The Great Stampede: Financial Crises and Unemployment Traps*

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Abstract

Standard Mortensen-Pissarides search and matching models feature pro-cyclical search intensity and quick recoveries. Both predictions are at odds with the US labor market after the Great Recession. In this paper, we show that an otherwise-standard MP model that incorporates multi-market simultaneous search predicts that large and temporary financial shocks can cause both higher search intensity and persistently high unemployment, like a stampede into an unemployment trap. The calibrated model can quantitatively account for most of the slow recovery. Productivity shocks are less likely to induce such stampedes. While other government policies are ineffective, a subsidy to firms' entry costs can bring quick recovery.

JEL codes: E24; J64

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1 Introduction

There are four challenges to understand the US labor market after the Great Recession. First, the unemployed increased search effort since the crisis despite high unemployment rate, whereas the canonical Mortensen-Pissarides (MP) search and matching framework predicts pro-cyclical search intensity. Second, given the higher search effort, it is more puzzling why the matching efficiency of the labor market deteriorated (e.g., shifted Beveridge curve). Third, it is yet to be understood why temporary shocks would cause such large and long-lasting effects ("jobless recovery").[1] Fourth, a satisfactory theory also need to explain why a recession caused by a financial crisis would be so different from other recessions, and why government policies used were not effective.

In this paper, we show that an otherwise-standard MP model that incorporates multi-market simultaneous search (MMSS) can explain these four challenges in a parsimonious framework. This model predicts that large and temporary financial shocks can cause both higher search intensity and higher persistent unemployment. In the model, a financial crisis triggers a stampede among unemployed workers and the labor market falls into a high-search-intensity unemployment trap, whereas productivity shocks are less likely to induce such stampede. We also explain why many traditional policies are not effective and propose a novel policy which is effective in restoring matching efficiency and bringing quick recovery to the labor market.

Our theory is motivated by a simple observation: an unemployed worker can increase the chance of being employed either by searching harder in a market (the intensive margin), or by simultaneously searching in more markets (the extensive margin). A market can be a geographical location, an industry, an occupation, or a combination of these dimensions. For example, when academic jobs are scarce, a researcher may start applying for industry jobs. Similarly, jobs in nearby counties which require relocation or longer commuting hours and thus are not considered in normal times can become attractive when conditions in the local market deteriorate.[2] We find that workers in states with higher unemployment rates indeed tend to increase the radius of their job searching radars, as showcased by a higher moving rate in Table 1.[3] Such an extensive margin of search intensity, or to say, MMSS (multi-market simultaneous

[1] Aguiar et al. (2013) and Mukoyama et al. (2018) document increased search intensity of the unemployed. Wealth effect cannot explain why search intensity increased immediately in the Great Recession. Barnichon and Figura (2015) and Sedláček (2016) estimate decreased matching efficiency. Christiano et al. (2010), Gertler and Karadi (2011), and Jermann and Quadrini (2012) emphasize the importance of incorporating financial shocks in macroeconomic models.

[2] Our readers may also be familiar with the fact that junior economists trained in the U.S., on average, search in more foreign locations after the Great Recession.

[3] In Table 1, the moving rate also decreases with distance. Marinescu and Rathelot (2018) document that people dislike distance and yet do send some job applications to firms far away.

search), has been overlooked by the literature.

Table 1: DID: The moving rate and the unemployment rate at the state level, CPS 2008-2014.

	(1)	(2)	(3)	(4)
Moving rates	Overall	within-county	within-state across-county	inter-state
Unemployment rate	0.247**	0.145*	0.059*	0.026
	(0.069)	(0.074)	(0.028)	(0.014)
# of observations	357	357	350	357
R-squared	0.78	0.77	0.73	0.75

Notes: *** p<0.01, ** p<0.05, * p<0.1. Robust standard errors (clustered at the year level) in parentheses. The dependent variable is the moving rate. Independent variables include state fixed effects and year fixed effects.

Allowing MMSS has two direct implications. First, a worker tends to increase search effort in bad times because the extra search effort in the second market is only useful if the worker cannot find a job in the first market. Second, a higher search intensity can discourage job creation. To see this, define a match as the best acceptable offer from a market (i.e., the result of the matching function in that market). Suppose a group of workers exert more effort by searching in more markets, so each of them has, on expectation, more matches.[4] Some matches that would have been accepted in the absence of MMSS are now rejected, slowing down hiring. Firms need to be compensated by a lower market tightness, so that other workers would also have to search harder. We call it the match-rejection externality. It is different from the usual congestion externaltiy, which is complementary to job creation.

Due to this match-rejection externality arising with MMSS search intensity is a strategic complement among workers. Multiple equilibria may arise. More interestingly, the high-intensity equilibrium (HIE) can feature lower matching efficiency – more can be less. Lower search cost can make everyone worse off as it may induce the less efficient HIE. Most importantly, hysteresis is possible and the labor market may fall into an unemployment trap. We calibrate a two-market model to the U.S. economy before the Great Recession and it features two steady-state equilibria, with unemployment rates 5.25% at the high-intensity equilibrium (LIE) and 9.67% at the HIE. Large and temporary financial shocks can lower firm entry and, thus, the job finding rate in a single market, eliminating the LIE by raising the value of MMSS.[5] The economy switches to the HIE as the unique dynamic equilibrium path, experiencing a large increase in unemployment.[6] The effects are also persistent because when the shocks are gone, the economy remains stuck in the less efficienty HIE.

[4]In continuous time, multiple matches are possible if firms give workers time to consider. (I don't understand why this sentence is relevant so I deleted it)

[5]If there are two markets and they are symmetric, then the value of MMSS is a quadratic function of the job finding rate in a market.

[6]The overall offer rejection may still be procyclical. The model abstracts from within-market offer rejection, which is likely to be procyclical.

Negative productivity shocks, however, are less likely to induce similar hysteresis because they also lower wages and thus offset the incentives of MMSS – financial shocks have minimal effects on wages because firms' financing costs are sunk in wage bargaining. These insights also shed light on policies: To overturn the less efficient equilibrium (i.e., the HIE), we need to boost firm entry without encouraging workers' search intensity. The traditional policy candidates would not work: Monetary policy has its limit due to the zero lower bound; government hiring only crowds out firm entry in the MP framework; the usual tax cuts are counter-productive because they increase wages and, thus, the incentives of MMSS. On the other hand, the model suggests a novel solution to halt the Great Stampede: government subsidy to firms' vacancy creation. It acts much like a favorable financial shock, and the mere expectation of it can induce immediate switch of equilibrium.

Last, to explain why the unemployment rate did eventually come down, we propose a simple extension: allowing heterogeneous moving costs. During bad times, only low-moving-cost workers adopt MMSS. The higher search effort makes them leave the unemployed pool faster. This dynamic composition implies that the magnitude of the match-rejection externality eventually decreases over time. It also means that during the transition, the unemployment rate tends to "overshoot." The preferred specification of the model can quantitatively account for both the rise and the fall of the unemployment rate.

There are many papers about the US labor market experience after the Great Recession. Some theories focus on the slow recovery itself. For example, Sterk (2016) and Acharya et al. (2018) extend the work of Pissarides (1992). In their models, multiple equilibria and hysteresis exist because skill losses associated with higher unemployment discourage hiring. However, such mechanism relies on the assumption that different workers look for jobs in the same labor market,[7] and Kroft et al. (2013) find evidence in favor of employer screening over human capital depreciation. Other papers explore channels that help to explain the lowered matching efficiency (see e.g. Barnichon and Figura (2015) Sedláček (2014)). Another related literature explores the effects of nominal or real wage rigidity (see discussion in Pissarides (2009) and Bils et al. (2014)). These studies provide useful insights but do not speak to the aforementioned four challenges. The main contribution of this paper is to provide a simple extention of the MP framework to answer the four challenges at once. It reconciles the endegenously increased search intensity and decreased matching efficiency, explains why financial shocks, rather than productivity shocks, can cause large and persistent effects on the labor market, and proposes a novel policy solution to slow

[7] Consider a model with two skill levels and two types of firms. More skill losses would imply less firm entry in the high-skill market but more firm entry in the low-skill market, breaking the complementarity between skills and aggregate hiring.

recoveries.

Our theory also provides new insights in interpreting existing research on mismatch. Şahin et al. (2014) document low levels of geographical mismatch after the Great Recession. Conventional wisdom says that mismatch is costly, and the less of it, the better. Marinescu and Rathelot (2018) argue that workers can easily use simultaneous search to overcome geographical mismatch so relocating workers would not be very useful in bringing down unemployment. Our theory predicts that simultaneous search can be bad and that the weak mismatch caused by MMSS is associated with low matching efficiency and hiring and high unemployment. We reach different conclusion because it allows endogenous search intensity and job creation, whereas they assume fixed supply of vacancies in each location and fixed number of applications by workers.

This paper is also the first to study endogenous MMSS. The match-rejection externaltiy is not present in previous MP models, because there a higher search intensity increases the chance of a match but does not involve simultaneous matches. Relatedly, many papers study multi-firm simultaneous search (MFSS), providing insights about the microfoundations of matching.[8] The main advantage of the multi-market approach is tractability. It also relies less on specific microfoundations of the matching process.[9] Moreover, the fixed costs of searching and moving are realistic features that are not yet considered in the MFSS literature and are important for our results. Another contribution of this paper is to advance a novel theory of multiple equilibria and hysteresis (see the survey in Kaplan and Menzio, 2016). Note that, unlike sunspot models, the switching of equilibria, in this paper, is the unique equilibrium path.

The rest of the paper is organized as follows. Section 2 demonstrates the mechanism of simultaneous search in a two-period-two-market setting, where productivity shocks and financial shocks are compared. We also discuss several N-market versions which are isomorphic to the two-market setting. Section 3 studies the dynamic version of the model. Section 4 carries out the calibration and simulation. Section 5 considers an extension with heterogeneous moving costs. Section 6 concludes.

[8]Shimer (2004) studies MFSS with random search. For MFSS with directed search, see Albrecht et al. (2006), Galenianos and Kircher (2009), and Kircher (2009). Later, Gautier et al. 2017 and Gautier and Moraga-González 2017 consider endogenous number of applications. These are static models, whereas Wolthoff (2017) studies a dynamic environment. The analyses are theoretically and computationally challenging. In Wolthoff (2017), for example, there exists a continuum of equilibria.

[9]Pissarides (2000) points out that no microfoundation for the matching function dominates all others.

2 The Two-period Model

2.1 Environment

There are two different markets (Market 1 and 2) and two types of workers (Type 1 and 2) with measures u_1 and u_2, respectively. Time is discrete and runs for two periods. In the first period, firms post vacancies and workers search for jobs. In the second period, labor market outcomes realize and workers produce and consume. Workers have linear utility.

Market i is called the "home" market of Type i workers and the "away" market of Type $j\,(\neq i)$ workers. For concreteness, think of a market as a geographical location. For any worker, the search cost in the home market is zero whereas it is $c_u\,(> 0)$ in the away market. If employed in the away market, workers need to pay a one-time cost $c\,(\geq 0)$, which represents the cost of relocation (e.g., monetary and psychological attachment).[10] A worker always searches in the home market but choose whether to search in the away market.

An employed worker produces y and an unemployed worker receives $y_u\;(< y)$ as unemployment payoff. Wage is determined through Nash bargaining with bargaining power ρ of workers. For simplicity, we assume bargaining happens after c is paid.[11] Thus, the wage must satisfy:

$$\max_{w}\,(w - y_u)^{\rho}\,(y - w)^{1-\rho}\,, \tag{1}$$

which gives the wage equation: $w = \rho y + (1-\rho)\,y_u$. The wages from both markets are the same (i.e., $w_i = w_j$). We do not allow for competition among firms when a worker gets matched in both markets, even though that would make our mechanism even stronger. See more discussion below.

The measure of matches in a market, is given by the matching function, $m\,(v,\, n)$, where v and n are the measure of vacancies and applicants in the market. The matching function is increasing in both arguments, constant-returns-to-scale and $m\,(v, n) \leq \min\{v, n\}$. Since workers might search in both markets, n could be different from u_i. An applicant (firm) can at most find one match in a market. Let the market tightness be $\theta = v/n$ in a market, then the chance of finding a match in this market is $q\,(\theta) = m\,(v, n)\,/v$ for a vacancy, and $f\,(\theta) = m\,(v, n)\,/n$ for an applicant. Both $f\,(\theta)$ and $q\,(\theta)$ are positive and smaller than unity.

[10] If we think a market as an occupation or an industry, then "home" means the one that a worker is familiar with, and the c would be the income loss due to training or preference differences.

[11] Suppose c is paid after the bargaining, then $w = \rho y + (1-\rho)\,(y_u + c)$. The relocation cost is shared by the worker and the firm, so that a worker still strictly prefer a job from the home market to a job from the away market. The results are similar.

If one searches in both market, then the job finding rate, λ_i, is given by

$$\lambda_i = 1 - [1 - f(\theta_i)]\left[1 - f\left(\theta_j\right)\right], \text{ with simultaneous search,} \tag{2}$$

where $i \neq j$. Notice λ_i is higher than $f(\theta_i)$. An individual worker takes the market tightness as given, so searching in more markets always increases one's chance of securing a job, though at the cost of c_u. Let Γ^i be the net surplus of simultaneously searching for a Type i worker, i.e., the difference between the payoffs of multi-market simultaneous search and home-market search only. Given the wage equation above, Γ^i is given by

$$\Gamma^i = -c_u + [1 - f(\theta_i)]\, f\left(\theta_j\right)[\rho(y - y_u) - c]. \tag{3}$$

An unemployed worker conducts MMSS if $\Gamma^i > 0$.

We emphasize two assumptions about the matching process. First, the matching function is the same as in the usual single-market models, but now a worker may simultaneously receive two matches, one from each market. This captures the idea that higher search intensity leads to more offers. To map into reality, a match here can be seen as the best acceptable offer in a market during a period of time. If time is continuous, then multiple matches are still possible if firms give workers time to consider the offers, which is true in reality. Wolthoff (2014) studies a continuous-time model in which workers can receive multiple offers because jobs do not start immediately. Second, we assume independence of matching in the two markets. That is, searching in the away market means extra search effort and does not directly affect a worker's chance of finding a match in the home market. Any microfoundation of the MP framework that is consistent with these two assumptions is also consistent with our model.

If a worker receives two matches, one from each market, then the analysis depends on c. If $c > 0$, then the worker always chooses the job from the home market. If $c = 0$, then we assume each job is accepted with $1/2$ probability. Our mechanism works in both cases. But for rigorousness, we separately study these two cases.

2.2 Symmetric Equilibria with Zero Relocation Costs

A. Entry Conditions

Firms borrow the vacancy creation cost c_v in the first period at real interest rate r and generate a profit in the second period. We assume free entry, which drives firms' profit to zero.[12] In symmetric equilibria, the free entry conditions for Market i firms

[12]We assume firms cannot conduct MMSS because by definition a vacancy in Market i promises a job opportunity in Market i (think it as a location or an industry) so the same vacancy should not appear in the matching function of another market. Of course, in reality firms in Market i can try to attract workers from other markets. But in our language that would mean MMSS by the workers. We abstract from these advertisement efforts by firms. See Pissarides (2000) for a discussion.

are given by the following two equations:

$$c_v = \frac{1}{1+r} q(\theta_i)(y-w), \text{ without MMSS;} \tag{4}$$

$$c_v = \frac{1}{1+r} q(\theta_i) \left\{ [1 - f(\theta_j)] + \frac{1}{2} f(\theta_j) \right\} (y-w), \text{ with MMSS.} \tag{5}$$

The second equation captures the idea that if a worker also finds a match from Market j, which happens with $f(\theta_j)$ probability, she only accepts the match in Market i with half of the chance. Since these equations applies to both markets, so we have $\theta_i = \theta_j$.

There are potentially two types of pure-strategy symmetric equilibria: the low-intensity equilibrium (LIE) where all workers only search in their home markets, and the high-intensity equilibrium (HIE) where all workers adopt MMSS and search in both markets. Note that $q(\theta)$ and $q(\theta)[1 - f(\theta)/2]$ are both monotonically decreasing in θ. Therefore either entry condition uniquely pins down a market tightness. We use θ_L and θ_H to denote the market tightness implied by (4) and (5) respectively. We immediately have the following result:

Lemma 1. *The market tightness with free entry is lower in the HIE than in the LIE:* $\theta_H < \theta_L$.

The result is straightforward because $1 - f(\theta)/2 < 1$. In the HIE, matches are rejected with the probability of $f(\theta)/2$, so firms require a higher arrival rate (i.e., lower market tightness: $\theta_H < \theta_L$) to offset these rejected matches. It is therefore harder for an applicant to find a match in a market in the HIE than in the LIE. We call this the *match-rejection externality* imposed by MMSS. Of course, the overall probability of finding a job is not necessarily lower in the HIE, because an unemployed worker searches in both markets instead of one.

B. Equilibrium

A symmetric pure-strategy equilibrium is defined as a combination of a search intensity level (i.e., one market or two markets) and a market tightness, which satisfy both the incentive condition for workers (i.e., searching in only home market requires $\Gamma \leq 0$ and searching in both markets requires $\Gamma \geq 0$) and the free-entry condition of firms (i.e., (4) or (5)). Note that (3) can be written as

$$\Gamma(\theta) = -c_u + \left\{ -\left[f(\theta) - \frac{1}{2} \right]^2 + \frac{1}{4} \right\} \rho (y - y_u). \tag{6}$$

To see the existence of the two types of equilibria, we only need to check if θ_L and θ_H satisfy the respective incentive conditions. The LIE exists iff (if and only if) $\Gamma(\theta_L) \leq 0$; and the HIE exists iff $\Gamma(\theta_H) \geq 0$. We have the following two situations depending on the parameter values:

Case	θ_L	θ_H	$\Gamma(\theta_L)$	$\Gamma(\theta_H)$	LIE	HIE
1	$\geq \bar{\theta}$	$> \bar{\theta}$	≤ 0	< 0	$\surd$	
2	$\geq \bar{\theta}$	$[\underline{\theta}, \bar{\theta}]$	≤ 0	≥ 0	$\surd$	$\surd$
3	$\geq \bar{\theta}$	$< \underline{\theta}$	≤ 0	< 0	$\surd$	
4	$(\underline{\theta}, \bar{\theta})$	$[\underline{\theta}, \bar{\theta}]$	> 0	≥ 0		$\surd$
5	$(\underline{\theta}, \bar{\theta})$	$< \underline{\theta}$	> 0	< 0		
6	$\leq \underline{\theta}$	$< \underline{\theta}$	≤ 0	< 0	$\surd$	

Table 2: Six combinations of θ_L and θ_H

- If $c_u > \rho(y - y_u)/4$, then $\Gamma(\theta)$ is always negative so only the LIE exists.
- If $c_u \leq \rho(y - y_u)/4$, then there exist two cutoffs $\underline{\theta}$ and $\bar{\theta}$, with $f(\underline{\theta}) \leq \frac{1}{2} \leq f(\bar{\theta})$, such that $\Gamma(\underline{\theta}) = \Gamma(\bar{\theta}) = 0$. Furthermore, if $\theta \in [\underline{\theta}, \bar{\theta}]$, then $\Gamma(\theta) \geq 0$, and otherwise $\Gamma(\theta) < 0$.

Intuitively, MMSS is not profitable if c_u is high enough. In the second situation, the θ space could be divided into three regions. Because $\theta_L > \theta_H$, we then have six cases which are summarized in Table 2. There are five cases with at least one pure-strategy symmetric equilibrium. Mixed strategy equilibrium is discussed in Appendix C. Most interestingly, in the second case we may have multiple equilibria. Panel (a) in Figure 1 shows an example of multiple equilibria with $\Gamma(\theta_L) < 0$ and $\Gamma(\theta_H) > 0$.

C. Search Intensity and Job Finding Rate

Now we compare the job finding rate in the HIE and the LIE. It is not surprising that we can find examples of the HIE that have higher overall job finding rates than the LIE – workers double their search effort in HIE. However, the opposite can also be true – more effort is less efficient.[13] This happens when the match-rejection externality is strong enough. We explore more on this issue in the calibrated dynamic model in Section 3.

Below we provide a useful lemma.

Lemma 2. *If multiple equilibria exist, the job finding rate is lower in the HIE than in the LIE if and only if $2\theta_H < \theta_L$ or, equivalently, $v_H/u_H < v_L/u_L$ is satisfied.*

Here we scatch the proof. From equations (2), (4), (5) and the fact that $f(\theta) = \theta q(\theta)$, the overall job finding rate in the two types of equilibria, λ_H and λ_L, satisfy

$$\frac{\lambda_H}{\lambda_L} = \frac{2\theta_H}{\theta_L}. \tag{7}$$

Note that the market tightness in our model is defined as the ratio of vacancies to applicants. In the HIE, the number of applicants is the sum of unemployed in both

[13] In other words, the MMSS model can provide a mechanism for generating a substitution relationship between search intensity and labor market tightness, as assumed in Mukoyama et al. (2018).

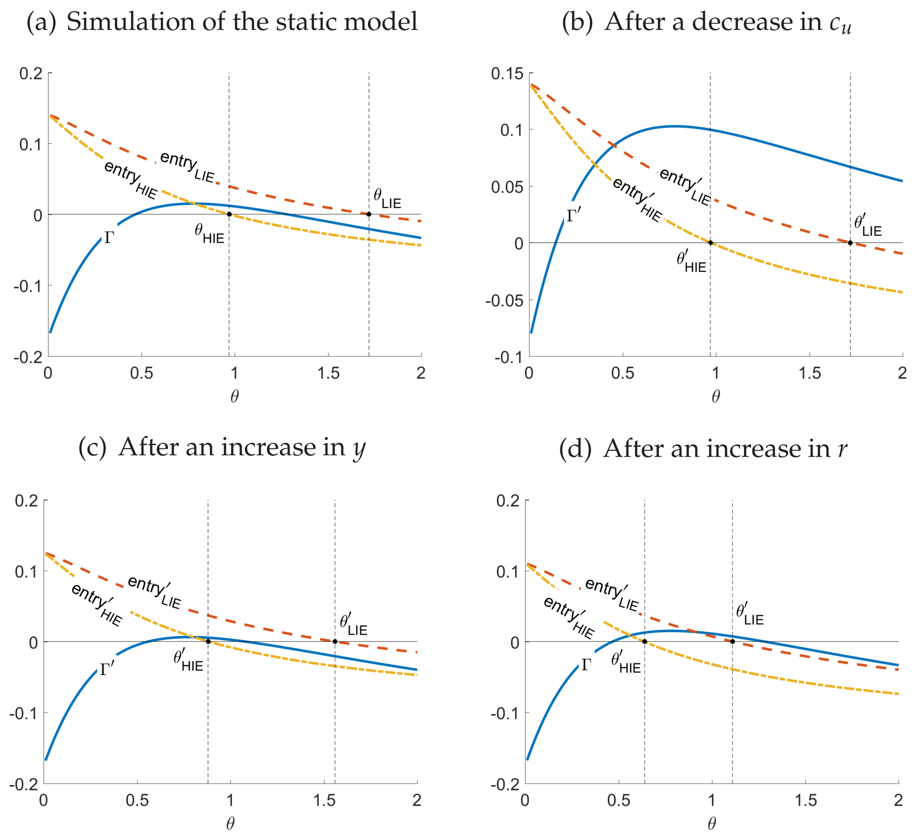

Figure 1: Two-period model

markets. The above lemma says observed overall market tightness (i.e., v/u) is enough for us to infer whether the overall job finding rate is higher or lower in the two types of equilibria.

D. Comparative Statics

Now we conduct comparative statics in this tractable two-period model to get insights about how the model works.

D1. Cost of Searching in Other Market(s)

Note that a decrease in the search cost c_u shifts up the $\Gamma(\theta)$ curve. Panel (b) of Figure 1 plots an example with the same parameters as in Panel (a), except for a lower c_u. Now the HIE is the unique equilibrium. An interesting observation is that the labor market can become less efficient in matching workers and firms (lower job finding rate) if the match-rejection externality is strong enough. So decreasing the search cost may not always be good.[14] One may apply the theory to think about the high unemployment rates in Europe since the 80's (see reference in Blanchard and Summers

[14]Of course, the development of IT can also increase the efficiency of the matching function itself, so the overall effect of IT can still be positive even if that means a less efficient equilibrium.

(1986) and Kennan (1988)). The European Union greatly reduced the costs of working and looking for jobs in a different country within the Union. Another implication is that equilibrium *selection* can be path dependent. If the search cost is initially high and gradually decreases over time, it is possible the LIE is the unique equilibrium at first and later the HIE also becomes feasible. However, the economy stays in the LIE because equilibrium switching requires strong coordination.

D2. Changes in Productivity

Panel (c) plots an example with the same parameters as in Panel (a), except for a lower y. Two things happen. First, the two entry curves shift downwards, because firms have less incentive to create vacancies. Second, the $\Gamma(\cdot)$ function also shifts downwards, because jobs are less valuable for workers (i.e., wages go down). In the example in Panel (c), the decrease in productivity eliminates the HIE and preserves the LIE. In fact, from (6) we know that if productivity is low enough such that $c_u > \rho(y - y_u)/4$, then the HIE is never feasible, because the entire $\Gamma(\cdot)$ is below zero for any θ.

D3. Changes in Financing Costs

During a financial crisis, firms finding it harder or more costly to borrow. We interpret negative financial shocks as an increase in r. Panel (d) in Figure 1 plots an example of higher r relative to Panel (a). Compared to a decrease in y, a higher r only shifts the entry curves downwards, leaving the $\Gamma(\cdot)$ function intact. This is because when matched firms and workers bargain over wages, the entry cost is sunk. If we start with the LIE as in Panel (a), then we can always find an increased value of r such that θ_L falls within the range of $(\underline{\theta}, \bar{\theta})$ and therefore $\Gamma(\theta_L) > 0$. In general, we have the following result.

Proposition 3. *If $c_u \leq \rho(y - y_u)/4$, then we can always find a $c_v(1+r)$ which makes the LIE feasible or infeasible. Similarly, we can always find a $c_v(1+r)$ which makes the HIE feasible or infeasible.*

See Appendix A for the proof. Suppose we start with the LIE and $\theta_L > \bar{\theta}$. It is clear from Figure 1 that a small increase in the financing cost would decrease θ and thus increase the incentives of MMSS. But an extremely large increase in financing cost would always make $\Gamma(\theta_L) < 0$. Using these insights, we can easily have the following corollary.

Corollary 4. *Suppose $c_u \leq \rho(y - y_u)/4$. If we let the value of $c_v(1+r)$ go from zero to infinity, then the set of equilibria would experience the six cases in Table 1.*

The proof is omitted. What is particularly interesting about this corollary is that given condition $c_u \leq \rho\,(y - y_u)\,/4$ and other parameters, we can always find a value of $c_v\,(1+r)$ such that only the LIE exists, or only the HIE exists, or both the LIE and the HIE exist. In other words, multiple equilibria are not only possible but also a generic feature of the model.

D4. Government Policies

In Panel (a) of Figure 1, suppose we are in the HIE and suppose it is less efficient than the LIE. It is hard for the economy to switch to the LIE because of the difficulty associated with coordination. Can the government policy improve the search efficiency? Consider the following three candidates: government hiring, reducing business income taxes, and subsidizing job creation. Conceptually, there are two ways for the government to hire people: set up vacancies and join the search and matching process, or skip the matching process and directly take people away from the unemployed pool (e.g. military draft). Neither would change anything on the graph of Panel (a). This is because if the government employs more people, the private sector would simply hire less so that the market tightness is unchanged. The free entry conditions does not differentiate between how many vacancies are created by the government and how many are created by private firms. Those conditions only pin down the market tightness. Because the matching process is constant return to scale, skipping the matching process and directly taking people out of the unemployed pool would not change the market tightness either.

Then consider a reduction in business income taxes. We can assume that the firms face some income tax rate τ in the model. Lower business income taxes means the joint surplus of a job is higher for workers and firms, which is much like an increase in productivity. The entry curve would shift up, but the Γ curve shifts up as well. That is, lower taxes also raises wages and the value of MMSS. So we might be still stuck in the HIE.

However, **subsidizing job creation** is different. This can be seen as a reduction of c_v in the model (e.g., tax rebate for every new job). The entry curve would shift up but the Γ curve is unchanged because the entry cost is sunk for wage bargaining. In fact, a subsidy for job creation is like a positive financial shock, but better. This is because, r can only go down so much (i.e., it has to be non-negative), which in some way reflects the "zero-lower-bound" problem in monetary economics. But there are large rooms in the c_v that can be adjusted if the government wants to get rid of the less efficient HIE. The following corollary shows that the government can always get rid of the HIE through changes in c_v. See Appendix A for the proof.

Corollary 5. *If $c_u \leq \rho\,(y - y_u)\,/4$, then there exists a cutoff $\bar{c}_v$ such that if $\forall c_v < \bar{c}_v$, then* *the HIE is infeasible.*

2.3 Symmetric Equilibria with $c > 0$

Now consider positive relocation cost. The analysis is much the same. The Γ function is the same as in (3). Of course, if c is large, then the HIE is not feasible because Γ is always negative. Next, consider the entry conditions. Notice that when $c = 0$, the entry condition (5) does not depend on u_1 or u_2. If $c > 0$, then the LIE entry condition is same as (4) whereas that for the HIE (in Market i) becomes

$$c_v (1 + r) = q(\theta_i)(y - w) \left\{ \left[1 - f(\theta_j)\right] + \frac{u_i}{u_i + u_j} f(\theta_j) \right\}, \text{ in the HIE.} \tag{8}$$

Conditional on receiving a match in Market i, a worker also receives a match from Market j with $f(\theta_j)$ probability. In this event, the match in Market i is accepted if and only if it is given to a Type i worker, which happens with $u_i / (u_i + u_j)$ probability. Interestingly, if $u_i = u_j$, which is the case that we focus on, then the entry condition of (8) is the same as (5).[15]

There are three obvservations. First, firm entry can offset the usual congestion externality (because wage is unaffected by market tightness) but not the match-rejection externality. This is because a higher v_i can affect θ_i and $q(\theta_i)$ but not the conditional match-rejection probability $u_i / (u_i + u_j)$. Second, a Type 1 worker's simultaneously search in Market 2 imposes negative externality only to the workers searching in Market 2 if $c > 0$, but also on workers searching in Market 1 if $c = 0$, because, in that case, she might also reject a match in Market 1. Of course, such differences are not important in symmetric equilibria with $u_1 = u_2$, because both deliver the same HIE entry condition. Interested readers are refered to Appendix B for more discussion of these two issues.

2.4 Extensions

The purpose of this subsection is to show that the economics of our benchmark model is preserved with these considerations: N-market, wage competition, and statistical market discrimination (employers may treat workers from different markets differently). Generally, wage competition among firms tends to strengthen the match-rejection externality, and statistical market discrimination tends to weaken it. See Appendix D for more discussion. Below we consider N-market versions.

[15] If u_1 and u_2 are very different, then it is important to consider the asymmetric equilibria (i.e., one type of workers only search in one market and the other type of workers search in both). This is left for future study.

N-Market Here we show that the two-market framework can be generalized. Specifically, the mathematics are exactly the same in the following two N-market environments. First, suppose the N identical markets lie in a circle. A worker can choose either to search only in the home market, or to search in both the home market and the market to the left/right. Second, consider any N-market environment. A worker from Market i can choose either to search only in the home market, or to search in the home market and randomly one of the $N-1$ away markets, with probability proportional to the size of that market (i.e., local job seekers, u_n). It is as if for any market, there is a mirror "away" market. In general, allowing workers to search in more than two markets simultaneously may change the math but not the economics: MMSS induces match-rejection externality. For tractability, we will continue to focus on the two-market setup for the rest of the paper.

3 The Dynamic Model

The previous section uses a two-period model to show the main mechanism of a MMSS model. Now we turn to its dynamic version with positive relocation cost, so we can interpret markets as geographical locations and study quantitative properties of the model. For tractability, we assume that if an agent has moved to a new market, then it becomes her home market. If she moves again, then she needs to pay the relocation cost again.

3.1 Model Setup

The value function of an employed worker in Market i can be written as:

$$W_t^i = w_t + \beta\delta E_t\left(U_{t+1}^i\right) + \beta\left(1-\delta\right)E_t\left(W_{t+1}^i\right), \tag{9}$$

where β is the common discount factor, δ is the exogenous separation probability, and U^i is the value function of an unemployed worker living in Market i, which is given by

$$U_t^i = y_u + \beta f\left(\theta_{it}\right)E_t\left(W_{t+1}^i\right) + \beta\left[1-f\left(\theta_{it}\right)\right]E_t\left(U_{t+1}^i\right) + \max\left(0, \Gamma_t^i\right), \tag{10}$$

which differs from those in single-market MP models because of the last term. Inside the max operator are zero and the dynamic version of the net surplus of MMSS:

$$\Gamma_t^i = \beta f\left(\theta_{jt}\right)\left[1-f\left(\theta_{it}\right)\right]\left[E_t\left(W_{t+1}^j\right) - E_t\left(U_{t+1}^i\right) - c\right] - c_u. \tag{11}$$

If $\Gamma_t^i < 0$, then the unemployed in Market i only search in Market i, in period t. if $\Gamma_t^i >$ 0, then they search in both markets. We have implicitly assumed that $E_t\left(W_{t+1}^j\right) - c \leq$

$E_t\left(W^i_{t+1}\right)$, which will be true if the two markets are symmetric, so that job seekers in Market i, upon receiving matches from both markets, will choose to work in Market i.[16] We formulate the problem in discrete time. Again, if time is continuous, then multiple matches are still possible if firms give workers time to consider.

Entry Conditions and Law of Motion of Unemployment

Similar to (4) and (8), the corresponding entry conditions in Market i can be written as

$$c_v = q\left(\theta_{it}\right) E_t\left(J_{t+1}\right), \text{ in the LIE;} \tag{12}$$

$$c_v = q\left(\theta_{it}\right)\left[1 - \frac{u_{jt}}{u_{it}+u_{jt}} f\left(\theta_{jt}\right)\right] E_t\left(J_{t+1}\right), \text{ in the HIE,} \tag{13}$$

where u_{it} and u_{jt} are endogenously determined, $\theta_{it} = v_{it}/n_{it}$, and $E_t\left(J_{t+1}\right)$ is the expected present value of the cash flow of a matched vacancy formed in period t. We assume that firms posting vacancies in period t sign fixed-interest-rate contracts with the financial market at interest rate r_t. So $E_t\left(J_{t+1}\right)$ can be written as

$$E_t\left(J_{t+1}\right) = \frac{1}{1+r_t} E_{t+1}\left[\sum_{k=1}^{\infty}\left(\frac{1-\delta}{1+r_t}\right)^{k-1}\left(y_{t+k} - w_{t+k}\right)\right]. \tag{14}$$

In the special case in which y and w are constant, we have

$$E_t\left(J_{t+1}\right) = \frac{y-w}{\delta + r_t}. \tag{15}$$

Note that we can interpret the interest rate r in the Pissarides type of free-entry conditions more broadly. Many firms face borrowing constraints, especially during crisis times. If such borrowing constraints become tighter at the same interest rate, then the effects on job creation would be similar if we assume free entry with a higher interest rate. In other words, the value of r_t, which is used to discount all future dividends, can also be seen as a measure of financial duress in period t. If one takes this interpretation, then the fixed-interest-rate contract is also easier to work with.[17]

[16] Theoretically, there could be cases where $E_t\left(W^j_{t+1}\right) - c \geq E_t\left(W^i_{t+1}\right)$. It can happen temporarily, for example, when some shocks hit one market more than the other. As workers flow to Market j, the difference would gradually decrease. These dynamics are interesting, but we leave it for future study and focus on the symmetric case.

[17] To get an idea of what a shock to r_t can do quantitatively in a fixed-interest-rate contract, suppose initially $r_t = 2\%$ and $\delta = 2.4\%$. If r_t increases by 1 percentage point, then $E_t\left(J_{t+1}\right)$ would decrease by more than 18%. Even an increase of 0.5 percentage points in r_t would reduce $E_t\left(J_{t+1}\right)$ by more than 10%. These numbers might appear big. In reality, the $v-u$ ratio in the U.S. economy was 0.71, 0.54, and 0.15 in April 2007, April 2008, and August 2009, respectively, while the separation rate remained broadly steady. The job creation process certainly experienced some major disturbances.

The main reason for assuming the fixed-interest-rate contract is due to tractability. The key question we ask is not what causes a crisis, but how the labor market responds to a crisis that makes firms temporarily harder to create jobs. If we assume adjustable-interest-rate contracts, then it matters when and what agents know about future interest rates and how to put such information into the contract.[18] That would complicate the analysis.

The laws of motion for the unemployment rate for Type i workers are as follows ($i \neq j$):

$$u_{i,t+1} = u_{it} + (1 - u_{it})\,\delta - u_{it} f\left(\theta_{it}\right), \text{ in the LIE,} \tag{16}$$

$$u_{i,t+1} = u_{it} + (1 - u_{it})\,\delta - u_{it}\left\{f\left(\theta_{it}\right) + f\left(\theta_{jt}\right)\left[1 - f\left(\theta_{it}\right)\right]\right\}, \text{ in the HIE.} \tag{17}$$

In both (16) and (17), the third term on the right-hand-side is the unemployment rate times the job finding rate in the respective equilibrium.

Wage Determination

We assume a matched worker and her firm bargain over the wage every period. The worker's and firm's outside options are as follows. The outside option for the worker is to take the unemployment income y_u and enter the next period still matched with the same firm; the outside option for the firm is to produce nothing and enter the next period still matched with the same worker (unless nature separates them with probability δ). In such a game, the wage outcome is the same as that from (1) in the two-period model. The outside options here certainly simplify the analysis compared to Pissarides (1985), Mortensen and Pissarides (1994) and many subsequent papers. But as argued by Hall and Milgrom (2008), the threat to terminate the relationship is not rational for both parties, and the credible threat point is to delay production. A strike would be an example of such delays. Our wage outcome is consistent with this view.[19]

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[18] If a firm sign an adjustable-interest-rate contract, then $E_t\left(J_{t+1}\right)$ can be written as

$$E_t\left(J_{t+1}\right) = \frac{1}{1+r_t} E_t\left[\sum_{k=1}^{\infty} \frac{(1-\delta)^{k-1}}{\Pi_{l=1}^{k-1}\left(1+r_{t+l}\right)}\left(y_{t+k} - w_{t+k}\right)\right].$$

A usual rationale for using adjustable-interest-rate contracts is that borrowers can later refinance the debt contract if interest falls. However, it is not common practice for a firm to refinance job positions, perhaps due to the transaction costs involved or potential information problems.

[19] Similar wage outcome is used in Kaplan and Menzio (2016).

3.2 Steady State Equilibria

An equilibrium is a sequence of search choices, wages and entry decisions that are consistent with workers' value functions, firms' entry conditions, and the laws of motion described above.[20] Below we focus on the LIE steady state and the HIE steady state. In a steady state, c_{vt} is constant over time, so are J_t, θ_{it}, W_t^i, W_t^j, U_t^i and u_{it}, where $i, j \in \{1, 2\}$, $i \neq j$. From (15), we can derive J. Then the entry conditions (12) and (13) uniquely pin down the market tightness in the respective type of equilibrium. We use θ_L and θ_H to denote the market tightness in the two types of equilibrium. Using subscript s to stand for H or L, from (9) and (10), we have

$$(1-\beta)\,W_s = w - \beta\delta\,(W_s - U_s)\,, \tag{18}$$

$$(1-\beta)\,U_s = y_u + \beta f\,(\theta_s)\,(W_s - U_s) + \max\left[0, \Gamma_s\,(\theta_s)\right], \tag{19}$$

where the subscript of the Γ function is there because the type of equilibrium affects value functions and thus the Γ function. Subtract (19) from (18) to derive the following:

$$W_s - U_s = \frac{w - \{y_u + \max\left[0, \Gamma_s\,(\theta_s)\right]\}}{1-\beta+\beta\delta+\beta f\,(\theta_s)}, \tag{20}$$

From (20) it is apparent that if the term $\max\left[0, \Gamma_s\,(\theta_s)\right]$ increases, it is as if the unemployed have a higher y_u. We can then write down U_s and W_s as weighted averages of w and $y_u + \max\left[0, \Gamma_s\,(\theta_s)\right]$:

$$(1-\beta)\,W_s = \frac{\left[1-\beta+\beta f\,(\theta_s)\right]w + \beta\delta\,\{y_u + \max\left[0, \Gamma_s\,(\theta_s)\right]\}}{1-\beta+\beta\delta+\beta f\,(\theta_s)}, \tag{21}$$

$$(1-\beta)\,U_s = \frac{\beta f\,(\theta_s)\,w + (1-\beta+\beta\delta)\,\{y_u + \max\left[0, \Gamma_s\,(\theta_s)\right]\}}{1-\beta+\beta\delta+\beta f\,(\theta_s)}. \tag{22}$$

For U_s, the weight of w depends on the probability of finding a job in the next period. The Γ function in (11) becomes

$$\Gamma_s = \beta f\,(\theta_s)\left[1 - f\,(\theta_s)\right](W_s - U_s - c) - c_u. \tag{23}$$

Low-Intensity Equilibrium (LIE)

In the low-intensity equilibrium (LIE), the market tightness is given by (12), which in steady state can be written as $c_v = q\,(\theta_L)\,J$. Of course, we need the incentive condi-

[20] For example, an equilibrium candidate could be: workers search in both markets in every even-numbered periods and in only one market otherwise. But this requires strong coordination.

tion: $\Gamma_L(\theta_L) < 0$. The steady state version of (11) is straightforward,

$$\begin{aligned}\Gamma_L(\theta_L) &= \beta f(\theta_L)[1 - f(\theta_L)][W_L - U_L - c] - c_u. \\ &= \beta f(\theta_L)[1 - f(\theta_L)]\left[\frac{w - y_u}{1 - \beta + \beta\delta + \beta f(\theta_L)} - c\right] - c_u \end{aligned} \tag{24}$$

We can see now that c_u does not directly affect the values of W_L and U_L, but affects the incentive condition of MMSS as shown in (11). Lastly, using (16), the unemployment rate in the LIE steady state is given by

$$u_L = \frac{\delta}{\delta + f(\theta_L)} \tag{25}$$

High-Intensity Equilibrium (HIE)

In the HIE, the market tightness is given by (13) which in the steady state can be written as (we assume the two markets are identical now):

$$c_v = q(\theta_H)\left[1 - \frac{1}{2}f(\theta_H)\right]J, \text{ in the HIE.} \tag{26}$$

The HIE requires the incentive condition: $\Gamma_H(\theta_H) > 0$. We can then derive the following,

$$\Gamma_H = \frac{\beta f(\theta_H)[1 - f(\theta_H)](w - y_u) - [1 + \beta\delta - \beta(1 - f(\theta_H))]\{c_u + \beta f(\theta_H)[1 - f(\theta_H)]c\}}{1 + \beta\delta - \beta[1 - f(\theta_H)]^2} \tag{27}$$

Lastly, using (17) and the fact that the two markets are identical, the unemployment rate in steady state is given by

$$u_H = \frac{\delta}{\delta + f(\theta_H)[2 - f(\theta_H)]}, \tag{28}$$

where $f(\theta_H)[2 - f(\theta_H)]$ is the job finding rate for the representative unemployed worker.

Multiple Equilibria

First, we present a lemma that is useful for checking the incentive conditions.

Lemma 6. *$\Gamma_H(\theta)$ and $\Gamma_L(\theta)$ always have the same sign.*

See Appendix A for the proof. Because $\Gamma_H(\theta)$ and $\Gamma_L(\theta)$ always have the same sign, we can simply put θ_H and θ_L from the two entry conditions into the same $\Gamma_L(\theta)$ function to check the incentive conditions. This means that even though we have a different Γ function for each type of equilibrium, the analysis of the existence of multiple equilibria is very similar to the two-period version, where we only have to work with one Γ function.

Our $\Gamma_L(\theta)$ is different from its counterpart in the two-period version: now θ also affects the size of the surplus if a worker is employed in the away market. However, it is straightforward to check the following results: $\Gamma_L(0) = \Gamma_L(1) = -c_u$, $\Gamma'_L(0) > 0$ and $\Gamma'_L(\theta) < 0$ if $f(\theta) > 1/2$. The following lemma is similar to our results in the two-period model. It will be useful to prove existence of multiple equilibria below.

Lemma 7. *In the steady state, $\theta_L > \theta_H$.*

This is because $q(\theta)$ is decreasing and from the two entry conditions we have $q(\theta_L) = q(\theta_H)\left[1 - \frac{1}{2}f(\theta_H)\right]$. Combining the above two lemmas we can have the following proposition:

Proposition 8. *If $c + 4c_u/\beta < \rho(y - y_u)/(1 - 0.5\beta + \beta\delta)$, then we can always find a $c_v(1+r)$ which makes the LIE steady state feasible or infeasible, and similarly for the HIE steady state.*

The proof is omitted because it is similar to Proposition 4 in the two-period version. There is one difference: now the condition $c + 4c_u/\beta < \rho(y - y_u)/(1 - 0.5\beta + \beta\delta)$ guarantees that $\Gamma_L(\tilde{\theta}) > 0$, where $f(\tilde{\theta}) = 1/2$. This condition says that *the relocation cost and the cost of MMSS should not be too high to make sure MMSS is profitable for at least some market tightness.* Because $\Gamma_L(1) = -c_u$ and $\Gamma'_L(\theta) < 0$ if $f(\theta) > 1/2$, we are sure to find a cutoff $\hat{\theta}$, such that $\Gamma_L(\theta) > 0$ for $\theta \in (\tilde{\theta}, \hat{\theta})$ and $\Gamma_L(\theta) < 0$ for $\theta \in (\hat{\theta}, 1)$. Then we only need to vary $c_v(1+r)$ to make sure θ_H and θ_L fall into the one of the regions to make the two types of equilibria feasible or infeasible. We also have a weaker version of Corollary 5 in the two-period as follows:

Corollary 9. *If $c + 4c_u/\beta < \rho(y - y_u)/(1 - 0.5\beta + \beta\delta)$, then we can always find a $c_v(1+r)$ to make sure that the LIE steady state is the unique steady state equilibrium or there is multiple steady state equilibria of both the HIE and the LIE.*

This is weaker than Corollary 5 because we know less about the $\Gamma_L(\theta)$ function. But it again shows that multiple equilibria are not only possible but also a generic feature of the model. Next, we can compare the properties of the LIE and the HIE steady states. Suppose we have multiple steady-state equilibria, the following proposition concerns the efficiency of the LIE and the HIE in the steady state.

Proposition 10. *The following two statements are identical:*

(a) The HIE steady state has a higher unemployment rate and a lower job finding rate.

(b) The LIE steady state has a higher $v - u$ ratio, which also means $\theta_L \geq 2\theta_H$.

See Appendix A for the proof. Intuitively, the HIE steady state has a higher unemployment rate if the match-rejection externality caused by MMSS is sufficiently strong. It should not come as a surprise if the HIE has a much smaller market tightness. What is perhaps surprising is that the observed v-u ratio does not need to be drastically lower in the HIE to have a higher steady-state unemployment rate. Note that even if the v-u ratio in the HIE is only slightly smaller, it already implies substantial match-rejection externality. After all, the unemployed workers double their search effort and yet still have a smaller job finding rate. Of course, whether we have $u_H > u_L$ is a quantitative question, which we address in the next section.

4 Quantitative Analysis

In this section, we calibrate the model to match features of the US economy from 1987 to 2007. We then ask whether the model economy features multiple equilibria and what happens in the labor market after some transitory productivity and financial shocks. Government interventions are also discussed.

4.1 Calibration Strategy

The properties and behaviors of the model are determined by a total of 12 parameters. Note that our model in the LIE behaves just like the standard one-market models. Thus we pick our parameter values so that when evaluated at the LIE steady state, the calibrated economy matches features of the U.S. economy between 1987 and 2007. So our calibration is consistent with previous studies without MMSS.

Without loss of generality, we normalize the productivity of a matched worker-firm pair to $y = 1$, and the unemployment benefit to $y_u = 0.2$, which lies in the range of $[0, 0.4]$, as indicated in Shimer (2005). We normalize a time period to be one month and set the discount factor to $\beta = 0.997$, which is equivalent to an annual discount factor of 0.965.[21] The implied monthly interest rate is $r_t = r = \frac{1}{\beta} - 1 =$ 0.3%, which is equivalent to an annual rate of 3.6%. The exogenous separation rate, $\delta = 0.024$, is chosen to match the average monthly transition rate from employment to unemployment in the U.S. between 1987 and 2007. We set the worker's bargaining

[21]This is derived from a time discount rate of 0.003, as used in Kaplan and Menzio (2016), $\beta = \exp(-0.003) = 0.997$.

power to $\rho = 0.74$.[22] Our matching function is CES as follows,[23]

$$m(u, v) = \kappa \left(u^{-\phi} + \alpha v^{-\phi}\right)^{-\frac{1}{\phi}}. \tag{29}$$

A special case of this matching function is used in Kaplan and Menzio (2016) by setting $\kappa = \alpha = 1$. When $\phi = 0$, it becomes a Cobb-Douglas matching function, which is also widely used. We set the parameter α in the matching function to 0.389. This is calculated as the ratio of the power terms in the Cobb-Douglas matching function, as in Shimer (2005).[24] The matching function parameters, κ and ϕ, are chosen to match the elasticity of the job finding rate with respect to the market tightness and the average v-u ratio. Specifically, we have

$$\phi = -\frac{\ln\left(\frac{\hat{\eta}}{\alpha(1-\hat{\eta})}\right)}{\ln \hat{\theta}}, \text{ and } \kappa = f\left(\hat{\theta}\right)(1-\hat{\eta})^{-\frac{1}{\phi}}, \tag{30}$$

where $\hat{\eta}$ and $\hat{\theta}$ are the targeted values of the elasticity and the v-u ratio, respectively, and $f\left(\hat{\theta}\right) = \hat{\theta} q\left(\hat{\theta}\right)$ is the job finding rate.[25] As noted in Menzio and Shi (2011) and Kaplan and Menzio (2016), after taking into account the on-the-job search, the elasticity of the job finding rate with respect to the market tightness is 65% in the data, which is our targeted value. We choose to target the v-u ratio at 0.8, which lies in the range of calculations in Hall and Schulhofer-Wohl (2015).[26] The job finding rate is measured as the average monthly transition rate from unemployment to employment, which is 0.433 in the U.S. between 1987 and 2007. Given the matching function, the vacancy costs, c_v, is chosen to match this average monthly transition rate from unemployment to employment. We choose the one time moving costs, $c = 0.5$ to match the median $c - w$ ratio of 65% in Kennan and Walker (2011).[27] We then set the worker's additional

[22] It is the same as in Kaplan and Menzio (2016). Shimer (2005) chooses $\rho = 0.72$.

[23] Note that as long as $\kappa \leq \min\left\{1, \alpha^{\frac{1}{\phi}}\right\}$, we have $f(\theta), q(\theta) \in [0, 1], \forall \theta \in \mathbf{R}^+$, where $\theta = \frac{v}{u}$ is the measure of market tightness. This condition is always satisfied in the calibrated baseline model and the variants.

[24] In Shimer (2005), the matching function is $1.355 u^{0.72} v^{1-0.72}$. We set $\alpha = (1 - 0.72)/0.72$.

[25] Knowing the job finding rate, $f\left(\hat{\theta}\right)$, we can back out the market tightness and the elasticity of the job finding rate with respect to the market tightness:

$$\hat{\theta} = \left\{\left[f\left(\hat{\theta}\right)/\kappa\right]^{-\phi} - 1\right\}^{-1/\phi} \alpha^{\frac{1}{\phi}}, \ \hat{\eta} = \frac{\partial \ln f(\theta)}{\partial \ln \theta}|_{f(\hat{\theta})} = 1 - \left[f\left(\hat{\theta}\right)/\kappa\right]^{\phi}.$$

Combining these two equations, we derive (30).

[26] In Figure 4 of Hall and Schulhofer-Wohl (2015), the v-u ratio ranges roughly between 0.7 and 0.85 in the United States between 2001 and 2008.

[27] The one-time moving costs $c = 0.5$ is taken from Table V in Kennan and Walker (2011). The average moving costs are \$18,686 in the case of moving back to the home state while the previous location is also the home state, which is about 40%-90% of the annual median wage income (\$20,166-\$42,850 in Table III). We pick the median value of 65%. Given the wage of 0.8 in our calibration, we have $c = 0.5$. This is likely the upper bound of moving costs in our model which also include less expensive within-

Table 3: Normalization and calibration outcome

Parameters		Values
Productivity of a matched pair	y	1.0
Unemployment benefit	y_u	0.2
Discount rate	β	0.997
Interest rate	r	0.3%
Exogenous separation rate	δ	0.024
Worker's bargaining power	ρ	0.74
Matching function: coef of v	α	0.389
Matching function: constant	κ	0.503
Matching function: elasticity	ϕ	7.007
Firm's vacancy costs	c_v	4.169
Worker's additional search costs	c_u	0.2
Worker's one time moving costs	c	0.5

search costs, $c_u = 0.2$. This parameter matters whether we can have multiple equilibria but not other properties of the model. In Appendix E, we show that for any relevant value of c, our results below holds for a wide range of values of c_u.

4.2 Calibration Results

Table 3 reports the normalized and calibrated parameter values. We find that the calibrated model features two steady states. In the LIE steady state, the unemployment rate is 5.25%. In the HIE steady state, the unemployment rate is 9.67%.[28]

Figure 2 illustrates the multiplicity of equilibria in the calibrated model. It is very similar to Panel (a) of Figure 1 in the two-period version of the model. If workers always only search in their local markets, then the firms' present value of profit (net of entry cost) as a function of the market tightness is plotted as the dashed line, labeled as "entry_L". Then the market tightness, θ_L, is needed to satisfy the free-entry condition. At θ_L, the workers' individual rationality condition is indeed satisfied, i.e., $\Gamma(\theta_L) < 0$. Thus, the LIE steady state exists. Similarly, we can plot the profit function and define θ_H if workers always search in both markets. The HIE steady state also exists because $\Gamma(\theta_H) > 0$. Since $2\theta_H < \theta_L$, the resulting unemployment rate is higher in the steady state with higher search intensity.

state moves. Kaplan and Schulhofer-Wohl (2016) use $c = 0.53$ for non-college graduates and $c = 0.22$ for college graduates.

[28]These number are exactly unchanged as long as c_u is in the range of $(0.19, 0.375)$, according to Appendix E.

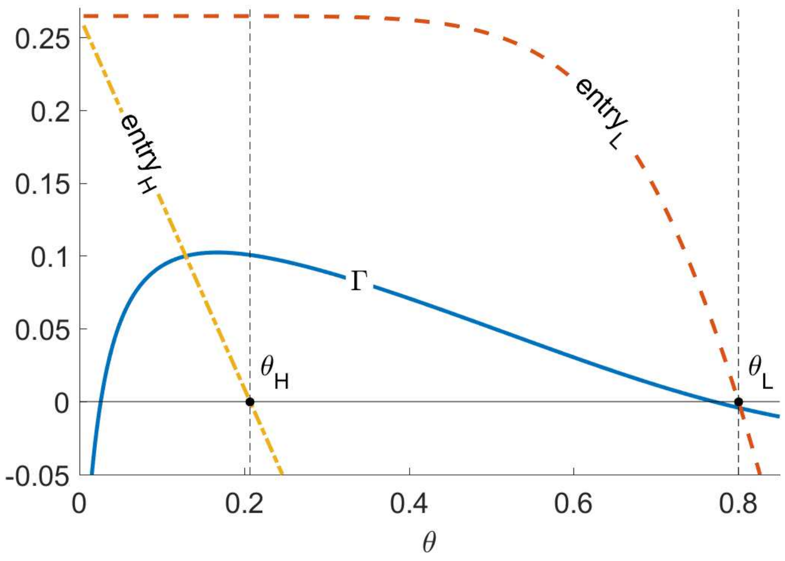

Figure 2: Multiple equilibria in the calibrated model

4.3 Response to Shocks

We have shown that our model allows for multiple equilibria for empirically relevant parameter values when calibrated to the U.S. economy. This suggests that the potential switching of equilibria may cause additional fluctuations in our model. We now study the response of our model to productivity and financial shocks quantitatively. Given the sluggish recovery of the labor market after the Great Recession, the key question for us is whether some temporary shocks during the crisis can cause long-lasting effects on the labor market. We shall pay special attention to whether and how a temporary shock may generate permanent switching of equilibria.

4.3.1 Productivity Shock

To quantitatively study the response of our model to productivity shocks during the Great Recession, we need first to understand the magnitude of these shocks. From 2008Q2 to 2009Q3, the U.S. GDP shrank by -3.75%. After taking into account the decreased employment, output per worker dropped by 2% relative to the long run trend. Using the same calculation, during 2008Q3 to 2009Q3 the shocks to productivity (annualized and relative to the 3% trend) were -2%, -3.4%, -2.3%, -2.3%, and -1.3%, respectively.[29] Panel (a) of Figure 3 shows the productivity series implied by data that

[29]The corresponding quarter-to-quarter rates were -0.5%, -0.9%, -0.6%, -0.6%, and -0.3%, respectively. From 2008Q2 to 2009Q3, the U.S. GDP shrank by -3.75%, according to the BEA. Suppose the crisis had never happened and suppose a 3% yearly growth rate, then by 2009Q3 the economy should have grown 3.75%. So we lost 7.5% of GDP directly due to the recession. However, during the same

we use in our quantitative exercise. Since our focus is how the model reacts to negative productivity shocks, for simplicity, we assume productivity resumes immediately back to unity after these five quarters.[30]

We simulate the responses of our model to these changes in y. The economy is assumed to be in the LIE steady state until agents learn about these shocks in the third quarter of 2008. We divide the time after 2008Q2 into two intervals: crisis periods (from 2008Q3 to 2009Q3) and post-crisis periods and allow workers to choose one search intensity during the crisis time and another intensity afterward. There are four symmetric equilibria candidates: High-Low (meaning unemployed workers choose high search intensity during the crisis but low intensity after the crisis), Low-Low, High-High, and Low-High.[31] To examine whether each of these four equilibrium candidates is incentive-compatible, one needs to look at the entire path of the net surplus function of MMSS. We plot the four paths of Γ_t in Panel (b) of Figure 3. It is clear that neither Low-High nor High-Low is a valid equilibrium path. For example, for Low-High, we need Γ_t to be negative during the crisis, which is not true. Only the High-High path and the Low-Low path are legitimate dynamic equilibrium paths. The evolutions of the v-u ratio and the unemployment rate are plotted in Panel (c) and Panel (d) of Figure 3. In the High-High path, the average unemployment rate between 2009 and 2011 is 9.64% in the model, compared to 9.50% in the data. Note again that the model is calibrated to match the pre-2008 U.S. economy. The model does a good job. However, the Low-Low path is also a dynamic equilibrium path, in which unemployment rate and the market tightness barely move.[32] This means that the observed productivity shocks alone might not be enough to explain the observed changes in the labor market because they do not predict the High-High path as the unique outcome. We need something else to explain the switch of equilibria. It could be the symbolic bankruptcy of Lehman Brothers, or the massive layoffs that appeared in the news that shook people's beliefs in "capitalism". It could also be a pessimistic self-fulfilling prophecy. If people believed that the shocks were permanent rather than temporary, then adopting MMSS could be the dominating strategy. Figure (4) demonstrates an example with a permanent negative productivity shock. After the shock, the HIE is the unique steady state equilibrium. Or, maybe financial shocks led to the inevitable switch of equilibria? We explore such a possibility below.

time, the employment to population ratio was also lower, dropping from 62.53% to 59.03%—there was a 5.6% decrease in the labor force. This implies that output per worker dropped only by 2%.

[30] In reality we did see some strong growth in productivity in the next four quarters, with average annual growth of 1.4% above the 3% trend. Of course, the growth rates were not as stark as we assume.

[31] Technically, workers can choose a distinct search intensity (how many markets to search) for each month, and even each day. We regard those equilibrium candidates as too exotic, because they require too much coordination.

[32] The Low-Low path is much like a standard single-market MP model. Note that our goal is not to resolve the issues raised in Shimer (2005) but to contrast the model with and without MMSS.

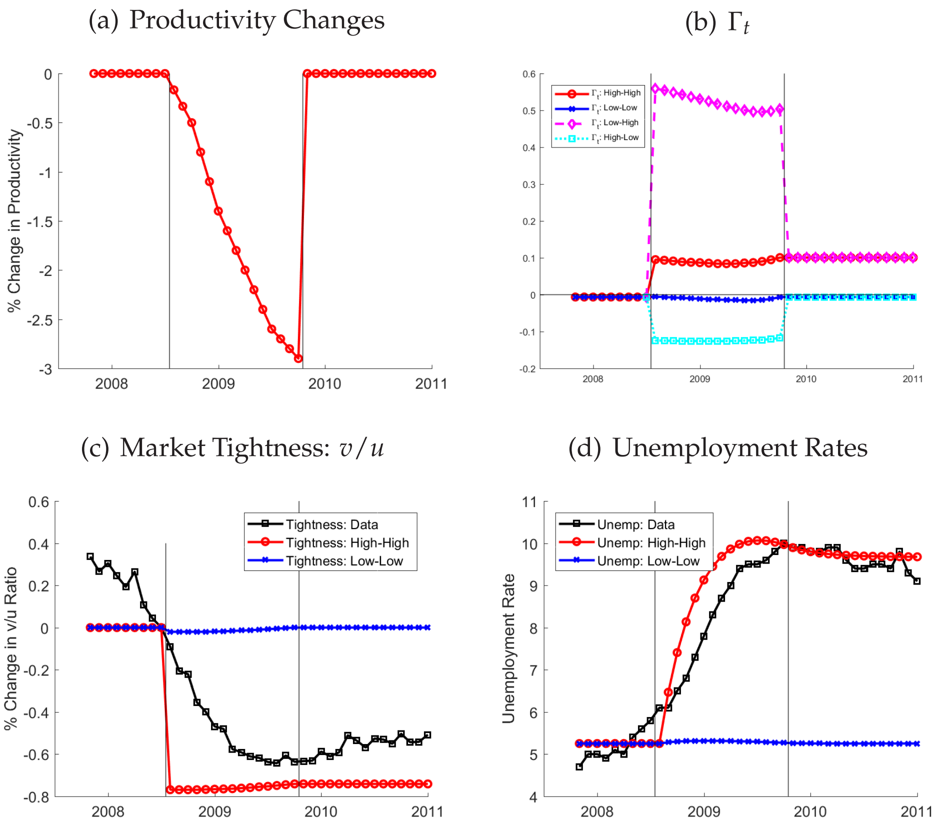

Figure 3: Responses to a productivity shock. (a) productivity changes; (b) MMSS net surplus Γ_t: level; (c) $\frac{v}{u}$ rates: deviation from 2008Q2; (d) unemployment rates.

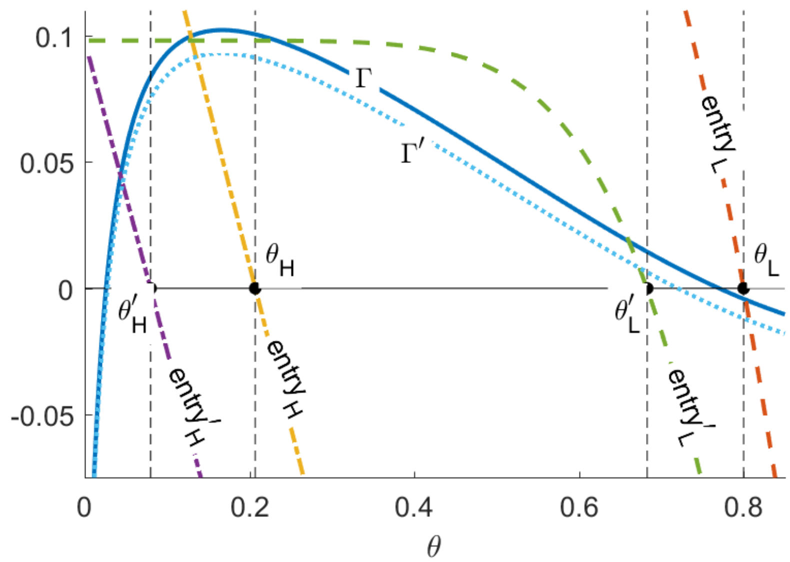

Figure 4: Response to a permanent productivity shock: steady states

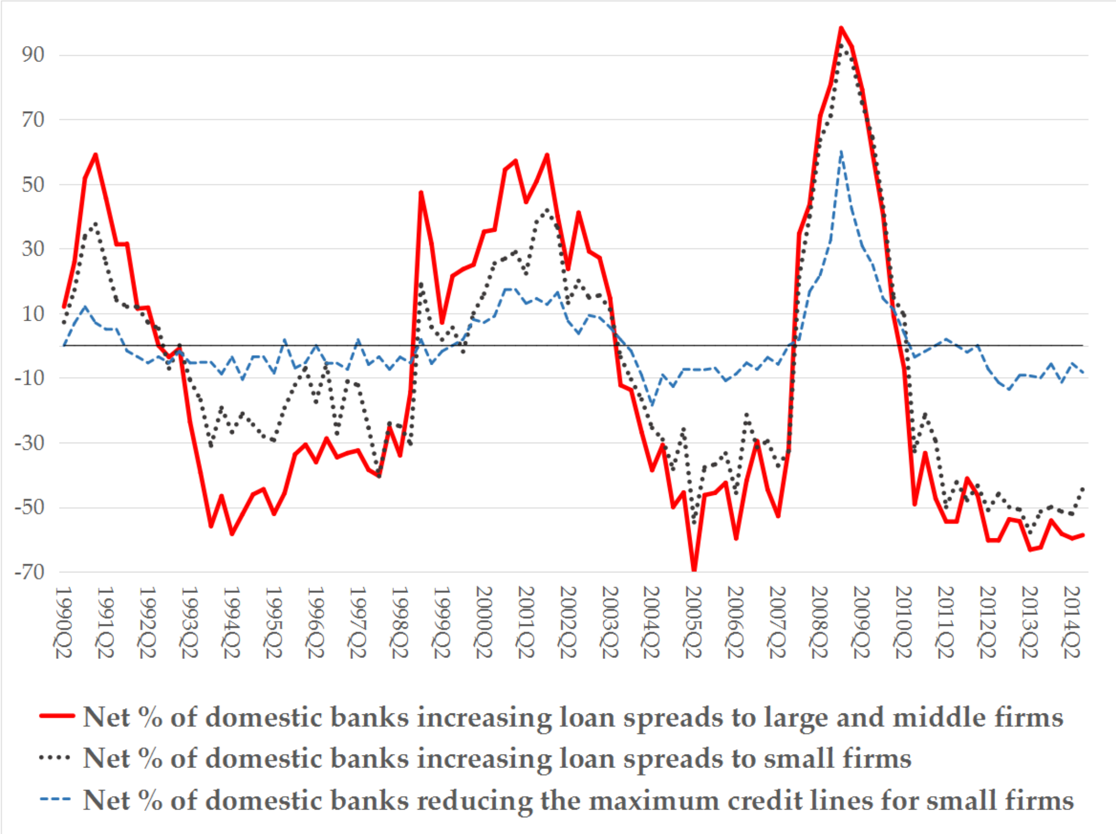

Figure 5: Senior Loan Officer Opinion Survey on Bank Lending Practices. Source FRB

4.3.2 Financial Shocks

There is little doubt that there were severe financial shocks during the 2008 financial crisis. Credit became more costly and harder to obtain. Figure (5) shows the results from the Senior Loan Officer Opinion Survey on Bank Lending Practices. In 2008Q4, the net percentage of domestic banks increasing spreads of loans to large and middle-market (small) firms was 98.2 (92.7), whereas the previous peak were 58.9 (41.8) in 2001Q4. The net percentage of domestic banks reducing the maximum credit lines for small firms was 69.1. The previous peak was only 17.3 in 2000Q4. Results are similar for big and middle-market firms. Apparently, U.S. firms experienced dramatic adverse financial shocks. However, this crisis was also relatively short-lived compared with the sluggish recovery of the labor market. With these observations in mind, we ask what happens in our model if there is a large and temporary financial crisis. In the simulation, we let the monthly interest rate increase by 40% ($r' = 1.4r = 0.42\%$) for eighteen months and resume to normal afterward (Panel (a) in Figure 6). There are two comments. First, the choice of eighteen months is just to showcase that even a short-lived financial crisis is able to trigger long-lasting effects. If we feed in longer financial shocks, the results would be exactly the same. Second, such a change is equivalent to changing the annual interest rate from 3.7% to 5.2%. For firms, this is equivalent to a

4.3% lower productivity permanently. So it is a large shock.[33] As with productivity shocks, we consider four symmetric equilibrium candidates: High-High, Low-Low, Low-High, and High-Low.

Unlike our simulation with productivity shocks, now the Low-Low path is no longer incentive compatible, as shown in Panel (b) of Figure 6. The intuition is as follows: Financial shocks reduce firm entry and make it harder for workers to find jobs in local markets, raising the value of MMSS. If these shocks are large and long enough (does not need to be permanent), then even if everyone else only looks for jobs locally, it is still profitable for an individual worker to adopt MMSS. That is why Γ_t is not always negative in the crisis in the Low-Low path. It is useful to digress and explain why the Low-Low path is not eliminated in our simulation of productivity shocks. There are mainly two reasons. First, we learned from the two-period model, productivity shocks tend to lower the incentives of MMSS due to lower wages. In our current setup, financial shocks have no effects on wages. Second, the observed productivity shocks might not be strong enough. During the five quarters of the Great Recession, productivity only dropped by 2%, and firms understood that these were transitory shocks. In comparison, our financial shocks are equivalent to a permanent productivity decrease of 1.8%.

The Low-High path and the High-Low path are similar to those in Panel (b) of Figure 3 and, thus, not shown here. Why are they always not incentive-compatible? Consider the Low-High path first. If workers anticipate that the HIE is played after the crisis, then they have high incentives to search for jobs to avoid being unemployed in the HIE, because that continuation value is very low. Conversely, if workers anticipate that the LIE will be played after the crisis, then they have low incentive to increase search intensity during the crisis, i.e., being unemployed in the LIE steady state is not so bad. So the High-Low path is also not incentive-compatible.

Using the similar logic, we know why High-High path is a feasible dynamic equilibrium path in both simulations: If workers anticipate that the HIE will be played after the crisis, then they have strong incentive to increase search intensity during the crisis to avoid being unemployed after the crisis.[34] The corresponding time series of the v-u ratio and the unemployment rate in the High-High path are plotted in Panels (c) and (d) of Figure 6. We also plot the Low-Low path because it can be interpreted as the standard one-market model (i.e., if agents are forbidden from MMSS). Financial shocks would barely affect the labor market in the standard model without MMSS.

[33]The basic idea of a negative financial shock is that it makes it harder for firms to finance vacancies. Theoretically either higher interest rates or higher vacancy costs can achieve this idea. Our approach is also reasonable if one interpret the interest rate as a measure of the difficulty of obtaining credit in the free entry conditions of firms as discussed in Section 3.1. See below for more discussions.

[34]Of course, if the financial shocks were too strong, then in theory Γ function could be negative because it is a quadratic function of $f(\theta)$. Such a possibility is not pursued in this paper because that

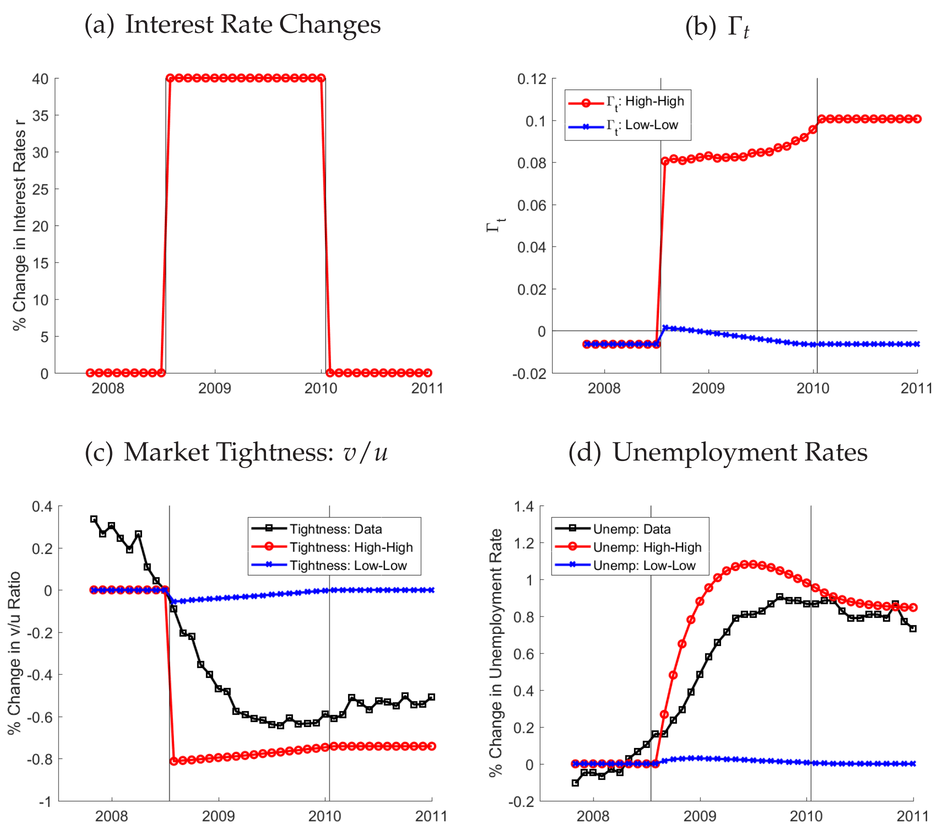

Figure 6: Responses to a negative financial shock: (a) Interest rate changes; (b) MMSS net surplus Γ_t; (c) $\frac{v}{u}$ rates: deviation from 2008Q2; (d) unemployment rates;

The High-High path, on the other hand, not only matches the evolution of the $v - u$ ratio and unemployment rate well, but many other properties of U.S. labor market experience. First, there was reportedly a record number of job applications by each worker, and Mukoyama et al. (2018) documents that the search intensity of the unemployed not only increased during the crisis but also has remained high afterward (their data continues to 2014). Existing theories have a hard time explaining this pattern. Second, despite higher search efforts by workers and unprecedented measures imposed by the US government and the Federal Reserve, the matching efficiency in the labor market is lower. The job finding rate appeared to be almost L-shaped between 2009 and 2014. Our High-High path is consistent with these patterns.

More Discussions

Of course, the High-High path might appear too extreme because it says the labor market will be stuck in the HIE forever. In Section 5, we show how to bring down the unemployment rate after the shocks are gone. But the key message here is the following: *there may well be many other reasons which could have caused the equilibrium switching, but financial shocks alone was enough to pull the trigger*. Such a conclusion is very different from the usual switch of equilibria with the help of sunspots, where there are usually many possible dynamic equilibrium paths. Our results suggest that a financial crisis, even a temporary one, can be particularly costly to society. It is more likely to cause long-lasting unemployment than productivity shocks.

Last, it is perhaps interesting to ask: why the switching of equilibria did not happen in many of the previous recessions? One possible explanation could be that the magnitude of shocks in many previous recessions was not as big. Another reason is that the types of shocks were different: many previous recessions were not caused by financial shocks. The third reason can be because the search cost, c_u, were too high for the HIE to exist during previous recession.[35]

4.3.3 Government Interventions

We also conduct another simulation which is not shown here: if the financial crisis only lasts for six months instead of eighteen, then the Low-Low path is also supported as equilibrium paths. This means the length of the crisis also matters. Swift government action to offset the effects of the financial crisis is desirable. But now suppose the economy is "stuck" in the HIE and the effects of monetary policy are limited due to the zero lower bound, what can we do?

would require some extremely large financial shocks.

[35]Learning and applying for jobs far away has never been easier, with the help of information technology; the costs of flying for an on-site interview have also considerably lowered; video interview over the internet was not possible until recent years.

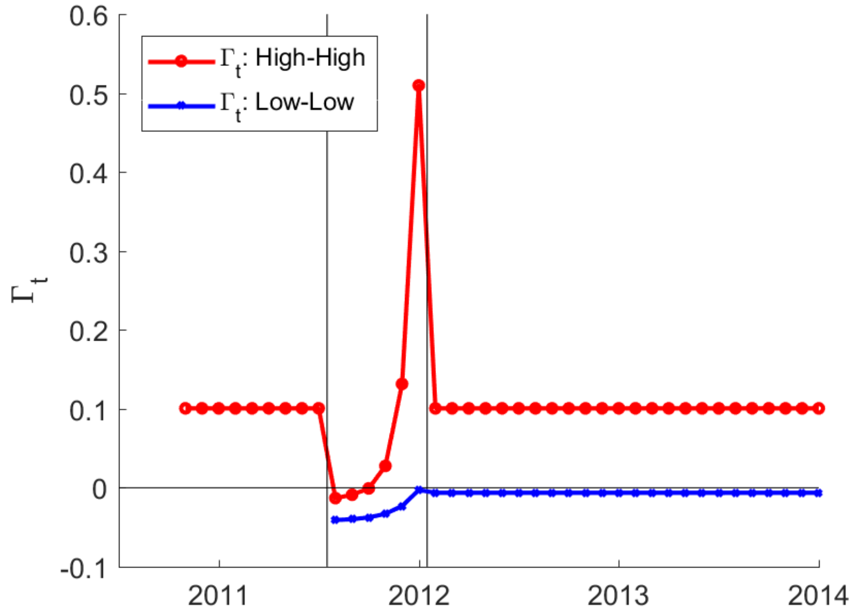

Figure 7: Response of Γ_t to Six-month Decreases in c_v

The goal is to obtain the LIE again. But equilibrium switching is hard in general as it requires changes in beliefs and coordination in both sides of the market. As discussed Section 2.2, government hiring would not work, and tax cuts may be counterproductive because they raise wages and the incentives of MMSS. Subsidizing job creation is the most effective. A large and permanent change in c_v would be enough to cause the equilibrium switch. But it is also costly. A more meaningful question is whether a temporary subsidy for job creation is enough to restore the LIE.

Specifically, let the economy starts with the HIE steady state, and we assume government subsidies 25% of the vacancy costs for six months. Again, we consider four different paths as studied above (High-High, Low-Low, Low-High, High-Low). For example, a High-Low equilibrium would mean MMSS during the subsidy periods and resume to the LIE afterward. We find that only the Low-Low path is supported as an equilibrium path. Figure (7) plots the MMSS incentive function, Γ_t, showing that the High-High path is not incentive compatible.[36] The idea is that during the subsidy periods the "local" job-finding rate is high enough, so agents give up MMSS. Another point to note is that even though the subsidy is six months, as long as such a plan is anticipated, the labor market responds immediately. Again, this is because a government subsidy of job creation is much like a reverse of financial shocks.

[36]The Low-High and the High-Low paths are not incentive-compatible and, thus, not plotted in the figure.

5 Extension: Heterogeneous Moving Cost

In the above analysis, the effects of a financial crisis is permanent. However, in reality, the unemployment rate did gradually come down. Now we introduce a way to bring down the unemployment rate while preserving the mechanism of MMSS. It is motivated by the fact that some workers are more attached to their "home" market than others. For example, living near one's family can be important to many people, possibly due to emotional or health reasons.[37] They may never consider MMSS, while others do. Such an extension is also useful for asking the following question: If only a small fraction of workers can conduct MMSS, can a financial crisis still generate large effects?

5.1 Setup

Specifically, assume $1 - \sigma_0 \in (0, 1)$ fraction of all the workers have moving cost $c_L = 0.5$, as in the previous section; but the rest σ_0 fraction of the workers have moving cost $c_H = \xi$. We call them LMC (low-moving-cost) workers and HMC (high-moving-cost) workers. The exact value of ξ does not matter, we assume the agents with c_H never consider MMSS (i.e., their Γ is always negative). For simplicity, we also assume that the types are permanent (i.e., a LMC/HMC worker is always a LMC/HMC worker), and the two markets are symmetric. Further assume these workers are initially identically distributed across the two markets, and across the employment status.

There are potentially two types of equilibria. In the LIE, every worker only searches in their local markets. In the HIE, the unemployed LMC workers search in both markets. The calibration of the model is unchanged, because the LIE still behave exactly as in the previous Section. The analysis of the Γ function is also unchanged, though it now only represents the incentives of the LMC workers.

We use σ_t^u and σ_t^e to denote the fraction of LMC workers in the unemployed and employed workers. In the LIE steady state, $\sigma_t^e = \sigma_t^u = \sigma_0$. But in the HIE, σ_t^e and σ_t^u can change over time, though the aggregate measure of LMC are constant:

$$\sigma_t^e (1 - u_t) + \sigma_t^u u_t = \sigma_0. \tag{31}$$

There are two more changes to the model. First, the entry condition of the HIE is now:

$$c_v = q(\theta_t) \left[1 - \frac{\sigma_t^u}{1 + \sigma_t^u} f(\theta_t) \right] E_t (J_{t+1}), \text{ in the HIE.} \tag{32}$$

[37]Similarly, preferences over occupations may be strong for some workers but not so for others.

σ_0	0.045	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
u_H	No HIE	5.38%	5.68%	6.31%	7.19%	8.03%	8.72%	9.19%	9.52%	9.65%

Table 4: Steady State Unemployment Rates in the HIE with Different σ_0

Second, we need to keep track the law of motion of σ_t^e and σ_t^u . Note that there are measure one of workers in each market, and u_t is the measure of unemployed workers in each market. The law of motion of u_t in the LIE is the same as (16), whereas that in the HIE becomes

$$u_{t+1} = [1 - f(\theta_t)](1 - \sigma_t^u) u_t + [1 - f(\theta_t)]^2 \sigma_t^u u_t + \delta(1 - u_t).$$

The law of motion of σ_{t+1}^u satisfies

$$\sigma_{t+1}^u u_{t+1} = \begin{cases} [1 - f(\theta_t)] \sigma_t^u u_t + \delta \sigma_t^e (1 - u_t), & \text{in the LIE} \\ [1 - f(\theta_t)]^2 \sigma_t^u u_t + \delta \sigma_t^e (1 - u_t), & \text{in the HIE} \end{cases}$$

where σ_t^e can be infered from the law of motion of u_t and σ_t^u, according to (31). These conditions capture the fact that in the HIE, a unemployed worker with or without MMSS remains unemployed with probability $1 - f(\theta_t)$ or $[1 - f(\theta_t)]^2$. Therefore, in the HIE, the LMC workers would leave the unemployment pool at a faster rate. The reason why heterogeneous moving cost is enough to bring down the unemployment rate is because the composition of the unemployed would change over time, dampening the match-rejection exteranlity.

Before we explore the dynamics of the model, below is a table summarizing how the value of σ_0 affect the unemployment rate in the HIE steady state. Remember that the unemployment rate in the LIE and HIE (with $\sigma_0 = 1$) is 5.25% and 9.67%. Note that even for very low σ_0, as long as it is above 5%, the HIE steady state exists. The unemployment rate in the HIE is increasing in σ_0: Not surprisingly, σ_0 can be seen as a measure of the match-rejection externality.

5.2 Dynamics

The purpose of this section is to show how the unemployment rate can gradually come down with heterogeneous moving costs. In our simulation, the financial shocks are according to Panel (a) of Figure 8. It captures the idea that the financial conditions first deteriorated, then resumed to the pre-crisis condition, and eventually becamed better thanks to the low interest rate policy and QE's. For the specific data when the negative financial shocks are gone, we use the fact that commercial and industrial loans in the U.S. resumed to its pre-crisis level only in July 2013. We pick $\sigma_0 = 0.3$.

Note that this value gives the HIE steady state an unemmployment rate of 6.31%.

There are a few observations. First, as with our simulation of financial shocks in Section 4, the Low-Low path is not incentive-compatible, as shown in Panel (b) of Figure 8. Second, we try all other reasonable equilibrium paths. For example, since there are total three stages of the financial conditions, we allow LMC workers to change search intensity twice after the shocks are gone at whenever they want. But the only incentive-compatible path is the High-High path: The economy settles in the HIE steady state. Third, the favorable financial conditions after 2016 is necessary for the unemployment rate to fall below the pre-crisis level – perhaps any model that is consistent with this observation will need something similar. Below are two more (perhaps more interesting) observations.

Fourth, our results with homogeneous moving cost is the same as long as the financials shocks are large and long enough, whereas here the magnitude and persistence of shocks matters for the reponses of the dynamic system. Fifth, note that to generate the magnitude of the reponses in unemployment rate, we only need 30% of the workers to be able to conduct MMSS, even though that implies an HIE unemployment rate of only 6.31% in the absence of shocks. The time-varying composition effect is shown in Panel (c) of Figure 8, and it causes the unemployment rate to overshoot. The reaons is simple. In the HIE steady state, there are roughly 22% of the unemployed who conduct MMSS, whereas at the beginning of the financial crisis, the number is 30%. The fraction of MMSS searchers is always lower in the HIE steady state than in the LIE steady state, so overshooting is a general feature of the model.

6 Concluding Remarks

We have demonstrated that search intensity at the extensive margin can induce strategic complementarity and multiple equilibria. More (search effort) can be less (efficient). This is due to the match-rejection externality caused by MMSS, which discourages job creation. The assumptions of the model in this paper is consistent with the standard Mortensen-Pissarides (MP) framework, yet the properties are very different from those of the textbook MP model with variable search intensity. Our MMSS model suggests that there is more to learn regarding the search effort of workers.

We also show that MMSS can have quantitatively large effects on the aggregate unemployment. In our calibrated model, transitory and large financial shocks alone can cause a permanent switch of equilibria as the unique dynamic outcome. Productivity shocks are different. Our model not only helps us to understand the labor market after the 2008 financial crisis, but also suggests a general message: due to the link between financial shocks and the labor market, the cost of a financial crisis can be much larger

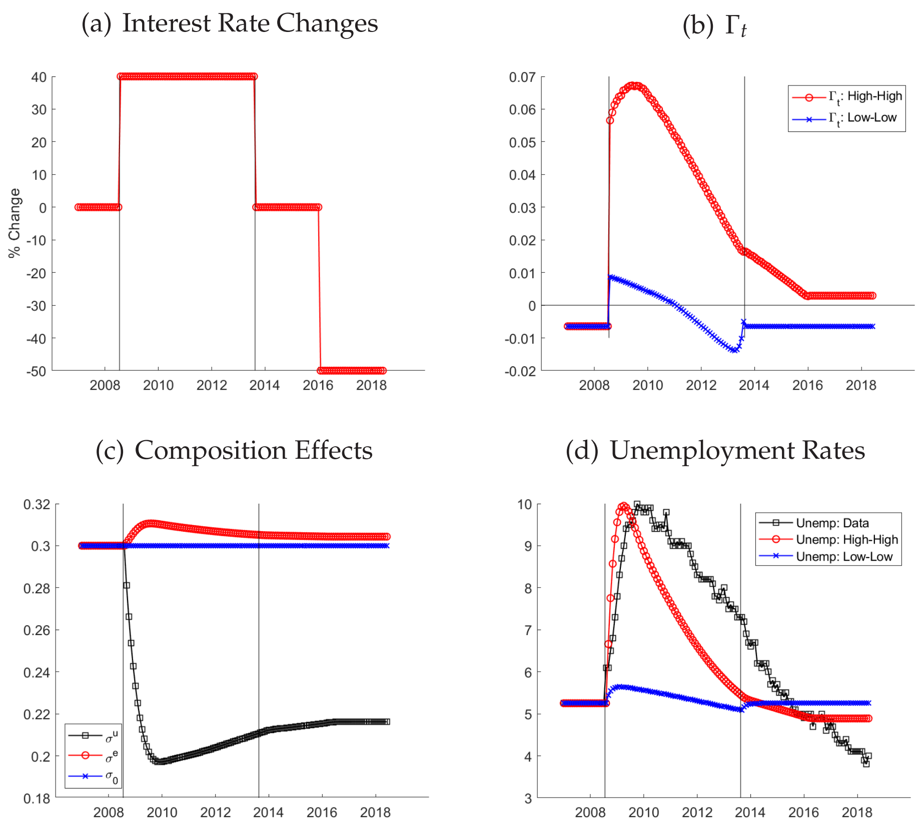

Figure 8: Responses to a negative financial shock: (a) Interest rate: deviation from the trend; (b) MMSS net surplus Γ_t; (c) unemployment rates: deviation from 2008Q2; (d) $\frac{v}{u}$ rates: deviation from 2008Q2.

than we thought.

The model in this papre is deliberately simple to best illustrate the mechanisms. The lack of realism in the current model opens many possibilities for future research. An interesting question is if we can generate a persistent but not permanent response of the labor market. We think it is possible with heterogeneous search cost: after the financial shocks, workers with lower search cost adopt MMSS and leave the unemployment pool faster so the composition of the remaining unemployed workers may gradually change over time. Another extension could be to endogenize the search effort within a market with the possibility of MMSS.

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Appendix

A Proofs

Proof of Proposition 4: Condition $c_u \leq \rho (y - y_u)/4$ guarantees that $(\underline{\theta}, \bar{\theta})$ is not empty so that there is at least some range such that $\Gamma(\theta) > 0$. From the entry conditions (4) and (5) we know both θ_H and θ_L are decreasing in $c_v (1+r)$. So adjusting the value of $c_v(1+r)$ can ensure $\Gamma(\theta_i) > 0$ or $\Gamma(\theta_i) < 0$. We have used the fact that $\Gamma(\theta) < 0$ for some range.

Proof of Corollary 6: Let $\bar{c}_v$ satisfy

$$(1+r)\,\bar{c}_v = q\left(\bar{\theta}\right)\left[1 - \frac{1}{2} f\left(\bar{\theta}\right)\right](1-\rho)(y - y_u),$$

where $\Gamma\left(\bar{\theta}\right) = 0$ and $\bar{\theta} \geq \frac{1}{2}$. Then $\forall c_v < \bar{c}_v$, we have $\theta_H > \bar{\theta}$. This implies $\Gamma(\theta_H) < \Gamma\left(\bar{\theta}\right) = 0$, which means the HIE at $\forall c_v < \bar{c}_v$ is infeasible.

Proof of Lemma 7: Re-organizing (27), we have

$$\Gamma_H(\theta_H)[1+\chi] = -c_u + \beta f(\theta_H)[1 - f(\theta_H)]\left[\frac{\rho(y-y_u)}{1-\beta+\beta\delta+\beta f(\theta_H)} - c\right] = \Gamma_L(\theta_H), \tag{A.1}$$

where $\chi = \frac{\beta f(\theta_H)[1-f(\theta_H)]}{1-\beta+\beta\delta+\beta f(\theta_H)}$. The right-hand-side is $\Gamma_L(\theta_H)$, that is, the net surplus of MMSS in the low-intensity equilibrium evaluated at θ_H. Because $\chi > 0$ and the above equation holds true for any θ_H, we know that $\Gamma_H(\theta)$ and $\Gamma_L(\theta)$ always have the same sign.

Proof of Proposition 11: Given $f(\theta) = \theta q(\theta)$, from (25) and (28) we have

$$u_H = \frac{\delta}{\delta + \frac{2\theta_H}{\theta_L} f(\theta_L)}. \tag{A.2}$$

Compare it with (25), we know that $u_H > u_L$ if and only if $2\theta_H \leq \theta_L$. In such case, the job finding rate is also lower in the HIE:

$$f(\theta_H)[2 - f(\theta_H)] \leq f(\theta_L). \tag{A.3}$$

As in the two-period version, the θ in our model is defined as vacancy over applicants, i.e., $\theta_H = v_H/(2u_H)$ and $\theta_L = v_L/u_L$. Thus $2\theta_H \leq \theta_L \Leftrightarrow \frac{v_H}{u_H} \leq \frac{v_L}{u_L}$.

B Match-rejection Externality

Although we are interested in firm entry, it is useful to shut down this channel for the moment to help understand the mechanisms of the model. Specifically, let $\tilde{v}$ be the exogenous number of vacancies in both markets. So $\tilde{\theta} = \tilde{v}/u$ and $\tilde{\theta}/2$ are the market tightness when workers search in one market and when they search in both markets, respectively. For multiple equilibria to exist, we need $\Gamma\left(\tilde{\theta}\right) \leq 0$ and $\Gamma\left(\tilde{\theta}/2\right) \geq 0$. This is stated in the following proposition:

Proposition 11. *With exogenous $\tilde{v}$ and u in both markets, if $c_u \leq \rho\left(y - y_u\right)/4$ and $\tilde{v}/2u \in \left(\underline{\theta}, \bar{\theta}\right)$ and $\tilde{v}/u > \bar{\theta}$, where $\underline{\theta}$ and $\bar{\theta} > \underline{\theta}$ satisfy $\Gamma\left(\underline{\theta}\right) = \Gamma\left(\bar{\theta}\right) = 0$, then $\Gamma\left(\tilde{\theta}\right) \leq 0$ and $\Gamma\left(\tilde{\theta}/2\right) \geq 0$, that is, we have both the LIE and the HIE.*

Multiple equilibria exist purely due to the usual congestion externality. If more workers from Market 2 adopt MMSS then Market 1 becomes more congested, making it harder to find jobs in Market 1 and thus MMSS becomes more attractive for workers in Market 1. However, in our setting (i.e., market tightness does not affect wage) firm entry can offset congestion externality. How can we still have multiple equilibria as in Panel (a) of Figure 1? This is because MMSS imposes another type of externality. We call it *the match-rejection externality*.

To illustrate, suppose measure a_1 of workers in Market 1 and measure a_2 of workers in Market 2 adopt MMSS. We have applicants $n_1 = u_1 + a_2$ in Market 1 and $n_2 = u_2 + a_1$ in Market 2. Then, a firm's expected profit in Market 1 is given by

$$\Pi_1 = \frac{1}{1+r} q\left(\frac{v_1}{u_1 + a_2}\right)\left[1 - \frac{1}{2} f\left(\frac{v_2}{u_2 + a_1}\right)\frac{a_1 + a_2}{u_1 + a_2}\right](y - w) - c_v. \tag{B.1}$$

If v_1 is fixed, then an increase of a_2 lowers the market tightness $\theta_1 = v_1/\left(u_1 + a_2\right)$ and raises the ratio $\left(a_1 + a_2\right)/\left(u_1 + a_2\right)$. The first (second) effect is the congestion (match-rejection) externality which makes profit Π_1 higher (lower). If firm entry is allowed but the match-rejection channel is absent, then zero profit implies a higher v_1 so that $v_1/\left(u_1 + a_2\right)$ is unchanged.[38] The congestion externality is completely offset.

But if the match-rejection channel is present as in (B.1), then as a_2 increases, the θ_1 implied by free entry would decrease. This is because an increase of v_1 can only offset the congestion externality, but not the match-rejection probability, as is clear from (B.1). Firms in Market 1 create more vacancies but not enough to make the market tightness unchanged – θ_1 must be lower to compensate firms for the higher match rejection rate. Type 1 workers find it harder to find jobs in Market 1 in equilibrium. Now strategic complementarity arises. According to (3), Γ^1 is decreasing in θ_1. The intuition is that

[38] It is as if some Type 2 workers move to Market 1 and only search in Market 1.

when one finds it harder to find a job in the home market, the extra search effort in other markets becomes more useful.

In our model, the source of strategic complementarity is the match-rejection externality of MMSS. Note that here the production technology mechanically has constant returns to scale, and both the market specific matching function $m(v, n)$ and the implied aggregate matching function (within the same equilibrium) have constant returns to scale in u and v.

Suppose $c = 0$. In (B.1), consider an increase of a_1. It has two effects: the ratio$(a_1 + a_2) / (u_1 + a_2)$ is higher whereas $f(\theta_2)$ is lower. The over effects on Π_1 and θ_1 are ambiguous. The idea of the first effect is that when Type 1 workers adopt MMSS, they might also reject matches from Market 1. Then suppose $c > 0$. The ex ante discounted present value of a firm's profit in Market 1 is given by

$$\Pi_1 = \frac{1}{1+r} q\left(\frac{v_1}{u_1 + a_2}\right)\left[1 - f\left(\frac{v_2}{u_2 + a_1}\right)\frac{a_2}{u_1 + a_2}\right](y - w) - c_v.$$

This effect is not present because in that case one prefer a match from home market to that from away market.

C Mixed Strategy Equilibrium

We only briefly describe how one should think of a mixed strategy equilibrium in this model. First, a mixed strategy equilibrium requires that the net surplus of simultaneous search is zero so that workers are indifferent between the two strategies, i.e. $\Gamma = 0$. For example, in Panel (a) of Figure 1, we have two values of θ that can make $\Gamma = 0$: one around 0.5 and the other around 1.3. Think first about the case of zero relocation cost, i.e. $c = 0$. Suppose in a mixed strategy equilibrium (thereafter MSE) ζ fraction of workers adopt the simultaneous search strategy. Let θ_M be the market tightness of the mixed strategy equilibrium and $\Gamma(\theta_M) = 0$, then the firm's entry condition can be written as

$$c_v(1+r) = q(\theta_M)(y - w)\left\{[1 - \zeta f(\theta_M)] + \frac{1}{2}\zeta f(\theta_M)\right\}, \text{ in MSE.}$$

As long as θ_H lies to the left of θ_M and θ_L lies to the right of it, we are sure that we have an MSE, such as the case in the Panel (a) of Figure 1. Notice that the MSE does not require the existence of multiple pure-strategy equilibria or even the existence of a pure-strategy equilibrium. For example, if θ_H is smaller than 0.5 in Figure 1, then we have the LIE and two MSE's. If both θ_H and θ_L are above 1.3, then we only have the LIE. There is another reason to focus on the pure-strategy equilibria: a MSE requires much more coordination than a pure-strategy equilibrium. It is natural to focus on the MSE when there does not exist a pure-strategy equilibrium. For example, if $\theta_L \in (0.5, 1.3)$ and $\theta_H < 0.5$, then the only equilibrium is the MSE. The rest of the paper focuses on the pure-strategy equilibria and leaves MSE to future studies.

D Statistical Market Discrimination and Wage Competition

Wage Competition When a worker receives two matches, Bertrand competition among firms would strengthen the match-rejection externality, because now firms always receive zero surplus whereas without the competition they still receive positive surplus. Workers' incentives of MMSS would be stronger because they receive higher surplus in such an event. Specifically, now the entry condition in the HIE (5) and the Γ function become

$$c_v = \frac{1}{1+r} q\left(\theta_i\right) \left[1 - f\left(\theta_j\right)\right] \left(y - w\right), \ HIE,$$
$$\Gamma^i = -c_u + \left[1 - f\left(\theta_i\right)\right] f\left(\theta_j\right) \left(y - c\right).$$

The properties of the model are similar.

Statistical Market Discrimination One might expect that employers could potentially treat the two types of workers differently, especially in the case of positive relocation costs. "Local" workers, who do not need to pay such cost, are more likely to accept a match in the home market. In reality, however, it requires that the employers in Market i can tell if a worker from Market j is searching simultaneously or if she is only looking for jobs in Market i, possibly due to some personal reasons. Any job candidate can claim the latter. Given the asymmetric information, the employers might choose to statistically discriminate against "outsiders". To accommodate such a possibility, we can assume that when Type i workers apply for jobs in Market j she has less search efficiency units, i.e. she counts as a fraction of an applicant in the matching function (e.g., her applications in Market j sometimes get overlooked). To give a concrete example, suppose when a worker searches in the away market, she counts as only $\gamma\ (< 1)$ fraction of an applicant. It can also be interpreted as saying the applications from the away market is overlooked with $1 - \gamma$ probability. Then the Γ function becomes

$$\Gamma^i = -c_u + \gamma f\left(\theta_j\right) \left[1 - f\left(\theta_i\right)\right] \left(w_j - y_u - c\right),$$

where $f(\theta_j)$ is the probability of a Type j worker finding a match in Market j and thus $\gamma f(\theta_j)$ is the probability of a Type i worker finding a match in Market j, because every Type i worker only counts as γ fraction of Type j worker in the matching process. Also $n_1 = u_1 + \gamma u_2$ and $n_2 = u_2 + \gamma u_1$. Of course, as γ decreases, Γ would be smaller, making it harder to achieve the HIE. But other conditions are exactly the same. As long as these simultaneous searchers sometimes receive matches from both markets, the match-rejection externality exists and so does the possibility of multiple equilibria.

E Robustness

The calibrated model features two steady states for a wide range of the worker's additional search costs, c_u, and the one time moving costs, c. For any given $c \in [0, 2]$, as long as the value of c_u is between the lower bound and the upper bound, as plotted in Figure 1, the worker's individual rationality constraint holds in both steady states. That is, $\Gamma(\theta) \leq 0$ at the steady state with low search intensity and $\Gamma(\theta) \geq 0$ at the steady state with high search intensity.

Specifically, given c, the lower bound c_u^L is derived from $\Gamma_L(\theta_L) \leq 0$, or

$$c_u \geq c_u^L = \beta\theta_L M(\theta_L)\left[1 - \theta_L M(\theta_L)\right]\left[\frac{w - y_u}{\beta\theta_L M(\theta_L) + 1 - \beta(1-\delta)} - c\right].$$

The upper bound, c_u^U, is derived from $\Gamma_H(\theta_H) \geq 0$, or

$$c_u \leq c_u^U = \frac{\beta\theta_H M(\theta_H)\left[1 - \theta_H M(\theta_H)\right]\left\{w - y_u - \left[\beta\theta_H M(\theta_H) + 1 - \beta(1-\delta)\right]c\right\}}{\beta\theta_H M(\theta_H) + 1 - \beta(1-\delta)}.$$

If $c_u < c_u^L$, then the steady state with low search intensity is not sustained; if $c_u > c_u^U$, then the steady state with high search intensity is not sustained. This implies that given fixed c, if c_u is initially high such that $c_u > c_u^U$, then there is a unique equilibrium which is the LIE. When c_u decreases to the level c_u' such that $c_u' \in (c_u^L, c_u^U)$, then both the LIE and the HIE are possible. When c_u' further decreases to c_u'' such that $c_u'' < c_u^L$ then the HIE is the unique equilibrium. This is consistent with the recent trend of increasing search intensity, before and after the Great Recession, accompanying the decreases in the additional search cost c_u as a result of the spreading of information technology.

It is worth noting that as the one-time moving cost, c, decreases, both the lower bound, c_u^L, and the upper bound, c_u^U, increases. This implies that given c_u, the HIE becomes more feasible as c decreases.

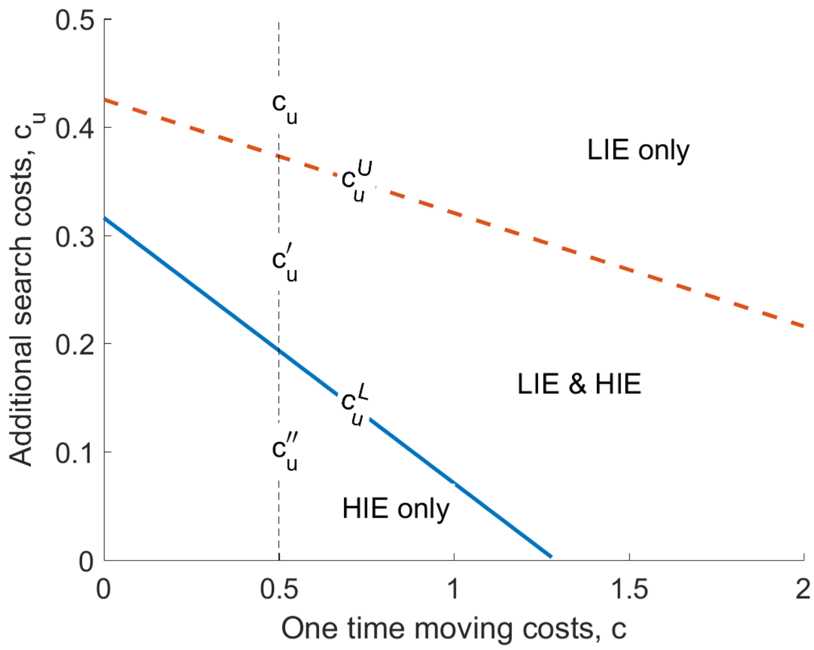

Figure 1: The range of c_u and c

F Computing the Case of Heterogeneous-Moving-Cost

F.1 Method 1

i) We assume a sequence of θ_t

ii) Backwards, solve for the sequence of W_t, U_t, and Γ_t

iii) Forwards, solve for the sequence of θ'_t, u_{t+1}, σ^u_{t+1} and σ^e_{t+1}.

iv) Compare $\{\theta'_t\}$ with $\{\theta_t\}$. If different enought, replace $\{\theta_t\}$ with $\{\theta'_t\}$ and repeat 2-4.

F.2 Method 2

i) Assume that the LMC workers start searching locally at period t. Thus,
   (a) from shock moment to period $t-1$, HIE
   (b) from period t onwards, LIE

ii) From period t onwards, it is in LIE. J_t is deterministic, so θ_t can be pinned down by the firm entry condition, 12.

iii) Between shock and period $t-1$, it is HIE. J_t is deterministic. However, u_t, σ^u_t, σ^e_t are unknown.

iv) Given the current u_t, σ^u_t, σ^e_t, we can solve the current θ_t from the firm entry condition. Then using the law of motions we can solve u_{t+1}, σ^u_{t+1}, σ^e_{t+1} forwards.

v) After that, we can, backwards, solve for the sequence of W_t, U_t, and Γ_t

vi) Check if the sequence of Γ_t is consistent with the statement in 1.

The key idea of the exercise is that during the crisis, the LMC workers start searching in both markets. However, because they search harder, so they leave the unemployment pool faster. Therefore the composition of the unemployed would change over time (i.e., σ_t^u). The σ_t^u should be decreasing after the crisis, so firm entry (i.e., θ) should gradually increase. At some point, we may even switch back to LIE if σ_t^u becomes small enough.

In fact, we may be able to do it right now: Given the Γ function in the steady state, we can find the range of θ such that $\Gamma < 0$. From (32), we know that we can pick σ_t^u to change θ. Therefore we have a relationship between the value of σ_t^u and Γ. Specifically, as σ_t^u becomes smaller, θ_t becomes larger according to (32) – therefore when σ_t^u is small enough, θ_t would be high enough so that Γ becomes negative, and we are back to the LIE.

Guess that the HIE will be played in the crisis, and afterwards the whole time. Then the hard question is when σ_t^u will be low enough so the LIE will be resumed. Maybe we can trial and error: suppose the LIE is resumed T periods after the crisis, see if everything hangs together; if not, try $T + 1$. Maybe we have a better way to do it?

Interested reader can see Appendix B for a discussion of how why such externality is not offset by free entry.

These changes in productivity in a two-period environment are analogous to permanent changes in productivity in an infinite horizon setting. We discuss the effects of transitory productivity shocks in an infinite-horizon calibrated version of the model in Section 4. But the insight we get from here is useful for that exercise as well: a change in productivity directly affects both the entry of firms and the $\Gamma(\cdot)$ function which describes the worker's incentive of MMSS.