

Accounting for Wealth Concentration in the US

Barış Kaymak^{*†}

Université de Montréal and CIREQ

David Leung[‡]

National Taiwan University

Markus Poschke[§]

McGill University and CIREQ

Abstract

The recent literature has argued for high concentration of earnings, differences in rates of return on assets and bequests as potential determinants of the high level of wealth concentration in the US. In this paper, we assess the relevance of different macroeconomic modeling approaches to wealth concentration, using the joint distribution of earnings, capital income and net worth, in combination with an overlapping generations model that features earnings heterogeneity, rate of return heterogeneity and bequest motives. We find the large disparities in labor earnings to be the primary source of US wealth concentration. This finding reflects the high correlation between earnings and wealth in the data, as well as the fact that earnings are a major source of income for top income and wealth groups.

1 Introduction

Wealth holdings in the US are highly concentrated, more so than income, with a fifth of the population holding almost all the assets and the wealthiest 1% alone holding over a third.

^{*} Department of Economics, Université de Montréal, C.P. 6128 succursale Centre-ville, Montréal, QC H3C 3J7, e-mail: baris.kaymak@umontreal.ca

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[‡] Department of Economics, National Taiwan University, Taiwan.

[§] Department of Economics, McGill University, Montréal, QC

To explain this, the literature has emphasized a set of competing factors. A first strand highlights labor income heterogeneity and risk, which lead to higher saving rates among high earning groups (Castañeda et al., 2003; Kindermann and Krueger, 2014; Kaymak and Poschke, 2016). A second strand emphasizes capital income heterogeneity, where some households have access to investment vehicles with persistently higher rates of return (Benhabib et al., 2011; Gabaix et al., 2016). A third strand points to dynastic accumulation of wealth through bequests (Galor and Zeira, 1993; De Nardi, 2004).¹

These approaches differ in their depictions of who the wealthy are and how they become wealthy. As a result, they reach different conclusions in their assessments of economic policies. For instance, using a model of labor income risk, Kindermann and Krueger (2014) prescribe an optimal marginal tax rate as high as 90% for top income groups, whereas Brüggemann (2020) calls for a top tax rate of 60% based on a model of entrepreneurship. Guvenen et al. (2019a) argue that wealth taxes may bring efficiency gains in models with rate of return heterogeneity. Similarly, Hubmer et al. (2020) attribute much of the rise in wealth concentration over the last 50 years to top income tax cuts, whereas Kaymak and Poschke (2016) find the rise in the dispersion of wage income to be the major factor behind the rise in wealth dispersion. Such variation in policy evaluation calls for a better understanding of the factors that shape the US wealth distribution.

Regrettably, a direct empirical assessment of how important labor and capital income are for building large fortunes in the US is infeasible due to the lack of long panel data on earnings, assets and their returns for households at the top of the income and wealth distribution. In this paper, we combine cross-sectional data on the joint distribution of assets and income with an overlapping-generations model of savings to assess the relevance of the different modeling approaches to wealth concentration.

The key difference between these approaches, which our analysis exploits, is their prediction for the factor composition of income among top income and wealth groups. If wealth concentration is driven by differences in the rate of return on assets, then these groups should rely heavily on capital income. If it is driven instead by earnings differences, then labor income should be the primary source of income. Data from administrative tax records show a substantial labor income component for high income households (Piketty et al., 2018; Smith et al., 2019). We reach a similar conclusion using data from the Survey

¹Realistic wealth distributions also arise in models of entrepreneurship (Quadrini, 2000; Cagetti and de Nardi, 2009). These models combine elements of labor income and capital income heterogeneity, as discussed further below.

of Consumer Finances (SCF). Earnings account for half to two thirds of total income for the top 1% of incomes, depending on the treatment of capital gains and proprietors' income. Households outside the top groups rely almost exclusively on labor income. These patterns suggest an indispensable role for earnings in shaping the wealth distribution. We show below that models that rely on differences in rates of return to generate wealth concentration predict counterfactually low labor shares for top income groups.

The somewhat lower labor income shares among top income and wealth groups nonetheless reflect the importance of capital income for these groups. For top wealth groups, our calculations indicate that the low shares are mostly explained by the large stocks of wealth rather than differences in rates of return on assets.² By contrast, our calculations suggest modest differences in asset returns across income groups. The likely effect of these differences on wealth concentration depends on variations in income and fluctuations in the rates of returns themselves over the course of a lifetime. But the cross-sectional data do not allow for direct measures of the persistence of those differences. To assess the contribution of different elements to wealth concentration, we therefore require them to be consistent with the joint cross-sectional distributions of earnings, capital income and wealth in a structural model of household savings. The emphasis on joint distributions is key to our approach relative to the macro literature on wealth distribution, which has focused exclusively on marginal distributions of income and wealth.

To that end, we employ a general equilibrium, life-cycle model of household saving behavior. The model features uninsurable shocks to earnings, heterogeneity in rates of return, a non-homothetic bequest motive, survival risk and retirement. These elements capture the three main motives for savings: the precautionary motive, the life-cycle consumption smoothing motive, and the bequest motive. We then calibrate the model to match the joint distribution of earnings, income and net worth observed in a cross-section of households in the SCF. When combined with our model of savings, these distributions are informative of the extent and persistence of rates of return on assets and top income dynamics, which are not directly observed in cross-sectional data. The calibrated model features realistic dispersion in earnings, earning dynamics with a high degree of kurtosis and negative skewness as documented by Guvenen et al. (2019b), as well as a realistic, modest correlation between assets and rates of return. It also accurately depicts the life-cycle profiles of average earnings, income and wealth, as well as their cross-sectional dispersions.

Next, we assess the relative contributions of the model elements to wealth concentration

²Saez and Zucman (2016) reach a similar conclusion using administrative tax data.

in two ways. First, we shut down different model components and compare the implied wealth concentration to the data. Eliminating top earning categories induces the largest drop in top wealth shares, by more than half. Eliminating bequest inequality reduces top wealth shares by ten percent to a third. Eliminating differences in the rates of return mostly affects the top 0.1% of the wealth distribution, reducing their share in total wealth by 7 to 43 percent. Other concentration measures and the Gini coefficient are affected much less.

The limited role of return heterogeneity in generating wealth concentration in a life-cycle setting reflects the slow transition dynamics of models with rate of return heterogeneity (Gabaix et al., 2016), combined with the fact that young people hold little wealth. The results instead highlight the importance of saving out of earnings, which leads to faster growth in household wealth compared to earnings over the life cycle. This, in turn, maps into a highly concentrated wealth distribution (Sargent et al., 2020).

Second, we recalibrate the model to match the observed levels of wealth concentration in the absence of either top earners or return heterogeneity. This reveals that models that rely only on differences in the rate of return to explain wealth concentration not only understate earnings concentration, but also predict a counterfactually high role for capital income for top income and wealth groups. Relative to the data, the implied correlation between earnings and wealth is much too low and that between income and wealth is too high, since wealth is the primary source of top incomes in this case.

Overall, these results suggest that concentration of labor earnings is the primary source of wealth dispersion in the US, reflecting the importance of labor earnings for top income and wealth groups in the data. Return heterogeneity and bequest inequality play significant but smaller roles.

In the next section, we give a brief overview of the related literature. In Section 3 we summarize the empirical distributions of earnings, income and wealth in the SCF, as well as the factor composition of income for different income and wealth groups. In Section 4, we present the model. The calibration procedure is described in Section 5 and the results for the benchmark economy are discussed in Section 6. Section 7 analyzes the relative roles of rate of return heterogeneity, labor income risk and bequests in determining the observed distribution of wealth in the US. Section 8 concludes.

2 Macroeconomics of the Wealth Distribution

The foundations of modern macroeconomic analysis of the wealth distribution are laid out in early work by Huggett (1993) and Aiyagari (1994), which eventually led to the “standard” incomplete markets model (Heathcote et al., 2009). In this setting, dispersion in asset holdings emerges from households’ motives to accumulate assets in order to insure themselves against fluctuations in their earnings. Early iterations of these models focused on the implications of household heterogeneity for aggregate macroeconomic outcomes, such as the role of precautionary savings for total capital accumulation or for business cycles. It was nonetheless noted that the observed differences in earnings and income risk as measured in household surveys (like e.g. the PSID) were not large enough to generate a highly skewed distribution of wealth. Subsequently, a separate literature emerged aiming to enhance the model for applications to questions related to wealth inequality. The macro literature on the wealth distribution now is vast, with applications to various economic questions. In our discussion of the literature below, we focus on the main modelling extensions and their implications for a subset of applications as an example.³

The main shortcoming in the original model was that wealthy households cared little about earnings risk and therefore limited their savings once their wealth was sufficiently high to shield consumption from future drops in earnings. The first modelling extensions that helped maintain continuing wealth accumulation, and thereby generate a more skewed wealth distribution, involved introducing differences in savings motives or rates of return on assets. This was achieved by explicitly introducing heterogeneity in preferences for saving (Krusell and Smith, 1998), in rates of return on assets (Benhabib et al., 2011; Gabaix et al., 2016; Nirei and Aoki, 2016; Cao and Luo, 2017), as well as bequest motives that are increasing in wealth (De Nardi, 2004). Benhabib et al. (2011) show analytically that idiosyncratic capital income risk can generate a Pareto tailed wealth distribution with a realistic tail index. Capital income risk is essential to a fat-tailed wealth distribution in some versions of the incomplete markets model, but is not generally necessary, e.g. if agents have finite lives as in an OLG setting (Jones, 2015; Stachurski and Toda, 2019; Sargent et al., 2020). Benhabib et al. (2019) and Cao and Luo (2017) provide quantitative assessments of the contribution of rate of return heterogeneity to wealth concentration. The common element among these models is that the main source of differences in wealth accumulation is capital income. High wealth concentration emerges because wealthy households enjoy

³See De Nardi and Fella (2017) for a more detailed review of the macro literature on wealth inequality.

higher rates of return on their assets and have higher saving rates out of income. As a consequence, capital income is essential to top income and wealth groups.

A second strand of the literature focused on better measurement of earnings. Household surveys typically provide an incomplete picture of the distribution of earnings and associated risks due to censoring of earnings above a certain level or limited sampling of high-earning households. Castañeda et al. (2003) were the first to show that the standard incomplete markets model can indeed generate a highly skewed wealth distribution if the earnings process is calibrated accordingly. This however required unrealistically high earnings levels for top income groups. Subsequent work refined this approach, using the recent progress in measurement of top earnings levels based on administrative data to discipline the extent of earnings dispersion and risk used as inputs in the model (Kindermann and Krueger, 2014; Kaymak and Poschke, 2016). The economic mechanism here is that households who temporarily have very high earnings anticipate lower future earnings (be it because of retirement or the vagaries of a top-level career), and therefore have a very strong saving motive. The explicit consideration of very high earnings levels is a key ingredient in these models, where the main source of wealth concentration consists in differences in labor income and the associated saving behavior.

Another mechanism that can generate high wealth concentration is entrepreneurship, which combines elements from both strands we have discussed, as profits reflect both the return on assets invested in the business and the value of entrepreneurial labor (Quadrini, 2000; Cagetti and De Nardi, 2006). Entrepreneurs in these models reap higher rates of returns on their investments if, or as long as, they are financially constrained (Buera, 2009; Moll, 2014).⁴ This may encourage them to save faster in order to bypass credit constraints. They may also save more because earnings on their entrepreneurial skills may be subject to significant fluctuations due to business risk.

All these approaches substantially improved the ability of the standard incomplete markets model to generate a realistic wealth distribution for the US, offering economists several modelling options. The existing literature has operated with either a model with capital income risk, one with high earnings dispersion, or one with entrepreneurship. Yet, the relative roles of earnings and capital income risk in generating the observed wealth concentration are not well understood, in part due to lack of data on the dispersion and persistence of rates of return on assets at the household level in the US.⁵ This paper combines these approaches

⁴Without credit constraints, models of entrepreneurship can be mapped into a model with earnings heterogeneity and a common return on assets (See Appendix C).

⁵Recent work by Fagereng et al. (2020) and Bach et al. (2016) provide empirical evidence for rate of

and is the first to use information on the joint distributions of earnings, capital income and assets to identify the relevance of different modelling approaches to wealth concentration.⁶

3 Income, Earnings and Wealth in the US

In this section we summarize the distributions of earnings, income and wealth, and discuss the role of capital income vis-à-vis earnings on labor for top income and wealth groups. The primary sources of data are the 2010 and 2016 waves of Survey of Consumer Finances (SCF), a triennial cross-sectional survey of US families on their assets, income, and demographic characteristics.⁷ The SCF is particularly suitable for our analysis since it oversamples high-income households and is commonly used in the macro literature to study upper tails of the income and wealth distribution (Diaz-Gimenez et al., 2011; Kuhn and Ríos-Rull, 2016; Kuhn et al., 2020). We also compare our results to those obtained from administrative tax records reported by Piketty and Saez (2003) and Smith et al. (2019).

3.1 Marginal and joint distributions

Since the objective is to use the joint distribution of income and wealth to identify the importance of different modeling components, we adopt a market-based notion of income that is compatible with the models of wealth distribution mentioned above. Our definition of market income includes wage and salary income, business and farm income, interest and dividend income, private pension withdrawals and capital gains, whereas it excludes income from fiscal sources, such as transfer income or social security income.

We distinguish between market income from labor and from capital. The SCF follows the tax filing guidelines for classifying sources of income. For most households, labor income consists of wage and salary income, which includes pay for work for an employer as well as any salary drawn from an actively managed business. For corporations, the IRS requires actively involved shareholders to explicitly report wage and salary. Some business organizations, such as partnerships and sole proprietorships are exempted from this

return heterogeneity using panel data from Norway and Sweden, respectively.

⁶Our decomposition approach distinguishes between heterogeneity in capital and labor incomes. Hence, it also encompasses entrepreneurs, but treats their capital and labor income components separately.

⁷We exclude the 2013 survey, which reports income from the 2012 calendar year and shows an unusual increase in realized capital gains. This is largely due to an anticipated increase in the capital gains tax scheduled by the Patient Protection and Affordable Care Act that was enacted in 2010 and provided for additional taxes on high income groups starting in 2013.

requirement. As a result, a small group of business owners report only business income. In such cases, we impute wage and salary income only if a household reports income from actively owned businesses, but does not report any wage income, or, if the respondent or their spouse reports explicitly that they did not draw salary from their actively managed business.⁸ This does not change the conclusions we draw from the empirical patterns below, reported with and without imputed salaries. For our quantitative analysis, we include imputed wages in labor income.

To determine the share of business income that is attributable to capital, we assume that the contribution of capital to active business income is proportional to the total value of equity held in the business. Consequently, we regress active business income on business equity, controlling for the quantity and quality of the labor input. Specifically, we include the number of hours worked by the household members that are actively involved in the business as well as demographic characteristics of the head of household, such as gender, age and education as control variables. The resulting coefficient on equity is 0.27 (s.e. 0.03), which we interpret as the capital income share.⁹ Accordingly, we allocate 73 percent of active business income to labor for those who do not report wage income from their business.

Our estimate is consistent with empirical work that relies on administrative tax records and variations in ownership that are more exogenous in nature. Smith et al. (2019), in particular, find that profits of a business decline substantially upon the owner's demise. Consequently, they attribute much of business income to human capital and estimate the labor share of business income to be 75 percent. Similarly, Piketty et al. (2018) attribute 70 percent of pass-through income to labor. Despite the differences in methodology and data sources, our estimate of 73 percent is very close to these studies.

The resulting labor income share potentially underestimates the true contribution of labor for three reasons. First, since it is less advantageous to report business income as wages for tax purposes, business owners who report wage income may underreport it. Second, for those who do not report wage income, we only impute wages for the spouse and the respondent. If other members of the household work for the business, their labor income is classified as part of the household's business income.¹⁰ Third, both years in the sample co-

⁸These households constitute 8.9% of all households and are overwhelmingly middle class—95% of them are outside the top 1% income and wealth groups. See Data Appendix for a discussion of the imputation sample and the procedure.

⁹This is the share in *net* income, since depreciation expenses are deducted from the reported business income. The share in gross income can be found by adding the rate of depreciation.

¹⁰The survey questions needed to ascertain if household members have claimed wage income from their

Table 1 – Cross-Sectional Distributions of Income, Earnings and Net Worth

Top percentile	0.1%	0.5%	1%	5%	10%	20%	40%	Gini
Net worth	0.14	0.28	0.37	0.63	0.76	0.88	0.97	0.85
Income	0.08	0.18	0.23	0.41	0.53	0.68	0.86	0.67
Earnings	0.06	0.14	0.19	0.36	0.49	0.66	0.86	0.66 [†]

[†] The Gini coefficient for households with a working-age head is 0.58.

Note.– Table shows the cumulative concentration shares for the top percentile groups. Income includes capital gains. Data comes from the SCF 2010 and 2016. Sample includes all households.

incide with the post-recession recovery period, where asset returns were above their typical average.

Table 1 shows the cross-sectional distributions of income, earnings and wealth. The distribution of net worth is far more skewed than the distributions of income and earnings: the Gini coefficient for net worth is 0.85, whereas it is 0.66 for earnings and 0.67 for income. This is driven by both the heavier concentration of wealth at the top and a larger fraction of households without assets relative to those without income. The top 1% of the net worth distribution has 37% of assets and the top 0.1% holds 14% of total wealth. Earnings are also concentrated, with the top 1% earners' share of 19% in total earnings and the top 0.1% share of 6%.

There is a strong correlation between wealth and earnings. The coefficient of correlation between earnings and net worth is 0.35 for households with a working-age head, and it is 0.30 for the entire sample. Similarly, the correlation between income and net worth is 0.52. This strong relationship can also be seen in Table 2, which shows the wealth shares of different earning and income groups. The top 1% of earners hold about 19% of wealth. Similarly, the households in the highest 1% of incomes hold 27% of total wealth in the US. If the correlation were zero, wealth shares would be equal to population shares when ranking groups by income or earnings. This suggests that savings out of earnings and income play a significant role for accumulation of wealth.

business are only available for the respondent and the spouse.

Table 2 – Shares of Net Worth by Income and Earning Groups

Top percentile of ...	0.1%	0.5%	1%	5%	10%	20%	40%
... income	0.09	0.20	0.27	0.51	0.61	0.71	0.81
... earnings	0.04	0.13	0.19	0.38	0.47	0.57	0.67

Note.– Table shows cumulative shares of net worth held by top income and earning groups. Income includes capital gains. Data comes from the SCF 2010 and 2016.

3.2 The share of income from labor

Figure 1 shows the factor composition of income for top income and wealth groups. The gray bars show the share of wage and salary in total income, as reported by the households. The red solid bars show the labor share of total income, including imputed earnings for those proprietors who do not report wage income from their businesses. The whisker ticks on each bar indicate the values when capital gains are included in or excluded from total income. The height of each bar represents the average of these two values.

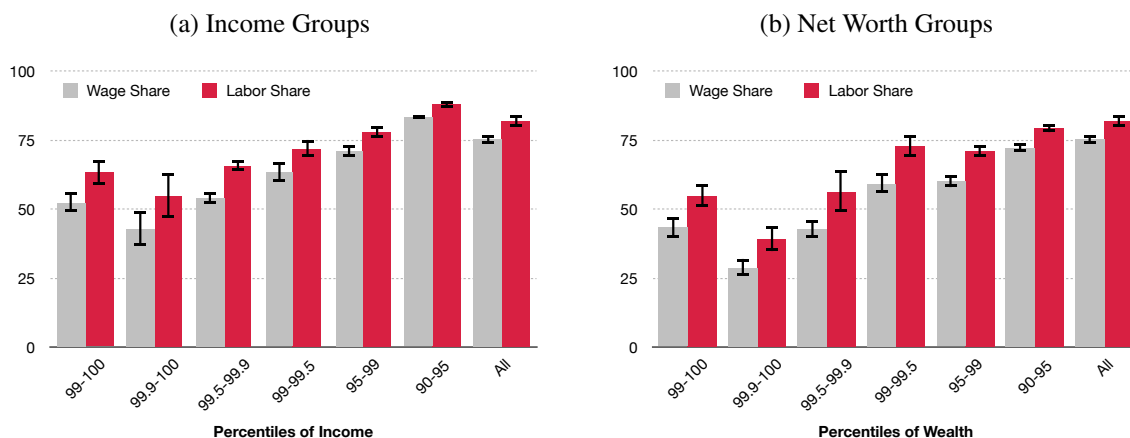
On the aggregate, 74 to 84 percent of net income is attributed to labor, depending on the treatment of capital gains and business income.¹¹ Panel (a) shows the labor shares by percentiles of total income. Most households rely primarily on wage and salary income. Outside the top 1 percent of the income distribution, labor income constitutes at least two thirds of total income. Since business income and capital gains are not an important source of income for these groups, the particular definition of income does not affect this result.

For the top 1 percent of the income distribution, labor income constitutes 59 percent of total income when capital gains are included, and 68 percent when they are excluded from income. The wage share, which excludes imputed wages for some proprietors, is roughly 10 points lower. Columns 2 to 4 show the percentiles of income within the top 1%. Income from labor is the major source of income, accounting for at least half of total income, with the exception of the top 0.1%.

A similar pattern is observed for top groups by net worth in Panel (b) of Figure 1. Labor’s share of income for the top 1% of wealth is 0.51 and 0.59, with and without capital gains. Excluding capital gains, income from labor is the main source of income for households outside of the top 0.1% of the net worth distribution. With capital gains, income from

¹¹Since the accounting convention is to report the net income from capital, i.e. excluding depreciation, the share of labor income in net income is higher than its share in gross income typically used to calibrate macro models. We use net capital income in our comparisons of the model predictions below with the data above.

Figure 1 – Labor Component of Income by Income and Wealth Groups (%)



Note.— Figure shows wage and labor shares of total income by percentiles of income and net worth. Labor income includes imputed wage income for active business owners who do not draw salary from their businesses. The whiskers show the shares with and without capital gains in total income. The bar heights show the average of the two values. See Appendix Table B.1 for the data values. Data comes from the 2010 and 2016 waves of the SCF.

capital dominates labor for those in the top 0.5%.

Table 3 compares our findings with statistics from IRS data. We use the 2015 update to the tables in Piketty and Saez (2003), who report the sources of income for finely defined top income groups. Since it is not possible to observe which tax units draw salary from their business, no imputation is made, and we report business income separately. These figures are comparable to the top rows of Figure 1. The share of wage income for the top 1 percent income group as reported by tax units in Table 3 is 49 percent when capital gains are included, and 56 percent when they are excluded – exactly as in our findings in the SCF data reported in Figure 1.¹² Columns 2 to 5 in Table 3 report the components of income within the top 1 percent of income. Wage income constitutes more than half the income for those outside the top 0.1 percent of top income earners. For the top 0.1 percent of the income distribution, the share of wage income drops and interest and dividend income becomes increasingly important. For the top 0.01 percent of the income distribution, interest and dividend income constitute 42 percent of total income when capital gains are included.

¹²There are two subtle but apparently inconsequential differences between the two sets of statistics. First, the income concept reported in Piketty and Saez (2003) includes fiscal income, such as social security payments and other transfer payments. Since transfer payments are not a significant source of income for top income groups, this does not affect the results. Second, the IRS data is based on tax units whereas the SCF data is based on primary economic units, which consists of the core members of the household. In most cases, this includes the respondent, their spouse, if any, and their dependent children.

Table 3 – Composition of Income for Top Income Groups (IRS)

without capital gains	Income Percentile Category				
	99-100	99-99.5	99.5-99.9	99.9-99.99	99.99-100
Wage	56	73	61	47	34
Business	30	20	29	37	37
Interest and Dividend	14	7	10	15	29

with capital gains	Income Percentile Category				
	99-100	99-99.5	99.5-99.9	99.9-99.99	99.99-100
Wage	49	68	54	40	27
Business	27	19	26	32	30
Int., Div. and Capital Gains	24	13	19	28	42

Note.– Figures in percentages and correspond to averages for 2010-2015. Income percentiles are determined excluding capital gains (KG). Figures come from 2015 data update to Piketty and Saez (2003).

Both the survey data from the SCF and the tax data from the IRS records agree on the relative roles of sources of income. For most households, earned income from labor services is the primary source of income. As we move up the income ladder, the share of labor income declines, and income from capital increases. Nonetheless, even among the top 1% of households (and tax units), the most conservative definition of labor income indicates that at least half the income can be attributed to labor. As the size of the top fractile is reduced, capital income becomes more important. The upshot of this is that labor income remains a non-negligible source of income throughout, and is a primary source of income for most households (or tax units) outside the highest income and net worth groups.

3.3 Implied heterogeneity in the rate of return on assets

Next, we demonstrate how labor’s share of income can help identify heterogeneity in the rate of return and discuss the limitations of inference based on cross-sectional data alone.

A group’s relative rate of return on assets can be inferred from its relative labor share of income. To see this, let λ_i denote the labor income share of a group of households i :

$$\lambda_i = \frac{e_i}{e_i + r_i k_i},$$

where e_i and k_i are average earnings and assets of a household in the group, and r_i is the

Table 4 – Labor Income Shares and the Implied Rate of Return on Assets

Income Percentile	0 - 90	90 - 95	95 - 99	99 - 99.5	99.5 - 99.9	99.9 - 100
<i>Data:</i>						
Relative Wealth	1	5	14	36	63	206
Relative Earnings	1	4	7	17	31	83
Labor Income Share (LIS)	0.91	0.88	0.78	0.72	0.66	0.55
<i>Inferred values:</i>						
Common Return LIS [†]	0.91	0.89	0.84	0.82	0.83	0.80

Note.— Data LIS comes from the 2010 and 2016 waves of the SCF and reflects the average labor share of income with and without capital gains. † Labor income share implied by the relative size of assets and earnings assuming that all households have the same return on their assets.

group-specific return on assets. Let $i = 0$ represent the base group, which we define below as the bottom 90% of the income or wealth distribution. Denote the earnings ratio of group i relative to the base group by $e_{i/0} = e_i/e_0$, and the asset ratio by $k_{i/0} = k_i/k_0$. Then the labor income share of any group can be expressed as:¹³

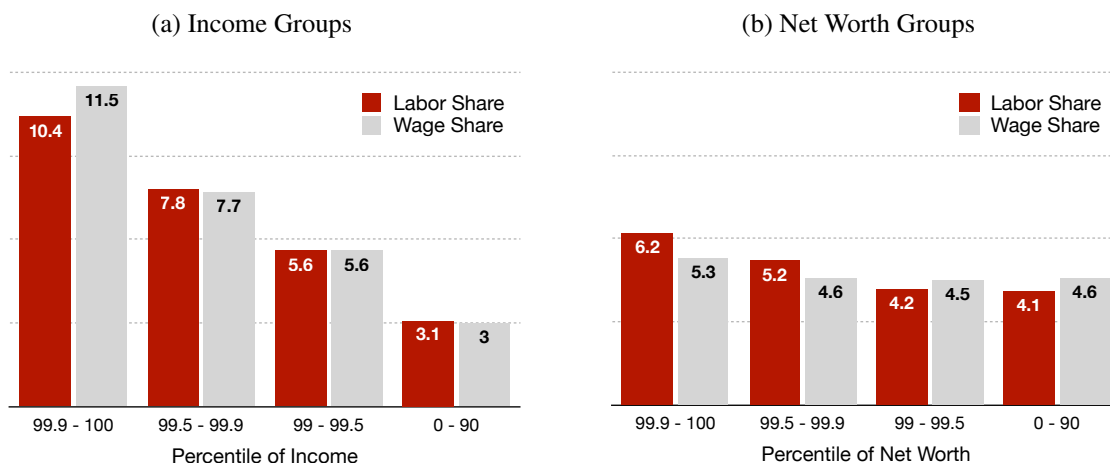
$$\lambda_i = \frac{\lambda_0}{\lambda_0 + \frac{k_{i/0}}{e_{i/0}} \frac{r_i}{r_0} (1 - \lambda_0)}. \quad (1)$$

Equation (1) relates the labor income share of top income groups to that of the base group. Top income groups have lower labor income shares in two situations. First, their relative wealth is higher than their relative earnings, $k_{i/0}/e_{i/0} > 1$, or, equivalently, their wealth-to-earnings ratio is relatively higher. This could arise if, for instance, the saving rate increases with earnings. Second, they have a higher rate of return on their assets: $r_i/r_0 > 1$. To isolate the role of the latter, we carry out two calculations. First, we compute the counterfactual labor share of income for top income groups implied by their relative wealth-to-earnings ratio, assuming that all income groups have the same rate of return on their assets. If returns are higher for higher income groups, then the labor share should be less than that implied by their assets alone.

Table 4 shows the results. The base income group is the bottom 90% of the income distribution. As expected from Table 1, relative wealth and earnings are higher for higher

¹³ $\lambda_i = \frac{e_{i/0}e_0}{e_{i/0}e_0 + r_i k_{i/0} k_0} = \frac{e_0}{e_0 + \frac{k_{i/0}}{e_{i/0}} r_i k_0} = \frac{e_0}{e_0 + r k_0} \frac{e_0 + r k_0}{e_0 + \frac{k_{i/0}}{e_{i/0}} \frac{r_i}{r_0} r_0 k_0} = \frac{\lambda_0}{\lambda_0 + \frac{k_{i/0}}{e_{i/0}} \frac{r_i}{r_0} (1 - \lambda_0)}.$

Figure 2 – Rates of Return Implied by Labor Shares (%)



Note.— Figure shows the synthetic rate of return on assets by household income and net worth implied by the labor share of income, assuming an annual average rate of return of 4.9%. Labor share includes imputed wage income for active business owners who do not draw salary from their businesses, whereas wage share excludes it (see Figure 1). Data comes from the 2010 and 2016 waves of the SCF.

income groups with a wealth-to-earnings ratio increasing by income. The third and fourth rows report the observed labor share and the labor share implied by equation (1), assuming common rates of return. The latter declines moderately from 0.91 for the base group to 0.80 for the top 0.1% reflecting the relatively larger asset holdings. However, in each category, the observed labor share is below the labor share implied by assets alone. This suggests that top income groups must also have experienced higher rates of return on their assets.

Next, we solve equation (1) for the relative rates of return, implied by the observed labor income shares for different groups:

$$\frac{r_i}{r_0} = \frac{e_{i/0}}{k_{i/0}} \cdot \frac{1/\lambda_i - 1}{1/\lambda_0 - 1}. \quad (2)$$

Equation (2) allows for calculation of the rate of return relative to the base group in cross-sectional data. Higher income groups have increasingly higher rates of return on their assets. The 90-95th percentile, for example, has 1.14 times the base return, whereas the top 0.1% of incomes have 3.36 times the base return. The dispersion is substantial. To translate the relative returns to actual returns, we assume an aggregate return of 4.9% per year, which corresponds to the rate in our quantitative analysis below. Panel (a) in Figure 2 shows the results. Using our preferred labor share measure, the dark columns show an annual rate of return of 3.0% for the base group and increasingly higher rates for top income

groups. The average return for the top 1% of incomes is 8.0%, which is 3.1 points higher than the aggregate return on assets. For the highest income category (top 0.1 percent), the implied rate of return is 10.4%, roughly twice the aggregate return. The implied dispersion in rates of return is robust to the definition of labor income. The gray bars show the rates implied by excluding imputed wages for some business owners (corresponding to gray bars in Figure 1). The estimated rates of return rise from 3.0% for the base group to 11.5% for the highest income group.

A similar analysis can be done using labor income shares by percentiles of the net worth distribution. This yields smaller differences in rates of return by wealth (Panel b), because the top wealth groups have dramatically more assets relative to the base group, which, for the most part, suffices to explain the higher share of income from capital.

While Figure 2 suggests a modest degree of *cross-sectional* heterogeneity in asset returns, it is not possible to accurately gauge how much this matters for wealth concentration. Since higher rates of return lead to higher income and, ultimately, higher wealth, the positive correlation between rates of return and income (or wealth) may be spurious. Moreover, the dynamic process for the rates of return cannot be estimated from cross-sectional data. But the persistence and predictability of returns are crucial for inferring the saving response to these rates by income and wealth. Below, we combine the cross-sectional information above with a model of household saving to quantify the role of earnings concentration and rate of return heterogeneity in shaping the wealth distribution in US. We require, in particular, that the stochastic process that governs the rates of return be consistent with the observed wealth concentration, conditional on the earnings distribution in the data.

4 A Life-Cycle Model of Wealth Accumulation

For the analysis, we employ an overlapping generations model of life-cycle wealth accumulation under incomplete markets (Imrohoroglu and Imrohoroglu, 1995; Huggett, 1996). We augment the model by incorporating idiosyncratic labor income with extraordinary earning levels, heterogeneity in the return to capital income and a non-homothetic bequest motive.

4.1 Environment

Each period, a continuum of new agents enter the economy, with a potential life-span of J periods, subject to survival probabilities $s(j)$ for each age j . The total population is

normalized to one.

Agents work for the first $J_r - 1$ periods of their lives, after which they retire. Workers earn income for their labor and on their assets. A worker's labor endowment is given by $z\varepsilon_j$, where z is a stochastic component following a first-order Markov process $F_z(z'|z)$, and ε_j is a deterministic component that captures age-dependent improvements in human capital, such as work experience. With this endowment, a worker generates a labor income of $wz\varepsilon_j h$, where w is the market wage per skill unit and $h \in [0, 1]$ is hours worked. Income from capital is $r\kappa k$, where k denotes assets, and κ is an idiosyncratic rate of return that follows a Markov process defined by $F_\kappa(\kappa'|\kappa)$. Once retired, agents collect a pension, $b(z)$, that depends on the last realization of the labor productivity shock z , and continue to earn income on their assets.¹⁴ Total income is denoted by y .

All income is subject to taxation. The tax system, outlined below in detail, distinguishes between different sources of income and features transfers. The disposable income after all taxes and transfers is denoted by y^d . Consumption is subject to sales tax at a rate τ_s . The government uses the tax revenue to finance an exogenously given level of expenditures, G , pension payments and other transfers.

The consumption goods are produced by a representative firm using aggregate capital K and total effective labor N with a Cobb-Douglas production function: $Y = F(K, N) = \Psi K^\alpha N^{1-\alpha}$. The firm hires capital and labor in a competitive market to maximize its profits.

4.2 The consumption-savings problem

Agents value consumption, leisure and assets they leave for their offspring. The problem of an agent is to choose labor supply, consumption, savings and bequests to maximize the expected present value of lifetime utility. At each period j , agents are informed of their labor endowment for the period, $z\varepsilon_j$, and their rate of return on assets, $r\kappa$, prior to taking their decisions. Future utility is discounted by a factor $\beta \in (0, 1)$. Formally, the Bellman equation for a worker's problem is

¹⁴The actual US social security benefits depend on a worker's average earnings over their career. Following Kindermann and Krueger (2014), we assume that pension benefits depend on the earnings of the last working age period. This allows us to capture the redistributive structure of the US pensions system while maintaining computational feasibility.

$$V(j, k, z, \kappa) = \max_{c, k' \geq 0, h \in [0, 1]} \left\{ \frac{c^{1-\sigma_c}}{1-\sigma_c} - \theta \frac{h^{1+\sigma_l}}{1+\sigma_l} + \beta(1-s(j))\phi(k') \right. \\ \left. + \beta s(j)\mathbb{E}[V(j+1, k', z', \kappa')|z, \kappa] \right\}$$

subject to

$$(1 + \tau_s)c + k' = y^d(zw\varepsilon_j h, r\kappa k) + k + Tr + \Phi(j, z, \kappa),$$

where $\phi(k) = \phi_1 [(k + \phi_2)^{1-\sigma_c} - 1]$ is the utility value of bequeathed assets, and $\Phi(j, z, \kappa)$ denotes assets received as a bequest. The expectation is taken over the future values of the labor endowment, z' , and the rate of return on assets, κ' , given the processes F_z and F_κ .

Since retirees do not work, the Bellman equation for a retiree's problem is given by

$$V(j, k, z, \kappa) = \max_{c, k' \geq 0} \left\{ \frac{c^{1-\sigma_c}}{1-\sigma_c} + \beta s(j)\mathbb{E}[V(j+1, k', z, \kappa')|\kappa] + \beta(1-s(j))\phi(k') \right\}$$

subject to

$$(1 + \tau_s)c + k' = y^d(b(z), r\kappa k) + k + Tr$$

4.3 Stationary equilibrium

Let $s = \{j, k, z, \kappa\} \in S$ be a generic state vector. The stationary equilibrium of the economy is given by a consumption function, $c(s)$, a savings function, $k'(s)$, labor supply, $h(s)$, a value function $V(s)$, a wage rate w and a distribution of agents over the state space $\Gamma(s)$, such that (i) functions $V(s)$, $c(s)$, $k'(s)$ and $h(s)$ solve the consumers' problems, (ii) firms maximize profits, factor markets clear:

$$K = \int k'(s)d\Gamma(s) \quad N = \int z\varepsilon_j h(s)d\Gamma_{j < J_r}(s),$$

the government's budget is balanced:

$$G + \int b(z)d\Gamma_{j \geq J_r}(s) = \tau_s \left[\int c(s)d\Gamma(s) \right] + \int [y - y^d(zw\varepsilon_j h, r\kappa k)]d\Gamma_{j < J_r}(s) \\ + \int [y - y^d(b(z), r\kappa k)]d\Gamma_{j \geq J_r}(s).$$

and $\Gamma(s)$ is consistent with the policy functions, and is stationary.

5 Calibration of the Model

To quantify the model parameters, we first choose a set of parameters based on information that is exogenous to the model. Then, we calibrate the remaining parameters so that the stationary equilibrium of the model economy is consistent with the empirical distributions of earnings, wealth and income, as well as other informative data moments. We do so by minimizing the equally weighted sum of squared deviations between model moments and data moments.

While our approach is broadly consistent with the standard for quantitative macro models of overlapping generations with idiosyncratic risk, it has some distinctive elements. From a modeling perspective, the main differences are in the earning process, where we allow some households the possibility of reaching an extraordinarily high labor productivity level in the spirit of Castañeda, Díaz-Giménez and Ríos-Rull (2003), Kindermann and Krueger (2014) and Kaymak and Poschke (2016), and in the rate of return risk in the spirit of Benhabib, Bisin and Luo (2019). From an empirical point of view, we differ from earlier studies in our explicit use of the joint distribution of earnings, income and wealth in addition to their marginal distributions to identify these modeling extensions.

In this section, we discuss the choice of target moments. In the next section, we present the fit of the model in terms of those moments as well as additional over-identifying moments that we do not target.

5.1 Demographics

The model period is five years. The first model period corresponds to ages 20 to 24. Death is certain after age $J = 16$, which corresponds to ages 95-99. Retirement is mandatory at age 65 ($J_r = 10$). Following Halliday et al. (2019), we assume that the survival probability is a logistic function of age: $s(j) = [1 + \exp(\omega_0 + \omega_1 j + \omega_2 j^2)]^{-1}$ and use the parameter values recommended therein.¹⁵

¹⁵Halliday et al. (2019) calibrate to three moment conditions: the dependency ratio (population aged 65 and over divided by population aged 20-64), which is 39.7% in the data, the age weighted death rate for 20 to 100 year olds of 8.24%, and the ratio of the change in the survival probability between ages 65-69 and 75-79 to the change in survival probability between ages 55-59 and 65-69, which is 2.27 in the data.

5.2 Preferences and production technology

Preferences are described by a discount factor, β , the inverse elasticity of intertemporal substitution, σ_c , the inverse elasticity of labor supply, σ_l , the disutility of work θ and the parameters of utility from bequests: ϕ_1 and ϕ_2 . We discuss the last two separately below. We set $\sigma_l = 1.22$, which implies a Frisch elasticity of 0.82, the average of the values of 0.68 for males and 0.96 for females reported by Blundell, Pistaferri and Saporta-Eksten (2016). We choose θ so that an average household allocates 35% of their time endowment to work in equilibrium. We set $\sigma_c = 1.5$, in the middle of the range typically used in the literature. The discount factor, β , is chosen so that the ratio of capital to annual income is 2.9 given an annual depreciation rate of 4.5%. This results in a value of $\beta = 0.90$, or 0.98 per annum. The implied (value-weighted) interest rate that clears the asset market is 4.88%. We normalize the equilibrium wage rate, $w = 1$, which requires an aggregate TFP of $\Psi = 1.55$, and calibrate the elasticity of output with respect to capital, α , to 0.27, to match the net labor income share observed in the SCF.

5.3 Labor productivity process

The stochastic component of labor productivity takes eight values. Six of these are ordinary states, and the other two are extraordinary states that generate exceptionally high earnings levels. The ordinary levels z_1 to z_6 consist in combinations of two components: a permanent component, $f \in \{f_H, f_L\}$, that is fixed over a household's career, and a transitory component, $a \in \{a_L, a_M, a_H\}$. Individuals randomly draw their value of f in the first period of their lives. Idiosyncratic fluctuations in labor income risk over the life-cycle are captured by a 3-by-3 matrix $A = [A_{ij}]$ with $i, j \in \{L, M, H\}$ and $\sum_j a_{ij} = 1 - \lambda_{in}$, as well as by λ_{in} , which represents the probability of entering an extraordinary state of productivity. The stochastic labor productivity process is summarized by the matrix in Table 5. The following additional assumptions are explicit in the formulation of the matrix. The probability of reaching an extraordinary status, λ_{in} , is independent of one's current productivity state and age. Likewise, if a household loses their extraordinary status, then it is equally likely to transition to any ordinary productivity state.¹⁶

Our working assumption is that the values for the ordinary states and the transitions among them can be inferred from survey data, whereas the transitions to, from and among extraordinary states can not. To calibrate values and transitions of ordinary states, we

¹⁶The effect of these assumptions on our quantitative analysis is negligible.

Table 5 – Transition Matrix for the Labor Productivity Process

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
	$f_L + a_L$	$f_L + a_M$	$f_L + a_H$	$f_H + a_L$	$f_H + a_M$	$f_H + a_H$		
$f_L + a_L$	A_{11}	A_{12}	A_{13}	0	0	0	λ_{in}	0
$f_L + a_M$	A_{21}	A_{22}	A_{23}	0	0	0	λ_{in}	0
$f_L + a_H$	A_{31}	A_{32}	A_{33}	0	0	0	λ_{in}	0
$f_H + a_L$	0	0	0	A_{11}	A_{12}	A_{13}	λ_{in}	0
$f_H + a_M$	0	0	0	A_{21}	A_{22}	A_{23}	λ_{in}	0
$f_H + a_H$	0	0	0	A_{31}	A_{32}	A_{33}	λ_{in}	0
z_7	λ_{out}	λ_{out}	λ_{out}	λ_{out}	λ_{out}	λ_{out}	λ_{ll}	λ_{lh}
z_8	0	0	0	0	0	0	λ_{hl}	λ_{hh}
initial dist.	$\zeta/4$	$(1 - \zeta)/2$	$\zeta/2$	$\zeta/4$	$(1 - \zeta)/2$	$\zeta/4$	0	0

Note.— The transition probabilities from the state in Column 1 to the states in Columns 2 to 9. The last row shows the initial distribution of young workers across the productivity states at the time of labor market entry.

assume that the transitory component of productivity follows an AR(1) process, with an annual persistence of 0.97, as estimated by Heathcote et al. (2010), and variance σ_a . Wage regressions in the PSID with fixed worker effects indicate that 60% of the total variance of wages reflect differences in the permanent component, and the remaining 40% reflect transitory shocks. Accordingly, we set $\sigma_a^2 = 0.4\sigma^2$, where σ^2 is the total variance. Normalizing $a_M = 0$ and setting $a_L = -\eta$ and $a_H = \eta$ then allows us to determine η and the elements of A in terms of σ using the Rouwenhorst approximation. To determine the levels of the fixed components, we set $f_L = -f_H$. Assuming an equal division of households between the two permanent states, we then express f_H in terms of σ such that the implied variance is $0.6\sigma^2$.

At this point, all ordinary productivity levels are expressed relative to σ . Note that σ^2 is the variance corresponding to the long-run stationary state associated with the transition matrix. Since the wage distribution is not stationary over the life-cycle, this object is not directly observed in the data. To determine σ , we parameterize the initial distribution of households over the ordinary productivity states at the beginning of their careers as in the last row of Table 5. By assumption, households are not born to extraordinary productivity. Then, given the age distribution implied by the survival function described in Section 5.1, we jointly calibrate the parameters ζ and σ such that the overall cross-sectional variance of wages equals 0.58 and the standard deviation of wages grows by 47 percent between the

ages of 22 and 57, as we estimate in the PSID. This requires that $\sigma^2 = 0.81$ and $\zeta = 0.18$.

This leaves the extraordinary productivity levels z_7 and z_8 , and the transition probabilities $(\lambda_{in}, \lambda_{out}, \lambda_{ll}, \lambda_{lh}, \lambda_{hl}, \lambda_{hh})$. Two of these are pinned down by adding-up constraints for probabilities. To identify the remaining parameters, we target moments on the marginal distribution of earnings, specifically, the top 0.1 and 1 percent shares, the labor income shares of the percentile groups 95-99 and 99-100 of the income distribution, the Gini coefficient for wealth, as well as the probability of remaining a top 1% earner as reported by Kopczuk et al. (2010) from administrative data.

The stochastic process for labor productivity is combined with a deterministic age profile of wages common to all workers. We calibrate this profile to that from the PSID.

5.4 Rate of return process

The rate of return on capital is stochastic and takes three distinct values, $\{r^{\kappa_L}, r^{\kappa_H}, r^{\kappa_{top}}\}$, where r is the equilibrium market rate of return, and the κ_i are the relative idiosyncratic returns. The transitions between these states are governed by the following transition matrix:

$$\Pi_{\kappa} = \left(\begin{array}{c|ccc} & \kappa_L & \kappa_H & \kappa_{top} \\ \hline \kappa_L & \pi_{ll} & 1 - \pi_{ll} - \pi_{in} & \pi_{in} \\ \kappa_H & 1 - \pi_{hh} - \pi_{in} & \pi_{hh} & \pi_{in} \\ \kappa_{top} & 0 & 1 - \pi_{top,top} & \pi_{top,top} \end{array} \right)$$

Since asset returns are not directly observed in the data, we target moments on wealth concentration and intergenerational wealth mobility to identify the κ_i and the transition probabilities π_{ll} , π_{hh} , $\pi_{top,top}$, and π_{in} . The targets are the top 0.1%, 1%, 5% and 10% wealth shares as well as the intergenerational probabilities of staying in the fourth and fifth quintiles of the age-adjusted wealth distribution. Using data from the PSID for the period from 1984 to 1999, Charles and Hurst (2003) report the latter two moments to be 0.26 and 0.36, indicating substantial persistence of wealth across generations.¹⁷ We replicate their estimation method in our model to compute the corresponding model moments.¹⁸

¹⁷Gayle et al. (2016) extend the analysis to more recent waves and find very similar numbers.

¹⁸We exclude model parent-child pairs where either the child or the parent is in the top 1% of wealth. Results are similar when they are included.

5.5 Tax and transfer system

The tax system consists of personal income taxes levied on capital and labor earnings, corporate taxes and a sales tax. The tax receipts are used to support exogenous government expenditures, transfers to households, and pensions.

Corporate taxes are modeled as a flat rate, τ_c , levied on a portion of capital earnings before households receive their income. We set $\tau_c = 23.6\%$, which is the average effective marginal tax rate on corporate profits in 2010 as estimated by Gravelle (2014) based on tax records. To reflect the fact that for most households, positive net worth takes the form of real estate and thus is not subject to corporate income taxes, we assume that corporate taxes only apply to capital income above a threshold d_c .¹⁹ We then choose d_c such that the corporate tax revenue as a share of GDP is 2.5%.²⁰ Households are subject to sales tax, which is set to 5% of consumption, following Kindermann and Krueger (2014).

Personal income taxes are applied to earnings, non-corporate capital income and pension income, if any. Taxable personal income is given by:

$$\begin{aligned} y_f &= zw\varepsilon_j h + \min\{r\kappa k, d_c\} & \forall j < J_r \\ y_f &= b + \min\{r\kappa k, d_c\} & \forall j \geq J_r. \end{aligned}$$

Total disposable income is obtained after applying corporate and personal income taxes and adding lump-sum transfers from the government:

$$y^d = \lambda \min\{y_b, y_f\}^{1-\tau} + (1 - \tau_{max}) \max\{0, y_f - y_b\} + (1 - \tau_c) \max(r\kappa k - d_c, 0) + Tr$$

The first two terms above represent our formulation of the current US income tax system, which can be approximated by a log-linear form for income levels outside the top of the income distribution (Benabou, 2002), augmented by a flat rate for the top income tax bracket. The power parameter $0 \leq \tau \leq 1$ controls the degree of progressivity of the tax system, while λ adjusts to meet the government's budget requirement.²¹

The second term in the maximum operator imposes a cap on the marginal tax rate, τ_{max}

¹⁹Only about 20% of US households hold stocks or mutual funds directly (Bover, 2010; Heaton and Lucas, 2000).

²⁰This figure, like those on government expenditure and pensions used below, comes from NIPA Tables and is an average for the years 2010 to 2016.

²¹This formulation of the income tax system captures net transfers that are non-monotone in income, such as the earned income tax credit and welfare-to-work programs. See Guner et al. (2014), Heathcote et al. (2017) and Bakış et al. (2015) for evidence on the fit of this function.

set to 39.6%, as reported by the IRS. y_b denotes the critical level of taxable income at which the top marginal tax rate is reached: $\lambda(1 - \tau)y_b^{-\tau} = 1 - \tau_{max}$. We calibrate the progressivity of income tax system, τ , to the difference between the average income tax rate paid by the top 1% and the bottom 99% of the income distribution. Piketty and Saez (2007) report this value to be 6.8%.

Tax revenue finances exogenous expenditures, pension payments and transfers. The expenditures are set at 15.5% of GDP to yield a sum of expenditure and transfers of 26.1% of GDP, as observed in the data. In addition, the government makes lump-sum transfers to all households. In the data, these transfers represent 2.7% of GDP in the form of disability benefits, veterans benefits etc. We set the transfers in the model Tr accordingly. In the last step, we choose λ in the personal income tax function to balance the government's budget.

Pension benefits are modeled after the US social security system as described in the US Social Security Bulletin (Social Security Administration, 2013). The benefit formula features two bend points (bp_1 and bp_2 expressed as multiples of average earnings), three replacement rate brackets (90%, 32%, and 15%), and a maximum receivable pension benefit (b^{cap}). The benefit for an individual retiring with productivity z is

$$b(z) = \xi \min\{b^{cap}, 0.9 \min(\tilde{e}(z), bp_1) + 0.32 \max[\min(\tilde{e}(z), bp_2) - bp_1, 0] + 0.15 \max(\tilde{e}(z) - bp_2, 0)\},$$

where $\tilde{e}(z)$ are average earnings of working age agents of productivity z in the model's stationary equilibrium. The formula reported by SSA is for an individual, whereas the model is based on households, which may contain non-working spouses or survivors. Therefore, we adjust benefits by a factor, ξ , and calibrate it to match the average ratio of social security expenditure to GDP in the data for the years 2010 to 2016.

5.6 Bequests

Recall the utility from bequests in Section 4.1: $\phi(k) = \phi_1[(k + \phi_2)^{1-\sigma_c} - 1]$. The parameter ϕ_2 represents the degree of non-homotheticity of bequests, while ϕ_1 controls the overall preference for bequests. We choose these parameters to match the bequest-to-wealth ratio reported by Guvenen et al. (2019a), as well as the share of all bequests accounted for by the top 2% largest bequests, which is 40% (Feiveson and Sabelhaus, 2018).

The model does not feature an explicit link between parents and their offspring, which requires a larger state space, and is computationally challenging. On the other hand, re-

distribution of all bequests among younger agents, a common simplification, curbs the model’s ability to capture the dynastic persistence of wealth. We proceed with a hybrid approach, which can be summarized as follows. We assume that at age 50, the average age of bequest receipt in the data (Feiveson and Sabelhaus, 2018), agents randomly draw a bequest from a mixture of the bequest distributions of the deceased in the model, where the weights in the mixture depend on the recipient’s state: a recipient with permanent productivity component i' and saving return j' draws from the distribution of bequests left by deceased agents with permanent productivity component i and return j ($i, j, i', j' = L, H$) with probability $\gamma(i', j'; i, j)$.²² To limit the number of parameters, we model $\gamma(i', j'; i, j)$ as $\gamma_z(i, i')\gamma_\kappa(j, j')\bar{\Gamma}(i, j)/\tilde{\Gamma}(i', j')$, where $\gamma_z(i, i')$ equals the parameter $\bar{\gamma}_z \in [0, 1]$ if $i = i'$ and $1 - \bar{\gamma}_z$ otherwise, and analogous for $\gamma_\kappa(j, j')$. $\bar{\Gamma}(i, j)$ denotes the fraction of deaths with states (i, j) , and $\tilde{\Gamma}(i', j') = \sum_{i,j} \gamma(i', j'; i, j)$ ensures that the probabilities sum to one.

These assumptions allow the model to capture intergenerational correlations by ensuring that the bequest received by a child is more likely to come from a parent with similar characteristics. Concretely, if $\bar{\gamma}_z (\bar{\gamma}_\kappa) > 1/2$, high-productivity (high-return) children are more likely to receive a bequest from a high-productivity (high-return) parent. We calibrate $\bar{\gamma}_z$ and $\bar{\gamma}_\kappa$ to match the intergenerational correlations of wages and wealth of 0.3 and 0.365 reported by Solon (1992) and Charles and Hurst (2003), respectively.

Table 6 shows the resulting values for parameters that are calibrated outside the model. Table 7 presents the parameters that are estimated internally. A summary list of all targeted moments is provided in the Appendix Table B.3. The following section discusses the fit of the model.

6 The Benchmark US Economy

In this section we discuss the fit of the model to the distributions of earnings, income and wealth, followed by a discussion of earning and rate of return processes implied by the calibration. As an overidentification check, we also compare the model’s implications for the evolution of earnings, income and net worth over the life-cycle.

²²For this purpose, we treat the top productivity states z_7, z_8 like f_H , and the top return state κ_{top} like κ_H .

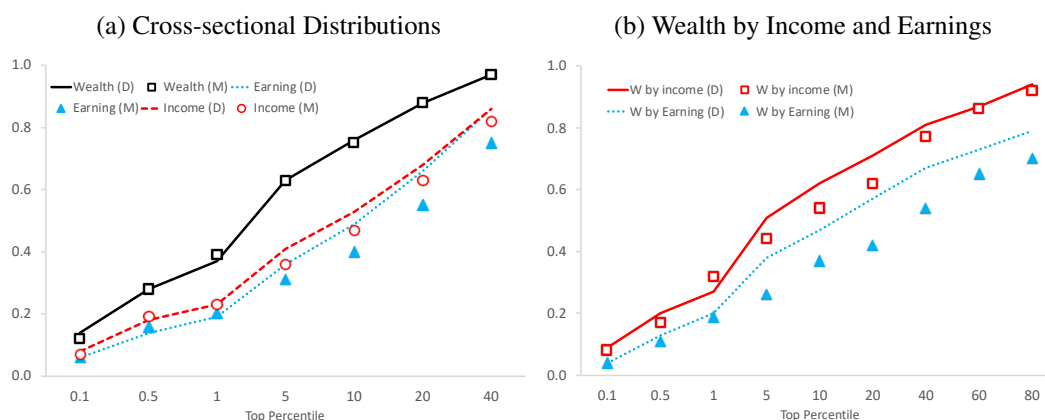
Table 6 – Calibration of the Model: Preset Parameters

Parameter	Description	Value	Source
<i>Demographics</i>			
J	Maximum life span	16	corresponds to age 100
J_R	Mandatory retirement age	10	corresponds to age 65
s_0, s_1, s_2	Survival probability by age	-5.49, 0.15, 0.016	Halliday et al. (2019)
<i>Preferences</i>			
σ_c	Risk aversion	1.5	
σ_l	Inverse Frisch elasticity	1.22	Blundell et al. (2016)
<i>Technology</i>			
δ	Depreciation (annual)	0.045	
<i>Labor productivity</i>			
See Section 5.3			
<i>Taxes and transfers</i>			
τ_c	Marginal corporate tax rate	0.236	Gravelle (2014)
τ_s	Consumption tax rate	0.05	Kindermann and Krueger (2014)
Tr	Government transfers/GDP	2.7%	NIPA Table 3.12

Table 7 – Calibration of the Model: Jointly Calibrated Parameters

Parameter	Description	Value	Parameter	Description	Value
β	Annual Discount rate	0.98	θ	Labor disutility	6.0
α	Capital elasticity	0.27	$\lambda_{in}, \lambda_{ll}, \lambda_{lh}, \lambda_{hh}$	Transition rates	Table B.2
z_7, z_8	Top productivity states	Table B.2	$\pi_{ll}, \pi_{hh}, \pi_{in}, \pi_{top, top}$	Transition rates	Table 9
$\kappa_L, \kappa_H, \kappa_{top}$	Rates of return	Table 9	d_c	Corporate asset threshold	2.21
τ_l	Tax progressivity	0.183	G/Y	Expenditures / GDP	15.5%
ξ	Pension / Earnings	0.62	$\bar{\gamma}_z, \bar{\gamma}_\kappa$	Bequest correlations	0.65, 0.9
ϕ_1, ϕ_2	Bequest utility	-0.42, 0.19			

Figure 3 – Distribution of Wealth, Income and Earnings



Note.— Panel (a) shows the cumulative shares for the top percentile groups. Panel (b) shows share of net worth held by top income and earning groups. Data values come from SCF 2010 and 2016. Income includes capital gains.

6.1 Distributions of earnings, income and net worth

Figure 3 presents the distributions of earnings, income and net worth in the calibrated model (markers) and compares them to the data (lines). Panel (a) shows the marginal distributions for top percentiles of each variable. The model captures the high concentration of net worth very well, even among the top fractiles, as the model markers are almost exactly on the data line. This implies that model replicates the Pareto tail of the empirical distribution of net worth. The overall Gini coefficient for net worth is 0.83 in the model, which is very close to the 0.85 in the data. Similarly, the concentration levels of income and earnings for top groups is in line with the data. Panel (b) shows the shares of net worth held by different income and earning groups, which is not directly targeted in the calibration. The model generates a strong correlation between income and net worth, as observed in the data, and closely matches their joint distribution. The model also captures the strong connection between earnings and net worth.

Next, we compare the factor composition of income for different income groups. Table 8 shows the share of labor income in total income for various income groups. The labor component of income is 64% in the model for the top 1% of incomes, identical to the data value. The labor share for the top 1% wealthiest households in the model, which is not targeted in the calibration, is 49%, somewhat below the data value of 55%.

Table 8 – Share of Income from Labor by Income Groups

	All	Top Percentiles			Quintiles				
	0-100	99-100	95-99	90-95	5th	4th	3rd	2nd	1st
Data	0.82	0.64	0.78	0.88	0.78	0.93	0.91	0.82	-0.46
Model	0.79	0.64	0.81	0.78	0.77	0.85	0.84	0.78	0.07

Notes.— Data comes from the 2010 and 2016 waves of the SCF. See text for details.

Overall, the model captures the distributions of earnings, income and net worth. In particular, it features a highly skewed tail of the net worth distribution, generates a realistic correlation between earnings and net worth and a realistic share of income from labor for top income groups. Next, we discuss the stochastic processes for labor efficiency and the rate of return on assets implied by the calibration procedure.

6.2 Labor productivity process

The extraordinary productivity states are critical for generating the concentration of earnings observed in the data. In the model, workers in these states (z_7 and z_8) are 29 and 288 times as productive as the average worker, and they represent 0.63% and 0.02% of the population at the stationary state. But earnings are a combination of productivity and hours worked. The earnings levels for the top 0.1, 0.5 and 1 percent of earners are 61, 30 and 19 times the average in the model, very close to the levels of 60, 28 and 19 in the data. The model therefore features realistic skewness of the earnings distribution.

Each period, an ordinary worker has a 0.2% chance to experience an extraordinary productivity boost. This state is about as persistent as ordinary productivity states. The probability of remaining among the top 1% of earners after 5 years is 57% in the model. Kopczuk et al. (2010) estimate this probability to be 62% at the individual level using data on earnings from the Social Security Administration.²³

Another way to test the dynamic properties of the productivity process is to compare the distribution of earnings growth in the model with that in the data. Guvenen et al. (2019b) document that earnings growth of the top 1% of individual earners is characterized by a

²³The transition matrix for the earnings process and the earnings levels implied by the calibration procedure are shown in the Appendix in Table B.2.

Table 9 – The Transition Matrix for Rates of Return on Assets

from \ to	κ_L	κ_H	κ_{top}
κ_L	0.99	9.75×10^{-3}	2.5×10^{-4}
κ_H	9.75×10^{-3}	0.99	2.5×10^{-4}
κ_{top}	0.0	0.10	0.90
population share (%)	49.4	50.5	0.1
annual rate of return	0.01	0.06	0.245

Note.– Table shows the transition probabilities in the benchmark economy from the rate of return in Column 1 to rates of returns in Columns 2-4. The annual rates of return associated with each state and the share of the population in each state are reported in the last two rows.

large standard deviation, a high degree of kurtosis, and negative skewness. The model replicates the standard deviation of earnings growth for this group of 1.7 exactly, even though this moment was not targeted. It generates a skewness of -3.0 and a kurtosis of 11.8, comparable to the estimates in Guvenen et al. (2019b) of -1.3 and 8.3.

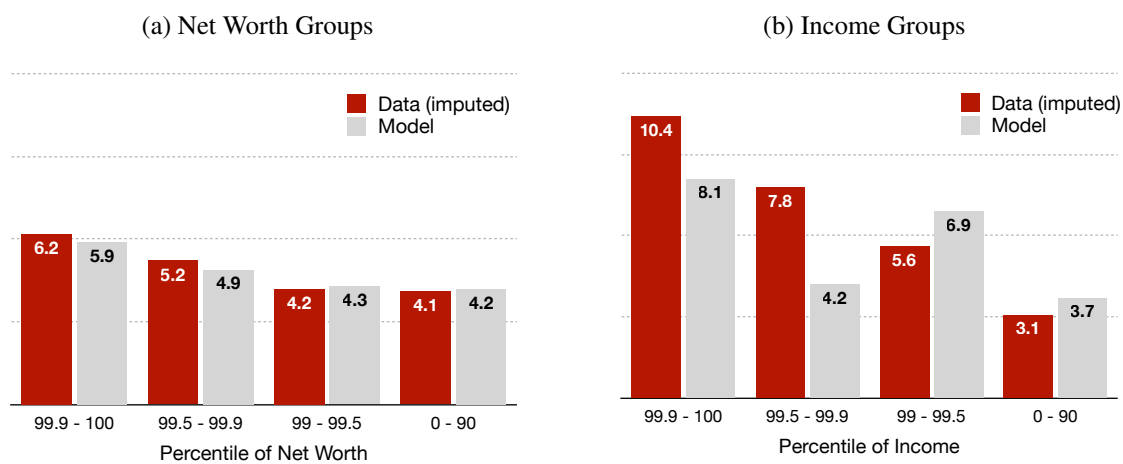
Overall, the estimated earning process captures fundamental properties of the earnings distribution well. It closely matches the cross-sectional distribution of earnings, while also capturing the dynamic aspects of earnings growth.

6.3 Rate of return heterogeneity

The rates of return on assets and the corresponding transition matrix are shown in Table 9. The three rates are 1%, 6% and 24.5%. 0.11% of households enjoy the top rate of return. The lower rates are highly persistent, with a 99% probability that the rate of return remains the same between periods. The persistence for the top category is 90%. Combined with the very high persistence, one could think of three types of households: one that invests in a savings account, a one that holds stocks, and a small third one that, for some time, has access to a very lucrative investment opportunity. While the rates of return are highly persistent, they are far from permanent. As a result, the dispersion in average, life-time rates of return across households is smaller than in the cross-section. Given the transition rates in Table 9, the average (unweighted) life-time rate of return is 3.55% with a standard deviation of 2.43%.

Figure 4 shows the average rates of return in the benchmark economy by wealth and income for a cross-section of households. The red bars report the corresponding data val-

Figure 4 – Rates of Return by Wealth and Income: Model vs. Data



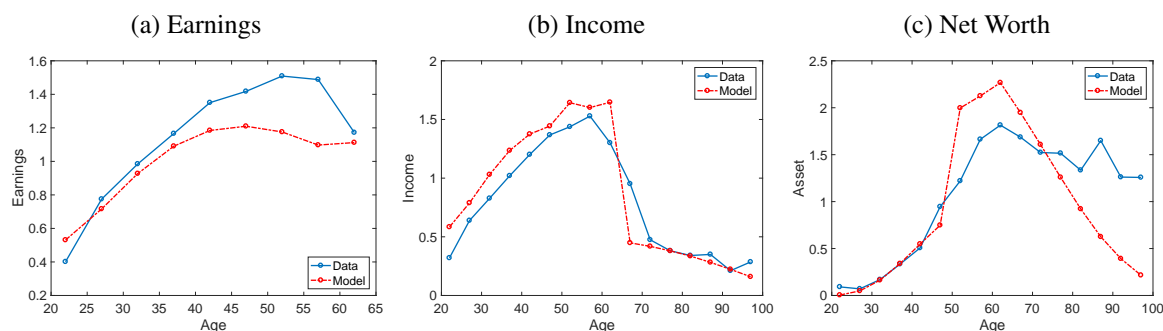
Note.— Figure shows the rates of return on assets by household income and net worth implied in the model. Data comes from the 2010 and 2016 waves of the SCF. See notes to Figure 2 for explanations.

ues from Figure 2. Model and data values are generally very close, in particular for the partition by wealth. Higher wealth (Panel a) and income (Panel b) groups have slightly higher rates of return. Because top labor productivity is limited to two states, the pattern by income is choppy in the model, which slightly overshoots the average return of households between the 99-99.5th percentiles and undershoots the next category. Note that this scale dependence is not hardwired in the model—it emerges endogenously as households with higher returns are more likely to be wealthy.

The difference in rates of return across wealth groups is modest. The main difference in wealth accumulation across wealth groups therefore comes from differences not in asset returns, but in saving rates. The saving rate out of income among the wealthiest 1% is 38% compared to the 18% for the aggregate economy. These findings are consistent with Saez and Zucman (2016) who report small differences in rates of returns but large differences in saving rates across wealth groups in the data.²⁴

²⁴Saez and Zucman (2016) compute synthetic saving rates based on the evolution of top wealth shares, composition of assets and the market returns on those assets, assuming no mobility across wealth groups. They find saving rates of 38% for the wealthiest 1% and 10% for the aggregate economy between 2010 and 2012, the latest reported years. The corresponding synthetic rates are 32% and 13% in our model.

Figure 5 – Earnings, Income and Wealth over the Life-Cycle



Note.— Solid lines depict the life-cycle profiles of average earnings, income and net worth implied by the benchmark calibration. Dashed lines show the data values from the SCF.

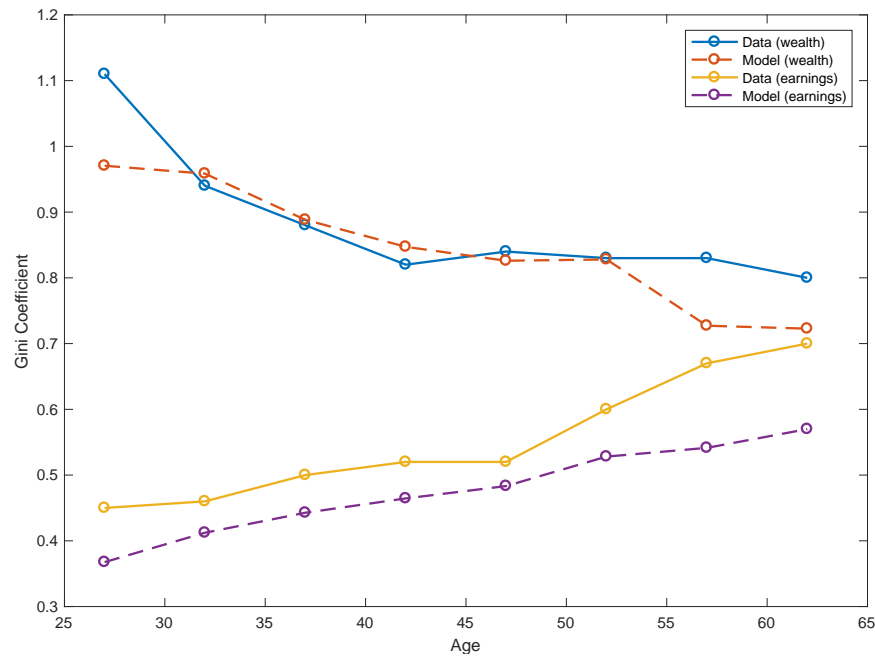
6.4 Implications for life-cycle dynamics

Next, we analyze the model’s implications for the evolution of income and wealth over the life-cycle, and compare it with the data. Note that age-dependent distributions of income and wealth are not targeted in the calibration. Therefore, this analysis provides an overidentification test of our model.

Figure 5 compares average earnings, income and wealth by age group in the models with data from the SCF. The age profile of labor productivity is calibrated to the age profile of wages in the PSID. The earnings profile depicted in Figure 5a is a result of households’ labor supply decisions given the wage rates and their assets. This is the primary source of income for young households, as their assets are initially close to zero. With age, households accumulate assets, and start generating investment income. Average wealth increases up until the retirement age. After retirement, agents start consuming out of their savings. The model accurately captures the salient features of the life-cycle dynamics of income and wealth (apart from the well-known issue that life-cycle models predict too quick wealth decumulation in retirement; see e.g. De Nardi et al. (2009) and Kopecky and Koreshkova (2014)). The fact that the calibrated model closely replicates earnings, income and wealth patterns over the life cycle demonstrates its ability to accurately capture the labor supply and savings behavior among households.

Figure 6 shows the evolution of earnings and wealth dispersion in the model in comparison with the data. The rise in the dispersion of earnings is governed by the productivity process described in Table B.2. Earnings inequality grows mainly because the wages of

Figure 6 – Earnings and wealth inequality over the Life-Cycle



young households are similar to each other. With age, some households move to higher earning states, and some to top earning states.

The Gini for wealth is initially very high, because most youngest households have little assets and weak saving motives in anticipation of earnings growth. The presence of many households without assets delivers a high Gini coefficient. With age, earnings grow and retirement approaches. As a result, asset accumulation becomes more prevalent among households. This reduces the Gini coefficient in the first part of the life-cycle. About 15-20 years after market entry, the reduction in the wealth Gini is counteracted by the increasing dispersion in earnings and income, which raises wealth dispersion. These two forces are more or less equivalent, resulting in a stable dispersion of wealth for middle-aged groups and older, as in the data.

The model also generates plausible age profiles of wealth across the distribution. For instance, the average age in the top 1% of wealth is 62.8 years in the model, compared to 61.1 in the data.

Overall, the benchmark economy provides an accurate description of the distributions of earnings, income and wealth. The productivity process captures the salient features of

earnings growth both in the short run and over the life-cycle. The factor composition of income is realistic, including at the top of the distribution. The implied wealth distribution is highly concentrated at the top and correlated with earnings and income, as in the data. Next, we provide a quantitative assessment of the relative importance of different modeling approaches to wealth concentration.

7 Determinants of Wealth Concentration

To quantify the relative roles of earning concentration, rate of return differences and bequests in shaping wealth concentration, we conduct two experiments. First, we shut down different model components and compare the implied wealth concentration with the data. Second, we force the model to match the observed wealth concentration by shutting down each model component and recalibrating the others. Then, we contrast the implied joint distribution of earnings and wealth with the data. This allows us to highlight the sources of identification in our benchmark calibration.

7.1 Decomposition analysis

Table 10 shows the decomposition results. The first two rows report measures of earnings and wealth concentration along with the top 1% labor income share in the data and in the benchmark model economy. Each of the remaining rows takes away critical model components and reports the counterfactual values of the same moments.

We begin by fully eliminating the heterogeneity in the rate of return by setting κ to its value-weighted average in the benchmark economy for all households. Doing so reduces the Gini coefficient for wealth from 0.83 to 0.79, the top 1% wealth share from 37% to 34%, and the top 0.1% share from 14% to 11%. Much of the wealth concentration remains intact, however, as a result of savings out of earnings. In this scenario, top incomes comprise mainly of labor income with its share among the top 1% income earners at 72%, above its benchmark value of 63%.

Next, we investigate the effects of the bequest motive and our modeling assumptions regarding the distribution of bequests and intergenerational links. After restoring the rate-of return heterogeneity to its benchmark, we conduct three experiments. First, we set ϕ_2 to zero, making the bequest motive homothetic. Next, we remove the correlation of bequests with parental wealth by setting both $\bar{\gamma}_z$ and $\bar{\gamma}_\kappa$ to 0.5, so that all recipients draw their bequest

Table 10 – Determinants of Wealth Concentration: A Decomposition Analysis

	wealth Gini	top wealth shares		top earnings shares		top 1% LIS
		0.1%	1%	0.1%	1%	
data	0.85	0.14	0.37	0.06	0.19	0.64
benchmark	0.83	0.14	0.37	0.06	0.18	0.64
counterfactual economies with ...						
... (1) no return differences	0.79	0.11	0.34	0.06	0.18	0.72
... (2) homothetic bequests	0.80	0.13	0.36	0.06	0.18	0.65
... (3) uncorrelated bequests	0.82	0.13	0.36	0.06	0.18	0.65
... (4) equal bequests	0.73	0.11	0.30	0.06	0.19	0.69
... (5) no top earners	0.74	0.07	0.16	0.004	0.04	0.47

Note.– Results from model simulations. LIS denotes labor income share of top 1% of incomes. Economy (1) in the third row sets κ to its value-weighted average in the benchmark economy. Economy (5) sets the extraordinary productivity levels to that of the highest “regular” earnings category ($z_8 = z_7 = z_6$).

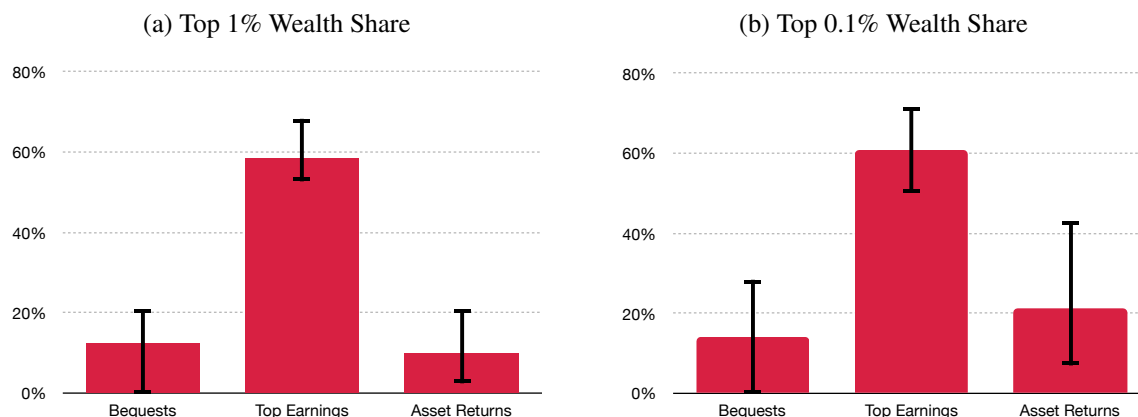
from the same distribution. Finally, we compute results for an economy where bequests are equally distributed, a common application in the quantitative literature on overlapping generations models.²⁵ In all three cases, we adjust ϕ_1 such that the overall importance of bequests in the economy, as measured by bequests relative to wealth, is unchanged.²⁶ The first two experiments lead to similar results, namely slightly lower top wealth shares and a small decline in the Gini coefficient for wealth compared to the benchmark economy. The largest changes occur when bequests are fully redistributed among recipients. In this case, the Gini coefficient drops to 0.73, and the top wealth shares fall by one fifth to one quarter. Overall, bequest inequality has a significant impact on the wealth distribution, as it perpetuates the wealth dispersion across generations. The labor income share of the top 1% of incomes is slightly higher at 69%, as fewer top income earners have had a large bequest.

Finally, we remove superearners from the benchmark economy by setting the productivity at the two extraordinary states to that at the highest “ordinary” state: $z_8 = z_7 = z_6$. This preserves the wage distribution among the remaining states. Earnings are much less concentrated in this scenario, with a top 1% earnings share of only 4%, compared to 19% in

²⁵Notable exceptions are De Nardi (2004) and De Nardi and Yang (2016) among others.

²⁶This change is minor and hardly affects our results.

Figure 7 – Factors of Wealth Concentration



Note.— Figure shows the marginal contribution of each factor to the concentration of net worth relative to the benchmark economy. The whiskers show the range of marginal effects obtained by permuting the order in which factors are eliminated from the benchmark economy. The column height represents the average across permutations.

the data. This immensely reduces wealth concentration. The wealth share of the wealthiest 1% is 16%, down from 37%, and that of the wealthiest 0.1% is 7%, down from 14%. These changes to the wealth distribution are markedly larger than those implied by eliminating the differences in rates of return or bequests. Because there is still substantial earnings dispersion outside of the top groups, the overall drop in wealth dispersion, while sizable, is less extreme—the Gini coefficient drops from 0.83 to 0.74. Without superearners, top incomes now comprise mainly of returns on assets. At 47%, the labor income share among the top 1% of incomes is significantly below its data value of 64%.

Eliminating the different factors individually may mask potential interactions between them. Differences in returns on assets or bequests, for instance, may matter more when earnings are highly concentrated. To measure the interaction effects, we remove multiple model components at once, permuting the order in which they are removed. We then compute the marginal effect of each factor across different permutations. For each channel, four different marginal effects can be computed. For example, top earners can be removed starting in a situation where all channels are active, where only one other channel is active (two permutations), or where only the top earner channel is active.

Figure 7 summarizes the range of marginal effects of each factor to wealth concentration across these permutations.²⁷ The left (right) panel shows the marginal effect on the top 1% (top 0.1%) wealth share. For each channel, the bar represents the average of four

²⁷The simulation results for all permutations are reported in Table B.4 in the Appendix.

marginal effects expressed as a fraction of the benchmark value. The smallest and largest marginal effects are shown as whiskers. By all measures, the contribution of top earners to wealth concentration is large, as removing top earners leads to declines in the top 0.1% and top 1% wealth shares by half or more in all scenarios. The contribution of asset returns to top wealth shares is moderate with the largest impact seen on the wealthiest top 0.1% at about 20 percent of their benchmark share in wealth on average. The marginal effect of unequal bequests ranges from zero to almost a third. Note that bequests on their own do not add much to wealth concentration. They mostly amplify the other channels, by perpetuating wealth inequality that is created within a generation.

The reason why differences in rates of returns alone do not generate a larger impact on wealth concentration is that life is too short for the calibrated rates of return to have a larger impact. Even an agent who permanently makes 3 times the average wage, corresponding to the highest “regular” productivity level z_6 , and enjoys a life-long 24.5% rate of return joins the wealthiest 1% only after age 50, and the 0.1% after 60, assuming that they receive an average bequest.²⁸ Agents with lower productivity levels or rates of returns do not join the wealthiest 1% within a lifetime. Since agents start their lives with little wealth, high returns have little effect in the short term, as they lack high incomes or asset holdings to operate on. As a result, accumulation of wealth by top-return agents remains limited unless they are also extremely productive or receive a large bequest at a young age. But the latter scenarios are inconsistent with life-cycle progression of earnings and wealth in the data, which the model replicates. These findings are in line with the inherent property of models with return heterogeneity that the wealth distribution fans out slowly (Gabaix et al., 2016).

High earnings states instead have a strong effect on wealth concentration. For top earners, the objective of accumulating a sizeable retirement nest egg dictates high saving rates, which results in a faster growth of wealth relative to earnings over the life cycle (Figure 5). This maps into a high concentration of wealth.²⁹

These experiments highlight the indispensability of earnings heterogeneity in models of wealth distribution. High earnings concentration not only directly manifests itself in wealth concentration but can also amplify the dispersion in capital income attributable to asset returns and bequests.

²⁸These figures are computed using the benchmark savings policy functions to compute paths for wealth by age for these groups.

²⁹This is consistent with Sargent et al. (2020) who theoretically show that saving out of earnings can lead to a wealth distribution with a fatter tail than the earnings distribution.

Table 11 – Alternative calibrations that match the top 0.1% wealth share

	top wealth shares		top earnings shares		top 1% LIS by		correlation of wealth with	
	0.1%	1%	0.1%	1%	income	wealth	earnings	income
data	0.14	0.37	0.06	0.19	0.64	0.55	0.35	0.52
benchmark	0.14	0.37	0.06	0.18	0.64	0.49	0.20	0.65
alternative calibrations:								
...(1) no return het.	0.14	0.35	0.07	0.19	0.72	0.57	0.36	0.49
...(2) no top earners	0.14	0.22	0.004	0.04	0.31	0.07	0.01	0.67

Note.— Table shows simulation results from an economy without return heterogeneity or without top earning states. In alternative calibration (1), κ is set to its asset-weighted mean in the benchmark economy for all households. z_8 is set to 550 (1.45 times its benchmark value) to match the top 0.1% wealth share of 14%. In alternative calibration (2), $z_8 = z_7 = z_6$. The top return is raised to 28% per year to match the top 0.1% wealth share.

7.2 Alternative calibrations and the labor income share

In this section, we consider two alternative calibrations where we remove either the return heterogeneity or the top earners, and partially re-calibrate the model to match the observed wealth concentration in the data. We then compare the implied correlations between income, earnings and wealth with the data to underline the sources of identification in our benchmark calibration.

In the first case, we set κ to its asset-weighted mean from the benchmark economy for all households. To ensure that the model still matches top wealth concentration, as measured by the top 0.1% wealth share, we adjust the value of the top productivity state z_8 . This requires raising it by 45%. In the second case, we eliminate the top productivity state by setting $z_8 = z_7 = z_6$, and adjust the value of the top return κ_{top} to match the top 0.1% wealth share. This requires a top return of 28% per year.

By construction, both calibrations match the top 0.1% wealth share in the data. But as shown in Table 11, they deviate from the benchmark calibration, and thus from the data, in other dimensions. This is particularly pronounced for the calibration without the top productivity state. In this case, the economy features very little concentration of earnings and an unrealistically low share of labor income for top income and wealth groups. Among top income groups, the implied labor share of income is 0.31, compared to 0.64 in the benchmark. Top wealth groups rely almost exclusively on capital income, with a labor

share of income of 7% compared to 49% in the benchmark. The correlation of wealth with earnings virtually drops to zero.

Counterfactually low labor income shares among the wealthy are common in quantitative work that relies primarily on dispersion in capital income to generate a highly skewed wealth distribution. This can be seen by employing equation (1) to calculate the labor income share for the wealthiest for such analyses. For Benhabib et al. (2019), for instance, we calculate the labor share of income among the wealthiest 1% of households to be between 8% and 21%, depending on the correlation between earnings and wealth in their simulations.³⁰ Their labor income share is low because the wealthiest 1% have 34 times the average assets and enjoy higher-than-average returns on those assets, but their earnings are at most three times the average earnings in their model economy (assuming perfect correlation of earnings and wealth). As a result, their capital income swamps their labor earnings. Similarly, in Hubmer et al. (2020), the implied labor share of the wealthiest is bounded between 6.5% and 26.2% depending, once again, on the correlation of earnings and wealth.³¹ In comparison, Piketty and Saez (2003) report the corresponding labor share in income as 62.5% for the wealthiest 1% in 1967, the target year in Hubmer et al.'s calibration.

In contrast, an economy with homogeneous returns features an excessively high labor income share at the top and a top 0.1% earnings share that, at 7%, slightly exceeds the benchmark and data values of 6%. It also features an intergenerational wealth correlation that, at 7%, lies significantly below its benchmark value. Otherwise, this alternative calibration does not deviate much from the benchmark.³²

Taken together, these exercises illustrate how our empirical approach allows identifying the quantitative drivers of wealth concentration. Results are unambiguous: while there is clear evidence of some importance of heterogeneous returns, stemming from the labor

³⁰Defining the aggregate economy as the benchmark group, equation (1) gives the labor share of the wealthiest 1% as a function of their relative assets, $k_{1pct/0} = 33.6$ (from Table 10 in Benhabib et al. (2019)), relative rates of return $r_{1pct/0} = 6.01\%/3.94\%$ (pg. 1638), aggregate labor share, $\lambda_0 = 0.82$ from the SCF, and relative earnings of the wealthiest 1%, $e_{1pct/0}$. The latter figure is not reported. It is bounded from above by the relative earnings of the top 1% *earners* (calculated as 3 from Table 1) – corresponding to perfect correlation of earnings and wealth – and from below by 1, which corresponds to zero correlation. The likely value of that correlation is much lower, as in our counterfactual exercise in Table 11, so we suspect that the labor income share is closer to its lower bound.

³¹There, the wealthiest 1% have 27.4 times the average assets holdings, and roughly twice the average rate of return, and the highest 1% of earners make 5.1 times the average.

³²Table B.5 in the Appendix shows that these results are similar when the alternative calibrations keep the top 1% wealth share as in the benchmark.

income share at the top, overall wealth concentration is to a large extent driven by the concentration of earnings.

8 Discussion

Our findings indicate a significant role for differences in labor income and earnings risk in explaining the observed dispersion in net worth in the US. This is driven essentially by the high concentration of earnings and the large share of earnings in total income among the top income and wealth groups in the data.

The relevance of earnings for wealth concentration warrants a deeper analysis of the determinants of the concentration of earnings. Routes of inquiry that appear promising include human capital accumulation by top earners (Huggett and Badel, forthcoming; Karahan et al., 2019), labor market frictions, in particular among low earnings groups (Karahan et al., 2019), and production complementarities and changes in the degree of assortative matching both between workers and firms and among workers across firms (Geerolf, 2016; Song et al., 2019).

Earnings concentration is also partly driven by the entrepreneurial incomes. In fact, in an economy where entrepreneurs are endowed with a diminishing-return-to-scale production function and do not face any credit restrictions when investing in their businesses, differences in entrepreneurial productivity are fully reflected in the labor component of income. This is because the optimal investment in the business requires that the marginal return to business investment be equal to the common market return on capital. Business income then simply is the sum of a capital income component with a common return and a labor income component that varies across entrepreneurs (see Appendix C for the derivation). From the perspective of our model and data analysis, productivity differences are then captured as earnings differences.

We also find evidence of differences in rates of return on assets at the household level. An unusually high rate of return for a small group of households is required to explain the very top tail of the wealth distribution. To some extent, higher rates of return could reflect the capacity to handle high exposure to risk among some households. They could also capture variation in the marginal return on capital arising from credit constraints on entrepreneurial investment. In our model, these are picked up as differences in the return to capital. Under such an interpretation, the extent of the variation in rates of return depends on the tightness of credit constraints and on productivity differences among con-

strained entrepreneurs. Changes in these two factors affect both entrepreneurs' labor and capital income (see Appendix C for a brief discussion). This interdependence between an entrepreneur's labor and capital income makes it impossible to counterfactually eliminate heterogeneity in only one income type for entrepreneurs. Eliminating credit constraints would eliminate rate of return differences. It would also amplify earnings differences. As a consequence, our estimate of the effect of return heterogeneity on wealth concentration, which kept earnings differences constant, could be considered an upper bound. Eliminating productivity differences would reduce both earnings differences and rate of return heterogeneity. Therefore, our results on the effect of top earnings on wealth concentration could be considered a lower bound. An analysis of wealth concentration that models earnings concentration and rate of return heterogeneity among entrepreneurs more explicitly while matching the joint distribution of income and wealth remains an open topic for future research.

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Online Appendix

A Data Appendix

The primary sources of data are the 2010 and 2016 waves of the Survey of Consumer Finances (SCF), sponsored by the Federal Reserve Board in cooperation with the Department of the Treasury.^{A1} The measure of net worth is the difference between total assets and total liabilities, as provided by the SCF. To compute total market income, we sum wage and salary income, business and farm income, interest and dividend income, private pension withdrawals and capital gains. Some tables exclude capital gains from income, as explained in the text and in the table notes. Income from fiscal sources, such as transfer income or social security income, is excluded.

Wage shares reported in the tables represent wage and salary income divided by total market income. The labor income share reported in the tables additionally includes part of business and farm income for a subset of households. The SCF distinguishes between business income from actively managed business or farm and that from non-actively owned businesses. Specifically, we impute wage income in two situations. First, when the entire household reports no wage and salary income at all, but report a positive income from an actively managed business or farm and positive equity invested in an active business. These households constitute 5.5% of the sample and 2.8% of the population. Second, we impute wage income when the respondent (R) or their spouse (SP) reports active business income, reports self-employment as their main job *and* reports not having drawn any wage and salary income from their business. These households constitute 8.9% of the sample and 6.1% of the population. Importantly, we do not modify any wage and salary income reported to the SCF by active business owners.

Our imputation sample also excludes situations where active involvement in the business is not the main job of the respondent or their spouse. It also excludes other members of the household, e.g. the children, who are actively involved in the business but do not report wage or salary income. Therefore, our measure of the labor's share of income likely understates the true labor share.

Most of the households with imputed wages do not belong to the top income or wealth groups. 2.8% of all the households in our imputation sample belong to the top 1% income

^{A1}The public use data files are available for download at <https://www.federalreserve.gov/econres/scf-previous-surveys.htm>.

group and 3.1% belong to the wealthiest 1% of the population.^{A2} These are better odds than in the general population. Consequently, roughly a quarter of the households in the top 1% income or wealth group have some imputed wages.

To estimate the capital share of business income we regress active business income on the equity invested in the business among households who report no wage and salary income, but report a positive income from an actively managed business or farm, our first imputation sample.^{A3} The regression specification is:

$$\ln Y_i = \text{cons.} + \alpha \ln K_i + \beta \ln L_i + \epsilon_i,$$

where Y_i is household's total income from the business, K_i is equity invested in the business and L_i is the effective labor input, including hours of work as well human capital or entrepreneurial acumen. The implicit assumption behind this regression is that the investment income from the business is distributed in proportion to the equity of the shareholders, so that the capital component of business income is αY_i . The SCF reports active business equity, which we include as K_i . The effective labor input, L_i , is not directly observed. To control for the labor input, we include the following variables as controls: indicators for categories of educational attainment, age, race, gender, major occupation category, survey year interacted with educational attainment and occupation, and (log) total hours worked by the respondent and their spouse for the business, interacted with educational attainment and occupation. These variables capture the quantity of labor input with hours worked and the quality of the labor input with demographic variables as well as education and occupation. The estimated value of α is 0.27 (s.e. 0.03), which is what we use to apportion business income into its capital and labor components.^{A4}

B Supplementary Tables

This appendix presents additional results mentioned in the main text. Table B.1 shows the values for wage and labor components of income (for different income and net worth

^{A2}95% belong in neither top group.

^{A3}We exclude households from the regression sample if anyone other than the respondent or their spouse is actively involved in the business since hours worked for the business is only available for R and SP.

^{A4}As common in the literature, we use the demographic measures, educational attainment and occupation for the head of the household as details are not available for each member of the household who actively participates in the business. Omitting those variables related to the quality of the labor input altogether and including only total number of hours worked results in an estimate 0.26 (0.03) instead.

Table B.1 – Labor Component of Income by Income and Wealth Group

Income Percentile	99-100	99.9-100	99.5-99.9	99-99.5	95-99	90-95	0-100
Wage Income							
with capital gains	0.49	0.37	0.52	0.60	0.69	0.83	0.74
without capital gains	0.56	0.49	0.56	0.67	0.73	0.84	0.77
Labor Income							
with capital gains	0.59	0.47	0.64	0.69	0.76	0.87	0.80
without capital gains	0.68	0.63	0.68	0.75	0.80	0.89	0.84
Net Worth Percentile	99-100	99.9-100	99.5-99.9	99-99.5	95-99	90-95	0-100
Wage Income							
with capital gains	0.40	0.26	0.40	0.56	0.58	0.71	0.74
without capital gains	0.47	0.32	0.46	0.63	0.62	0.74	0.77
Labor Income							
with capital gains	0.51	0.35	0.49	0.69	0.69	0.78	0.80
without capital gains	0.59	0.44	0.64	0.77	0.73	0.81	0.84

Note.– Table shows wage and labor shares of total income by percentiles of the income and net worth distribution. Labor income includes imputed wage income for active business owners who do not draw salary from their businesses. Data comes from the 2010 and 2016 waves of the SCF.

groups) that are used to plot Figure 1.

Table B.2 reports the calibrated values for the labor productivity states and the corresponding transition probabilities. The initial distribution represents the share of workers in each productivity state at labor market entry. Since the initial distribution of young workers is different from the invariant distribution, and because agents have finite lives, the population shares of workers across productivity states is different from the invariant distribution. The population shares for the working age population are reported in the last row of the table. Retired agents have zero labor productivity.

Table B.3 shows a summary list of calibration targets along with their sources and the associated values obtained in the benchmark economy.

Table B.4 shows the details of the counterfactual economies used to calculate the marginal contributions of superearners, bequests and asset returns depicted in Figure 7. The benchmark economy (0) is reported in the first row. Each counterfactual economy removes the factors of wealth concentration in different combinations (economies 1-9). Economies 1, 4 and 5 are repeated from Table 10 in the main text. For each factor, four different marginal

Table B.2 – Productivity Transitions in the Benchmark Economy

	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8
$z_1 = 1.00$	0.874	0.119	0.004	0	0	0	0.002	0
$z_2 = 1.97$	0.060	0.878	0.060	0	0	0	0.002	0
$z_3 = 3.89$	0.004	0.119	0.874	0	0	0	0.002	0
$z_4 = 3.24$	0	0	0	0.874	0.119	0.004	0.002	0
$z_5 = 6.39$	0	0	0	0.060	0.878	0.060	0.002	0
$z_6 = 12.61$	0	0	0	0.004	0.119	0.874	0.002	0
$z_7 = 137.36$	0.021	0.021	0.021	0.021	0.021	0.021	0.850	0.021
$z_8 = 1349.46$	0	0	0	0	0	0	0.242	0.758
initial distribution	0.044	0.412	0.044	0.044	0.412	0.044	0	0
population share	0.089	0.318	0.089	0.089	0.318	0.089	0.0063	0.0002

Notes.— Table shows the calibrated productivity levels and the corresponding transition probabilities. The last row shows the fraction of working age population in each productivity state.

effects were computed. The marginal contribution of rate of return heterogeneity to top 1% wealth share, for instance, were computed as follows: by the difference between the benchmark economy (0) and the counterfactual economy (1) where only the rate of return differences are eliminated, which gives $0.03 = 0.37 - 0.34$; by the difference between economy (6) where superearners are absent and economy (5) where both superearners are absent and asset returns are common, which gives, $0.08 = 0.16 - 0.08$; by the difference between economy (4) with equal bequests and economy (8) with equal bequests and common asset returns, which gives $0.01 = 0.30 - 0.29$; and, finally, by the difference between economy (7) with equal bequests and without superearners and economy (9) where all three factors are inactive, which gives $0.02 = 0.10 - 0.08$. The whiskers in Figure 7 represent the minimum and the maximum values of the four different marginal effects, namely 0.01 and 0.08, relative the benchmark top 1% wealth share of 0.38. The height of the bar represents the average marginal effect across the four marginal effects: $(0.03 + 0.01 + 0.08 + 0.02)/4 = 0.035$. The marginal effects for other factors are calculated in a similar fashion.

Table B.5 shows the alternative calibrations where superearners are excluded from the benchmark economy and the top rate of return, κ_{top} is recalibrated so that the economy matches the share of the wealthiest 1% in wealth (as opposed to matching that of the wealth-

Table B.3 – Summary of Target Moments

Moment	Source	Data Value	Model Fit	Moment	Source	Data Value	Model Fit
Mean hours worked		0.35	0.35	Soc. Sec. Pay / GDP	NIPA, 2010-16 average	7.9%	8.1%
Top 0.1%, 1% earning shares	SCF 2010 & 2016	Figure 3	Figure 3	Log wealth correlation between parents and kids	Charles and Hurst (2003)	0.30	0.32
Top 0.1%, 1%, 5%, 10% wealth shares	SCF 2010 & 2016	Figure 3	Figure 3	Gini coefficient of wealth	SCF 2010 & 2016	0.85	0.83
Bequest/Wealth	Guvenen et al. (2019a)	~ 1%	1.7%	Top 2% bequest dist.	Feiveson and Sabelhaus (2018)	40%	45%
Difference between average income tax rate for top 1% and 99%	Piketty and Saez (2007)	6.8%	6.5%	Corporate income tax revenue/GDP	NIPA	2.5%	2.0%
Probability of staying in top 1% earners	Kopczuk et al. (2010)	0.62	0.57	Overall labor income share	SCF 2010 & 2016	0.82	0.79
Top 1% labor income share	SCF 2010 & 2016	0.64	0.64	P95-99 labor income share	SCF 2010 & 2016	0.78	0.81
Intergenerational wealth persistence at 4th quintile	Charles and Hurst (2003)	0.26	0.22	Intergenerational wealth persistence at 5th quintile	Charles and Hurst (2003)	0.36	0.32

Table B.4 – Determinants of Wealth Concentration: A Decomposition Analysis

	wealth Gini	top wealth shares		top earnings shares		top 1% LIS
		0.1%	1%	0.1%	1%	
... (0) benchmark	0.83	0.14	0.37	0.06	0.18	0.63
counterfactual economies with ...						
... (1) no return heterogeneity	0.79	0.11	0.34	0.06	0.18	0.72
... (4) equal bequests	0.73	0.11	0.30	0.06	0.19	0.69
... (5) no top earners	0.74	0.07	0.16	0.004	0.04	0.47
... (6) no top earn. & no ret. heterogeneity	0.67	0.01	0.08	0.004	0.04	0.83
... (7) no top earn. & equal bequests	0.65	0.03	0.10	0.005	0.04	0.53
... (8) no ret. het. & equal bequests	0.71	0.10	0.29	0.06	0.18	0.76
... (9) all three channels removed	0.62	0.01	0.08	0.01	0.04	0.84

Note.– Results from model simulations. LIS denotes labor income share of top 1% of incomes. Counterfactual economy (1) sets κ to its value-weighted average in the benchmark economy. Economy (5) sets $z_8 = z_7 = z_6$. Economy (4) fully redistributes all bequests among recipients. The remaining counterfactual economies represents different combinations of these actions.

Table B.5 – Alternative calibrations that match the top 1% wealth share

	top wealth shares		top earnings shares		top 1% LIS by		correlation of wealth with	
	0.1%	1%	0.1%	1%	income	wealth	earnings	income
data	0.14	0.37	0.06	0.19	0.64	0.55	0.35	0.52
benchmark	0.14	0.37	0.06	0.18	0.64	0.48	0.20	0.65
alternative calibrations:								
...(1) no het. returns	0.17	0.38	0.08	0.20	0.70	0.53	0.34	0.49
...(2) no top productivity	0.30	0.38	0.005	0.04	0.20	0.05	0.00	0.62

Note.— Table shows simulations results from an economy without return heterogeneity or without top earning states. In alternative calibration (1), κ is set to its wealth-weighted mean in the benchmark economy for all households. z_8 is set to 703 (1.85 times its benchmark value) to match the benchmark top 1% wealth share of 38%. In alternative calibration (2), $z_8 = z_7 = z_6$. The top return κ_{top} is set to match the benchmark top 1% wealth share. This implies a top return $r\kappa_{top}$ of 31% p.a..

iest 0.1% in Section 7.2. The results are broadly similar in these alternative recalibrations, where the correlation of earnings and wealth plummets to zero and the labor share of income among top income and wealth groups is substantially below its data counterparts.

C Entrepreneurship and the Distribution of Labor and Capital Income

Consider the following portfolio allocation problem for an entrepreneur endowed with a units of assets and a diminishing-return-to-scale business income production function $y^b = \theta k^\alpha$, where θ represents the productivity of the entrepreneur. We implicitly assume that a unit of entrepreneurial labor is supplied inelastically as long as the business is in operation.

$$\max_k y = \theta k^\alpha + r(a - k),$$

where the first term is business income and the second term is market income on excess assets (or debt service if $a < k$ in equilibrium). The optimal business investment k^* solves $\theta \alpha k^{\alpha-1} = r$. Substituting the optimality condition back into the objective function gives:

$$y^* = ra + \underbrace{(1 - \alpha)\theta_i^{\frac{1}{1-\alpha}}(r/\alpha)^{\frac{\alpha}{\alpha-1}}}_{\text{non-capital income}}$$

From the perspective of our approach, this setting is observationally equivalent to a version of our model with a common return on assets and labor income heterogeneity, which here is driven by differences in entrepreneurial ability, θ . Our calibration procedure interprets this as labor income heterogeneity. So to the extent that top income and wealth groups consist in unconstrained entrepreneurs, the cause of wealth concentration is correctly attributed to labor income, which include the return to entrepreneurial labor.^{A5}

Next, consider the case where entrepreneurs are constrained by their assets when investing in their business: $k \leq a$. For entrepreneurs with sufficient assets, given their productivity θ , this constraint does not bind, and the argument above applies all the same. If an entrepreneur is constrained, then the optimal investment is $k^* = a$. Let $r_i = \theta \alpha a^{\alpha-1} > r$ denote the marginal return on business capital of a constrained entrepreneur. Then total income of an entrepreneur can be written as:

$$y^* = r_i a + \underbrace{(1 - \alpha)\theta_i^{\frac{1}{1-\alpha}}(r_i/\alpha)^{\frac{\alpha}{\alpha-1}}}_{\text{non-capital income}}$$

From the perspective of the model, variation in the first term across households is captured

^{A5}Note that, given the production function, capital's share of business income is αy^b , and that $\alpha y^b + r(a - k) = ra$.

as differences in the return on assets, and variation in the second term is captured as differences in labor productivity. The relative shares of labor and capital income are correctly identified. Note, however, that for constrained entrepreneurs, heterogeneity in the rate of return affects not only the capital income component, but also the labor income component of income. In particular, constrained entrepreneurs have lower earnings conditional on productivity, since they cannot scale up their ideas to full capacity. Therefore, eliminating differences in asset returns also raises labor income dispersion. As a consequence, eliminating rate of return differences while keeping earnings heterogeneity unchanged, as we do in our analysis, may overstate the importance of rate of return differences. Similarly, eliminating differences in calibrated productivity levels reduces dispersion in rates of return across households, given the definition of r_i . This implies that eliminating earnings differences while keeping rate of return differences unchanged, as we do in our analysis, may understate the importance of productivity differences.