An Equilibrium Labor Market Model with Internal and External Referrals

Youze Lang and Youzhi Yang*

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Abstract

More than one-third of workers find their jobs through friends, relatives and acquaintances. Based on Job Search Survey, we show that the jobs found through informal methods pay more (less), compared to the ones found through formal methods, if the informant knows someone (does not know anyone) at the firm. In order to account for this new observation, a new mechanism is introduced into the classical Burdett and Mortensen (1998) on-the-job search model, through which workers can share job openings information with each other. In equilibrium, the distribution of wages offered is non-degenerate. The model, when calibrated to the monthly U.S. labor market data, is able to account for the percentage of jobs through informal methods, and a large part of the corresponding effects on wages. By introducing informal methods, the model's ability to account for the observed wage dispersion also improves substantially. The use of informal methods enlarges wage dispersion and wage differentials across firm size and search methods. Restricting informal methods would make both unemployed and employed workers worse off. Furthermore, the current unemployment insurance system reduces the wage differential between the jobs found through formal and informal methods, and improves social welfare.

1 Introduction

The role of social network in the labor market has been long recognized by economists and it is well known that more than one-third of workers find their jobs through friends, relatives and acquaintances (Ioannides and Loury, 2004; Topa, 2011). In the seminal work by Granovetter (1995), 56% of respondents find their current jobs through informal methods. Corcoran et al. (1980) study a dataset from the Panel Study of Income Dynamics (PSID) and put that figure between 52% to 58% for male workers under the age of 45. For the European data, Pellizzari (2010) analyses a panel dataset of European households and documents that between 25% and 45% of workers find their jobs through family, friends or other contacts.

*School of Economics, Shanghai University of Finance and Economics; Lang: falelang@163.shufe.edu.cn; Yang: yang.youzhi@mail.shufe.edu.cn;

However, whether the jobs found through informal methods, compared to the ones found through formal methods, pay more or less varies across empirical studies. On one hand, most studies argue that job seekers who use informal methods earn higher wages, such as Burks et al. (2015), Brown et al. (2016) and Dustmann et al. (2016). Such informal methods serve as an information transmission mechanism and, therefore, have the potential to enhance the efficiency of labor market by reducing uncertainty about match quality. Besides, Heath (2018) indicates that firms use referrals from their employees to mitigate moral hazard problems. Referrals can help firms select unobservably good workers, thus being offered higher wages. On the other hand, some studies emphasize the negative effect of informal search methods on wages (Bentolila et al., 2010; Loury, 2006; Chen et al., 2018). The use of informal methods sends a negative signal for workers who rely on informal information sources as a last resort, by implying that those workers have limited access to jobs through formal methods.

Our paper provides a new perspective to discuss wage differential, making a distinction between two different types of informal methods according to the status of informant. Based on Job Search Survey, conducted by Federal Reserve Bank of New York as a supplement to Survey of Consumer Expectations, informal search methods can be further divided into two types: one where the referee is a worker or a manager at this employer, which we call internal referral; and one where the referee does not know anyone at this employer but knows about the job opening, which we call external referral.

In comparison with the jobs found through formal methods, wage premium of 4.75% occurs for the jobs found through internal referral, while wage penalty of 11.6% occurs for the ones through external referral. For the overall effect of informal methods on wages, it depends on the proportion of workers who use internal and external referral to find jobs, respectively. Our survey data, in which the jobs found through referral account for the majority of 93.48%, shows that the jobs found through informal methods exhibit 3.76% higher wages than the ones found through formal means, which is consistent with the estimation of the wage premium of 3.3% in Dustmann et al. (2016).

In order to account for this new observation, we introduce a two-stage search process in our model. In the first stage, the workers (both unemployed and employed) and firms are matched through a constant-return-to-scale matching function, as in a standard search model. In the second stage, the workers randomly run into each other, then share the job opening information obtained in the first stage. This information sharing mechanism is incentive-compatible since the workers only share the information not valued by themselves. Specifically, an employed worker who knows about a job opening (paying the same wage) at his/her own employer has incentives to share this information with whoever he/she meets in the second stage. And, an employed worker who receives information (through on-the-job search) about a job opening paying less than his current job also has incentives to share this information. The formal corresponds to internal referral, and the latter corresponds to external referral, as defined in the data. The firms post fixed-wages contracts to recruit workers.

We show that frictional wage dispersion arises in equilibrium. The increase in wages offered by firms reduces the flow profit per period, however prolongs the duration of employment relationship and shortens the duration of vacancies. Such a tradeoff guarantees that the steady state equilibrium can be characterized by non-degenerate distributions of wages offered and earned.

In theoretical literature, the effect of informal methods has been widely studied from the perspective of network and graph theory. Calvó-Armengol and Jackson (2004, 2007) and Zenou (2015) develop a partial equilibrium framework in which the implications of exogenous job finding rate and information networks are explored. Calvó-Armengol and Zenou (2005) and Fontaine (2008) study the network effect in a general equilibrium framework where the number of firms is endogenously determined by free entry condition. These models discuss the outcomes of informal methods under fixed wage distributions. On the contrary, the distribution of wages offered in our model is endogenously determined by the firms, who fully take into consideration the fact that the workers could share the job opening information with each other. Hence, our model is able to discuss how the fact that the workers use informal methods to find jobs affects the distribution of wages offered by the firms.

Our model, when calibrated to the monthly U.S. data, is able to account for the percentage of jobs through informal methods, and a large part of the corresponding effects on wages. In the calibrated model, 40.34% of jobs are found through informal method, and pay 3.01% more than jobs through formal methods, which accounts for about 80% of wage premium observed in Job Search Survey. Compared to jobs found through formal methods, jobs through internal referral pay 3.53% more, while jobs through external referral pay 1.95%. By introducing informal methods, the model's ability to account for the observed wage dispersion also improves substantially. The mean-min ratio is 1.46, which is larger than 1.27 as attained in the standard on-the-job search models.

Hornstein et al. (2011) use the mean-min ratio to measure frictional wage dispersion and show that existing search models cannot generate a reasonable frictional wage dispersion. Even though considering the model with on-the-job search, the mean-min ratio is between 1.16 and 1.27, which is substantially lower than that of the one between 1.5 and 2 as observed in the data. The key reason is that when unemployed workers face large wage dispersion in the labor market, they would perceive higher option values of waiting for higher paying jobs. However, in the data, the observed duration of unemployment spells is very short: only about 2.5 months. To solve this puzzle, Wang and Yang (2018) develop a dynamic wage-tenure contract to discuss frictional wage dispersion analytically in which outside offers are public or private information imposed on the firm, respectively. Incorporating wage tenure contract and information friction can produce frictional wage dispersion that resembles the data not only with mean-min ratio but also with the shape for the observed distributions.

This paper tackles the puzzle by introducing informal methods into the classical Burdett and Mortensen (1998) on-the-job search model. The use of informal methods gets the wage support stretched, and causes asymmetric effects between the highest and lowest paying jobs. Specifically, firms paying the highest obtain strictly positive profits due to an increase in the acceptance probability, pushing up the upper bound of the wage support. Firms paying the lowest obtain strictly negative profits mainly caused by the reduction in both the acceptance and retention probability, pulling down the lower bound of the wage support. Allowing for informal methods, employed workers climb up the job ladder faster, leading to a higher average wage earned. As a result, the model arises a higher mean-min ratio of 1.46 compared to the standard on-the-job search model.

Counterfactual analyses indicate that the increase in the contact probability enlarges the wage dispersion and wage differentials across firm size and search methods. Restricting informal methods would decline the E-E transition probability and pull down the average wage earned, making both unemployed and employed workers worse off. Furthermore, the current unemployment insurance system substantially reduces the wage differentials between the jobs found through formal and informal methods, and improves social welfare.

This paper is also related to the literature on combining a search model with social network, such as Galenianos (2014), Igarashi (2016) and Schmutte (2016). Differing from Galenianos (2014) where vacancies are created through the expansion of producing firms, in our model, each worker has an opportunity to share job information with the individuals he/she meets. Igarashi (2016) extends Galenianos (2014) with heterogeneous workers (networked and non-networked) and argues that referral-restricting policy on non-networked agents would make all workers worse off. This is consistent with the policy implication in our calibrated model, but our approach is to extend these studies to a setting where the job information can be transmitted among employed workers and frictional wage dispersion arises in equilibrium. Furthermore, our model is tractable and easily extended to a general environment to discuss the effect of informal contacts and the wage differential between jobs found through formal and informal methods.

The rest of this paper is organized as follows. In Section 2, we estimate the effects of informal search methods on wages based on Job Search Survey. Section 3 introduces informal search methods into a two-stage search model and characterizes the equilibrium patterns. Section 4 shows the equilibrium results. In section 5, we calibrate the model to the monthly U.S. labor market data, and then investigate the effects of informal methods on wage dispersion and wage differentials. Section 6 discusses two counter-factual analyses about informal methods and unemployment insurance. Finally, Section 7 contains the concluding remarks. All proofs and tables for detail are relegated to the Appendix.

2 Empirical Evidence

In this section, we first review the literature on the usage of informal search methods and their impact on wage earnings. We then describe the data from Job Search Survey, specially how the informal methods are divided into two categories: internal and external referrals. Finally, we estimate the effect of informal search methods on wage earnings.

2.1 Related Literature

Rees (1966) first points out that both employers and job seekers prefer to use informal search methods rather than formal ones. Since then, an extensive body of literature has developed about the effect of informal methods on job search process. Ioannides and Loury (2004) review both theoretical and empirical research and organize stylized facts about how the outcomes are influenced by social interaction. The literature has established that there is widespread use of friends, relatives and acquaintances to search for jobs (Loury, 2006; Pellizzari, 2010; Topa, 2011). Not only for job seekers, employers also use social network to recruit workers and fill job vacancies. Holzer (1987b) reports that 36% of firms surveyed by the Employment Opportunity Pilot Project (EOOP) use referrals from employees in their recruiting. Marsden and Gorman (2001) find that between 37% and 53% of establishments use referrals from current employees when posting vacancies.

Many studies also explore that the usage of informal methods varies by demographic characteristics. There exists a robust consensus in the literature that less educated job seekers are more likely to use friends and relatives to search for jobs (Ioannides and Loury, 2004). Job seekers with lower education have more incentives to join a network in order to obtain information about job openings by virtue of the fact that they are associated with higher probability of job loss (Elsby et al., 2010). With regard to gender and racial differences, the conflicting patterns are explored. Men are more likely to use informal methods to find jobs than women (Corcoran et al., 1980; Ports, 1993; Smith, 2000), while Moore (1990) suggests that women are equally likely to find jobs through informal methods relative to men. Many studies, such as Corcoran et al. (1980) and Datcher (1983), report higher usage of informal methods by Blacks than Whites, yet Holzer (1987a) indicates that the fraction of using social networks does not differ significantly between Blacks and Whites. Finally, with regard to age, Ports (1993) documents that about 20% of 16-24 year-olds use friends or relatives to find jobs compared to 26.5% of 45-64 year-olds. On the other hand, Corcoran et al. (1980) and Marsden and Hurlbert (1988) argue that the probability of using informal methods declines with age.

The empirical results about the effect of informal search methods on wages are also mixed. The bulk of the evidence goes in the direction of supporting the positive wage effect. Some studies focus on learning theory in which the use of friend and relatives can reduce uncertainty about match quality, such as Simon and Warner (1992), Galenianos (2013) and Brown et al. (2016). By linking two German household surveys, Dustmann et al. (2016) find that referrals raise employees' staring wages by around 3.3%. This positive wage effect declines with tenure at an annual rate of 1.7%. On the other hand, Montgomery (1991) and Casella and Hanaki (2008) suggest that the positive effect is consistent with homophily theory, the pervasive tendency of workers to associate with those like themselves.

In contrast to the literature that emphasizes the positive impact on wages, Bentolila et al. (2010) argue that the use of informal search methods give rise to low match quality and negative externalities on aggregate productivity. Job seekers would accept lower wages since they would sacrifice their productive advantages in order to obtain information about job openings quickly. Chen et al. (2018) examine the wage effect of utilizing informal methods for rural migrants in urban China and find that informal methods send negative signals of workers' ability to be hired, lending support to the mechanism hypothesized by Bentolila et al. (2010).

Furthermore, the mixed wage effect has also been emphasized. Loury (2006) argues that better matches and limited choices are simultaneously valid for different types of contacts, therefore giving rise to both positive and negative effects. Pellizzari (2010) uses a panel dataset of European households and indicates that the wage premium and wage penalty to finding a job through informal methods are equally frequent across countries. Zaharieva (2013) shows that the direction of wage effects are associated with the bargaining power between employers and job seekers.

2.2 Job Search Survey

Our data comes from Job Search Survey, which is a supplement to Survey of Consumer Expectation (SCE) conducted by Federal Reserve Bank of New York. The SCE is a monthly national survey of roughly 1,300 individuals, while the supplement in the labor market has been fielded every October since 2013. We use the available data from 2013 to 2016 on job search behavior, employment status, current earnings and the search methods that job seekers have used.

Respondents are asked how they acquire their current jobs. We categorize the search methods into two by whether the job information comes through personal contacts. Informal sources include searching for jobs through friends, relatives, former co-workers and business associates, etc. Formal resources include employment agencies, employers' websites, career centers and professional registers, etc. A prominent feature of this survey is that it probes further to collect the information on informants. Therefore, we can make a further distinction about informal search methods based on the status of informants. One is called internal referral, where the informant is a manager or worker at this employer or knows someone at this employer personally. And the other one is called external referral, where the informant does not know anyone at this employer personally, but knows about the job opening.

The samples are restricted to currently employed individuals aged 18-64 with valid information on their search methods and reported current wages, excluding the self-employed and someone working part-time. For respondents' demographic data, we match them with the samples through the monthly portion of SCE survey, including gender, age, education dummies and race dummies.

Table 1 summaries the usage of search methods reported by respondents. In the survey, 59.57% of respondents use formal methods to acquire information on their current jobs. For informal methods, 37.80% of respondents report that their informants have social ties with someone at the employers, therefore finding their jobs through internal referral according to our definitions, while 2.64% of respondents are hired through external referral. For our quantitative analysis, we will calibrate the model to these key features.

The summary statistics in Table 1 show that there exists many differences in the way of the usage of search methods across demographic characteristics. First, less educated workers are more likely to find jobs through informal methods, both internal and external referrals. With regard to gender and race, higher usage of formal methods and external referral by females and Blacks, while males and Whites are more likely to use internal referral. Finally, with regard to age, respondents who are aged with between 18 and 30 year-olds are more preferable with external referral and those with between 41 and 60 year-olds are more likely to be hired through internal referral. For those with between 41 and 64 year-olds, formal methods are most commonly used[1].

In the Job Search Survey, respondents report their nominal earnings either as hourly, weekly or annual. In order to convert into hourly earnings, we divide weekly earnings by reported usual hours per week and divide the annual earnings by 52 weeks and reported usual hours. Real hourly earnings are then calculated by using the annual Consumer Price Index (CPI). Furthermore, we calculate the tenure of current jobs by linking the

[1]More details about the usage of search methods across demographic characteristics are available in Appendix Table H.1.

Table 1: The Usage of Search Methods (%)

	Formal	Internal Referral	External Referral
Sample freq.	59.57	37.80	2.64
Education			
High school	55.75	40.45	3.8
Some college	62.8	35.22	1.99
College	60.24	37.24	2.51
Graduate	59.84	38.3	1.85
Gender			
Females	61.49	35.1	3.41
Males	57.77	40.32	1.91
Race			
Whites	58.76	39.01	2.23
Blacks	61.17	35.27	3.56
Others	63.61	31.79	4.6
Age			
$18 \sim 30$	55.1	41.56	3.35
$31 \sim 40$	55.76	41.72	2.52
$41 \sim 60$	62.93	34.42	2.65
$61 \sim 64$	60.89	38.04	1.07
Observations	1335	808	52

Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. The sample frequency is calculated by sampling weights.

reported date that the respondent starts working at his current job and the date surveyed.

Table 2 mainly reports the summary statistics about wage differential by search methods. We focus on the comparison of the average wages obtained by respondents finding jobs through different methods. The basic statistics describes that jobs found through internal (external) referral pay more (less) than jobs found through formal methods. However, this new observation is mostly robust across demographics. Specifically, among 13 subgroups across education, gender, race and age, the jobs found through internal referral pay more in 9 subgroups, and the jobs found through external referral pay less in 12 subgroups.

Finally, the survey also documents the impact of informal methods on the duration of matches and usual hours. Longer duration of matches for jobs obtained through informal methods, both internal and external referrals, relative to formal methods. Respondents found jobs through external referral work 2.29 less hours per week than those through formal methods, whereas there is no apparent difference between those found jobs through internal referral and formal methods.

Table 2: Descriptive Evidence on Wage Differential

	Formal	Referral			Full Samples
		All	Internal	External	
Current Job					
Hourly wages	25.65	26.45	26.88	20.24	25.97
Usual hours	42.75	42.61	42.76	40.46	42.69
Tenure year	8.04	9.27	9.06	12.32	8.53
Education					
High school	18.03	17.76	17.94	15.93	17.91
Some college	21.61	24.10	24.32	20.24	22.54
College	30.26	32.25	32.87	23.15	31.05
Graduate	39.05	39.17	39.59	30.43	39.10
Gender					
Female	22.43	21.24	21.47	18.91	21.97
Male	28.85	30.89	31.29	22.46	29.71
Race					
Whites	25.57	26.70	27.10	19.65	26.04
Blacks	21.31	21.44	20.88	26.91	21.36
Others	29.72	29.17	30.83	17.71	29.52
Age					
$18 \sim 30$	21.64	22.64	23.12	16.64	22.08
$31 \sim 40$	26.20	25.27	25.65	18.92	25.78
$41 \sim 60$	26.82	28.45	28.89	22.67	27.43
$61 \sim 64$	23.77	28.89	29.25	16.10	25.77
Observations	1335	860	808	52	2195

Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. Hourly wages are adjusted by CPI. Usual hours are hours per week working at the current jobs reported by respondents.

2.3 Empirical Strategy

In the survey, more than 40% of workers find their current jobs through informal methods. And, the jobs found through internal referral pay more than 4% more, while the jobs found through external referral pay more than 20% less, compared to those through formal methods. To formally test the prediction about the wage effects, we estimate the following regression:

$$y_i = \beta_0 + \beta_1 IntR_i + \beta_2 ExtR_i + \beta_3 IntR_i \cdot Tenure_i + \beta_4 ExtR_i \cdot Tenure_i + X_i'\gamma + \varepsilon_i \quad (1)$$

where y_i is the log real hourly wage of worker i, $IntR_i$ is an indicator variable which equals to 1 if the worker learns about information on the current job through internal referral. $ExtR_i$ is an indicator variable which equals to 1 if the worker learns through external referral. X_i is a vector of control variables, including tenure, tenure squared, education dummies, demographic controls and year dummies, and ε_i is an unobserved error term.

Table 3: The Impact of Internal and External Referrals on Wages

	(1)	(2)	(3)	(4)
Int. Referral	0.0483*	0.0450**	0.0475**	0.0908***
	(0.0262)	(0.0226)	(0.0225)	(0.0327)
Ext. Referral	-0.1458**	-0.115*	-0.116*	-0.188**
	(0.0688)	(0.0675)	(0.0676)	(0.0952)
Tenure		0.0110***	0.0232***	0.0245***
		(0.0015)	(0.0037)	(0.0038)
$(\text{Tenure})^2/100$			-0.0433***	-0.0412***
			(0.0124)	(0.0121)
Tenure × Int. Referral				-0.0051*
				(0.0028)
Tenure × Ext. Referral				0.0073
				(0.0067)
Demographic Controls	No	Yes	Yes	Yes
Year Dummies	No	Yes	Yes	Yes
Observations	2195	2195	2195	2195

Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. Hourly wages are adjusted by CPI. Demographic controls include gender, age, race and education categories.

For the internal referral method, the key parameters of our interest are β_1 and β_3, where β_1 measures the impact through internal referral on the worker's starting wages (tenure=0), and β_3 measures how this impact changes with tenure. Likewise, for the external referral method, β_2 measures the impact through external referral on the starting wages and β_4 measures how the impact of external referral on wages changes with tenure.

Results are documented in Table 3. We start by estimating the wage effect of informal search methods without considering the impact of tenure. Ignoring the demographic controls and year dummies, column (1) shows that workers who find jobs through internal referral earn 4.83% more, whereas workers who find jobs through external referral earn 14.58% less than those through formal methods. In column (3), after controlling tenure and tenure squared, the average wage premium for internal referral is 4.75% and wage penalty for external referral is 11.6%.

The result in column (4) provides an evidence that jobs found through internal referral exhibit 9.08% higher starting wages than jobs found through formal methods, which is positive and statistically significant. The wage effect of internal referral declines with tenure at the rate of 0.51%. On the other hand, the use of external referral reduces starting wages about 18.8% compared to formal methods. The wage effect of external referral is negative and diminishes with tenure at the rate of 0.73%. The reduction in the wage differential could come from the fact that firms and workers learn each other over time. But the effect is long-lasting since it takes about 17.8 (25.75) years on the job for the effect of internal (external) referral to disappear.

Therefore, the evidence shows that jobs found through internal (external) referral pay more (less) than jobs found through formal methods, after controlling the tenure and demographics. Next, we integrate the samples of jobs found through internal and external referrals into one group and investigate the effect of informal methods on wages as follows.

$$y_i = \beta_0 + \beta_1 Informal_i + \beta_2 Informal_i \cdot Tenure_i + X_i'\gamma + \varepsilon_i \tag{2}$$

In the regression (2), y_i represents the log real hourly wages of worker i. $Informal_i$ is an indicator variable which equals to 1 if the worker learns about information on the current job through informal methods, either internal or external referral.

Table 4 shows that on average, workers finding jobs through informal methods earn 3.76% more than those through formal methods. By considering the interaction between the tenure and informal indicator, the result in column (3) shows that the jobs found through informal methods pay 7.55% higher starting wages than the ones found through formal methods. The positive effect on wages diminishes with tenure at the rate of 0.45%.

Differing from the existing literature about the impact of informal methods on wages, a new observation in the survey is that informal search methods can be further divided into two types: internal and external referrals. Compared to the jobs found through formal methods, positive wage effect of 4.75% occurs for the ones found through internal referral, while negative wage effect of 11.6% occurs for the ones found through external referral. Therefore, for the overall effect of informal methods on wages, it depends on the proportion of workers who use internal and externals to find jobs, respectively. In the samples of larger proportion of those through internal referral, workers hired through informal methods earn higher wages than those through formal methods, which is consistent with the results in most studies, such as Loury (2006), Brown et al. (2016) and Dustmann et al. (2016). Our survey data, in which those who use internal referral methods account for the majority of 93.48%, shows that the jobs found through informal

Table 4: The Impact of Informal Methods on Wages

	(1)	(2)	(3)
Informal	0.0457**	0.0376*	0.0755**
	(0.0224)	(0.0221)	(0.0321)
Tenure		0.0231***	0.0244***
		(0.0037)	(0.0038)
$(\text{Tenure})^2/100$		-0.0430***	-0.0411**
		(0.0123)	(0.0121)
Tenure × Informal			-0.0045*
			(0.0027)
Demographic Controls	Yes	Yes	Yes
Year Dummies	Yes	Yes	Yes
Observations	2195	2195	2195

Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. Hourly wages are adjusted by CPI. Demographic controls include gender, age, race and education categories.

methods pay 3.76% higher wages, very close to the estimation of 3.3% in Dustmann et al. (2016).

To consolidate the results, we make two robust checks in Table H.3. First, we drop the samples associated with other search methods in Table H.1, including temporary or part-time job converted into full-time job, within-company promotion or transfer and family business, etc. The result in column (2) shows that compared to workers finding jobs through formal methods, those through internal referral earn 9.17% higher starting wages, whereas those through external referral earn 17.82% lower starting wages. Both wage effects diminish with tenure.

The second robust check is that we use starting wages reported by respondents as the dependent variable instead of current wages. After dropping invalid samples about starting wages, the observations fall to 1972. The result in column (3) shows that wage premium occurs for the jobs through internal referral, while wage penalty occurs for the ones through external referral. The direction of wage effects still holds, although the measurement errors of long backtracking data give rise to a reduction in the magnitude and significance about the effect of informal methods on wages.

3 Model

Time $t = 0, 1, 2\ldots$ is discrete. The economy is populated by a continuum, with unit-mass, of identical workers who can be either unemployed or employed. Workers are risk averse. There is a single perishable consumption good in the economy. All workers have the following preferences:

$$\mathbb{E}_\tau \left[\sum_{t=\tau}^{\infty} \beta^{t-\tau} u(c_t) \right]$$

where $\mathbb{E}_\tau$ denotes the worker's expectation conditional on information available at the beginning of period τ and $\beta \in (0, 1)$ denotes the discount factor. $c_t \in \mathbb{R}_+$ denotes the worker's consumption. We assume that the utility function is bounded, strictly increasing, strictly concave, twice differentiable and satisfies the Inada conditions.

The economy also has a continuum of identical firms who share the same discount factor β with the workers. Firms are risk neutral and maximize their expected profits. Any firm must incur a fixed cost of $k \geq 0$ units of the good to post a vacancy.

Labor market opens at the beginning of each period. All the workers and firms enter the labor market. Firms post vacancies to recruit workers, while unemployed and employed workers search for information about job openings. Time line is described in Figure 1.

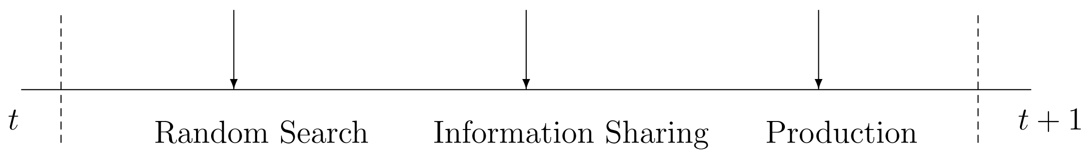

Figure 1: Timing of events in a single period

In each period, the search process is divided into two stages. The first stage is random search. The workers and firms are matched through a constant-return-to-scale matching function $M(1, v) \in [0, \min\{1, v\}]$, where 1 is the measure of workers (both unemployed and employed) and v is the measure of vacancies posted by firms. The second stage is information sharing, in which the environment is just like a network of social relationships. The workers randomly run into each other[2], then share the job openings information obtained in the first stage.

After that, each worker could have learned about at most two types of job openings information: one from the firm he/she is matched with in the first stage of search, and one from the worker he/she meets in the second stage of search. The worker then decides which job to accept. If the worker accepts the job learned directly from the firm, then he/she finds the job through formal methods. If the worker accepts the job learned from another worker, then he/she finds the job through informal methods.

When the labor market closes, the firm that has recruited a worker begins to produce a constant output, denoted as $\theta > 0$. Each unemployed worker receives unemployment benefit $b \geq 0$ and each employed worker receives the compensation $w \geq 0$ committed by the wage contract. At the beginning of the next period, each employed worker is separated from his current job with an exogenous probability $\lambda \in [0, 1]$ and reverts to an unemployed worker.

Assumption 1. *(Limited Liability) Compensation to the worker must be non-negative.*

[2]This setting is consistent with the view of weak ties that are more useful for transmitting about job information than strong ties, as in Granovetter (1995). The worker has no idea about which of his contacts would share job opening information and hence have contacts with each other at random.

Assumption 2. *(Limited Commitment) In each period, the worker is free to walk away from the contract. But the firm is fully committed to the terms of any contract it offers.*

Assumption 3. *The firm does not respond to any outside offer received by its employee.*

Note that the assumptions are consistent with Burdett and Mortensen (1998) and Burdett and Coles (2003). To simplify the analysis and pay attention to the implication of informal search methods, our model abstracts away from counteroffers[3]. In such an environment, the employment relationship could be terminated in two scenarios. Associating with on-the-job search, the employed worker can quit voluntarily to take a higher paying outside offer. The other one is involuntary when the exogenous separation occurs.

3.1 The First Stage of Search

The labor market is frictional. Let $u \in [0, 1]$ denote the measure of unemployed workers and $1 - u \in [0, 1]$ denote that of employed workers. In the first stage of search process, matchings are random. An unemployed (employed) worker is matched with a vacancy with probability p_w^u (p_w^e). The total number of matches is determined by the matching function $M(1, v)$, which satisfies,

$$M(1, v) = up_w^u + (1 - u)p_w^e \tag{3}$$

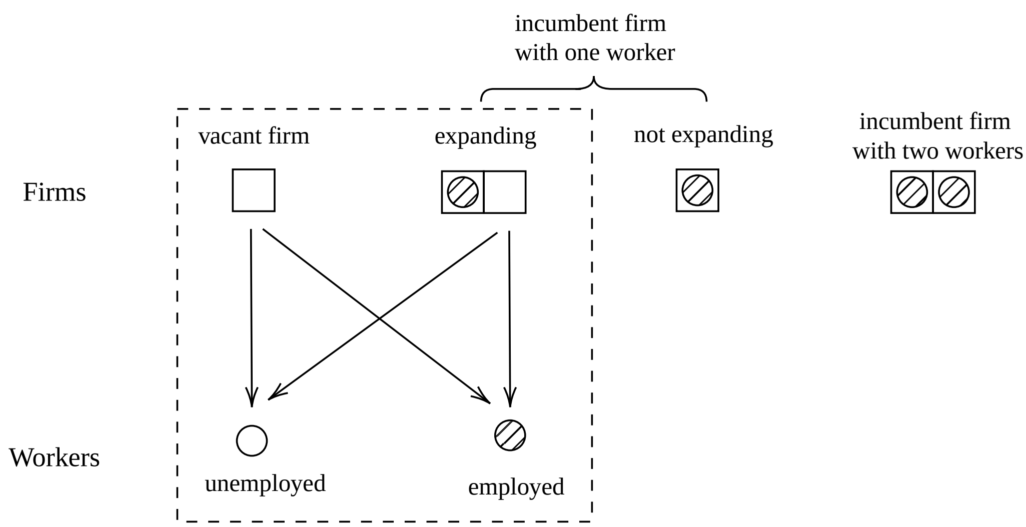

Figure 2: The flow of job openings information in the first stage

[3]This restriction is relaxed in Wang and Yang (2018), where both workers and firms are homogenous. Wang and Yang (2018) construct a dynamic wage-tenure contract to discuss wage dispersion analytically in which outside offers are public or private information imposed on the firm, respectively.

Each firm can hire at most two workers, but post only one in each period[4]. The probability with which a vacancy is matched with a worker is

$$p_f = \frac{M(1, v)}{v} \in [0, 1] \tag{4}$$

As shown in Figure 2, the vacancies are posted by both vacant and part of incumbent firms. The latter ones are only filled with one worker and choose to expand[5], which we call expanding firms. Let v_0 denote the measure of vacant firms and v_1 denote the measure of expanding firms. For employed workers, let $e_1(e_2) \in [0, 1-u]$ denote the measure of employed workers at incumbent firms with one worker (two workers), respectively.

Before the labor market opens in each period, there are two distributions of vacant and expanding firms in the starting wages offered for new hires. The support of those offered is denoted by Φ, which is the set of wages that firms are able to offer and deliver in equilibrium. For each $w \in \Phi$, let $F_0(F_1) : \Phi \to [0, 1]$ denote the fraction of vacant firms (expanding firms) that post wages no greater than w. Assume both F_0 and F_1 have density functions $f_0, f_1 : \Phi \to \mathbb{R}_+$. Therefore, the distribution of wages offered by both vacant and expanding firms, denoted as $F : \Phi \to [0, 1]$, is determined by

$$F(w) = \frac{v_0}{v_0 + v_1} F_0(w) + \frac{v_1}{v_0 + v_1} F_1(w) \tag{5}$$

Before the labor market opens, there are also two distributions of employed workers in the wages that their employers have promised to deliver. Let $G_1(G_2) : \Phi \to [0, 1]$ denote the fraction of employed workers at incumbent firms with one worker (two workers) in terms of wages no greater than w, for all $w \in \Phi$. Assume both G_1 and G_2 have density functions $g_1, g_2 : \Phi \to \mathbb{R}_+$. Therefore, the distribution of wages earned in the labor market, denoted as $G : \Phi \to [0, 1]$, is determined by

$$G(w) = \frac{e_1}{e_1 + e_2} G_1(w) + \frac{e_2}{e_1 + e_2} G_2(w) \tag{6}$$

At the end of the first stage, if not matched with any worker, the expanding firms can ask their employees to refer links and release the job openings information into the labor market.

3.2 The Second Stage of Search

In the second stage of search, each worker is in direct contact with another worker with probability $\mu \in [0, 1]$. When a worker is paired with another worker successfully, they are going to play a one-shot game to decide whether to share the job openings information they have obtained.

Consider one currently unemployed worker who is matched with a vacancy in the first stage of search. The unemployed worker always has incentives to retain the job information, provided the wage offered is no less than the reservation. That is, not

[4] This restriction can be relaxed to an environment, in which each vacant firm can post either one or two vacancies. In the Appendix G, we show that posting one vacancy in each period strictly dominates posting two vacancies for any vacant firm in equilibrium.

[5] Within the firm, the workers receive the same compensation and follow equal treatment.

passing any acceptable information is the weak dominant strategy for the unemployed worker.

An employment worker can receive at most two types of job openings information in the first stage. For the internal one (paying the same wage) received from his/her own employer's asking for referral, the employed worker has incentives to pass the job information on to the worker he/she meets. For the external one received through on-the-job search, if its value is below the employed worker's current earned, then the job information would also be passed on to the worker contacts. Otherwise, the employed worker has incentives to retain the higher paying job information and recommend his/her current job opportunity to the worker contacts[6].

This information sharing mechanism is incentive-compatible since the workers only share the information not valued by themselves. As a result, in the second stage, a job seeker can definitely acquire job information if having contact with an employed worker who has received job information, either internal or external. In other words, the job seeker would receive job opening information through informal methods since the job learned from another worker he/she meets.

During information sharing, informal methods can be further divided into two types according to the status of informant, as defined in the data: one where the informant works at the hiring firm, which we call internal referral; and one where the informant does not work at the hiring firm, but knows about the job opening information, which we call external referral.

Figure 3 exhibits the flow of job openings information in the second stage. Specifically, an employed worker who knows about a job opening at his/her own employer shares this information with whoever he/she meets. And, an employed worker who receives information (through on-the-job search) about a job opening paying more than his/her earned, however, retains this valued information and in turn shares his/her current job information. Both the two channels correspond to internal referral since the informant works at the hiring firm. An employed worker receives information (through on-the-job search) about a job opening paying less than his/her current earned, and then shares the information with whoever he/she meets, which corresponds to external referral.

Let $p_w^R(p_w^C)$ denote the probability with which a worker receives job opening information through internal (external) referral in the second stage of search. For the internal referral, the matching probability is

$$\begin{aligned} p_w^R = & \mu e_1 \iint \Big\{ \eta(w_2)(1-p_f) + [1-\eta(w_2)(1-p_f)] p_w^e I(w_2' > w_2) \Big\} dG_1(w_2) dF(w_2') \\ & + \mu e_2 \iint p_w^e I(w_2' > w_2) dG_2(w_2) dF(w_2') \end{aligned} \tag{7}$$

where I is the indicate function and $\eta(w) = v_1 f_1(w)/(e_1 g_1(w)) \in [0,1]$ is the probability of choosing to expand for incumbent firms with one worker in terms of wages w. In the second stage of search, two types of job openings information can be shared as internal referral. As shown in Figure 3, one originates from the expansion of firms. An expanding

[6] If the worker contacts finally accepts the job, then the vacancy after the employed worker quits will be filled directly without incurring posting cost. In the literature, this feature is called replacement hiring.

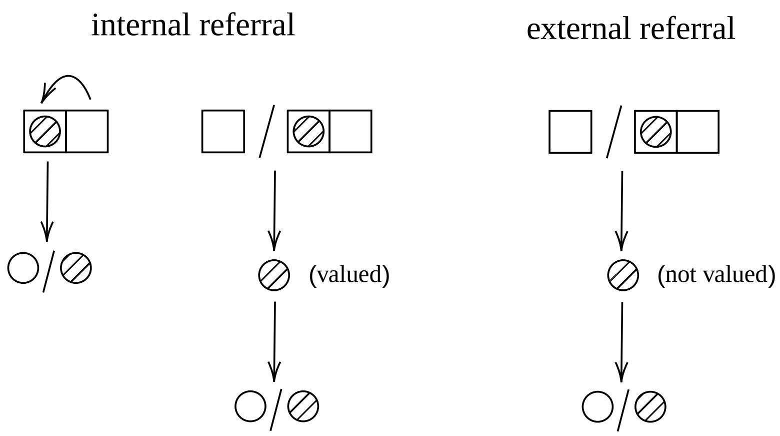

Figure 3: The flow of job openings information in the second stage

firm is not matched with any worker in the first stage, and then asks its employee to pass the job information on to another worker he/she meets. The other one comes from the employee's current job. If an employed worker receives information (through on-the-job search) about a job opening paying more than his/her earned, then his/her current job information is inferior and passed on to whoever he/she meets.

For the external referral, the matching probability is

$$\begin{aligned} p_w^C =& \mu e_1 \iint [1-\eta(w_2)(1-p_f)]\, p_w^e I(w_2' \leq w_2) dG_1(w_2) dF(w_2') \\ &+ \mu e_2 \iint p_w^e I(w_2' \leq w_2) dG_2(w_2) dF(w_2') \end{aligned} \tag{8}$$

According to the information sharing protocol demonstrated above, a job seeker knows about the external information by having contact with an employed worker who receives information (through on-the-job search) about a job opening paying less than his/her earned. Meanwhile, the employed worker does not receive information about job expansion from his/her hiring firm.

We now define the conditional distributions of wages offered through internal and external referrals. For each $w \in \Phi$, $F_R, F_C : \Phi \to [0,1]$ represent the distributions of wages offered no greater than w, conditional on the job openings information is received through internal and external referrals respectively, which are determined by the following equations.

$$\begin{aligned} p_w^R F_R(w) =& \mu e_1 \iint \Big\{ \eta(w_2)(1-p_f) + [1-\eta(w_2)(1-p_f)] p_w^e I(w_2' > w_2) \Big\} I(w_2 \leq w) \\ & dG_1(w_2) dF(w_2') + \mu e_2 \iint p_w^e I(w_2' > w_2) I(w_2 \leq w) dG_2(w_2) dF(w_2') \end{aligned} \tag{9}$$

and

$$p_w^C F_C(w) = \mu e_1 \iint [1-\eta(w_2)(1-p_f)]\, p_w^e I(w_2' \le w_2) I(w_2' \le w) dG_1(w_2) dF(w_2') \\ + \mu e_2 \iint p_w^e I(w_2' \le w_2) I(w_2' \le w) dG_2(w_2) dF(w_2') \tag{10}$$

Both the conditional distributions of wages offered through internal and external referrals are endogenous with a combination of the distributions of wages offered $F(w)$ and earned $G_1(w)$ and $G_2(w)$. This differs from Mortensen and Vishwanath (1994) in which an offer obtained through a contact is merely a draw from the distribution of wages earned.

3.3 Workers

The model is studied in the case of fixed wage contract, in which the compensation does not depend on an employee's tenure at the firm.

We denote the expected utility of an unemployed worker in equilibrium by V_U and the expected utility of being an employed worker with wages earned w by $V_E(w)$. Considering informal search methods, an unemployed worker's value function is

$$\begin{aligned} V_U = u(b) &+ \underbrace{\beta(1-p_w^u)(1-p_w^R-p_w^C)V_U}_{\text{staying unemployed}} \\ &+ \underbrace{\beta p_w^u \int_{\underline{w}}^{\overline{w}} \Big[1-p_w^R(1-F_R(w_1'))-p_w^C(1-F_C(w_1'))\Big] V_E(w_1') dF(w_1')}_{\text{finding a job through formal methods}} \\ &+ \underbrace{\beta p_w^R \int_{\underline{w}}^{\overline{w}} \Big[1-p_w^u(1-F(w_2))\Big] V_E(w_2) dF_R(w_2)}_{\text{finding a job through internal referral}} \\ &+ \underbrace{\beta p_w^C \int_{\underline{w}}^{\overline{w}} \Big[1-p_w^u(1-F(w_2'))\Big] V_E(w_2') dF_C(w_2')}_{\text{finding a job through external referral}} \end{aligned} \tag{11}$$

The unemployed worker receives and consumes the unemployment benefit b in the current period. Then, in the next period, there are four scenarios:

1. The workers stays unemployment if he does not obtain any job opening information from either a firm (in the first stage of search) or another worker (in the second stage of search).

2. The worker finds a job through formal channel if he obtains a job opening information directly from a firm, and he does not obtain any information from another worker (or the information obtained from another worker is inferior to the one from a firm in terms of wage).

3. The worker finds a job through internal referral if he obtains a job opening information from another worker who works at the hiring firm, and he does not obtain any information directly from another firm (or the information obtained is inferior).

4. The worker finds a job through external referral if he obtains a job opening information from another worker who does not work at the hiring firm, and he does not obtain any information directly from another firm (or the information obtained is inferior).

Here, the probability of receiving the job information through internal and external referrals are disjoint. External referral occurs only if the job information obtained by the worker contacts is inferior to his current earned in terms of wage, and meanwhile he does not obtain the internal job information from the hiring firm. Otherwise, the worker would share the information from either expansion or directly his current job through internal referral.

For a worker being employed with the contract promised fixed wage w, the value function after the labor market closes is as follows.

$$\begin{aligned}
&V_E(w) = u((1-\tau)w) \\
&+\beta \underbrace{\left[1 - p_w^e(1-F(w)\right]\left[1 - p_w^R(1-F_R(w)) - p_w^C(1-F_C(w))\right] V_E(w)}_{\text{staying at the current job}} \\
&+\beta \underbrace{(1-\lambda)p_w^e \int_{w}^{\overline{w}} \Big[1 - p_w^R(1-F_R(w_1')) - p_w^C(1-F_C(w_1'))\Big] V_E(w_1') dF(w_1')}_{\text{finding a new job through formal methods}} \\
&+\beta \underbrace{(1-\lambda)p_w^R \int_{w}^{\overline{w}} \Big[1 - p_w^e(1-F(w_2))\Big] V_E(w_2) dF_R(w_2)}_{\text{finding a new job through internal referral}} \\
&+\beta \underbrace{(1-\lambda)p_w^C \int_{w}^{\overline{w}} \Big[1 - p_w^e(1-F(w_2'))\Big] V_E(w_2') dF_C(w_2')}_{\text{finding a new job through external referral}} + \lambda\Big(V_U - u(b)\Big)
\end{aligned} \tag{12}$$

where $\tau \in [0, 1]$ represents the payroll tax rate and is used for financing unemployment insurance by the government.

The employed worker receives and consumes the disposable compensation $(1-\tau)w$ in the current period. Then in the next period, if the matching is not separated with probability $1-\lambda$, then there are also four scenarios:

1. The employed worker stays at the current job if he doe not obtain any job opening information (or the information obtained is inferior to his current earned in terms of wage) from either a firm in the first stage of search or another worker in the second stage of search.

2. The employed worker finds a higher paying job through formal channel if he obtains a superior job opening information directly from a firm, and he does not obtain any information from another worker (or the information obtained from another worker is inferior to the one from a firm).

3. The employed worker finds a higher paying job through internal referral if he obtains a superior job opening information from another worker who works at the hiring firm, and he does not obtain any information directly from another firm (or the information obtained is inferior).

4. The employed worker finds a higher paying job through external referral if he obtains a superior job opening information from another worker who does not work at

the hiring firm, and he does not obtain any information directly from another worker (or the information obtained is inferior).

3.4 Firms

In what follows, we consider the problem of firms. All the value functions are defined before labor market opens. Let $U_0(w)$ denote the value of a vacant firm of posting an employment contract with fixed wage w. For an incumbent firm with one worker, let $U_1(w)$ denote the value of choosing not to expand, and $U_{10}(w)$ denote the value of choosing to expand. Furthermore, let $U_{11}(w)$ denote the value of an incumbent firm with two workers. For the vacant firm, the value $U_0 : \Phi \to \mathbb{R}$ is

$$\begin{aligned} U_0(w) = & - k + a_0(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda U_0(w)\Big] \\ & + (1 - a_0(w))\left[0 + \beta U_0(w)\right] \end{aligned} \tag{13}$$

where $a_0(w) \in [0, 1]$ denotes the probability with which the job vacancy can be accepted by a worker after the labor market closes. The vacant firm incurs a positive cost k to post a vacancy. On one hand, if the vacancy is accepted, then the firm hires the worker and obtains the profit $\theta - w$. In the next period, if the match is not separated, the firm can choose whether to post the other vacancy. On the other hand, if the vacancy is not accepted by any worker, the vacant firm yields zero profit and has to recruit workers in the next period.

The acceptance probability is determined by the following equation.

$$\begin{aligned} a_0(w) = & \underbrace{\left[\frac{up_w^u}{v} + \frac{e_1 p_w^e}{v} G_1(w) + \frac{e_2 p_w^e}{v} G_2(w)\right]\left[1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))\right]}_{\text{being accepted through formal methods}} \\ & + \underbrace{\left\{\frac{e_1 p_w^e}{v}\int_w^{\overline{w}} [1 - \eta(w_2) + \eta(w_2)p_f]dG_1(w_2) + \frac{e_2 p_w^e}{v}(1 - G_2(w))\right\}\phi(w)}_{\text{being accepted through external referral}} \end{aligned} \tag{14}$$

where $\phi(w)$ represents the probability of meeting a worker who is willing to accept job information in terms of wage w.

$$\phi(w) = \mu\Big\{u[(1 - p_w^u) + p_w^u F(w)] + [e_1 G_1(w) + e_2 G_2(w)]\left[(1 - p_w^e) + p_w^e F(w)\right]\Big\} \tag{15}$$

Conditional on the vacant firm is matched with a worker, the first line of (14) represents the probability of being accepted through formal methods. In the first stage, the vacancy is matched with a worker, unemployed or employed, who earns no greater than the job paying w. Under this circumstance, the vacancy with wage w would be accepted, provided the worker does not receive a higher paying job information through informal methods from the worker contacts in the second stage.

If the vacancy is matched with an employed worker who earns greater than the job paying w, then the job information is lost when considering the canonical Burdett-Mortensen search model. However, associating with the informal methods, the worker has an access to share the job information that could have lost with another worker contacts. For those working at the expanding firms, the job information can be transmitted through external referral only if the internal job vacancy is matched with a worker in the first stage of search. Otherwise, during information sharing, the worker contacts will prefer the internal job instead of the one received through external referral.

For the incumbent firm that filled with one worker and not choose to expand, the value $U_1 : \Phi \to \mathbb{R}$ is

$$\begin{aligned} U_1(w) =& r(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda U_0(w)\Big] \\ &+ (1 - r(w))\,[0 + \beta U_0(w)] \end{aligned} \tag{16}$$

where $r(w)$ represents the probability with which the position with fixed wage contract w is retained after the labor market closes. The retention probability is due to the distributions of wages offered. That is,

$$r(w) = [1 - p^e_w(1 - F(w)]\left[1 - p^R_w(1 - F_R(w)) - p^C_w(1 - F_C(w))\right] + p^e_w[1 - F(w)]\phi(w) \tag{17}$$

As on-the-job search is included in the model, employed workers can move from lower to higher paying jobs. Therefore, the employed worker does not accept any job information in terms of wage below his current earned w. The second part of (17) represents the probability of replacement hiring. The employee receives a higher paying job information and shares his current job with the worker he/she meets as internal referral. Provided the worker contacts would accept this job information, the vacancy after the employee quits can be directly filled by information sharing through internal referral. As a result, such an incumbent firm incurs no posting cost to fill the vacancy. Note that the retention probability is simplified to $r(w) = 1 - p^e_w + p^e_w F(w)$ if $\mu = 0$, as in Burdett-Mortensen search model.

The value of an expanding firm is determined by the following.

$$\begin{aligned} U_{10}(w) =& -k + \sigma_{11}(w)\left\{\begin{array}{c} 2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + \beta\lambda^2 U_0(w) \\ +2\beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\} \end{array}\right\} \\ &+\sigma_1(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda U_0(w)\Big] \\ &+\sigma_0(w)\,[0 + \beta U_0(w)] \end{aligned} \tag{18}$$

If the vacancy is not matched with any job seeker in the first stage of search, the expanding firm has an access to ask its employee to share the job opening information with the worker contacts. $\sigma_{11}(w)$ measures the probability with which the vacancy is accepted and meanwhile the incumbent position is also retained. $\sigma_1(w)$ measures the probability with which only one position is filled, either the incumbent one or the expanding one. If both two positions are separated with probability $\sigma_0(w)$, the expanding firm yields zero profit, and in the next period returns to the vacant firm.

Lemma 1. *The probability with which two positions are both filled is*

$$\sigma_{11}(w) = r(w)a_0(w) + \underbrace{(1 - p_f)[1 - p^e_w + p^e_w F(w)]\phi(w)}_{\textit{internal referral premium}} \tag{19}$$

Proof. See Appendix A □

where $\sigma_0(w) = (1 - r(w))(1 - a_0(w))$ and $\sigma_1(w) = 1 - \sigma_{11}(w) - \sigma_0(w)$. Compared to a vacant firm, the expanding firm can reduce search friction and increase the acceptance probability by making use of internal referral.

The value of an incumbent firm filled with two workers is

$$\begin{aligned} U_{11}(w) = r(w)^2 & \left\{ \begin{array}{c} 2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + \beta\lambda^2 U_0(w) \\ +2\beta(1-\lambda)\lambda \max\{U_{10}(w), U_1(w)\} \end{array} \right\} \\ +2(1 - r(w))r(w) & \Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda U_0(w)\Big] \\ +(1 - r(w))^2 & \left[0 + \beta U_0(w)\right] \end{aligned} \tag{20}$$

For the incumbent firm filled with two workers, if both two positions are retained, then the firm yields the flow profit $2(\theta - w)$ and then moves into the next period to obtain the value that contingents on how many workers still work at the firm. If only one job position is retained, then it becomes an incumbent firm filled with one worker, whereupon the flow profit is $\theta - w$. Finally, if both two positions are separated, the firm has to be a vacant firm.

3.5 Equilibrium Contract

To describe the equilibrium system, let $Q_U(w)$ denote the probability with which an unemployed worker finds job information of whose wage no greater than w.

$$Q_U(w) = p_w^u F(w)\Big[1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))\Big] + (1 - p_w^u)\Big[p_w^R F_R(w) + p_w^C F_C(w)\Big] \tag{21}$$

Note that, incorporating informal methods can increase the UE transition probability by passing job information on to unmatched unemployed workers. Therefore, informal methods has a positive effect on the job finding probability $Q_U(w)$. For an unemployed worker, U-E transition probability is

$$\sigma_{UE} = p_w^u + (1 - p_w^u)(p_w^R + p_w^C) \tag{22}$$

In our model, if an unemployed worker is not matched with a vacancy in the first stage of search, he also has a chance with probability $p_w^R + p_w^C$ to meet an employed worker and obtain job opening information through informal methods.

To ensure the stationary equilibrium, we need to provide two stationary conditions from the side of firms. For the numbers of firms filled with one worker, the stationary condition is described as follows.

$$\begin{aligned} e_1 G_1(w) &= (1-\lambda)\left[v_0 \int_{\underline{w}}^{w} a_0(x)dF_0(x) + e_1 \int_{\underline{w}}^{w} (1 - \eta(x))r(x)dG_1(x)\right] \\ &+ (1-\lambda)e_1 \int_{\underline{w}}^{w} \eta(x)\sigma_1(x)dG_1(x) + 2\lambda(1-\lambda)e_1 \int_{\underline{w}}^{w} \eta(x)\sigma_{11}(x)dG_1(x) \\ &+ 2\lambda(1-\lambda)\frac{e_2}{2}\int_{\underline{w}}^{w} r(x)^2 dG_2(x) + (1-\lambda)\frac{e_2}{2}\int_{\underline{w}}^{w} 2(1 - r(x))r(x)dG_2(x) \end{aligned} \tag{23}$$

The first line of RHS in (23) represents the flow-in from vacant firms to incumbent firms that filled with one worker and not making an expansion. The second line is the number of expanding firms that still hire one worker in the next period. The third line represents the flow-in from incumbent firms filled with two workers, in which one position is separated by an exogenous probability λ or an endogenous probability with voluntary employment relationship termination. For the numbers of firms filled with two workers, the stationary condition is

$$\frac{e_2}{2}G_2(w) = (1-\lambda)^2 \left[e_1 \int_{\underline{w}}^{w} \eta(x)\sigma_{11}(x)dG_1(x) + \frac{e_2}{2}\int_{\underline{w}}^{w} r(x)^2 dG_2(x) \right] \tag{24}$$

In stationary equilibrium, the first part of RHS is the number of flow into incumbent firms with two workers from expanding firms and the second part is the number of staying at the original incumbent state with two positions.

Furthermore, we introduce the payroll tax at a constant rate $\tau \in [0,1]$ to finance the unemployment benefit. For the budget balance of the government in each period, we have

$$u(1-\sigma_{UE})b = \tau\Big[1 - u(1-\sigma_{UE})\Big]\int_{\underline{w}}^{\overline{w}} wdG(w) \tag{25}$$

Note that in our model, u is the measure of unemployed workers before the labor market opens. During the course of job finding, a fraction of unemployed workers are being employed and obtain the wages written in the employment contract. Therefore, only the unemployed workers who are about to stay on, which is $u(1-\sigma_{UE})$, have qualified for unemployment insurance.

We now define a stationary equilibrium of the model. Since we assume the agents and firms are both homogenous, the aggregate state of the economy now includes: (1) productivity θ (2) matching probability p_w^u, p_w^e and p_f (3) the distributions of wage earned in equilibrium G.

Definition 1. *A stationary equilibrium consists of matching probability p_w^u, p_w^e and p_f, an allocation set, including unemployment u, the number of employed workers e_1, e_2, the number of vacancies v_0 posted by vacant firms and vacancies v_1 posted by expanding firms. For the distributions, the distributions of starting wages offered F_0 and F_1: $\Phi \to [0,1]$ and distributions of wages earned G_1 and G_2: $\Phi \to [0,1]$,. Such that, given the aggregate state of the economy,*

(a) An unemployed worker accepts job information, formal or informal, if and only if $V_E(w) \geq V_U$, where V_U given by (11).

(b) An employed worker accepts an outside offer, formal or informal, if and only if the expected utility of the outside offer is greater than his current offer, where $V_E(w)$ given by (12).

(c) The acceptance function satisfies (14) and the retention function satisfies (17).

(d) Free entry and exit. That is, $U_0(w)$=0, for all $w \in \Phi$.

(e) Matching function $M(1,v) = up_w^u + (1-u)p_w^e = vp_f$, where p_f satisfies (4).

(f) The distributions G_1 and G_2 of the employed workers' wages earned are both consistent with F, generating from the the stationary conditions (23) and (24).

(g) The distribution of wages offered through internal referral $F_R(w)$, which is determined by (9), is consistent with F and G.

(h) The distribution of wages offered through external referral $F_C(w)$, which is determined by (10), is consistent with F and G.

(i) Government budget balanced is satisfied by (25).

Note that $V_E(w) \geq V_U$ is the self-enforcing constraint. The worker is free to terminate the employment contract and becomes unemployed. Any expected utility promised below V_U would never be taken, thus never be offered. Hence, to retain an employed worker, the contract must offer the employed worker at least as great as his reservation.

4 Results

Proposition 1. *In equilibrium, for all $w \in \Phi$, we have $U_{10}(w) > U_1(w)$. That is, expanding is the best response for any incumbent firm with one worker, which implies the expanding probability $\eta(w) = 1$.*

Proof. See Appendix B □

Proposition 1 arises from free entry condition $U_0(w) = 0$, under which the value of any vacant firm to post a vacancy is zero. Otherwise, new vacant firms have incentives to enter the labor market to pursue strictly positive market profits. The intuition for this results is that the expanding firm can have an information advantage by asking its employee to pass along the job information. For a vacant firm, the vacancy can be accepted through formal methods and external referral, as indicated in the acceptance function (14). However, the vacancy posted by an expanding firm can also be accepted through internal referral when it is not matched with a job seeker in the first stage of search, indicating a higher acceptance probability. Free entry condition implies the profit of any vacant firm in equilibrium must be zero, and thus the increases in the acceptance probability through internal referral leads to a strictly positive profit for an expanding firm. As a result, all incumbent firms filled with one worker choose to expand and obtain higher values compared to those not expanding.

$$\frac{U_{10}(w) - U_1(w)}{U_1(w)} = \frac{\left[1 - \beta(1-\lambda)^2 r(w)^2\right]/r(w)}{1 - \beta(1-\lambda)^2 r(w)^2 + 2\beta(1-\lambda)^2\sigma_{11}(w)} \underbrace{(1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)]}_{\text{internal referral premium}} \tag{26}$$

Using the adjoint of value functions (16) and (20), we obtain

$$\left[1 - \beta(1-\lambda)^2 r(w)^2\right]\left[U_{11}(w) - 2U_{10}(w)\right] = 2U_1(w) - 2U_{10}(w) < 0 \tag{27}$$

Although the production displays constant return to scale, $U_{11}(w) < 2U_{10}(w)$ implies that the value of firms exhibit decreasing return to scale. For the simplicity, we assume each firm has at most two positions to be filled. However, once considering multi-workers and impose no restrictions on the firm size, the incumbent firms will not expand production scale without limitation. Since the posting cost is constant, the optimal number of vacancies posted can be pinned down.

Proposition 2. *The equilibrium unemployment is determined by*

$$u = \frac{\lambda}{\lambda + (1-\lambda)\left[p_w^u + (1-p_w^u)(p_w^R + p_w^C)\right]} \tag{28}$$

where $p_w^R + p_w^C = \mu e_1(1 - p_f + p_w^e p_f) + \mu e_2 p_w^e$.

Proof. See Appendix C □

In addition to the separation probability and the probability with which an unemployed worker is matched with a vacancy, the equilibrium unemployment is also dependent with the probability of meeting an informant, either through internal or external referral. The equilibrium unemployment is pinned down by the Beveridge curve (28), which is downwards sloping between unemployment and vacancies. A reduction in search friction of information obtained through informal methods indicates a lower equilibrium unemployment rate.

Proposition 3. *For the equilibrium contract, suppose* $k > 0$*, then the following holds.*

(a) F *is continuous and* Φ *is connected.*

(b) $\Phi = [\underline{w}, \overline{w}]$ *with* $V(\underline{w}) = V_U$*.*

Proof. See Appendix D. □

Proposition 3 describes the basic property of the equilibrium fixed wage contract. The distribution of starting wages offered has no mass point and the domain of wages offered by vacant firms Φ is a closed interval in equilibrium. In fact, if there is a mass point in the distribution F, the vacant firm can deviate the equilibrium path by raising wages a little bit. If so, the firm can improve the value with a significantly larger retention probability and acceptance probability but only a slightly smaller flow profit. Hence, this proposition rules out a single market wage or noncontinuous distribution of wages offered as an equilibrium possibility.

The steady state equilibrium is a continuous wage distribution that yields the same value to vacant firms posting different wages in equilibrium. The maximum profit is zero due to free entry and exit condition. Firms offering higher wages obtain less profits per period than those firms offering lower wages. On-the-job search, however, implies those firms offering higher wages attract more employed workers and in turn implies not only a higher retention probability but also a higher acceptance probability once the current employment relationship is terminated. As a result, those firms offering higher wages have longer duration of the employment contract and shorter duration of being vacant to make recruitment.

For employed workers, self-enforcing constraint implies that wages below outside option V_U is never be offered. For unemployed workers, the wage contract can be accepted only when the expected utility from the job information is greater than the unemployed one V_U. Hence, the reservation wage $\underline{w}$ satisfies $V(\underline{w}) = V_U$.

After the algebraic operations, we finally obtain the equilibrium zero profit condition[7].

$$a_0(w)\frac{\theta - w}{1 - \beta(1-\lambda)(r(w) + \Delta(w))} = k \tag{29}$$

where

$$\Delta(w) = \frac{[1-\beta(1-\lambda)^2 r(w)^2]}{1-\beta(1-\lambda)^2 r(w)^2 + 2\beta(1-\lambda)^2\sigma_{11}(w)}(1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)] \tag{30}$$

5 Calibration

We calibrate the model to the monthly U.S. labor market data. The monthly interest rate is set to be 0.00417 to obtain an annual interest rate of 5%. The discount factor is then set to be $\beta = 1/(1 + 0.00417) = 0.9959$. The output θ is normalized to 1. The separation probability λ is set to be 0.03 to target the average monthly E-U transition probability of 3%, as in Shimer (2012).

The worker's CARA utility function is

$$u(c) = 1 - e^{-\eta c}, \quad \forall c \geq 0$$

where η is positive. While the absolute risk aversion coefficient is η, the relative risk aversion coefficient is ηc which, given the worker's compensation is always less than the period output $\theta = 1$ in equilibrium, is less than η. Shimer (2012) indicates that the monthly U-E transition probability equals to 43%. Hence, the equilibrium unemployment rate is calculated to be 6.71% from (22) and (28).

Given these above, the following moments are targeted. We pin down p_w^u and p_w^e by targeting the U-E transition probability of 43% as estimated in Shimer (2012), and E-E transition probability of 3.2% as estimated in Nagypal (2008) respectively. We choose the unemployment benifit b to target the aggregate replacement ratio of 41% according to Shimer (2005). The contact probability μ is chosen to target the percentage of jobs found through informal methods, which is 40.43% based on Job Search Survey.

The payroll tax τ is used to finance the unemployment benefit and determined by the budget balance of the government (25). The outcomes of our calibration is shown in Table 5 and Table 6.

Table 5: Calibrated parameters

	μ	p_w^u	p_w^e	b	p_f	τ	η	k
Value	0.3	0.386	0.048	0.381	0.133	0.016	3	0.12

The calibration does well in matching the targets. The mean-min ratio is 1.46, which is larger than 1.27 as attained in the standard on-the-job search models. More importantly,

[7] See Appendix E for the derivation.

Table 6: Calibration outcomes

Variable	Calibration	Data	Source
U-E transition prob.	43%	43%	Shimer (2012)
E-U transition prob.	3%	3%	Shimer (2012)
E-E transition prob.	3.18%	3.2%	Nagypal (2008)
Replacement ratio	41%	41%	Shimer (2005)
Mean-min ratio	1.46	1.5–2	Hornstein et al. (2011)
Wage Differential			
Internal Referral/Formal	3.53%	4.75%	Job Search Survey
External Referral/Formal	-1.95%	-11.6%	Job Search Survey
Informal/Formal	3.01%	3.76%	Job Search Survey
Informal Prevalence			
Jobs finding (informal)	40.34%	40.43%	Job Search Survey
Jobs finding (internal referral)	37.29%	37.8%	Job Search Survey
Jobs acceptance (informal)	39.68%	37%	Marsden and Gorman (2001)

it is close to the range from 1.5 to 2, as estimated by Hornstein et al. (2011). In our calibration, 40.34% of jobs are found through informal methods, and pay 3.01% more than jobs through formal methods, consistent with what we observe in the data. Specifically, compared to jobs found through formal methods, jobs through internal referral pay 3.53% more (which accounts for about 75% of wage premium observed in the data), while jobs through external referral pay 1.95% less (which accounts for about 17% of wage penalty observed in the data). That is, the model is able to match the differential effects of search methods on wages.

Figure 4a and 4b exhibit the probability density functions of wages offered and earned respectively. In the stationary equilibrium, the distributions are non-degenerate and continuous. Employed workers move from lower to higher paying jobs through on-the-job search. Hence, the average wage earned by employed worker is higher than the average wage offered by vacancies.

Figure 4c an 4d exhibit the probability density functions of wages offered by vacant firms and expanding firms respectively. Firms paying more are more likely to find workers, and less likely to lose workers (to other firms). Hence, the average wage paid by firms without any worker is less than the average wage paid by firms with one worker (which is in turn less than the average wage paid by firms with two workers as shown in Figure 4f). That is, our model is consistent with the firm size wage premium as widely observed in the data.

Interestingly, the probability density function of wages offered by vacant firms is unimodal, instead of increasing and convex as commonly obtained in the literature.

Finally, Figure 4g and 4h exhibit the probability density functions of wages offered through internal and external referrals respectively. Note that all offers originate from the same distribution of wages offered by both vacant firms and expanding firms. However, some of them would be passed on through internal referral, and some through external referral. Specifically, if an expanding firm does not meet any worker in the first stage of search, then it would ask its current worker to pass the job information on to any

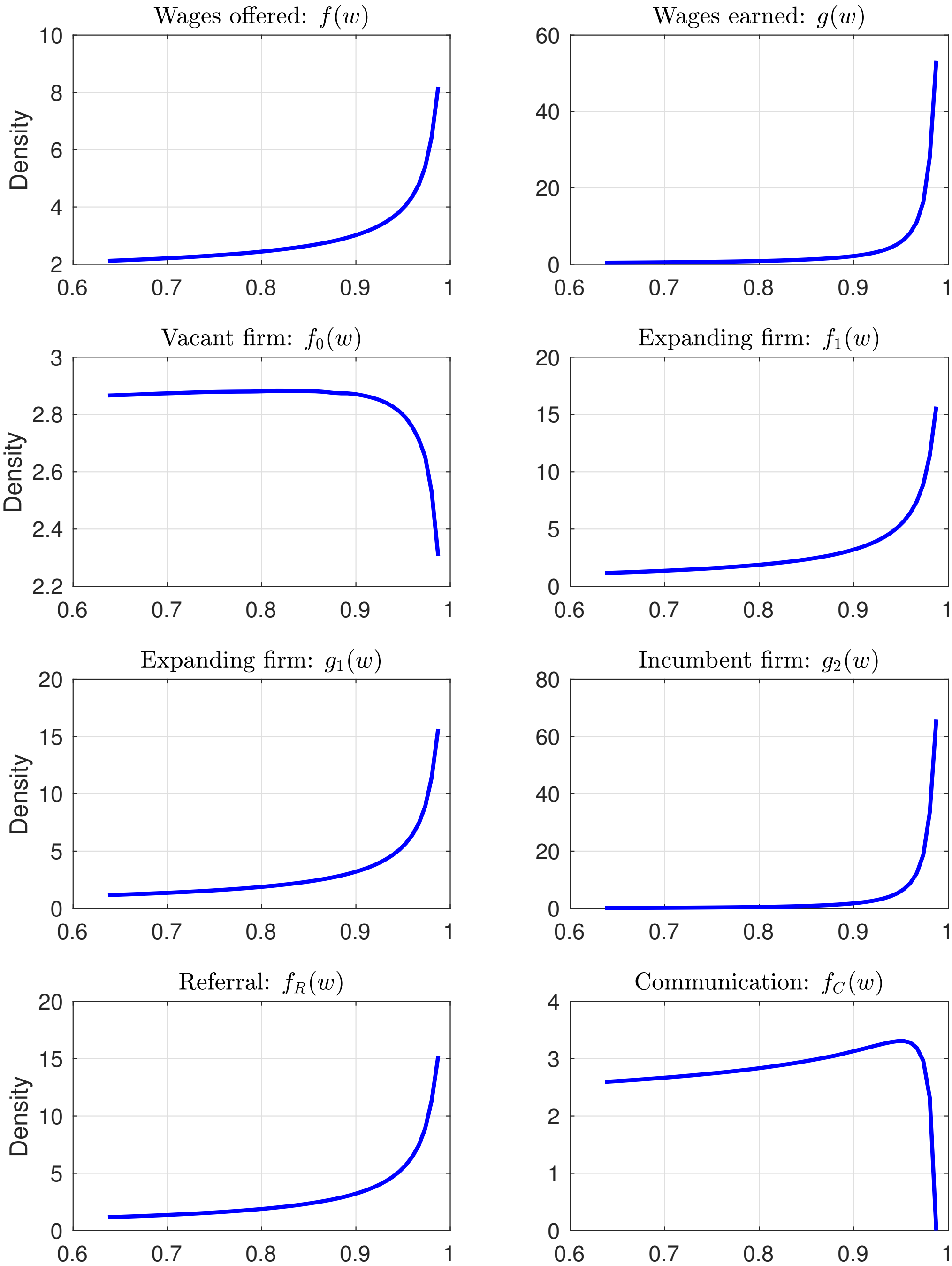

Figure 4: The density functions of wages offered

worker he/she meets in the second stage. Since the average wage paid by expanding firms is higher than the average wage paid by vacant firms as shown above, jobs found through internal referral pay more on average, as observed in the data. If an employed worker receives a job paying less than his/her current one through on-the-job search in the first stage, then he/she would pass the job information on to any worker he/she meets in the second stage. In this case, employed workers only share the job information not valuable to themselves. Hence, jobs found through external referral pay less on average, as observed in the data.

5.1 Wage Dispersion

Hornstein et al. (2011) use mean-min ratio to measure the wage dispersion and show that search models cannot generate a reasonable observed wage dispersion. The estimation of mean-min ratio that resembles data is between 1.5 to 2. To the best of our knowledge, the on-the-job search model exhibits the best performance, in which the mean-min ratio can be attained to 1.27. By incorporating informal methods, however, our calibrated model yields the mean-min ratio of 1.46 and substantially improves the ability of search models to account for the observed wage dispersion.

In our calibrated model, the use of informal methods gets the wage support stretched, and causes asymmetric effects between the highest and lowest paying jobs. Specifically, firms paying the highest obtain strictly positive profits due to an increase in the acceptance probability, pushing up the upper bound of wage support. Firms paying the lowest obtain strictly negative profits mainly caused by the reduction in both the acceptance and retention probability, pulling down the lower bound of the wage support.

$$\overline{w} = \theta - \frac{k}{p_f}\left[1 - \beta(1-\lambda)(1+\Delta(\overline{w}))\right] \tag{31}$$

$$\Delta(\overline{w}) = \frac{1-\beta(1-\lambda)^2}{1-\beta(1-\lambda)^2 + 2\beta(1-\lambda)^2[p_f + (1-p_f)\mu]}(1-p_f)\mu \tag{32}$$

The use of informal methods increases the upper bound of wage support, which is determined by (31) and (32). At the highest wage, the employed worker whom the firm employs would stay on the job until the exogenous separation occurs. The job paying highest will be accepted only through formal methods, since the external referral channel does not work in equilibrium. Any worker who learns about the highest paying job in the first stage would not pass the job information on to the worker contacts.

There are two reinforced effects of informal methods on the value of firms paying the highest. On the one hand, the internal referral directly leads to a higher acceptance probability for those expanding firms paying the highest. Once the vacancy is not matched with any worker in the first stage of search, the expanding firm would ask its current employee to pass along the job information to any worker contacts in the second stage. Therefore, such an acceptance premium through internal referral $(1-p_f)\mu$ increases the value of firms, as well as the labor demand.

$$\mu \Rightarrow \begin{cases} \text{direct effect: } \Delta(\overline{w}) > 0 \;\Rightarrow\; \overline{w}\uparrow \\ \\ \text{equilibrium effect: } v\downarrow \;\Rightarrow \begin{cases} p_f\uparrow \;\Rightarrow\; a_0(\overline{w})\uparrow \;\Rightarrow\; \overline{w}\uparrow \text{(dominant)} \\ \partial\Delta(\overline{w})/\partial p_f < 0 \;\Rightarrow\; \Delta(\overline{w})\downarrow \;\Rightarrow\; \overline{w}\downarrow \end{cases} \end{cases}$$

On the other hand, informal methods provide an extra opportunity for workers and firms to match with each other in the second stage. Such a reduction in search frictions requires less vacancies to generate the same amount of job creations, reducing the number of firms in equilibrium. Since the labor market is frictional, the equilibrium effect of informal methods increases the job filling probability p_f, which implies an increase in the acceptance probability through formal methods. This positive effect on the upper bound $\overline{w}$ dominates the negative effect caused by the decrease in the acceptance probability premium through internal referral ($\partial\Delta(\overline{w})/\partial p_f < 0$). Overall, both the direct effect and equilibrium effect increase the value of firms and the upper bound of the wage support.

$$\underline{w} = \theta - \frac{k}{a_0(\underline{w})}\left[1 - \beta(1-\lambda)(r(\underline{w}) + \Delta(\underline{w}))\right] \tag{33}$$

For the lowest paying job, an unemployed worker matched with the vacant firm is in direct contact with another worker to obtain job opening information in the second stage, giving rise to a lower acceptance probability for the vacant firm. Likewise, the worker whom the firm employs at the lowest wage is more likely to receive higher paying jobs through informal methods, leading to a lower retention probability and higher replacement cost. Both the two negative effects dominate the positive effect on the acceptance premium through internal referral, thereby pulling down the lower bound of wages support.

The equilibrium effect alleviates the reduction in the lower bound by way of increasing the job filling probability p_f and decreasing the job finding probability p_w^u and p_w^e. An increase in p_f indicates a higher acceptance probability, shortening the vacant duration. Due to the lower job finding probability, the employed worker is less likely matched with a vacancy through on-the-job search, which implies a decline in the retention probability.

Table 7: The effect of informal methods on the lower bound of wages offered

	Acceptance $a_0(\underline{w})$	Retention $r(\underline{w})$	Internal Referral $\Delta(\underline{w})$	Lower Bound $\underline{w}$
direct effect (μ)	−	−	+	−
equilibrium effect ($v \downarrow$)	+	+	+	+

The total effect of informal methods finally decreases the lower bound of wage support. As shown in Table 7, the rise in the lower bound caused by the equilibrium effect is offset by the the substantial fall caused by the direct decrease in the acceptance probability $a_0(\underline{w})$ and retention probability $r(\underline{w})$. The higher replacement cost gives rise to a negative profit at the lowest paying job. As a result, the vacant firm has more incentives to move for lower wages offered.

5.2 Wage Differential across Search Methods

Figure 5 exhibits the cumulative distribution functions of wages earned for jobs found through different search methods. As shown in Figure 5a and 5b, for both unemployed and employed workers who find jobs, the equilibrium distribution of wages earned through internal referral first-order stochastically dominates that through formal methods, which in turn first-order stochastically dominates that through external referral.

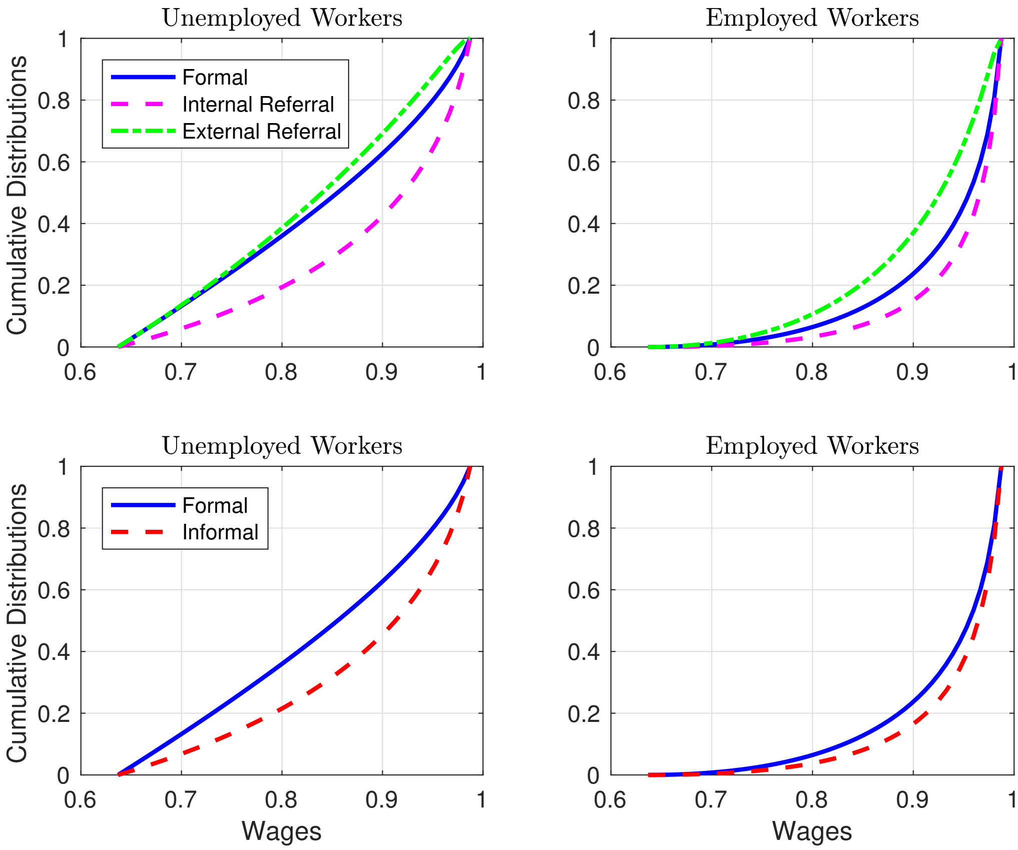

Figure 5: The equilibrium distributions of wages earned through informal methods

An employed worker learns about job openings information through on-the-job search in the first stage of search. On the one hand, for the job paying more than his/her earned, the employed worker would retain it, and in turn pass his/her current job (accompanied with information about job expansion from his/her employer, if possible) on to any worker he/she meets, as internal referral. The average wage paid by expanding firms is more than that paid by vacant firms due to the firm size wage premium, as shown in Figure 4c and 4d. Therefore, jobs found through internal referral pay more than those found through formal methods. On the other hand, for the job paying less than his/her earned, the employed worker would share the information with whoever he/she meets, which corresponds to external referral. Since the employed worker only shares the inferior information, external referral channel becomes inactive in the domain of higher wages offered, as shown in Figure 4h. As a result, jobs found through external referral pay less than those found through formal methods.

Quantitatively, for unemployed (employed) workers, jobs through internal referral pay 5.55% (1.67%) more, whereas jobs through external referral pay 1.38% (2.70%) less, compared to those found through formal methods. To identify the overall effect of informal methods on wages, it depends on the proportion of jobs found through internal and external referrals respectively. In our calibration, 37.29% of jobs are found through internal referral, which accounts for 92.45% of those found through informal methods. Therefore, as shown in Figure 5c and 5d, the average wage paid for jobs found through informal methods is higher than those through formal methods. The positive wage effect of 4.77% occurs for unemployed workers, and that of 1.37% occurs for employed workers.

Furthermore, the wage differential across search methods differs between unemployed and employed workers. Specifically, unemployed workers earn higher wage premium (5.55%) than employed workers (1.67%) between internal referral and formal methods, but earn less wage penalty (1.38%) than employed workers (2.70%) between external referral and formal methods. As shown in Figure 5a and 5b, for those jobs found through formal methods, the distribution of wages earned by employed workers dominates that by unemployed workers, leading to higher wages paid, since employed workers move from lower to higher paying jobs through on-the-job search. Therefore, higher wage premium between internal referral and formal methods occurs for unemployed workers, whereas higher wage penalty between external referral and formal methods occurs for employed workers.

6 Counterfactual

6.1 Informal Methods

We first discuss the effects of informal methods on the steady state outcomes, starting from the impact on the distributions of wages offered and earned.

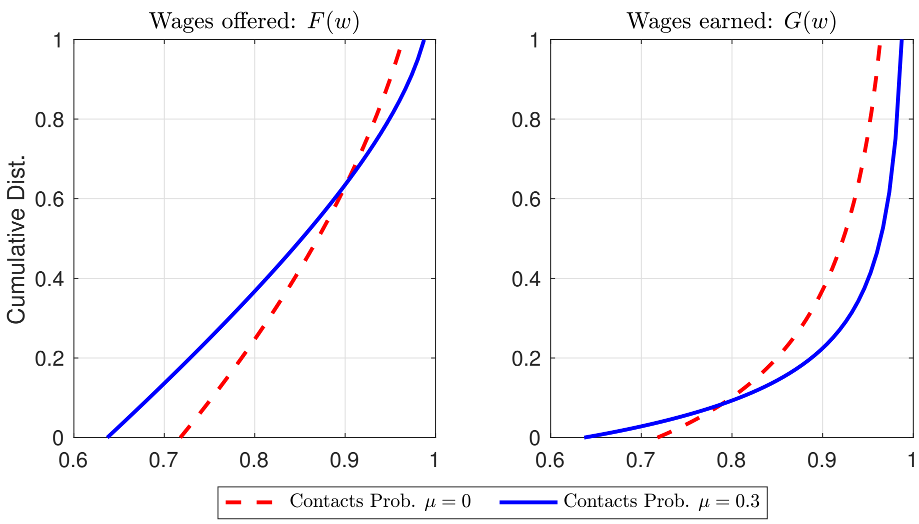

Figure 6: The distribution of wages offered and earned

The distribution of wages offered: Following the logic in Section 5.1, the use of informal methods enlarges wage dispersion. The upper bound is pushed up by the increases in the acceptance probability, whereas the lower bound is pulled down by the decrease in both the acceptance and retention probability. As a result, the support of wages offered spreads extensively by the total effect. As shown in Figure 6a, in the economy restricting informal methods, the distribution of wages offered second-order stochastically dominates that in the economy allowing for informal methods ($\mu = 0.3$). Hence, risk averse workers (those with increasing and concave utility functions) learn about job information with lower average wages offered if they have access to informal methods.

The distribution of wages earned and E-E transition: The use of informal methods reduces search friction and provides an opportunity for the worker to share job opening information with each other. The employed worker can find a job through internal or external referrals in the second stage of search, which increases the probability of receiving higher paying jobs information, thus accelerating E-E transition.

Allowing for informal methods, the employed worker moves from lower to higher paying jobs through on-the-job search and climbs up the job ladder faster. Hence, in the stationary equilibrium, the distribution of wages earned is pushed rightwards, leading to higher average wage earned. Figure 6b exhibits that the distribution of wages earned second-order stochastically dominates that when restricting informal methods. Contrary to the distribution of wages offered $F(w)$, risk-averse workers earn higher average wages and are better off.

Table 8 displays the counterfactual outcomes of informal methods. Compared to the economy restricting informal methods, the use of informal methods substantially raises the E-E transition probability from 1.94% to 3.18%. More workers flow into the firms paying higher. The number of employed workers in expanding firms decreases by 39.2%, whereas that in incumbent firms with two workers increases by 26.44%.

Table 8: The effect of informal methods

	$\mu = 0$	$\mu = 0.3$	Increase by
Panel A: Unemployment and Vacancy			
Unemployment	0.0618	0.0671	8.58%
Number of vacant firms	0.3657	0.2993	-18.16%
Number of firms	1.0279	0.8832	-14.08%
Job Creation/Matching	55.98%	81.46%	45.52%
Panel B: Turnover			
U-E Transition Prob.	46.99%	43%	-8.49%
E-E Transition Prob.	1.94%	3.18%	63.92%
Number of employed workers e_1	0.3862	0.2348	-39.2%
Number of employed workers e_2	0.5521	0.6981	26.44%
Panel C: Welfare			
Consumption Equivalence (U)	0.8332	0.8455	1.48%
Consumption Equivalence (E)	0.8473	0.8633	1.89%
Consumption Equivalence (ALL)	0.8468	0.8626	1.87%

Unemployment and vacancy: The increase in the acceptance probability improves the matching efficiency in the labor market, where it requires less vacancies to generate the same number of job creations. Moreover, the rise in the average wage paid increases the cost of labor input, pulling down the labor demand and job creations. Hence, the number of vacancies will further decrease. In the stationary equilibrium, as shown in Table 8, the number of vacancies has dropped by 18.16%, and the total number of firms in the labor market has also declined by 14.08%.

The use of informal methods induces a dominant distribution of wages earned. The rise in average wage earned increases the unemployment insurance paid by the payroll tax under the constant replacement ratio of 41%, leading to a higher value of staying unemployed. Since the unemployed worker has more incentives to wait for higher paying jobs, the U-E transition declines by 8.49% and the unemployment rate goes up from 6.29% to 6.71%.

Due to the faster E-E transition, unemployed workers would accept lower reservation wage. The gap that reservation wage goes beyond unemployment insurance b becomes smaller because the unemployed worker is willing to flow into the employment status quickly, and then climbs up the job ladder faster.

Welfare: Because of higher unemployment insurance and average wage earned, both unemployed and employed workers are better off. The consumption equivalence[8] increases by 1.48% and 1.89% respectively. The total welfare goes up by 1.87% if allowing for informal search methods.

Table 9: The effect of informal methods on wage dispersion

	$\mu = 0$	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.7$	$\mu = 0.9$
Mean-min ratio	1.255	1.46	1.54	1.605	1.657
Length of support	0.246	0.350	0.382	0.405	0.421
S.D. of $F(w)$	0.073	0.108	0.119	0.124	0.128
Average wage	0.901	0.930	0.938	0.944	0.949
Reservation wage	0.718	0.637	0.609	0.588	0.573

Wage disperison: In the calibrated benchmark model ($\mu = 0.3$), the use of informal methods pushes up average wage slightly by 3.21%, but pulls down the minimum wage offered by 11.26%, compared to the economy restricting informal methods ($\mu = 0$). Therefore, the mean-min ratio goes up from 1.255 to 1.46, which approaches to the one between 1.5 and 2 as observed in the data indicated in Hornstein et al. (2011)

As the contact probability increases from 0.3 to 0.9, the equilibrium support of wages offered by vacant firms spreads extensively. Firms paying the highest obtain strictly positive profits due to an increase in the acceptance probability, pushing up the upper bound of the wage support. Firms paying the lowest obtain strictly negative profits caused by a reduction in both the acceptance and retention probability, pulling down the lower bound of the wage support. Figure 7 exhibits the probability functions of wages offered by vacant firms. The distributions of wages offered with a lower contact probability dominates that with a higher contact probability. As the contact probability μ increases, the equilibrium distribution goes in the direction of lower average wages offered.

[8] Since the welfare depends on its form of utility function, we calculate the consumption equivalence to normalize the social welfare.

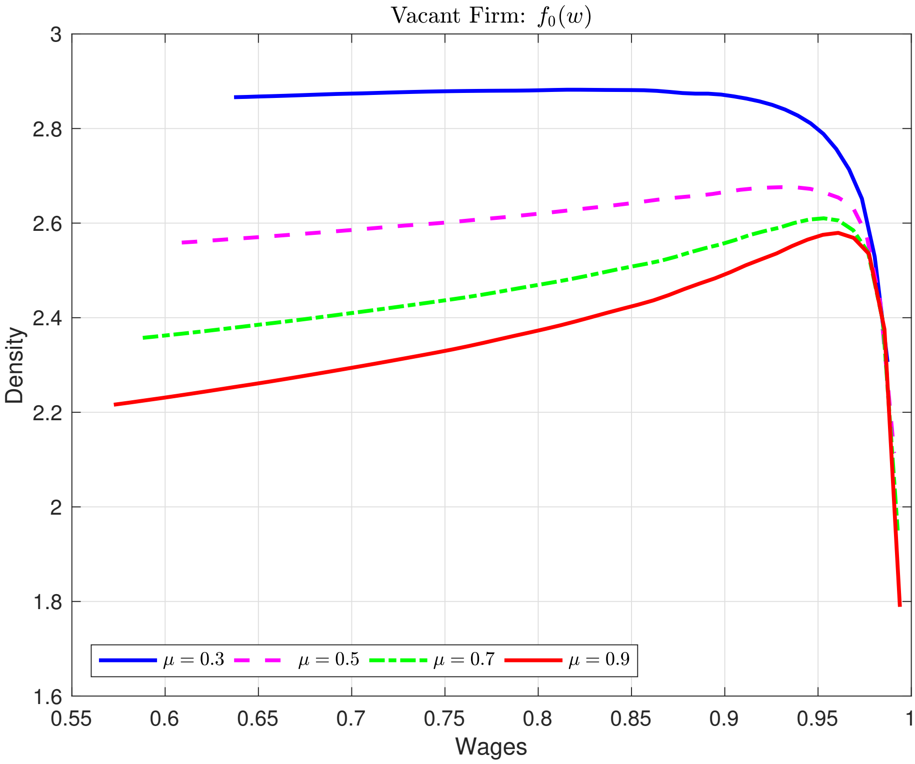

Figure 7: The probability density functions of wages offered by vacant firms

We measure wage dispersion based on three indicators: The mean-min ratio, length of wage support and standard deviation of wages offered. Table 9 displays that the mean-min ratio rises from 1.255 to 1.657 as contact probability goes up from 0 to 0.9. Wage dispersion enlarges regardless of the indicator we use to measure. Although the calibrated benchmark model ($\mu = 0.3$) arises a moderate mean-min ratio of 1.46, the model can generate larger wage dispersion that resembles the data, if calibrated to a higher percentage of jobs found through informal methods.

Table 10: The effect of informal methods on wage differentials (unit:%)

	$\mu = 0$	$\mu = 0.3$	$\mu = 0.5$	$\mu = 0.7$	$\mu = 0.9$
Firm Size					
Expanding/Vacant	4.86	8.19	9.48	10.17	10.72
Incumbent/Expanding	4.09	8.28	9.19	9.91	10.45
Search Methods					
Internal Referral/Formal		3.53	3.79	3.95	4.03
External Referral/Formal		-1.95	-1.98	-2.01	-2.04
Informal/Formal		3.01	3.14	3.2	3.2

Wage differentials: In the calibrated benchmark model ($\mu = 0.3$), the average wage paid by incumbent firms with two workers is 8.28% higher than the average wage paid by expanding firms, which is in turn 8.19% higher than that paid by vacant firms. The first part of Table 10 exhibits that the increase in the contact probability raises the firm size wage premium. Firms paying more are more likely to find workers, and less likely to lose workers. The use of informal methods provides an opportunity for the employed worker to learn about job information through information sharing in the second stage, increasing the probability with which the employed worker receives a higher paying job. The employment relationship between firms and workers are more likely to be terminated by more frequent use of informal methods. Hence, in the stationary equilibrium, the wage differential across firm sizes enlarges as the contact probability increases.

In addition to the firm size wage premium, the increase in the contact probability also enlarges the wage differential between the jobs found through informal and formal methods. On the one hand, if an expanding firm is not matched with any worker in the first stage, then it would ask its current employee to pass the expansion information on to the worker he/she meets in the second stage. Since the wage premium between expanding and vacant firms has been enlarged, the average wage must be paid by a growing premium for the jobs found through internal referral. On the other hand, if an employed worker receives a job paying less than his/her earned through on-the-job search, then he/she would pass the information on to the worker contacts. The use of informal methods induces a dominant distribution of wages earned, but a dominated distribution of wages offered. Hence, as the contact probability increases, the wage penalty between the jobs found through external referral and formal methods has also been growing.

The second part of Table 10 exhibits the quantitative outcomes of wage differentials across search methods. As the contact probability increases from 0.3 to 0.9, compared to the jobs found through formal methods, the wage premium for the jobs found through internal referral goes up from 3.53% to 4.03%, whereas the wage penalty through external referral raises from 1.95% to 2.04%. For the overall wage effect, the jobs found through informal methods pay 3.2% more than those through formal methods when the contact probability rises to 0.9.

6.2 Unemployment Insurance

In what follows, we summarize the effects of unemployment insurance (hereafter UI) on the steady state outcomes, starting from its impact on unemployed workers.

Unemployment and U-E transition: Without UI, workers do not have enough incomes to stay unemployed. Therefore, unemployed workers are willing to sacrifice their compensations in order to find jobs more quickly. On the contrary, for those unemployed workers who receive UI, they have incentives to spend more time to search for higher paying job openings information, thus prolonging unemployment duration and resulting in higher unemployment rate. The increase in the value of staying unemployed raises reservation wages and pulls down the U-E transition probability. Table 11 displays the counterfactual results of UI. In the calibrated model, UI can substantially increase the unemployment rate by 28.05% and decreases U-E transition probability by 23.17%.

The numbers of firms and wages earned: UI also increases the option value of employed workers. The vacancies posted by firms must pay higher to recruit workers. Therefore, in the steady state equilibrium with UI, average wages earned by employed workers increase 4.21% and the number of firms falls from 1.17 to 0.88.

Wage dispersion: The effect of UI directly affects the distribution of wages offered. UI provides more incentives on firms to post higher wages because UI substantially increases the reservation wage of workers. As a result, the support of wages offered shifts rightwards and wages offered grow up dramatically around the reservation wage. As is shown in Figure 8, since UI raises the reservation wage and induces a stochastically dominant distribution of wages offered, in the steady state equilibrium, employed workers' earnings are more likely to concentrate on the higher wages support, resulting in a dominant distribution of wages earned.

UI pushes up average wages earned, while the length of the support shrinks by 46.61%. With UI, wage dispersion measured by mean-min ratio falls from 2.71 to 1.46, and the standard deviation of the distribution of wages offered drops by 45.62%. This is consistent with the implication of Hornstein et al. (2011), in which a large replacement ratio associates with a small wage dispersion.

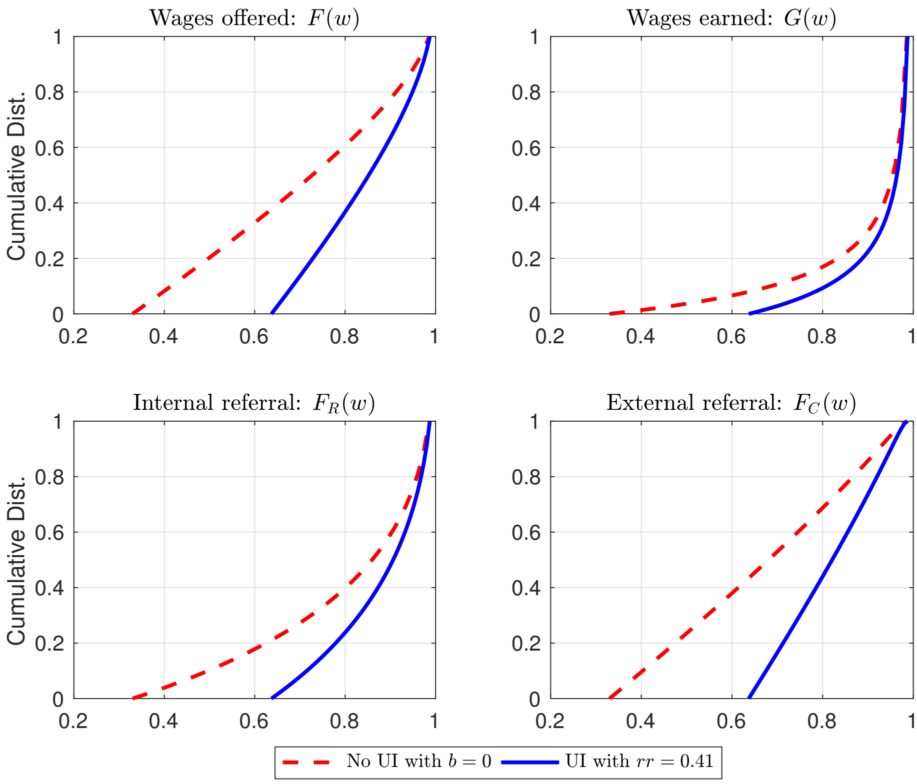

Figure 8: The distributions of wages offered and earned

Wage differential: UI can reduce the wage differentials across firms size and search methods. Allowing for the UI, The support of the equilibrium distribution of the wages offered shrinks, as shown in Figure 8. Since employed workers move to higher paying jobs, the average wage earned by employed worker is higher than the average wage offered by vacancies. However, the firm size wage premium falls dramatically due to the substantial rise in the reservation wage and the lower E-E transition probability.

Without UI (with UI), compared to those found through formal methods, the jobs found through internal referrals pay 9.74% (3.53%) more, whereas the jobs through external referral pay 2.38% (1.95%) less. To compare the overall effect of informal methods on wages, Table 11 shows that the jobs found through informal methods pay 8.47% (3.01%) higher wages than the ones found through formal methods.

The intuition is as follows: Jobs found through external referral pay less than those through formal methods, since employed workers only share the jobs paying less than their current earned. Jobs found through internal referral pay more than those through formal methods, since the average wage offered by vacant firms is higher than that offered by expanding firms. UI raises the reservation wage and reduces the magnitude of wage dispersion. Jobs found through both informal and formal methods pay more in a less dispersed domain. Therefore, the wage differential becomes lower, and falls from 8.47% to 3.01% between informal and formal methods. The wage premium through internal referral falls to 3.53%, and the wage penalty through external referral reduces to 1.95%.

Welfare: Unemployed workers are better off with UI. The consumption equivalence increases by 13.54%. Since UI raises the average wage earned and pushes up the support rightwards, employed workers are also better off, which goes up by 11.38%. Overall, consumption equivalence of all workers increases by 11.39% in the steady state equilibrium with UI.

7 Conclusion

This paper introduces informal search methods into a two-stage search model. In the first stage, the workers and firms are matched randomly through a constant-return-to-scale matching function. The second stage is information sharing, in which the workers randomly run into each other and share the job openings information obtained. In equilibrium, the distribution of wages offered is continuous and has no mass point. We calibrate the model to the monthly U.S. labor market data. The mean-min ratio, a measure of wage dispersion in Hornstein et al. (2011), is 1.46, which approaches to the one between 1.5 to 2 as observed in the data. In the calibrated model, 40.34% of jobs are found through informal methods, and pay 3.01% more than jobs through formal methods, consistent with what we observe in Job Search Survey. Our model is also consistent with the firm size wage premium as widely observed in the data. Counterfactual analyses indicate that the use of informal methods enlarges frictional wage dispersion and wage differentials across firm size and search methods. Restricting informal methods would make both unemployed and employed workers worse off. Furthermore, the current unemployment insurance system reduces the wage differentials between the jobs found through formal and informal methods and improves social welfare.

In the future research, the two assumptions in our model can be relaxed. The first one is that the firm does not respond to any outside offer received by its employee. This restriction is not in line with real economics activities and has been relaxed in the search model of Postel-Vinay and Robin (2002) and Moscarini (2005). In recent literature, Wang and Yang (2018) develop a dynamic wage-tenure contract to discuss frictional wage dispersion analytically in which outside offers are public or private information imposed on the firm, respectively. The other one is the assumption of fixed wage contract. In the evidence, we find that the effect of wage premium by internal referral, as well as that of wage penalty by external referral, dissipates with tenure by the fact that firms and workers learn over time. This is an interesting issue to incorporate informal methods into a wage tenure contract framework, where we can address the effect of informal search methods on the distribution of starting wages offered, and discuss wage growth and wage differential as the tenure increases.

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A Proof of Lemma 1

Define two random events X_1 and X_2. In the second stage of search,

X_1: The worker does not obtain any job information (or the information obtained is no greater than w) through informal methods.

X_2: The worker meets another worker who is willing to accept the job information with wage w.

The random event X_1 occurs with probability $P(X_1) = 1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))$ and the random event X_2 occurs with probability $P(X_2) = \mu\Big\{u[(1 - p_w^u) + p_w^u F(w)] + [e_1 G_1(w) + e_2 G_2(w)]\,[(1 - p_w^e) + p_w^e F(w)]\Big\} \equiv \phi(w)$.

Step 1: $X_2 \subset X_1$.

Step 2: For the expanding firm, we denote the random event "the incumbent position is retained" as A and denote the random event "the vacant position is accepted" as B.

Specifically, A consists of two disjoint sub-events: (A1) the employed worker does not obtain any job opening information (or the information obtained is inferior to his current earned). (A2) replacement hiring. The employed worker finds a higher paying job, and shares his current job information with the worker who is willing to accept it. That is

$$\begin{aligned} P(A) &= \underbrace{[1 - p_w^e(1 - F(w))]\left[1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))\right]}_{\text{(A1)}} + \underbrace{p_w^e[1 - F(w)]\phi(w)}_{\text{(A2)}} \\ &= r(w) \end{aligned}$$

The random event B also consists of two disjoint sub-events: (B1) the vacancy is accepted through formal methods and external referral, $a(w)$, which is the same acceptance function compared to the vacant firm. (B2) the vacancy can be accepted through referral.

$$\begin{aligned} P(B) =& \underbrace{\left[\frac{u p_w^u}{v} + \frac{e_1 p_w^e}{v} G_1(w) + \frac{e_2 p_w^e}{v} G_2(w)\right]\left[1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))\right]}_{\text{being accepted through formal methods}} \\ &+ \underbrace{\left\{\frac{e_1 p_w^e}{v}\int_w^{\overline{w}} [1 - \eta(w_2) + \eta(w_2) p_f] dG_1(w_2) + \frac{e_2 p_w^e}{v}(1 - G_2(w))\right\}\phi(w)}_{\text{being accepted through external referral}} \\ &+ \underbrace{(1 - p_f)\mu\Big\{u[(1 - p_w^u) + p_w^u F(w)] + [e_1 G_1(w) + e_2 G_2(w)]\,[(1 - p_w^e) + p_w^e F(w)]\Big\}}_{\text{(B2) being accepted through internal referral}} \\ =& a_0(w) + (1 - p_f)\phi(w) \end{aligned}$$

Hence, the transition probability with which both positions are filled, is

$$P(AB) = P(A \cap (B1 \cup B2)) = P(A \cap B1) + P(A \cap B2)$$

The second equality follows that the random event $B1$ and $B2$ are disjoint. For the first part, $P(A \cap B1) = P(A)P(B1) = r(w)a_0(w)$ since A and $B1$ is independent. For the other part,

$$\begin{aligned}P(A\cap B2) &= P(A1\cap B2)+P(A2\cap B2)\\ &=(1-p_f)\left[1-p_w^e(1-F(w))\right]P(X1\cap X2)+P(\emptyset)\\ &=(1-p_f)\left[1-p_w^e(1-F(w))\right]\phi(w)\end{aligned}$$

where $A1$ and $A2$ are disjoint. The replacement hiring $A2$ is not compatible with acceptance through internal referral $B2$, thus indicating $A2\cap B2=\emptyset$. Finally, in the first stage of search, the sub-events $A1$ and $B2$ are independent. The employed worker does not obtain information whose wage is greater than w, and the vacancy also is not matched with any worker. In the second stage of search, $X1\cap X2 = X2$.

Therefore, $\sigma_{11}(w) = r(w)a_0(w) + (1-p_f)\left[1-p_w^e(1-F(w))\right]\phi(w)$.

Step 3: We derive the probability with which both positions are separated.

$$\begin{aligned}\sigma_0(w) =&[1-p_w^e+p_w^eF(w)][p_w^R(1-F_R(w))+p_w^C(1-F_C(w))][p_f-a_0(w)+1-p_f]\\ &+p_w^e(1-F(w))(1-\phi(w))[p_f-a_0(w)+1-p_f]\\ =&(1-r(w))(1-a_0(w))\end{aligned}$$

B Proof of Proposition 1

Proof. Let $U(w)=(\theta-w)+\beta(1-\lambda)\max\{U_{10}(w),U_1(w)\}$. Free entry condition implies that for any $w\in\Phi$, we have $U_0(w)=0$. Hence, $a_0(w)U(w)=k$ and $U_1(w)=r(w)U(w)=r(w)k/a_0(w)$.

Step 1: $U_{11}(w)\le 2\max\{U_{10}(w),U_1(w)\},\forall w\in\Phi$.

According to (20), the value function can be rearranged

$$\begin{aligned}U_{11}(w) =&r(w)^2\Big[2(\theta-w)+2\beta(1-\lambda)\lambda\max\{U_{10}(w),U_1(w)\}\Big]+r(w)^2\beta(1-\lambda)^2U_{11}(w)\\ &+2r(w)[1-r(w)]U(w)\end{aligned}$$

which implies

$$\begin{aligned}&\left[1-\beta(1-\lambda)^2r(w)^2\right]U_{11}(w)\\ =&2r(w)^2U(w)-2r(w)^2\beta(1-\lambda)^2\max\{U_{10}(w),U_1(w)\}+2r(w)[1-r(w)]U(w)\\ =&2r(w)U(w)-2\beta(1-\lambda)^2r(w)^2\max\{U_{10}(w),U_1(w)\}\end{aligned}$$

Finally,

$$\left[1-\beta(1-\lambda)^2r(w)^2\right]\Big[U_{11}(w)-2\max\{U_{10}(w),U_1(w)\}\Big]=2U_1(w)-2\max\{U_{10}(w),U_1(w)\} \tag{B.1}$$

Hence, $\forall w\in\Phi$, if $U_{10}(w)>U_1(w)$, then the righthand of (B.1) is negative, which implies $U_{11}(w)<2\max\{U_{10}(w),U_1(w)\}=2U_{10}(w)$. If $U_{10}(w)\le U_1(w)$, then the righthand of (B.1) is zero, which implies $U_{11}(w)=2\max\{U_{10}(w),U_1(w)\}=2U_1(w)$.

Step 2: We prove $U_{10}(w)>U_1(w)$, $\forall w\in\Phi$.

For expanding firms, the value function is

$$\begin{aligned}U_{10}(w)=-k+&\sigma_{11}(w)\Big[2(\theta-w)+\beta(1-\lambda)^2U_{11}(w)+2\beta(1-\lambda)\max\{U_{10}(w),U_1(w)\}\Big]\\ &+\sigma_1(w)U(w)-2\sigma_{11}(w)\beta(1-\lambda)^2\max\{U_{10}(w),U_1(w)\}\end{aligned}$$

which implies

$$U_{10}(w) = -k + [2\sigma_{11}(w) + \sigma_1(w)]U(w) + \beta(1-\lambda)^2\sigma_{11}(w)\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big] \tag{B.2}$$

where

$$2\sigma_{11}(w) + \sigma_1(w) = a_0(w) + r(w) + (1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)] \tag{B.3}$$

Suppose $U_{10}(w) \leq U_1(w)$, which implies $\max\{U_{10}(w), U_1(w)\} = U_1(w)$. Hence, the righthand of equation (B.1) equals zero, thus indicating that $U_{11}(w) = 2\max\{U_{10}(w), U_1(w)\} = 2U_1(w)$. Substituting this result into the value function (B.2), it yields

$$\begin{aligned}U_{10}(w) =& -k + \Big\{a_0(w) + r(w) + (1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)]\Big\}U(w)\\ =& U_1(w) + (1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)]U(w)\\ >& U_1(w)\end{aligned}$$

The inequality follows that frictional labor market implies $p_f < 1$. The result contradicts with $U_{10}(w) \leq U_1(w)$. Therefore, we conclude that $U_{10}(w) > U_1(w)$.

Finally, according to (B.1), $U_{11}(w) < 2\max\{U_{10}(w), U_1(w)\} = 2U_{10}(w)$.

□

C Proof of Proposition 2

Proof. In the stationary equilibrium, the number of firms that flows into and out of the set of employed workers at wages no greater than w are equal. The stationary condition from workers' side satisfies

$$\begin{aligned}&(1-u)G(w) = (1-\lambda)uQ_U(w) \\ +\quad &(1-\lambda)(1-u)G(w)\Big[(1-p_w^e) + p_w^e F(w)\Big]\Big[1 - p_w^R(1-F_R(w)) - p_w^C(1-F_C(w))\Big]\end{aligned} \tag{C.1}$$

The first part of RHS of (C.1) represents the flow from unemployment to employment at wages no greater than w. The second part represents the number of employed workers stay on or move from lower to higher paying jobs but still below wages w. Both the two parts are not separated with probability $1-\lambda$. Hence, let $w = \overline{w}$, which implies that the equilibrium unemployment is

$$u = \frac{\lambda}{\lambda + (1-\lambda)\left[p_w^u + (1-p_w^u)(p_w^R + p_w^C)\right]} \tag{C.2}$$

In equilibrium, substituting the probability of expanding $\eta(w) = 1$ and $e_1 = v_1$ into the matching probability through informal methods yields that

$$\begin{aligned}p_w^R + p_w^C &= \mu\left[e_1 p_w^e + v_1(1-p_f)(1-p_w^e) + e_2 p_w^e\right]\\ &= \mu e_1(1 - p_f + p_f p_w^e) + \mu e_2 p_w^e\end{aligned} \tag{C.3}$$

□

D Proof of Proposition 3

Step 1: We prove $\phi(w) \leq 1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))$.

Under Proposition 1, the matching probability with which a worker obtains job information through informal methods can be simplified as following.

$$p_w^R + p_w^C = \mu e_1(1 - p_f + p_f p_w^e) + \mu e_2 p_w^e \tag{D.1}$$

and

$$\begin{aligned} p_w^R F_R(w) + p_w^C F_C(w) =& \mu e_1 \Big\{ (1 - p_f) G_1(w) + p_f p_w^e [F(w) + G_1(w) - F(w) G_1(w)] \Big\} \\ &+ \mu e_2 [F(w) + G_2(w) - F(w) G_2(w)] \end{aligned} \tag{D.2}$$

Hence,

$$1 - p_w^R - p_w^C = 1 - \mu + \mu u + \mu(1 - p_w^e) e_1 p_f + \mu e_2 (1 - p_w^e)$$

and

$$\begin{aligned} & 1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w)) \\ =& 1 - \mu + \mu u + \mu e_2 [1 - p_w^e + p_w^e F(w)] + \mu e_2 p_w^e [1 - F(w)] G_2(w) \\ &+ \mu e_1 p_f [1 - p_w^e + p_w^e F(w)] + \mu e_1 (1 - p_f) G_1(w) + \mu e_1 p_f p_w^e [1 - F(w)] G_1(w) \\ \geq& \mu u + \mu e_2 [1 - p_w^e + p_w^e F(w)] + \mu e_1 p_f [1 - p_w^e + p_w^e F(w)] + \mu e_1 (1 - p_f) G_1(w) \end{aligned}$$

Rearranging the last two terms yields that

$$\begin{aligned} & \mu e_1 p_f [1 - p_w^e + p_w^e F(w)] + \mu e_1 (1 - p_f) G_1(w) \\ \geq& \mu e_1 p_f G_1(w) [1 - p_w^e + p_w^e F(w)] + \mu e_1 (1 - p_f) G_1(w) [1 - p_w^e + p_w^e F(w)] \\ =& \mu e_1 G_1(w) [1 - p_w^e + p_w^e F(w)] \end{aligned}$$

Hence, we obtain

$$\begin{aligned} & 1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w)) \\ \geq& \mu u + \mu e_2 [1 - p_w^e + p_w^e F(w)] + \mu e_1 G_1(w) [1 - p_w^e + p_w^e F(w)] \\ \geq& \mu u [1 - p_w^u + p_w^u F(w)] + \mu e_2 G_2(w) [1 - p_w^e + p_w^e F(w)] + \mu e_1 G_1(w) [1 - p_w^e + p_w^e F(w)] \\ =& \phi(w) \end{aligned}$$

Step 2: F has no mass point.

Suppose F is discontinuous at $w = w_0$, which implies w_0 is a mass point of F. Without loss of generality, let $F(w_0 + \epsilon) = F(w_0) + \delta$, where $\delta > 0$ is the mass of firms offering wage w_0.

If a firm posts a fixed wage contract with $w_0 < \underline{w}$, no employee would accept this offer. If $\underline{w} \leq w_0 < \overline{w}$, the employer can improve his value by offering wage slightly greater than w_0. We prove that the firm has a significantly larger retention probability and acceptance probability and only a slightly smaller flow profit than offering w_0.

Suppose the firm posts a slightly higher wage $w_0 + \epsilon$, the retention probability is

$$\begin{aligned} r(w_0+\epsilon) =& [1 - p_w^e(1 - F(w_0+\epsilon)] \left[1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon))\right] \\ &+ p_w^e[1 - F(w_0+\epsilon)]\phi(w_0+\epsilon) \\ =& [1 - p_w^e(1 - F(w_0)) + p_w^e\eta] \left[1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon))\right] \\ &+ p_w^e[1 - F(w_0) - \eta]\phi(w_0+\epsilon) \\ =& [1 - p_w^e(1 - F(w_0))] \left[1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon))\right] \\ &+ p_w^e\eta \left[1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon))\right] \\ &+ p_w^e[1 - F(w_0)]\phi(w_0+\epsilon) - p_w^e\eta\phi(w_0+\epsilon) \\ \geq& [1 - p_w^e(1 - F(w_0))] \left[1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon))\right] \\ &+ p_w^e[1 - F(w_0)]\phi(w_0+\epsilon) \end{aligned}$$

The last inequality follows from the result of step 1 $\phi(w) \leq 1 - p_w^R(1 - F_R(w)) - p_w^C(1 - F_C(w))$. Next, equation (D.2) implies

$$\begin{aligned} &1 - p_w^R(1 - F_R(w_0+\epsilon)) - p_w^C(1 - F_C(w_0+\epsilon)) \\ =&1 - p_w^R(1 - F_R(w_0)) - p_w^C(1 - F_C(w_0)) + \mu e_1 p_f p_w^e \delta(1 - G_1(w_0)) + \mu e_2 p_w^e \delta(1 - G_2(w_0)) \\ =&1 - p_w^R(1 - F_R(w_0)) - p_w^C(1 - F_C(w_0)) + \nu_1\delta \\ >&1 - p_w^R(1 - F_R(w_0)) - p_w^C(1 - F_C(w_0)) \end{aligned}$$

where $\nu_1 = \mu e_1 p_f p_w^e(1 - G_1(w_0)) + \mu e_2 p_w^e(1 - G_2(w_0)) \in (0, 1)$. According to equation (15), $\phi(w_0 + \epsilon)$ is determined by

$$\phi(w_0+\epsilon) = \phi(w_0) + \mu u p_w^u \delta + \mu[e_1 G_1(w_0) + e_2 G_2(w_0)]p_w^e \delta = \phi(w_0) + \nu_2\delta > \phi(w_0)$$

where $\nu_2 = \mu u p_w^u + \mu[e_1 G_1(w_0) + e_2 G_2(w_0)]p_w^e \in (0, 1)$. Therefore, substituting the two inequalities into retention function yields $r(w_0 + \epsilon) > r(w_0)$.

Similarly, the acceptance function for vacant firms (**??**) implies

$$\begin{aligned} a_0(w_0+\epsilon) =& a_0(w_0) + \left[\frac{u p_w^u}{v} + \frac{e_1 p_w^e}{v} G_1(w_0) + \frac{e_2 p_w^e}{v} G_2(w_0)\right]\nu_1\delta \\ &+ \left[\frac{e_1 p_w^e}{v} p_f(1 - G_1(w_0)) + \frac{e_2 p_w^e}{v}(1 - G_2(w_0))\right]\nu_2\delta > a_0(w_0) \end{aligned}$$

The firm has significant larger retention probability and acceptance probability if offering a slightly higher wage $w_0 + \epsilon$. Therefore, the vacant firm has an incentive to deviate the equilibrium and offer a slightly higher wage, which contradicts with the definition of equilibrium.

If there were a mass of F at $w_0 \geq \overline{w}$, firms make a nonpositive flow profit and value. Free entry condition implies that such firms voluntarily withdraw from the market, thus indicating that it is not an equilibrium.

Step 3: Φ is connected and $\Phi = [\underline{w}, \overline{w}]$.

Suppose Φ is not connected. Since F has no mass point, there exists $w_1, w_2 \in \Phi$, such that $w_1 < w_2$ and $F(w_1) = F(w_2)$. Similarly, it is straightforward to show that $r(w_1) = r(w_2)$ and $a_0(w_1) = a_0(w_2)$.

Hence, a vacant firm posting wage w_2 has an incentive to deviate its strategy and instead post the lower wage w_1, indicating that $w_2 \notin \Phi$. This is a contradiction.

Finally, since noncontinuous distributions of wages offered have been ruled out, it can be shown that F is continuous. From self-enforcing constraint $V_E \geq V_U$, it follows that $\Phi = [\underline{w}, \overline{w}]$.

E The Derivation of Zero Profit Condition (29)

Since $U_{10}(w) > U_1(w)$, it follows that $\max\{U_{10}(w), U_1(w)\} = U_{10}(w)$. Substituting the above into the value function (B.1), it yields that $U_{11}(w) < 2U_{10}(w)$. Next, we rearrange the value function of expanding firms.

$$\begin{aligned} U_{10}(w) =& r(w)U(w) + (1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)]U(w) \\ &+ \frac{2\beta(1-\lambda)^2\Big\{r(w)a_0(w) + (1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)]\Big\}}{1-\beta(1-\lambda)^2 r(w)^2}[r(w)U(w) - U_{10}(w)] \end{aligned}$$

which implies

$$\begin{aligned} & U_{10}(w) - r(w)U(w) \\ =& \frac{[1-\beta(1-\lambda)^2 r(w)^2](1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)]U(w)}{1-\beta(1-\lambda)^2 r(w)^2 + 2\beta(1-\lambda)^2\Big\{r(w)a_0(w) + (1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)]\Big\}} \end{aligned}$$

Hence, we find that $U_{10}(w) = [r(w) + \Delta(w)]U(w)$, where $\Delta(w)$ follows equation (30). According to the free entry condition $U_0(w) = 0$, it follows that

$$\begin{aligned} k =& a_0(w)U(w) \\ =& a_0(w)(\theta - w) + \beta(1-\lambda)a_0(w)U_{10}(w) \\ =& a_0(w)(\theta - w) + \beta(1-\lambda)a_0(w)[r(w)U(w) + \Delta(w)U(w)] \\ =& a_0(w)(\theta - w) + \beta(1-\lambda)[r(w) + \Delta(w)]k \end{aligned}$$

Finally, we can obtain the equilibrium zero profit condition as follows.

$$\Big\{1 - \beta(1-\lambda)[r(w) + \Delta(w)]\Big\}k = a_0(w)(\theta - w)$$

F Appendix on Calibration

F.1 Reservation wage

For an employed worker, the value at reservation wage $\underline{w}$ is indifferent with that of remaining unemployed in equilibrium, indicating that $V_E(\underline{w}) = V_U$. Hence, the employed worker's value function at reservation wage is

$$\begin{aligned}V_E(\underline{w}) &= u((1-\tau)\underline{w}) + \beta(1-p_w^e)(1-p_w^R-p_w^C)V_E(\underline{w})\\ &+\beta(1-\lambda)p_w^e\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^R(1-F_R(w_1'))-p_w^C(1-F_C(w_1'))\Big]V_E(w_1')dF(w_1')\\ &+\beta(1-\lambda)p_w^R\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2))\Big]V_E(w_2)dF_R(w_2)\\ &+\beta(1-\lambda)p_w^C\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2'))\Big]V_E(w_2')dF_C(w_2')+\lambda\Big(V_U-u(b)\Big)\end{aligned}$$

Hence, we subtract $\beta(1-\lambda)V_E(\underline{w})$ on both sides, it yields

$$\begin{aligned}&[1-\beta(1-\lambda)]V_E(\underline{w}) = u((1-\tau)\underline{w}) + \lambda\Big(V_U-u(b)\Big)\\ &+\beta(1-\lambda)p_w^e\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^R(1-F_R(w_1'))-p_w^C(1-F_C(w_1'))\Big]\left[V_E(w_1')-V_E(\underline{w})\right]dF(w_1')\\ &+\beta(1-\lambda)p_w^R\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2))\Big]\left[V_E(w_2)-V_E(\underline{w})\right]dF_R(w_2)\\ &+\beta(1-\lambda)p_w^C\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2'))\Big]\left[V_E(w_2')-V_E(\underline{w})\right]dF_C(w_2')\end{aligned}$$

which implies

$$\begin{aligned}[1-\beta(1-\lambda)]V_E(\underline{w}) =&u((1-\tau)\underline{w}) + \lambda\Big(V_U-u(b)\Big)+\beta(1-\lambda)p_w^e\int_{\underline{w}}^{\overline{w}}[1-F(w_1')]dV_E(w_1')\\ &+\beta(1-\lambda)p_w^R\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2))\Big][1-F_R(w_2)]dV_E(w_2)\\ &+\beta(1-\lambda)p_w^C\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^e(1-F(w_2'))\Big][1-F_C(w_2')]dV_E(w_2')\end{aligned} \tag{F.1}$$

Using the method of Integration by parts, the unemployed worker's value function can be reduced to the following equation.

$$\begin{aligned}(1-\beta)V_U =&u(b)+\beta p_w^u\int_{\underline{w}}^{\overline{w}}[1-F(w_1')]dV_E(w_1')\\ &+\beta p_w^R\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^u(1-F(w_2))\Big][1-F_R(w_2)]dV_E(w_2)\\ &+\beta p_w^C\int_{\underline{w}}^{\overline{w}}\Big[1-p_w^u(1-F(w_2'))\Big][1-F_C(w_2')]dV_E(w_2')\end{aligned} \tag{F.2}$$

Hence, combining equation (F.1) with (F.2), the unemployment benefit is determined by

$$\begin{aligned}&[1-\beta(1-\lambda)](V_E(\underline{w})-V_U)\\ =&u((1-\tau)\underline{w})-u(b)-\beta(1-\lambda)(p_w^u-p_w^e)\int_{\underline{w}}^{\overline{w}}[1-F(w_1')]dV_E(w_1')\\ &+\beta(1-\lambda)(p_w^u-p_w^e)p_w^R\int_{\underline{w}}^{\overline{w}}[1-F(w_2)][1-F_R(w_2)]dV_E(w_2)\\ &+\beta(1-\lambda)(p_w^u-p_w^e)p_w^C\int_{\underline{w}}^{\overline{w}}[1-F(w_2')][1-F_C(w_2')]dV_E(w_2')\end{aligned}$$

In equilibrium, since $V_E(\underline{w}) = V_U$, this equation can be reduced to

$$u((1-\tau)\underline{w}) = u(b) + wedge \tag{F.3}$$

where

$$wedge = \beta(1-\lambda)(p_w^u - p_w^e)\int_{\underline{w}}^{\overline{w}}[1-F(x)]\Big[1-p_w^R(1-F_R(w)) - p_w^C(1-F_C(w))\Big]V_E'(w)dw \tag{F.4}$$

Finally, to solve $V_E'(w)$, differentiating the value function (12) with respect to w yields that

$$V_E'(w) = \frac{(1-\tau)u'((1-\tau)w)}{1-\beta(1-\lambda)\Big[1-p_w^e(1-F(w))\Big]\Big[1-p_w^R(1-F_R(w))-p_w^C(1-F_C(w))\Big]} \tag{F.5}$$

The wedge between the reservation wage $\underline{w}$ and unemployment benefit b depends on the distributions of wages offered, payroll tax τ and the difference of matching probability p_w^u and p_w^e. The positive option value of waiting for a better outside offer yields higher reservation wage relative to the unemployment benefit.

F.2 Equations used in the Calibration

This appendix shows how to solve for the values of $\underline{w}$, $\overline{w}$ and the functions $F(w)$, $F_0(w)$, $F_1(w)$, $G(w)$, $G_1(w)$, $G_2(w)$, $F_R(w)$, $F_C(w)$, $r(w)$ and $a_0(w)$.

First, according to the equilibrium zero profit condition (29), we can solve for the values of $\underline{w}$ and $\overline{w}$.

$$\underline{w} = \theta - \frac{1-\beta(1-\lambda)(r(\underline{w})+\Delta(\underline{w}))}{a_0(\underline{w})}k \tag{F.6}$$

and

$$\overline{w} = \theta - \frac{1-\beta(1-\lambda)(r(\overline{w})+\Delta(\overline{w}))}{a_0(\overline{w})}k \tag{F.7}$$

where the retention function $r(w)$, acceptance function $a_0(w)$ and premium function of referral $\Delta(w)$ are determined by (30).

Next, we solve a set of differential equations for the seven functions $F(w)$, $G(w)$, $F_0(w)$, $G_1(w)$, $G_2(w)$, $r(w)$ and $a_0(w)$. For all $w \in [\underline{w}, \overline{w}]$, according to Proposition 1, we have

$$p_w^R + p_w^C = \mu e_1(1-p_f+p_f p_w^e) + \mu e_2 p_w^e$$

and

$$\begin{aligned} p_w^R F_R(w) + p_w^C F_C(w) =& \mu e_1\Big\{(1-p_f)G_1(w) + p_f p_w^e[F(w)+G_1(w)-F(w)G_1(w)]\Big\} \\ &+ \mu e_2[F(w)+G_2(w)-F(w)G_2(w)] \end{aligned}$$

Hence, the function $r(w)$ and $a_0(w)$ are both determined by the distributions $F(w)$, $G_1(w)$ and $G_2(w)$. Since $\eta(w) = 1$, all firms with one position choose to expand, indicating that $F_1(w) = G_1(w)$ and

$$(v_0+v_1)F(w) = v_0F_0(w) + v_1G_1(w) \tag{F.8}$$

Furthermore, differentiating the stationary condition (24) with respect to w yields

$$\begin{aligned} e_2 g_2(w) =& 2(1-\lambda)^2 e_1 \Big\{ r(w) a_0(w) + (1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)] \Big\} g_1(w) \\ &+ (1-\lambda)^2 e_2 r(w)^2 g_2(w) \end{aligned} \tag{F.9}$$

where $\phi(w)$ is determined by (15). Differentiating the sum of stationary conditions (23) and (24), we obtain

$$\begin{aligned} (1-u)g(w) =& (1-\lambda)(v_0+v_1)a_0(w)f(w) + (1-\lambda)(1-u)r(w)g(w) \\ &+ (1-\lambda)e_1(1-p_f)\phi(w)[1-p_w^e + p_w^e F(w)]g_1(w) \end{aligned} \tag{F.10}$$

The system consists of above equations (F.8), (F.9) and (F.10), together with the aggregate distribution of wages earned (6), equilibrium zero profit condition (29), plus the retention function (17) and acceptance function (14), a total of seven equations. Hence, we can apply the numerical method of Differential-Algebraic Equations to solve the seven equilibrium functions $F(w)$, $G(w)$, $F_0(w)$, $G_1(w)$, $G_2(w)$, $r(w)$ and $a_0(w)$.

G Model II

A vacant firm can post either one vacancy or both two vacancies. The value of a vacant firm, denoted as $\overline{U} : \Phi \to \mathbb{R}$, is defined as follows.

$$\overline{U}(w) = \max\{U_0(w), U_{00}(w)\} \tag{F.11}$$

where $U_0(w)$ and $U_{00}(w)$ denote the value of a vacant firm to post one vacancy and two vacancies with fixed wage contract w respectively. For an incumbent firm filled with one vacancy, let $U_1(w)$ denote the value of that chooses not to expand, and $U_{10}(w)$ denote the value of that chooses to expand. Furthermore, let $U_{11}(w)$ denote the value of an incumbent firm filled with two vacancies.

Hence, the value of the vacant firm to post one vacancy is

$$\begin{aligned} U_0(w) =& -k + a_0(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda\overline{U}(w)\Big] \\ &+ (1-a_0(w))\left[0 + \beta\overline{U}(w)\right] \end{aligned} \tag{F.12}$$

The value of a vacant firm to post two vacancies is

$$\begin{aligned} U_{00}(w) =& -2k + a_0(w)^2 \left\{ \begin{array}{c} 2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + \beta\lambda^2\overline{U}(w) \\ +2\beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\} \end{array} \right\} \\ &+2a_0(w)(1-a_0(w))\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda\overline{U}(w)\Big] \\ &+(1-a_0(w))^2\left[0 + \beta\overline{U}(w)\right] \end{aligned} \tag{F.13}$$

To post two vacancies, the vacant firm must incur posting cost $2k$. If both vacancies are accepted, the firm yields the profit of $2(\theta - w)$. In the next period, there are three scenarios that are contingent on the number of matches separated: (1) The firm obtains the value of $U_{11}(w)$ if none of matches is separated. (2) The firm decides whether to expand if one matches is separated. (3) The firm turns into a vacant firm if two matches are both separated. The second line represents the value of being filled with only one vacancy. The third line represents the value of being vacant if any vacancy is not filled.

For the incumbent firm that filled with one vacancy and not choose to expand, the value $U_1 : \Phi \to \mathbb{R}$ is

$$\begin{aligned} U_1(w) =& r(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda\overline{U}(w)\Big] \\ &+ (1 - r(w))\left[0 + \beta\overline{U}(w)\right] \end{aligned} \tag{F.14}$$

The value of an expanding firm is determined by the following.

$$\begin{aligned} U_{10}(w) =& -k + \sigma_{11}(w)\left\{\begin{array}{c} 2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + \beta\lambda^2\overline{U}(w) \\ +2\beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\} \end{array}\right\} \\ &+\sigma_1(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda\overline{U}(w)\Big] \\ &+\sigma_0(w)\left[0 + \beta\overline{U}(w)\right] \end{aligned} \tag{F.15}$$

where $\sigma_{11}(w), \sigma_1(w), \sigma_0(w)$ denotes the transition probability. Specifically, after the labor market closes, there are 3 states about the positions of incumbent firms.

1. Both two positions are filled: $\sigma_{11}(w) = r(w)a_0(w) + (1-p_f)\phi(w)[1 - p_w^e + p_w^e F(w)]$.
2. Only one position is filled: $\sigma_1(w) = r(w)(1 - a_0(w)) + (1 - r(w))a_0(w) - (1 - p_f)[1 - p_w^e + p_w^e F(w)]\phi(w)$.
3. Both two positions are separated: $\sigma_0(w) = (1 - r(w))(1 - a_0(w))$.

Finally, the value of an incumbent firm filled with two workers is

$$\begin{aligned} U_{11}(w) =& r(w)^2\left\{\begin{array}{c} 2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + \beta\lambda^2\overline{U}(w) \\ +2\beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\} \end{array}\right\} \\ &+2(1 - r(w))r(w)\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\} + \beta\lambda\overline{U}(w)\Big] \\ &+(1 - r(w))^2\left[0 + \beta\overline{U}(w)\right] \end{aligned} \tag{F.16}$$

Proposition 4. *In equilibirum,*

(a) Expanding is the best response for any incumbent firm filled with one vacancy $U_{10}(w) > U_1(w)$*, which implies* $\forall w \in \Phi$*, the probability of expanding* $\eta(w) = 1$*.*

(b) For any vacant firm, posting one vacancy is the best response $U_{00}(w) < U_0(w) = 0$*.*

(c) $\forall w \in \Phi$*,* $U_{11}(w) < 2U_{10}(w)$*.*

Proof.

Step 1. $U_{11}(w) \leq 2\max\{U_{10}(w), U_1(w)\}, \forall w \in \Phi$.

Let $U(w) = (\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\}$. Free entry condition implies that for any $w \in \Phi$, we have $\overline{U}(w) = 0$. Substituting the free entry condition into (16), it yields

$$U_1(w) = r(w)U(w) \tag{F.17}$$

According to (20), the value function can be rearranged

$$\begin{aligned}U_{11}(w) =& r(w)^2\Big[2(\theta - w) + 2\beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\}\Big] + r(w)^2\beta(1-\lambda)^2U_{11}(w)\\ &+ 2r(w)[1-r(w)]U(w)\end{aligned}$$

which implies

$$\begin{aligned}&\left[1-\beta(1-\lambda)^2r(w)^2\right]U_{11}(w)\\ =&2r(w)^2U(w) - 2r(w)^2\beta(1-\lambda)^2\max\{U_{10}(w), U_1(w)\} + 2r(w)[1-r(w)]U(w)\\ =&2r(w)U(w) - 2\beta(1-\lambda)^2r(w)^2\max\{U_{10}(w), U_1(w)\}\end{aligned}$$

Finally,

$$\left[1-\beta(1-\lambda)^2r(w)^2\right]\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big] = 2U_1(w) - 2\max\{U_{10}(w), U_1(w)\} \tag{F.18}$$

Hence, $\forall w \in \Phi$, if $U_{10}(w) > U_1(w)$, then the righthand of (F.18) is negative, which implies $U_{11}(w) < 2\max\{U_{10}(w), U_1(w)\} = 2U_{10}(w)$. If $U_{10}(w) \leq U_1(w)$, then the righthand of (F.18) is zero, which implies $U_{11}(w) = 2\max\{U_{10}(w), U_1(w)\} = 2U_1(w)$.

Step 2. $U_{00}(w) \leq U_0(w) = 0$

Free entry condition $\overline{U}(w) = \max\{U_0(w), U_{00}(w)\} = 0$ implies $U_0(w) \leq 0$ and $U_{00}(w) \leq 0$. Using the adjoint of (13) and (**??**), it follows that

$$\begin{aligned}&U_{00}(w) - U_0(w)\\ =& -k + 2a_0(w)^2\Big[(\theta - w) + \beta(1-\lambda)\lambda\max\{U_{10}(w), U_1(w)\}\Big] + a_0(w)^2\beta(1-\lambda)^2U_{11}(w)\\ &+ 2a_0(w)(1-a_0(w))\Big[(\theta - w) + \beta(1-\lambda)\max\{U_{10}(w), U_1(w)\}\Big] - a_0(w)U(w)\\ =& -k + a_0(w)U(w) + \beta(1-\lambda)^2a_0(w)^2\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big]\\ =&U_0(w) + \beta(1-\lambda)^2a_0(w)^2\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big]\end{aligned} \tag{F.19}$$

where the last equality follows from the value function (13). Suppose $U_0(w) < U_{00}(w) = 0$, which implies that

$$U_0(w) = \frac{1}{2}\beta(1-\lambda)^2a_0(w)^2\Big[2\max\{U_{10}(w), U_1(w)\} - U_{11}(w)\Big] \geq 0 \tag{F.20}$$

The inequality follows from the result of Step 1, which yields a contradiction. Hence, we obtain $U_{00}(w) \leq U_0(w) = 0$.

Step 3. $U_{10}(w) > U_1(w)$, $U_{11}(w) < 2U_{10}(w)$.

For expanding firms, the value function is

$$U_{10}(w) = -k + \sigma_{11}(w)\Big[2(\theta - w) + \beta(1-\lambda)^2 U_{11}(w) + 2\beta(1-\lambda)\max\{U_{10}(w), U_1(w)\}\Big]$$
$$+\sigma_1(w)U(w) - 2\sigma_{11}(w)\beta(1-\lambda)^2 \max\{U_{10}(w), U_1(w)\}$$

which implies

$$U_{10}(w) = -k + [2\sigma_{11}(w) + \sigma_1(w)]U(w) + \beta(1-\lambda)^2\sigma_{11}(w)\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big] \tag{F.21}$$

where

$$2\sigma_{11}(w) + \sigma_1(w) = r(w) + a_0(w) + (1-p_f)[1 - p_w^e + p_w^e F(w)]\phi(w) > 0 \tag{F.22}$$

Suppose $U_{10}(w) \le U_1(w)$, which implies $\max\{U_{10}(w), U_1(w)\} = U_1(w)$. Hence, the righthand of equation (F.18) equals zero, thus indicating that $U_{11}(w) = 2\max\{U_{10}(w), U_1(w)\} = 2U_1(w)$. Substituting this result into the value function (F.21), it yields

$$\begin{aligned}
U_{10}(w) =& - k + [2\sigma_{11}(w) + \sigma_1(w)]\, U(w) \\
=& - k + a_0(w)U(w) + r(w)U(w) + (1-p_f)[1 - p_w^e + p_w^e F(w)]\phi(w)U(w) \\
=& U_0(w) + U_1(w) + (1-p_f)[1 - p_w^e + p_w^e F(w)]\phi(w)U(w) \\
>& U_1(w)
\end{aligned}$$

The result $U_0(w) = 0$ implies that $a_0(w)U(w) = k$. The last inequality follows that frictional labor market implies $p_f < 1$. The result contradicts with $U_{10}(w) \le U_1(w)$. Therefore, we conclude that $U_{10}(w) > U_1(w)$.

Finally, substituting $U_{10}(w) > U_1(w)$ into (F.18) yields that $U_{11}(w) < 2U_{10}(w)$.

Step 4. $U_{00}(w) < 0$.

(F.19) implies that

$$\begin{aligned}
U_{00}(w) &= 2U_0(w) + \beta(1-\lambda)^2 a_0(w)^2\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big] \\
&= \beta(1-\lambda)^2 a_0(w)^2\Big[U_{11}(w) - 2\max\{U_{10}(w), U_1(w)\}\Big] \\
&< 0
\end{aligned}$$

The second equality follows from $U_0(w) = 0$ and the last inequality follows from $U_{10}(w) > U_1(w)$ and $U_{11}(w) < 2U_{10}(w)$. □

For those vacant firms, posting one vacancy is superior to posting two vacancies. The intuition is as follows: The vacancy posted by the expanding firm can be accepted through internal referral. As a result, the expanding firm obtain a higher transition probability to fill with two workers. By posting one vacancy per period, the vacant firm yields an opportunity cost of the value $U_{11}(w)$, however, yields a higher benefit of the value $2U_{10}(w)$ through internal referral.

$$U_0(w) - U_{00}(w) = 2\beta(1-\lambda)^2 a_0(w)^2[2U_{10}(w) - U_{11}(w)] > 0 \tag{F.23}$$

H Table Appendix

Table 11: The effect of UI

	Without UI	UI ($rr = 41\%$)	Increase by
Panel A: Unemployment and Vacancy			
Unemployment	0.0524	0.0671	28.05%
Number of vacant firms	0.5639	0.2993	-46.92%
Number of firms	1.1723	0.8832	-24.66%
Job creation/Matching	71.59%	81.46%	13.79%
Panel B: Turnover			
U-E Transition Prob.	55.97%	43%	-23.17%
E-E Transition Prob.	3.69%	3.18%	-13.82%
Number of employed workers e_1	0.2691	0.2348	-12.75%
Number of employed workers e_2	0.6785	0.6981	2.89%
Panel C: Welfare			
Consumption equivalence (U)	0.7447	0.8455	13.54%
Consumption equivalence (E)	0.7751	0.8633	11.38%
Consumption equivalence (ALL)	0.7744	0.8626	11.39%
Panel D: Wage Dispersion			
Mean-min ratio	2.71	1.46	-46.13%
Length of support	0.6561	0.3503	-46.61%
S.D. of $F(w)$	0.1986	0.108	-45.62%
Average wage	0.8921	0.9297	4.21%
Reservation wage	0.3298	0.6369	93.12%
Panel E: Wage Differentials			
Expanding/Vacant	20.40%	8.19%	
Incumbent/Expanding	16.96%	8.28%	
Internal referral/formal	9.74%	3.53%	
External referral/formal	-2.38%	-1.95%	
Informal/formal	8.47%	3.01%	

Table H.1: Methods of Job Search

Category	1	2	3	4	5	6	7	ALL
	emplr	public agency	friend acquai.	online	ads	un-solicited	other	
Sample freq.	27.05	11.10	40.43	11.27	8.66	9.80	12.66	
Gender								
Female	49.16	52.34	46.05	56.02	53.14	36.87	50.69	48.35
Male	50.84	47.66	53.95	43.98	46.86	63.13	49.31	51.65
Age								
$18 \sim 30$	20.35	19.44	19.14	23.25	3.13	13.26	23.20	17.23
$31 \sim 40$	23.23	28.22	28.80	30.68	21.48	27.80	28.28	26.32
$41 \sim 60$	50.80	46.70	46.04	39.80	68.02	50.93	44.32	50.22
$61 \sim 65$	5.61	5.64	6.02	6.27	7.36	8.01	4.20	6.23
Education								
High school	28.95	23.63	33.03	19.42	40.35	15.13	32.21	30.18
Some college	28.73	23.66	26.96	28.71	34.40	30.35	25.54	29.30
College	22.40	31.23	22.98	30.10	17.06	31.63	25.00	23.37
Graduate	19.92	21.47	17.03	21.77	8.19	22.90	17.25	17.14
Race								
White	80.75	78.91	79.90	72.04	88.66	75.55	76.65	78.34
Black	9.73	9.77	9.67	9.05	3.46	7.32	14.00	10.06
Others	9.53	11.32	10.43	18.91	7.88	17.14	9.35	11.59

Table H.2: Summary Statistics

	Full sample	Formal	Internal Referral	External Referral	Difference R - F	Difference C - F
Current Job						
Hourly wages	25.97	25.65	26.88	20.24	1.23	-5.41***
	(0.572)	(0.783)	(0.864)	(1.636)		
Usual hours	42.69	42.75	42.76	40.46	0.01	-2.29**
	(0.227)	(0.310)	(0.340)	(0.976)		
Tenure year	8.53	8.04	9.06	12.32	1.02**	4.28**
	(0.247)	(0.305)	(0.416)	(1.679)		
Demographics: Proportion (%)						
Male	51.65	50.10	55.10	37.46	5.00*	-12.64
	(0.014)	(0.018)	(0.023)	(0.084)		
White	78.34	77.28	80.86	66.19	3.58	-11.09
	(0.012)	(0.016)	(0.018)	(0.097)		
High school	30.18	28.25	32.30	43.54	4.05	15.29
	(0.017)	(0.022)	(0.027)	(0.102)		
College	40.51	40.86	40.40	34.36	-0.46	-6.5
	(0.012)	(0.016)	(0.020)	(0.074)		
Aged 18-40	43.55	40.58	48.00	47.00	7.42**	6.42
	(0.014)	(0.018)	(0.023)	(0.096)		
Aged 41-60	50.22	53.05	45.73	50.46	-7.32**	-2.59
	(0.014)	(0.018)	(0.023)	(0.095)		
Observations	2195	1335	808	52		

Standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. Hourly wages are adjusted by CPI. Demographic controls include gender, age, race and education categories.

Table H.3: Robust Check

	(1)	(2)	(3)
Int. Referral	0.0515**	0.0917***	0.0459*
	(0.0230)	(0.0335)	(0.0247)
Ext. Referral	-0.1112*	-0.1782*	-0.1659**
	(0.0667)	(0.0921)	(0.0809)
Tenure	0.0210***	0.0222***	
	(0.0038)	(0.0040)	
$(\text{Tenure})^2/100$	-0.0378***	-0.0355***	
	(0.0130)	(0.0128)	
Tenure × Int. Referral		-0.0048*	
		(0.0028)	
Tenure × Ext. Referral		0.0067	
		(0.0067)	
Demographic Controls	Yes	Yes	Yes
Year Dummies	Yes	Yes	Yes
Observations	2040	2040	1972

Robust standard errors in parentheses. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Data source: 2013-2016 Job Search Survey, for full-time individuals aged 18-64 with valid information on their search methods and reported current wages. Hourly wages are adjusted by CPI. Demographic controls include gender, age, race and education categories.