The Global Business Cycle: Measurement and Transmission*

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Abstract

This paper examines the role of both technology and non-technology shocks in international business cycle comovement. Using industry-level data on 30 countries and up to 28 years, we first provide estimates of utilization-adjusted TFP shocks, and an approach to infer non-technology shocks. We then set up a quantitative model calibrated to the observed international input-output and final goods trade, and use it to assess the contribution of both technology and non-technology shocks to international comovement. We show that unlike the traditional Solow residual, the utilization-adjusted TFP shocks are virtually uncorrelated across countries. Transmission of TFP shocks across countries also cannot generate noticeable comovement in GDP in our sample of countries. By contrast, non-technology shocks are correlated across countries, and the model simulation with only non-technology shocks generates substantial GDP comovement. While shocks transmit across countries through production networks, the contribution of trade openness to comovement depends also on whether more or less correlated sectors get larger as the country opens to trade. We conclude that in order to understand international comovement, it is essential to both model and measure non-TFP shocks in a framework with international production networks.

Keywords: TFP shocks, non-technology shocks, international comovement, input linkages
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1 Introduction

Real GDP growth is positively correlated across countries. In spite of a large amount of research into the causes of international comovement, we still lack a comprehensive account of this phenomenon. Two related themes cut through the literature. First, is international comovement driven predominantly by technology (Backus, Kehoe, and Kydland, 1992) or non-technology (Stockman and Tesar, 1995) shocks? Second, does comovement occur because shocks are transmitted across countries (e.g. Frankel and Rose, 1998; di Giovanni, Levchenko, and Mejean, 2018), or because the shocks themselves are correlated across countries (Imbs, 2004)?

This paper uses sector-level data for 30 countries and up to 28 years to provide a forensic account of the sources of international comovement. To clarify the mechanisms at play and objects of interest for measurement, we start by setting up a simple accounting framework that expresses the GDP covariance between two countries as a function of covariances of primitive shocks and the elements of the global influence matrix. This influence matrix collects the general equilibrium elasticities of GDP movements at home and abroad to sector-country-specific shocks, and thus translates the variances and covariances of the primitive shocks into comovements of GDP. In particular, two countries can experience positive comovement if influential sectors in the two economies have correlated shocks. Comovement also arises if shocks in one country influence another country's GDP in another through trade and production linkages. This accounting framework reveals an underappreciated mechanism through which trade opening affects GDP comovement: it changes the relative influence of domestic sectors. Thus, whether trade opening increases or lowers GDP comovement depends in part on whether it leads to the expansion or contraction of sectors with more correlated shocks.

The accounting framework provides a road map for the measurement and quantification exercises that follow. First, we must measure both technology and non-technology shocks in order to understand their international comovement properties. Second, we must impose sufficient structure and bring sufficient data on international trade linkages to recover the global influence matrix. This will allow us to establish both how they interact with the shock correlation, and lead to transmission. Finally, to fully understand the contribution of trade to GDP comovement, we must use our theoretical structure to infer how the autarky influence matrix would differ from the one in the current trade equilibrium.

To provide a theoretical foundation for shock measurement and quantification exercises, we set up a multi-country, multi-sector, multi-factor DSGE model of world production and trade. Final consumption in each country and sector is an Armington aggregate of the goods coming from different source countries. Each sector uses labor, capital, and intermediate inputs that can come from any sector and country in the world. Between periods, capital and employees can be accumulated in each sector. However, within a period, labor and capital supply to each sector and country are upward-sloping in the real prices of labor and capital, respectively, and subject to shocks. Thus, our framework captures the notion of variable factor utilization: even conditional on the observed number of installed machines and employee-hours, the utilization rate of those machines and the employees' effort can vary within a period in response to shocks. Therefore, true factor usage is not perfectly observed and must be accounted for in the estimation of shocks.

We estimate utilization-adjusted TFP growth rates in our sample of countries, sectors, and years. Our methodology is based on our theoretical framework and uses the insights of Basu, Fernald, and Kimball (2006, henceforth BFK). BFK estimate TFP shocks for the United States controlling for unobserved input utilization and industry-level variable returns to scale. Importantly, they show that doing so produces a TFP series with dramatically different properties than the traditional Solow residual. We bring this insight into the international context by estimating utilization adjusted TFP series for a large sample of countries, and analyzing the international correlations in these series.

Having measured the TFP shocks at the country-sector level, we next use our framework to infer non-technology shocks. The objective is to obtain a shock that rationalizes the change in primary factor inputs conditional on the technology shock. In our model, sectors use capital, labor, and intermediates. For each sector, real output growth is therefore moved by (i) its TFP shock; (ii) the change in the use of its intermediate inputs; and (iii) the non-technology shock to the supply of the primary factors to this sector. It is this non-technology shock that we are interested in measuring. Using data on productivity shocks, sectoral prices, and the world input-output matrix, we back out the non-technology shock that rationalizes the data on output and input growth in each country, sector, and year.

Our first main finding is about the properties of the shocks themselves. We show that TFP growth is virtually uncorrelated across countries, implying that TFP covariance has a small direct contribution to observed GDP comovement in our sample of countries. In contrast to TFP, the aggregated non-technology shocks are quite correlated across countries, with the correlation coefficients about one-third of the correlation in real GDPs.

Of course, actual GDP growth is endogenous as factor and intermediate inputs respond to both domestic and foreign shocks. Thus, the finding that TFP growth is uncorrelated does not necessarily imply TFP shocks do not contribute to international comovement. It could be that correlated observed input growth is driven by the propagation of TFP shocks. To develop the full picture of the role of different types of shocks, correlated shocks, and international transmission, we perform model-based counterfactuals. The model features standard international transmission mechanisms. A positive foreign shock lowers the prices of intermediate inputs coming from that country, stimulating demand in countries and sectors that use those inputs in production. At the same time, a positive shock in a foreign country makes final goods supplied by that country cheaper, reducing demand for final goods produced by countries competing with it in final goods markets. Prior to simulating the model, we first structurally estimate two key elasticities – the final demand elasticity and the elasticity of substitution between intermediate inputs. Estimates of these elasticities vary substantially in the literature, and any assessment of the role of transmission vs. correlated shocks will be influenced by these parameters. Our estimates imply an elasticity of substitution between intermediate inputs that is not statistically different from 1. On the other hand, we obtain a range of estimates for the elasticity of substitution in final demand. Given the uncertainty in the appropriate value of this elasticity, our quantitative analysis uses two values, 1 and 2.75, reflecting our range of estimates.

We simulate the world economy's responses to shocks in two ways. The first is a static setting akin to the network literature (e.g. Acemoglu et al., 2012; Baqaee and Farhi, 2019). Static frameworks of shock propagation through a network write the change in real GDP as an inner product of the vector of sectoral shocks and the influence vector. We extend this approach to an international setting, and write the change in GDP of a single country as an inner product of the vector of shocks to all countries and sectors in the world and the country-specific influence vector that collects the elacticities of that country's GDP to every sectoral shock in the world. For the first time to our knowledge, we provide an analytical solution for the first-order approximation to this influence vector in a multi-country general equilibrium setting. This analytical solution expresses the influence matrix in terms of observables that can be measured, and structural elasticities.

The network propagation approach captures intra-temporal propagation, but shuts down dynamic factor accumulation responses to shocks. Our second set of exercises thus simulates the fully dynamic version of our model, in which sectoral capital and labor can respond to both foreign and domestic shocks, subject to adjustment costs. We show that the analytical influence matrix characterizes the impact response of GDP in all countries to shocks in the dynamic setting.

To focus on the distinction between technology and non-technology shocks, we simulate the model with only one type of shock at a time. It turns out that a model with only TFP shocks cannot generate almost any international comovement, whereas the model with only non-technology shocks can produce a correlation that is 60% of the correlation in the data. Thus, our second main finding is that non-technology shocks are much more successful at generating the observed comovement than technology shocks. This result is insensitive to the choice of elasticities.

To assess the role of trade and input linkages in the transmission of shocks, we perform several related exercises. First, we compute impulse responses to a hypothetical shock abroad on each country's GDP. We simulate three kinds of shocks: a 1% increase in U.S. productivity and the non-technology shocks, a 1% increase in those parameters in every other country in the world (rest of the world or ROW shock), and a 1% increase in these parameters in all countries simultaneously. The impulse responses point to positive comovement in response to shocks: real GDP in all countries increases following a shock to the U.S., though the effect is much more pronounced under a low substitution elasticity. In response to a 1% ROW shock, the real GDP in the mean country increases by 0.7% under the low elasticity of substitution, and by 0.2% under the high one, suggesting substantial responsiveness of countries to developments in the world economy. Using our simple accounting framework, we decompose the effects of the shocks in the third exercise in the the direct "influence" of own-country shocks and the transmission of other countries' shocks. We find that the direct effects are much more important than transmission, in particular for larger economies like the US.

We then simulate the model under the observed shocks, but in which every country is in autarky. This counterfactual reveals how much comovement would occur purely due to correlated shocks, and without any transmission of shocks through trade. However, when shocks are correlated and input linkages propagate sectoral shocks within a country, the choice of autarky production structure will matter. We report results for three autarky models. In the first, we shut down all input linkages and consider a value-added only model. In the second, we keep the domestic input coefficients exactly the same as in the data, and increase the value added share by the total cost share of imported inputs. This amounts to the assumption that if inputs were not imported from abroad, they would be produced as value added by the using sector. Hence, the importance of the domestic inputs is not changed. In the final case, we increase the domestic input coefficients at each input-output sector pair by the amount of observed foreign purchases. That is, if inputs weren't imported, they would be sourced from the corresponding domestic supplying sector such that the total input spending is unchanged.

The main result is that among the G7 countries autarky correlations in the value-added-only autarky model are actually quite a bit *higher* than the correlations under trade. Allowing for domestic input linkages lowers the autarky correlations substantially, bringing them more in line with the baseline trade model.

To understand this result, we write the difference in GDP comovement between the trade and autarky equilibria as a sum of two terms: the international transmission of shocks through trade linkages, and the changes in the weights assigned to the domestic shocks times the covariance of those shocks. The latter term captures the notion that the relative importance of domestic shocks in different sectors will change when going from autarky to trade, and the overall change in GDP comovement will depend on whether the fundamentally more or less correlated sectors are increasing in importance. It turns out that moving from trade to autarky increases the relative importance of sectors whose shocks are more correlated, producing the paradoxical outcome that autarky correlations are higher than the trade ones. These results reveal the unexpected role of input linkages in cross-border comovement – input linkages provide significant diversification away from the most internationally correlated sectors.

All in all, the quantitative importance of transmission depends both on the assumptions about the input-output structure and on the elasticity of substitution. We provide clear evidence of transmission of shocks through input-output networks, however, such transmission appears less important for GDP growth than the direct effects of correlated shocks. The input-output structure does play a very important role in regulating comovement. Absent the observed input-output structure, the directly measured correlated sectoral shocks would lead to international comovement substantially higher than the data. Here, there is not much distinction between amplified domestic input linkages or cross-border linkages – both play an important role in regulating international comovement.

Our paper contributes to the literature on international comovement. There is a small number of papers dedicated to documenting international correlations in productivity shocks and inputs (Imbs, 1999; Ambler, Cardia, and Zimmermann, 2004). Also related is the body of work that identifies technology and demand shocks in a VAR setting and examines their international propagation (e.g. Canova, 2005; Corsetti, Dedola, and Leduc, 2014; Levchenko and Pandalai-Nayar, 2018). Relative to these papers, we use sector-level data to provide novel estimates of both utilization-adjusted TFP and non-technology shocks, and expand the sample of countries. A large literature builds models in which fluctuations are driven by productivity shocks, and asks under what conditions those models can generate observed international comovement (see, among many others, Backus, Kehoe, and Kydland, 1992; Kose and Yi, 2006; Johnson, 2014). A smaller set of contributions adds non-technology shocks (Stockman and Tesar, 1995; Wen, 2007). In these analyses, productivity shocks are proxied by the Solow residual, and non-technology shocks are not typically measured based on data. Our quantitative assessment benefits from improved measurement of both types of shocks. Finally, our paper generalizes some of the insights in the literature studying the transmission of shocks through networks (see, among many others, Acemoglu et al., 2012; Baqaee, 2018; Baqaee and Farhi, 2019) to an international setting.

The rest of the paper is organized as follows. Section 2 lays out a basic GDP accounting framework and presents some decompositions underlying the sources of comovement. Section 3 introduces the dynamic multi-country, multi-sector model of production and trade necessary to back out non-technology shocks. The approach to measuring the shocks and estimating key elasticities is detailed in Section 4, together with the results. Section 5 uses the model to perform static counterfactuals and illustrate the role of the network for comovement. Dynamic counterfactuals are in Section 6. Section 7 concludes.

2 Accounting Framework

Let there be J sectors indexed by j and i, and N countries indexed by n, m, and k. Time is indexed by t. Gross output in sector j country n is:

$$Y_{njt} = Z_{njt}\left[\left(K_{njt}(\mathbf{Z},\boldsymbol{\xi})^{\alpha_j}L_{njt}(\mathbf{Z},\boldsymbol{\xi})^{1-\alpha_j}\right)^{\eta_j}X_{njt}^{1-\eta_j}\right]^{\gamma_j}. \tag{2.1}$$

Total output is a Cobb-Douglas aggregate of primary factor inputs K_{njt} and L_{njt} and materials inputs X_{njt}, with possibly non-constant returns to scale ($\gamma_j \neq 1$). The sector is directly affected by two shocks: a TFP shock Z_{njt}, and a non-TFP shock that shifts factor supply ξ_{njt}. The vectors $\mathbf{Z}$, $\boldsymbol{\xi}$ of length NJ collect all the TFP and non-TFP shocks in the world. Because the economy is interconnected through trade, output in every sector and country is in principle a function of all the worldwide shocks. The non-technology shocks ξ_{njt} can have either a literal interpretation as exogenous shifts in intra-temporal factor supply curves, or more broadly as business cycle shocks that are orthogonal to contemporaneous productivity, such as news shocks (e.g. Beaudry and Portier, 2006), or sentiment shocks (e.g. Angeletos and La'O, 2013). For parsimony, there is only a single non-technology shock ξ_{njt} that affects both capital and labor, though it does not need to move the two factors of production in the same way. When it comes to measurement, it will be important that K_{njt} and L_{njt} are true, utilization-adjusted inputs that may not be directly observable. The bundle of inputs X_{njt} can include foreign imported intermediates.

Define real GDP at time t, evaluated at base prices (prices at $t-1$) by:

$$Y_{nt} = \sum_{j=1}^{J}\left(P_{njt-1}Y_{njt}(\mathbf{Z},\boldsymbol{\xi}) - P^X_{njt-1}X_{njt}(\mathbf{Z},\boldsymbol{\xi})\right),$$

where P_{njt-1} is the gross output base price, and P^X_{njt-1} is the base price of inputs in that sector-country.

A first order approximation to the log change in GDP of country n can be written as:

$$d\ln Y_{nt} \approx \sum_m\sum_i s^Z_{mnit}d\ln Z_{mit} + \sum_m\sum_i s^\xi_{mnit}d\ln\xi_{mit}, \tag{2.2}$$

where $s^Z_{mnit} \equiv \left.\frac{\partial\ln Y_{nt}}{\partial\ln Z_{mit}}\right|_{\mathbf{Z_{t-1}},\boldsymbol{\xi_{t-1}}}$ and $s^\xi_{mnit} \equiv \left.\frac{\partial\ln Y_{nt}}{\partial\ln\xi_{mit}}\right|_{\mathbf{Z_{t-1}},\boldsymbol{\xi_{t-1}}}$ are the elements of the global influence matrix, that give the elasticity of the GDP of country n with respect to TFP and non-TFP shocks in sector i, country m. Notice that these elasticites are general equilibrium objects, and capture the full effect of a shock through direct and indirect input-output links and general equilibrium effects.

[1]

To highlight the sources of international GDP comovement, focus on the comovement driven by one type of shock (TFP without loss of generality). Real GDP growth can be written approximately as

$$d\ln Y_{nt} = \underbrace{\sum_j s^Z_{nnj} d\ln Z_{njt}}_{\mathcal{D}_n} + \underbrace{\sum_j s^Z_{mnj} d\ln Z_{mjt}}_{\mathcal{P}_n} + \underbrace{\sum_{n'\neq n,m} \sum_j s^Z_{n'nj} d\ln Z_{n'jt}}_{\mathcal{T}_n}. \tag{2.3}$$

This equation simply breaks out the double sum in (2.2) into the component due to country n's own shocks ($\mathcal{D}_n$), the component due to a particular trading partner's m shocks ($\mathcal{P}_n$), and the impact of "third" countries that are neither n nor m ($\mathcal{T}_n$).

Then, the GDP covariance between country n and country m is:

$$\begin{aligned} Cov(d\ln Y_{nt}, d\ln Y_{mt}) &= \underbrace{Cov(\mathcal{D}_n, \mathcal{D}_m)}_{\text{Shock Correlation}_{mn}} && (2.4)\\ &+ \underbrace{Cov(\mathcal{D}_n, \mathcal{P}_m) + Cov(\mathcal{P}_n, \mathcal{D}_m) + Cov(\mathcal{P}_n, \mathcal{P}_m)}_{\text{Bilateral Transmission}_{mn}} && (2.5)\\ &+ \underbrace{Cov(\mathcal{D}_n + \mathcal{P}_n + \mathcal{T}_n, \mathcal{T}_m) + Cov(\mathcal{T}_n, \mathcal{D}_m + \mathcal{P}_m)}_{\text{Multilateral Transmission}_{mn}} && (2.6) \end{aligned}$$

This equation underscores sources of international comovement. Economies might be correlated even in the absence of trade, if the underlying shocks themselves are correlated ($Cov(d\ln Z_{njt}, d\ln Z_{mit}) > 0$), especially in sectors influential in the two economies (large $s^Z_{nnj} s^Z_{mmi}$). This is captured by the first term:

$$Cov(\mathcal{D}_n, \mathcal{D}_m) = \sum_j \sum_i s^Z_{nnj} s^Z_{mmi} Cov(d\ln Z_{njt}, d\ln Z_{mit}).$$

Thus, a full account of international comovement would have to start with a reliable estimation of the shock processes hitting the economies.

The second term captures bilateral or direct transmission. If the GDP of country n has an elasticity with respect to the shocks occurring in country m ($s^Z_{mnit} > 0$), that would contribute to comovement as well. Taking one of the terms of the Bilateral Transmission component:

$$\begin{aligned} Cov(\mathcal{D}_n, \mathcal{P}_m) &= \sum_j \sum_i s^Z_{nnj} s^Z_{nmi} Cov(d\ln Z_{njt}, d\ln Z_{nit}) \\ &= \mathbf{s^{Z\prime}_{nn}} \mathbf{\Sigma^Z_n} \mathbf{s^Z_{nm}}, \end{aligned} \tag{2.7}$$

[1] The form of s^Z_{mnit} is known for some simple economies. For instance, if country n is in autarky, factors of production are supplied inelastically, and returns to scale are constant, $s^Z_{nnit} = P_{nit-1}Y_{nit-1}/P_{nt-1}Y_{nt-1}$ are the Domar weights (Hulten, 1978; Acemoglu et al., 2012), and $s^Z_{mnit} = 0 \ \forall m \neq n$. In general for an economy with international trade there will not be closed-form solutions for s^Z_{mnit}. We derive closed-form solutions in some special cases in Section 5.

where $\boldsymbol{\Sigma}_n^{\boldsymbol{Z}}$ is the $J \times J$ covariance matrix of shocks in country n, and $\mathbf{s}_{\mathbf{nm}}^{\mathbf{Z}}$ is the $J \times 1$ influence vector collecting the impact of shocks in n on GDP in m. This expression underscores that one source of comovement is that under trade, both country n and country m will be affected by shocks in n.

Finally, the Multilateral Transmission term collects all the other sources of comovement between n and m that do not come from shocks to either n or m, such as shocks in other countries.

We can now write the difference in covariances between autarky and trade as a sum of two terms:

$$\begin{aligned}\Delta Cov(d\ln Y_{nt}, d\ln Y_{mt}) \quad &= \quad \underbrace{\sum_j \sum_i \left(s_{nnj}^Z s_{mmi}^Z - s_{AUT,nj}^Z s_{AUT,mi}^Z\right) Cov(d\ln Z_{njt}, d\ln Z_{mit})}_{\Delta \text{Shock Correlation}_{mn}} \quad (2.8)\\ &+ \quad \text{Bilateral} \quad \text{Transmission}_{mn} + \text{Multilateral} \quad \text{Transmission}_{mn},\end{aligned}$$

where $s_{AUT,mi}^Z$ are the elements of the influence vectors in autarky. This expression shows that trade opening can affect GDP covariance in two ways. First, it can make countries sensitive to foreign shocks, as captured by the Bilateral Transmission$_{mn}$ and Multilateral Transmission$_{mn}$ terms. Second, and more subtly, opening to trade can re-weight sectors in the two economies either towards, or away, from sectors with more correlated fundamental shocks. This is captured by the first line of the equation above.

While the exposition above is focused on TFP shocks, the same line of reasoning applies to the non-TFP shocks ξ. To summarize, in order to provide an account of international comovement, we must (i) measure TFP and non-TFP shocks in order to understand their comovement properties; (ii) assess how sectoral composition (the distribution of s_{nnjt}'s) translates sectoral comovement of the primitive shocks into GDP comovement. Further, in order to understand the contribution of international trade to international comovement, we must (iii) capture not only the cross-border elements of the influence vectors (the s_{nmjt}'s), but also how going from autarky to trade changes the sectoral composition of the economy (the differences between s_{nnj} and $s_{AUT,nj}$ for both shocks).

3 Quantitative Framework

While the decomposition above is general and would apply in any model with trade, any attempt to measure the technology and non-technology shocks and assess the importance of correlated shocks and transmission will be conditional on a specific model framework, within which the elements of the influence matrix can be calculated. We now provide one such framework and use it to understand the role of correlated shocks and transmission through networks.

Preliminaries Each country n is populated by $\overline{L}_n$ households. Each household consumes the final consumption good available in country n and supplies labor and capital to firms. Trade is subject to iceberg costs τ_{mnj} to ship good j from country m to country n (throughout, we adopt the convention that the first subscript denotes source, and the second destination).

Our benchmark model assumes financial autarky. As highlighted by Heathcote and Perri (2002), models featuring financial autarky outperform complete and incomplete markets models when explaining business cycle comovement. We therefore assume that there are only goods flows across countries, and further, trade is balanced period by period.[2]

Households We assume that there is a continuum of workers in a representative household who share the same consumption. The problem for households is

$$\max_{\substack{\{m_{njt}\},\{N_{njt}\},\\ \{h_{nj}\},\{e_{njt}\},\{u_{njt}\}}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(c_{nt} - \sum_j \xi_{njt} N_{njt} G(h_{njt}, e_{njt}, u_{njt}) - \sum_j \Psi_N\left(N_{njt}\right)\right),$$

subject to

$$P_{nt}\left(c_{nt} + \sum_j I_{njt}\right) = \sum_j w_{njt} N_{njt} h_{njt} e_{njt} + \sum_j r_{njt} u_{njt} m_{njt} + \sum_j \pi_{njt}$$
$$m_{njt+1} = (1-\delta) m_{njt} + I_{njt} + \Psi_m\left(m_{njt}, m_{njt-1}\right)$$

where c_{nt} is per-capita consumption, N_{njt} is the number of workers employed in sector j, h_{njt} is the number of hours per worker, e_{njt} is the amount of effort per worker, u_{njt} is the capital utilization rate, I_{njt} is investment, and m_{njt} is the amount of machines.

Let $C_{nt} \equiv c_{nt} + \sum_j I_{njt}$ denote the final goods consumption, which is a Cobb-Douglas aggregate across sectors. The functional form and its associated price index are given by

$$C_{nt} = \prod_j C_{njt}^{\omega_{jn}}, \qquad P_{nt} = \prod_j \left(\frac{P_{njt}^c}{\omega_{jn}}\right)^{\omega_{jn}},$$

where C_{njt} is the consumption of sector j in country n, and P_{njt}^c is the consumption price index in sector j and country n. Within each sector, aggregation across source countries is Armington, and the sector price index is defined in a straightforward way:

$$C_{njt} = \left[\sum_m \vartheta_{mnj}^{\frac{1}{\rho}} C_{mnjt}^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \qquad P_{njt}^c = \left[\sum_m \vartheta_{mnj} {P_{mnjt}^c}^{1-\rho}\right]^{\frac{1}{1-\rho}},$$

where C_{mnjt} is consumption in n of sector j goods coming from country m, and P_{mnjt}^c is the price

[2]Alternatively we can incorporate deficits in a manner similar to Dekle, Eaton, and Kortum (2008), without much change in our results.

of C_{mnjt}.

Different from a standard international business cycle model, we allow households also choose efforts that change the effective amount labor supply and utilization rates that change the effective amount of capital supply. These margins capture the idea that utilization rates of factor inputs typically vary over the business cycle. Let $L_{njt} = e_{njt}h_{njt}N_{njt}$ denote the total labor supply and $K_{njt} = u_{njt}m_{njt}$ denote the total capital supply, both of which are sector-specific. While households can freely adjust hours, effort, and utilization rates within a period, the number of employed workers and machines in a sector is predetermined. Specifically, the number of workers in a particular sector has to be chosen in the previous period before observing current shocks. The adjustment costs of employment and capital are given by $\Psi_N(N_{njt})$ and $\Psi_m(m_{njt}, m_{njt-1})$.

Our framework implies that within a period, labor and capital supply are upward-sloping, which is similar to using the Greenwood, Hercowitz, and Huffman (1988) preferences for labor and a similar isoelastic formulation of the utilization cost of capital (e.g. Christiano, Motto, and Rostagno, 2014). The GHH preference mutes the interest rate effects and income effects on the choice of hours, effort, and utilization rates, which helps to study the properties of the static equilibrium where the number of machines and employees are treated as exogenous variables.

This specification also helps to infer the factor utilization shocks ξ_{njt}, as will become clear below. As mentioned earlier, these shocks can have either a literal interpretation as exogenous shifts in intra-temporal factor supply curves, or more broadly as business cycle shocks that are orthogonal to contemporaneous productivity.

Static Decision Notice that the households' intra-temporal optimization problem leads to

$$h_{njt}G_h(t) = e_{njt}G_e(t).$$

This condition implies that the unobserved choice of effort is a function of the observed choice of hours. This property facilitates the estimation of the utilization-adjusted TFP process in Section 4. A similar expression can be derived for the relationship between the optimal choice of unobserved capital utilization and the optimal choice of hours:

$$\frac{h_{njt}G_h(t)}{u_{njt}G_u(t)} = \frac{w_{njt}N_{njt}h_{njt}e_{njt}}{r_{njt}u_{njt}m_{njt}}.$$

From the firms' problem, we will show that the right-hand side of the equation above is equal to the ratio of output elasticities $\alpha_j/(1-\alpha_j)$, which is a constant. As a result, the utilization rate is also a function of hours worked. These conditions capture the idea that the variations of flexible inputs tend to move jointly in the same direction.

In order to place a structural interpretation for the coefficients estimated in next section, we assume the following functional form for $G(.)$:

$$G\left(h_{njt}, e_{njt}, u_{njt}\right) = \left(\frac{h_{njt}}{\psi^h}\right)^{\psi_h} + \left(\frac{e_{njt}}{\psi^e}\right)^{\psi_e} + \left(\frac{u_{njt}}{\psi^u}\right)^{\psi_u}. \tag{3.1}$$

The responsiveness of effort and utilization rates depend on elasticities, which can be partially identified in the next section. We also define a composite elasticity term

$$\tilde{\psi} = \frac{1}{\psi^h} + \frac{1}{\psi^e}$$

for notational convenience later.

Dynamic Decision Households also face intertemporal decisions determining capital accumulation and labor allocation over time. The first-order condition with respect to capital accumulation is

$$U'_{njt} = \beta \mathbb{E}_t \left[U'_{njt+1} \frac{r_{njt+1}}{P_{nt+1}} u_{njt+1}\right], \tag{3.2}$$

which is similar to the standard Euler equation but is sector specific and adjusted by the utilization rate.

As mentioned earlier, employment is chosen one period ahead before observing current shocks. The optimality condition for N_{njt} is

$$\mathbb{E}_{t-1}\left[u'_{njt}\left(\frac{w_{njt}}{P_{nt}} h_{njt} e_{njt} - \xi_{njt} G(h_{njt}, e_{njt}, u_{njt}) - \Psi'_N(N_{jt})\right)\right] = 0. \tag{3.3}$$

Firms A representative firm in sector j in country n operates a CRS production function

$$y_{njt} = Z_{njt}\Theta_{njt}\left(k_{njt}^{\alpha_j} \ell_{njt}^{1-\alpha_j}\right)^{\eta_j} x_{njt}^{1-\eta_j}, \tag{3.4}$$

where the total factor productivity $Z_{njt}\Theta_{njt}$ is taken as given. The intermediate input usage x_{njt} is an aggregate of inputs from potentially all countries and sectors:

$$x_{njt} \equiv \left(\sum_{m,i} \mu_{mi,nj}^{\frac{1}{\varepsilon}} x_{mi,njt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where $x_{mi,njt}$ is the usage of inputs coming from sector i in country m in production of sector j in country n, and $\mu_{mi,nj}$ is the input coefficient.

The total factor productivity consists of two parts: the exogenous shocks Z_{njt} and the endogenous part Θ_{njt}. The latter is assumed to be

$$\Theta_{njt} = \left(\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right)^{\gamma_j - 1}, \tag{3.5}$$

where γ_j controls possible congestion or agglomeration effects. As a result, the sectoral aggregate production function is then

$$Y_{njt} = Z_{njt} \left[\left(K_{njt}^{\alpha_j} L_{njt}^{1-\alpha_j} \right)^{\eta_j} X_{njt}^{1-\eta_j} \right]^{\gamma}, \tag{3.6}$$

which is the same as in equation (2.1).

Let P_{njt} denote the price of output produced by sector j in country n,[3] and let $p_{mi,njt}$ be the price paid in sector n, j for inputs from m, i. Due to the competitiveness assumption, the prices "at the factory gate" and the price at the time of consumption or intermediate usage are related by:

$$p_{mi,njt} = P_{mnit}^{c} = \tau_{mni} P_{mit}.$$

In a competitive market, primary factors and inputs receive compensation proportional to their share in total input spending. This implies:

$$\begin{aligned} r_{njt} K_{njt} &= \alpha_j \eta_j P_{njt} Y_{njt} \\ w_{njt} L_{njt} &= (1 - \alpha_j) \eta_j P_{njt} Y_{njt} \\ p_{mi,njt} X_{mi,njt} &= \pi_{mi,njt}^{x} (1 - \eta_j) P_{njt} Y_{njt}, \end{aligned} \tag{3.7}$$

where $\pi_{mi,njt}^{x}$ is the share of intermediates from country m sector i in total intermediate spending by n, j, given by:

$$\pi_{mi,njt}^{x} = \frac{\mu_{mi,nj} \left(\tau_{mni} P_{mit} \right)^{1-\varepsilon}}{\sum_{k,l} \mu_{kl,nj} \left(\tau_{knl} P_{klt} \right)^{1-\varepsilon}}.$$

The sectoral level price needs to satisfy

$$P_{njt} = Z_{njt}^{-\frac{1}{\gamma_j}} Y_{njt}^{\frac{1-\gamma_j}{\gamma_j}} \left(\frac{r_{njt}}{\alpha_j \eta_j} \right)^{\alpha_j \eta_j} \left(\frac{w_{njt}}{(1-\alpha_j) \eta_j} \right)^{(1-\alpha_j)\eta_j} \left(\frac{\sum_{m,i} \mu_{mi,nj} p_{mi,njt}^{1-\varepsilon}}{1-\eta_j} \right)^{\frac{1-\eta_j}{1-\varepsilon}}.$$

Equilibrium An intertemporal equilibrium in this economy is a set of goods and factor prices $\{P_{njt}, w_{njt}, r_{njt}\}$, factor allocations $\{L_{njt}, K_{njt}\}$, and goods allocations $\{Y_{njt}\}$, $\{C_{mnjt}\}$, and $\{X_{mi,njt}\}$ for all countries and sectors such that (i) households maximize utility; (ii) firms maximize profits;

[3] Note this is not the same as the ideal price index P_{njt}^{c} of sector j final consumption in n, which aggregates imports from the other countries.

and (iii) all markets clear.

Total expenditure on exports from n to m in sector j is the sum of final consumption expenditure and expenditure on intermediates by all sectors i in m:

$$EX_{nmjt} \quad = \quad \pi^c_{nmjt}\omega_{jm}P_{mt}C_{mt} + \sum_i \pi^x_{nj,mit}\left(1-\eta_j\right)P_{mit}Y_{mit},$$

where $\pi^c_{nmjt} \equiv \frac{\vartheta_{nmj}(\tau_{nmj}P_{njt})^{1-\rho}}{\left(P^c_{mjt}\right)^{1-\rho}}$ is the share of country n in the total final consumption expenditure of sector j, country m, and thus $\pi^c_{nmjt}\omega_{jm}P_{mt}C_{mt}$ is the total final consumption expenditure in m on j sector goods from n, and $\sum_i \pi^x_{nj,mit}\left(1-\eta_j\right)P_{mit}Y_{mit}$ is the intermediate spending. Then, total spending on output produced by country n, sector j is:

$$\Upsilon_{njt} \quad = \quad \sum_m \left[\pi^c_{nmjt}\omega_{jm}P_{mt}C_{mt} + \sum_i \pi^x_{nj,mit}\left(1-\eta_j\right)\Upsilon_{mit}\right],$$

where we defined $\Upsilon_{njt} \equiv P_{njt}Y_{njt}$ as the total revenue in sector j, country m, which will be expositionally convenient.

3.1 Analytical Influence Matrix

Within this framework, we can derive an analytical expression for the global influence matrix. In general, closed-form expressions for influence vectors cannot be obtained in multi-country multi-sector elastic factor and variable returns to scale models such as the one here. We derive a closed-form solution for a first order approximation of the influence vector in our model. In Appendix C, we assess the fit of this first-order approximation relative to the full model and illustrate that the first-order approximation is reasonable here.

Let $\mathbf{p}_t$ and $\mathbf{y}_t$ be vectors of length NJ containing sector-country prices and quantities at time t. Linearizing the market clearing conditions above,we obtain

$$\mathbf{p}_t + \mathbf{y}_t = \underbrace{\Big(\mathbf{\Psi}^c + \mathbf{\Psi}^x\Big)(\mathbf{p}_t + \mathbf{y}_t)}_{\text{destination country output variation}} \quad + \tag{3.8}$$

$$\underbrace{(1-\rho)\Big(\text{diag}(\mathbf{\Psi}^c\mathbf{\Pi}^c\mathbf{1}) - \mathbf{\Psi}^c\mathbf{\Pi}^c\Big)\mathbf{p}_t}_{\text{consumption goods relative price variation}} + \underbrace{(1-\epsilon)\Big(\text{diag}(\mathbf{\Psi}^x\mathbf{\Pi}^x\mathbf{1}) - \mathbf{\Psi}^x\mathbf{\Pi}^x\Big)\mathbf{p}_t}_{\text{intermediate goods relative price variation}} \tag{3.9}$$

where $\mathbf{\Pi}^x$ and $\mathbf{\Pi}^c$ are matrices containing the steady state import shares of intermediate and final goods,[4] $\mathbf{\Psi}^x$ and $\mathbf{\Psi}^c$ are matrices containing the steady state export shares of intermediate and final

[4] A typical element of $\Pi^x_{(n-1)N+j,(m-1)N+i}$ is the share of intermediate goods imported from sector j country n to

goods.[5]

Then, equation 3.8 implies that we can express the vector of country-sector price changes in terms of output changes and known parameters: $\mathbf{p}_t = \mathbf{A}\mathbf{y}_t$.

Let further the labor output elasticity, adjusted for utilization and returns-to-scale be $\mathbf{F}^h$ and the intermediates output elasticity be $\mathbf{F}^x$. Combining equation 3.8 with linearized versions of the production function 2.1, labor market clearing and the demand for intermediate goods, the analytical influence matrix is:

$$\mathbf{y}_t = \left\{ \mathbf{I} - (\psi_h^{-1}\mathbf{F}^h + \mathbf{F}^x)(\mathbf{I} + \mathbf{A}) + (\psi_h^{-1}\mathbf{F}^h\mathbf{\Pi}^c + \mathbf{F}^x\mathbf{\Pi}^x)\mathbf{A} \right\}^{-1} (\mathbf{z}_t + \psi_h^{-1}\mathbf{F}^h\boldsymbol{\xi}_t) \tag{3.10}$$

where Π_c and Π_x are matrices collecting the steady state consumption shares π^c_{mnj}s and intermediate input shares π^x_{minj} respectively.

The influence matrix encodes the general equilibrium response to sectoral output in a country to shocks in any sector-country, taking into account the full model structure and all direct and indirect links between the countries and sectors. This is particularly evident in equation xx, which pins down the matrix $\mathbf{A}$ relating changes in quantities to changes in prices. The first term contains the response of GDP that arises from output changes in every country and sector in response to a shock in a sector-country. The second term contains the relative price changes of final goods and the final term the relative price changes of intermediate inputs.

Equation 3.10 illustrates that all we need to understand the GDP elasticity to various sector-country shocks in this quantitative framework are measures of steady state final goods consumption and production shares, as well as model elasticites. Since every country's output also responds to shocks in every other country and sector, to understand comovement we further need information about the covariance structure of the shocks.

Notice that this influence vector contains the full response of GDP in all countries to measured shocks if our model were treated as static (fixing the capital stock and the number of employees in each sector). In the DSGE framework, it corresponds to the *impact* response of the GDP of all countries in response to a set of shocks. The response of GDP in later periods will depend on the persistence of the shock and the capital and labor accumulation decisions, which are not encoded in this vector. Unfortunately, it is not possible to derive a dynamic influence vector even with a linear approximation in this framework.

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sector i country m, and a typical element of $\Pi^c_{(n-1)N+j,(m-1)N+i}$ is the share of final goods imported from sector j country n to country m which is independent of i.

[5] A typical element of $\mathbf{\Psi}^x$ is $\Psi^x_{(n-1)N+j,(m-1)N+i} = \Pi^x_{(n-1)N+j,(m-1)N+i}\frac{(1-\eta_i)P_{mi}Y_{mi}}{P_nC_n}$, and a typical element of $\mathbf{\Psi}^c_{(n-1)N+j,(m-1)N+i}$ is $\Psi^c_{(n-1)N+j,(m-1)N+i} = \Pi^c_{(n-1)N+j,(m-1)N+i}\frac{\eta_i P_{mi}Y_{mi}}{P_nC_n}$.

GDP Movement To see the impact of a sector-country shock on another country's GDP, we need to aggregate the changes of sector-country quantity change and adjust the associated price change.[6] Define the Domar weight matrix as $\mathbf{D}$ where $D_{n,j} = \frac{P_{nj}Y_{nj}}{P_n D_n}$ is the Domar weight for secotr j in country n. Also define the vector of value added ratio $\boldsymbol{\eta}$ with the j-th element being η_j.

The real output changes are given by

$$\mathbf{O}_t = \left[\mathbf{D}\boldsymbol{\eta} \circ \mathbf{I} + (\mathbf{D}(\mathbf{1} - \boldsymbol{\eta}) \circ \mathbf{I})(\mathbf{I} - \boldsymbol{\Pi}^x)\mathbf{A}\right]\mathbf{y}_t \tag{3.11}$$

where $\circ$ stands for the transposed Khatri-Rao product. The first term in equation (3.11) captures the changes in quantity which is aggregated according to Domar weight. The second term capture the relative changes between the prices of domestically produced goods and the prices of imported intermediate goods.

4 Measurement

4.1 Estimating TFP Shocks

Unobserved Factor Utilization As emphasized by BFK, measuring TFP innovations is difficult because the intensity with which factors are used in production varies over the business cycle, and cannot be directly observed by the econometrician. As unobserved factor utilization will respond to TFP innovations, it is especially important to account for it in estimation, otherwise factor usage will appear in estimated TFP. BFK develop an approach to control for unobserved factor utilization which leads to a TFP series in the United States that has very different properties than the Solow residual. Our approach in this paper is similar in spirit.

In the model above, the true factor inputs were $K_{njt} \equiv u_{njt} m_{njt}$ and $L_{njt} \equiv e_{njt} h_{njt} N_{njt}$. The true capital input is the product of the quantity of capital input ("machines") m_{njt} that can be measured in the data, and capital utilization u_{njt} that is not directly observable. Similarly, the true labor input is the product of the number of workers N_{njt}, hours per worker h_{njt}, and labor effort e_{njt}. While N_{njt} and h_{njt} can be obtained from existing datasets, e_{njt} is unobservable.

Log-differencing (3.6), and writing input usage breaking up the observed and the unobserved com-

[6] Using steady state's fixed price, the log-deviation of country n's real GDP in period t can be expressed as

$$O_{nt} = \sum_j \left(\frac{\eta_j P_{nj} Y_{nj}}{P_n D_n} y_{njt} - \frac{(1-\eta_j) P_{nj} Y_{nj}}{P_n D_n} \left(p_{njt} - \sum_{k,l} \Pi_{kl,nj} p_{k\ell t} \right) \right).$$

ponents yields:

$$\begin{aligned} d\ln Y_{njt} &= \gamma_j \underbrace{\left(\alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln\left(h_{njt}N_{njt}\right) + (1-\eta_j)d\ln X_{njt}\right)}_{\text{Observed Inputs}} \\ &\quad + \gamma_j \underbrace{\left(\alpha_j\eta_j d\ln u_{njt} + (1-\alpha_j)\eta_j d\ln e_{njt}\right) + d\ln Z_{njt}}_{\text{Unobserved Inputs}}. \end{aligned} \tag{4.1}$$

To derive an estimating equation for the true TFP process, we use the simple general equilibrium framework above. We begin by considering the profit maximization problem of the firm with the production function given by equation 3.4. The first-order conditions of this problem imply that the cost shares of the composite labor and capital inputs are $(1-\alpha_j)\,\eta_j$ and $\alpha_j\eta_j$ respectively. Given a wage w_{njt} or a rental rate r_{njt}, the firm is indifferent between increasing effort/hours or employees holding other inputs constant, and similarly between utilization and machines. However, we assumed that the household faces increasing disutility from supplying more on any individual margin (effort, hours, or utilization of capital). In our full model in section 3, sectoral allocations of N_{mjt} and m_{mjt} are predetermined within a period. The market-clearing wages and rental rates therefore pin down the equilibrium choices of effort, hours, and utilization in a period. This further implies that the household's optimal choices of unobserved utilization and effort will be proportional to its choice of observed hours. The intra-temporal first-order conditions for the household therefore allow us express unobserved effort and capital utilization as a log-linear function of observed hours:

$$\gamma_j\left(\alpha_j\eta_j d\ln u_{njt} + (1-\alpha_j)\eta_j d\ln e_{njt}\right) = \zeta_j d\ln h_{njt}, \tag{4.2}$$

where $\zeta_j = \eta_j\left((1-\alpha_j)\,\frac{\psi^h}{\psi^u} + \alpha_j\frac{\psi^h}{\psi^e}\right)$.

Notice that this structure is similar to assuming that firms face an upward-sloping cost schedule for increasing effort, hours, or utilization holding other factors constant, which is the model in BFK. While our framework is somewhat less general, an advantage is that we do not have to assume ad-hoc convex cost functions for firm choices.

Plugging these relationships into (4.1) yields the following estimating equation:

$$\begin{aligned} d\ln Y_{njt} &= \delta_j^1\left(\alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln\left(h_{njt}N_{njt}\right) + (1-\eta_j)d\ln X_{njt}\right) \\ &\quad + \delta_j^2 d\ln h_{njt} + \delta_{nj} + d\ln Z_{njt}, \end{aligned} \tag{4.3}$$

where we also added country×sector fixed effects δ_{nj} to allow for country-sector specific trend output growth rates. The estimation proceeds to regress real output growth on the growth of the composite observed input bundle and the change in hours. The coefficient δ_1^j is clearly an estimate of returns-to-scale γ_j. In Section 5 we provide specific functional forms under which we can provide a structural interpretation for the constant $\delta_j^2 = \zeta_j$.[7]

We use a strategy similar to BFK when implementing this estimation. First, input usage will move with TFP shocks $d\ln Z_{njt}$, and thus the regressors in this equation are correlated with the residual. To overcome this endogeneity problem, we use potentially three instruments. The first is oil shocks, defined as the difference between the log oil price and the maximum log oil price in the preceding four quarters. This oil price shock is either zero, or is positive when this difference is positive, reflecting the notion that oil prices have an asymmetric effect on output. The annualized oil shock is the sum over the four quarters of the preceding year. The second instrument is the growth rate in real government defense spending, lagged by one year. Finally, the third instrument is the foreign monetary policy shock interacted with the exchange rate regime. This instrument follows di Giovanni and Shambaugh (2008) and di Giovanni, McCrary, and von Wachter (2009), who show that major country interest rates have a significant effect on countries' output when they peg their currency to that major country. The assumption in specifications that use this instrument is that for many countries, interest rates in the US, Germany, or the UK are exogenous.

In practice, we estimate two separate sets of regressions. The first is confined to only the G7 countries, and uses only the first two instruments (oil and military spending). This tends to lead to the strongest instruments and most precisely estimated coefficients. Since these are the major world economies, the foreign interest rate instrument is not appropriate here. Second, we estimate this equation on the full sample of countries excluding the "base interest rate" countries of the US, Germany, and the UK, in which case we use all three instruments.

Finally, following BFK, to reduce the number of parameters to be estimated, we restrict δ_j^2 to take only three values, according to a broad grouping of sectors: durable manufacturing, non-durable manufacturing, and all others.

Conditional on these estimates and the log changes in the observed inputs, we obtain the TFP shocks $d\ln Z_{njt}$ as residuals. We use the estimate of ζ_j in two places, as we need it to construct the $d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j}X_{njt}^{1-\eta_j}\right]$ term:

$$d\ln\left[\left(K_{njt}^{\alpha_j}L_{njt}^{1-\alpha_j}\right)^{\eta_j}X_{njt}^{1-\eta_j}\right] \quad = \quad d\ln\left(m_{njt}^{\alpha_j\eta_j}N_{njt}^{(1-\alpha_j)\eta_j}h_{njt}^{(1-\alpha_j)\eta_j+\frac{\zeta_j}{\gamma_j}}X_{njt}^{1-\eta_j}\right),$$

where we substituted for unobserved inputs using (4.2).

[7]The setting in BFK implies the same reduced-form estimating equation, derived under different assumptions. The structural interpretation of the estimated parameters differs slightly from BFK in our framework, but we can still recover estimates of returns-to-scale and adjust for unobserved utilization.

4.2 Extracting Non-Technology Shocks

Given data on gross revenues Υ_{njt} and its deflators P_{njt}, we have estimates of real output Y_{njt} (indeed, these are the same data needed to estimate TFP). Denote by a "$\widehat{\ }$" the gross change in a variable: $\widehat{x}_{t+1} \equiv x_{t+1}/x_t$. Then we can write the gross change in real output as:

$$\widehat{Y}_{njt+1} \quad = \quad \widehat{Z}_{njt+1} \left(\left(\widehat{K}_{njt+1}^{\alpha_j} \widehat{L}_{njt+1}^{1-\alpha_j} \right)^{\eta_j} \widehat{X}_{njt+1}^{1-\eta_j} \right)^{\gamma_j}.$$

Plugging the gross proportional change versions of equation (3.7) and the household's optimal choices of variable factor utilization, we obtain the following expression:

$$\widehat{Y}_{njt+1} \quad = \quad \widehat{Z}_{njt+1} \left(\left(\widehat{m}_{njt+1}^{\alpha_j} \widehat{N}_{njt+1}^{\left(1-\tilde{\psi}\right)(1-\alpha_j)-\frac{\alpha_j}{\psi^u}} \left(\frac{\widehat{P}_{njt+1}\widehat{Y}_{njt+1}}{\widehat{P}_{nt+1}} \right)^{\tilde{\psi}(1-\alpha_j)+\frac{\alpha_j}{\psi^u}} \right)^{\eta_j} \widehat{X}_{njt+1}^{1-\eta_j} \right)^{\gamma_j} \widehat{\xi}_{njt+1}^{\left(-\tilde{\psi}(1-\alpha_j)-\frac{\alpha_j}{\psi^u}\right)\eta_j\gamma_j} \tag{4.4}$$

Our dataset, discussed below, has the information on all the elements of equation (4.4) required to back out the composite factor supply shock $\widehat{\xi}_{njt+1}$ except for the consumption price index $\widehat{P}_{n,t+1}$. That is, we know real output growth $\widehat{Y}_{njt+1}$, real input growth $\widehat{X}_{njt+1}$, TFP growth $\widehat{Z}_{njt+1}$, as well as the changes in the price indices $\widehat{P}_{njt+1}$. If we knew $\widehat{P}_{nt+1}$, we could back out $\widehat{\xi}_{njt+1}$.

We rely on the model structure and the observed final expenditure shares to compute the model-implied $\widehat{P}_{nt+1}$. Standard steps yield the following expressions for the changes in price indices:

$$\widehat{P}_{nt+1} \quad = \quad \prod_j \left(\widehat{P}_{njt+1}^c \right)^{\omega_{jn}} \tag{4.5}$$

$$\widehat{P}_{njt+1}^c \quad = \quad \left[\sum_m \widehat{P}_{mjt+1}^{1-\rho} \pi_{mnjt}^c \right]^{\frac{1}{1-\rho}}. \tag{4.6}$$

Since we know the gross output price indices for each country and sector $\widehat{P}_{mjt+1}$, and the final consumption shares of each source country in each destination and sector π_{mnjt}^c and ω_{jn}, we can simply construct $\widehat{P}_{nt+1}$ directly.[8]

4.3 Data

The data requirements for estimating equation (4.3) is growth of real output and real inputs for a panel of countries, sectors, and years. The dataset with the broadest coverage of this information is

[8]The construction of $\widehat{P}_{nt+1}$ requires information on ρ. We structurally estimate ρ using our model, and provide a detailed discussion of the estimating equation and instruments in Appendix B.

KLEMS 2009 (O'Mahony and Timmer, 2009).[9] This database contains gross output, value added, labor and capital inputs, as well as output and input deflators. In a limited number of instances, we supplemented the information available in KLEMS with data from the WIOD Socioeconomic Accounts, which contains similar variables. After data quality checking and cleaning, we retain a sample of 30 countries, listed in Appendix Table A1. The database covers all sectors of the economy at a level slightly more aggregated than the 2-digit ISIC revision 3, yielding, after harmonization, 30 sectors listed in Appendix Table A2. In the best cases we have 28 years of data, 1970-2007, although the panel is not balanced and many emerging countries do not appear in the data until the mid-1990s.

The oil price series is the West Texas Intermediate, obtained from the St. Louis Fed's FRED database. We have also alternatively used the Brent Crude oil price, obtained from the same source. Military expenditure comes from the Stockholm International Peace Research Institute (SIPRI). The exchange rate regime classification along with information on the base country comes from Shambaugh (2004), updated in 2015. Finally, base country interest rates are proxied by the Money Market interest rates in these economies, and obtained from the IMF International Financial Statistics.

The extraction of the non-technology shocks and the quantitative analysis require additional information on the input linkages at the country-sector-pair level, as well as on final goods trade. This information comes from the 2013 WIOD database (Timmer et al., 2015), which contains the global input-output matrix.

4.4 Empirical Results

TFP Estimation Table 1 reports the results of estimating equation (4.3). The returns to scale parameters vary from about 0.7 to 0.9 in durable manufacturing, from 0.3 to 1 in non-durable manufacturing, and from 0.1 to nearly 2 in the quite heterogeneous non-manufacturing sector. Thus, the estimates show departures from constant returns to scale in a number of industries, consistent with existing evidence. The coefficient on hours per worker ($d \ln h_{njt}$) is significantly different from zero in two out of three industry groups, indicating that adjusting for unobserved utilization is important in the manufacturing industries.

Appendix Table A3 provides more detailed results for all industries within each of these three broad groups. Appendix Figure A1 also plots the estimated TFP series against the Solow residual for all the countries in the sample.

[9] This is not the latest vintage of KLEMS, as there is a version released in 2016. Unfortunately, however, the 2016 version has a shorter available time series, as the data start in 1995, and also has many fewer countries. A consistent concordance between the two vintages is challenging without substantial aggregation.

Table 1: Summary of Production Function Parameter Estimates

Industry Group	Median Returns to Scale	Utilization Adjustment
Durables	0.771	2.939
	[0.701,0.895]	(1.767)
Non-durable manufacturing	0.806	1.419
	[0.289,0.988]	(0.389)
Non-durable non-manufacturing	1.221	0.245
	[0.121,1.863]	(0.649)

Notes: This table reports the range of estimates of γ_j in the three broad groups of sectors, and the estimates of ζ_j along with their standard errors in parentheses.

Non-Technology Shocks Having estimated these production function parameters and TFP shocks, we simply back out the implied non-technology shocks using our data and equation (4.4). Appendix Figure A2 plots these shocks against our estimated TFP shocks for all countries.

Cross-Country Correlations With these estimates in hand, we are ready to examine cross-country correlations. The estimates of the TFP shocks alone deliver some insights about the direct effects of these shocks relative to the Solow residual (the traditional measure of TFP). We present results for two subsamples: the G7 countries and the full sample. The G7 countries have less variation among them, making patterns easier to detect. In addition, the production function coefficient estimates are most reliable for the G7 sample, and we use them as the baseline coefficients to be applied to all other countries, implying that TFP and inputs in other countries are likely measured with greater error.

In the first instance, we are interested in the proximate drivers of comovement between countries, and in particular whether aggregate comovement occurs because of correlated TFP or inputs. Appendix A shows that GDP growth can be written a sum of two components:

$$d\ln Y_{nt} \quad \approx \quad d\ln Z_{nt} + d\ln \mathcal{I}_{nt}, \tag{4.7}$$

where aggregate TFP is denoted by:

$$d\ln Z_{nt} = \sum_{j=1}^{J} w^D_{njt-1} d\ln Z_{njt}, \tag{4.8}$$

and $d\ln \mathcal{I}_{nt}$ is the log change in the scale-adjusted primary factor inputs (see equation A.4). According to (4.8), aggregate TFP growth is thus a weighted average of sectoral TFP growth rates, with w^D_{njt-1} being the Domar weights.

We begin by presenting in Table 2 the basic summary statistics for the elements of the GDP decomposition in equation (4.7). While the non-technology shocks do not appear in this decomposition, these results are useful for highlighting the role of the TFP shocks and comparing them to the Solow residual. The top panel reports the correlations among the G7 countries. The average correlation of real GDP growth among these countries is 0.38. The second line summarizes correlations of the TFP shocks. Those are on average zero, if not negative. By contrast, input growth is positively correlated, with a 0.24-0.25 average. We then correlate the components of $d\ln\mathcal{I}_{nt}$ in equation (A.4) separately. The primary inputs and the scale effect term are both positively correlated across countries, with an order of magnitude that is similar to the correlation in the overall $d\ln\mathcal{I}_{nt}$.

Appendix A also shows that the Solow residual can be written as a sum of the aggregate TFP growth and the aggregated variable utilization change $d\ln\mathcal{U}_{nt}$:

$$d\ln S_{nt} = d\ln Z_{nt} + d\ln\mathcal{U}_{nt}, \tag{4.9}$$

with the expression for $d\ln\mathcal{U}_{nt}$ provided in (A.7).

Thus, it is an empirical question to what degree correlations in the Solow residual reflect true technology shock correlation as opposed to endogenous input adjustments. Table 2 shows that the Solow residual has an average correlation of 0.15 in this sample of countries. If Solow residual was taken to be a measure of TFP shocks, we would have concluded that TFP is positively correlated in this set of countries. As we can see, this conclusion would be misleading. Indeed, the correlation in the utilization term $\mathcal{U}_{nt}$, which is the difference between the TFP shock $d\ln Z_{nt}$ and the Solow residual, accounts for for the entirety of the correlation in the Solo residual, on average. This indicates that the correlation in the Solow residual is in fact driven by unobserved input utilization and scale adjustments. The top panel of Figure 1 depicts the kernel densities of the correlations of real GDP, TFP, and inputs. There is a clear hierarchy, with the real GDP being most correlated, and the TFP being least correlated and centered on zero.

The bottom panel of Table 2 repeats the exercise in the full sample of countries. The basic message is the same as for the G7 but quantitatively the picture is not as stark and the variation is greater. It is still the case that $d\ln Z_{nt}$ has a very low average correlation, with the mean and median of 0.014 and 0.043, respectively. It is also still the case that the inputs $d\ln I_{nt}$ have greater correlation, and that their correlation is on average about half of the average real GDP correlation. The Solow residuals are also more correlated than $d\ln Z_{nt}$, and part of the difference is accounted for by the fact that the unobserved inputs are positively correlated. The bottom panel of Figure 1 displays the kernel densities of the correlations in the full sample.

To summarize, real GDP growth is significantly positively correlated in our sample of countries,

Table 2: Correlations Summary Statistics

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
$d\ln Y_{nt}$	0.396	0.378	0.252	0.549
$d\ln Z_{nt}$	-0.014	0.015	-0.199	0.200
$d\ln I_{nt}$	0.248	0.235	0.157	0.382
$Primary\ inputs$	0.264	0.258	0.153	0.352
$Scale\ effect$	0.172	0.152	0.122	0.262
$d\ln S_{nt}$	0.151	0.144	0.060	0.314
$d\ln U_{nt}$	0.125	0.165	-0.014	0.277
	All countries (N. obs. = 406)			
$d\ln Y_{nt}$	0.175	0.201	-0.038	0.442
$d\ln Z_{nt}$	0.011	0.032	-0.197	0.230
$d\ln I_{nt}$	0.089	0.098	-0.121	0.330
$Primary\ inputs$	0.112	0.132	-0.092	0.314
$Scale\ effect$	0.092	0.091	-0.083	0.285
$d\ln S_{nt}$	0.054	0.072	-0.160	0.302
$d\ln U_{nt}$	0.036	0.055	-0.165	0.238

Notes: This table presents the summary statistics of the correlations in the sample of G7 countries (top panel) and full sample (bottom panel). Variable definitions and sources are described in detail in the text.

especially in the G7. TFP growth adjusted for utilization has an order of magnitude lower average correlation than GDP growth. Indeed, average TFP correlation is essentially zero. By contrast, correlations in input growth have the same order of magnitude as real GDP correlations. Finally, using Solow residuals as a proxy for TFP growth can be quite misleading. In our sample of countries, it would lead us to conclude that productivity growth is strongly positively correlated across countries, whereas in fact correlation in the Solow residuals appear to be driven mostly by correlation in the unobserved inputs.

This is of course only an accounting decomposition. Input usage will respond to TFP shocks at home and abroad. Since the growth in inputs has not been cleaned of the impact of technology shocks, it cannot be thought of as driven exclusively by non-technology shocks. We next turn to assessing the unconditional Domar-weighted correlation of non-technology shocks across countries as we did for TFP shocks. Then, in section 5 we use our full model and the decompositions outlined in Section 2 to perform a number of exercises aimed at understanding the full role of these shocks in international comovement.

Patterns in Non-Technology Shocks Across Countries Unlike the decomposition of GDP growth into TFP and inputs in (4.7), there is no decomposition that isolates the non-technology shocks $\widehat{\xi}_{njt+1}$ as an additive component in the GDP growth rate. Nonetheless, to provide a simple illustration of the correlations of $\widehat{\xi}_{njt+1}$ across countries, we construct a Domar-weighted non-technology shock, to parallel the Domar-weighted TFP shock in (4.8):

$$d \ln \xi_{nt} = \sum_{j=1}^{J} w_{njt-1}^{D} d \ln \xi_{njt}. \tag{4.10}$$

Table 3 reports the correlations in $d \ln \xi_{nt}$ among the G7 and in the full sample. As was evident from equation 4.4, the values of the ξ shocks depend on several model elasticities. We calibrate the factor supply elasticities (Table 4), and structurally estimate the trade elasticities (Appendix B). Therefore, we report those correlations under both values of ρ that we consider, 2.75 and 1, and for two values of ψ_u, 4 and 1.01 (see below for the description of calibration). The non-technology shocks are positively correlated across countries, unlike TFP. The correlation be non-technology shocks is around 0.12-0.15 on average in the G7 countries, which is well short of the observed GDP correlation, but substantially higher than the average TFP correlation in this set of countries, which is essentially zero. In the full sample, aggregated non-technology shocks have about a 0.04 correlation on average, which is not very different from the TFP correlation. This suggests that, when considering the G7 group of countries alone, non-technology shocks have a better chance of producing positive output correlations observed in the data. The average correlations in $d \ln \xi_{nt}$ are insensitive to the values of ρ and ψ_u. Appendix Table A5 shows that the pattern of correlations remains very similar when using values of ψ_u that differ across industries.

Table 3: Correlations in $d \ln \xi_{nt}$ Summary Statistics

	Mean	Median	25th pctile	75th pctile
		G7 Countries (N. obs. = 21)		
$\rho = 2.75$				
$\psi_u = 4$	0.139	0.125	0.023	0.253
$\psi_u = 1.01$	0.118	0.126	-0.004	0.230
$\rho = 1$				
$\psi_u = 4$	0.148	0.207	0.044	0.389
$\psi_u = 1.01$	0.141	0.254	-0.022	0.336
		All countries (N. obs. = 406)		
$\rho = 2.75$				
$\psi_u = 4$	0.038	0.059	-0.178	0.254
$\psi_u = 1.01$	0.019	0.040	-0.187	0.230
$\rho = 1$				
$\psi_u = 4$	0.029	0.034	-0.210	0.267
$\psi_u = 1.01$	0.009	0.013	-0.243	0.259

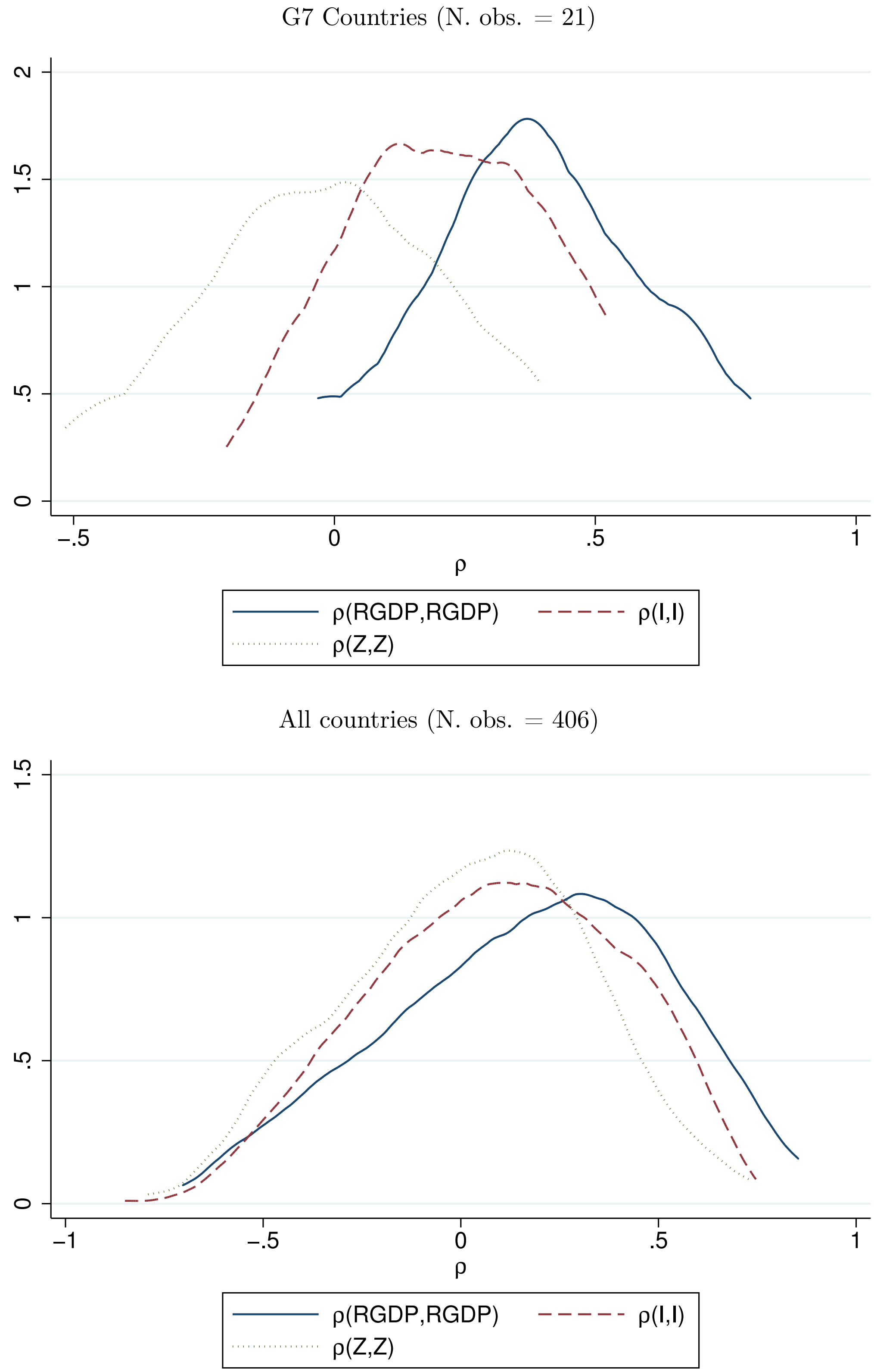
Notes: This table presents the summary statistics of the correlations of $d \ln \xi_{nt}$ defined in (4.10) in the sample of G7 countries (top panel) and full sample (bottom panel). Variable definitions and sources are described in detail in the text.

5 Quantitative Assessment

Shocks in our model can affect aggregate outcomes due to their contemporaneous impact – their correlation and the intratemporal transmission through the network – as well as their dynamic impact driven by the response of capital accumulation and intertemporal labor adjustment to the shocks.

To understand and separate the mechanisms in the model that generate comovement, it is useful to first consider a "static" version of the model, where both capital accumulation within a sector and movement of workers across sectors is not permitted. This approach emphasizes the role of the input-output structure of the model in amplifying or dampening the underlying contemporaneous

Figure 1: Correlations: Kernel Densities



Notes: This figure displays the kernel densities of real GDP growth, the utilization-adjusted TFP, and input correlations in the sample of G7 countries (top panel) and full sample (bottom panel). Variable definitions and sources are described in detail in the text.

correlations of the sectoral shocks. We therefore begin by conducting several counterfactuals in a static version of our model, before moving to the fully dynamic setup where we develop intuition about the intertemporal propogation in the model.

5.1 "Static" Counterfactuals

Having recovered both technology and non-technology shocks in each sector and country, we would like to simulate output growth rates in the counterfactuals in which one of these shocks is turned off and machines m_{njt} and employees N_{njt} are held constant. The analytical solution expressed as a global influence matrix is in Section 3.1. The linear solution is useful as it permits decompositions of changes in GDP into intuitive additive terms. However, for the static model, we can obtain the exact solution using the hat algebra approach of Dekle, Eaton, and Kortum (2008). The details of the exact solution to the model are in Appendix C. It turns out that in our setting, the exact and first-order approximation solutions are very close to each other. Below, we will present the correlations coming from both.

5.2 Calibration

In implementing this static approach, we only need to take a stand on the value of a small number of parameters, and use our data to provide the required quantities. Table 4 summarizes the assumptions for the static model and data sources. The final consumption Armington elasticity ρ is set to either 2.75 or 1 based on our estimation results. Two parameters ψ_e and ψ_h govern the elasticity of different margins of labor supply (hours and effort). As we lack evidence that the elasticity with respect to hours should vary from that for effort, we set them both to 4, implying the Frisch labor supply elasticity is 0.5 as advocated by Chetty et al. (2013). We have less guidance to set the capital supply parameter ψ_u. Our TFP estimation procedure coupled with our choices of ψ_e and ψ_h provides an overidentification restriction for ψ_u, which we outline in Appendix D.1. However, the range of values that satisfy this restriction is large, and includes values that imply very elastic and inelastic capital supply. We therefore choose a baseline value of 4, implying a relatively inelastic capital supply, but also assess the performance of the model for a value of 1.01 – a highly elastic capital supply.

All other parameters in the static model have close counterparts in basic data and thus we compute them directly. Capital shares in total output α_j come from KLEMS, and are averaged in each sector across countries and time. The scale parameters γ_j come from our own production function estimates reported in Appendix Table A3. We initialize both the static and dynamic models in the same steady state. Steady state input shares π^x_{nmj} and final consumption shares π^c_{mnj} are computed as averages from WIOD. Appendix C outlines our algorithm for solving the model and constructing counterfactuals.

Table 4: Parameter values

Param.	Value	Source	Related to
ρ	2.75 or 1	Our estimates	final substitution elasticity
ε	1	Our estimates	intermediate substitution elasticity
$\overline{\psi_e}$, $\overline{\psi_h}$	4	Chetty et al. (2013)	Frisch elasticity
$\overline{\psi_u}$	4 or 1.01	Our estimates	capital supply elasticity
α_j, β_j		KLEMS	labor and capital shares
γ_j		own estimates	returns to scale
π^c_{mnjt}		WIOD	final use trade shares
π^x_{mnjt}		WIOD	intermediate use trade shares
ω_{nj}		WIOD	final consumption shares
Additional Parameters for Dynamic Model			
δ			
β			
Ψ_N			
Ψ_m			
ϑ_{mnj}			
τ_{mnj}			

Our second implementation will be the full dynamic solution of the model. For this we must calibrate a larger group of parameters. The additional parameters for the dynamic model are in the second panel of table 4.

5.3 Impulse Responses

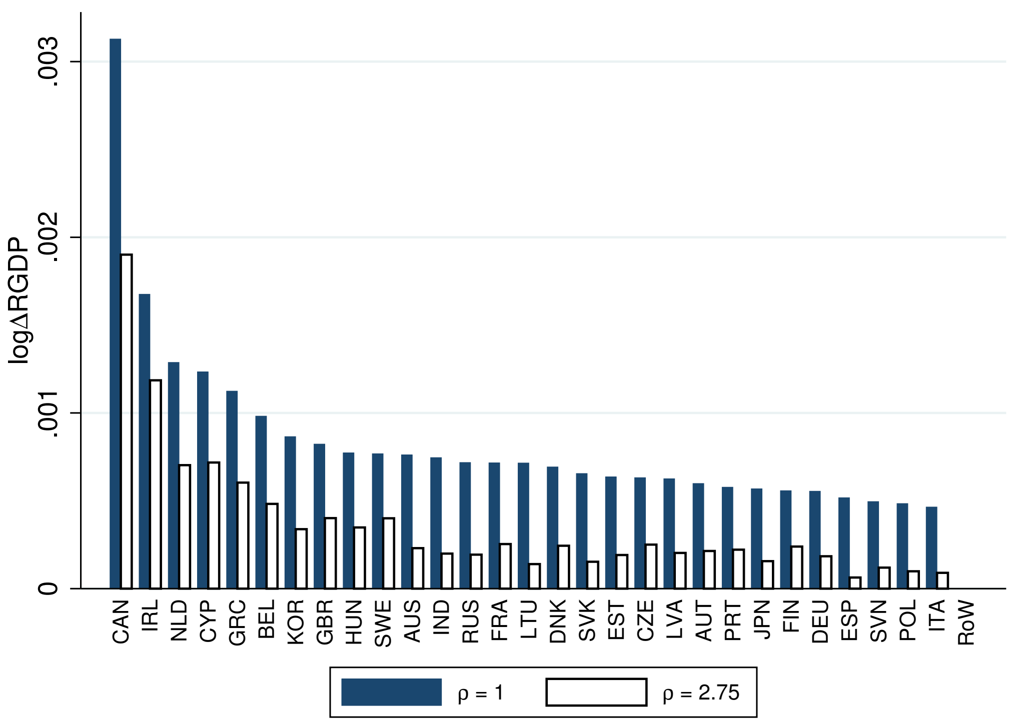
Analytical results or intuition about the transmission of shocks in our framework are complicated by the large country and sector dimension of the model. Prior to simulating the model with the observed shocks, we therefore conduct the following exercises to understand the transmission in our model:

1. a hypothetical U.S. shock in all sectors,
2. a hypothetical rest-of-the-world shock in all sectors from the perspective of each country, and,
3. a symmetric shock in each sector in every country of the world.

In each exercise, we simulate a hypothetical 1% shock – technology and non-technology. The rest-of-the-world exercise assumes that the country in question is not shocked, but all other possible countries and sectors are, and as a result this exercise has to be conducted country by country. Examining the static equilibrium equations (C.1)-(C.4) reveals that up to a scaling parameter the technology and non-technology shocks do not have differential transmission properties in this model. The impact of these two shocks in the exercises identical by construction, an thus to conserve space we only report the impulse responses to TFP shocks.

Figure 2 displays the change in real GDP in every other country in the world following a 1% U.S. shock in each sector. The white bars depict the GDP responses under $\rho = 2.75$, while the dark bars depict the response under $\rho = 1$.

FIGURE 2: Impulse Responses to a US 1% Shock



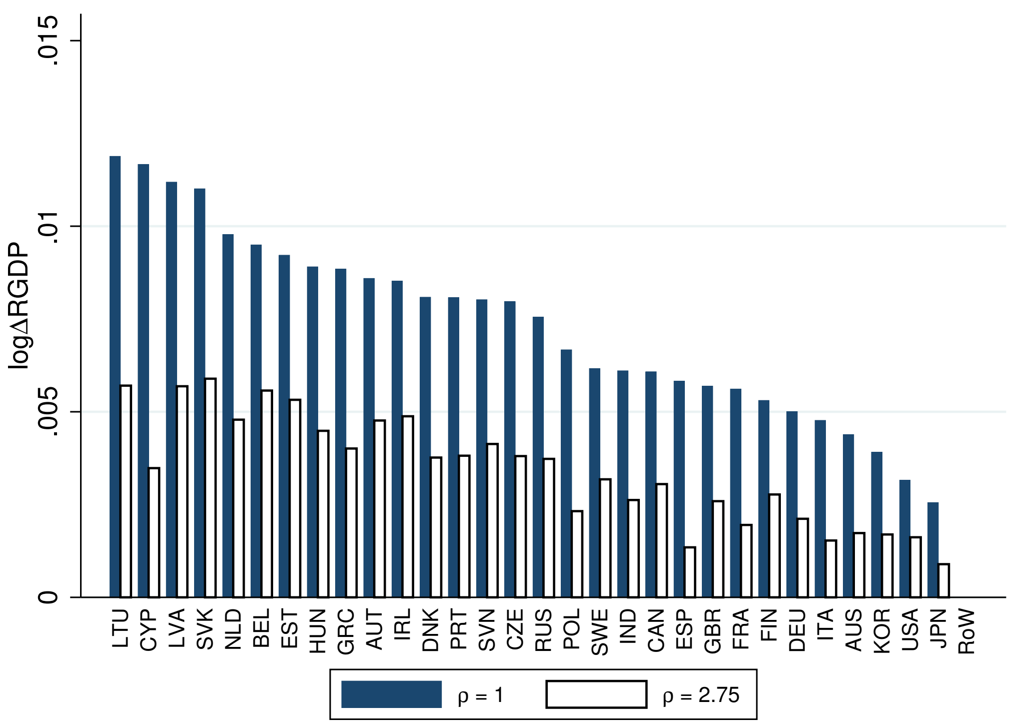
Notes: This figure displays the change in log real GDP of every other country in the sample when the United States experiences a productivity shock of 0.01 in every sector.

The results show that the observed trade linkages do result in transmission. Smaller economies with large trade linkages to the U.S., such as Canada, are the most strongly affected by the U.S. shocks. Under the low elasticity, the mean response of foreign GDP is 0.08%, and the maximum response – Canada – is about 0.3%. On the other hand, the final substitution elasticity matters a great deal for the size of the effects: the response of foreign GDP to the US shocks is about three times as high for

$\rho = 1$ than for $\rho = 2.75$.

Next, we simulate the real GDP responses of each country n in the sample when all other countries (excluding n) experience a 1% technology shock. The exercise answers the question, if there is a 1% world shock outside of the country, how much of that shock will manifest itself in the country's GDP? Figure 3 displays the results. In response to a 1% world TFP shock, under the low elasticity of substitution the mean country's GDP increases by 0.7%, with the impact ranging from less than 0.2% in the U.S. and Japan to 1.1-1.2% in Latvia and Lithuania. Smaller countries are not surprisingly more affected by shocks in their trade partners. The magnitude of transmission is uniformly lower with the higher elasticity. In this case, the mean impact is about 0.2% for the 1% technology shock. All in all, these results suggest that world shocks have a significant impact on most countries.

FIGURE 3: Impulse Responses to Rest of the World 1% Shocks



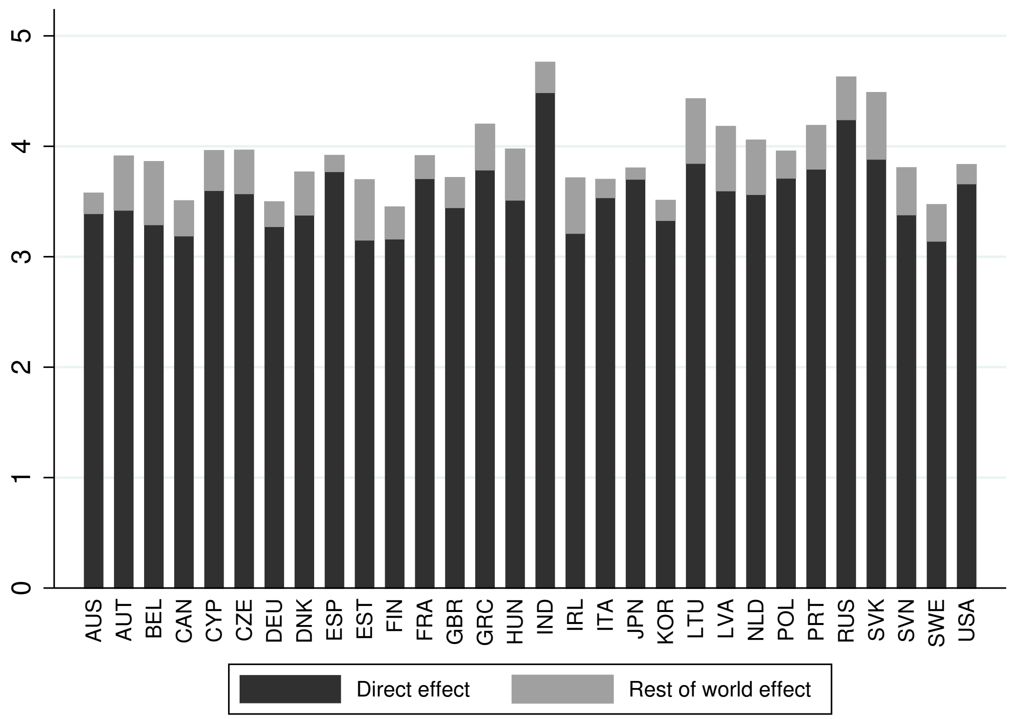
Notes: This figure displays the change in log real GDP of every country in the sample when the rest of the world excluding the country experiences a TFP shock of 0.01 in every sector.

Figure 4 illustrates the results of our third impulse response exercise, a 1% productivity shock to every country and sector in the world. In this exercise, we are most interested in the share of the total GDP change that comes from the shocks to the country's own productivity, and how much comes from foreign shocks. Thus, we use the linear approximation to a country's GDP growth (2.3), and separate the overall impact into the own term $\mathcal{D}_n$ and the rest.

The figure highlights that for all countries, shocks to domestic sectors matter much more for GDP growth than foreign sector shocks. The relative importance of foreign sector shocks is larger for smaller, more open economies like Ireland or Latvia. For Ireland, the direct effect of a 1% shock in each sector is 3.6% and the rest-of-the-world effect (grouping the partner and rest-of-the world terms) is 0.8%. In contrast, for the US the numbers are 4.6% and 0.07%.

FIGURE 4: Impulse Responses to 1% shock in every sector in every country

$$d\ln Z_{m,j,t} = 0.01, \forall m, j$$



Notes: This figure displays the change in log real GDP of every country in the sample, decomposed into a direct effect and a rest of world effect, when all sectors in every country experiences a productivity shock of 0.01.

5.4 GDP Correlations in the Model

We next simulate the full "static" model by feeding in the estimated shocks. Tables 5 and 6 report correlations in our model simulated with both technology and non-technology shocks, as well as counterfactual economies featuring only technology or non-technology shocks, under $\rho = 2.75$ and

$\rho = 1$, respectively. Trade is balanced in every period.[10] The first two lines report the summary statistics for the real GDP correlations in the data and in the baseline model in which both shocks are as measured in the data. Our static model generates correlations that are about half of what is observed in the data, for both the G7 and the full sample. The tables also include results under a higher Frisch elasticity of 2. Predictably, the correlations generated by the model rise when the Frisch elasticity is higher, but the relative contributions of the two types of shocks do not change (results available on request). Figure 5 compares the full distribution of bilateral correlations in our model with both shocks for $\rho = 2.75$ to the data.

TABLE 5: Model Fit and Counterfactuals: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$, $\psi_u = 4$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.380	0.378	0.265	0.533
Model	0.190	0.187	-0.061	0.488
Model (Frisch elasticity=2)	0.252	0.290	0.064	0.493
Non-Technology Shocks Only	0.267	0.291	0.194	0.383
Technology Shocks Only	0.087	0.092	-0.200	0.381
	All countries (N. obs. = 406)			
Data	0.171	0.205	-0.078	0.428
Model	0.100	0.114	-0.157	0.373
Model (Frisch elasticity=2)	0.117	0.144	-0.145	0.397
Non-Technology Shocks Only	0.049	0.065	-0.184	0.291
Technology Shocks Only	0.028	0.046	-0.192	0.245

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) under the different assumptions on shocks and trade linkages. Variable definitions and sources are described in detail in the text.

Next, we simulate the model under only non-technology and only TFP shocks. It is immediately apparent that the non-technology shocks are responsible for much of the comovement in the model. For the G7 group, the model with only non-technology shocks generates 47-66% of the average correlations implied by the model with both shocks, while the model with only technology shocks

[10] Appendix Table A4 reports the fit of the model and counterfactual exercises where deficits are allowed to evolve as in the data.

TABLE 6: Model Fit and Counterfactuals: Correlations of $d\ln Y_{nt}$, $\rho = 1$

	Mean	Median	25th pctile	75th pctile
	G-7 countries (N. obs. = 21)			
Data	0.380	0.378	0.265	0.533
Model	0.190	0.187	-0.061	0.488
Model (Frisch elasticity=2)	0.252	0.290	0.064	0.493
Non-Technology Shocks Only	0.267	0.291	0.194	0.383
Technology Shocks Only	0.087	0.092	-0.200	0.381
	All countries (N. obs. = 406)			
Data	0.171	0.205	-0.078	0.428
Model	0.100	0.114	-0.157	0.373
Model (Frisch elasticity=2)	0.117	0.144	-0.145	0.397
Non-Technology Shocks Only	0.049	0.065	-0.184	0.291
Technology Shocks Only	0.028	0.046	-0.192	0.245

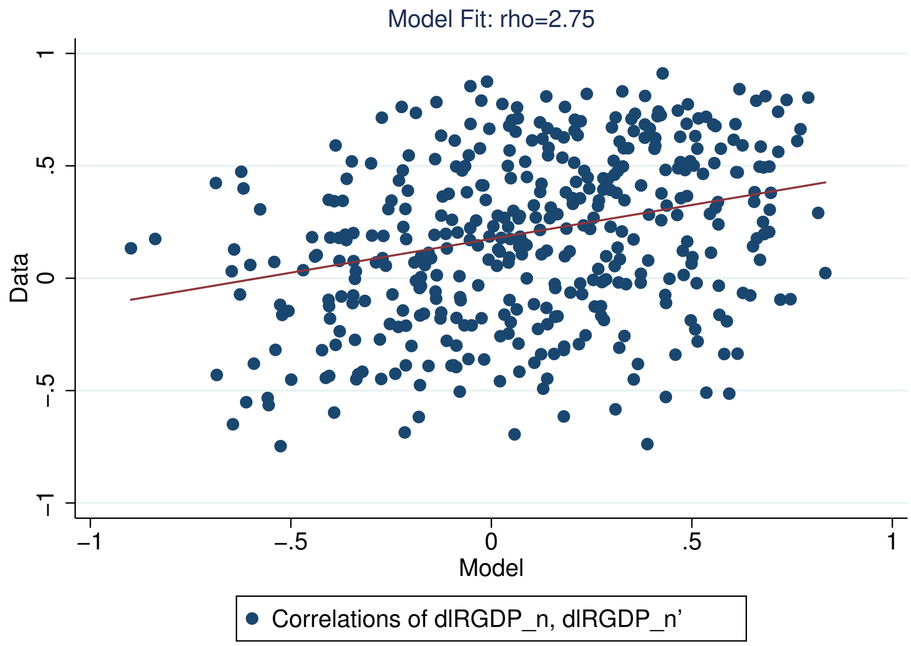
Notes: This table presents the summary statistics of the correlations of $d\ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) under the different assumptions on shocks and trade linkages. Variable definitions and sources are described in detail in the text.

generates only 25% of the comovement on average. The results for all countries are similar in terms of relative magnitudes, though even non-technology shocks account for less comovement: technology shocks generate 14-19% of the comovement of the full model on average, while the non-technology shocks generate 24-33% of the comovement. These relative magnitudes are not sensitive to the two alternative values of ρ.[11]

To assess the importance of correlated shocks relative to transmission in the model with the estimated shocks, we decompose bilateral correlations along the lines of equation (2.4), rewritten in correlations. Table 7 illustrates the results for the estimated shocks. The line labeled "Baseline" reproduces the correlations generated by the model. Under "Approximation" we report the correlations computed based on the first-order approximation to the GDP growth rate in (2.2). It is clear that the first-order approximation delivers correlations that are virtually the same as the fully-solved model. The "Decomposition" lines break down the overall correlation into the terms in equation (2.4). For the

[11] As we have emphasized throughout, the relative importance of technology and non-technology shocks in this framework can at this point only arise through differences in correlation patterns of the underlying shocks. For the non-G7 countries, the non-technology shocks are less correlated than for the G7 countries. So their smaller contribution to cross-country correlations is not surprising.

FIGURE 5: Bilateral correlations: Model vs. Data



Notes: This figure displays the bilateral correlations for all country-pairs in our baseline model with $\rho = 2.75$ to those in the data. The slope estimate of a regression of the empirical correlations on the model correlations, weighted by partner-country GDP, is 0.35. Capital and the number of workers are kept fixed at steady state levels.

G7 countries, the correlation of shocks is responsible for around two-thirds of the model correlations in the simulation with both shocks. Nonetheless, the bilateral and multilateral transmission terms are a non-negligible component of the overall correlation.

5.4.1 The Role of the Input Network

Another way to quantify the role of transmission in generating observed comovement is to compare the correlations in the baseline model to correlations that would obtain in an autarky counterfactual. However, when shocks are correlated and input linkages propagate sectoral shocks within a country, what one assumes about the autarky counterfactual input-output matrix is not innocuous. As emphasized in Section 2, an important determinant of GDP comovement is whether sectors with more correlated shocks are relatively more influential (high s_{nnj}'s) in the two economies. When comparing comovement in the trade equilibrium to autarky, we must take a stand on the autarky influence vectors $\mathbf{s_{AUTn}}$. The assumptions put on the counterfactual autarky input-output matrix will determine the shape of $\mathbf{s_{AUTn}}$. We only observe the full global input-output matrix, which in

Table 7: Transmission of shocks, G7 countries, $\rho = 2.75$

	Both Shocks			
Baseline:	0.190	0.187	-0.061	0.488
Approximation	0.186	0.209	-0.084	0.489
Decomposition:				
Shock Correlation	0.150	0.115	-0.137	0.509
Bilateral Transmission	0.012	0.010	0.005	0.015
Multilateral Transmission	0.022	0.019	0.009	0.033
	Only TFP Shocks			
	Mean	Median	25th pctile	75th pctile
Baseline	0.087	0.092	-0.200	0.381
Approximation	0.086	0.091	-0.201	0.382
Decomposition:				
Shock Correlation	0.052	0.072	-0.216	0.312
Bilateral Transmission	0.013	0.009	0.004	0.018
Multilateral Transmission	0.007	0.004	0.000	0.018
	Only Non-Technology Shocks			
Baseline:	0.267	0.291	0.194	0.383
Approximation	0.274	0.298	0.196	0.404
Decomposition:				
Shock Correlation	0.221	0.239	0.138	0.367
Bilateral Transmission	0.017	0.009	0.005	0.016
Multilateral Transmission	0.028	0.027	0.009	0.039

Notes: This table presents the decomposition of the transmission of observed shocks into direct effects, the direct transmission and the multilateral transmission based on the influence vector approximation.

our analysis is taken as given in steady state. Theory does not offer clear guidance on what the autarky counterfactual input-output structure would look like.

We report results of 3 autarky counterfactuals. The first is a value added-only model: $\eta_j^{AUT1} = 1$

$\forall j$. In this model, there are no input-output linkages, domestic or international.

The second is a model in which the domestic input coefficients are unchanged as a share of gross output, whereas the sum total of the observed foreign input coefficients is reapportioned to value added:

$$\pi^{x,AUT2}_{ni,nj,t} = \pi^{x}_{ni,nj,t} \tag{5.1}$$

$$\eta^{AUT2}_{nj} = \eta_j + \sum_{i;m\neq n} \pi^{x}_{mi,nj,t}. \tag{5.2}$$

In other words, the second autarky counterfactual assumes that in each sector and country, the intermediates that in the data are imported will be replaced by value added.[12] This counterfactual keeps the propagation of shocks through the domestic linkages unchanged.

Finally, the third autarky counterfactual reassigns foreign input coefficients to the domestic inputs, while keeping the value added share of gross output the same as in the baseline:

$$\pi^{x,AUT3}_{ni,nj,t} = \pi^{x}_{ni,nj,t} + \sum_{m\neq n} \pi^{x}_{mi,nj,t} \tag{5.3}$$

$$\eta^{AUT3}_{j} = \eta_j. \tag{5.4}$$

As an example, suppose that the US Apparel sector spent 10 cents on US textile inputs and 5 cents on Chinese textile inputs per dollar of Apparel output, the remaining 85 cents being accounted for by value added. The second autarky counterfactual assumes that this sector still spends 10 cents on US textile inputs, while its value added rises to 90 cents per dollar of output. The third autarky counterfactual assumes instead that value added is still 85 cents per dollar of gross output, but now the sector spends 15 cents on US textile inputs. The third autarky counterfactual thus raises the domestic input coefficients for each sector by the amount of lost foreign input coefficients. As a result, it increases the scope for propagation of domestic shocks even as it rules out propagation of shocks from abroad. By construction, all autarky counterfactuals assume that there is no input trade: $\pi^{x,AUT1}_{mi,nj,t} = \pi^{x,AUT2}_{mi,nj,t} = \pi^{x,AUT3}_{mi,nj,t} = 0 \; \forall m \neq n$.

The value-added-only autarky model has an exact analytical solution for the vector $\mathbf{s_{AUTn}}$:

$$d \ln Y_{nt} = \underbrace{\boldsymbol{\omega}'_{\boldsymbol{n}} \left[\mathbf{I}_J - \left(\left(\frac{\boldsymbol{\alpha}}{\psi_u} + \tilde{\psi} \left(1 - \boldsymbol{\alpha}\right) \right) .\boldsymbol{\gamma} \right) \boldsymbol{\omega}'_{\boldsymbol{n}} \right]^{-1}}_{\mathbf{s_{AUTn}}} d \ln \widehat{\mathbf{Z}}_{nt}, \tag{5.5}$$

[12] The input spending shares $\pi^{x}_{mi,nj,t}$ are not parameters when the aggregation is CES. However, the quantitative implementation uses a unitary elasticity, and thus the $\pi^{x}_{mi,nj,t}$ can be treated as parameters with no ambiguity.

where $\boldsymbol{\omega}$ and $\boldsymbol{\gamma}$ are vectors of length $J \times 1$ and $\mathbf{I}_J$ is the $J \times J$ identity matrix.

This expression makes it clear that even in this simple economy, the influence vector differs from final expenditure shares $\boldsymbol{\omega_n}$. The difference comes from non-constant returns to scale and elastic factor supply. In other words, shocks to sectors with high returns to scale can be amplified both by increasing returns in the sector and the response of factors. Notice also that the size of the sector's share in final consumption ω_{nj} matters for the impact of its shocks: relative to a world with symmetric ω_{nj}, shocks to larger sectors in final consumption will have a larger aggregate impact.

The influence vectors in the $AUT2$ and $AUT3$ models do not have a closed-form exact analytical expression, but they will generally differ from the vector in equation (5.5). Similarly, the direct influence vector in the open economy will also generally differ from the vectors in autarky. While an exact analytical solution for this influence vector does not exist, we can utilize the analytical influence vector obtained from the first-order approximation in Section 3.[13] The changes in GDP comovement between autarky and trade trade will depend on how these influence vectors differ across models, as emphasized by Equation (2.8).

Tables 8-9 report the GDP correlations in the three autarky counterfactuals. The row labeled "VA Only" summarizes the correlations in the $AUT1$ model, with no domestic input linkages. Strikingly, in the G7 sample the autarky value-added-only model produces much *higher* GDP correlations than the model with the full international input linkages. This model generates around 0.31 average correlations in the G7 countries, compared to the 0.165 averages in the baseline with the high ρ. The lines labeled "Same Dom. Links" report the correlations under the $AUT2$ autarky counterfactuals. These correlations fall relative to the $AUT1$ scenario, but do not fall all the way to the baseline averages for the G7. Finally, the $AUT3$ counterfactuals are reported under "Increased Dom. Links." This scenario generates averages that are by and large the same as in the baseline with trade in the G7. This pattern holds for both the high and low ρ, though when $\rho = 1$ – the fully Cobb-Douglas model – the correlations are less sensitive to the assumed input-output structure.[14] Outside of the G7 sample, the comparison of the autarky and trade correlations does not reveal a clear ranking.

Equation (2.8) helps understand these results. The change in GDP comovement between autarky and trade is actually a sum of two terms: the re-weighting of sectors towards or away from those with more correlated shocks (ΔShock Correlation$_{mn}$), and the international transmission terms. We established above that the international transmission terms are generally positive. Thus, to observe the lower average correlation under trade, it must be that the change in the shock correlation term is

[13] Appendix C assess the fit of the first order approximation by comparing GDP growth rates in the full model with those implied by the influence vector. In our model, the first order approximation matches the full model well.

[14] This would be expected as in this case the model is closest to the standard model for instance in Acemoglu et al. (2012).

TABLE 8: Autarky Counterfactuals: Correlations of $d \ln Y_{nt}$, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
Data	0.380	0.378	0.265	0.533
Baseline	0.190	0.187	-0.061	0.488
Autarky Models:				
VA Only	0.296	0.350	0.167	0.397
Same Dom. Links	0.239	0.219	0.122	0.401
Increased Dom. Links	0.153	0.157	-0.112	0.468
	All countries (N. obs. = 406)			
	Mean	Median	25th pctile	75th pctile
Data	0.171	0.205	-0.078	0.428
Baseline	0.100	0.114	-0.157	0.373
Autarky Models:				
VA Only	0.079	0.062	-0.147	0.310
Same Dom. Links	0.068	0.057	-0.163	0.306
Increased Dom. Links	0.084	0.100	-0.156	0.343

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) under the different assumptions on shocks and trade linkages. Variable definitions and sources are described in detail in the text.

Table 9: Autarky Counterfactuals: Correlations of $d \ln Y_{nt}$, $\rho = 1$

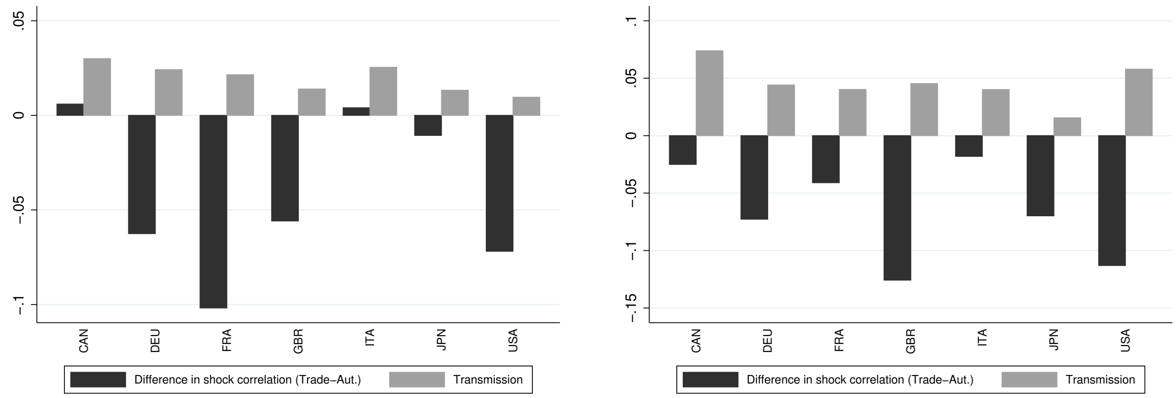
	Mean	Median	25th pctile	75th pctile
Data	0.380	0.378	0.265	0.533
Baseline	0.190	0.187	-0.061	0.488
Autarky Models:				
VA Only	0.296	0.350	0.167	0.397
Same Dom. Links	0.239	0.219	0.122	0.401
Increased Dom. Links	0.153	0.157	-0.112	0.468
	All countries (N. obs. = 406)			
	Mean	Median	25th pctile	75th pctile
Data	0.171	0.205	-0.078	0.428
Baseline	0.100	0.114	-0.157	0.373
Autarky Models:				
VA Only	0.079	0.062	-0.147	0.310
Same Dom. Links	0.068	0.057	-0.163	0.306
Increased Dom. Links	0.084	0.100	-0.156	0.343

Notes: This table presents the summary statistics of the correlations of $d \ln Y_{nt}$ in the sample of G7 countries (top panel) and full sample (bottom panel) under the different assumptions on shocks and trade linkages. Variable definitions and sources are described in detail in the text.

much more strongly negative than the positive transmission terms. This will happen when primitive shocks are more correlated in sectors with a higher influence in autarky. Figure 6 illustrates this by plotting the average ΔShock Correlation$_{mn}$ and the transmission terms for the G7. On average, there is non-negligible positive transmission of shocks in the model with trade, but it is more than offset by the negative ΔShock Correlation$_{mn}$ terms (meaning that in the trade equilibrium, less correlated sectors receive on average higher weight).

Figure 7 plots the average changes in the influence vectors in the G7 sample, by sector. The figure reveals which sectors receive a higher influence in the full baseline model, compared to each of the autarky models. It is clear that the largest changes are for the non-tradeable sectors (Machinery and Equipment Rentals and Other Business Services; and Real Estate Activities). These sectors have a much larger influence in the trade model compared to the value-added only model ($AUT1$). By contrast, the influence vectors change much less between the trade model and the autarky model

FIGURE 6: Decomposing Changes in Correlations: Trade vs Autarky



TFP shocks ξ shocks

Notes: This figure illustrates how the pattern of correlations changes between the full model and the autarky value added only model for the G7. The gray bars illustrate the average bilateral and multilateral transmission terms (all positive) and the dark bars illustrate the decreasing direct correlation effect (mostly negative).

with increased domestic linkages ($AUT3$). The intermediate model ($AUT2$) is in-between those two extremes.

The reason that these services sectors have a much higher influence in the model with IO linkages relative to the value added-only model is that these sectors are important input suppliers to other sectors. The left panel of figure 8 reports the scatterplot of the change in the influence of a sector against the intensity with which other sectors use it as inputs. The correlation between the two is 0.75: sectors used as inputs experience an increase in influence as we move from a value added-only model to the full IO model.

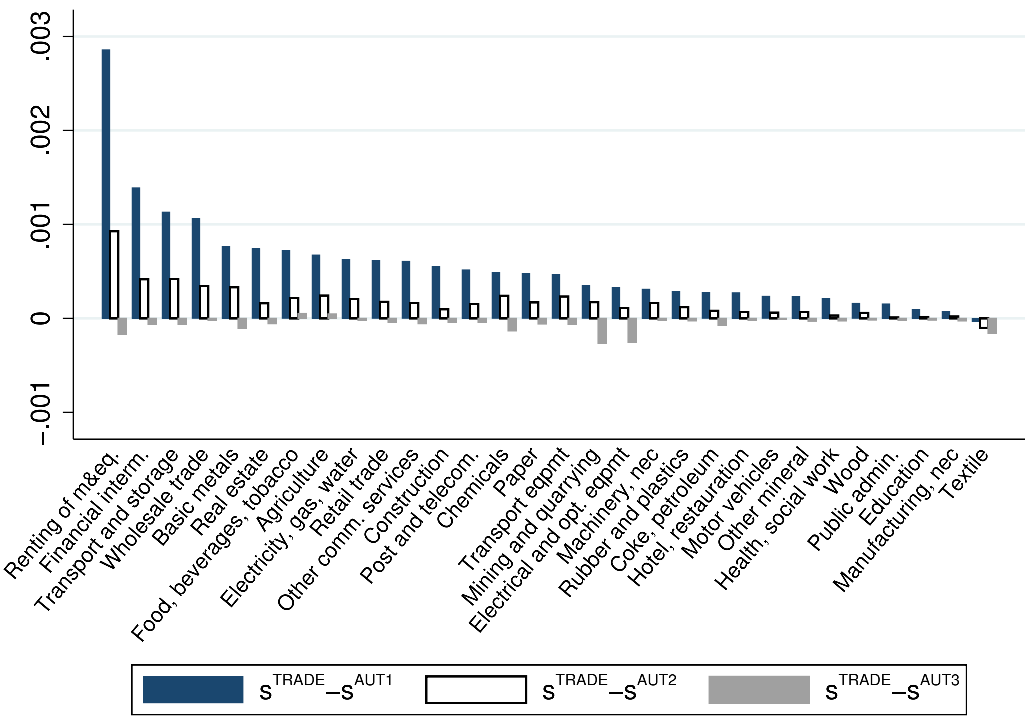
At the same time, shocks in these sectors are on average less correlated with the foreign shocks. The right panel in Figure 8 presents the scatterplot of the average correlation of the TFP shocks in a sector with foreign shocks. The observations are weighted by the size of the sector in the baseline trade model (more precisely, by the influence vector in the trade model). The negative correlation (about -0.25) is evident.[15]

Figures 7-8 illustrate the mechanics behind the finding that GDP comovement can actually fall when going from autarky to an equilibrium with input trade. The reason is that the introduction of input trade can dramatically change the influence of some sectors. What matters is whether introducing input trade increases the influence of sectors with more or less correlated shocks. In our model

[15]There change in the influence vectors is virtually uncorrelated with the correlation in the non-technology shocks, and we do not report that plot to conserve space.

economy, the sectors whose importance increases the most are non-tradeable service sectors whose shocks are actually relatively less correlated. Thus, adding input trade lowers GDP comovement.

FIGURE 7: Average Changes in the Influence Vectors: Trade vs. Autarky Models



Notes: This figure displays the average change in the direct influence vectors between the baseline model and each of the autarky models.

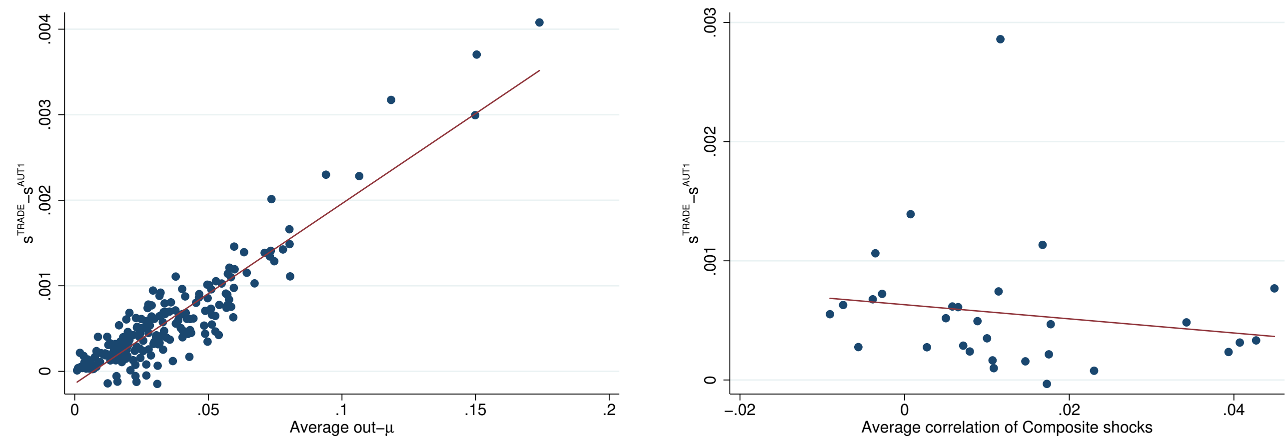
6 Dynamic Counterfactuals

Coming soon.

7 Conclusion

We set out to provide a comprehensive account of international comovement in real GDP. At the heart of our exercise is measurement of both technology and non-technology shocks for a large sample of countries, sectors, and years. Having measured these two types of shocks, we answer two questions. First, is comovement primarily due to TFP or non-technology shocks? The answer here is quite clear: non-technology shocks generate most of the observed international comovement. Second, to what extent do countries comove due to correlated shocks vs. transmission of shocks

Figure 8: Changes in the Influence Vectors, Intensity of Use as an Input, and Shock Correlation



Intensity of Use as Input | Average Correlation w. Foreign Shocks

Notes: This figure displays the change in the influence vectors against the intensity with which a sector is used as an intermediate input (left panel), and the correlation of shocks between that sector and foreign sectors (right panel). The line through the data is the OLS fit.

across countries? One clear answer is that correlated (non-technology) shocks are responsible for the bulk of observed comovement. However, there is also evidence of transmission, especially under low substitution elasticities and with a rich input-output network. Most interestingly however, we find that the input-output network can also play an important role in regulating comovement, by diversifying the economy towards sectors whose shocks are less correlated across countries.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica* 80 (5):1977–2016.

Adao, Rodrigo, Costas Arkolakis, and Federico Esposito. 2017. "Trade, Agglomeration, and Labor Markets: Theory and Evidence." Mimeo, University of Chicago Booth and Yale University and Tufts University.

Ambler, Steve, Emanuela Cardia, and Christian Zimmermann. 2004. "International business cycles: What are the facts?" *Journal of Monetary Economics* 51 (2):257–276.

Angeletos, George-Marios and Jennifer La'O. 2013. "Sentiments." *Econometrica* 81 (2):739–779.

Atalay, Enghin. 2017. "How Important Are Sectoral Shocks?" *American Economic Journal: Macroeconomics* 9 (4):254–280.

Backus, David K, Patrick J Kehoe, and Finn E Kydland. 1992. "International Real Business Cycles." *Journal of Political Economy* 100 (4):745–75.

Baqaee, David Rezza. 2018. "Cascading Failures in Production Networks." CEPR Discussion Papers 12922, C.E.P.R. Discussion Papers.

Baqaee, David Rezza and Emmanuel Farhi. 2019. "Macroeconomics with Heterogeneous Agents and Input-Output Networks." Mimeo, UCLA and Harvard.

Bartelme, Dominick, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare. 2017. "External Economies of Scale and Industrial Policy: A View from Trade." Mimeo, University of Michigan.

Basu, Susanto, John G. Fernald, and Miles S. Kimball. 2006. "Are Technology Improvements Contractionary?" *American Economic Review* 96 (5):1418–1448.

Beaudry, Paul and Franck Portier. 2006. "Stock Prices, News, and Economic Fluctuations." *American Economic Review* 96 (4):1293–1307.

Boehm, Christoph, Aaron Flaaen, and Nitya Pandalai-Nayar. 2017. "Input Linkages and the Transmission of Shocks: Firm Level Evidence from the 2011 Tōhoku Earthquake." Forthcoming, *Review of Economics and Statistics*.

Canova, Fabio. 2005. "The transmission of US shocks to Latin America." *Journal of Applied Econometrics* 20 (2):229–251.

Chamberlain, Gary. 1987. "Asymptotic efficiency in estimation with conditional moment restrictions." *Journal of Econometrics* 34 (3):305–334.

Christiano, Lawrence J., Roberto Motto, and Massimo Rostagno. 2014. "Risk Shocks." *American Economic Review* 104 (1):27–65.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2014. "The International Dimension of Productivity and Demand Shocks in the US Economy." *Journal of the European Economic Association* 12 (1):153–176.

Dekle, Robert, Jonathan Eaton, and Samuel Kortum. 2008. "Global Rebalancing with Gravity: Measuring the Burden of Adjustment." *IMF Staff Papers* 55 (3):511–540.

di Giovanni, Julian, Andrei A. Levchenko, and Isabelle Mejean. 2018. "The Micro Origins of International Business Cycle Comovement." *American Economic Review* 108 (1):82–108.

di Giovanni, Julian, Justin McCrary, and Till von Wachter. 2009. "Following Germany's Lead: Using International Monetary Linkages to Estimate the Effect of Monetary Policy on the Economy." *Review of Economics and Statistics* 91 (2):315–331.

di Giovanni, Julian and Jay C. Shambaugh. 2008. "The impact of foreign interest rates on the economy: The role of the exchange rate regime." *Journal of International Economics* 74 (2):341–361.

Frankel, Jeffrey A. and Andrew K. Rose. 1998. "The Endogeneity of the Optimum Currency Area Criteria." *Economic Journal* 108 (449):1009–25.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle." *American Economic Review* 78 (3):402–17.

Hamilton, James D. 1994. *Time Series Analysis*. Princeton, NJ: Princeton University Press.

Heathcote, Jonathan and Fabrizio Perri. 2002. "Financial autarky and international business cycles." *Journal of monetary Economics* 49 (3):601–627.

Hulten, Charles R. 1978. "Growth Accounting with Intermediate Inputs." *The Review of Economic Studies* 45 (3):511–518.

Imbs, Jean. 1999. "Technology, growth and the business cycle." *Journal of Monetary Economics* 44 (1):65 – 80.

———. 2004. "Trade, Finance, Specialization, and Synchronization." *Review of Economics and Statistics* 86 (3):723–34.

Johnson, Robert C. 2014. "Trade in Intermediate Inputs and Business Cycle Comovement." *American Economic Journal: Macroeconomics* 6 (4):39–83.

Kose, M. Ayhan and Kei-Mu Yi. 2006. "Can the Standard International Business Cycle Model Explain the Relation Between Trade and Comovement." *Journal of International Economics* 68 (2):267–295.

Levchenko, Andrei A. and Nitya Pandalai-Nayar. 2018. "TFP, News, and "Sentiments": The International Transmission of Business Cycles." Forthcoming, *Journal of the European Economic Association*.

O'Mahony, Mary and Marcel P. Timmer. 2009. "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database." *The Economic Journal* 119 (538):F374–F403.

Shambaugh, Jay C. 2004. "The Effect of Fixed Exchange Rates on Monetary Policy." *The Quarterly Journal of Economics* 119 (1):301–352.

Stockman, Alan C and Linda L Tesar. 1995. "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements." *American Economic Review* 85 (1):168–85.

Timmer, Marcel P., Erik Dietzenbacher, Bart Los, Robert Stehrer, and Gaaitzen J. de Vries. 2015. "An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production." *Review of International Economics* 23 (3):575–605.

Wen, Yi. 2007. "By force of demand: Explaining international comovements." *Journal of Economic Dynamics and Control* 31 (1):1–23.

Young, Alwyn. 2017. "Consistency without Inference: Instrumental Variables in Practical Application." Mimeo, LSE.

Appendix A TFP, GDP, and the Solow Residual

Aggregate GDP Growth This appendix presents the derivation of the decomposition of GDP growth into the movement in aggregate TFP and aggregate factor inputs. Using the definition of real GDP, we can express the change in real GDP between $t-1$ and t as:

$$\Delta Y_{nt} = \sum_{j=1}^{J} \left(P_{njt-1} \Delta Y_{njt} - P^X_{njt-1} \Delta X_{njt} \right),$$

and the proportional change:

$$\begin{aligned} \frac{\Delta Y_{nt}}{Y_{nt-1}} &= \frac{\sum_{j=1}^{J} \left(P_{njt-1} \Delta Y_{njt} - P^X_{njt-1} \Delta X_{njt} \right)}{Y_{nt-1}} \\ &= \sum_{j=1}^{J} w^D_{njt-1} \left(\frac{\Delta Y_{njt}}{Y_{njt-1}} - \frac{\Delta X_{njt}}{X_{njt-1}} \frac{P^X_{njt-1} X_{njt-1}}{P_{njt-1} Y_{njt-1}} \right), \end{aligned}$$

where $w^D_{njt-1} \equiv \frac{P_{njt-1} Y_{njt-1}}{Y_{nt-1}}$ is the Domar weight of sector j in country n, that is, the weight of the sector's gross sales in aggregate value added. Approximate the growth rate with log difference:

$$\begin{aligned} d \ln Y_{nt} &\approx \sum_{j=1}^{J} w^D_{njt-1} \left(d \ln Y_{njt} - \frac{P^X_{njt-1} X_{njt-1}}{P_{njt-1} Y_{njt-1}} d \ln X_{njt} \right) \\ &= \sum_{j=1}^{J} w^D_{njt-1} \Bigg(d \ln Z_{njt} + \gamma_j \alpha_j \eta_j d \ln K_{njt} + \gamma_j (1-\alpha_j) \eta_j d \ln L_{njt} \\ &\quad + \gamma_j \left(1 - \eta_j\right) d \ln X_{njt} - \frac{P^X_{njt-1} X_{njt-1}}{P_{njt-1} Y_{njt-1}} d \ln X_{njt} \Bigg). \end{aligned} \tag{A.1}$$

Under the assumption that the share of payments to inputs in total revenues is the same as in total costs, the growth in real GDP can be written as:[16]

$$d\ln Y_{nt} \approx \sum_{j=1}^{J} w^{D}_{njt-1} \left\{ \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1)d\ln\left[\left(K^{\alpha_j}_{njt}L^{1-\alpha_j}_{njt}\right)^{\eta_j} X^{1-\eta_j}_{njt}\right]}_{\text{Scale Effect}} + \underbrace{\alpha_j\eta_j d\ln K_{njt} + (1-\alpha_j)\eta_j d\ln L_{njt}}_{\text{Primary Inputs}} \right\}. \tag{A.2}$$

Then, the growth rate of GDP can be expressed in terms of observable and estimated values:

$$d\ln Y_{nt} \approx \sum_{j=1}^{J} w^{D}_{njt-1} \left\{ \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1)\left[d\ln\left(m^{\alpha_j\eta_j}_{njt} N^{(1-\alpha_j)\eta_j}_{njt} h^{(1-\alpha_j)\eta_j+\xi_j}_{njt} X^{1-\eta_j}_{njt}\right)\right]}_{\text{Scale Effect}} + \underbrace{(\alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln h_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}) + \zeta_j d\ln h_{njt}}_{\text{Utilization-adjusted Primary Inputs}} \right\}, \tag{A.3}$$

leading to equations (4.7) and (4.8) in the main text, with the input-driven component of GDP growth defined as:

$$d\ln \mathcal{I}_{nt} \equiv \sum_{j=1}^{J} w^{D}_{njt-1} \left\{ \underbrace{(\gamma_j - 1)d\ln\left[\left(K^{\alpha_j}_{njt}L^{1-\alpha_j}_{njt}\right)^{\eta_j} X^{1-\eta_j}_{njt}\right]}_{\text{Scale Effect}} + \underbrace{\alpha_j\eta_j d\ln K_{njt} + (1-\alpha_j)\eta_j d\ln L_{njt}}_{\text{Primary Inputs}} \right\}. \tag{A.4}$$

Further, we can also express the covariance in real GDP growth between two countries as

$$\begin{aligned} Cov(d\ln Y_{nt}, d\ln Y_{n\prime t}) &= Cov(d\ln Z_{nt}, d\ln Z_{n\prime t}) + Cov(d\ln I_{nt}, d\ln I_{n\prime t}) \\ &\quad + Cov(d\ln Z_{nt}, d\ln I_{n\prime t}) + Cov(d\ln I_{nt}, d\ln Z_{n\prime t}). \end{aligned} \tag{A.5}$$

[16] Recall that, regardless of the nature of variable returns to scale or market structure, under cost minimization $\alpha_j\eta_j$ is the share of payments to capital in the total costs, while $(1-\alpha_j)\eta_j$ is the share of payments to labor. We do not observe total costs, only total revenues. We assume that $\alpha_j\eta_j$ also reflects the share of payments to capital in total revenues. Under our assumption that sector j is competitive and the variable returns to scale are external to the firm, this assumption is satisfied. In that case, these can be taken directly from the data as $\alpha_j\eta_j = r_{njt}K_{njt}/P_{njt}Y_{njt}$ and $(1-\alpha_j)\eta_j = w_{njt}L_{njt}/P_{njt}Y_{njt}$, where $P_{njt}Y_{njt}$ is total revenue, r_{njt} is the price of capital, and w_{njt} is the wage rate.

Relationship to Solow residual The expression in equation (4.7) is useful to compare the estimated TFP series to the traditional measure of technology, the Solow residual. The Solow residual S_{njt} takes factor shares and nets out the observable factor uses. It has the following relationship to gross output and observed inputs:

$$d\ln Y_{njt} = d\ln S_{njt} + \alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln h_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt} + (1-\eta_j)\, d\ln X_{njt}.$$

Plugging this way of writing output growth into the real GDP growth equation (A.1), we get the following expression:

$$\begin{aligned} d\ln Y_{nt} &\approx \sum_{j=1}^{J} w^D_{njt-1}\left(d\ln S_{njt} + \alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln h_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}\right. \\ &\quad \left. + (1-\eta_j)\, d\ln X_{njt} - d\ln X_{njt}\frac{p^X_{njt-1}X_{njt-1}}{p_{njt-1}Y_{njt-1}}\right) \\ &= \sum_{j=1}^{J} w^D_{njt-1}\left(d\ln S_{njt} + \alpha_j\eta_j d\ln m_{njt} + (1-\alpha_j)\eta_j d\ln h_{njt} + (1-\alpha_j)\eta_j d\ln N_{njt}\right) \end{aligned} \tag{A.6}$$

Comparing (A.2) to (A.6), the Solow residual contains the following components:

$$\begin{aligned} d\ln S_{njt} &= \underbrace{d\ln Z_{njt}}_{\text{True TFP}} + \underbrace{(\gamma_j - 1)d\ln\left[\left(K^{\alpha_j}_{njt}L^{1-\alpha_j}_{njt}\right)^{\eta_j} X^{1-\eta_j}_{njt}\right]}_{\text{Scale Effect}} \\ &\quad + \underbrace{\alpha_j\eta_j d\ln u_{njt} + (1-\alpha_j)\eta_j d\ln e_{njt}}_{\text{Unobserved Utilization}}. \end{aligned}$$

This expression makes it transparent that in this setting, the Solow residual can diverge from the true TFP shock for two reasons: departures from constant returns to scale at the industry level, and unobserved utilization of inputs.

Let aggregate Solow residual be denoted by:

$$\begin{aligned} d\ln S_{nt} &= \sum_{j=1}^{J} w^D_{njt-1} d\ln S_{njt} \\ &= d\ln Z_{nt} + d\ln \mathcal{U}_{nt}, \end{aligned}$$

where in the second equality, $d\ln\mathcal{U}_{nt}$ is the aggregate utilization adjustment:

$$d\ln\mathcal{U}_{nt} \equiv \sum_{j=1}^{J} w^D_{njt-1}\left\{(\gamma_j - 1)d\ln\left[\left(K^{\alpha_j}_{njt}L^{1-\alpha_j}_{njt}\right)^{\eta_j} X^{1-\eta_j}_{njt}\right]\right. \tag{A.7}$$

$$\left. +\alpha_j\eta_j d\ln u_{njt} + (1-\alpha_j)\eta_j d\ln e_{njt}\right\}. \tag{A.8}$$

It is immediate that the observed Solow residual can be correlated across countries both due to correlated shocks to true TFP, and due to correlated unobserved input adjustments:

$$\begin{aligned} Cov(d\ln S_{nt}, d\ln S_{n't}) &= Cov(d\ln Z_{nt}, d\ln Z_{n't}) + Cov(d\ln \mathcal{U}_{nt}, d\ln \mathcal{U}_{n't}) \\ &\quad + Cov(d\ln Z_{nt}, d\ln \mathcal{U}_{n't}) + Cov(d\ln \mathcal{U}_{nt}, d\ln Z_{n't}). \end{aligned}$$

Appendix B Estimating Model Elasticities

Our framework offers a straightforward approach to estimating ρ and ε. To introduce an error term in the estimating equations, assume that iceberg trade costs, final consumer taste shocks, and input share shocks have a stochastic element, and denote their gross proportional changes by $\widehat{\tau}_{mnjt+1}$, $\widehat{\vartheta}_{mnjt+1}$, and $\widehat{\mu}_{mj,ni,t+1}$, respectively. Straightforward manipulation of CES consumption shares yields the following relationships between shares and prices:

$$\ln\left(\frac{\widehat{\pi}^{c}_{mnj,t+1}}{\widehat{\pi}^{c}_{m'nj,t+1}}\right) = (1-\rho)\ln\left(\frac{\widehat{P}_{mj,t+1}}{\widehat{P}_{m'j,t+1}}\right) + \ln\left(\frac{\widehat{\vartheta}_{mnjt+1}\widehat{\tau}^{1-\rho}_{mnjt+1}}{\widehat{\vartheta}_{m'njt+1}\widehat{\tau}^{1-\rho}_{m'njt+1}}\right) \tag{B.1}$$

and

$$\ln\left(\frac{\widehat{\pi}^{x}_{mj,nit+1}}{\widehat{\pi}^{x}_{m'j,nit+1}}\right) = (1-\varepsilon)\ln\left(\frac{\widehat{P}_{mjt+1}}{\widehat{P}_{m'jt+1}}\right) + \ln\left(\frac{\widehat{\mu}_{mj,ni,t+1}\widehat{\tau}^{1-\varepsilon}_{mnjt+1}}{\widehat{\mu}_{m'j,ni,t+1}\widehat{\tau}^{1-\varepsilon}_{m'njt+1}}\right). \tag{B.2}$$

We express the final consumption share change $\widehat{\pi}^{c}_{mnj,t+1}$ relative to the final consumption share change in a reference country m'. This reference country is chosen separately for each importing country-sector n, j as the country with the largest average expenditure share in that country-sector. (Thus, strictly speaking, the identity of the reference country m' is distinct for each importing country-sector, but we suppress the dependence of m' on n, j to streamline notation.) Furthermore, we drop the own expenditure shares $\widehat{\pi}^{c}_{nnj,t+1}$ from the estimation sample, as those are computed as residuals in WIOD, whereas final import shares from other countries are taken directly from the international trade data. Dropping the own expenditure shares has the added benefit of making the regressions less endogenous, as the domestic taste shocks are much more likely to affect domestic prices.

We use two estimation approaches for (B.1)-(B.2). We first show the results with OLS. To absorb as much of the error term as possible, we include source-destination-reference country-time ($n \times m \times m' \times t$) fixed effects. These absorb any common components occurring at the country 3-tuple-time level, such as exchange rate changes and other taste and transport cost changes, and thus the coefficient is estimated from the variation in the relative sectoral price indices and relative sectoral share movements within that cell. The identifying assumption is then that price change ratio $\widehat{P}_{mj,t+1}/\widehat{P}_{m'j,t+1}$ is uncorrelated with the residual net of the $n \times m \times m' \times t$ fixed effects. The remaining errors would be largely measurement error. If this measurement error is uncorrelated with the price change ratios, then the OLS estimates are unbiased, and if not, we would expect a bias towards zero. In the latter case, the IV estimates (described below) should be larger than the OLS estimates, assuming the measurement error in (B.1) and (B.2) is independent of the measurement error in the technology shock ratios.

The estimation amounts to regressing relative share changes on relative price changes. A threat to identification would be that relative price changes are affected by demand shocks (e.g. $\widehat{\vartheta}_{mnjt+1}$), and thus correlated with the residual. As a way to mitigate this concern, we also report estimates based on the subsample in which destination countries are all non-G7, and the source and reference countries are all G7 countries. In this sample it is less likely that taste shocks in the (smaller) destination countries will affect relative price changes in the larger G7 source countries. Finally, to reduce the impact of small shares on the estimates, we report results weighting by the size of the initial shares ($\pi^c_{mnj,t}$ and $\pi^x_{mj,ni,t}$).

We also implement IV estimation. We use the TFP shocks $\widehat{Z}_{mjt+1}/\widehat{Z}_{m'jt+1}$ as instruments for changes in relative prices. The exclusion restriction is that the technology shocks are uncorrelated with taste and trade cost shocks, and thus only affect the share ratios through changing the prices. Even if the shock ratio $\widehat{Z}_{mjt+1}/\widehat{Z}_{m'jt+1}$ is a valid instrument for observed prices, it does not include the general-equilibrium effects on prices in the model. To use all of the information –both the direct and indirect GE effects –incorporated in the model, we also use the model-optimal IV approach to construct the instrument. In our context this simply involves computing the model using only the estimated technology shocks, and solving for the sequence of equilibrium prices in all countries and sectors. The model-implied prices are then the optimal instrument for the prices observed in the data. See Chamberlain (1987) for a discussion of optimal instruments, and Adao, Arkolakis, and Esposito (2017) and Bartelme et al. (2017) for two recent applications of this approach. The results from the model-optimal IV are very similar to simply instrumenting with the TFP shock ratio, and we do not report them to conserve space.

Model Elasticities Table A1 presents the results of estimating equations (B.1) and (B.2). Columns 1-3 report the OLS estimates of ρ (top panel) and ε (bottom panel). The OLS estimates of ρ are all significantly larger than zero, and we cannot rule out a Cobb-Douglas final demand elasticity. The OLS estimates for ρ are also not very sensitive to restricting the sample to non-G7 destinations and G7 sources, or to weighting by the initial share. The IV estimates in columns 4-6 are substantially larger than the OLS coefficients, ranging from 2.27 to 3.04, and significantly different from 1 in most cases. This difference between OLS and IV could suggest either measurement error in (B.1), or greater noise in the IV estimator (Young, 2017). Given the substantial disagreement between OLS and IV estimates of ρ, we report the results under two values: $\rho = 1$, corresponding to the OLS estimates, and $\rho = 2.75$ based on the IV.

The OLS and IV estimates of ε display somewhat greater consensus. The OLS point estimates are in the range 0.68, and not sensitive to the sample restriction or weighting. The IV estimates are less stable. While the full sample (column 4) yields an elasticity of 2.8, either restricting to the non-G7 destinations/G7 sources, or weighting by size reduces the coefficient dramatically and renders it not statistically different from 1. Such evidence for the low substitutability of intermediate inputs is consistent with the recent estimates by Atalay (2017) and Boehm, Flaaen, and Pandalai-Nayar (2017), who find even stronger complementarity. We therefore

TABLE A1: Elasticity Estimates

	(1) OLS	(2) OLS (G7 m, m', non-G7 n)	(3) OLS (weighted)	(4) IV	(5) IV (G7 m, m', non-G7 n)	(6) IV (weighted)
ρ	0.775	0.730	1.051	2.881	2.273	3.037
SE	(0.055)	(0.146)	(0.082)	(0.584)	(0.966)	(0.470)
First stage K-P F				92.117	30.539	89.669
FE	Yes	Yes	Yes	Yes	Yes	Yes
ε	0.698	0.686	0.682	2.838	0.382	1.322
SE	(0.051)	(0.120)	(0.143)	(0.578)	(0.872)	(0.856)
First stage K-P F				94.863	16.188	86.631
FE	Yes	Yes	Yes	Yes	Yes	Yes

Notes: Standard errors clustered at the destination-source-reference country level in parentheses. This table presents results from the OLS and IV estimation of (B.1) and (B.2). The fixed effects used in each regression are $n \times m \times m' \times t$. The instruments are the relative productivity shocks $\widehat{Z}_{mjt+1}/\widehat{Z}_{m'jt+1}$, with the Kleibergen-Papp first stage F-statistic reported. The weights in columns 3 and 6 are lagged share ratios $\pi^c_{mnj,t}$ and $\pi^x_{mj,ni,t}$.

set $\varepsilon = 1$ for all implementations of the model.

Appendix C Exact Solution to "Static" Counterfactuals

In response to TFP and non-TFP shocks, the price in sector j, country n experiences the change:

$$\begin{aligned}\widehat{P}_{njt} \quad = \quad & \widehat{Z}_{njt}^{-1}\widehat{\Upsilon}_{njt}^{1-\gamma_j+\alpha_j\eta_j\left(\frac{\psi^u-1}{\psi^u}\right)\gamma_j+\left(1-\tilde{\psi}\right)(1-\alpha_j)\eta_j\gamma_j}\widehat{m}_{njt}^{-\alpha_j\eta_j\gamma_j}\left(\widehat{\xi}_{njt}\widehat{P}_{nt}\right)^{\left(\frac{\alpha_j}{\psi^u}+\tilde{\psi}(1-\alpha_j)\right)\eta_j\gamma_j} \\ & \widehat{N}_{njt}^{\left(\tilde{\psi}-1\right)(1-\alpha_j)\eta_j\gamma_j+\frac{\alpha_j\eta_j}{\psi^u}\gamma_j}\left(\sum_{m,i}\pi_{mi,nj,t}\widehat{P}^{1-\varepsilon}_{mi,njt}\right)^{\frac{1-\eta_j}{1-\varepsilon}\gamma_j}.\end{aligned} \tag{C.1}$$

This, together with the dependence of $\widehat{P}_{nt+1}$ on the constituent $\widehat{P}_{njt+1}$'s stated in (4.5)-(4.6) defines a system of $J \times N$ equations in prices, conditional on known initial-period data quantities (such as π^c_{mnjt}), a vector of $\widehat{\Upsilon}_{njt+1}$'s, and the assumption that $\widehat{m}_{njt}$ and $\widehat{N}_{njt}$ are 1. The price changes in turn determine next period's shares:

$$\pi^c_{nmjt+1} \quad = \quad \frac{\widehat{P}^{1-\rho}_{njt+1}\pi^c_{nmjt}}{\sum_k \widehat{P}^{1-\rho}_{kjt+1}\pi^c_{kmjt}}, \tag{C.2}$$

$$\pi^x_{nj,mit+1} = \frac{\widehat{P}^{1-\varepsilon}_{njt}\pi^x_{nj,mit}}{\sum_{k,l}\widehat{P}^{1-\varepsilon}_{klt}\pi^x_{kl,mit}}. \tag{C.3}$$

These trade shares have to be consistent with market clearing at the counterfactual $t+1$, expressed using proportional changes as:

$$\begin{aligned} \widehat{\Upsilon}_{njt+1}\Upsilon_{njt} &= \sum_m \Bigg[\pi^c_{nmjt+1}\omega_{jm}\left(\sum_i \eta_i \widehat{\Upsilon}_{mit+1}\Upsilon_{mit}\right) \\ &\quad + \sum_i \pi^x_{nj,mit+1}(1-\eta_i)\widehat{\Upsilon}_{mit+1}\Upsilon_{mit}\Bigg]. \end{aligned} \tag{C.4}$$

The sets of equations (C.1)-(C.4) represent a system of $2\times N\times J+N^2\times J+N^2\times J^2$ unknowns, $\widehat{P}_{njt+1}$ $\forall n,j$, $\widehat{\Upsilon}_{njt+1}$ $\forall n,j$, π^c_{nmjt+1} $\forall n,m,j$, and $\pi^x_{nj,mit+1}$ $\forall n,j,m,i$ that is solved under given parameter values and under a set of shocks $\widehat{Z}_{njt+1}$ and $\widehat{\xi}_{njt+1}$.

C.0.1 Algorithm for solving the model

To solve the model, we use an initial guess for $\widehat{\Upsilon_{nj,t+1}}$ together with data on $\pi^c_{mnj,t}$ and $\pi^x_{mjni,t}$. Given these variables, the algorithm is as follows:

- Solve for $\widehat{P_{nj,t+1}}$ given the guess of $\widehat{\Upsilon_{nj,t+1}}$ and the data on $\pi^c_{mnj,t}$ and $\pi^x_{mjni,t}$. This step uses equations (4.6), (4.5) and (C.1).
- Update $\pi^c_{mnj,t+1}$ and $\pi^x_{mj,ni,t+1}$ given the solution to (1) and the guess of $\widehat{\Upsilon_{nj,t+1}}$ using equations (C.2) and (C.3).
- Solve for $\widehat{\Upsilon'_{nj,t+1}}$ using equation (C.4) given the prices $\widehat{P_{nj,t+1}}$ obtained in step (1) and the updated shares $\pi^c_{mnj,t+1}$ and $\pi_{minj,t+1}$ from step (2).
- Check if max$|(\widehat{\Upsilon'_{nj,t+1}}$-$\widehat{\Upsilon_{nj,t+1}})| < \delta$, where δ is a tolerance parameter that is arbitrarily small. If not, update the guess of $\widehat{\Upsilon_{nj,t+1}}$ and repeat steps (1)-(4) until convergence.

Appendix D Empirical Appendix

D.1 TFP Estimation Appendix

The following discussion is an abbreviated outline of the model and estimation method in BFK. For a more detailed discussion, please see Basu, Fernald, and Kimball (2006).

D.1.1 Model

BFK estimate a utilization adjusted TFP series for $n = US$ only. In this Appendix, we therefore drop the n subscript for compact notation. Sector i produces gross output Y_i with a production function

$$Y_i = F^i \left(A_i M_i, E_i H_i N_i, X_i, Z_i\right) \tag{D.1}$$

where the definitions of $A_i, M_i, E_i, H_i, N_i, X_i$ and Z_i are as in Section 2. BFK make the following assumptions to derive the estimating equation (4.3) for sectoral TFP:

Assumptions

- The physical capital stock M_i and number of employees N_i are quasi-fixed, and firms face adjustment costs to changing M_i or N_i.
- Firms do not face adjustment costs for varying capital utilization A_i, labor hours H_i or effort E_i, but firms must pay a shift premium or higher wages for increasing utilization of the quasi-fixed factors.
- F^i is a generalized Cobb-Douglas production function that is locally homogeneous of degree γ_i.
- Firms are price-takers in the factor markets

The first two assumptions are necessary for a meaningful notion of variable utilization, as if firms could costlessly adjust utilization or the physical capital stock or number of workers, they would always fully utilize all factors and vary M_i and N_i instead. Within a period, the firm's costs of labor are given by $WG^i(H_i, E_i)V(A_i)$, where W is the base wage, the function G^i relates labor compensation to the hours worked and the effort, and $V(A_i)$ is the shift premium paid to workers as a function of the workweek of capital A_i.

The first-order conditions of the intra-temporal cost-minimization problem of the firms in sector i with production function (D.1) relevant for expressing variations in unobserved utilization as a function of observed hours are:

$$\lambda F_1^i M_i = W_i N_i G^i \left(H_i, E_i\right) V^{i'} \left(A_i\right), \tag{D.2}$$

$$\lambda F_2^i E_i N_i = W_i N_i G_H^i \left(H_i, E_i\right) V^i \left(A_i\right) \tag{D.3}$$

and

$$\lambda F_2^i H_i N_i = W_i N_i G_E^i \left(H_i, E_i\right) V^i \left(A_i\right), \tag{D.4}$$

where λ is the multiplier (marginal cost) in the firm's cost minimization problem, and $F_i, i = 1, 2, 3$ denotes derivatives of production function D.1 with respect to argument j. The function $G(.)$ is assumed to satisfy conditions such that there is one optimal value of E for every value of H. Note that we only need to focus on the firm's intra-temporal cost-minimization problem here, which is conditional on the levels of the state variables M_i and N_i. The intertemporal conditions which govern the firm's choices of the capital stock and number of workers do not affect the derivation of the estimating equation.

D.1.2 Estimation

Combining equations (D.3) and (D.4), and since we assumed a unique $E = E(H), E'(H) > 0$, we can express $de = \zeta dh$, where $\zeta = H^* E'(H^*) / E(H^*)$. Similarly, combining equations (D.2) and (D.3) and log-linearizing, we can express $da = \frac{\epsilon}{\iota} dh$ where ϵ is elasticity of labor costs with respect to H_i in steady-state, and ι is the elasticity of the ratio of the marginal to average shift premium with respect to A_i. Combining these results, we obtain the estimating equation 4.3.

In this paper, we estimate purified TFP series for a large number of countries and sectors as discussed in the main text. Table A1 lists the countries and Table A2 the sectors in our sample. We require instruments orthogonal to the TFP shocks in our panel that have predictive power for movements in inputs. BFK use a monetary policy shock identified in a VAR, an oil price shock and the growth in real defense spending. We use instruments similar in spirit: the lagged growth in real defense spending in each country, an oil price shock defined using the approach in Hamilton (1994) and a version of a monetary policy shock that relies on the exogenous movements in base-country interest rates affecting countries that are pegged to a base country. This last instrument cannot be used for large countries like the U.S., U.K. or Germany.

Table A1: Countries in Estimation Sample

Australia	Germany	Netherlands
Austria	Greece	Poland
Belgium	Hungary	Portugal
Canada	India	Russian Federation
Cyprus	Ireland	Slovak Republic
Czech Republic	Italy	Slovenia
Denmark	Japan	Spain
Estonia	Republic of Korea	Sweden
Finland	Latvia	U.K.
France	Lithuania	U.S.A.

Comparison to BFK's estimates: While the point estimates of both the returns to scale for our sectors and the coefficients on the utilization adjustment term naturally vary from those in BFK, they are not significantly different from the estimates in that paper in many cases. For instance, we estimate coefficients on the utilization adjustment term of 1.419(0.389), 2.939(1.767) and 0.245(0.649) for durables, non-durables and non-manufacturing respectively. The comparable estimates in BFK Table 1 are 1.34(0.22), 2.13(0.38) and 0.64(0.34) respec-

Table A2: Sectors in Estimation Sample

agriculture hunting forestry and fishing	basic metals and fabricated metal	financial intermediation
mining and quarrying	machinery nec	real estate activities
food beverages and tobacco	electrical and optical equipment	renting of m&eq and other business activities
textiles textile leather and footwear	transport equipment	public admin and defence; compulsory social security
wood and of wood and cork	manufacturing nec; recycling	education
pulp paper paper printing and publishing	electricity gas and water supply	health and social work
coke refined petroleum and nuclear fuel	construction	other community social and personal services
chemicals and chemical products	hotels and restaurants	sale maintenance and repair of motor vehicles
rubber and plastics	transport and storage	wholesale trade and commission trade
other nonmetallic mineral	post and telecommunications	retail trade except of motor vehicles

tively.

D.2 Properties of the TFP series

Figure A1 contrasts the Solow residual with the utilization-adjusted TFP series for all the countries in our sample. While we do find that the utilization-adjusted TFP series is less volatile than the Solow residual for the U.S., as in BFK, for the large majority of other countries the adjusted TFP series is more volatile. In fact, the mean (median) volatility of the TFP series is .0006 (0.0005), while for the Solow residual it is 0.0002(0.0002).

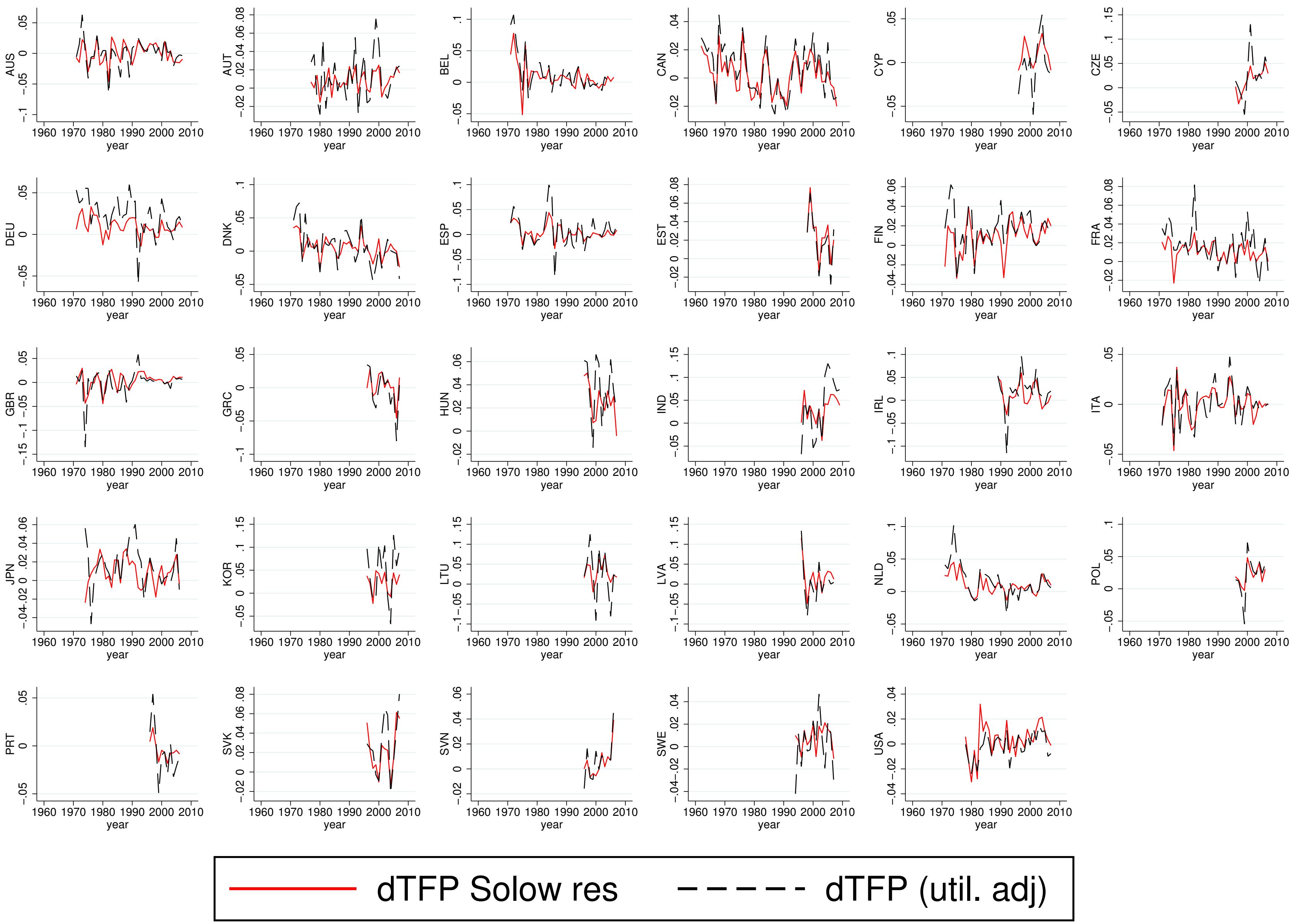
TABLE A3: Estimation Results

Industry	Returns to Scale $(\widehat{\eta}_j^1)$	Utilization $(\widehat{\eta}_j^2)$	Industry	Returns to Scale $(\widehat{\eta}_j^1)$	Utilization $(\widehat{\eta}_j^2)$
Durables			**Non-durable non-manufacturing**		
wood and of wood and cork	0.750***		sale maintenance and repair of motor	1.653***	
	(0.133)		vehicles and motorcycles; retail sale of fuel	(0.317)	
basic metals and fabricated metal	0.701**		wholesale trade and commission trade	1.472***	
	(0.329)		except of motor vehicles and motorcycles	(0.168)	
machinery nec	0.791***		retail trade except of motor vehicles	0.873*	
	(0.241)	1.419***	and motorcycles; repair of household goods	(0.454)	
electrical and optical equipment	0.711**	(0.389)	transport and storage	1.081***	
	(0.300)			(0.164)	
transport equipment	0.844***		post and telecommunications	0.632***	
	(0.207)			(0.150)	
manufacturing nec; recycling	0.895***		real estate activities	0.456	
	(0.129)			(0.332)	
			renting of m&eq and other business activities	1.221***	
				(0.226)	0.245
			agriculture hunting forestry and fishing	1.787*	(0.649)
				(0.959)	
Non-durable manufacturing			mining and quarrying	0.121	
				(0.757)	
food beverages and tobacco	0.988***		electricity gas and water supply	1.825	
	(0.174)			(1.387)	
textiles textile leather and footwear	0.289		construction	1.041***	
	(0.524)			(0.225)	
pulp paper paper printing and publishing	0.504		hotels and restaurants	1.267***	
	(0.430)			(0.429)	
coke refined petroleum and nuclear fuel	0.894	2.939*	financial intermediation	1.335***	
	(1.088)	(1.767)		(0.368)	
chemicals and chemical products	0.806*		public admin and defence;	1.863	
	(0.426)		compulsory social security	(1.504)	
rubber and plastics	0.925***		education	0.674***	
	(0.279)			(0.246)	
other nonmetallic mineral	0.695		health and social work	1.374	
	(0.487)			(2.173)	
			other community social and personal services	0.752***	
				(0.198)	

Notes: This table contains the results from the production function estimation described in Section 4. Standard errors in parentheses. Significance levels are indicated by *** p<0.01, ** p<0.05, * p<0.1.

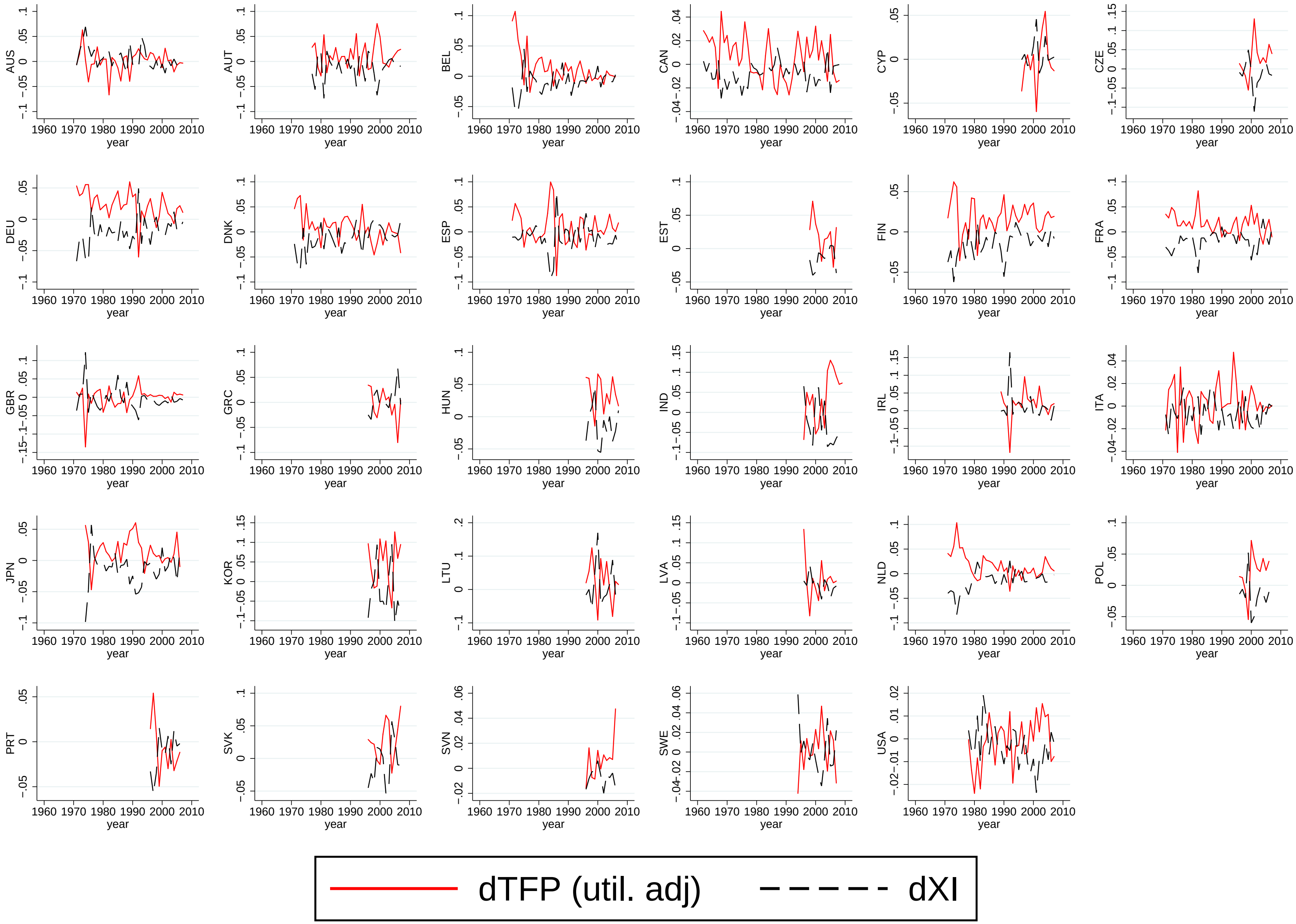
† These coefficients are estimated in the specification that includes three instruments.

Figure A1: Comparison between Utilization-Adjusted TFP and the Solow Residual



Notes: This figure displays the Solow residual and the utilization-adjusted TFP series for every country in our sample.

Figure A2: Comparison between Utilization-Adjusted TFP and the non-technology shocks



Notes: This figure displays the utilization-adjusted TFP and the recovered non-technology shocks series for every country in our sample.

Table A4: Model Fit and Counterfactuals with Deficits: $d\ln Y_{nt}$

	Mean	Median	25th pctile	75th pctile
		G7 Countries (N. obs. = 21)		
Data		Coming soon		
Model				
No Technology Shocks				
No Input Shocks				
		All countries (N. obs. = 435)		
Data				
Model				
No Technology Shocks				
No Input Shocks				

Table A5: Correlations in $d\ln\xi_{nt}$ summary statistics

	Mean	Median	25th pctile	75th pctile
	G7 Countries (N. obs. = 21)			
$\rho = 2.75$	0.169	0.195	0.043	0.326
$\rho = 1$	0.161	0.203	-0.008	0.376
	All countries (N. obs. = 406)			
$\rho = 2.75$	0.010	0.046	-0.215	0.238
$\rho = 1$	0.017	0.029	-0.215	0.262

Notes: This table presents the summary statistics of the correlations of $d\ln\xi_{nt}$ defined in (4.10) in the sample of G7 countries (top panel) and full sample (bottom panel), when ψ_u varies by industry. Variable definitions and sources are described in detail in the text.

Table A6: Correlations of unweighted shock country averages summary statistics, $\rho = 2.75$

	Mean	Median	25th pctile	75th pctile
$\rho = 2.75$	G7 Countries (N. obs. = 21)			
TFP	0.041	0.076	-0.141	0.262
Non-technology ($\psi_u = 4$)	0.132	0.115	-0.025	0.281
Non-technology ($\psi_u = 1.01$)	0.133	0.154	-0.037	0.257
	All countries (N. obs. = 406)			
TFP	-0.001	0.023	-0.239	0.214
Non-technology ($\psi_u = 4$)	0.036	0.061	-0.158	0.254
Non-technology ($\psi_u = 1.01$)	0.023	0.042	-0.183	0.245

Notes: This table presents the summary statistics of the correlations of the average TFP and non-technology shocks in the sample of G7 countries (top panel) and full sample (bottom panel).