

Diversification and Systemic Bank Runs*

Xuewen Liu[†]

Hong Kong University of Science and Technology

This draft: December 2018

Abstract

Diversification through pooling and tranching securities was supposed to mitigate creditor runs in financial institutions by reducing their credit risk, yet many financial institutions holding diversified portfolios experienced creditor runs in the recent financial crisis of 2007-2009. We present a theoretical model to explain this puzzle. In our model, because financial institutions all hold similar (diversified) portfolios, their behavior in the asset market is clustered: they either sell their assets at the same time or collectively do not sell. Such clustering behavior reduces market liquidity after an adverse shock and increases the probability of panic runs by creditors. Diversification, while making the financial system more robust against small shocks, increases the possibility of systemic runs in the case of a larger shock. Diversification, inducing stronger strategic complementarities across institutions, makes self-fulfilling systemic runs (multiple equilibria) more likely. Because individual institutions either over-diversify or under-diversify in the competitive equilibrium compared with the social optimum, there is room for regulation.

*I am grateful to Deniz Okat for detailed feedback and suggestions. I thank Heng Chen, Itay Goldstein, Zheng Song, Tianxi Wang, Liyan Yang, and seminar participants at City University of Hong Kong, Chinese University of Hong Kong, University of Hong Kong, HKUST, Society for Economic Dynamics (SED) Annual Meeting 2018, SFS Cavalcade Asia Pacific 2018, and Econometric Society China Meeting 2018 for helpful comments.

[†]Associate Professor of Finance, Department of Finance, Hong Kong University of Science and Technology (HKUST), Clear Water Bay, Hong Kong. Tel: (+852) 2358 7679. Email: xuewenliu@ust.hk.

1 Introduction

Shadow banks were at the center of the recent financial crisis of 2007-2009. On the eve of the crisis, shadow banks as well as many other financial institutions all held diversified portfolios. These institutions were believed to be safe thanks to the diversification of their portfolios through pooling and tranching securities. Despite their high credit ratings, however, these institutions experienced runs by their creditors during the crisis.¹ Remarkably, the systemic bank runs coincided with a “sudden dry-up” of asset market liquidity. Fire-sale discounts and repo rates exhibited striking spikes at the peak of the runs.² Commentators have attributed the sudden discrete drops in market liquidity, in the absence of any apparent large exogenous disturbance to the economy, to multiplicity jumps from one equilibrium to another under self-fulfilling beliefs (see, e.g., Brunnermeier (2009), Brunnermeier and Pedersen (2009), Gertler, Kiyotaki and Prestipino (2017)).

This paper proposes a theory to explain why diversification at financial institutions can be an important cause of systemic bank runs and why diversification can contribute to *self-fulfilling* systemic bank runs. Diversification results in a higher degree of similarity in asset portfolio holdings across financial institutions (because they all hold a similar “diversified” portfolio) and thus induces their clustering behavior in the asset market. These institutions either (fire) sell their assets at the same time or collectively do not sell. Such clustering behavior reduces market liquidity when an adverse aggregate shock is realized. Because market liquidity and hence the asset liquidation value determine the extent to which an institution can withstand its creditors’ withdrawals, lower market liquidity means a higher coordinating risk among its creditors and thus a higher probability of a creditor run. At the system level, there is further feedback: a higher number of creditor runs in the system, meaning more fire sales, in turn decreases market liquidity. As a result, diversification leads to a vicious spiral of lower market liquidity and more creditor runs in the system when an adverse aggregate shock is realized.

To illustrate the idea, consider the following simple example. A New York bank and a Los Angeles bank make mortgage loans in their own cities. The realized loan value will depend on the aggregate (nation-wide) economic shock as well as the local shock. Suppose there are three possible states of the aggregate shock: “Normal”, “Bad” and “Very Bad”, the economic fundamentals under which are assumed to be 18, 14 and 6, respectively. On top of the aggregate shock, the local (idiosyncratic) shock has two equal-probability states, $\{-5, +5\}$, and the local shock is perfectly negatively correlated between the two cities. Assume that each bank holds one unit of loans, and if a bank’s realized loan value is below the threshold 10, the bank will fail. We consider two cases:

¹Modern-day bank runs occurred in the repo market, money market mutual funds (MMMFs), and the ABCP market. See evidence in Gorton and Metrick (2011), Copeland et al. (2014), Krishnamurthy et al. (2014), Duygan-Bump et al. (2013), Kacperczyk and Schnabl (2010), and Covitz et al. (2013), among others.

²See evidence in Gorton and Metrick (2011) and Covitz et al. (2013), among others.

without and with diversification. In the first case, each bank holds local loans only. Then, in the “Normal” state, the realized loan values of the two banks are $23 = 18 + 5$ and $13 = 18 - 5$, which implies both banks will survive. In contrast, in the “Bad” state, the realized loan values of the two banks are $19 = 14 + 5$ and $9 = 14 - 5$, which implies one bank will survive and the other will fail. In the “Very Bad” state, again one bank will survive and the other will fail. In the second case, for simplicity, assume that each bank diversifies by holding a half unit of local loans and a half unit of loans from the other city.³ Hence, the local shock is perfectly diversified away. It is clear that in the “Normal” state, the loan value of both banks is 18, which implies both banks will survive. In the “Bad” state, again both banks will survive. In contrast, in the “Very Bad” state, the loan value of both banks is 6 and thus both will fail. The above example illustrates that diversification, while having no effect on the outcome in the “Normal” state, results in fewer of the banks failing in the “Bad” state but more of the banks failing in the “Very Bad” state. Our model will further endogenize the failure threshold of banks — by endogenizing creditor runs and market liquidity — and demonstrate how diversification impacts on their interplay.

In our model, there is a continuum of financial institutions (“banks”). At the initial date, banks decide which assets to include in their portfolios. Their decisions determine the degree of heterogeneity across banks: the greater the diversification, the lower the heterogeneity. In the extreme case, for example, if all banks perfectly diversify (hold the “market portfolio”), there would be no heterogeneity at all across banks. At the interim date, creditors of a bank learn and receive noisy signals about the fundamental value of its portfolio assets (i.e., the payoff at the final date) and decide whether or not to run. In equilibrium, banks in the higher range of fundamental values do not suffer a creditor run and survive, while banks in the lower range of fundamental values are run by their creditors and fail. The failing banks liquidate their assets in the market, where the liquidation price of a bank depends on its fundamentals as well as on the market liquidity in the system which in turn endogenously depends on aggregate fire sales. Importantly, the diversification of banks affects market liquidity at the interim date. When banks hold similar portfolios, their selling decisions are clustered; so in a relatively bad state (a large shock), there is an increased number of banks under fire sales and market liquidity is thus reduced for every bank, while in a relatively good state (a small shock) there is a decreased number of banks under fire sales and market liquidity is thus lifted for every bank. This in turn triggers a feedback loop in the system. That is, in a relatively bad state diversification results in a downward spiral of a lower market liquidity and more creditor runs, while an upward spiral occurs in a relatively good state.

Our model derives three implications. First, diversification, while making the financial system more robust against small shocks, increases the possibility of a systemic crisis in the case of a larger shock. Systemic crises in terms of systemic bank runs become less likely for small shocks but more

³The portfolio choice will be endogenous in the full model.

likely for a larger shock. That is, diversification amplifies aggregate risk. Second, diversification increases the likelihood of a self-fulfilling systemic crisis (multiple equilibria). When banks diversify, their portfolios look alike. Lack of heterogeneity in their portfolios induces stronger strategic complementarities among creditors of different banks and makes multiple equilibria more likely. Third, we show that individual financial institutions either over-diversify or under-diversify in the competitive equilibrium compared with the social optimum. When a bank increases its diversification, it imposes externality on other banks as its action has an impact on market liquidity. In a relatively good state, the externality is positive because increasing diversification of a particular bank reduces the likelihood of its suffering a creditor run and thus reduces the pressure on market liquidity. However, when a relatively bad state occurs, the force becomes opposite and the externality is negative. If market liquidity is more important in the bad times than in the good times, the positive externality in the good times can be outweighed by the negative externality in the bad times, in which case individual banks over-diversify in the competitive equilibrium.

Our study is related to a few theoretical papers studying diversification at financial institutions and systemic crises. Brunnermeier (2009) provides an excellent survey on financial crises and systemic risk.⁴ Wagner (2010) (as well as Wagner (2011)) builds a simple and clean model showing that individual institutions over-diversify from the social perspective because diversification makes banks more similar. In his model, when two banks fail jointly, there are additional costs over and above the cost of individual failures for each bank. Therefore, when an individual bank diversifies more, this increases the probability of the joint failure and thus imposes negative externality on the other bank. Ibragimov, Jaffee and Walden (2011) develop a model also showing that diversification that is optimal for individual intermediaries may be suboptimal for the society. The negative externality in their model originates from the mechanism that when a shock disrupts all institutions simultaneously, it takes time for the financial system and the economy to recover. Their focus is on the distributional properties of risks and the number of risk classes in the economy. Allen, Babus and Carletti (2012) develop a model showing that asset commonality interacted with short-term debt of banks can generate excessive systemic risk. Their mechanism concerns information contagion and the effect of debt maturity. These extant studies neither consider coordination risk nor model the creditor runs of financial institutions. Compared with these earlier contributions, modeling the relation between diversification at financial institutions and systemic bank runs is our focus. There is an *endogenous threshold* of institutions' failure in our model. We show that diversification can have either a stabilizing or a destabilizing effect depending on the size of the aggregate shock. In particular, we show that diversification can make the system more fragile by increasing the possibility of *multiple equilibria*. In addition, different from these prior studies, our model also shows that diversification can introduce either a negative or a positive externality,

⁴See also Brunnermeier and Oehmke (2013) and Freixas and Rochet (2008).

depending on the aggregate state of the economy, so there can be either too much or too little diversification in the competitive equilibrium.

Our paper is related to the literature that uses global games methods to model creditor runs. Rochet and Vives (2004) and Goldstein and Pauzner (2005) are the pioneering works of this literature.⁵ Morris and Shin (2009) build an analytical framework to decompose creditor risk in a financial institution into insolvency risk and illiquidity risk. The recent papers of Liu (2016) and Eisenbach (2017) study the interplay between asset prices and creditor runs. Our current paper follows the approach of Liu (2017) in endogenizing liquidation prices of bank assets and showing the interaction with creditor runs. Our paper adds to the global games literature by showing that diversification can restore equilibrium multiplicity of global games: even when the precision of creditors' private signals approaches infinity, multiple equilibria can still emerge. This is because diversification induces stronger strategic complementarities among the creditors of different banks.

The paper is organized as follows. Section 2 lays out the model setting. Sections 3 and 4 present the model equilibria. Section 5 extends the model. Section 6 concludes.

2 Model Setting

There are three dates of the model: $t = 0, 1$ and 2 . All agents are risk-neutral. We discuss banks, assets, the asset market, and creditor runs, in order.

2.1 Banks and Assets

Consider an economy that consists of a continuum of structured investment vehicles (SIVs or simply banks) with unit mass, indexed by j , and a continuum of asset types with unit mass, indexed by i . The assets may correspond to asset-backed securities (ABS) in reality.

The unit cost of investing in any asset i at $t = 0$ is 1. One unit of asset i has payoff

$$x_i = \theta_i + e_i$$

at $t = 2$. The term e_i has the ex ante distribution $e_i \sim N(0, \sigma_e^2)$ and resolves its uncertainty at $t = 2$. We will consider two cases of e_i : being perfectly correlated or imperfectly correlated across different assets i . At this stage, we assume the former case and denote $e \equiv e_i$ for all i . The term θ_i , interpreted as asset quality, has its realization value at $t = 1$, which is *independently* drawn from an identical distribution across assets. There are three possible states of the *aggregate* economy at

⁵Some other related papers in this literature include Dasgupta (2004), Liu and Mello (2011), He and Xiong (2012), Xavier (2014), Bouvard et al. (2015), Ahnert (2016), Zhou (2016), Allen et al. (2017), Leonello (2017), and Li and Ma (2017). See Angeletos and Lian (2017) for a recent extensive survey on global games and their application.

$t = 1$: Normal state ($\omega = N$), Bad or small-shock state ($\omega = B$), and Very bad or large-shock state ($\omega = E$). In state ω , the distribution of asset quality is given by $\theta_i \sim N(\mu_\theta^\omega, \sigma_\theta^2)$, where $\omega = N, B$ and E , and $\mu_\theta^N > \mu_\theta^B > \mu_\theta^E$. Ex ante, at $t = 0$, the probability that state ω will occur is π_ω , where $\sum_\omega \pi_\omega = 1$. Denote $\tau_e \equiv \frac{1}{\sigma_e^2}$ and $\tau_\theta \equiv \frac{1}{\sigma_\theta^2}$. Figure 1 illustrates the timeline of the cash flow realization.

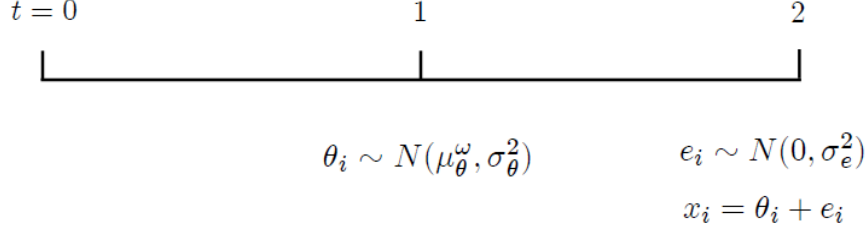


Figure 1: Timeline of cash flow realization

A bank has one unit of capital at $t = 0$, of which an amount F comes from a continuum of its creditors (debtholders) with F mass, each of them contributing 1 unit, and an amount $1 - F$ comes from its equityholder (bankowner).⁶ At $t = 1$, a creditor of a bank has the right to decide whether to roll over his lending to his bank. If he decides not to roll over, his claim is the par value 1 at $t = 1$; if, instead, he decides to roll over, the (promised) notional claim to him is R at $t = 2$, where $R > 1$ is the gross interest rate.⁷ The term R in the deposit contract will be endogenized. A creditor's reserve return (opportunity cost) of lending is R_0 , where $R_0 \geq 1$.

At $t = 0$, each bank needs to make its portfolio choice. Bank j 's portfolio is denoted by

$$X_j = (1 - \eta) x_j + \eta \int_0^1 x_i di \quad \text{for } \eta \in [0, 1],$$

where η measures the degree of diversification. Essentially, a bank's portfolio can be regarded as having two components: one is "local" asset x_j and the other is "aggregate" asset $\int_0^1 x_i di$. The "aggregate" asset may correspond to pooled and well-diversified assets in reality, like CDOs, CDO-squared, CDO-cubed, and so on. Thus, the portfolio payoff for bank j at $t = 2$, conditional on the realization of asset qualities $\{\theta_i\}$ at $t = 1$, follows the distribution

$$X_j \sim N((1 - \eta) \theta_j + \eta \mu_\theta^\omega, \sigma_e^2).$$

We denote $\Theta_j \equiv (1 - \eta) \theta_j + \eta \mu_\theta^\omega$, interpreted as the portfolio quality of bank j . Diversification potentially also reduces asset-specific risk e_i realized at $t = 2$, which will be examined in Section 5.

⁶We assume that each bank has its own creditor base (for example, these banks are regional banks).

⁷Without loss of generality, we normalize the interim notional claim to 1. What matters to the model is the interest rate between $t = 1$ and $t = 2$, i.e., the R .

Although the portfolio quality of a bank is realized at $t = 1$, creditors are not informed of it. Nevertheless, a creditor of a bank receives imperfect information (a signal) at $t = 1$ about the portfolio quality of his bank. Specifically, the signal (about portfolio quality Θ_j) for creditor h of bank j at $t = 1$ is $s_j^h = \Theta_j + \sigma_s \epsilon^h$, where $\sigma_s > 0$ is constant (denote $\tau_s \equiv \frac{1}{\sigma_s^2}$), and the individual-specific noise $\epsilon^h \sim N(0, 1)$. ϵ^h is i.i.d. across creditors of a bank, and each is independent of Θ_j . We assume that before creditors make their rollover decision at $t = 1$, they are informed of the status of the aggregate economy, $\omega = N$ or B or E . Since a creditor of bank j knows μ_θ^ω , his private signal about Θ_j is equivalent to a signal about the quality of local asset θ_j . In addition, it is assumed that the aggregate states are *uncontractible* ex ante as in the incomplete literature.

2.2 Asset Market

We follow the setting of the asset market in Liu (2017). If a bank suffers a creditor run (to be elaborated), its assets must be liquidated at $t = 1$ in a competitive asset market, which consists of a continuum of competitive investors with unit mass. Investor m has utility function

$$U(W^m) = -\exp(-\gamma W^m),$$

where W^m is the end-of-period wealth at $t = 2$, and γ is the risk-aversion (CARA) coefficient. The risk-free (gross) interest rate between $t = 1$ and $t = 2$ is normalized to 1.

Investors have private information (signals) about banks' asset portfolio qualities. Specifically, the signal for investor m about asset portfolio quality Θ_j at $t = 1$ is $\rho_j^m = \Theta_j + \sigma_\rho \epsilon_j^m$, where $\sigma_\rho \geq 0$ is constant, and the individual-specific noise $\epsilon_j^m \sim N(0, 1)$. ϵ_j^m is independent across asset portfolios for a given m and independent across investors for a given j , and each ϵ_j^m for a given j is independent of Θ_j .

Suppose that the system has, in total, a mass of φ ($\in [0, 1]$) of banks suffering creditor runs. Then, there are φ units of assets in the system under fire sales.⁸ Denote by L_j the liquidation (fire-sale) price per unit of bank j 's asset portfolio.

2.3 Creditor Runs

Let us consider a typical bank j . If it has greater than $\frac{L_j}{F}$ proportion of its creditors declining to roll over their lending at $t = 1$, its liquidation value will not be sufficient to cover the creditors' claims, leading to its failure (we call this scenario "a creditor run").

A creditor's payoff crucially depends on the actions of other creditors of the same bank. Let

⁸We will focus on the limiting case of $\sigma_s \rightarrow 0$. In equilibrium, then, if a bank suffers a creditor run, it will liquidate its assets *entirely* (i.e., there is no partial liquidation).

λ denote the proportion of creditors of a bank that choose not to roll over (i.e., choose to call) at $t = 1$. Then, the payoff for a particular creditor is given in Table 1.

	Total calling proportion $\lambda \in [0, \frac{L_j}{F})$ (bank survives)	Total calling proportion $\lambda \in [\frac{L_j}{F}, 1]$ (bank fails)
Hold	$\min \left[R, \frac{\left(1 - \frac{F\lambda}{L_j}\right)}{(1-\lambda)F} X_j \right]$	$\frac{L_j}{F} - \Delta$
Call	1	$\frac{L_j}{F}$

Table 1: Creditor-run payoff structure

If $\lambda \in [\frac{L_j}{F}, 1]$, a creditor run occurs and the bank fails at $t = 1$. In this case, *all* creditors share the liquidation value L_j at $t = 1$, but those who have not called will pay an extra fee Δ (e.g., legal cost) to get their money back. This setup of a first-mover advantage of withdrawing (calling) follows the setup in Eisenbach (2017). As Eisenbach argues, the first-mover advantage, not depending on the sequential service constraint inherent in deposit contracts, is more representative of market-based funding without a sequential service constraint as in Cole and Kehoe (2000).

If $\lambda \in [0, \frac{L_j}{F})$, the bank needs to liquidate $\frac{F\lambda}{L_j}$ units of its assets to raise cash to pay its $F\lambda$ calling creditors. Thus, at $t = 2$, $1 - \frac{F\lambda}{L_j}$ units of assets remain. Since the number of staying creditors at $t = 2$ is $(1 - \lambda)F$, these creditors' total notional claim is $(1 - \lambda)FR$. Hence, a staying creditor will have payoff $\frac{\min \left[(1-\lambda)FR, \left(1 - \frac{F\lambda}{L_j}\right) X_j \right]}{(1-\lambda)F} = \min \left[R, \frac{\left(1 - \frac{F\lambda}{L_j}\right)}{(1-\lambda)F} X_j \right]$ at $t = 2$. A creditor who calls obtains the par value 1 at $t = 1$.

For a cleaner and simpler analysis, we follow Morris and Shin (2009) to simplify the payoff structure of the creditor-run game. Morris and Shin (2009) assume that “if there is not a run, new creditors will eventually be found and the balance sheet reverts to its initial state after the failed run.” Basically, they are assuming that “the partial liquidation of assets has no long-run effect” (in the language of Vives (2014)). Concretely, if a bank has less than $\frac{L_j}{F}$ proportion of its creditors withdrawing, *partial liquidation* will occur but the bank can still survive to $t = 2$, in which case Morris and Shin (2009) assume that the bank’s balance sheet reverts to its initial state. Essentially, after an *unsuccessful run*, the asset side of the balance sheet of the bank is restored to X_j and the liability side reverts to the total notional debt value FR claimed by F creditors.⁹ In short, the

⁹For example, as long as the bank is still alive (after an unsuccessful run), it can buy back its (partially) liquidated assets with refinancing from its original withdrawing creditors or new creditors. Alternatively, as long as the total amount of early withdrawals at $t = 1$ is less than L_j , the bank is able to raise this amount of cash temporarily (for instance, from some outside deep-pocketed investors) by using its assets as collateral; new creditors will then eventually be found to replace the withdrawing creditors and the refinancing amount from the new creditors is used to repay the temporary borrowing.

assumption of Morris and Shin (2009) gives the simplified payoff structure in Table 2.

	Total calling proportion $\lambda \in [0, \frac{L_j}{F})$ (bank survives)	Total calling proportion $\lambda \in [\frac{L_j}{F}, 1]$ (bank fails)
Hold	$\min \left[R, \frac{X_j}{F} \right]$	$\frac{L_j}{F} - \Delta$
Call	1	$\frac{L_j}{F}$

Table 2: Simplified creditor-run payoff structure

Note that Table 2 is identical to Table 1 if we set $\lambda = 0$ for the payoff of holding in the case of $\lambda \in [0, \frac{L_j}{F})$. That is, as long as $\lambda \in [0, \frac{L_j}{F})$, the bank continues as if it had not experienced any withdrawals in Table 2. Liu (2017) shows that the simplification of Morris and Shin (2009) does not change model results qualitatively, only quantitatively.

2.4 Timeline

At $t = 0$, the liability side of the balance sheet of a bank is given by $(F, 1 - F)$. On the asset side, the degree of diversification, η , is decided by a bank. Moreover, a bank chooses the (notional) interest rate R in the deposit contract subject to creditors' participation. At date $t = 1$, for a given η , creditors move *first* by making their rollover decisions, and banks move *later* by conducting asset sales in the asset market based on the total withdrawals requested by their creditors.

We solve the equilibrium by backward induction: from $t = 1$ to $t = 0$.

3 Equilibrium at $t = 1$

We are interested in the equilibrium where every creditor uses a threshold (monotone) strategy. The strategy is given by

$$s_j^h \mapsto \begin{cases} Call & s_j^h < s^* \\ Hold & s_j^h \geq s^* \end{cases},$$

where s_j^h is the signal of creditor h in bank j and s^* is the rollover threshold. Because banks are identical ex ante, we naturally consider the *symmetric* equilibrium in which creditors of all banks use a common strategy, i.e., the threshold s^* is not bank-specific.

We will show that *an upper dominance region* exists for the bank-run game in our model. An upper dominance region exists because in our model the interim liquidation value of a bank is fundamentals-dependent as in Rochet and Vives (2004).¹⁰

¹⁰Goldstein and Pauzner (2005), following the original model of Diamond and Dybvig (1983), assume that the

3.1 Equilibrium Definition

Because creditors are also informed at $t = 1$ of the state of the aggregate economy, $\omega = N$ or B or E , before they make their rollover decision, the equilibrium can be state-dependent. Formally, we define the equilibrium at $t = 1$ as follows.

Definition 1 *The equilibrium at $t = 1$ is characterized by triplet $(s^{*\omega}, \{L_j^\omega\}, \varphi^\omega)$ for state ω , where $s^{*\omega}$ is the rollover threshold for creditors, L_j^ω is the liquidation price of bank j 's assets, and φ^ω is the total mass of banks under fire sales, such that 1) given creditors' rational expectations of φ^ω , they set their rollover threshold as $s^{*\omega}$; 2) given the rollover threshold $s^{*\omega}$, the mass of failed banks in the system is φ^ω ; and 3) given the total fire sales φ^ω , the equilibrium price of bank j 's assets in the asset market is L_j^ω .*

We derive the equilibrium in three steps.

Asset market equilibrium To reduce notational clutter, we drop the state index superscript ω for now. The portfolio choice of an investor in the asset market at $t = 1$ is given by

$$\begin{aligned} \max_{\{q_j^m\}} \mathbb{E} [-\exp(-\gamma W^m) | \{\rho_j^m\}, \{L_j\}] \\ \text{s.t. } W^m = \int q_j^m (X_j - L_j) dj, \end{aligned} \quad (1)$$

where q_j^m is the quantity of demand for bank asset j for investor m . For simplicity, we follow the trading game (mechanism) in Vives (2014b) and Benhabib, Liu and Wang (2016) to focus on the *fully-revealing equilibrium* of the asset market, i.e., the equilibrium in which financial prices fully reveal the fundamentals of the trading assets. In our context, it is the equilibrium in which $\{\Theta_j\}$ is fully revealed to the investors through the financial prices.¹¹

The first-order condition of (1) implies that $q_j^m = q_j$ for any m (i.e., a representative investor exists and demands d_j), and that

$$\int q_j dj = \frac{\Theta_j - L_j}{\gamma \sigma_e^2}.$$

Given that the total mass of banks suffering creditor runs is φ^ω in state ω , the market clearing condition dictates

$$\int q_j dj = \varphi.$$

Lemma 1 follows.

interim liquidation value of a bank is fundamentals-independent, so an upper dominance region does not exist in their model.

¹¹Alternatively, instead of $\{\Theta_j\}$ being fully-revealed through the financial prices, we can assume that the precision of investors' private signals approaches the limit $\sigma_\rho \rightarrow 0$ as in Morris and Shin (2004b), just as $\sigma_s \rightarrow 0$ for the precision of creditors' private signals. Note that this part of the model is not a focus of our paper, and we can adopt either alternative.

Lemma 1 *The liquidation price of bank j 's assets in state ω is given by $L_j^\omega = \Theta_j - \varphi^\omega k$, where $k \equiv \gamma\sigma_e^2$ and $\varphi^\omega k$ measures market liquidity in state ω .*

Proof. See Appendix. ■

The result of Lemma 1 is in the spirit of Grossman and Miller (1988). When the risk-averse market maker sector is forced to absorb more risky assets, the price of each risky asset is affected and reduced because of the limited risk-absorbing capacity of the market maker sector. As in Brunnermeier and Pedersen (2009), market liquidity is measured as the degree to which the market price of an asset is depressed below its fundamental value. Market liquidity in our model is thus measured by the term $\varphi^\omega k$.

Creditor-run equilibrium for an individual bank Considering that L_j^ω is fundamentals (Θ_j)-dependent by Lemma 1, when Θ_j is sufficiently high, bank j will survive even if everyone of its creditors withdraws. That is, *an upper dominance region* exists. Therefore, we only need to focus on threshold equilibria (see Morris and Shin (2003) and Vives (2014a)).

Denote by $D(\Theta_j; R) \equiv \mathbb{E} \left(\min \left[R, \frac{X_j}{F} \right] \mid \Theta_j \right)$ the expected payoff of the debt at $t = 2$ conditional on the realization of Θ_j at $t = 1$, where $X_j \sim N(\Theta_j, \sigma_e^2)$. Clearly, $\frac{\partial D}{\partial \Theta_j} > 0$.

We consider the limiting case of signal precision: $\sigma_s \rightarrow 0$. The threshold equilibrium of the creditor run game is given by

$$(D(s^{*\omega}) - 1) \cdot \frac{L_j^\omega}{F} = \Delta \cdot \left(1 - \frac{L_j^\omega}{F} \right), \quad (2)$$

where $L_j^\omega = s^{*\omega} - \varphi^\omega k$. The proof is provided in Appendix. The intuition is the following. To the marginal creditor who receives signal $s_j^h = s^{*\omega}$, he perceives that λ (i.e., the proportion of his peer creditors choosing to call) is uniformly distributed within $[0, 1]$. Hence, in his eyes, the probability of bank survival is $\frac{L_j^\omega}{F}$ and that of bank failure is $1 - \frac{L_j^\omega}{F}$. In the case of bank survival, holding has an advantage over calling with the additional payoff being $D(s^{*\omega}) - 1$, which is the LHS of (2). On the contrary, in the case of bank failure, calling has an advantage over holding with the additional payoff being Δ , which is the LHS of (2). Because the marginal creditor is indifferent with calling and holding, the equality of (2) follows. Moreover, under the limit $\sigma_s \rightarrow 0$, the marginal creditor's assessment of L_j^ω is $L_j^\omega = s^{*\omega} - \varphi^\omega k$. We can rewrite (2) as

$$\frac{s^{*\omega} - \varphi^\omega k}{F} = \frac{\Delta}{D(s^{*\omega}) - 1 + \Delta}. \quad (3)$$

Let $\Theta^{*\omega}$ denote the threshold such that if and only if a bank's realized portfolio quality $\Theta_j < \Theta^{*\omega}$ will the bank fail at $t = 1$ in state ω . Under the limit $\sigma_s \rightarrow 0$, it is easy to show that

$$\Theta^{*\omega} = s^{*\omega}.$$

The limit $\sigma_s \rightarrow 0$ also implies that in equilibrium all creditors of a bank are in the same position ex post, i.e., either all of them decide to roll over or none of them does so. This in turn implies that in equilibrium a bank either completely liquidates its assets or does not liquidate any fraction of it, i.e., there is no partial liquidation.

Bank failures in the system Given η , the portfolio quality distribution across banks in the system at $t = 1$ (conditional on the realization of state ω) is

$$\Theta_j \sim N(\mu_\theta^\omega, (1 - \eta)^2 \sigma_\theta^2).$$

Banks with realized portfolio quality $\Theta_j \geq s^{*\omega}$ survive at $t = 1$ while others fail. Hence, the total mass of failing banks in the system is given by

$$\varphi^\omega = \Phi\left(\frac{s^{*\omega} - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right), \quad (4)$$

where $\Phi(\cdot)$ stands for the c.d.f. of the standard normal and $\phi(\cdot)$ denotes its p.d.f.

Lemma 2 *The equilibrium at $t = 1$ is given by the system of equations (3) and (4) for $\omega = N, B$ and E under the limiting case of $\sigma_s \rightarrow 0$. Two-way feedback exists between market liquidity ($\varphi^\omega k$) and the creditor-run threshold ($s^{*\omega}$): $\frac{\partial s^{*\omega}}{\partial \varphi^\omega} > 0$ in (3) and $\frac{\partial \varphi^\omega}{\partial s^{*\omega}} > 0$ in (4).*

Proof. See Appendix. ■

The two-way feedback between market liquidity and creditor runs has been studied in Liu (2017). Here we are interested in the effect of diversification on the feedback. Specifically, we have the following properties of (4):

$$\frac{\partial \varphi^\omega}{\partial \eta} \begin{cases} > 0 & \text{if } s^{*\omega} > \mu_\theta^\omega \\ < 0 & \text{if } s^{*\omega} < \mu_\theta^\omega \end{cases}. \quad (5)$$

3.2 Characterization of the Equilibrium

Combining (3) and (4) yields one equation:

$$\left\{ \frac{1}{F} \left[s^{*\omega} - \Phi\left(\frac{s^{*\omega} - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right) k \right] \right\} \frac{D(s^{*\omega}) - 1 + \Delta}{\Delta} = 1. \quad (6)$$

The equilibrium at $t = 1$ is fully characterized by equation (6). When the liquidation value L_j^ω is exogenously given, the creditor-run game, characterized by equation (2), has a unique equilibrium. Similarly, when the market liquidity $\varphi^\omega k$ is exogenously given, the creditor-run game, characterized by equation (3), also has a unique equilibrium. Under endogenous market liquidity, the creditor-run game may have multiple equilibria (Liu (2017)). Mathematically, the first term on the LHS of (6)

may be decreasing in $s^{*\omega}$ while the second term is increasing in $s^{*\omega}$, so the function on the LHS with respect to $s^{*\omega}$ may be non-monotonic and thus multiple solutions to equation (6) are possible.

In particular, η affects the likelihood that multiple equilibria exist. Write the LHS of (6) as function $V(s^*; \eta, \mu_\theta)$, where

$$V(s^*; \eta, \mu_\theta) = \left\{ \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) k \right] \right\} \frac{D(s^*) - 1 + \Delta}{\Delta}.$$

Figure 2 plots the function under a set of parameter values, where $\mu_\theta = 1.5$, $\sigma_\theta = 4$, $k = 2$, $F = 0.6$, $R = 1.2$, $\sigma_e = 0.1$, and $\Delta = 0.1$.

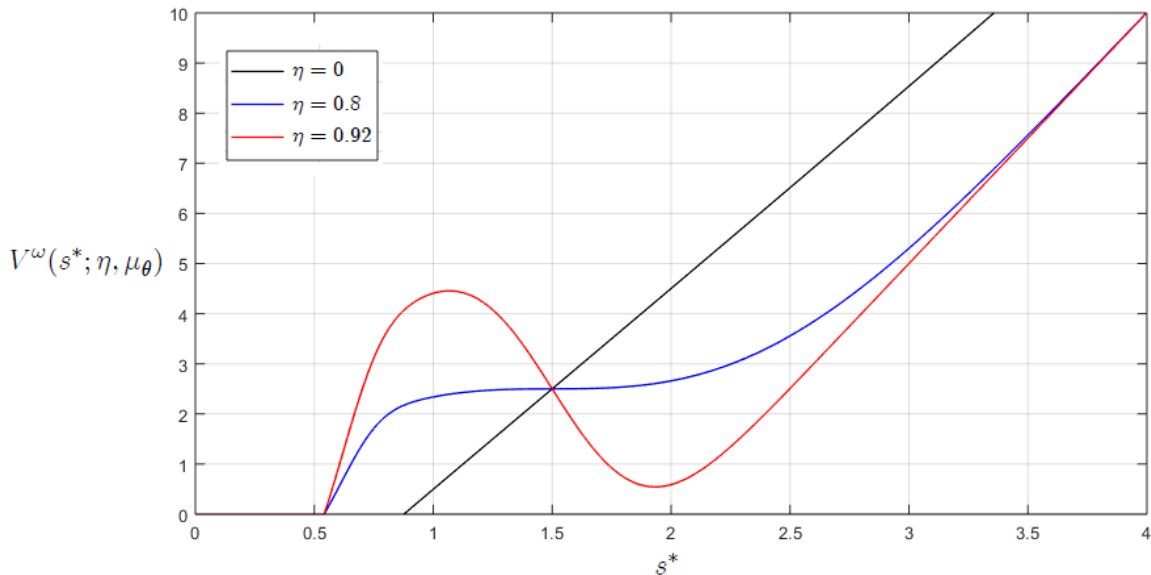


Figure 2: Function $V(s^*; \eta, \mu_\theta)$

Lemma 3 follows.

Lemma 3 *Consider the limiting case of $\sigma_s \rightarrow 0$. For a given η , when σ_θ is high enough, the equilibrium at $t = 1$ is always unique. For a given σ_θ , when η is high enough, multiple equilibria may exist.*

Proof. See Appendix. ■

Even when the precision of creditors' private signals approaches infinity ($\sigma_s \rightarrow 0$), multiple equilibria can still emerge. Equilibrium multiplicity arises because diversification makes banks more similar and thus introduces stronger strategic complementarities among creditors of different banks. When market liquidity is endogenous, a creditor of a bank effectively faces a coordination problem not only with creditors of the same bank but also with creditors of some other *similar* banks. When η increases, there are more of such similar banks and thus the coordination becomes more difficult, making multiple equilibria more likely.

3.3 Equilibrium Outcome

For a given σ_θ , we conduct analysis on two ranges of η . We start the analysis from $\eta = 0$ (i.e., no diversification). To make the research question interesting, we focus on a sufficiently high σ_θ such that when $\eta = 0$, equation (6) admits a unique solution to $s^{*\omega}$ for $\omega = N, B$ and E (see Lemma 3).

We have two results, summarized in Propositions 1 and 2 below.

Proposition 1 *When η is small enough, ceteris paribus, the equilibrium at $t = 1$ is unique. In state $\omega = E$, greater diversification can lead to the feedback loop of a lower market liquidity and more creditor runs (i.e., $\frac{\partial(\varphi^E k)}{\partial\eta} > 0$, $\frac{\partial s^{*E}}{\partial(\varphi^E k)} > 0$, and $\frac{\partial(\varphi^E k)}{\partial s^{*E}} > 0$). In state $\omega = B$, the feedback loop can be in the opposite direction (i.e., $\frac{\partial(\varphi^B k)}{\partial\eta} < 0$, $\frac{\partial s^{*B}}{\partial(\varphi^B k)} > 0$, and $\frac{\partial(\varphi^B k)}{\partial s^{*B}} > 0$).*

Proof. See Appendix. ■

The condition for realizing the equilibrium outcome in Proposition 1 is that μ_θ^B is sufficiently high and μ_θ^E is sufficiently low such that when $\eta = 0$, the unique solution $s^{*\omega}$ to equation (6) for $\omega = B$ and E satisfies $s^{*B} < \mu_\theta^B$ and $s^{*E} > \mu_\theta^E$.¹² We also choose a sufficiently high μ_θ^N such as $s^{*N} < \mu_\theta^N - 3\sigma_\theta$ for the unique solution to equation (6) when $\eta = 0$.¹³

Proposition 1 states that diversification results in more creditor runs under a large shock and fewer creditor runs under a small shock. The mechanism is the following. First, diversification results in a higher degree of “similarity” among banks and thereby their “clustering” actions in the asset liquidation/fire sale market. This is characterized by $\Theta_j \sim N(\mu_\theta^\omega, (1 - \eta)^2 \sigma_\theta^2)$, where an increase in η leads to a reduction in variance. Hence, in the large-shock state, there is an *increased* number of banks under fire sales while in the small-shock state there is a *decreased* number of banks under fire sales. Therefore, diversification results in market liquidity becoming higher in the small-shock state but lower in the large-shock state, i.e., $\frac{\partial(\varphi^E k)}{\partial\eta} > 0$ and $\frac{\partial(\varphi^B k)}{\partial\eta} < 0$ by (5). Second, there is a further feedback loop between market liquidity and creditor runs as shown in Lemma 2. Figure 2 illustrates the effects in Proposition 1.

¹²This means that in equilibrium there are less than 50% of banks failing for state $\omega = B$ while there are more than 50% of banks failing for state $\omega = E$.

¹³We are mainly interested in states $\omega = N$ and E . As for the state $\omega = B$, although qualitatively the positive feedback loop as in state $\omega = B$ also applies, the magnitude of the feedback is small because we have chosen a sufficiently high μ_θ^N such as $\mu_\theta^N - s^{*N} > 3\sigma_\theta$ in equilibrium for $\eta = 0$. That is, even under no diversification ($\eta = 0$), almost no banks will suffer a creditor run (i.e., $\Phi\left(\frac{s^{*N} - \mu_\theta^N}{\sigma_\theta}\right)$ is close to 0). Hence, diversification has almost no effects.

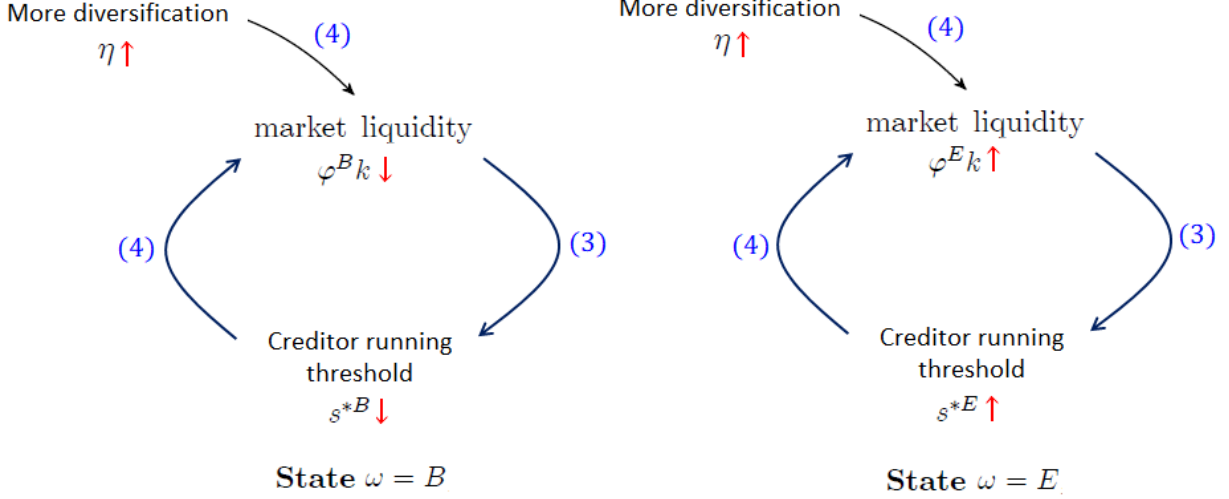


Figure 3: Amplification loop triggered by diversification in states $\omega = B$ and E

When η increases further, equilibrium multiplicity becomes possible. Proposition 2 follows.

Proposition 2 *When η is high enough, ceteris paribus, the equilibrium at $t = 1$ can be such that there is a unique equilibrium for state $\omega = B$ and there are multiple (typically three) equilibria for state $\omega = E$.*

Proof. See Appendix. ■

Proposition 2 highlights that when diversification is high enough, a sufficient deterioration in fundamentals (μ_θ) of the aggregate state can trigger a self-fulfilling systemic crisis (multiple equilibria). That is, under a high degree of diversification, the economy can experience a discrete jump when the fundamentals (μ_θ) worsen to a certain point: there is a regime change from equilibrium uniqueness to equilibrium multiplicity and the “bad” equilibrium among the multiple equilibria can be realized. The jump corresponds to the evidence discussed in the introduction that a discontinuous drop in market liquidity is accompanied by systemic creditor runs on banks.

Figure 4 illustrates the effect in Proposition 2. Rewrite the LHS of (6) as $V(s^{*h}; s^*, \mu_\theta) = \left\{ \frac{1}{F} \left[s^{*h} - \Phi \left(\frac{s^* - \mu_\theta}{(1-\eta)\sigma_\theta} \right) k \right] \right\} \frac{D(s^{*h}) - 1 + \Delta}{\Delta}$. Hence, the solution with respect to s^{*h} to $V(s^{*h}; s^*, \mu_\theta) = 1$ gives the best response function $s^{*h} = r(s^*; \mu_\theta)$. Figure 4 plots function $s^{*h} = r(s^*; \mu_\theta)$. Under $\omega = B$ (a small shock), the unique equilibrium is represented by point A ; under $\omega = E$ (a large shock), the equilibrium can be either B or B' .¹⁴

¹⁴The curve for state $\omega = E$ has three intersections with the 45° line. The middle intersection corresponds to an unstable equilibrium. The other two correspond to stable equilibria.

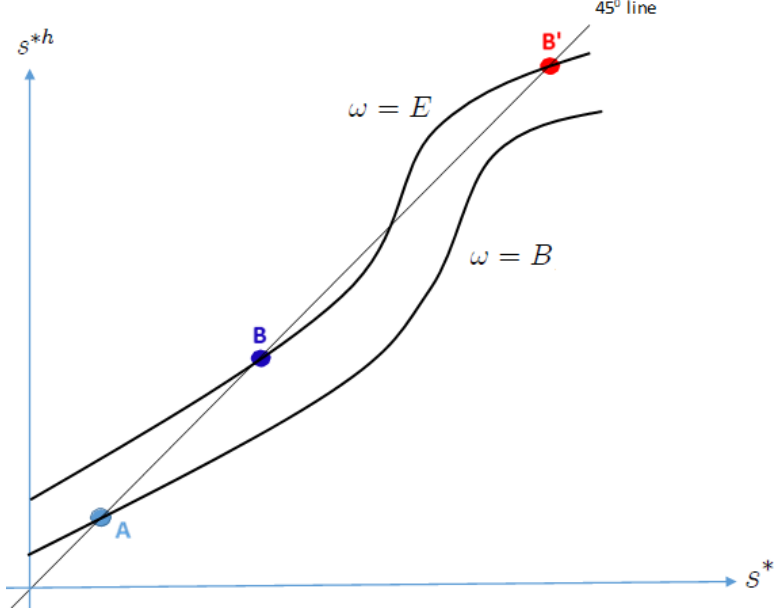


Figure 4: Equilibrium multiplicity in state $\omega = E$

4 Equilibrium at $t = 0$

We move on to study the banks' decisions at $t = 0$. We analyze the competitive equilibrium and the constrained second-best equilibrium, in order.

4.1 Competitive Equilibrium

In the competitive equilibrium, every bank takes the market liquidity ($\varphi^\omega k$) as *given* (their rational expectations) in state $\omega = N, B$ and E , while the market liquidity ($\varphi^\omega k$) is the equilibrium outcome of the diversification (η) decisions of other banks.

First, let us consider the diversification decision of an individual bank j . Its objective is to maximize its expected equity value (or, equivalently, bank value) subject to the participation condition (IR) of its creditors.

In the limiting case of $\sigma_s \rightarrow 0$, all creditors of a bank are in the same position *ex post*, i.e., either all of them run on the bank or none of them does so. We find the participation condition of a creditor as follows:

$$R_0 = \sum_{\omega} \pi_{\omega} \left[\underbrace{\int_{\Theta_j = -\infty}^{s^{*\omega}} \frac{\Theta_j - \varphi^\omega k}{F} \cdot d\Phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1 - \eta) \sigma_{\theta}} \right)}_{\text{bank fails at } t=1} + \underbrace{\int_{\Theta_j = s^{*\omega}}^{\infty} \mathbb{E} \left[\min \left[R, \frac{X_j}{F} \right] | \Theta_j \right] \cdot d\Phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1 - \eta) \sigma_{\theta}} \right)}_{\text{bank survives to } t=2} \right]. \quad (7)$$

The first and second terms within the outermost brackets in (7) correspond to the two cases of $\Theta^j < s^{*\omega}$ and $\Theta^j > s^{*\omega}$, respectively. In the case of $\Theta^j < s^{*\omega}$, the bank fails at $t = 1$ and creditors divide the liquidation value $L_j^\omega = \Theta_j - \varphi^\omega k$; that is, each creditor obtains $\frac{L_j^\omega}{F}$. In the case of $\Theta^j > s^{*\omega}$, the bank survives to $t = 2$ and the debt value for each creditor is $\min\left[R, \frac{X_j}{F}\right]$. The expected debt value conditional on a realized Θ_j is $\mathbb{E}\left[\min\left[R, \frac{X_j}{F}\right] \mid \Theta_j\right]$ with $X_j \sim N(\Theta_j, \sigma_e^2)$. Ex ante, at $t = 0$, the bank knows that Θ_j will be drawn from the distribution $\Theta_j \sim N(\mu_\theta^\omega, (1 - \eta)^2 \sigma_\theta^2)$, so the term $d\Phi\left(\frac{\Theta_j - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right)$ represents the (unconditional) probability density of Θ_j .

Similarly, the expected equity value of the bank is given by

$$\sum_{\omega} \pi_{\omega} \left[\underbrace{\int_{\Theta_j = s^{*\omega}}^{\infty} \mathbb{E}[\max(X_j - FR, 0) \mid \Theta_j] \cdot d\Phi\left(\frac{\Theta_j - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right)}_{\text{bank survives to } t=2} \right], \quad (8)$$

where the term $\max(X_j - FR, 0)$ represents the equity value at $t = 2$. Note that the bank equityholder obtains nothing if the bank fails at $t = 1$ (i.e., when $\Theta_j < s^{*\omega}$).

Bank j 's optimal diversification choice, η^{*j} , is therefore given by the following program:

$$\begin{aligned} \eta^{*j} &= \arg \max_{\eta} \sum_{\omega} \pi_{\omega} \left[\int_{\Theta_j = s^{*\omega}}^{\infty} \mathbb{E}[\max(X_j - FR, 0) \mid \Theta_j] \cdot d\Phi\left(\frac{\Theta_j - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right) \right] \\ \text{s.t.} \quad &\left. \begin{array}{l} (3) \text{ (creditor run)} \\ (7) \text{ (IR of creditors)} \end{array} \right\} \text{ (bank } j\text{'s specific } (s^{*\omega}, R) \text{ for a given } \varphi^\omega k). \end{aligned} \quad (9)$$

The objective function in (9) is to maximize the bank's equity value. As for the constraint, creditors of bank j take φ^ω as given, and choose *bank-specific* $(s^{*\omega}, R)$ in response to each η that bank j (its owner) chooses. In other words, the constraint gives the mapping $(\eta, \varphi^\omega) \rightarrow (s^{*\omega}, R)$.

We can rewrite the objective function in (9) and transform Program (9) to an equivalent optimization problem:

$$\begin{aligned} \eta^{*j} &= \arg \max_{\eta} \sum_{\omega} \pi_{\omega} \left[\int_{\Theta_j = -\infty}^{s^{*\omega}} (\Theta_j - \varphi^\omega k) \cdot d\Phi\left(\frac{\Theta_j - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right) + \int_{\Theta_j = s^{*\omega}}^{\infty} \mathbb{E}(X_j \mid \Theta_j) \cdot d\Phi\left(\frac{\Theta_j - \mu_\theta^\omega}{(1 - \eta)\sigma_\theta}\right) \right] \\ \text{s.t.} \quad &\left. \begin{array}{l} (3) \text{ (creditor run)} \\ (7) \text{ (IR of creditors)} \end{array} \right\} \text{ (bank } j\text{'s specific } (s^{*\omega}, R) \text{ for a given } \varphi^\omega k). \end{aligned} \quad (10)$$

The objective function in (10) is to maximize the total value of bank j (i.e., its debt value plus equity value). This is equivalent to maximizing the bank's equity value, because creditors of a bank, in total, claim a *constant residual* value, FR_0 . In fact, by adding the RHS of (7) multiplied

by F to (8), we have the bank value, which is exactly the term in the objective function of (10).

Second, suppose all other banks $\ell \neq j$ choose η as η^* . The market equilibrium then determines the triplet $(s^{*\omega}, \varphi^\omega, R)$; in particular, market liquidity $\varphi^\omega k$ is determined. That is,

$$\left. \begin{array}{l} (3) \\ (4) \\ (7) \end{array} \right\} \Big|_{\eta=\eta^*} \quad (\text{other banks determine } \varphi^\omega k \text{ given their } \eta^*). \quad (11)$$

(11) is identical to (3)-(4) and (7) with η being replaced by η^* .

Finally, by symmetric equilibrium across banks, we have

$$\eta^{*j} = \eta^*. \quad (12)$$

Lemma 4 *The competitive equilibrium at $t = 0$ that determines the optimal diversification η^* is given by (10)-(12).*

The first-order condition of (10) implies

$$\sum_{\omega} \pi_{\omega} \left[\int_{\Theta_j = -\infty}^{s^{*\omega}} (\Theta_j - \varphi^\omega k) \cdot \left[\frac{1}{(1-\eta)^2 \sigma_{\theta}} \left(\phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) + \frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \phi' \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \right) \right] d\Theta_j + \frac{s^{*\omega} - \varphi^\omega k}{(1-\eta)\sigma_{\theta}} \phi \left(\frac{s^{*\omega} - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \frac{ds^{*\omega}}{d\eta} \right. \\ \left. + \int_{\Theta_j = s^{*\omega}}^{\infty} \Theta_j \cdot \left[\frac{1}{(1-\eta)^2 \sigma_{\theta}} \left(\phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) + \frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \phi' \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \right) \right] d\Theta_j - s^{*\omega} \cdot \frac{1}{(1-\eta)\sigma_{\theta}} \phi \left(\frac{s^{*\omega} - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \frac{ds^{*\omega}}{d\eta} \right] = 0, \quad (13)$$

where $\frac{ds^{*\omega}}{d\eta}$ is the first-order derivative of $s^{*\omega}(\eta)$, and $s^{*\omega}(\eta)$ is the solution to the system of equations (3) and (7) in the constraint of Program (9) for a given φ^ω . In the first-order condition of (13), individual bank j takes φ^ω as given (i.e., not a function of its chosen η).

4.2 Constrained Second-best Equilibrium

In the constrained second-best equilibrium, the social planner takes the market liquidity $\varphi^\omega k$ as *endogenous* and she recognizes that diversification, η , endogenously impacts on the market liquidity $\varphi^\omega k$. Her optimal diversification choice, η^{*SB} , is given by

$$\eta^{*SB} = \arg \max_{\eta} \sum_{\omega} \pi_{\omega} \left[\underbrace{\int_{\Theta_j = -\infty}^{s^{*\omega}} (\Theta_j - \varphi^\omega k) \cdot d\Phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right)}_{\text{failing banks in the system at } t=1} + \underbrace{\int_{\Theta_j = s^{*\omega}}^{\infty} \mathbb{E}(X_j | \Theta_j) \cdot d\Phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right)}_{\text{surviving banks at } t=2} \right] \quad (14)$$

s.t. (3), (4), and (7).

In Program (14), the objective function is to maximize the *aggregate value* of all banks in the economy, including the failing banks at $t = 1$ (the first term) and the surviving banks at $t = 2$ (the second term). The appearance of the objective function in (14) is exactly the same as that of (10) (i.e., the two objective functions take the same form), which makes the comparison between the competitive equilibrium and the constrained second-best equilibrium meaningful. The constraint of Program (14) gives the mapping $\eta \rightarrow (s^{*\omega}, \varphi^\omega, R)$.

Lemma 5 *The constrained second-best equilibrium at $t = 0$ that determines the optimal diversification η^{*SB} is given by (14).*

The first-order condition of (14) implies

$$\sum_{\omega} \pi_{\omega} \left[\begin{aligned} & \int_{\Theta_j = -\infty}^{s^{*\omega}} (\Theta_j - \varphi^\omega k) \cdot \left[\frac{1}{(1-\eta)^2 \sigma_{\theta}} \left(\phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) + \frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \phi' \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \right) \right] d\Theta_j + \frac{s^{*\omega} - \varphi^\omega k}{(1-\eta)\sigma_{\theta}} \phi \left(\frac{s^{*\omega} - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \frac{ds^{*\omega}}{d\eta} \\ & + \int_{\Theta_j = s^{*\omega}}^{\infty} \Theta_j \cdot \left[\frac{1}{(1-\eta)^2 \sigma_{\theta}} \left(\phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) + \frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \phi' \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \right) \right] d\Theta_j - s^{*\omega} \cdot \frac{1}{(1-\eta)\sigma_{\theta}} \phi \left(\frac{s^{*\omega} - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right) \frac{ds^{*\omega}}{d\eta} \\ & + \underbrace{\int_{\Theta_j = -\infty}^{s^{*\omega}} (-k) \frac{d\varphi^\omega}{d\eta} \cdot d\Phi \left(\frac{\Theta_j - \mu_{\theta}^{\omega}}{(1-\eta)\sigma_{\theta}} \right)}_{\text{internalizing the impact on market liquidity}} \end{aligned} \right] = 0, \quad (15)$$

where $\frac{ds^{*\omega}}{d\eta}$ and $\frac{d\varphi^\omega}{d\eta}$ are respectively the first-order derivatives of $s^{*\omega}(\eta)$ and $\varphi^\omega(\eta)$, the solutions to the system of equations (3), (4) and (7) in the constraint of Program (14) for a given η .

Let us compare the two first-order conditions. In (15), the social planner internalizes the price impact when deciding her η , so there is an additional term in (15) relative to (13).

Now we can compare η^* and η^{*SB} . Proposition 3 follows.

Proposition 3 *The competitive equilibrium η^* can be higher or lower than the constrained optimum η^{*SB} . That is, an individual bank either over-diversifies or under-diversifies.*

Proof. *See Appendix.* ■

We discuss the intuition behind Proposition 3.¹⁵ Based on the discussion in Proposition 1, it is easy to see that diversification has almost no effect on a bank's expected value for state $\omega = N$. So we only need to consider the two other aggregate states ($\omega = B$ and E). When an individual bank increases its diversification, it imposes an externality on other banks as its action has an impact on market liquidity. When state $\omega = B$ occurs, the externality is positive because greater diversification of this particular individual bank reduces its likelihood of suffering a creditor run and

¹⁵In both the competitive equilibrium and the second-best constrained equilibrium, we assume that agents will coordinate on the efficient equilibrium in the case that there are multiple equilibria ex post at $t = 1$.

hence its probability of undergoing fire sales, which in turn reduces the pressure on market liquidity and thereby decreases the probability of creditor runs for all other banks. However, when state $\omega = E$ occurs, the force becomes opposite and the externality is negative.¹⁶ Essentially, greater diversification of an individual bank increases market liquidity in the good times but reduces market liquidity in the bad times, while market liquidity affects the probability of creditor runs for all other banks. Therefore, ex ante at $t = 0$, an individual bank either over-diversifies or under-diversifies in the competitive equilibrium compared with the constrained optimum.

5 Model Extension - Imperfect Correlation of e_i

In the main model, for simplicity, we have assumed that e_i is perfectly correlated across assets i . In this subsection, we show that as long as e_i cannot be completely diversified away, our model result holds true. It is important to emphasize that in reality the number of assets (and banks) is not infinite and, therefore, e_i cannot possibly be completely diversified away even when e_i is independent across assets.

Now we consider the case where e_i is correlated across i to some degree. Specifically, we assume that e_i has two components: $e_i = \hat{e} + \tilde{e}_i$, where $\hat{e} \sim N(0, \sigma_{\hat{e}}^2)$, $\tilde{e}_i \sim N(0, \sigma_{\tilde{e}}^2)$ which is independent across i , $\sigma_{\hat{e}}^2 = \beta \sigma_e^2$, and $\sigma_{\tilde{e}}^2 = (1 - \beta) \sigma_e^2$. That is, the first component, \hat{e} , is undiversifiable while the second component is diversifiable, and β measures the proportion of the former. We can interpret \hat{e} as the systemic risk of the economy, which is undiversifiable. Note that the benchmark case in which e_i is perfectly correlated across i is a special case of the current setup by setting $\beta = 1$ (i.e., both parts of e_i are undiversifiable).

This alternative assumption has two impacts on the main model. First, investors in the asset market who hold the diversified portfolio can diversify away a part of the risk associated e_i . That is, in Lemma 1, we have $L_j^\omega = \Theta_j - \varphi^\omega k$, where $k \equiv \gamma(\beta \sigma_e^2)$ instead of $k \equiv \gamma \sigma_e^2$. Second, banks' asset portfolios also have a lower variance of payoff. It is easy to show that the payoff distribution of bank j 's portfolio, conditional on its portfolio quality Θ_j , becomes

$$X_j \sim N\left(\Theta_j, \sigma_{\tilde{e}}^2 + (1 - \eta)^2 \sigma_{\hat{e}}^2\right). \quad (16)$$

Clearly, greater diversification (a higher η) still decreases the conditional variance of X_j (where

¹⁶Wagner's (2010) model assumes that there are only two assets and the support of an asset's payoff is specified as $[0, s]$; a bank fails when its realized asset value is less than the (deposit) liability d . In addition, his model implicitly assumes parameter condition $s \geq 2d$. Under these conditions, the externality in his model is always negative. Technically, our model generalizes his model by considering a continuum of banks and a continuum of assets. Consequently, the externality in our model can be positive or negative, depending on the realization of the aggregate state.

$Var(X_j|\Theta_j) \in [\beta\sigma_e^2, \sigma_e^2]$. With this new setup, equation (3) is replaced by

$$\frac{s^{*\omega} - \varphi^\omega k}{F} = \frac{\Delta}{\hat{D}(s^{*\omega}) - 1 + \Delta}, \quad (17)$$

where $\hat{D}(\Theta_j; \eta, \beta) \equiv \mathbb{E} \left(\min \left[R, \frac{X_j}{F} \right] | \Theta_j \right)$ with the distribution of X_j being given by (16). Equation (4) does not change. The system of equations (17) and (4) gives the creditor-run equilibrium.

Lemma 6 *Suppose e_i is imperfectly correlated across assets. The equilibrium at $t = 1$ is given by the system of equations (17) and (4) under the limiting case of $\sigma_s \rightarrow 0$. Under the sufficient condition that β is high enough, ceteris paribus, the conclusions in Propositions 1 and 2 do not change.*

Proof. See Appendix. ■

6 Conclusion

In the recent financial crisis of 2007-2009, the banking system experienced systemic runs by creditors. This paper explains why diversification at financial institutions can be an important cause of systemic bank runs. We show that diversification, while making the financial system more robust against small shocks, increases the possibility of systemic creditor runs when a larger shock hits. In particular, diversification increases the likelihood of a self-fulfilling systemic crisis (multiple equilibria). The underlying mechanism is that diversification makes financial institutions more similar in terms of their asset portfolio holdings and thus induces their clustering behavior in the asset liquidation market. Such clustering behavior reduces market liquidity after an adverse aggregate shock and contributes to a vicious spiral of a lower market liquidity and more creditor runs in the system. Our model suggests that ex post intervention measures to mitigate systemic creditor runs include the injection of liquidity into the financial system, which is crucial to breaking the vicious cycle of feedback.¹⁷ Ex-ante policy measures can include regulations on financial institutions' asset portfolio holdings to prevent their over-diversification which can impose a negative externality on the entire system.

¹⁷See also Liu (2016).

Appendix

A Proofs

Proof of Lemma 1: Considering that the price L_j fully reveals the fundamentals Θ_j (see the trading mechanism in Vives (2014) and Benhabib, Liu and Wang (2016)), an investor does not rely on his private information in trading. Hence, all investors are basically the same (as a representative investor). Thus, the objective function of (1) can be transformed into maximizing

$$\begin{aligned} & \int q_j(\Theta_j - L_j)dj - \frac{1}{2}\gamma Var\left(e \int q_j dj\right) \\ = & \int q_j(\Theta_j - L_j)dj - \frac{1}{2}\gamma\sigma_e^2\left(\int q_j dj\right)^2. \end{aligned}$$

The first-order condition with respect to q_j implies

$$(\Theta_j - L_j)dj - \gamma\sigma_e^2\left(\int q_j dj\right)dj = 0,$$

that is, $\int q_j dj = \frac{\Theta_j - L_j}{\gamma\sigma_e^2}$. Because $\int q_j dj = \varphi$ by the market clearing condition, we have $L_j^\omega = \Theta_j - \varphi^\omega k$, where $k \equiv \gamma\sigma_e^2$.

Proof of Lemma 2: The distribution of X_j under a higher Θ_j has first order stochastic dominance over that under a lower Θ_j . Because function $\min\left[R, \frac{X_j}{F}\right]$ is non-decreasing in X_j and strictly increasing for some ranges of X_j , $D(\Theta_j; R) \equiv \mathbb{E}\left(\min\left[R, \frac{X_j}{F}\right]|\Theta_j\right)$ is increasing in Θ_j .

To reduce notational clutter, we drop the state index superscript ω for now. Given that all other creditors of a bank use threshold s^* , the bank when realizing asset quality as Θ_j has a $\Phi\left(\frac{s^* - \Theta_j}{\sigma_s}\right)$ proportion of its creditors withdrawing. Moreover, the bank with realized asset quality Θ_j will have its asset liquidation value as $L_j = \Theta_j - \varphi k$. Hence, by the nature of credit runs, the threshold of the bank's failure, denoted by Θ^* , is given by

$$\frac{\Theta^* - \varphi k}{F} = \Phi\left(\frac{s^* - \Theta^*}{\sigma_s}\right). \tag{A.1}$$

That is, the bank fails if and only if $\Theta_j < \Theta^*$ and individual creditors rationally anticipate this. Given the bank's failure threshold Θ^* , an individual creditor's threshold s^* is given by the following

indifference condition:

$$\int_{\Theta_j=\Theta^*}^{+\infty} (D(\Theta_j) - 1) d\Phi \left(\frac{\Theta_j - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}} \right) = \int_{\Theta_j=-\infty}^{\Theta^*} \Delta d\Phi \left(\frac{\Theta_j - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}} \right), \quad (\text{A.2})$$

where the prior of Θ_j is $\Theta_j \sim N(\mu_\theta, (1 - \eta)^2 \sigma_\theta^2)$ and $\tau_\Theta \equiv \frac{1}{(1 - \eta)^2 \sigma_\theta^2}$.

Combining (A.1) and (A.2) means that Θ^* in (A.2) is given by the implicit function (A.1) or $s^* = \Theta^* + \sigma_s \Phi^{-1} \left(\frac{\Theta^* - \varphi k}{F} \right)$.

Write the LHS of (A.2) as $Y^L(s^*; \sigma_s)$. We transform $Y(s^*; \sigma_s)$ by changing variables to $z = \frac{\Theta_j - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}}$ and obtain

$$Y^L(s^*; \sigma_s) = \int_{z=z_0}^{\infty} \left[D \left(\sqrt{\frac{1}{\tau_\Theta + \tau_s}} z + \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right) \right) - 1 \right] \cdot \phi(z) dz,$$

where z_0 satisfies the joint equations

$$\begin{aligned} s^* &= \Theta^* + \sigma_s \Phi^{-1} \left(\frac{\Theta^* - \varphi k}{F} \right) \Big|_{\Theta^*=\Theta_j} \\ z &= \frac{\Theta_j - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}} \Big|_{z=z_0}. \end{aligned} \quad (\text{A.3})$$

By (A.3), we have

$$\begin{aligned} s^* - \Theta^* &= \sigma_s \Phi^{-1} \left(\frac{\Theta^* - \varphi k}{F} \right) \\ \Leftrightarrow s^* - \left[z_0 \sqrt{\frac{1}{\tau_\Theta + \tau_s}} + \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right) \right] &= \sigma_s \Phi^{-1} \left(\frac{\Theta^* - \varphi k}{F} \right) \\ \Leftrightarrow -z_0 \sqrt{\frac{1}{\tau_\Theta + \tau_s}} + \frac{\tau_\Theta}{\tau_\Theta + \tau_s} (s^* - \mu_\theta) &= \sigma_s \Phi^{-1} \left(\frac{\Theta^* - \varphi k}{F} \right) \\ \Leftrightarrow z_0 = \frac{\frac{\tau_\Theta}{\tau_\Theta + \tau_s} (s^* - \mu_\theta)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}} - \frac{\sigma_s}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}} \Phi^{-1} \left(\frac{\left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right) + z_0 \sqrt{\frac{1}{\tau_\Theta + \tau_s}} - \varphi k}{F} \right). \end{aligned}$$

So it follows that

$$\lim_{\sigma_s \rightarrow 0} z_0 = -\Phi^{-1} \left(\frac{s^* - \varphi k}{F} \right).$$

Thus, under the limit $\sigma_s \rightarrow 0$ for a given τ_Θ , we have $\Theta^* = s^*$ and

$$\begin{aligned} \lim_{\sigma_s \rightarrow 0} Y^L(s^*; \sigma_s) &= (D(s^{*\omega}) - 1) \cdot \int_{-\Phi^{-1}\left(\frac{s^* - \varphi k}{F}\right)}^{\infty} \phi(z) dz \\ &= (D(s^{*\omega}) - 1) \cdot \frac{s^* - \varphi k}{F}. \end{aligned}$$

Similarly, writing the RHS of (A.2) as $Y^R(s^*; \sigma_s)$, we have $\lim_{\sigma_s \rightarrow 0} Y^R(s^*; \sigma_s) = \Delta \cdot \left(1 - \frac{s^* - \varphi k}{F}\right)$. Therefore, (2) is proved.

We also need to prove that a creditor rolls over when his signal is higher than s^* and otherwise withdraws. An individual creditor takes Θ^* as given. Writing the LHS minus the RHS of (A.2) as $\tilde{Y}(s^*)$ and changing variables to $z = \frac{\Theta_j - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^*\right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}}$, we obtain

$$\tilde{Y}(s^*) = \left\{ \begin{array}{l} \int_{z = \frac{\Theta^* - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^*\right)}^{+\infty} \left[D \left(\sqrt{\frac{1}{\tau_\Theta + \tau_s}} z + \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^* \right) \right) - 1 \right] \cdot \phi(z) dz \\ - \int_{z = -\infty}^{\frac{\Theta^* - \left(\frac{\tau_\Theta}{\tau_\Theta + \tau_s} \mu_\theta + \frac{\tau_s}{\tau_\Theta + \tau_s} s^*\right)}{\sqrt{\frac{1}{\tau_\Theta + \tau_s}}}} \Delta \cdot \phi(z) dz \end{array} \right\}.$$

We have $\partial \tilde{Y}(s^*) / \partial s^* > 0$.

It is straightforward to show that $\frac{\partial \varphi}{\partial s^*} > 0$ in (4). As for $\frac{\partial s^*}{\partial \varphi} > 0$ in (3), write (3) as

$$\hat{Y}(s^*, \varphi) \equiv (s^* - \varphi k) (D(s^*) - 1 + \Delta) - F\Delta = 0,$$

and we have $\frac{\partial \hat{Y}}{\partial s^*} > 0$ and $\frac{\partial \hat{Y}}{\partial \varphi} < 0$, and thus $\frac{\partial s^*}{\partial \varphi} = -\frac{\partial \hat{Y}}{\partial \varphi} / \frac{\partial \hat{Y}}{\partial s^*} > 0$.

Proof of Lemma 3: Write the LHS of (6) as function $V(s^*; \eta, \mu_\theta)$, where

$$V(s^*; \eta, \mu_\theta) = \left\{ \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) k \right] \right\} \frac{D(s^*) - 1 + \Delta}{\Delta}.$$

It is easy to show

$$\begin{aligned} & \frac{\partial V(s^*; \eta, \mu_\theta)}{\partial s^*} \\ &= \frac{1}{F} \left[1 - \phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) \frac{k}{(1 - \eta) \sigma_\theta} \right] \frac{D(s^*) - 1 + \Delta}{\Delta} + \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) k \right] \frac{D'(s^*)}{\Delta}. \end{aligned} \tag{A.4}$$

For a given η , when $\sigma_\theta \rightarrow +\infty$, $\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial s^*} > 0$ holds for any s^* ; when σ_θ is high enough, $\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial s^*} > 0$ also holds for any s^* , so $V(s^*; \eta, \mu_\theta) = 1$ has a unique solution with respect to s^* .

For a given σ_θ , when η is close enough to 1, $\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial s^*} < 0$ around $s^* = \mu_\theta$, considering that

$$\lim_{\eta \rightarrow 1} -\phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) \frac{k}{(1 - \eta) \sigma_\theta} \Big|_{s^* = \mu_\theta} = -\infty.$$

Also, under η being close enough to 1, $\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial s^*} > 0$ when s^* is sufficiently higher or lower than μ_θ . That is, when η is close enough to 1, $V(s^*; \eta, \mu_\theta)$ is non-monotonic in s^* : it is increasing in s^* initially, and then decreasing in s^* around $s^* = \mu_\theta$, and then increasing in s^* again, as the non-monotonic curve in Figure 2 depicts. Thus, equation $V(s^*; \eta, \mu_\theta) = 1$ typically admits three solutions.

Proof of Proposition 1: By Lemma 3, under $\eta = 0$, $V(s^*; \eta, \mu_\theta) = 1$ has a unique solution with respect to s^* as long as σ_θ is high enough. Hence, given such a σ_θ , when η is sufficiently close to 0, $V(s^*; \eta, \mu_\theta) = 1$ still admits a unique solution with respect to s^* .

We also have

$$\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial \eta} = \frac{1}{F} \left[-\phi \left(\frac{s^* - \mu_\theta}{(1 - \eta) \sigma_\theta} \right) \frac{s^* - \mu_\theta}{(1 - \eta)^2 \sigma_\theta} k \right] \frac{D(s^*) - 1 + \Delta}{\Delta}.$$

So

$$\frac{\partial V(s^*; \eta, \mu_\theta)}{\partial \eta} \begin{cases} > 0 & \text{if } s^* < \mu_\theta \\ < 0 & \text{if } s^* > \mu_\theta \end{cases}.$$

Applying the implicit function theorem, $\frac{\partial s^*}{\partial \eta} = -\frac{\partial V(s^*; \eta, \mu_\theta)/\partial \eta}{\partial V(s^*; \eta, \mu_\theta)/\partial s^*}$. Hence, the following comparative statics is obtained:

$$\frac{\partial s^*}{\partial \eta} \begin{cases} < 0 & \text{if } s^* < \mu_\theta \\ > 0 & \text{if } s^* > \mu_\theta \end{cases}.$$

Note that we focus on the equilibrium such that when $\eta = 0$, $s^{*B} < \mu_\theta^B$ and $s^{*E} > \mu_\theta^E$. Hence, when η increases, s^{*B} is decreasing and moves further below μ_θ^B while s^{*E} is increasing and move further above μ_θ^E .

Proof of Proposition 2: Note that $V(s^*; \eta, \mu_\theta)$ is increasing in μ_θ . Hence, when μ_θ is increasing, the non-monotonic curve in Figure 2 shifts upward. Therefore, by Lemma 3, when η is high enough, it is a possible that for a lower μ_θ equation $V(s^*; \eta, \mu_\theta) = 1$ admits multiple solutions with respect to s^* , and for a higher μ_θ equation $V(s^*; \eta, \mu_\theta) = 1$ admits a unique solution with respect to s^* .

Proof of Proposition 3: The two first-order conditions, (15) and (13), are different. So the two programs have different optimum η . η^* can be higher or lower than η^{*SB} . For some set of parameter values, η^* is higher than η^{*SB} .

Proof of Lemma 6: Combining (17) and (4), we have

$$\left\{ \frac{1}{F} \left[s^{*\omega} - \Phi \left(\frac{s^{*\omega} - \mu_\theta^\omega}{(1-\eta)\sigma_\theta} \right) k \right] \right\} \frac{\hat{D}(s^{*\omega}) - 1 + \Delta}{\Delta} = 1, \quad (\text{A.5})$$

where $\hat{D}(\Theta_j; \eta, \beta) \equiv \mathbb{E} \left(\min \left[R, \frac{X_j}{F} \right] | \Theta_j \right)$ with $X_j \sim N \left(\Theta_j, \left[\beta + (1-\beta)(1-\eta)^2 \right] \sigma_\epsilon^2 \right)$, and $k \equiv \gamma(\beta\sigma_\epsilon^2)$. Denote the LHS of (A.5) by function $V(s^*; \eta, \beta)$, where

$$V(s^*; \eta, \beta) = \left\{ \frac{1}{F} \left[s^* - \Phi \left(\frac{s^* - \mu_\theta}{(1-\eta)\sigma_\theta} \right) k \right] \right\} \frac{\hat{D}(s^*) - 1 + \Delta}{\Delta}.$$

Considering that $V(s^*; \eta, \beta)$ is continuous in β , the properties of $V(s^*; \eta, \beta = 1)$ hold for $V(s^*; \eta, \beta)$ when β is sufficiently close to 1. Therefore, when β is high enough, the conclusions in Propositions 1 and 2 do not change.

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