d\hat{B}_{D,t} + \nu(t)\,\phi_x(t)\,\frac{\hat{q}(t)}{\sigma_s}d\hat{B}_{s,t} - \phi_\theta(t)\,\nu(t)\,\sigma_\theta dB_{\theta,t}. \end{aligned}$$

We define $\Delta_t \equiv \hat{x}_t - \tilde{x}_t^\kappa$ as the difference in belief, using equation (9), we have:

$$d\Delta_t = -\left[a_\Delta(t)\,\Delta_t + b_\Delta(t)\right]dt - \sigma_{\Delta D}(t)\,d\hat{B}_{D,t} + \sigma_{\Delta s}(t)\,d\hat{B}_{s,t} + \sigma_{\Delta\theta}(t)\,dB_{\theta,t}, \tag{53}$$

where the coefficients are:

$$a_{\Delta}(t) = b + \frac{\hat{q}(t) + \tilde{q}(t)}{\sigma_D^2} + \nu(t)\, m_x(t), \tag{54}$$

$$b_{\Delta}(t) = \left[\frac{\tilde{q}(t)}{\sigma_D^2} + \frac{\hat{q}(t)}{\sigma_s^2}\left(\phi_x(t)\,\nu(t) - 1\right)\right] K(t), \tag{55}$$

$$\sigma_{\Delta D}(t) = \frac{\tilde{q}(t)}{\sigma_D},$$

$$\sigma_{\Delta s}(t) = \frac{\hat{q}(t)}{\sigma_s}\left[1 - \phi_x(t)\,\nu(t)\right],$$

$$\sigma_{\Delta\theta}(t) = \phi_\theta(t)\,\nu(t)\,\sigma_\theta.$$

Note that compared to Han (2020), $d\Delta_t$ has a downward trend. The uninformed think that the informed are over optimistic, and therefore, $\tilde{x}_t$ typically increases faster than what $\hat{x}_t$ actually is.

Filtering of the uninformed investors who acquire information Consider a generation i investor who observes $\phi_x(t)\,\hat{x}_t - \phi_\theta(t)\,\theta_t$ and tries to learn $\hat{x}_t$. Therefore, $\phi_x(t)\,\hat{x}_t - \phi_\theta(t)\,\theta_t = \phi_x(t)\,\tilde{x}_t^i - \phi_\theta(t)\,\tilde{\theta}_t^i$. The pricing function can be written as: $P_t = \phi(t) + \phi_D D_t - \phi_\theta(t)\,\tilde{\theta}_t^i + \bar{\phi}_x \tilde{x}_t^i - \phi_\Delta(t)\,\Delta_t^i$, where

$$\Delta_t^i \equiv \tilde{x}_t^i - \tilde{x}_t. \tag{56}$$

Therefore, to understand the price dynamics from the perspective of the generation i investor, we need to understand $\tilde{x}_t^i = \tilde{\mathbb{E}}^i[\hat{x}_t]$, $\tilde{\theta}_t^i = \tilde{\mathbb{E}}^i[\theta_t]$ and $\Delta_t^i = \tilde{\mathbb{E}}^i\left[\tilde{x}_t^i - \tilde{x}_t^\kappa\right] = \tilde{x}_t^i - \tilde{x}_t$.

First, let's characterize the dynamics of $\tilde{x}_t^i$. The learning problems remain the same. The only difference is, at time i, the initial condition for $\tilde{x}^i(i) = \hat{x}(i)$, and $\tilde{q}^i(i) = 0$. Hence, the filtering equation for $i < t < T$ is written as

$$d\tilde{x}_t^i = \left[b\left(\bar{x} - \tilde{x}_t^i\right) - \hat{q}(t)\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right) K(t)\right] dt + \frac{\hat{q}_t + \tilde{q}_t^i}{\sigma_D} d\tilde{B}_{D,t}^i + \nu^i(t)\,\sigma_\xi(t)\, d\tilde{B}_{\xi,t}^i, \tag{57}$$

where $\nu^i(t) = \frac{\frac{\phi_x(t)\hat{q}_t^2}{\sigma_s^2} + m_x(t)\tilde{q}_t^i}{\sigma_\xi^2(t)}$, $d\tilde{B}_{D,t}^i = \frac{1}{\sigma_D}\left[dD_t - \left(\tilde{x}_t^i - K_t - D_t\right)dt\right]$, and $d\tilde{B}_{\xi,t}^i = \frac{1}{\sigma_\xi(t)}\Big\{d\xi_t - \Big[b\bar{x}\phi_x(t) - a\bar{\theta}\phi_\theta(t) + m_x(t)\,\tilde{x}_t^i + \left(\frac{\phi_\theta'(t)}{\phi_\theta(t)} - a\right)\xi_t + m_D(t)\,D_t - \frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\,K(t)\Big]dt\Big\}$. The posterior variance is,

$$d\tilde{q}_t^i = \left[\left(\frac{1}{\sigma_D^2} + \frac{1}{\sigma_s^2}\right)\hat{q}_t^2 - 2b\tilde{q}_t^i - \frac{\left(\hat{q}_t + \tilde{q}_t^i\right)^2}{\sigma_D^2} - \left(\frac{m_x(t)\,\tilde{q}_t^i + \frac{\phi_x(t)\hat{q}_t^2}{\sigma_s^2}}{\sigma_\xi(t)}\right)^2\right] dt. \tag{58}$$

Also, we can write $d\tilde{B}_{D,t} = \frac{1}{\sigma_D}\Delta_t^i dt + d\tilde{B}_{D,t}^i$, and $d\tilde{B}_{\xi,t} = \frac{1}{\sigma_\xi(t)} m_x(t)\,\Delta_t^i dt + d\tilde{B}_{\xi,t}^i$. These imply that dD_t can be written as

$$dD_t = \left(\tilde{x}_t^i - K_t - D_t\right) dt + \sigma_D d\tilde{B}_{D,t}^i. \tag{59}$$

Then, let's turn to the calculation of $\tilde{\theta}_t^i$. Similarly, $\phi_\theta(t)\tilde{\theta}_t^i = \phi_x(t)\tilde{x}_t^i - \xi_t - \frac{\hat{q}(t)}{\sigma_D^2}\phi_x(t)D_t$.Therefore,

$$d\tilde{\theta}_t^i = a\left(\bar{\theta}^\kappa - \tilde{\theta}_t^i\right)dt + \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}^i(t)}{\sigma_D}d\tilde{B}_{D,t}^i + \left[\phi_x(t)\nu^i(t) - 1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}_{\xi,t}^i. \tag{60}$$

Last, we need to calculate the law of motion for Δ_t^i. We first need to represent $\tilde{x}_t$ as the BM with respect to generation i's belief. Therefore,

$$d\Delta_t^i = -a_\Delta(t)\Delta_t^i dt + \frac{\tilde{q}_t^i - \tilde{q}_t}{\sigma_D}d\tilde{B}_{D,t}^i + \frac{m_x(t)}{\sigma_\xi(t)}\left(\tilde{q}_t^i - \tilde{q}_t\right)d\tilde{B}_{\xi,t}^i \tag{61}$$

Excess Returns In this subsection, we use the results from the filtering problem derived above to derive the excess return of the stock as diffusion processes under three types of investors' beliefs. We have conjectured that the equilibrium price is of the form (11). In order to solve for the optimal portfolio choice, we need to compute investors' belief about the return process. In the interior, this means we need to represent instantaneous excess return $dQ_t = dP_t + D_t dt - rP_t dt$ as functions of investors' own BM. On the boundary, we need to compute the conditional distribution of $P_T^+ - P_T^-$ from investors' own belief. Consider first the informed investors. Equations (33), (6), (9), and (15) represent the variables D_t, θ_t, $\hat{x}_t$, and Δ_t in terms of Brownian motions with respect to their information set. These give

$$\begin{aligned} dQ_t = {} & \{e_0(t) + [1-(1+r)\phi_D(t)]D_t + e_\theta(t)\theta_t + [\phi_D - (b+r)\phi_x]\hat{x}_t + e_\Delta(t)\Delta_t\}dt \\ & + \varrho_D(t)d\hat{B}_{D,t} + \varrho_s(t)d\hat{B}_{s,t} + \varrho_\theta(t)dB_{\theta,t}, \end{aligned} \tag{62}$$

where

$$e_0(t) = \phi'(t) - r\phi(t) + b\bar{x}\bar{\phi}_x + \phi_\Delta(t)b_\Delta(t) - a\bar{\theta}\phi_\theta(t) \tag{63}$$

$$e_\theta(t) = (a+r)\phi_\theta(t) - \phi_\theta'(t), \tag{64}$$

$$e_\Delta(t) = (a_\Delta(t)+r)\phi_\Delta(t) - \phi_\Delta'(t), \tag{65}$$

$$\varrho_D(t) = \phi_D\sigma_D + \bar{\phi}_x\frac{\hat{q}(t)}{\sigma_D} + \phi_\Delta(t)\sigma_{\Delta D}(t), \tag{66}$$

$$\varrho_s(t) = [1+\phi_\Delta(t)\nu(t)]\phi_x(t)\frac{\hat{q}_t}{\sigma_s}, \tag{67}$$

$$\varrho_\theta(t) = -[1+\phi_\Delta(t)\nu(t)]\phi_\theta(t)\sigma_\theta. \tag{68}$$

Further define the variance of excess return as

$$\sigma_P(t) = \varrho_D^2(t) + \varrho_s^2(t) + \varrho_\theta^2(t). \tag{69}$$

The market clearing condition implies that the expected return of the stock cannot depend on D_t, $\hat{x}_t$ and the constant. As a result, the coefficients them must be 0, implying

$$\phi_D = \frac{1}{1+r}, \text{ and } \bar{\phi}_x = \frac{\phi_D}{b+r}. \tag{70}$$

Similarly, we can use equations (19), and (13) to write the excess return in terms of Brownian motions with respect to the uninformed investor's information set. This gives

$$dQ_t = \left[e_1(t) + e_\theta(t)\tilde{\theta}_t\right]dt + \varrho_D(t)\,d\tilde{B}_{D,t} + \varrho_\xi(t)\,d\tilde{B}_{\xi,t}. \tag{71}$$

where

$$\begin{aligned} e_1(t) &= e_0(t) - \left[\varrho_D(t)\frac{1}{\sigma_D} + \varrho_\xi(t)\frac{1}{\sigma_\xi(t)}\frac{\hat{q}(t)}{\sigma_s^2}\phi_x(t)\right]K_t \quad &(72)\\ \varrho_\xi(t) &= -\frac{\sigma_\xi(t)}{\sigma_\theta\phi_\theta(t)}\varrho_\theta(t). &(73)\end{aligned}$$

Third, for the uninformed investor who acquires information, we can obtain,

$$dQ_t = \left[e_1(t) + e_\theta(t)\tilde{\theta}_t^i + e_\Delta(t)\Delta_t^i\right]dt + \varrho_D(t)\,d\tilde{B}_{D,t}^i + \varrho_\xi(t)\,d\tilde{B}_{\xi,t}^i. \tag{74}$$

6.2 Optimal portfolio choice decisions

Portfolio demand for the informed: interior The optimization problem for the informed investor in the interior is written as

$$\begin{aligned} \hat{V}\left(t,\hat{W},\theta,\Delta\right) &= \max_{\alpha,\hat{C}_t}\mathbb{E}\left[\int_0^{T-t} -e^{-\rho s-\gamma C_{t+s}^i}ds + e^{-\rho(T-t)}\hat{V}^-\left(T,\hat{W}_T,\theta_T,\Delta_T\right)\right]\\ s.t.\ d\hat{W}_t &= \left(\hat{W}_t r - \hat{C}_t\right)dt + \alpha_t dQ_t\\ dQ_t &= \left[e_0(t) + e_\theta(t)\theta_t + e_\Delta(t)\Delta_t\right]dt + \varrho_D(t)\,d\hat{B}_{D,t} + \varrho_s(t)\,d\hat{B}_{s,t} + \varrho_\theta(t)\,dB_{\theta,t},\\ d\theta_t &= a\left(\bar{\theta} - \theta_t\right)dt + \sigma_\theta dB_{\theta,t}\\ d\Delta_t &= -\left[a_\Delta(t)\Delta_t + b_\Delta(t)\right]dt - \sigma_{\Delta D}(t)\,d\hat{B}_{D,t} + \sigma_{\Delta s}(t)\,d\hat{B}_{s,t} + \sigma_{\Delta\theta}(t)\,dB_{\theta,t}.\end{aligned}$$

Conjecture the informed investor's value function takes the form of $\hat{V}\left(t,\hat{W},\theta,\Delta\right) = -e^{-r\gamma\hat{W} - g(t,\theta,\Delta)}$, where

$$g(t,\theta,\Delta) = g(t) + g_\theta(t)\theta_t + \frac{1}{2}g_{\theta\theta}(t)\theta_t^2 + g_\Delta(t)\Delta_t + \frac{1}{2}g_{\Delta\Delta}(t)\Delta_t^2 + g_{\theta\Delta}(t)\theta_t\Delta_t. \tag{75}$$

Using Ito's Lemma, the HJB equation is:

$$\begin{aligned}\rho J &= -e^{-\gamma\hat{C}} + \hat{V}_t + \hat{V}_W\left[r\hat{W} - \hat{C} + \alpha\left(e_0(t) + e_\theta(t)\theta + e_\Delta(t)\Delta\right)\right] + \frac{1}{2}\hat{V}_{WW}\alpha^2\sigma_P(t) + \alpha\hat{V}_{W\theta}\sigma_\theta\varrho_\theta(t)\\ &\quad + \alpha\hat{V}_{W\Delta}\sigma_{Q\Delta}(t) + \hat{V}_\theta a\left(\bar{\theta} - \theta\right) - \hat{V}_\Delta\left(a_\Delta(t)\Delta + b_\Delta(t)\right) + \frac{1}{2}\hat{V}_{\theta\theta}\sigma_\theta^2 + \frac{1}{2}\hat{V}_{\Delta\Delta}\sigma_\Delta(t) + \hat{V}_{\Delta\theta}\sigma_\theta\sigma_{\Delta\theta}(t)\end{aligned} \tag{76}$$

where

$$
\begin{aligned}
\sigma_{\Delta}(t) &= \sigma_{\Delta D}^{2}(t)+\sigma_{\Delta s}^{2}(t)+\sigma_{\Delta\theta}^{2}(t) \\
\sigma_{Q\Delta}(t) &= -\varrho_{D}(t)\,\sigma_{\Delta D}(t)+\varrho_{s}(t)\,\sigma_{\Delta s}(t)+\varrho_{\theta}(t)\,\sigma_{\Delta\theta}(t), \qquad (77)
\end{aligned}
$$

Under the guessed value function form, the first order condition (FOC) with respect to $\hat{C}$ and α are

$$
\begin{aligned}
\hat{C} &= r\hat{W}+\frac{1}{\gamma}\left[g(t,\theta,\Delta)-\ln r\right], \qquad (78) \\
\alpha &= \frac{\left[\begin{array}{c} e_{0}(t)+e_{\theta}(t)\,\theta+e_{\Delta}(t)\,\Delta-\left(g_{\theta}(t)+g_{\theta\theta}(t)\,\theta_{t}+g_{\theta\Delta}(t)\,\Delta_{t}\right)\sigma_{\theta}\varrho_{\theta}(t) \\ -\left(g_{\Delta}(t)+g_{\Delta\Delta}(t)\,\Delta_{t}+g_{\theta\Delta}(t)\,\theta_{t}\right)\sigma_{Q\Delta}(t) \end{array}\right]}{r\gamma\sigma_{P}(t)} \qquad (79)
\end{aligned}
$$

substituting expressions in (77) yields the demand function of the form:

$$
\alpha_{t}=\alpha_{0}(t)+\alpha_{\theta}(t)\,\theta_{t}+\alpha_{\Delta}(t)\,\Delta_{t}, \qquad (80)
$$

where

$$
\begin{aligned}
\alpha_{0}(t) &= \frac{e_{0}(t)-g_{\theta}(t)\,\sigma_{\theta}\varrho_{\theta}(t)-\sigma_{Q\Delta}(t)\,g_{\Delta}(t)}{r\gamma\sigma_{P}(t)} \qquad (81) \\
\alpha_{\theta}(t) &= \frac{e_{\theta}(t)-\varrho_{\theta}(t)\,\sigma_{\theta}g_{\theta\theta}(t)-\sigma_{Q\Delta}(t)\,g_{\theta\Delta}(t)}{r\gamma\sigma_{P}(t)} \qquad (82) \\
\alpha_{\Delta}(t) &= \frac{e_{\Delta}(t)-\varrho_{\theta}(t)\,\sigma_{\theta}g_{\theta\Delta}(t)-\sigma_{Q\Delta}(t)\,g_{\Delta\Delta}(t)}{r\gamma\sigma_{P}(t)}. \qquad (83)
\end{aligned}
$$

Matching coefficients of the value function, and use $\alpha_{0}(t)$, $\alpha_{\theta}(t)$ and $\alpha_{\Delta}(t)$ to simplify, we have the following odes system,

$$
\begin{aligned}
g'(t) &= r-\rho-r\ln r+rg(t)-\frac{1}{2}r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{0}^{2}(t)+b_{\Delta}(t)\,g_{\Delta}(t)+\frac{1}{2}\sigma_{\theta}^{2}\left[g_{\theta}^{2}(t)-g_{\theta\theta}(t)\right] \\
&\quad +\frac{1}{2}\sigma_{\Delta}(t)\left[g_{\Delta}^{2}(t)-g_{\Delta\Delta}(t)\right]+\sigma_{\theta}\sigma_{\Delta\theta}(t)\left[g_{\theta}(t)\,g_{\Delta}(t)-g_{\theta\Delta}(t)\right]-a\bar{\theta}g_{\theta}(t), \qquad (84) \\
g_{\theta\theta}'(t) &= rg_{\theta\theta}(t)-r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{\theta}^{2}(t)+2ag_{\theta\theta}(t)+\sigma_{\theta}^{2}g_{\theta\theta}^{2}(t)+\sigma_{\Delta}(t)\,g_{\theta\Delta}^{2}(t)+2\sigma_{\theta}\sigma_{\Delta\theta}(t)\,g_{\theta\theta}(t)\,g_{\theta\Delta}(t), \\
g_{\Delta\Delta}'(t) &= rg_{\Delta\Delta}(t)-r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{\Delta}^{2}(t)+2a_{\Delta}(t)\,g_{\Delta\Delta}(t)+\sigma_{\theta}^{2}g_{\theta\Delta}^{2}(t)+\sigma_{\Delta}(t)\,g_{\Delta\Delta}^{2}(t) \\
&\quad +2\sigma_{\theta}\sigma_{\Delta\theta}(t)\,g_{\theta\Delta}(t)\,g_{\Delta\Delta}(t), \qquad (85) \\
g_{\theta\Delta}'(t) &= rg_{\theta\Delta}(t)-r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{\theta}(t)\,\alpha_{\Delta}(t)+ag_{\theta\Delta}(t)+a_{\Delta}(t)\,g_{\theta\Delta}(t)+\sigma_{\theta}^{2}g_{\theta\theta}(t)\,g_{\theta\Delta}(t) \\
&\quad +\sigma_{\Delta}(t)\,g_{\Delta\Delta}(t)\,g_{\theta\Delta}(t)+\sigma_{\theta}\sigma_{\Delta\theta}(t)\left[g_{\theta\theta}(t)\,g_{\Delta\Delta}(t)+g_{\theta\Delta}^{2}(t)\right]; \qquad (86) \\
g_{\theta}'(t) &= rg_{\theta}(t)-r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{0}(t)\,\alpha_{\theta}(t)+ag_{\theta}(t)+b_{\Delta}(t)\,g_{\theta\Delta}(t)+\sigma_{\theta}^{2}g_{\theta}(t)\,g_{\theta\theta}(t) \\
&\quad +\sigma_{\Delta}(t)\,g_{\Delta}(t)\,g_{\theta\Delta}(t)+\sigma_{\theta}\sigma_{\Delta\theta}(t)\left[g_{\theta}(t)\,g_{\theta\Delta}(t)+g_{\theta\theta}(t)\,g_{\Delta}(t)\right]-a\bar{\theta}g_{\theta\theta}, \qquad (87) \\
g_{\Delta}'(t) &= rg_{\Delta}(t)-r^{2}\gamma^{2}\sigma_{P}(t)\,\alpha_{0}(t)\,\alpha_{\Delta}(t)+a_{\Delta}(t)\,g_{\Delta}(t)+b_{\Delta}(t)\,g_{\Delta\Delta}(t)+\sigma_{\theta}^{2}g_{\theta}(t)\,g_{\theta\Delta}(t) \\
&\quad +\sigma_{\Delta}(t)\,g_{\Delta}(t)\,g_{\Delta\Delta}(t)+\sigma_{\theta}\sigma_{\Delta\theta}(t)\left[g_{\theta}(t)\,g_{\Delta\Delta}(t)+g_{\theta\Delta}(t)\,g_{\Delta}(t)\right]-a\bar{\theta}g_{\theta\Delta}. \qquad (88)
\end{aligned}
$$

Portfolio demand for the uninformed investors who never acquire information: interior
The optimization problem of the uninformed investor who never acquire information is :

$$
\begin{aligned}
V\left(t,\tilde{W},\tilde{\theta}^{\kappa}\right) &= \max_{\beta_t,\tilde{C}_t}\mathbb{E}\left[\int_0^{T-t} -e^{-\rho s-\gamma \tilde{C}_{t+s}}ds + e^{-\rho(T-t)}V^{-}\left(T,\tilde{W}_T,\tilde{\theta}_T\right)\right] \\
s.t.\ d\tilde{W}_t &= \left(\tilde{W}_t r-\tilde{C}_t\right)dt+\beta_t dQ_t \\
dQ_t &= \left[e_1(t)+e_\theta(t)\tilde{\theta}_t\right]dt+\varrho_D(t)\,d\tilde{B}_{D,t}+\varrho_\xi(t)\,d\tilde{B}_{\xi,t}. \\
d\tilde{\theta}_t &= a\left(\bar{\theta}^{\kappa}-\tilde{\theta}_t\right)dt+\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}d\tilde{B}_{D,t}+\left[\phi_x(t)\nu(t)-1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}d\tilde{B}_{\xi,t}.
\end{aligned}
$$

Conjecture the uninformed investor's value function would be of the form: $V\left(t,\tilde{W},\tilde{\theta}\right)=-e^{-r\gamma\tilde{W}-f\left(t,\tilde{\theta}\right)}$, where

$$f\left(t,\tilde{\theta}\right)=f(t)+f_\theta(t)\tilde{\theta}_t+\frac{1}{2}f_{\theta\theta}(t)\tilde{\theta}_t^2. \tag{89}$$

The HJB of the above problem is written as:

$$
\begin{aligned}
\rho V &= -e^{-\gamma\tilde{C}}+V_t+V_W\left[r\tilde{W}-\tilde{C}+\beta\left(e_1(t)+e_\theta(t)\tilde{\theta}^{\kappa}\right)\right] \\
&\quad +\frac{1}{2}V_{WW}\beta^2\sigma_P(t)+\beta V_{W\theta}\sigma_{Q\theta}(t)+V_\theta a\left(\bar{\theta}^{\kappa}-\tilde{\theta}^{\kappa}\right)+\frac{1}{2}V_{\theta\theta}\sigma_{\theta\theta}(t)
\end{aligned}
$$

where

$$\sigma_{Q\theta}(t) = \frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\varrho_D(t)+\left(\phi_x(t)\nu(t)-1\right)\frac{\sigma_\xi(t)}{\phi_\theta(t)}\varrho_\xi(t) \tag{90}$$

$$\sigma_{\theta\theta}(t) = \left[\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}(t)}{\sigma_D}\right]^2+\left[\left(\phi_x(t)\nu(t)-1\right)\frac{\sigma_\xi(t)}{\phi_\theta(t)}\right]^2. \tag{91}$$

Under the guessed value function form, the FOCs are

$$\tilde{C} = rW^u+\frac{1}{\gamma}\left[f\left(t,\tilde{\theta}_t\right)-\ln r\right]. \tag{92}$$

$$\beta = \frac{e_1(t)+e_\theta(t)\tilde{\theta}_t-\left(f_\theta(t)+f_{\theta\theta}(t)\tilde{\theta}_t\right)\sigma_{Q\theta}(t)}{r\gamma\sigma_P(t)} \tag{93}$$

We then obtain the demand function of the form

$$\beta_t=\beta_0(t)+\beta_\theta(t)\tilde{\theta}_t. \tag{94}$$

where

$$\beta_0(t) = \frac{e_1(t) - \sigma_{Q\theta}(t) f_\theta(t)}{r\gamma\sigma_P(t)} \tag{95}$$

$$\beta_\theta(t) = \frac{e_\theta(t) - \sigma_{Q\theta}(t) f_{\theta\theta}(t)}{r\gamma\sigma_P(t)}. \tag{96}$$

Substituting this into HJB and matching coefficients of the value function, we have

$$f'(t) = r - \rho - r\ln r + rf(t) - r\gamma\beta_0 - \frac{1}{2}r^2\gamma^2\sigma_P(t)\beta_0^2(t) + \frac{1}{2}\sigma_{\theta\theta}\left[f_\theta^2(t) - f_{\theta\theta}(t)\right] - a\bar{\theta}^\kappa f_\theta(t) \tag{97}$$

$$f'_{\theta\theta}(t) = rf_{\theta\theta}(t) - r^2\gamma^2\sigma_P(t)\beta_\theta^2(t) + 2af_{\theta\theta}(t) + \sigma_{\theta\theta}f_{\theta\theta}^2(t) \tag{98}$$

$$f'_\theta(t) = rf_\theta(t) - r^2\gamma^2\sigma_P(t)\beta_0(t)\beta_\theta(t) + af_\theta(t) + \sigma_{\theta\theta}f_\theta(t)f_{\theta\theta}(t) - a\bar{\theta}^\kappa f_{\theta\theta}(t) \tag{99}$$

Portfolio demand for the uninformed who acquire information: interior The optimization problem of the uninformed investor who acquire information is characterized by

$$\begin{aligned}
V^i\left(t, W^i, \tilde{\theta}^i, \Delta^i\right) &= \max_{\varepsilon_t, C_t^i} \tilde{\mathbb{E}}^i\left[\int_0^{T-t} -e^{-\rho s - \gamma C_{t+s}^i} ds + e^{-\rho(T-t)} V^{i-}\left(T, W_T^i, \tilde{\theta}_T^i, \Delta_T^i\right)\right] \\
s.t.\ dW_t^i &= \left(W_t^i r - C_t^i\right) dt + \varepsilon_t dQ_t \\
dQ_t &= \left[e_1(t) + e_\theta(t)\tilde{\theta}_t^i + e_\Delta(t)\Delta_t^i\right] dt + \varrho_D(t) d\tilde{B}_{D,t}^i + \varrho_\xi(t) d\tilde{B}_{\xi,t}^i. \\
d\tilde{\theta}_t^i &= a\left(\bar{\theta} - \tilde{\theta}_t^i\right) dt + \frac{\phi_x(t)}{\phi_\theta(t)} \frac{\tilde{q}^i(t)}{\sigma_D} d\tilde{B}_{D,t}^i + \left[\phi_x(t)\nu^i(t) - 1\right] \frac{\sigma_\xi(t)}{\phi_\theta(t)} d\tilde{B}_{\xi,t}^i. \\
d\Delta_t^i &= -a_\Delta(t)\Delta_t^i dt + \frac{\tilde{q}_t^i - \tilde{q}_t}{\sigma_D} d\tilde{B}_{D,t}^i + \frac{m_x(t)}{\sigma_\xi(t)}\left(\tilde{q}_t^i - \tilde{q}_t\right) d\tilde{B}_{\xi,t}^i
\end{aligned}$$

Conjecture the informed investor's value function takes the form of $V^i\left(t, W^i, \tilde{\theta}^i, \Delta^i\right) = -e^{-r\gamma W^i - h\left(t, \tilde{\theta}^i, \Delta^i\right)}$, where

$$h\left(t, \tilde{\theta}_t^i, \Delta_t^i\right) = h(t) + h_\theta(t)\tilde{\theta}_t^i + \frac{1}{2}h_{\theta\theta}(t)\tilde{\theta}_t^{i2} + h_\Delta(t)\Delta_t^i + \frac{1}{2}h_{\Delta\Delta}(t)\Delta_t^{i2} + h_{\theta\Delta}(t)\tilde{\theta}_t^i\Delta_t^i. \tag{100}$$

Using Ito's Lemma, the HJB equation is:

$$\begin{aligned}
\rho V^i = &-e^{-\gamma C^{ui}} + V_t^i + V_W^i\left[rW^i - C^i + \varepsilon\left(e_1(t) + e_\theta(t)\tilde{\theta}^i + e_\Delta(t)\Delta^i\right)\right] + \frac{1}{2}V_{WW}^i\varepsilon^2\sigma_P(t) + \varepsilon V_{W\theta}^i\sigma_{Q\theta}^i(t) \\
&+\varepsilon V_{W\Delta}^i\sigma_{Q\Delta}^i(t) + V_\theta^i a\left(\bar{\theta}^\kappa - \tilde{\theta}^i\right) - V_\Delta^i a_\Delta(t)\Delta^i + \frac{1}{2}V_{\theta\theta}^i\sigma_{\theta\theta}^i(t) + \frac{1}{2}V_{\Delta\Delta}^i\sigma_{\Delta\Delta}^i(t) + V_{\Delta\theta}^i\sigma_{\Delta\theta}^i(t),
\end{aligned} \tag{101}$$

where

$$\begin{aligned}
\sigma^i_{Q\theta}(t) &= \varrho_D(t)\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}^i(t)}{\sigma_D}+\varrho_\xi(t)\left[\phi_x(t)\nu^i(t)-1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}\\
\sigma^i_{Q\Delta}(t) &= \left[\frac{\varrho_D(t)}{\sigma_D}+\frac{\varrho_\xi(t)\,m_x(t)}{\sigma_\xi(t)}\right]\left(\tilde{q}^i_t-\tilde{q}_t\right)\\
\sigma^i_{\theta\theta}(t) &= \left(\frac{\phi_x(t)}{\phi_\theta(t)}\frac{\tilde{q}^i_t}{\sigma_D}\right)^2+\left(\left[\phi_x(t)\nu^i(t)-1\right]\frac{\sigma_\xi(t)}{\phi_\theta(t)}\right)^2\\
\sigma^i_{\Delta\Delta}(t) &= \left(\frac{1}{\sigma_D^2}+\frac{m_x^2(t)}{\sigma_\xi^2(t)}\right)\left(\tilde{q}^i_t-\tilde{q}_t\right)^2\\
\sigma^i_{\Delta\theta}(t) &= \frac{\tilde{q}^i_t-\tilde{q}_t}{\phi_\theta(t)}\left[\frac{\tilde{q}^i_t}{\sigma_D^2}+m_x(t)\left(\phi_x(t)\nu^i(t)-1\right)\right]
\end{aligned}$$

The FOCs are therefore

$$\begin{aligned}
C^i &= rW^i+\frac{1}{\gamma}\left[h\left(t,\tilde{\theta}^i,\Delta^i\right)-\ln r\right],\\
\varepsilon &= \frac{\left[\begin{array}{c} e_1(t)+e_\theta(t)\,\tilde{\theta}^i+e_\Delta(t)\,\Delta^i-\left(h_\theta(t)+h_{\theta\theta}(t)\,\tilde{\theta}^i+h_{\theta\Delta}(t)\,\Delta^i\right)\sigma^i_{Q\theta}(t)\\ -\left(h_\Delta(t)+h_{\Delta\Delta}(t)\,\Delta^i+h_{\theta\Delta}(t)\,\tilde{\theta}^i\right)\sigma^i_{Q\Delta}(t)\end{array}\right]}{r\gamma\sigma_P(t)}
\end{aligned}$$

so that the demand function can be written as

$$\varepsilon_t=\varepsilon_0(t)+\varepsilon_\theta(t)\,\tilde{\theta}^i_t+\varepsilon_\Delta(t)\,\Delta^i_t, \tag{102}$$

where

$$\varepsilon_0(t) = \frac{e_1(t)-\sigma^i_{Q\theta}(t)\,h_\theta(t)-\sigma^i_{Q\Delta}(t)\,h_\Delta(t)}{r\gamma\sigma_P(t)} \tag{103}$$

$$\varepsilon_\theta(t) = \frac{e_\theta(t)-\sigma^i_{Q\theta}(t)\,h_{\theta\theta}(t)-\sigma^i_{Q\Delta}(t)\,h_{\theta\Delta}(t)}{r\gamma\sigma_P(t)} \tag{104}$$

$$\varepsilon_\Delta(t) = \frac{e_\Delta(t)-\sigma^i_{Q\theta}(t)\,h_{\theta\Delta}(t)-\sigma^i_{Q\Delta}(t)\,h_{\Delta\Delta}(t)}{r\gamma\sigma_P(t)}. \tag{105}$$

Matching coefficients of the value function, and use $\varepsilon_0(t)$, $\varepsilon_\theta(t)$ and $\varepsilon_\Delta(t)$ to simplify, we have

$$\begin{aligned}
h'(t) &= r-\rho-r\ln r+rh(t)-\frac{1}{2}r^2\gamma^2\sigma_P(t)\varepsilon_0^2(t)+\frac{1}{2}\sigma^i_{\theta\theta}\left[h_\theta^2(t)-h_{\theta\theta}(t)\right]\\
&\quad+\frac{1}{2}\sigma^i_{\Delta\Delta}(t)\left[h_\Delta^2(t)-h_{\Delta\Delta}(t)\right]+\sigma^i_{\Delta\theta}(t)\left[h_\theta(t)h_\Delta(t)-h_{\theta\Delta}(t)\right]-a\bar{\theta}^\kappa h_\theta(t), \qquad (106)\\
h'_{\theta\theta}(t) &= rh_{\theta\theta}(t)-r^2\gamma^2\sigma_P(t)\varepsilon_\theta^2(t)+2ah_{\theta\theta}(t)+\sigma^i_{\theta\theta}h^2_{\theta\theta}(t)+\sigma^i_{\Delta\Delta}(t)h^2_{\theta\Delta}(t)+2\sigma^i_{\Delta\theta}(t)h_{\theta\theta}(t)h_{\theta\Delta}(t),\\
h'_{\Delta\Delta}(t) &= rh_{\Delta\Delta}(t)-r^2\gamma^2\sigma_P(t)\varepsilon_\Delta^2(t)+2a_\Delta(t)h_{\Delta\Delta}(t)+\sigma^i_{\theta\theta}h^2_{\theta\Delta}(t)+\sigma^i_{\Delta\Delta}(t)h^2_{\Delta\Delta}(t)\\
&\quad+2\sigma^i_{\Delta\theta}(t)h_{\theta\Delta}(t)h_{\Delta\Delta}(t), \qquad (107)\\
h'_{\theta\Delta}(t) &= rh_{\theta\Delta}(t)-r^2\gamma^2\sigma_P(t)\varepsilon_\theta(t)\varepsilon_\Delta(t)+ah_{\theta\Delta}(t)+a_\Delta(t)h_{\theta\Delta}(t)+\sigma^i_{\theta\theta}h_{\theta\theta}(t)h_{\theta\Delta}(t)\\
&\quad+\sigma^i_{\Delta\Delta}(t)h_{\Delta\Delta}(t)h_{\theta\Delta}(t)+\sigma^i_{\Delta\theta}(t)\left[h_{\theta\theta}(t)h_{\Delta\Delta}(t)+h^2_{\theta\Delta}(t)\right]; \qquad (108)\\
h'_\theta(t) &= rh_\theta(t)-r^2\gamma^2\sigma_P(t)\varepsilon_0(t)\varepsilon_\theta(t)+ah_\theta(t)+b_\Delta(t)h_{\theta\Delta}(t)+\sigma^i_{\theta\theta}h_\theta(t)h_{\theta\theta}(t)\\
&\quad+\sigma^i_{\Delta\Delta}(t)h_\Delta(t)h_{\theta\Delta}(t)+\sigma^i_{\Delta\theta}(t)\left[h_\theta(t)h_{\theta\Delta}(t)+h_{\theta\theta}(t)h_\Delta(t)\right]-a\bar{\theta}^\kappa h_{\theta\theta}, \qquad (109)\\
h'_\Delta(t) &= rh_\Delta(t)-r^2\gamma^2\sigma_P(t)\varepsilon_0(t)\varepsilon_\Delta(t)+a_\Delta(t)h_\Delta(t)+b_\Delta(t)h_{\Delta\Delta}(t)+\sigma^i_{\theta\theta}h_\theta(t)h_{\theta\Delta}(t)\\
&\quad+\sigma^i_{\Delta\Delta}(t)h_\Delta(t)h_{\Delta\Delta}(t)+\sigma^i_{\Delta\theta}(t)\left[h_\theta(t)h_{\Delta\Delta}(t)+h_{\theta\Delta}(t)h_\Delta(t)\right]-a\bar{\theta}^\kappa h_{\theta\Delta}. \qquad (110)
\end{aligned}$$

Market clearing conditions Using the market clearing conditions, the ODEs for $\phi_0(t)$, $\phi_\theta(t)$ and $\phi_\Delta(t)$ can be characterized as follows

$$\begin{aligned}
\phi'(t) &= r\phi(t)+a\bar{\theta}\phi_\theta-b\bar{x}\bar{\phi}_x+\omega\left[\varrho_\theta\left(\frac{K_t\hat{q}\left(\phi_\Delta-\bar{\phi}_x\right)}{\sigma_\theta\phi_\theta\sigma_s^2}-g_\theta\sigma_\theta-\eta\right)+\frac{K_t\varrho_D}{\sigma_D}\right]\\
&\quad+f_\theta\sigma_{Q\theta}(\omega-\omega_t)+(1-\omega)g_\Delta\sigma_{Q\Delta}-b_\Delta\phi_\Delta+g_\theta\sigma_\theta\varrho_\theta+\omega_t\left(h_\Delta\sigma^i_{Q\Delta}+h_\theta\sigma^i_{Q\theta}\right) \qquad (111)\\
\phi'_\theta(t) &= (a+r)\phi_\theta-r\gamma\sigma_P+f_{\theta\theta}\sigma_{Q\theta}(\omega_t-\omega)-(1-\omega)\left(g_{\theta\theta}\sigma_\theta\varrho_\theta+g_{\theta\Delta}\sigma_{Q\Delta}\right)-\omega_t\left(h_{\theta\Delta}\sigma^i_{Q\Delta}+h_{\theta\theta}\sigma^i_{Q\theta}\right)\\
\phi'_\Delta(t) &= \frac{\left[\begin{array}{c}\phi_\theta\Big[\phi_\Delta\left((a-a_\Delta)(\omega-\omega_t)+a_\Delta+r\right)+(a+r)(\omega_t-\omega)\bar{\phi}_x\\
+(\omega-1)g_{\theta\Delta}\sigma_\theta\varrho_\theta-\omega_t\left(h_{\Delta\Delta}\sigma^i_{Q\Delta}+h_{\theta\Delta}\sigma^i_{Q\theta}\right)\Big]\\
-f_{\theta\theta}\sigma_{Q\theta}(\omega-\omega_t)\left(\phi_\Delta-\bar{\phi}_x\right)+(\omega-1)g_{\Delta\Delta}\phi_\theta\sigma_{Q\Delta}-\phi'_\theta(t)(\omega-\omega_t)\left(\phi_\Delta-\bar{\phi}_x\right)\end{array}\right]}{\phi_\theta(1-\omega+\omega_t)} \qquad (112)
\end{aligned}$$

Portfolio demand for the informed: boundary First, we derive boundary conditions for the informed investor's value function coefficients. The informed investor's optimization problem at the boundary can be written as

$$\begin{aligned}
-e^{-r\gamma\hat{W}^- - g(T,\theta_T,\Delta_T)} &= \max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-r\gamma\hat{W}^+ - g(0,\theta_T,0)}\right]\right\}\\
&= e^{-r\gamma\hat{W}^-}\max_{\alpha_T}\left\{-\hat{\mathbb{E}}_T\left[e^{-r\gamma\alpha_T\left(P_T^+-P_T^-\right)-g(0,\theta_T,0)}\right]\right\}, \qquad (113)
\end{aligned}$$

where $x_T\sim\mathcal{N}(\hat{x}_T,\hat{q}_T)$. Solving the exponent part within the expectation operator yields:

$$-r\gamma\alpha_T\left(P_T^+-P_T^-\right)-g(0,\theta_T,0)=-\Phi_0-\Phi_1x_T,$$

where $\Phi_0 = r\gamma\alpha_T \left\{ [\phi(0) - \phi(T)] - [\phi_\theta(0) - \phi_\theta(T)]\theta_T - \bar{\phi}_x \hat{x}_T + \phi_\Delta(t)\Delta_T \right\} + g(0) + g_\theta(0)\theta_T + \frac{1}{2} g_{\theta\theta}(0)\theta_T^2$ and $\Phi_1 = r\gamma\alpha_T\bar{\phi}_x$. Then

$$\hat{\mathbb{E}}_T \left[e^{-r\gamma\alpha_T\left(P_T^+ - P_T^-\right) - g(0,\theta_T,0)} \right] = e^{-\Phi_0 - \left(\Phi_1 \hat{x}_T - \frac{1}{2}\Phi_1^2 \hat{q}_T\right)} = e^{Term^i},$$

where

$$\begin{aligned} Term^i &= -r\gamma\alpha_T \left\{ [\phi(0) - \phi(T)] - [\phi_\theta(0) - \phi_\theta(T)]\theta_T + \phi_\Delta(t)\Delta_T \right\} \\ &\quad - g(0) - g_\theta(0)\theta_T - \frac{1}{2} g_{\theta\theta}(0)\theta_T^2 + \frac{1}{2} r^2\gamma^2\alpha_T^2\bar{\phi}_x^2\hat{q}_T. \end{aligned}$$

Optimization implies

$$\alpha_T = \alpha_0(T) + \alpha_\theta(T)\theta_T + \alpha_\Delta(T)\Delta_T, \tag{114}$$

where

$$\alpha_0(T) = \frac{\phi(0) - \phi(T)}{r\gamma\bar{\phi}_x^2\hat{q}_T},\ \alpha_\theta(T) = \frac{\phi_\theta(T) - \phi_\theta(0)}{r\gamma\bar{\phi}_x^2\hat{q}_T},\ \text{and } \alpha_\Delta(T) = \frac{\phi_\Delta(T)}{r\gamma\bar{\phi}_x^2\hat{q}_T}. \tag{115}$$

Therefore, $g(T, \theta_T, \Delta_T) = -Term^i$ gives

$$\begin{aligned} & g(T) + g_\theta(T)\theta_T + \frac{1}{2} g_{\theta\theta}(T)\theta_T^2 + g_\Delta(T)\Delta_T + \frac{1}{2} g_{\Delta\Delta}(T)\Delta_T^2 + g_{\theta\Delta}(T)\theta_T\Delta_T \\ = & \frac{[\phi(0) - \phi(T) + \phi_\Delta(T)\Delta_T + (\phi_\theta(T) - \phi_\theta(0))\theta_T]^2}{\hat{q}_T\bar{\phi}_x^2} + \frac{1}{2} g_{\theta\theta}(0)\theta_T^2 + g_\theta(0)\theta_T + g(0). \end{aligned}$$

Matching the coefficients yields the boundary conditions summarized as follows

$$\begin{aligned} g(T) - g(0) &= \frac{[\phi(T) - \phi(0)]^2}{2\hat{q}_T\bar{\phi}_x^2},\ g_{\theta\theta}(T) - g_{\theta\theta}(0) = \frac{[\phi_\theta(T) - \phi_\theta(0)]^2}{\hat{q}_T\bar{\phi}_x^2}, \\ g_\theta(T) - g_\theta(0) &= \frac{-[\phi(T) - \phi(0)][\phi_\theta(T) - \phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2},\ g_{\Delta\Delta}(T) = \frac{\phi_\Delta^2(T)}{\hat{q}_T\bar{\phi}_x^2}, \\ g_\Delta(T) &= -\frac{[\phi(T) - \phi(0)]\phi_\Delta(t)}{\hat{q}_T\bar{\phi}_x^2},\ g_{\theta\Delta}(T) = \frac{\phi_\Delta(t)[\phi_\theta(T) - \phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2}. \end{aligned} \tag{116}$$

Portfolio demand for the uninformed who never acquire information: boundary Second, we derive boundary conditions for the uninformed investor's value function coefficients. The uninformed investor's optimization problem at the boundary is

$$\begin{aligned} -e^{-r\gamma\tilde{W}^- - f\left(T,\tilde{\theta}_T\right)} &= \max_{\beta_T} \left\{ \tilde{\mathbb{E}}_T \left[-e^{-r\gamma\tilde{W}_T^+ - f(0,\theta_T)} \right] \right\} \\ &= e^{-r\tilde{W}^-} \max_{\beta_T} \tilde{\mathbb{E}}_T \left[-e^{-r\gamma\beta_T\left(P_T^+ - P_T^-\right) - f(0,\theta_T)} \right], \end{aligned} \tag{117}$$

where $\begin{pmatrix} x_T \\ \theta_T \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \tilde{x}_T - K_T \\ \tilde{\theta}_T \end{pmatrix}, \begin{pmatrix} \hat{q}_T + \tilde{q}_T & \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \\ \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T & \frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T \end{pmatrix}\right)$, in which we use the variance-covariance relationship derived before: $\tilde{\mathbb{E}}(x_t) = \tilde{x}_t - K(t)$, $\tilde{Var}(x_t) = \hat{q}_t + \tilde{q}_t$, and $\tilde{Cov}(x_t, \theta_t) = \frac{\phi_{x,t}}{\phi_{\theta,t}}\tilde{q}_t$. Solving the exponent part within the expectation operator gives:

$$\begin{aligned} & -r\gamma\beta_T\left\{[\phi(0) - \phi(T)] - \left[\phi_\theta(0)\theta_T - \phi_\theta(T)\tilde{\theta}_T^\kappa\right] + \bar{\phi}_x\left(x_T - \tilde{x}_T^\kappa\right)\right\} - f(0) - f_\theta(0)\theta_T - \frac{1}{2}f_{\theta\theta}(0)\theta_T^2 \\ = \; & \Psi_0 + \Psi_1 x_T + \Psi_2\theta_T - \frac{1}{2}f_{\theta\theta}(0)\theta_T^2, \end{aligned}$$

where $\Psi_0 = r\gamma\beta_T\left[\phi(T) - \phi(0) + \bar{\phi}_x\tilde{x}_T^\kappa - \phi_\theta(T)\tilde{\theta}_T^\kappa\right] - f(0)$, $\Psi_1 = -r\gamma\beta_T\bar{\phi}_x$, and $\Psi_2 = r\gamma\beta_T\phi_\theta(0) - f_\theta(0)$. Given $\phi_\theta(t) > 0$, log multivariate normal distribution implies

$$\tilde{\mathbb{E}}_T\left[e^{-r\beta_T\left(P_T^+ - P_T^-\right) - f(0,\theta_T)}\right] = \frac{\phi_{\theta,T}}{\sqrt{\phi_{\theta,T}^2 + f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T}} e^{\Psi_0 + \frac{\bar{\Psi}}{2\left(\phi_{\theta,T}^2 + f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T\right)}} = e^{Term^u}, \quad (118)$$

where $Term^u = \Psi_0 + \frac{\bar{\Psi}}{2\left(\phi_{\theta,T}^2 + f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T\right)} + \Psi_3$, $\Psi_3 = \ln\phi_\theta(T) - \frac{1}{2}\ln\left(\phi_\theta^2(T) + f_{\theta\theta,0}\phi_x^2(T)\tilde{q}_T\right)$, and

$$\begin{aligned} \bar{\Psi} = \; & -2\Psi_1 K_T\left(f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T + \phi_{\theta,T}\right) + \phi_{\theta,T}\left(-f_{\theta\theta,0}\tilde{\theta}_T^2 + \Psi_1\left(\Psi_1\left(\tilde{q}_T + \hat{q}_T\right) + 2\tilde{x}_T\right) + 2\Psi_2\tilde{\theta}_T\right) \\ & +\phi_{x,T}^2\tilde{q}_T\left(\Psi_1 f_{\theta\theta,0}\left(2\tilde{x}_T + \Psi_1\hat{q}_T\right) + \Psi_2^2\right) + 2\Psi_1\phi_{\theta,T}\phi_{x,T}\tilde{q}_T\left(\Psi_2 - f_{\theta\theta,0}\tilde{\theta}_T\right) \\ = \; & r^2\gamma^2\beta_T^2\left[\phi_{x,T}^2\tilde{q}_T\left(f_{\theta\theta,0}\hat{q}_T\bar{\phi}_x^2 + \phi_{\theta,0}^2\right) + \phi_{\theta,T}^2\bar{\phi}_x^2\left(\tilde{q}_T + \hat{q}_T\right) + 2\phi_{\theta,0}\phi_{\theta,T}\phi_{x,T}\bar{\phi}_x\tilde{q}_T\right] \\ & +2r\gamma\beta_T\Big[-K_T\bar{\phi}_x\left(f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T + \phi_{\theta,T}^2\right) + \phi_{\theta,T}\tilde{\theta}_T\left(\phi_{\theta,0}\phi_{\theta,T} - f_{\theta\theta,0}\phi_{x,T}\bar{\phi}_x\tilde{q}_T\right) \\ & -f_{\theta,0}\phi_{x,T}\tilde{q}_T\left(\phi_{\theta,T}\bar{\phi}_x + \phi_{\theta,0}\phi_{x,T}\right) + \bar{\phi}_x\tilde{x}_T\left(f_{\theta\theta,0}\phi_{x,T}^2\tilde{q}_T + \phi_{\theta,T}^2\right)\Big] \\ & +f_{\theta,0}^2\phi_{x,T}^2\tilde{q}_T - 2f_{\theta,0}\phi_{\theta,T}^2\tilde{\theta}_T - f_{\theta\theta,0}\phi_{\theta,T}^2\tilde{\theta}_T^2 \end{aligned}$$

The FOC with respect to β_T gives

$$\beta_T = \beta_0(T) + \beta_\theta(T)\tilde{\theta}_T^\kappa = \beta_0(T) + \beta_\theta(T)\tilde{\theta}_T^\kappa - \beta_\theta(T)\frac{\phi_{x,T}}{\phi_{\theta,T}}\Delta_T, \quad (119)$$

where

$$\beta_0(T) = \frac{1}{\beta_q}\left\{\phi_{x,T}\tilde{q}_T\left[f_{\theta,0}\left(\phi_{\theta,0}\phi_{x,T} - \phi_{\theta,T}\bar{\phi}_x\right) - f_{\theta\theta,0}\phi_{x,T}\left(K_T\bar{\phi}_x + \phi_T - \phi_0\right)\right] - \phi_{\theta,T}^2\left(K_T\bar{\phi}_x + \phi_T - \phi_0\right)\right\} \quad (120)$$

$$\beta_\theta(T) = \frac{\phi_{\theta,T}}{\beta_q}\left[f_{\theta\theta,0}\phi_{x,T}\tilde{q}_T\left(\phi_{x,T} - \bar{\phi}_x\right) + \phi_{\theta,T}\left(\phi_{\theta,T} - \phi_{\theta,0}\right)\right] \quad (121)$$

Define $\beta_q = r\gamma\tilde{q}_T\left[f_{\theta\theta,0}\hat{q}_T\phi_{x,T}^2\bar{\phi}_x^2 + \left(\phi_{\theta,0}\phi_{x,T} - \phi_{\theta,T}\bar{\phi}_x\right)^2\right] + r\gamma\hat{q}_T\phi_{\theta,T}^2\bar{\phi}_x^2$. Substituting this into $f\left(T, \tilde{\theta}_T^\kappa\right) = -Term^u$ and matching the coefficients yields the boundary conditions summarized as follows

$$
\begin{aligned}
f(T)-f(0) &= \frac{r\gamma}{2\beta_q}\Big\{\tilde{q}_T\Big[f_{\theta\theta,0}\phi_{x,T}^2\left(\left(K_T\bar{\phi}_x+\phi_T-\phi_0\right)^2-2\Psi_3\hat{q}_T\bar{\phi}_x^2\right)-f_{\theta,0}^2\hat{q}_T\phi_{x,T}^2\bar{\phi}_x^2 \\
&\quad -2\left(\phi_{\theta,0}\phi_{x,T}-\phi_{\theta,T}\bar{\phi}_x\right)\left(f_{\theta,0}\phi_{x,T}\left(K_T\bar{\phi}_x+\phi_T-\phi_0\right)+\Psi_3\left(\phi_{\theta,0}\phi_{x,T}-\phi_{\theta,T}\bar{\phi}_x\right)\right)\Big] \\
&\quad +\phi_{\theta,T}^2\left[\left(K_T\bar{\phi}_x+\phi_T-\phi_0\right)^2-2\Psi_3\hat{q}_T\bar{\phi}_x^2\right]\Big\} \qquad (122) \\
f_\theta(T) &= \frac{r\gamma\phi_{\theta,T}}{\beta_q}\Big\{\tilde{q}_T\left(\bar{\phi}_x-\phi_{x,T}\right)\left[f_{\theta\theta,0}\phi_{x,T}\left(K_T\bar{\phi}_x+\phi_T-\phi_0\right)+f_{\theta,0}\left(\phi_{\theta,T}\bar{\phi}_x-\phi_{\theta,0}\phi_{x,T}\right)\right] \\
&\quad +\phi_{\theta,T}\left[f_{\theta,0}\hat{q}_T\bar{\phi}_x^2-\left(\phi_{\theta,T}-\phi_{\theta,0}\right)\left(K_T\bar{\phi}_x+\phi_T-\phi_0\right)\right]\Big\} \qquad (123) \\
f_{\theta\theta}(T) &= \frac{r\gamma\phi_{\theta,T}^2}{\beta_q}\left[f_{\theta\theta,0}\left(\tilde{q}_T\left(\phi_{x,T}-\bar{\phi}_x\right)^2+\hat{q}_T\bar{\phi}_x^2\right)+\left(\phi_{\theta,0}-\phi_{\theta,T}\right)^2\right] \qquad (124)
\end{aligned}
$$

Portfolio demand for the uninformed who acquire information: boundary Now we turn to derive boundary conditions for the uninformed investor who acquire information.The optimization problem at the boundary is

$$
\begin{aligned}
-e^{-r\gamma W^{i-}-h\left(T,\tilde{\theta}_T^i,\Delta_T^i\right)} &= \max_{\varepsilon_T}\left\{-\tilde{\mathbb{E}}_T^i\left[e^{-r\gamma W^{i+}-h\left(0,\tilde{\theta}_T^i,0\right)}\right]\right\} \\
&= e^{-r\gamma W^{i-}}\max_{\varepsilon_T}\left\{-\tilde{\mathbb{E}}_T^i\left[e^{-r\gamma\varepsilon_T\left(P_T^+-P_T^-\right)-h\left(0,\tilde{\theta}_T^i,0\right)}\right]\right\}, \qquad (125)
\end{aligned}
$$

where $\begin{pmatrix} x_T \\ \theta_T \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \hat{x}_T-K_T \\ \theta_T \end{pmatrix}, \begin{pmatrix} \hat{q}_T & 0 \\ 0 & 0 \end{pmatrix}\right)$. Solving the exponent part within the expectation operator yields:

$$
-r\gamma\varepsilon_T\left(P_T^+-P_T^-\right)-h\left(0,\tilde{\theta}_T^i,0\right)=-\Psi_0^i-\Psi_1^i x_T,
$$

where $\Psi_0^i = r\gamma\varepsilon_T\left\{\left[\phi(0)-\phi(T)\right]-\left[\phi_\theta(0)-\phi_\theta(T)\right]\tilde{\theta}_T^i-\bar{\phi}_x\tilde{x}_T^i+\phi_\Delta(t)\Delta_T^i\right\}+h(0)+h_\theta(0)\tilde{\theta}_T^i+\frac{1}{2}h_{\theta\theta}(0)\tilde{\theta}_T^{i2}$ and $\Psi_1^i = r\gamma\varepsilon_T\bar{\phi}_x$. Then

$$
\tilde{\mathbb{E}}_T^i\left[e^{-r\gamma\varepsilon_T\left(P_T^+-P_T^-\right)-h\left(0,\tilde{\theta}_T^i,0\right)}\right] = e^{-\Psi_0^i-\left[\Psi_1^i\left(\hat{x}_T-K_T\right)-\frac{1}{2}\Psi_1^{i2}\hat{q}_T\right]}=e^{Term^{ui}},
$$

where

$$
\begin{aligned}
Term^{ui} &= -r\gamma\varepsilon_T\left\{\left[\phi(0)-\phi(T)\right]-\left[\phi_\theta(0)-\phi_\theta(T)\right]\tilde{\theta}_T^i+\phi_\Delta(t)\Delta_T^i-\bar{\phi}_x K_T\right\} \\
&\quad -h(0)-h_\theta(0)\theta_T-\frac{1}{2}h_{\theta\theta}(0)\theta_T^2+\frac{1}{2}r^2\gamma^2\varepsilon_T^2\bar{\phi}_x^2\hat{q}_T.
\end{aligned}
$$

Optimization implies

$$
\varepsilon_T=\varepsilon_0(T)+\varepsilon_\theta(T)\tilde{\theta}_T^i+\varepsilon_\Delta(T)\Delta_T^i, \qquad (126)
$$

where

$$\varepsilon_0(T)=\frac{\phi(0)-\phi(T)-\bar{\phi}_x K_T}{r\gamma\bar{\phi}_x^2\hat{q}_T},\ \varepsilon_\theta(T)\ =\ \frac{\phi_\theta(T)-\phi_\theta(0)}{r\gamma\bar{\phi}_x^2\hat{q}_T},\ \text{and}\ \varepsilon_\Delta(T)=\frac{\phi_\Delta(T)}{r\gamma\bar{\phi}_x^2\hat{q}_T}. \quad (127)$$

Therefore, $h\left(T,\tilde{\theta}_T^i,\Delta_T^i\right)=-Term^{ui}$ gives

$$\begin{aligned}
&h(T)+h_\theta(T)\tilde{\theta}_T^i+\frac{1}{2}h_{\theta\theta}(T)\tilde{\theta}_T^{i2}+h_\Delta(T)\Delta_T^i+\frac{1}{2}h_{\Delta\Delta}(T)\Delta_T^{i2}+h_{\theta\Delta}(T)\tilde{\theta}_T^i\Delta_T^i\\
=\ &\frac{\left[\phi(0)-\phi(T)-\bar{\phi}_x K_T+\phi_\Delta(T)\Delta_T^i+(\phi_\theta(T)-\phi_\theta(0))\tilde{\theta}_T^i\right]^2}{\hat{q}_T\bar{\phi}_x^2}+\frac{1}{2}h_{\theta\theta}(0)\tilde{\theta}_T^{i2}+h_\theta(0)\tilde{\theta}_T^i+h(0).
\end{aligned}$$

Matching the coefficients yields the boundary conditions summarized as follows

$$\begin{aligned}
h(T)-h(0) &= \frac{\left[\phi(T)-\phi(0)+\bar{\phi}_x K_T\right]^2}{2\hat{q}_T\bar{\phi}_x^2},\ h_{\theta\theta}(T)-h_{\theta\theta}(0)=\frac{[\phi_\theta(T)-\phi_\theta(0)]^2}{\hat{q}_T\bar{\phi}_x^2},\\
h_\theta(T)-h_\theta(0) &= \frac{-\left[\phi(T)-\phi(0)+\bar{\phi}_x K_T\right][\phi_\theta(T)-\phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2},\ h_{\Delta\Delta}(T)=\frac{\phi_\Delta^2(T)}{\hat{q}_T\bar{\phi}_x^2},\\
h_\Delta(T) &= -\frac{\left[\phi(T)-\phi(0)+\bar{\phi}_x K_T\right]\phi_\Delta(t)}{\hat{q}_T\bar{\phi}_x^2},\ h_{\theta\Delta}(T)=\frac{\phi_\Delta(t)[\phi_\theta(T)-\phi_\theta(0)]}{\hat{q}_T\bar{\phi}_x^2}. \quad (128)
\end{aligned}$$

Market Clearing Note that market clearing requires: $(1-\omega)\alpha_T+(\omega-\Omega_T)\beta_T+\Omega_T\varepsilon_T=\theta_T$. This implies

$$(1-\omega)\alpha_0(T)+(\omega-\Omega_T)\beta_0(T)+\Omega_T\varepsilon_0(T)\ =\ 0, \quad (129)$$

$$(1-\omega)\alpha_\theta(T)+(\omega-\omega_T)\beta_\theta(T)+\Omega_T\varepsilon_\theta(T)\ =\ 1, \quad (130)$$

$$(1-\omega)\alpha_\Delta(T)-(\omega-\Omega_T)\beta_\theta(T)\frac{\phi_{x,T}}{\phi_{\theta,T}}+\Omega_T\varepsilon_\Delta(t)\ =\ 0. \quad (131)$$

Substituting expressions in equations (115) and (121) eventually pins down the boundary conditions for the pricing function coefficients.

Let's solve the above equilibrium in steps. First, equations (130) and (131) gives

$$\phi_\Delta(T)=\frac{r\gamma\hat{q}_T\bar{\phi}_x^3-(1-\omega+\omega_T)\bar{\phi}_x(\phi_{\theta,T}-\phi_{\theta,0})}{r\gamma\hat{q}_T\bar{\phi}_x^2+\phi_{\theta,0}(1-\omega+\omega_T)}, \quad (132)$$

Further put back to $f_{\theta\theta}(0)$ gives

$$f_{\theta\theta}(0)=\frac{r\gamma\hat{q}_T\bar{\phi}_x^2\left(r\gamma-\frac{\phi_{\theta,T}(\omega-\omega_T)}{\phi_{\Delta,T}\tilde{q}_T\left(\bar{\phi}_x-\phi_{\Delta,T}\right)}\right)+\frac{\phi_{\theta,T}(1-\omega+\omega_T)}{\tilde{q}_T\left(\bar{\phi}_x-\phi_{\Delta,T}\right)^2}}{\omega-\omega_T-1} \quad (133)$$

Finally, (129) gives

$$\phi(T)-\phi(0) = \frac{\phi_{\Delta,T}\tilde{q}_T\left(\bar{\phi}_x-\phi_{\Delta,T}\right)\left[f_{\theta,0}\left(1-\omega+\omega_T\right)-r\gamma\left(1-\omega\right)K_T\bar{\phi}_x\right]}{\phi_{\theta,T}\left(\omega-\omega_T-1\right)}-\omega K_T\bar{\phi}_x \quad (134)$$

6.3 Implied volatility and trading volume

Implied variance We would like to compute $Var_0[P_t-P_0]=Var_0[P_t]$. First consider the case in which $t<T$. We solve the three components separately. First, we solve for $\tilde{x}_t$. Using the law of motion (13), we have:

$$\tilde{x}_t=e^{-bt}\int_0^t e^{bs}\left[b\bar{x}-\hat{q}(s)\left(\frac{1}{\sigma_D^2}+\frac{1}{\sigma_s^2}\right)K_s\right]ds+e^{-bt}\int_0^t e^{bs}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}d\tilde{B}_{D,s}+e^{-bt}\int_0^t e^{bs}\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}.$$

Therefore, with an abuse of notation, we use $DF[X]$ to denote the diffusion part of X, we have:

$$DF\left[\bar{\phi}_x\tilde{x}_t\right]=\bar{\phi}_x\int_0^t e^{b(s-t)}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}d\tilde{B}_{D,s}+\bar{\phi}_x\int_0^t e^{b(s-t)}\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}. \quad (135)$$

Next, we compute D_t.

$$dD_t=\left(\tilde{x}_t-K_t-D_t\right)dt+\sigma_D d\tilde{B}_{D,t},$$

and therefore

$$D_t=e^{-t}\int_0^t e^s\left(\tilde{x}_s^\kappa-K_s\right)ds+e^{-t}\int_0^t e^s\sigma_D d\tilde{B}_{D,s}.$$

The term

$$\int_0^t e^u\tilde{x}_u du=\int_0^t e^{(1-b)u}\int_0^u\left\{e^{bs}\left[b\bar{x}-\hat{q}(s)\left(\frac{1}{\sigma_D^2}+\frac{1}{\sigma_s^2}\right)K(s)\right]ds+\int_0^u e^{bs}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}d\tilde{B}_{D,s}\right.$$
$$\left.+\int_0^u e^{bs}\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}\right\}du.$$

We focus on the diffusion part:

$$\begin{aligned}\int_0^t\int_0^u e^{bs+(1-b)u}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}d\tilde{B}_{D,s}du &= \int_0^t\int_s^t e^{bs+(1-b)u}\frac{\hat{q}_s+\tilde{q}_s}{\sigma_D}dud\tilde{B}_{D,s}\\ &= \frac{1}{(1-b)\sigma_D}\int_0^t\left[e^{(1-b)t+bs}-e^s\right]\left(\hat{q}_s+\tilde{q}_s\right)d\tilde{B}_{D,s}.\end{aligned}$$

Similarly, we have:

$$\int_0^t\int_0^u e^{bs+(1-b)u}\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}du=\frac{1}{(1-b)}\int_0^t\left[e^{(1-b)t+bs}-e^s\right]\nu(s)\sigma_\xi(s)d\tilde{B}_{\xi,s}.$$

We have:

$$D_t=-e^{-t}\int_0^t K_s ds+e^{-t}\left[\int_0^t e^u\tilde{x}_u^\kappa du+\int_0^t e^s\sigma_D d\tilde{B}_{D,s}\right]$$

The diffusion part is

$$DF\left[\phi_D D_t\right] = \phi_D \int_0^t \left[\left(e^{b(s-t)} - e^{s-t}\right)\frac{\hat{q}_s + \tilde{q}_s}{(1-b)\,\sigma_D} + e^{s-t}\sigma_D\right] d\tilde{B}_{D,s} + \phi_D \int_0^t \left[e^{b(s-t)} - e^{s-t}\right]\frac{\nu(s)\,\sigma_\xi(s)}{1-b} d\tilde{B}_{\xi,s} \tag{136}$$

Finally, we deal with $\tilde{\theta}_t$:

$$DF\left[-\phi_\theta(t)\,\tilde{\theta}_t\right] = -\phi_\theta(t)\left\{\int_0^t e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_D} d\tilde{B}_{D,s} + \int_0^t e^{a(s-t)}\left[\phi_x(s)\,\nu(s) - 1\right]\frac{\sigma_\xi(s)}{\phi_\theta(s)} d\tilde{B}_{\xi,s}\right\}. \tag{137}$$

Summing up (135), (136), and (137), we can represent price in the form of

$$DF\left[P_t\right] = \int_0^t Term_D(s)\, d\tilde{B}_{D,s} + \int_0^t Term_\xi(s)\, d\tilde{B}_{\xi,s}, \tag{138}$$

where

$$\begin{aligned} Term_D(s) &= \phi_D\left[\left(e^{b(s-t)} - e^{s-t}\right)\frac{\hat{q}_s + \tilde{q}_s}{(1-b)\,\sigma_D} + e^{s-t}\sigma_D\right] - \phi_\theta(t)\, e^{a(s-t)}\frac{\phi_x(s)}{\phi_\theta(s)}\frac{\tilde{q}(s)}{\sigma_D} + \bar{\phi}_x e^{b(s-t)}\frac{\hat{q}_s + \tilde{q}_s}{\sigma_D} && (139)\\ Term_\xi(s) &= \phi_D\left[e^{b(s-t)} - e^{s-t}\right]\frac{\nu(s)\,\sigma_\xi(s)}{1-b} - \phi_\theta(t)\, e^{a(s-t)}\left[\phi_x(s)\,\nu(s) - 1\right]\frac{\sigma_\xi(s)}{\phi_\theta(s)} + \bar{\phi}_x e^{b(s-t)}\nu(s)\,\sigma_\xi(s) && (140) \end{aligned}$$

and compute the variance as:

$$Var_0\left[P_t\right] = \int_0^t Term_D^2(s)\, ds + \int_0^t Term_\xi^2(s)\, ds. \tag{141}$$

Next, consider the general case where we need to compute $Var_t\left[P_{t+\tau}\right]$. If $t+\tau < T$, that is, if we compute implied variance within an announcement cycle, we use the above formula. If $t+\tau > T$. We first compute $Var_t\left[P_{T-}\right]$ using the above formula. It is also easy to compute $Var_{T^-}\left[P_{T^+} - P_{T^-}\right]$. We can then compute $Var_{T^+}\left[P_{t+\tau}\right]$. The reason we can just add up variance is because these different components are independent.

$$Var_t\left[P_{t+\tau}\right] = \int_t^{t+\tau} Term_D^2(s)\, ds + \int_t^{t+\tau} Term_\xi^2(s)\, ds. \tag{142}$$

$$\begin{aligned} P_T^+ - P_T^- &= \bar{\phi}_x\left(x_T - \tilde{x}_T^\kappa\right) - \phi_\theta(0)\left[\theta_T - \tilde{\theta}_T^\kappa\right] - \left[\phi_\theta(0) - \phi_\theta(T)\right]\tilde{\theta}_T^\kappa, \\ &= \bar{\phi}_x x_T - \phi_\theta(0)\,\theta_T - \bar{\phi}_x\tilde{x}_T^\kappa + \phi_\theta(T)\,\tilde{\theta}_T^\kappa, \end{aligned}$$

where $\begin{pmatrix} x_T \\ \theta_T \end{pmatrix} \sim \mathcal{N}\left(\left(\begin{array}{c} \tilde{x}_T^{\kappa} - K_T \\ \tilde{\theta}_T^{\kappa} \end{array}\right), \left(\begin{array}{cc} \hat{q}_T + \tilde{q}_T & \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \\ \frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T & \frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T \end{array}\right)\right)$,

$$Var_{T^-}\left[P_{T^+} - P_{T^-}\right] = \bar{\phi}_x^2\left(\hat{q}_T + \tilde{q}_T\right) + \phi_\theta^2(0)\frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T - 2\bar{\phi}_x\phi_\theta(0)\frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \tag{143}$$

Therefore, the total variance is obtained by

$$\begin{aligned} & Var_t\left[P_{T^-}\right] + Var_{T^-}\left[P_{T^+} - P_{T^-}\right] + Var_{T^+}\left[P_{t+\tau}\right] \\ = & \int_t^{T^-} Term_D^2(s)\,ds + \int_t^{T^-} Term_\xi^2(s)\,ds + \int_{T^+}^{t+\tau} Term_D^2(s)\,ds + \int_{T^+}^{t+\tau} Term_\xi^2(s)\,ds \\ & + \bar{\phi}_x^2\left(\hat{q}_T + \tilde{q}_T\right) + \phi_\theta^2(0)\frac{\phi_x^2(T)}{\phi_\theta^2(T)}\tilde{q}_T - 2\bar{\phi}_x\phi_\theta(0)\frac{\phi_x(T)}{\phi_\theta(T)}\tilde{q}_T \end{aligned} \tag{144}$$

Trading volume Define the trading volume from t to $t+\delta$ as the turnover rate:

$$M_{t,t+\delta} = \frac{1}{2}\left[(1-\omega)\left|\alpha_{t+\delta} - \alpha_t\right| + (\omega - \Omega_t)\left|\beta_{t+\delta} - \beta_t\right| + \Omega_t\left|\varepsilon_{t+\delta} - \varepsilon_t\right|\right] \tag{145}$$

On announcement days, the trading volume can be calculated as

$$M_T = \frac{1}{2}\left[(1-\omega)\left|\alpha_T^+ - \alpha_T^-\right| + (\omega - \Omega_T)\left|\beta_T^+ - \beta_T^-\right| + \Omega_T\left|\varepsilon_T^+ - \varepsilon_T^-\right|\right] \tag{146}$$

6.4 Policy functions and simulation

First, the calibrated parameters are:

Table 1: Parameters

Para.	Value	Description	Para.	Value	Description
r	0.03	risk-free rate	σ_x	0.55	volatility of hidden state
ρ	0.05	time discount factor	σ_θ	0.5	volatility of total equity supply
$\bar{x}$	15	mean level of dividend flow	κ	3.5	ambiguity aversion
b	0.2	persistence of hidden state	γ	1	risk aversion
a	0.01	persistence of total equity supply	$\bar{\theta}$	0	unconditional mean of aggregate supply
σ_d	1	dividend flow volatility	ω	0.99	fraction of uninformed investor
σ_s	0.02	inverse of signal precision			

This table displays annualized parameter values used in the simulations.

Second, to understand the announcement premium, it is convenient define the change of variable:

$$\hat{\Delta}_t = \Delta_t + \mathbb{E}\left[\Delta_t\right], \tag{147}$$

where $\mathbb{E}[\Delta_t] \equiv G(t)$ is defined as $G(t) = e^{-\int_0^t a_\Delta(u)du} \int_0^t e^{\int_0^s a_\Delta(u)du} b_\Delta(s)\, ds$. Note that

$$dG(t) = [-a_\Delta(t) \cdot G(t) + b_\Delta(t)]\, dt \tag{148}$$

Therefore, if we use the new state variable, the pricing equation can be written as

$$P_t = \hat{\phi}(t) + \phi_D D_t - \phi_\theta(t)\,\theta_t + \bar{\phi}_x \hat{x}_t - \phi_\Delta(t)\,\hat{\Delta}_t, \tag{149}$$

where

$$\hat{\Delta}_t = \Delta_t + G(t)\,, \quad \hat{\phi}(t) = \phi(t) + \phi_\Delta(t)\, G(t)\,. \tag{150}$$

With the above change of variable, the equilibrium price is written

$$P_t = \hat{\phi}(t) + \phi_D D_t - \phi_\theta(t)\,\theta_t + \bar{\phi}_x \hat{x}_t - \phi_\Delta(t)\,\hat{\Delta}_t \tag{151}$$

Before announcements, $P_T^- = \hat{\phi}(T) + \phi_D D_T - \phi_\theta(T)\,\theta_T + \bar{\phi}_x \hat{x}_T - \phi_\Delta(T)\,\hat{\Delta}_T$, and after announcement, at T^+, $P_T^+ = \hat{\phi}(0) + \phi_D D_T - \phi_\theta(0)\,\theta_T + \bar{\phi}_x x_T$. Therefore, the capital gain at the announcement is:

$$P_T^+ - P_T^- = \left[\hat{\phi}(0) - \hat{\phi}(T)\right] - [\phi_\theta(0) - \phi_\theta(T)]\,\theta_T + \bar{\phi}_x(x_T - \hat{x}_T) + \phi_\Delta(T)\,\hat{\Delta}_T. \tag{152}$$

The expected capital gain is therefore:

$$\hat{\mathbb{E}}_T^-\left[P_T^+ - P_T^-\right] = \left[\hat{\phi}(0) - \hat{\phi}(T)\right] - [\phi_\theta(0) - \phi_\theta(T)]\,\theta_T + \phi_\Delta(T)\,\hat{\Delta}_T. \tag{153}$$

Note that the unconditional mean of θ is $\bar{\theta}$ and the unconditional mean of $\hat{\Delta}_T$ is 0. Therefore, the unconditional capital gain at the announcement is

$$\mathbb{E}\left[P_T^+ - P_T^-\right] = \left[\hat{\phi}(0) - \hat{\phi}(T)\right] - [\phi_\theta(0) - \phi_\theta(T)]\,\bar{\theta}. \tag{154}$$

We define the unconditional level of price before announcement to be

$$\bar{P}_T = \hat{\phi}(T) + \left(\phi_D + \bar{\phi}_x\right)\bar{x} - \phi_\theta(T)\,\bar{\theta}. \tag{155}$$

We can similarly define the unconditional level of price after announcement as

$$\bar{P}_0 = \hat{\phi}(0) + \left(\phi_D + \bar{\phi}_x\right)\bar{x} - \phi_\theta(0)\,\bar{\theta}. \tag{156}$$

The unconditional announcement premium is roughly:

$$\frac{1}{\bar{P}_T}\mathbb{E}\left[P_T^+ - P_T^-\right] = \frac{1}{\bar{P}_T}\left\{\left[\hat{\phi}(0) - \hat{\phi}(T)\right] - [\phi_\theta(0) - \phi_\theta(T)]\,\bar{\theta}\right\}. \tag{157}$$