The Cyclical Behavior of Factor Shares

Michele Boldrin Yong Wang Lijun Zhu*

February 16, 2019

Abstract

We review the empirical evidence about factor shares and show that, apart from a varying trend, they are characterized by a strong and persistent cyclicality and cycles. We then provide a theory under competitive conditions which firms choose how many workers to hire, how much to invest, and which production technique to use. New productive capacity, embodying labor saving techniques, is costly. Central to our theory are endogenous movements in relative factor prices creating incentives for replacing old technologies with new ones. The endogenous interaction between labor-saving innovations and changes in the relative price of labor is the source of both growth and cycles.

Note to reviewer(s): The model, augmented with heterogenous firms, in this draft is preliminary and incomplete. We have shown that our model, in a version omitted from firm heterogeneity and not included here, generates endogenous growth and cycles and countercyclical factor shares. To the best of our knowledge, ours is the first to provide such a theory that builds upon the interaction of factor prices and firms' endogenous technology adoption in a competitive framework.

*Boldrin: Department of Economics, Washington University in St Louis, boldrin@wustl.edu. Wang: Institute of New Structural Economics, Peking University, yongwang@nsd.pku.edu.cn. Zhu: Institute of New Structural Economics, Peking University, lijunzhu@nsd.pku.edu.cn.

1 Introduction

This is a theoretical investigation of the role that endogenous decentralized technological change play in determining growth and cycles. In our model, factor prices change over time due to the accumulation of productive capacity; such changes create incentives for variations in the rate of utilization of productive capacity, the introduction of new technologies and the scrapping of old ones. This process brings about oscillations in growth rates and in the pace of capital accumulation. We focus on decentralized technological change: neither aggregate technological progress nor aggregate productivity/taste shocks are assumed. All decisions - crucially: the adoption of new methods of production - take place at the firm's level. They come about as profit maximizing responses to changes in relative prices and in the equilibrium conditions of competitive markets. Heterogeneous firms, as opposed to an aggregate production set, are the elementary units of analysis. We stress the endogenous nature of growth and cycles: the exogenous uncertainty in the viability of production techniques is modeled as idiosyncratic shocks acting at the firm-level. Aggregate and oscillatory growth is shown to persist even when such sources of uncertainty are set to zero and the model becomes fully deterministic.

Put differently, we study endogenous technological progress that is "biased" by changes in the relative price of inputs, and derive a model in which persistent growth, persistent business fluctuations, and persistent movements in factor share are simultaneously determined as the outcome of dynamic competitive equilibrium. We claim this makes for a model of growth and the business cycle that fits the facts better than existing alternatives.

In our model, growth in total factor productivity is endogenous and results from the adoption of new technologies - capital deepening - their subsequent expansion - capital widening - and their eventual replacement with with better ones - capital scrapping. The duration of each phase is endogenous, and determined by the equilibrium movements in the relative prices of labor and (different kinds of) capital. Recessions occur when capital widening has reached its upper limit and scrapping, followed by deepening, becomes economically beneficial; expansions set in when capital deepening is successful and widening may be undertaken at a higher than normal rate.
[1]

[1] Maybe some "new growth theory" model can give us same theoretical result.

Maybe heterogeneity of firms will kill the aggregate effect.

What comes first, recession or innovation? And what "causes" what? Is it only labor that matters? How about "credit capacity": can't we tell a similar story (credit is cheap at beginning of expansion, hence firms take a lot of credit, credit becomes scarce eventually and it price goes up (fast? why?) killing lots of bad projects (those that use more credit than average, and have lower value added) and inducing recession ... but here, how do you "create" new credit capacity after the recession cleans up bad projects?)

1.1 Empirical Motivation

Apart from the goal of building a theoretical model in which growth and cycles are joint equilibrium outcomes, our motivations are also empirical. For a theory of this kind to stand the chance of turning into a quantitative model of actual growth and cycles, its equilibrium paths must display a fairly long list of qualitative features. A relatively stable long run trend should obtain, around which cycles of varying length (between three and ten years) are observed. Quarterly growth rates are positive most of the time, but negative growth rates in GNP may occur at a fairly infrequent rate. Growth rates are positively auto-correlated. Income, consumption, investment, labor productivity are co-integrated series. Positive TFP obtains when quality adjustments in K and L are not made. Consumption and investment are pro-cyclical, but the latter oscillates more than GNP while the former substantially less. Productivity of labor is mostly procyclical while real wages are only weakly so. Factor shares follow a cyclical pattern, with delays: the share of capital is procyclical but peaks before total ouput does while the labor share is countercyclical but bottoms out after the recession ends.

While most of these stylized facts are extensively documented in the literature[2], some are worthy of a few additional words. First, the shares of income accruing to capital and labor move in a systematic way with the business cycle. Second, profits and the growth rate of labor productivity, beside being correlated, are pro-cyclical, but peak substantially earlier than the cycle does, i.e. when recessions set in, profits have been decreasing and labor productivity has stopped growing for a few quarters already. Third, while there is very little short term substitutability between capital and labor, the substitutability is substantial in the longer run, and most technological improvements appear to be labor saving. Fourth: statistical evidence suggests that employment drops on impact when a permanent technological improvement arrives, and start raising again only quite a few quarters later.
[3]

Standard business cycle models have a difficult time explaining one or more of these facts, even when assuming that growth is exogenous and that aggregate and autocorrelated exogenous technology and taste shocks are the main driving forces. That this is the case for any model working with an aggregate Cobb-Douglas production function, should be obvious; retaining a Cobb-Douglas production function and making the share parameter stochastic (as, e.g., Young, 2004), beyond generating counter-factually high correlation between capital share and output, is dangerously close to assuming a trivial answer (exogenous movements) to the empirical puzzle. Less obvious is the fact that the easy fix (a CES production function with elasticity of substitution different from one) is actually not a fix. If we compute a standard RBC model with CES production function and an elasticity of substitution similar to the ones reported in the literature (for example, around 0.8 as in Antràs, 2004 or Young, 2005)[4], we find that such models predict movements in factor

[2] Add basic references, for US and also for EU.

[3] References and relevant figures to be added

[4] More recent estimates should be added.

shares quite smaller than the observed ones. Moreover, if we assume that technological progress is Harrod-neutral (as required to have a stationary capital-output ratio in models of exogenous growth), wages become strongly countercyclical, contrary to empirical evidence.

Models with sticky prices and/or sticky wages do not have an easier time at capturing the facts. In response to a monetary shock, wages will go up because of a higher demand for labor, and labor productivity will go down as labor increases faster than capital, hence the capital income share will go down during an expansion driven by a positive monetary shock. Only in the, empirically unlikely, event that nominal wages are completely rigid and prices adjust very rapidly to the monetary shock, would real wages decrease. Only in the, even more unlikely, event that real wages decrease more than labor productivity does as demand for labor increases, will profit display a procyclical tendency. Leaving aside the fact that this does not seem to have ever happened in the business cycles of the real world, to achieve this we would need wage rigidity to last many quarters in the face of continuous monetary supply surprises and raising prices, an improbability to say the least. The same argument applies, without the latter caveat, if the expansion is driven by some non-ricardian fiscal "stimulus."
[5]

Focusing upon the cyclical movements in factor shares, labor productivity, wages and profit rates, beyond clarifying the problems of existing business cycle models, also tells us the element that a successful theory requires: a mechanism to increase labor productivity faster than wages at the beginning and slower at the end of the expansion. Our paper focuses on one possible channel: the endogenous adoption of labor saving technology by competitive firms. At the beginning of the expansion, firms will pick new technologies that are labor-saving relatively to previous ones. As the latter are scrapped and the new capital that embodies the more efficient technology is accumulated, labor moves accordingly and its productivity increases faster than wages, hence the capital share and output increase rapidly. However, as the replacement process completes and more and more labor is employed, wages will eventually go up, drying the corporate profits, reducing investment, and finishing with it the expansion. Only at the bottom of the recession, after old and inefficient productive capacity has been scrapped, a new technology is introduced and the whole cycle starts again.

The index of sections follows. Section 2 discusses the related literature. Section 3 outlines the basic model. Section 4 characterizes the competitive equilibrium, while section 5 defines the central planner problem and uses its properties to provide further insights into the dynamics predicted by our theory. Section 6 is concerned with some extensions, which are particularly important in light of empirical applications. Section 7 concludes.

[5]Document the other fact (canova and co?)

2 Related Literature (TO BE REWRITTEN)

We are not the first to deal with some of the issues discussed above. To the best of our knowledge, though, we are the first to make the claim that a sound theory of endogenous growth and cycles can be built upon the observation that firms expand productive capacity when they expect the adopted techonology to yield a profit in future periods, while they reduce capacity and try changing their technology when they realized the latter is no longer profitable at the expected equilibrium prices.

Let us leave aside, for the time being, the very vast literature concerned with endogenous growth and cycles; it suffices here to say that nowhere in that literature one can find a model in which both growth and cycles obtain.[6]

Further, nowhere in this literature has a model been built that is capable of facing the data. We will also spare the reader a long survey of the century-long debate on the nature of technological progress, its biasedness in one direction or another and the extent to which Harrod-neutral exogenous productivity does or does not mimic the data in a satisfactory form. To us, that technological progress must be labor - more generally: natural resource - saving is almost tautological beside being blatantly evident. The relevant issues are how to best model this fact, and if the pace at which technological change advances should or should not be made responsive to movements in factor prices. We refer to XXX and YYY for recent discussions of this issues, and survey of the literature.

Coming next to our first observation - that factor shares are strongly cyclical - we begin by distinguish three branches of the literature interested in the evolution of the input income shares and the business cycle. First, there have been papers that focused on the distribution of risk over the cycle. Boldrin and Horvath (1995) present a real business cycle model of contractual arrangements between employees and employers when the former are prevented from accessing capital markets and are more risk-adverse than the latter. The paper characterizes an optimal contract that maps the aggregate states of the economy into wages and labor market outcomes. Similarly, Gomme and Greenwood (1995) build a model where workers purchase insurance from the entrepreneurs through optimal contracts. Since our model assumes complete markets, none of the considerations used in those papers is directly pertinent to the mechanism explored here, even if, as it will be clear later, the introduction of risk-sharing contractual arrangements would reinforce some of the conclusions.

The second branch of the literature has focused on explanations based on models with imperfect competition and/or increasing returns to scale. Hornstein (1993) developed a

[6] Explicit mention should be made, though, of the papers by Goodwin (1968) and Reichlin (1986). The economic intuition underlying the endogenous oscillations these two models display being quite akin to ours. Technological innovation being absent, there is no growth, either exogenous or endogenous, in either model.

model of monopolistic competition where the capital income share is procyclical. However, the correlation between output and capital share is perfect, hence the cyclical "hump-shape" pattern for profits cannot be replicated. Other examples include Ambler and Cardia (1998), Bils (1987), and the models surveyed in the Rotemberg and Woodford (1999). Hansen and Prescott [2005] is an additional contribution along the same lines, which does not make use of monopolistic competition but, instead, of fixed capacity at the plant level.

Finally, and the most relevant for us, is the third branch of the literature, spearheaded by Blanchard (1997) and Caballero and Hammour (1998). These papers have explored the dynamics over the middle-run induced by exogenous changes in real wages. After an initial increase in wages, due for example to an exogenous strengthening of the bargaining power of workers, the capital share goes down. What happens over time depends on the long-run elasticity of substitution, either with a permanent fall on capital share or with a return to the initial level. Blanchard (1997) suggests that changes in efficiency induced by the original increase in wages may even increase the long-run share of capital income. Some of the intuitive arguments given by Blanchard, inspired by the European experience in the 1970s and 1980s, are close in spirit to the model we suggest here, in particular the idea that, facing a persistently high exogenous wage, firms may strive to adopt technologies that reduce the labor input per unit of output, thereby leading to an eventual decrease in the share of labor income.

The key differences in our investigations is that we do not begin with an initial, exogenously given shock to wages (due to a change in technology, bargaining power or markup) and explore also the aggregate dynamics after such shock. Further, we view the changes in capital income share as a systematic and recurrent feature of the economy: the main driving force behind the introduction of new technologies and, therefore, of sustained growth. To put it plainly, we posit that growth must come through oscillations in the rate of technology adoption, that such oscillations are endogenous, and that their main source is the ongoing - and sometime very explicit - "conflict" over the shares of income going to different factors.

Finally, we note the similarities between some points of our model and the literature on directed technological change surveyed by Acemoglu (2002). A more careful comparison of intuitions, models, and results will be added in future versions of this paper. In any case, three macroscopic differences are that (i) we claim business cycles are "caused" by labor-saving technological change, (ii) we focus on the fundamental bias (labor vs capital) in a perfectly competitive environment, and, (iii) we make the bias endogenous and not exogenous.

3 Stylized Facts

In this section we discuss some of the U.S. evidence pertinent to the variation of the capital share of income over the business cycle[7]. We contend that expansions begin with increases in the capital share, that this share peaks substantially earlier than the expansion in output, and that the last phase of the expansion is correlated with a fall in the capital share.

We illustrate our assertion by computing the capital share of the U.S. economy in three different ways. First, we compute the capital share for the whole economy. This measure has the advantage of comprehensiveness but the drawback that it includes the household and government sectors whose output is not sold in the market and which have a fixed capital share by construction. Moreover, we need to handle the distribution of proprietors income between (imputed) wages and capital income. To overcome some of these difficulties, we compute the capital share for the corporate sector. Finally, we compute the capital share for the non-financial corporate sector.

3.1 Overall Economy

Our first take at evaluating the capital share in the U.S. economy uses aggregate data from the whole economy. As explained before, following this route faces the basic difficulty of how to divide Proprietors Income between labor and capital. A common solution to this problem (Cooley and Prescott, 1995) is to split the Proprietors Income according to the share of capital income observed in the non-proprietors part of the economy. To do so, we can subtract from our measure of output the Proprietors Income and include as capital income only the unambiguous capital income.

This strategy implies, first, that capital income includes income coming from two different sources:

1. Unambiguous capital income, equal to Rental Income of persons, Corporate Profits, Net Interest and miscellaneous payments, and the current surplus of government enterprises. We diverge from Cooley and Prescott (1995) in our definition of unambiguous capital income only in our inclusion of the current surplus of government enterprises as capital income. This surplus can be considered an income of the capital used by those firms. However, the quantitative importance of this number is quite small, less than 0.05% of output.

2. The consumption of fixed capital by the non-proprietors private sector and the government. We do not include the consumption of fixed capital by proprietors to be consistent with our exclusion of their income from our computations.

[7]Abundant evidence is also available for pretty much each and every EU country, which we will summarize in subsequent versions. The stylized facts reported here are, if possible, even more clearly visible in the European post-WWII data, which is what motivated Blanchard and Caballero-Hammour initial work. Further, a cross-country comparison may be useful in assessing the empirical relevance of the model's main predictions.

Second, we define output as Gross Nation Product less Proprietors Income. In addition, we subtract the difference between Net National Product and National Income (since this statistical discrepancy is also of difficult imputation between capital and labor) and Net Taxes on Production and Imports, since again this item cannot be divided between capital and labor.

As a consequence, our capital share α is defined as Capital share is then defined as

$$\alpha = \frac{\text{Unambiguous Capital Income} + \text{Depreciation}}{\text{National Inc} + \text{Depreciation} - \text{Proprietors Income} - \text{Taxes on P\&I}}$$

Correspondingly, the net capital share is defined as

$$\alpha = \frac{\text{Unambiguous Capital Income}}{\text{National Inc} - \text{Proprietors Income} - \text{Taxes on P\&I}}$$

Our different measures are taken directly from NIPA, Table 1.7.5 (Relation of Gross Domestic Product, Gross National Product, Net National Product, National Income, and Personal Income) and Table 1.12. (National Income by Type of Income). Since we only need percentages, we take nominal quantities that avoid distortions induced by price indexes. Our sample, of quarterly data, goes from 1947:2 to 2018:2.

Figure 7.1 plots the gross capital income share in the whole economy. Clearly, capital share fluctuates quite a bit. Also plotted is a Hodrick-Prescott trend with $\lambda = 1600$ and the NBER dating of recessions. The capital share tends to go up at the beginning of the expansion, pick at the middle, and drop in the second half of the expansion. Figure 7.2 plots the non-depreciation components of gross capital income share. Corporate profits are largely responsible for the cyclical pattern of the overall capital share. Net interest are relatively acyclical and we only need to notice the big increase in the 80's associated with the high real rates of interest of the time. Finally, the last line corresponds to the rental income, which is also relatively smooth over time. Depreciation, as shown in Figure 7.3, is also a relatively smooth series. Last, the cyclical pattern remains if we focus on net capital income share, as plotted in Figure 7.4.

3.2 Corporate Sector and Nonfinancial Corporate Sector

Our next measure of the capital share uses data form the corporate sector. These measure is closer to the main theoretical thrust of the paper.

We define the output of the corporate sector to be equal to the Gross value added of corporate business less the Taxes on production and imports net of subsidies. As capital income we add the Net operating surplus plus the Consumption of fixed capital. We repeat the exercise with the same concepts for the Nonfinancial Corporate Sector.

Table 3.1: Evolution of the Net Operating Surplus

	Initial Value	Max. Value	Increase	Final Value	Decrease
Expansion 60s	18.4%	23.6%	28.4%	18.0%	30.6%
Expansion 80s	15.4%	19.0%	23.5%	16.6%	14.5%
Expansion 90s	16.6%	19.9%	19.8%	15.8%	25.9%

Our different measures are taken directly from NIPA, Table 1.14. Gross Value Added of Domestic Corporate Business in Current Dollars and Gross Value Added of Nonfinancial Domestic Corporate Business in Current and Chained Dollars. As in the previous subsection, we employ nominal quantities.

Figure 7.5 and 7.6 plots respectively the gross and net capital income share in the corporate business sector. Again we can see how, even if the capital share fluctuates around a mean, these fluctuations are not trivial. Figures 7.7 and 7.8 reproduces the same series for the nonfinancial corporate business sector.

An interesting pattern appears if we study the three longest expansions in the U.S. after the second world war. We will call these expansions the 60s expansion, the 80s expansion, and the 90s expansions. These three episodes can be thought as particularly interesting because the length of the expansion allows one to identify more clearly the type of phenomena we are concerned with. We observe a common structure: both measures of capital income go up at the beginning of the expansion by a considerable amount, peaks roughly at the middle of the expansion, and decreases afterwards.

Table 3.1 summarizes the information. The first column corresponds to the value of the Net Operating Surplus at the beginning of the expansion, the second column to the maximum value in the expansion, the third column to the percentage increase, the fourth column is the final value at the end of the expansion, and finally, the fifth column reports the percentage decrease.
The main message of this table is the sizable changes in the Net Operating Surplus over the business cycle. If we take a benchmark capital-output ratio of 3, we can transform this numbers into rates of return dividing them by 3. With this back-of-the-envolope calculation, we see, for example, that the rate of return of the Corporate sector in the 60s went from 6.1% to 7.9% and then fell to 6.0% at the start of the recession.

Table 3.2 reports the same information as table 1, except that now we measure the evolution of corporate profits. The increases in Corporate Profits are of an even more substantial magnitude, especially for the 80s and 90s cycles. For example in the 80s, Corporate Profits went up by a 43% and in the 90s by a 42.2%; while this is not the topic of the present paper, one may want to consider how much these dramatic increases in profitability account for the large stock market rallies witnessed during those two expansions, and for

Table 3.2: Evolution of Corporate Profits

	Initial Value	Max. Value	Increase	Final Value	Decrease
Expansion 60s	17.6%	22.4%	27.0%	15.4%	44.9%
Expansion 80s	9.6%	13.7%	43.0%	11.5%	18.8%
Expansion 90s	11.9%	16.9%	42.2%	10.9%	54.6%

the early 1970*s* and 2000 crashes as well.
We can also repeat the back-of-the-envolope calculation of our previous paragraph, considering now the leverage implied by debt. We can see then how the profitability rate of the Corporate sector went in the 90's from 4% to 5.6% and then fell again to 3.6%.

3.3 Further Evidence

Net operating surplus and depreciation shows an opposite cyclical pattern, as presented in Figure 7.9. Depreciation, or consumption of fixed capital, [8] peaks during recession. The correlation coefficient between the two series is -0.7.

Based on trough and peak in gross capital share in the non-financial corporate business sector (or labor share index in the NFC sector), divide all periods into two episodes: KSUP (equivalently LSDOWN), with KS from a trough to peak, and KSDOWN (or LSUP), the opposite. Table 3.3 presents the growth rate (i.e. percentage change from previous quarter at annual rate) of 5 variables, output (real gross value added), labor productivity, hours, employment, and real wage (real hourly compensation).

[8] According to the 2008 manual of the United Nations System of National Accounts, *consumption of fixed capital* is the decline, during the course of the accounting period, in the *current* value of the stock of fixed assets owned by a producer as a result of physical deterioration, normal obsolescence or normal accidental damage. Therefore it measures economic rather than only physical depreciation.

Table 3.3: Statistics for KS up and down episodes

var	Obs	Mean	Std. Dev.	Min.	Max.
KSUP					
vadd_gr	136	5.12	6.79	-21.3	28.3
lp_gr	136	3.42	4.49	-11	19.6
emp_gr	136	1.45	4.07	-11	14.4
hours_gr	136	1.65	5.03	-13.9	18.4
realwage_gr	136	1.00	3.75	-8.5	17.2
KSDOWN					
vadd_gr	149	2.88	5.72	-17.4	30.5
lp_gr	149	1.25	3.47	-10.7	15.6
emp_gr	149	2.10	3.17	-12.3	11
hours_gr	149	1.57	3.67	-12.2	12.9
realwage_gr	149	1.71	3.06	-5.2	12.3

Note: growth rate is percent change from previous quarter at annual rate for the nonfinancial corporate sector from 1947q2 to 2018q2.

4 The Model

We start with the basic analytical framework, retaining the assumptions of a representative agent and of recursively complete financial markets. Extensions, with the unavoidable additional complications, are discussed later.

Preferences There is a representative agent whose preference is represented by the standard expected utility

$$\max E_t \Sigma_{t=0}^{\infty} \delta^t [u(C_t) + v(1 - L_t)].$$

Both $u(C_t)$ and $v(1 - L_t)$ are monotone increasing, strictly concave and continuously differentiable real valued functions.

Technology Production takes places in two different sectors, $s = a, b$, composed of heterogeneous firms. Firms belong to a finite number of types, indexed by $f \in F^s, s = a, b$, and they differ along two dimensions, one persistent and one temporary. Firms have persistently different capital productivities and sizes, summarized by the pairs (μ^a, A) and (μ^b, B), where μ^s is a measure, and A, B are closed intervals on the real line. Temporary heterogeneity is due to idiosyncratic technology shocks. Both sources of heterogeneity are discussed more carefully later. Description of the two sectors follow.

Firms in the first sector use their stock of active technologies to produce aggregate consumption, C_t, through labor, L, and productive capacity, Π, according to a neoclassical production function $G^f(\Pi, L)$. In each period, starting with a given productive capacity, they hire labor in a competitive market, [9] produce and sell output, and purchase investment goods to modify future productive capacity. The latter can be done in two ways: by augmenting the capital stock that embodies already active technologies, or by innovating, i.e. by using investment goods to introduce a not-yet-active technology, embodied in a new capital stock. Once a firm introduces a new technology,[10] we label the latter as active for that firm.

Firms in the second sector also use labor and productive capacity to produce aggregate investment, I_t, again according to a neoclassical production function $H^f(\Pi, L)$. Firms use aggregate investment either to accumulate active technologies or to introduce new ones. In the next two subsections we illustrate the baseline specifications for the production functions, $G()$ and $H()$.

The total endowment of leisure/labor time is fixed at one in all periods. There are as many capital goods, K_t^j, as there are technologies, $j = 0, 1, 2, ...$ Abstracting from the time dimension, there are three homogeneous goods (consumption, investment and labor) and a countable infinity of technology-specific capital goods. The current value prices are denoted, respectively, by $\{p_t^a\}_{t=0}^{\infty}$ for the (dated) consumption good, $\{p_t^b\}_{t=0}^{\infty}$ for the (dated) investment good, $\{w_t\}_{t=0}^{\infty}$ for the (dated) labor, and $\{q_t^j\}_{t,j=0}^{\infty}$ for the (dated and technology-specific) capital goods. We set $p_0^a = 1$ as our numeraire. Because of the presence of uncertainty, to be introduced momentarily, prices are also functions of the state of the world that realizes in period t; we omit such dependence, when not strictly necessary, to save on notation.

Consumption Sector

Production Firms operating in sector $s = a$ may invest, under the conditions specified below, in a countable number of technologies, indexed by the superscript $j = 0, 1, ...$ We say that a technology j is active for firm $f \in F^a$ in period t if $K_t^{f,j} > 0$, i.e. the firm owns a positive amount of capital stock of type j. Denote with $J_t^f = j_1(f), ..., j_t(f)$ the set of all technologies that are active at time t for firm f.[11]

Using technology (plant) $j \in J_t^f$, a firm $f \in F^a$ obtains output[12]

$$Y^{f,j} = \min\{K^{f,j}, \alpha^{f,j} L^{f,j}\},$$

[9] In the baseline model a competitive labor market is assumed to be active in each period. In subsequent extensions, different labor market arrangements are examined.

[10] Or capital good, as the latter embodies the former: the two terms are synonymous in this paper. With "productive capacity" we refer, instead, to the aggregation of all existing capital goods-technologies.

[11] Think of technologies as plants, with constant returns to scale. Decreasing returns are at the firm level (span of control by managers).

[12] Our notation is, unfortunately, somewhat cumbersome in super- and sub-scripts, as we need to keep track of different firms (f), sectors (s), technologies (j), and time periods (t). Whenever we feel there is no danger of confusing the reader, as in the following formula, some, or all, of these super/sub-scripts will be omitted.

where $K^{f,j}$ and $L^{f,j}$ are capital and labor used by firm f in technology (plant) j, and $\alpha^{f,j}$ is the labor productivity parameter, the details of which we illustrate momentarily. Aggregating over plants owned by the same firm

$$Y^f = \Sigma_{j \in J_t^f} Y^{f,j},$$

from which we define final (marketable) output of firm f as

$$C^f = A^f (Y^f)^\theta,$$

where $A^f \in A = [0, A]$ is a firm-specific productivity parameter, while $\theta \in (0, 1)$ captures the decreasing returns induced by limited span of control at the firm level. As mentioned, there is a measure $\mu^a(f)$ of firms of type f, i.e. there are $\mu^a(f)$ firms with productivity parameter equal to A^f. [13]

Labor productivity is a crucial component of our theory, and it should therefore be discussed carefully. We assume that each technology j comes with an average labor productivity parameter α^j, where $\alpha > 1$ and j is an exponent. Hence, technological progress is, on average, labor-saving because $\alpha^j > \alpha^{j-1}$. The choice of technologies is endogenous, and carried out at the firm level: each firm knows α^j when adopting technology j. A degree of idiosyncratic uncertainty is introduced in the model by assuming that a random shock $\epsilon_t^{f,j}$ affects labor productivity, i.e. the actual productivity of labor firms of type f face in period t when using technology j is

$$\alpha_t^{f,j} = \alpha^j + \epsilon_t^{f,j},$$

where $\epsilon_t^{f,j}$ is an N-state, first order Markov process with transition matrix Θ. The latter plays a crucial role in our analysis. From a theoretical perspective, we intend to show that this economy exhibits endogenous growth and persistent oscillations even when Θ is set equal to the null matrix and the $\epsilon_t^{f,j}$ are therefore independent draws of an N-state stationary random variable, with measure of dispersion σ_ϵ. From an applied perspective, the challenge we face is to show that, when properly calibrated, this economy generates time series with statistical properties matching those observed in aggregate data. In particular, we are interested in assessing how large Θ and σ_ϵ need to be, given realistic values of the other parameters, for the moments of our simulated economy to come close to those in the data. The contention being that, for Θ "near" zero and σ_ϵ not "too large", our model does at least as well as standard Real Business Cycle models do along the quantitative dimension.

Back to the description of the first sector. At the beginning of period t, due to past investment decisions, firm f owns a vector of capital stocks $K_t^f = K_t^{j_1(f)}, ..., K_t^{j_t(f)}$, with $j_1(f) \leq j_t(f)$. This allows the definition of potential productive capacity

$$\Pi_t^f = \Sigma_{j=j_1(f)}^{j=j_t(f)} A^f (K_t^{f,j})^\theta.$$

[13] Returns are decreasing in capital and labor; hence A^f summarizes the effect of firm-specific fixed factors, reconciling our model with constant return to scale in the full list of productive factors.

and potential employment for firm f in period t is

$$\Lambda_t^f = \Sigma_{j=j_1(f)}^{j=j_t(f)} \frac{K_t^{f,j}}{\alpha_t^{f,j}},$$

We introduce next a few additional concepts that may turn out useful later on. Let $\phi_t^{f,j} \in [0,1]$ denote the degree of capacity utilization for technology j, in firm f, in period t,

$$\phi_t^{f,j} = (L_t^{f,j})/(K_t^{f,j}/\alpha_t^{f,j})$$

implying that when the degree of capacity utilization is equal to one a firm operates at the efficient L/K ratio. Marginal productivity of labor therefore is

$$((\partial Y_t^{f,j})/(\partial L_t^{f,j})) = \alpha_t^{f,j}, \textit{ for } \phi_t^{f,j} < 1, \textit{ andzerootherwise.}$$

Expansion of Productive Capacity A firm f, starting period t with productive capacity equal to $K_t^f = K_t^{j_t(f)}, ..., K_t^{j_t(f)}$, and scrapping the amounts $S_t^f = S_t^{j_t(f)}, ..., S_t^{j_t(f)}$, is left, at the end of the same period, with a productive capacity of $(1-\mu)K_t^f - S_t^f$. Let $I_t^{f,j}$ be the amount of investment goods it allocates to active technology j_t^f. We set

$$K_{t+1}^{f,j} = (1-\mu)K_t^{f,j} + \gamma^j I_t^{f,j} - S_t^{f,j}, I_t^f = \Sigma_{j_t^f} I_t^{f,j}$$

where $1/\alpha < \gamma < 1$, i.e. machines embodying more advanced technologies are costlier to accumulate but still convenient, in expected value, relative to those embodying less advanced technologies. We assume that investment decisions are taken at the end of the period, i.e. after production has been carried out, but before the shock $\epsilon_{t+1}^{f,j}$ is realized. Because $\alpha\gamma > 1$, in the deterministic version the only active technology with positive gross investment will be the best available technology j_t^f.[14]

We define the marginal technology j_t^f, in period t, as the lowest indexed technology for which $L_t^{f,j} > 0$. At the end of each period a firm may also purchase investment goods to innovate, i.e. introduce the new technology $j_t^f + 1$. Let D_t^f be the total amount allocated to this purpose, we assume that

$$K_{t+1}^{f,j_t^f+1} = [(\zeta)^{j_t^f+1}]D_t^f,$$

with $\zeta < \gamma$, i.e. it is costlier to introduce a new technology than to accumulate any among the old ones.[15] This implies that innovation does not take place automatically, instead

[14]In the stochastic version, a technology is the best available only in an expected value sense, and positive investment in active technologies other than the best one is an equilibrium outcome when shocks have some degree of persistence.

[15]Later on we will consider the impact that a minimum size constraint in the innovation technology may have on the model's dynamics.

new technologies are introduced along an equilibrium path only when their labor saving effect is strong enough, i.e. the cost of labor is high enough, as discussed below.

Investment Sector

Production The structure of the second sector parallels that of the first, hence we can describe it more succintly. Again, let $J_t^f = j_1(f), ..., j_t(f)$ be the set of all technologies that are active at time t for firm $f \in F^b$. Using technology $j \in J_t^f$, a firm $f \in F^b$ obtains output

$$Y^{f,j} = \min\{K^{f,j}, \beta^{f,j} L^{f,j}\},$$

where $\beta^{f,j}$ is the labor productivity parameter. As before, aggregating over plants owned by the same firm

$$Y^f = \Sigma_{j \in J_t^f} Y^{f,j},$$

final (marketable) output of firm f is

$$I^f = B^f (Y^f)^\theta,$$

where $B^f \in B = [0, B]$, is a firm-specific productivity parameter. Again, there are $\mu^b(f)$ firms with a capital productivity parameter equal to B^f. Also in this sector, the labor productivity parameters satisfy

$$\beta_t^{f,j} = \beta^j + \epsilon_t^{f,j}.$$

From a theoretical perspective, both $\alpha > \beta > 1$ and $1 < \alpha < \beta$ are admissible, but the data from last century suggest the second is the realistic case. Potential productive capacity is

$$\Pi_t^f = \Sigma_{j=j_1(f)}^{j=j_t(f)} B^f (K^{f,j})^\theta.$$

and potential employment is

$$\Lambda_t^f = \Sigma_{j=j_1(f)}^{j=j_t(f)} \frac{K_t^{f,j}}{\beta_t^{f,j}}.$$

The rest is defined in analogy with the first sector; in particular, marginal productivity of labor is

$$((\partial Y_t^{f,j})/(\partial L_t^{f,j})) = \beta_t^{f,j}, \textit{ for } \phi_t^{f,j} < 1, \textit{ andzerootherwise},$$

and total output of sector two is

$$I_t + D_t = Y_t^b = \Sigma_{f \in F^b} Y_t^f \mu^b(f),$$

where I_t and D_t are obviously defined by aggregating across firms in both sectors.

Expansion of Productive Capacity The law of motion of the capital stock is still

$$K_{t+1}^{f,j} = (1-\mu) K_t^{f,j} + \gamma^j I_t^{f,j} - S_t^{f,j},$$

where $1/\beta < \gamma < 1$ also holds. The best available and the marginal technology are also defined identically to those for firms in the consumption sector, and a new technology may be obtained according to

$$K_{t+1}^{f,j_t^f+1} = [(\zeta)^{j_t^f+1}]D_t^f.$$

The notion of equilibrium we adopt is completely standard. In each period, given productive capacity and the realization of idiosyncratic shocks, firms maximize their market value by hiring labor [16] and selling their output in competitive markets; given initial wealth, and the realization of the shocks the representative agent supplies labor, receives factor payments, and makes intertemporal consumption-saving decisions. Next, firms maximize their expected value by investing in either active or new technologies for next period. Because we assume that financial markets are sequentially complete, we will write the competitive equilibrium for the baseline model as the solution to a dynamic programming problem, and compute it accordingly.

Markets At time t, the state of the world x_t encompasses the collection of vectors

$$\{K_t^f{}_{f\in F^a,F^b},\ \alpha_t^{f,j}{}_{f\in F^a,j\in J_t^f},\ \beta_t^{f,j}{}_{f\in F^b,j\in J_t^f}\}.$$

Recall that, for all $f \in F^a, F^b$ and for all $j \in J_t^f$, the random variable $\epsilon_t^{f,j}$ is an N-state, first order Markov process with transition matrix Θ. We make the, somewhat heroic, assumption that financial markets are sequentially complete: in each period t there exists a set of $(|F^a| + |F^b|) * |J_t| * N$ independent securities to which the continuum of identical agents have access. This means that, given x_t and the set $X_{t+1}(x_t)$ of possible future states, for all $x \in X_{t+1}(x_t)$ there exists, at time t, a competitive market in which contingent claims $A(x)$ are traded, with payoff $\xi[A(x), x_{t+1}] = 1$ if $x_{t+1} = x$, and zero otherwise. Let $m(x, x_t)$ be the price, in units of current consumption, of asset $A(x)$ in period t and state x_t. To save on notation, $A_t(x)$ indicates also the quantity of the Arrow security "x" acquired in period t.

Firms' and Households' Problem Given initial wealth $A_0(x_0)$, the representative agent maximizes the intertemporal expected utility given above under the sequence of budget constraints

$$p_t c_t + \Sigma_{x\in X_{t+1}(x_t)} m(x, x_t) A_t(x) \leq w_t L_t + p_t A_{t-1}(x_t).$$

A firm $f \in F^s$ begins period t with capacity K_t^f and labor productivity vector σ_t^f, where, from here onward, $\sigma = \alpha$ when $s = a$ and $\sigma = \beta$ when $s = b$. The problem of the firm consists, first, of maximizing period's profits by choosing the current level of capacity utilization, and, second, of maximizing its expected market value by choosing tomorrow's productive capacity.

[16] We allow for inaction, i.e. $L_t^{f,j} = 0$, without requiring the firm to disband. We consider later the case in which inactive firms are shut down forever, i.e. $L_t^{f,j} = 0$ implies $K_t^{f,j} = S_t^{f,j}$.

For $s = a, b$, the firm's static optimization problem is

$$\max_{L_t^{f,j}} \pi_t^f = \Sigma_{j \in J_t^f} (p_t^s * \min S^f \{K_t^{f,j}, \sigma_t^{f,j} L_t^{f,j}\}^\theta - w_t L_t^{f,j}).$$

The inter-temporal optimization problem is, instead,

$$\max_{K_{t+1}^{f,j}} E_t(V_{t+1}^f) = \Sigma_{j \in J_{t+1}^f} [E_t(q_{t+1}^j K_{t+1}^{f,j}) - p_t^b I_t^{f,j} + q_t^j \psi^j S_t^{f,j}]$$

subject to

$$K_{t+1}^{f,j} = (1-\mu) K_t^{f,j} + \gamma^j I_t^{f,j} - S_t^{f,j}, j \in J_t^f$$

and

$$K_{t+1}^{f,j} = (\zeta)^j * D_t^{f,j}, j \in J_{t+1}^f \setminus J_t^f.$$

where $0 \leq \psi < 1$.

Market Clearing Sectoral output corresponds, respectively, to aggregate consumption and aggregate investment. In the baseline model we assume that capital goods are technology and firm specific, hence, the laws of motion at the firm level are enough, together with the definition of potential productive capacity, to characterize market clearing in the markets for machines.[17]

Aggregate labor demand is $L_t = L_t^a + L_t^b \leq 1$, where

$$L_t^s = \Sigma_{f \in F^s} \mu^s(f) [\Sigma_{j \in J_t^f} (\phi_t^{f,j} K^{f,j} / s_t^{f,j})], \quad s = \alpha, \beta.$$

Equilibrium in the financial markets means

$$\Sigma_{f \in F^a, F^b} V_t^f(x_t) = p_t A_{t-1}(x_t)$$

and

$$\Sigma_{x \in X_{t+1}(x_t)} m(x, x_t) A_t(x) = \Sigma_{f^a, F^b} E_t(V_{t+1}^f)$$
$$\Sigma_{x \in X_{t+1}(x_t)} m(x, x_t) A_t(x) = \Sigma_{f^a, F^b} V_{t+1}^f$$

For future usage, write as S_t the vector of scrapped stocks $S_t^{f,j}$, for all relevant j and f. Finally, there are the obvious non-negativity constraints

$$K_t^{f,j} \geq 0,\ I_t^{f,j} \geq 0,\ D_t^f \geq 0,\ S_t^{f,j} \geq 0,\ C_t \geq 0,\ L_t^{f,j} \geq 0.$$

An equilibrium for this economy is defined in the customary way as a collection of prices, consumption, work, and production plans such that firms maximize (expected) profits,

[17] Among the extensions considered below is the one in which capital goods can be traded between firms and are only technology specific.

the representative consumer maximizes expected utility, the resource constraints are satisfied and markets clear in all periods.

The price at time t of additional machines for an active technology is equal to the price of output/consumption, P_t, as the two are perfectly substitutable in the aggregate resource constraint; the price of a machine for the new technology is P_t/ρ. On the other hand, because installed capital is "technology-specific" (the model is "putty-clay") each existing machine has its own price, q_t^j. Here we look at the equilibrium relations among these prices (these are present value prices), as they are determined by the zero profit conditions. The present value of output/consumption in period t is $P_t = \delta^t u'(C_t)$.

Zero profits for active technology j in the production of aggregate output, gives

$$P_t = q_t^j/A^j + w_t/\gamma^j \Rightarrow q_t^j = A^j P_t - a_j w_t$$

for $j = 1, ..., j_t$. Zero profit for the innovation technology gives

$$q_{t+1}^{j_t+1} = P_t/\zeta.$$

Notice that

$$q_t^j = A^j[P_t - w_t/\gamma^j] > A^{j-1}[P_t - ((w_t)/(\gamma^{j-1}))] = q_t^{j-1}$$

for $j = 1, ..., j_t$. So, assuming that $A^j = (A)^j \gamma^j = (\gamma)^j$, the prices of machines embodying active technologies, when they are active, satisfy

$$q_t^j/q_t^{j-1} = (A[P_t - w_t/\gamma^j])/(P_t - w_t/\gamma^{j-1}) = A[(\gamma^j P_t - w_t)/(\gamma^j P_t - \gamma w_t)] > 1.$$

Clearly, for some $j \in 1, ..., j_t$ we may have $\gamma^j P_t \leq w_t$, and then $q_t^j = 0$, meaning that technology j is not used in period t.[18]

Notice also that, as long as $I_t^{j_t} > 0$, the price of investment in machine j_t today must equal the market value of the machine tomorrow

$$q_{t+1}^{j_t} = P_t, \textit{ if } I_t^{j_t} > 0, \textit{ and}$$

$$q_{t+1}^{j_t} < P_t, \textit{ if } I_t^{j_t} = 0,$$

meaning that Tobin Q is always less or equal to one in this version of the model. This implies that $I_t^j = 0$ for all $j = 1, ..., j_t - 1$, and only $I_t^{j_t} \geq 0$ (this is also be set equal to zero, in periods when a new machine is produced.) From these considerations and the zero profit condition for technology j_t in period t we conclude that, when $I_t^{j_t} > 0$

$$q_{t+1}^{j_t} = q_t^{j_t}/A^{j_t} + w_t/\gamma^{j_t} = P_t,$$

[18]Once the condition $q_t^j = 0$ is realized, technology j is scrapped for ever in this economy. That is, it may not come back in later periods.

which gives the first order process followed by the prices of the best installed machines, as long as it does not drop to zero. For the other active technologies $j = 1, ..., j_t - 1$, we know that $q_t^j < q_t^{j_t}$ and that

$$q_t^j = A^j P_t - a_j w_t$$

Manipulate the latter under the assumption of exponential productivity parameters, to find the marginal technology in period t. This is the lowest index j for which $q_t^j \geq 0$; from the zero profit condition we have that

$$w_t \geq \gamma^{\gamma^{j_t}} P_t.$$

Next, consider the hypothetical case in which a new machine gets introduced, i.e. $D_t > 0$, and there is also positive investment in the best available technology, i.e. $I_t^{j_t} > 0$. Then it must be true that, in this particular circumstances,

$$((q_{t+1}^{j_t+1})/(q_{t+1}^{j_t})) = \zeta^{-1}.$$

Clearly this is not a generic case. In general, you either innovate (and then you do not invest anything in any of the active technology) or you do not, and then you invest only in the most efficient among the active technologies. You innovate when

$$\zeta q_{t+1}^{j_t+1} > q_{t+1}^{j_t}$$

or, more properly, we innovate in period t, i.e. $D_t > 0$ and $I_t^{j_t} = 0$, when

$$\zeta[A^{j_t+1} P_{t+1} - a_{j_t+1} w_{t+1}] > [A^{j_t} P_{t+1} - a_{j_t} w_{t+1}].$$
$$\zeta[A^{j_t+1} P_{t+1} - a_{j_t+1} w_{t+1}] > [A^{j_t} P_{t+1} - a_{j_t} w_{t+1}].$$
$$[a_{j_t} - a_{j_t+1}] w_{t+1} > [A^{j_t} - \zeta A^{j_t+1}] P_{t+1}.$$
$$((1-\zeta)/\zeta) < ((w_{t+1})/(P_t))[((\gamma - 1)/(\gamma^{j_t+1}))].$$

Notice that, with a constant factor $\gamma > 1$, the denominator of the right hand side goes to zero, hence the wage rate (in unit of current consumption) must grow, on average, at a rate of $\gamma - 1$ per period to maintain a stable pattern over time. When the real wage grows at a rate lower than $\gamma - 1$, technological innovation is delayed; the reverse when the real wage grows at a rate higher than $\gamma - 1$, then new machines are introduced more rapidly.[19]

Conditions for innovation Because, in the deterministic case, only $I_t^{j_t} > 0$, it suffices to compare investment in the best available technology j_t with investment in the new technology $j_t + 1$. The latter is more profitable than the former if

$$(1-\zeta)/\zeta < P_{t+1}/P_t[A^{j_t+1} - A^{j_t}] - w_{t+1}/P_t[a^{j_t+1} - a^{j_t}].$$

[19]This formalizes the intuition according to which in Europe labor saving innovations are more often adopted, labor productivity is higher and the capital/output ratio is also higher, because wage rates are "artificially" kept high by union power, political considerations, and, more generally, labor and product market regulations.

In the exponential case, this simplifies to

$$(1-\zeta)/\zeta < A^{j_t}(P_{t+1}/P_t)[A-1] - a^{j_t}(w_{t+1}/P_t)[a-1].$$

Interestingly, this shows that it matters, in general, if the new technology is relatively more capital or more labor saving, not just if they are overall "better".

When they are more capital saving, we have $a = A/\gamma > 1$, hence the higher is the expected wage rate the less profitable it is to invest in the new machinery - alternatively, the higher is the relative price of tomorrow's consumption, the more valuable is the new technology. When the new machine is more labor saving, i.e. $a = A/\gamma < 1$, a high expected wage next period makes the new technology profitable. This is our benchmark case.

Planner's Problem Here, we exploit the two welfare theorems to cast the study of our baseline model economy in the form of a solution to a stochastic dynamic programming problem. We proceed in steps from the bottom up.

Static Step One: allocation of labor across firms and sectors We consider first a sequence of static problems. The state vector x_t is composed of

$$\{K_t^f{}_{f\in F^a,F^b},\ \alpha_t^{f,j}{}_{f\in F^a,j\in J_t^f},\ \beta_t^{f,j}{}_{f\in F^b,j\in J_t^f}\}$$

To save on notation, let $x_t^a = K_t^f,\ \alpha_t^{f,j}{}_{f\in F^a,j\in J_t^f},\ x_t^b = K_t^f,\ \beta_t^{f,j}{}_{f\in F^b,\,j\in J_t^f}$, and $x_t = [x_t^a, x_t^b]$. Let $L_t^s, s = a, b$, be the total amount of labor allocated to each sector. Our first task is to figure out how the planner will split L_t^s into the $L_t^{f,j,s}, f \in F^s, j \in J_t^f$.

Notice first, the effective productive capacity of technology j in firm f is $\kappa_t^{f,j} = A^f(K_t^{f,j})^\theta$. Because there are $\mu^s(f)$ firms of type f and their period t productivity of labor is $\alpha_t^{f,j}$- respectively $\beta_t^{f,j}$- potential employment in technology j, in firms of type f, in sector s is defined as

$$\Lambda_t^{f,j,s} = \mu^s(f)((\kappa_t^{f,j})/(\sigma_t^{f,j})),\ s = a, b;\ \sigma = \alpha, \beta,$$

and total potential employment in technology j in sector s is

$$\Lambda_t^{j,s} = \Sigma_{f\in F^s}\Lambda_t^{f,j,s}, s = a, b.$$

Assume $\Sigma_{j\in J_t^s}\Lambda_t^{j,s} \geq L_t^s$, otherwise $L_t^{f,j,s} = \Lambda_t^{f,j,s}$ is the optimal choice.

Notice second, in either sector labor will be assigned first to the firm-technology pair with the highest marginal productivity of labor, i.e. with the highest $\alpha_t^{f,j}$- respectively $\beta_t^{f,j}$- until $\phi_t^{f,j} = 1$ obtains. After this, labor is assigned to the firm-technology pair with the second highest marginal productivity of labor, and so on until L_t^s is exhausted. Write $L^{f,j,s}(x_t^s, L_t^s), s = a, b$, for the efficient allocation of labor across firms and technologies; it is straightforward to see that this is a stationary continuous function, which is increasing and concave in the second. Then

$$Y^s(x_t^s, L_t^s) = \Sigma_{f \in F^s} \mu^s(f) [\Sigma_{j \in J_t^f} \sigma_t^{f,j} L^{f,j,s}(x_t^s, L_t^s)], s = a, b$$

is total output in either sector. Again, we have a stationary and continuous function of x_t^s and L_t^s, increasing and concave in the second

Consider next the problem of efficiently allocating aggregate labor supply, L_t, between L_t^a and L_t^b. This is equivalent to

$$\max_{0 \leq L_t^a \leq L_t} Y^a(x_t^a, L_t^a)$$

subject to

$$Y_t^b \leq Y^b(x_t^b, L_t^b),$$
$$L_t^a + L_t^b \leq L_t.$$

This is a continuous and concave maximization problem with a convex and compact feasible set. The unique solution $L_t^s = L^s(x_t, L_t, Y_t^b), s = a, b$, is a stationary and continuous function, monotone increasing in the second argument and decreasing (increasing) in the third for $s = a (s = b)$. Less straightforward, but still true and useful properties, are that L^s is concave in L_t for $s = a, b$, and that L^a is concave while L^b is convex in Y_t^b.

Static Step Two: labor supply Consider next the problem of determining the supply of labor as a function $L(x_t, Y_t^b)$ of x_t and Y_t^b. Mathematically,

$$\max_{0 \leq L_t \leq 1} [u(C_t) + v(1 - L_t)]$$

subject to

$$C_t = Y^a[x_t^a, L^a(x_t, L_t, Y_t^b)].$$

This is again a concave optimization problem, under a convex and compact constraint. Under the maintained assumptions all functions are (almost everywhere) differentiable, hence interior solutions are characterized by the First order condition

$$u'[Y_t^a(x_t^a, L^a(x_t, L_t, Y_t^b))](\partial Y^a / \partial L_t^a)(\partial L^a / \partial L_t) = v'[1 - L(x_t, Y_t^b)].$$

Notice that $\partial Y^a / \partial L_t^a = \alpha_t^{f,j}$, which is the productivity of labor in the marginal technology of the consumption sector, and that $\partial L^a / \partial L_t = 1$, as Y_t^b is treated parametrically here. Hence, we have

$$\alpha_t^{f,j} = v'(1 - L_t) / u'(C_t),$$

implying that when labor productivity in the marginal technology improves, either because of an idiosyncratic shock or endogenously, either labor increases or consumption increases or both.

This maximization problem has a well behaved and continuous solution $L_t = L(x_t, Y_t^b)$, from which, by repeated substitution, one derives that $C_t = Y^a[x_t^a, L^a(x_t, L_t, Y_t^b)] = T(x_t, Y_t^b)$.

The function $C_t = T(x_t, Y_t^b)$ is the Production Possibility Frontier (PPF) of our two sector economy. It is known to be increasing in x_t, decreasing in Y_t^b, and concave in both. Under our assumptions about $u(), v(), G^f$ and H^f it is also strictly concave in Y_t^b.

Dynamic Problem The planner solves the following inter-temporal optimization problem

$$\max_{I_t^{f,j}, D_t^{f,j}, S_t^{f,j}} E_t \Sigma_{t=0}^{\infty} \delta^t [u(C_t) + v(1 - L(x_t, Y_t^b))]$$

subject to

$$C_t = T(x_t, Y_t^b)$$
$$I_t + D_t \leq Y_t^b = Y^b[x_t^b, L^b(x_t, L(x_t, Y_t^b))]$$
$$K_{t+1}^{f,j} = (1-\mu)K_t^{f,j} + \gamma^j I_t^{f,j} - S_t^{f,j}, j \in J_t^f,$$
$$I_t = \Sigma_{s=a,b}[\Sigma_{j \in J_t^f, f \in F^s} I_t^{f,j}], D_t = \Sigma_{s=a,b}[\Sigma_{j \in J_{t+1}^f{}_t^f, f^s} D_t^{f,j}],$$
$$K_{t+1}^{f,j} = (\zeta)^j * D_t^{f,j}, j \in J_{t+1}^f \setminus J_t^f.$$

FOC for $I_t^{f,j} > 0$,

$$((u'(C_t)\alpha_t)/(\delta\beta_t\gamma^j\theta A^f(K_{t+1}^{f,j})^{\theta-1})) = E_t[u'(C_{t+1})((\alpha_{t+1}^{f,j} - \alpha_{t+1})/(\alpha_{t+1}^{f,j}))]$$

FOC for $D_t^{f,j} > 0$

$$((u'(C_t)\alpha_t)/(\delta\beta_t\zeta^j\theta A^f(K_{t+1}^{f,j})^{\theta-1})) = E_t[u'(C_{t+1})((\alpha_{t+1}^{f,j} - \alpha_{t+1})/(\alpha_{t+1}^{f,j}))]$$

5 Quantitative Results-TBA

6 Reference

[1] Acemoglu, D. (2002), ”Directed Technical Change”. Review of Economic Studies 69, 781-810.

[2] Ambler, S. and E. Cardia (1998), ”The Cyclical Behavior of Wages and Profits under Imperfect Competition”. Canadian Journal of Economics 31, 148-164.

[3] Antràs, P. (2004), ”Is the U.S. Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution.” Contributions to Macroeconomics, 4-1.

[4] Bils, M. (1987), ”The Cyclical Behavior of Marginal Cost and Price.” American Economic Review,

[5] Blanchard, O. J. (1997), ”The Medium Run”, Brookings Papers in Economic Activity, Macroeconomics, 89-158.

[6] Boldrin, M. and M. Horvath (1995), ”Labor Contracts and Business Cycles”. Journal of Political Economy 103, 972-1004.

[7] Boldrin, M. and D.K. Levine (2001), ”Factor Saving Innovation”, Journal of Economic Theory

[8] Caballero, R. and M. Hammour (1998), ”Jobless Growth: Appropriability, Factor Substitution and Unemployment”, Carnegie-Rochester Conference Proceedings 48, 51-94.

[9] Cooley, T.F. and E.C. Prescott (1995), ”Economic Growth and Business Cycles” in T. F. Cooley. (ed), Frontiers of Business Cycle Research. Princeton University Press.

[10] Gomme, P. and J.Greenwood (1995), ”On the Cyclical Allocation of Risk”. Journal of Economic Dynamics and Control 19, 91-124.

[11] Reichlin, P. (1986), ”Euilibrium cycles in an overlapping generations economy with production”, Journal of Economic Theory

[12] Rotemberg, J.J. and M. Woodford (1999), ”The Cyclical Behaviour of Prices and Costs,”, in J. Taylor and M. Woodford (eds.) The Handbook of Macroeconomics, volume 1B, North-Holland Publ. Press.

[13] Solow, R. (1960), ”Investment and technological progress.” In K. Arrow, S. Karlin and P. Suppes (eds.), Mathematical Methods in Social Sciences 1959, 89-104, Stanford University Press.

[14] Young, A.T. (2004), ”Labor’s Share Fluctuations, Biased Technical Change, and the Business Cycle”. Review of Economic Dynamics 7, 916-931.

[15] Young, A.T. (2005), ”One of the Things We Know that Ain’t So: Why U.S. Labor’s Share is not Relatively Stable”, mimeo, Economics Department, University of Mississippi.

7 Appendix

A Leontief production function is used in the baseline model. Here we collect some algebra that is useful to understand the subtleties of the general CES case.

Definition of active technologies, best available technology and marginal technology is as in the fixed coefficient case. The production functions now are

$$Y_t^j = A^j[\eta(K_t^j)^\rho + \gamma^j(L_t^j)^\rho]^{1/\rho} + \nu^j S_t^j,$$

$0 < \nu < A^j < \infty, -\infty < \rho < 1, \eta > 0, \gamma > 1$. For each technology $j = 0, 1, ...,$ we assume that there exist x^j and x^j such that $x^j \leq ((K_t^j)/(L_t^j)) \leq x^j$ must hold; i.e. for each technology there exists a minimum and a maximum admissible capital/labor ratio. Rewrite the production functions in term of capital intensities $x = K/L$.

$$Y_t^j = A^j L_t^j[\eta(x_t^j)^\rho + \gamma^j]^{1/\rho} + \nu^j S_t^j,$$

Marginal productivities, for technology j, are

$$((\partial Y_t)/(\partial L_t^j)) = A^j \gamma^j [\eta(x_t^j)^\rho + \gamma^j]^{(1-\rho)/\rho}, \textit{ for } x^j \leq x_t^j \leq x^j, \textit{ andzerootherwise;}$$

$$((\partial Y_t)/(\partial K_t^j)) = A^j \eta (x_t^j)^{\rho-1}[\eta(x_t^j)^\rho + \gamma^j]^{(1-\rho)/\rho}, \textit{ for } x^j \leq x_t^j \leq x^j, \textit{ andzerootherwise.}$$

Potential labor demand Λ_t is the sum, over technologies, of maximum achievable employment given installed capacity

$$\Lambda_t = \Sigma_{j\in J_t} L_t^j = \Sigma_{j\in J_t}((K_t^j)/(x^j)),$$

while productive capacity Π_t is

$$\Pi_t = \Sigma_{j\in J_t} A^j[\eta + ((\gamma^j)/((x^j)^\rho))]^{1/\rho} + v^j K_t^j = \Sigma_{j\in J_t}(\kappa^j + \nu^j)K_t^j.$$

As before, the aggregate constraints are

$$\begin{aligned}
&Y_t = \Sigma_{j\in J_t} Y_t^j \leq \Pi_t,\\
&C_t + \Sigma_{j\in J_t} I_t^j + D_t Y_t,\\
&L_t^d = \Sigma_{j\in J_t} L_t^j \leq \Sigma_{j\in J_t} L_t^j = \Lambda_t,\\
&K_{t+1}^j = (1-\mu)K_t^j + I_t^j - S_t^j,
\end{aligned}$$

and

$$K_t^j \geq 0,\ I_t^{j_t} \geq 0,\ D_t \geq 0,\ S_t^j \geq 0,\ C_t \geq 0,\ L_t^j \geq 0.$$

Let the sequences $X_t = \Sigma_{j\in J_t} I_t^j + D_t{}_{t=0}^{\infty}$ and $S_t^1, ..., S_t^{j_t}{}_{t=0}^{\infty}$ be given, and characterize the period by period allocation of total labor supply across sectors, for given productive capacity installed. As before, the planner will

$$\max_{L^1,...,L^{j_t}} u(C) + v(1-L)$$

subject to

$$C + X = \Sigma_{j \in J_t} A^j L_t^j [\eta (x_t^j)^\rho + \gamma^j]^{1/\rho} + \nu^j S_t^j,$$
$$L = \Sigma_{j \in J_t} L_t^j$$

First order conditions

$$A^j \gamma^j [\eta (x_t^j)^\rho + \gamma^j]^{(1-\rho)/\rho} = w_t, \textit{ forall } j \in J_t \textit{ suchthat } L_t^j > 0.$$

In the exponential case the latter implies that, for all pairs $j, i \in J_t, j > i$, for which $L_t^j, L_t^i >$ 0

$$(A\gamma)^{((\rho(j-i))/((1-\rho)))}[\eta (x_t^j)^\rho + \gamma^j] = \eta (x_t^i)^\rho + \gamma^i.$$

This requires $x_t^j < x_t^i$, independently from the sign of ρ; hence, we should always observe a lower capital intensity in the most advanced sectors. This holds also in the special, but important, case in which technological progress is purely labor saving, i.e. when $A^j = A^i = A$.

Then, employment at time t is uniquely determined by the $x^{j_t} \leq x_t^{j_t} \leq x^{j_t}$ solving

$$v'(1 - \Sigma_{j=j_t+1}^{j_t}((K_t^j)/(x^j)) + ((K_t^{j_t})/(x_t^{j_t}))) =$$
$$u'(\Sigma_{j=j_t+1}^{j_t} \kappa^j K_t^j + A^{j_t}[\eta (K_t^{j_t})^\rho + \gamma^{j_t}(((K_t^{j_t})/(x_t^{j_t})))^\rho]^{1/\rho} + \Sigma_{j \in J_t} v^j S_t^j - X_t) A^{j_t} \gamma^{j_t} [\eta (x^{j_t})^\rho + \gamma^{j_t}]^{(1-\rho)/\rho}.$$

Efficient Dynamic Allocations Begin with simple case in which only capital of type 1 is active, and capital of type 2 may or may not be introduced. We have

$$\max \Sigma_{t=0}^{\infty} [u(C_t) + v(1 - L_t)] \delta^t$$

subject to

$$C_t + I_t + D_t = A^1 [\eta (K_t^1)^\rho + \gamma^1 (L)^\rho]^{1/\rho} + ?^1 S_t^1$$
$$K_{t+1}^1 = (1 - \mu) K_t^1 + I_t$$
$$K_{t+1}^2 = \zeta D_t$$

FOC for labor supply

$$u'(C_t)((\partial C_t)/(\partial L_t)) = v'(1 - L_t)$$
$$A^1 \gamma^1 [\eta (((K_t^1)/(L_t)))^\rho + \gamma^1]^{(1-\rho)/\rho} = ((v'(1 - L_t))/(u'(Y_t - I_t - D_t)))$$

which has a unique solution $L(K_t^1, K_{t+1}^1, D_t, S_t^1)$.

FOCs for the two investments (Note, $I_t > 0 \Leftrightarrow S_t^1 = 0$ and vice versa)

$$u'(C_t) = \delta [u'(C_{t+1})((\partial Y_{t+1}^1)/(\partial K_{t+1}^1)) - v'(1 - L_{t+1})((\partial L_{t+1})/(\partial K_{t+1}^1))]$$
$$u'(C_t) = \delta \zeta [u'(C_{t+1})((\partial Y_{t+1}^2)/(\partial K_{t+1}^2)) - v'(1 - L_{t+1})((\partial L_{t+1})/(\partial K_{t+1}^2))]$$

Notice that, contrary to the Fixed Coefficients case, BOTH these FOCs may be satisfied with equality as long as x_{t+1}^1 and x_{t+1}^2 are chosen to satisfy the equality of rate of returns condition, given in the appendix.

7.1 Additional Algebra on CES

First, in what sense are we modeling labor saving technological progress? Assume exponential productivity parameters, and compare two technologies, $j > i$, along isoquants at which they produce the same amount of aggregate output and use one unit of capital stock.

$$A^j[\eta + \gamma^j (L_t^j)^\rho]^{1/\rho} = A^i[\eta + \gamma^i (L_t^i)^\rho]^{1/\rho}$$
$$(A^{j-i})^\rho[\eta + \gamma^j (L_t^j)^\rho] = \eta + \gamma^i (L_t^i)^\rho$$
$$(A^\rho \gamma)^{j-i} (L_t^j)^\rho = (L_t^i)^\rho - (\eta/(\gamma^i))[(A^{j-i})^\rho - 1]$$

Capital is immobile, and investment flows only to the best available technology (in the deterministic case), hence it is not obvious that the marginal productivity of capital should ever be equalized across sectors. In any case, let's compute the conditions under which capital productivity is equalized across sectors.

$$(A^{j-i})^{\rho/(1-\rho)} \eta (x_t^j)^{-\rho}[\eta (x_t^j)^\rho + \gamma^j] = \eta (x_t^i)^{-\rho}[\eta (x_t^i)^\rho + \gamma^i]$$
$$(A^{j-i})^{\rho/(1-\rho)} \gamma^{j-i} (x_t^j)^{-\rho} = (x_t^i)^{-\rho} - (\eta/(\gamma^i))[(A^{j-i})^{\rho/(1-\rho)} - 1]$$

Special case, in which $A^j = A^i = A$, gives

$$(x_t^j)^{-\rho}[\eta (x_t^j)^\rho + \gamma^j] = (x_t^i)^{-\rho}[\eta (x_t^i)^\rho + \gamma^i]$$
$$\gamma^j (x_t^j)^{-\rho} = \gamma^i (x_t^i)^{-\rho}$$
$$((\gamma^j)/(\gamma^i)) = (((x_t^j)/(x_t^i)))^\rho$$

Notice that when $\rho > 0$ the capital labor ratio is higher in the most efficient sector, but the opposite is true when $\rho < 0$.

Zero profit in investment activity has no additional implications. A unit of investment, no matter where it is allocated, always costs a unit of consumption today, i.e. p_t. Its payoff in sector j is $p_{t+1}((\partial Y_{t+1})/(\partial K_{t+1}^j))$. Movements in labor shares.

$$((wL^j)/(Y^j)) = ((L^j A^j \gamma^j [\eta (x^j)^\rho + \gamma^j]^{(1-\rho)/\rho})/(A^j L^j [\eta (x^j)^\rho + \gamma^j]^{1/\rho})) =$$
$$= ((\gamma^j)/(\eta (x^j)^\rho + \gamma^j)) = (1/(1 + ((\eta (x^j)^\rho)/(\gamma^j))))$$

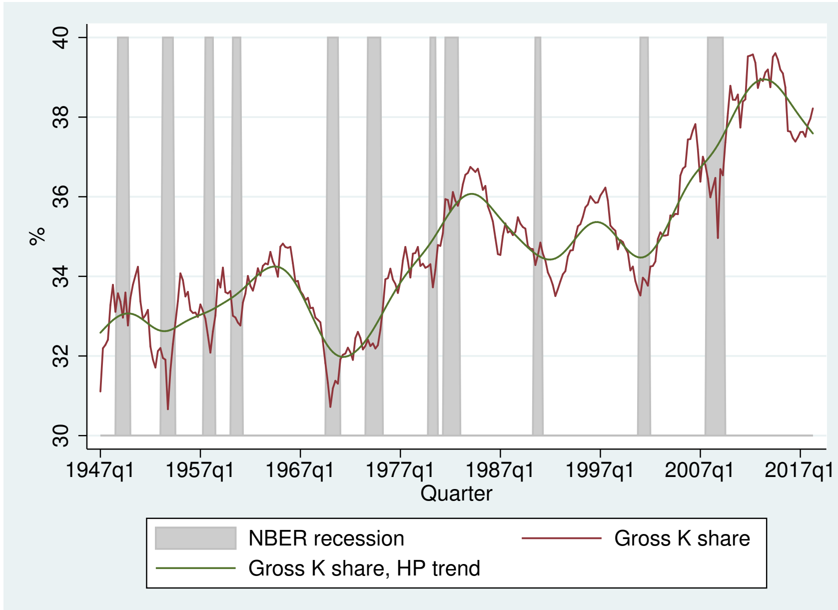
Hence, we have the two following cases

1. $\rho > 0$ $\quad (1/(1 + ((\eta (x^j)^\rho)/(\gamma^j))))$ is a decreasing function of x^j and an increasing function of γ^j, hence, for a given technology, the labor share decreases as the capital intensity increases. Recall that, at least in principle, along an expansion the capital intensity decreases in all sectors, hence the labor share should increase during an expansion, when $\rho > 0$.

   When there is an innovation, i.e. a technology with higher index is adopted, then, ceteris paribus, the labor share would increase. This suggests that, if our intuition works, right after an innovation the capital intensity of all sectors, and of the most recent ones in particular, should increase more than proportionally, thereby lowering the labor share of income.

2. $\rho < 0$ The opposite is true. Notice that, in this case, in order for the labor share to increase along an expansion we would need the capital intensity to increase along an expansion. That is to say, employment increases but investment increases more than proportionally driving up the labor share of income.

Figure 7.1: Gross capital share in the whole economy



Appendix: Figures

Figure 7.2: Gross capital share in the whole economy, non-depreciation components

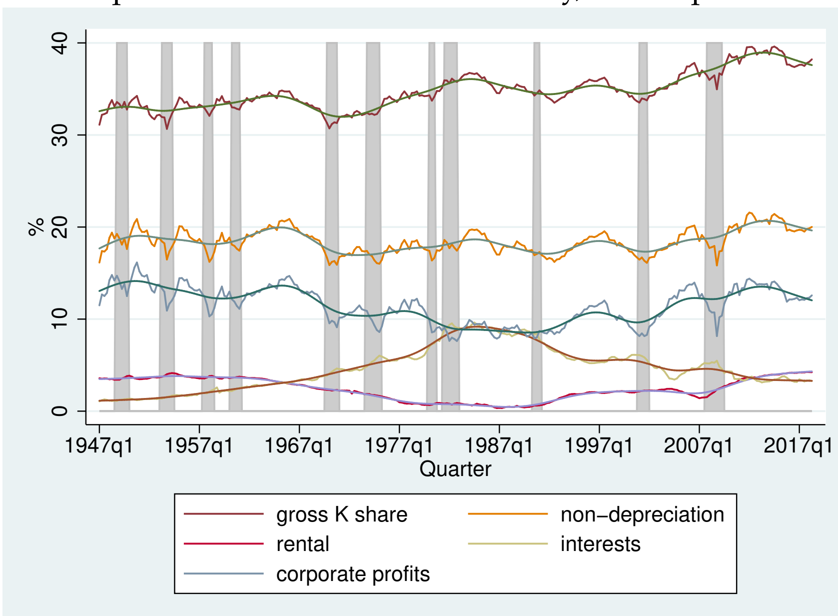


Figure 7.3: Gross capital share in the whole economy, depreciation

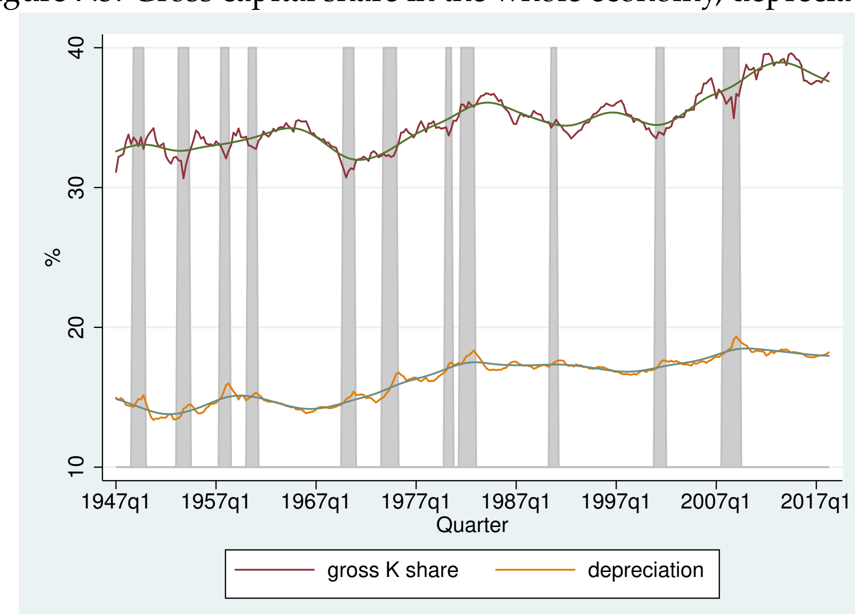


Figure 7.4: Net capital share in the whole economy

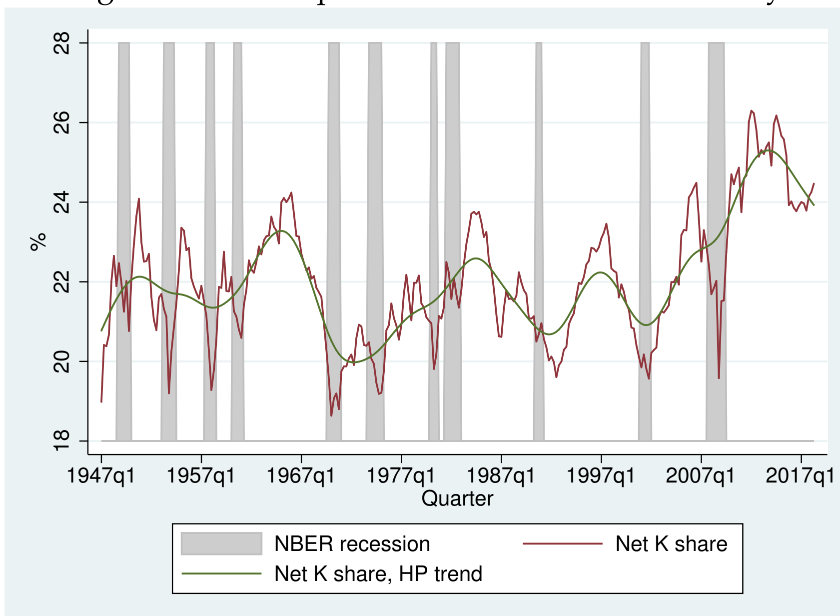


Figure 7.5: Gross capital share in the corporate business sector

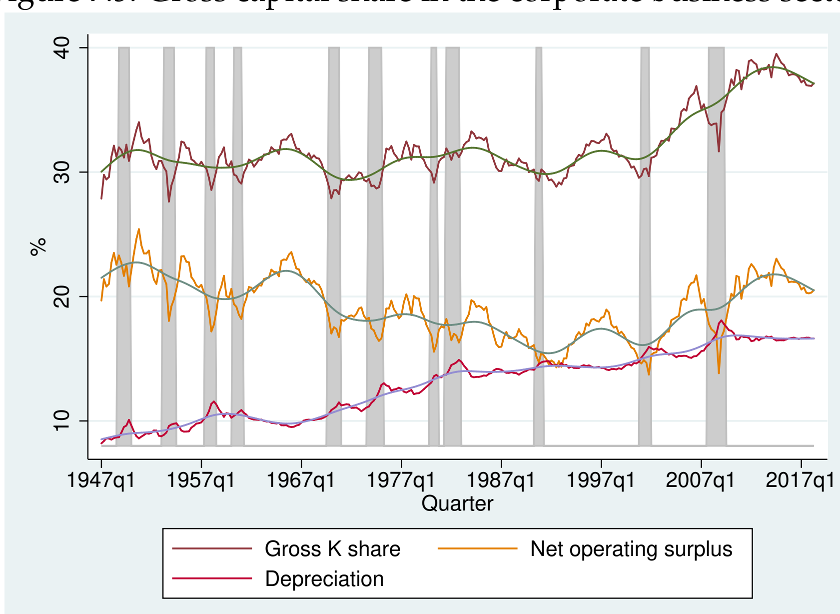


Figure 7.6: Net capital share in the corporate business sector

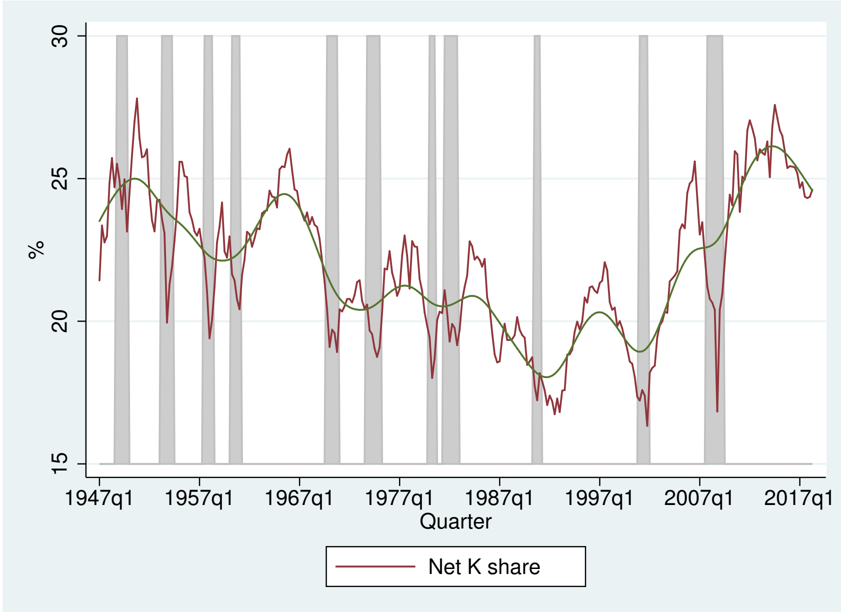


Figure 7.7: Gross capital share in the nonfinancial corporate business sector

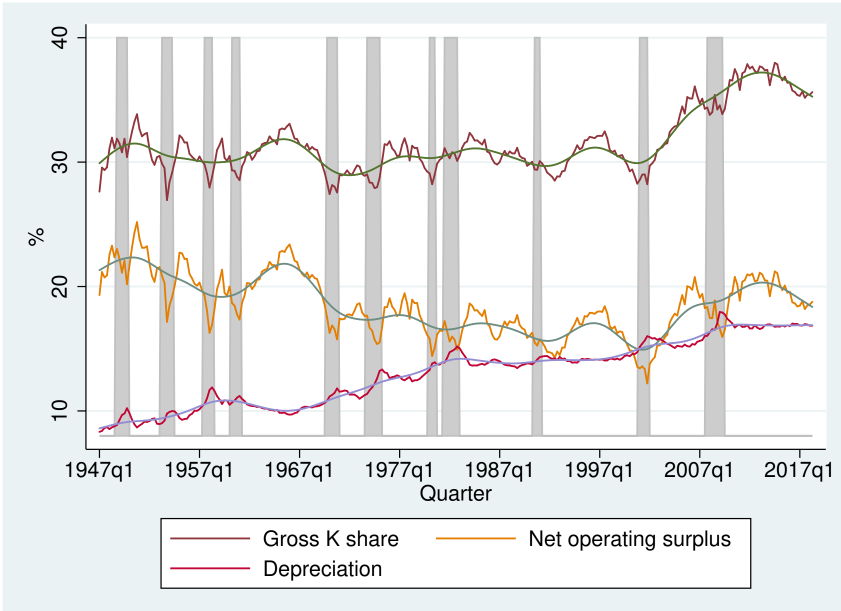


Figure 7.8: Net capital share in the nonfinancial corporate business sector

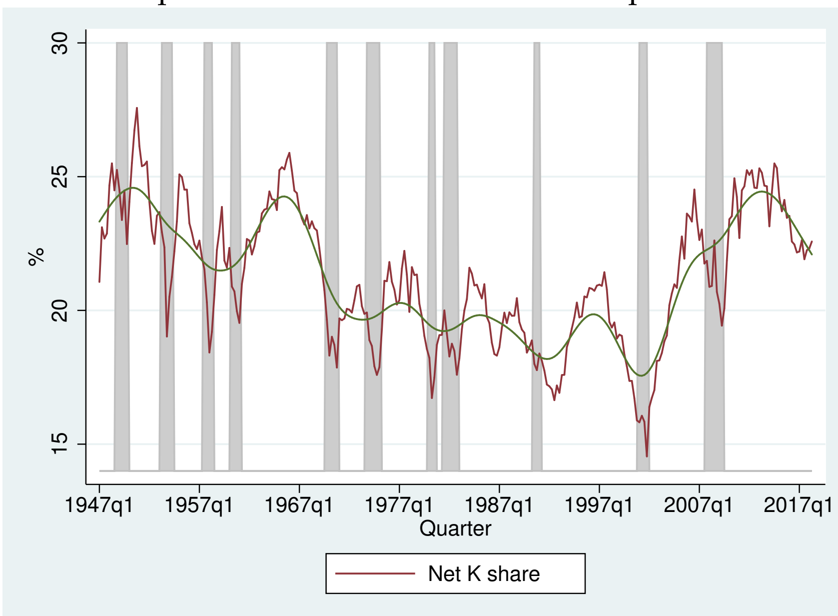
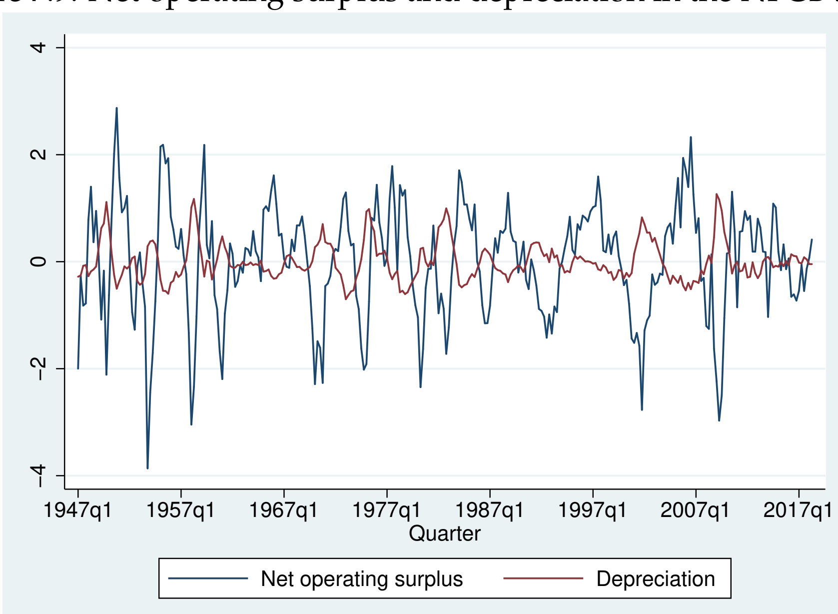


Figure 7.9: Net operating surplus and depreciation in the NFCB sector



Note: both series are HP detrended. NFCB-nonfinancial corporate business