

Low-Frequency Fiscal Uncertainty*

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ABSTRACT

Long-run fiscal levels, or synonymously, fiscal targets, are usually assumed to be known to households inside the economy. This paper investigates the effects of unknown fiscal targets in an incomplete information, anticipated utility (IIAU) environment. Highly persistent fiscal movements cause households to suspect time-varying targets and misperceptions impact both their expectation formation and decision making. Perceived targets enter models as non-linear state variables. Utilizing the conditional linearity of the model structure and estimating the model using a time-varying Kalman filter, data strongly prefer an IIAU model specification to its full information, rational expectation (FIRE) counterpart in an RBC framework with a detailed fiscal sector. Estimation results suggest the increasing transfer payments (Social Security, Medicare, and Medicaid spending) over the last half-century drives low-frequency fiscal uncertainty and shifts households' perceived long-run tax, government spending, and debt levels. The IIAU model implies smaller real frictions and weaker debt responses of fiscal instruments than the FIRE model. The IIAU model also introduces state-dependent, time-varying fiscal multipliers.

Keywords: State-Dependent Fiscal Policy; Incomplete Information; Signal Extraction; Expectation Formation; Belief-driven Fluctuations; Bayesian Estimation;

JEL Classification: C11; D83; E32; E62; H68;

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1 Introduction

The Great Recession and the resulting large-scale expansionary fiscal policies (i.e., the Economic Stimulus Act of 2008 and the American Recovery and Reinvestment Act of 2009) led to a rapid increase in U.S. debt-to-GDP ratios. Ten years after the crisis, the U.S. debt-to-GDP ratio is well above its post-WWII average (see Figure 8). The Tax Cuts and Jobs Act (TCJA) of 2017, which avoids automatic spending cuts under the Statutory Pay-as-You-Go Act of 2010, further widens public deficit.¹ Elevated public debt levels raise concerns among the public², policy makers³ and economists.⁴ Deficit hawks and doves argue when and how the budget deficit should be reduced. But how much debt is too much and to what extent should public debt be reduced or controlled? Different parties have different opinions regarding the long-run level of government debt.⁵ At the same time, independent of the recent fiscal stimulus packages, policy advisors warn that future debt-to-GDP ratios will continue to rise, mostly due to the aging population and larger demand on Social Security, Medicaid and Medicare payments.⁶ Policy advisers also raise questions on fiscal sustainability.⁷ Given possible instability in transfers and the potential reforms of the current social welfare programs, what is the long-run government transfers-to-GDP ratio? There is a huge degree of uncertainty with respect to these long-run fiscal levels.

Long-run fiscal levels are important: First, they govern the behavior of fiscal instruments in the long term. Without knowing them, the private sector’s long-run expectation of fiscal behavior is not anchored. Firms and households’ short-run expectations of the current fiscal stance could also be misspecified. Secondly, long-run fiscal levels are inherently connected due to the government

¹According to CBO (2018), the budget deficit increased from \$666 billion (3.5% of GDP) in fiscal year 2017 to \$779 billion (3.9% of GDP) in 2018. In January 2017, CBO forecast tax revenues would be \$3.60 trillion if laws in place as of January 2017 continued. The actual 2018 tax revenue is \$3.33 trillion. The \$270 billion (or 7.5%) forecast underestimation is primarily due to TCJA.

²Based on GALLUP polls, from March 2011 to March 2018, when asked “How much do you personally worry about federal spending and the budget deficit?”, between 49% and 64% responders worry about this problem “a great deal”, between 21% and 27% worry about it “a fair amount”. Only 3% to 7% responders think the budget deficit is “not at all” a problem. The “no opinion” rates are 1%. Source: <https://news.gallup.com/poll/147626/federal-budget-deficit.aspx>;

³At the Economic Club of Washington on January 10th, 2019, the current Federal Reserve Chairman Jerome Powell mentioned “*I’m very worried about [the ballooning amount of U.S. debt]. From the Fed’s standpoint, we’re really looking at a business cycle length: that’s our frame of reference. The long-run fiscal, non-sustainability of the U.S. federal government isn’t really something that plays into the medium term that is relevant for our policy decisions;...it’s a long-run issue that we definitely need to face, and ultimately, will have no choice but to face.*”

⁴In response to a survey conducted by the University of Chicago’s Initiative on Global Markets, 38 economists unanimously agree or strongly agree the US debt-to-GDP ratios will be substantially higher a decade from now, while there is no clear consensus on whether TCJA would benefit the US GDP—assuming no other changes in tax or spending policy. Source: <http://www.igmchicago.org/surveys/tax-reform-2>;

⁵In CBO’s 2012 Long-Term Budget Outlook, five different U.S. debt reduction proposals suggest various long-run debt targets, ranging from 59% to 90%. See Figure 1 of Richter and Throckmorton (2015).

⁶Recent long-term projections of debt-to-GDP ratios by the CBO display an “exploding” pattern. For visualizations, see Figure 1 of Leeper (2015) and Figure 1 and 2 of Davig et al. (2010).

⁷In IMF (2013) and IMF (2015), the international organization claims many advanced economies “face a lengthy, difficult and uncertain path to fiscal sustainability” and “Bringing public debt ratios to safer levels is an important long-term challenge in advanced economies”.

budget constraint.¹ Without knowing the debt target, long-run tax liabilities and/or government spending levels are also unclear. Thirdly, long-run fiscal levels directly enter the aggregate demand and thus affect the steady state of the real economy. In contrast to monetary policy where a neutrality result holds—the Phillips Curve is vertical in the long run and the choice of inflation target eventually will have no real effects—the long-run government spending level and distortionary tax rates directly affect the natural rates of output and unemployment.² Fourthly, expected long-run fiscal levels also influence forward-looking households’ consumption and saving decisions today—a central theme played by expectations in modern macroeconomic theories. Unknown long-run fiscal levels introduce prevalent uncertainties that operate at “both business-cycle and much lower frequencies” (see [Leeper \(2015\)](#)). I call uncertainties originating from unknown long-run fiscal levels *low-frequency fiscal uncertainty* (LFFU).

It is normal that uncertainty surrounding the long-run fiscal levels arises: The current legislation does not target long-term fiscal levels explicitly and the incumbent government cannot pre-commit future administrations’ fiscal actions. These long-run levels are synonymous to the fiscal variables’ steady-state values in models. Alternatively and by analogy with the inflation target, I call them *fiscal targets*. In stark contrast, while uncertainties surrounding long-run fiscal levels are pervasive in reality, this type of uncertainty is largely ignored or overlooked in academia up until recently. Common exercises in the literature are to calibrate the steady states of fiscal variables (i.e., fiscal targets) to their historical means³ or some exogenous Markov-switching processes and assume households inside the economy know the steady-state values or structure perfectly. This paper considers an incomplete information environment where households need to confront unknown fiscal targets directly. Without perfect information, not well-anchored beliefs open up the possibility that households’ perceived fiscal targets may be different than the actual ones underlying the economy.

I test such possibility qualitatively in a neoclassical growth model with income taxes and simple fiscal financing schemes and quantitatively in an RBC model with a rich fiscal sector featuring government spending, lump-sum transfers, risk-free debt and distortionary (i.e., consumption, capital and labor) taxes. To facilitate comparison with the existing literature, I model the actual fiscal targets as time-invariant and calibrate them to historical means. Households do not have perfect information on targets and need to learn them to form expectations and make decisions. Due to high persistence of fiscal movements, a crucial misspecification households make is they always suspect fiscal targets are time-varying: Households’ perceived laws of motion of fiscal targets follow

¹[Chung and Leeper \(2007\)](#) pointed out that long-run budget balance will imply restrictions on the VAR coefficients and impact calculations of fiscal multipliers.

²Long-run debt and transfers levels don’t *directly* affect natural rates of output as they are not part of the GDP. However, different long-run fiscal financing schemes indicate they affect the steady-state of the economy *indirectly*.

³Calibrating fiscal targets to historical means is not accidental due to very limited information on a country’s long-run fiscal stance. Data are also not informative as aggregate fiscal series are slow-moving and display “trends” (See [Figure 8](#)). Such “trends” cannot last forever due to non-negativity constraints and/or fiscal sustainability. “Incorporating such features into a model with fiscal policy is nontrivial” (This quotation is taken from [Leeper et al. \(2010\)](#)). This leads to another common exercise in the literature—detrending fiscal series before taking them to models. Detrending takes away (part of) low-frequency movements of fiscal series and is inconsistent with calibration of fiscal targets. In this paper fiscal series are not detrended.

unit root processes as in standard time-varying coefficient models in empirical macroeconomics (see [Cogley and Sargent \(2005\)](#) and [Primiceri \(2005\)](#)). These perceived laws of motion define not-well anchored beliefs precisely and break rational expectation: If beliefs were well-anchored, then perceived laws of motion should coincide with actual laws of motion of fiscal targets, which are essentially some time-invariant parameters in this paper. A signal extraction problem is established where households use past realizations of economic variables and the Kalman filter to infer their perceived fiscal targets. Households then form expectations conditional on the perceived fiscal targets using anticipated utility (see [Kreps \(1998\)](#) and [Cogley and Sargent \(2008\)](#)). The anticipated utility approach is a common way of forming expectations in the learning literature. It yields fast solution procedure and greatly facilitates the estimation of the RBC model. I call the incomplete information, anticipated utility environment IIAU. I explore consequences of low-frequency fiscal uncertainty by comparing equilibrium dynamics in the IIAU model with the full information, rational expectation (FIRE) model. Without LFFU, all IIAU models considered in this paper become FIRE.

Recent literature highlights another type of fiscal uncertainty by looking at stochastic volatilities of policy instruments. Their messages are mixed. While [Fernández-Villaverde et al. \(2015\)](#) claims a sizable adverse effect on economic activity due to unexpected changes in fiscal volatility shocks, [Born and Pfeifer \(2014\)](#) argues such policy risk is unlikely to play a major role in business cycle fluctuations since volatility shocks are too small and not sufficiently amplified. I follow [Hollmayr and Matthes \(2015\)](#) and [Richter and Throckmorton \(2015\)](#) and consider uncertainty about the first-order moments (i.e., levels), rather than the second-order moments (i.e., variances) of fiscal policy.¹ Both papers simulate costs of fiscal uncertainty and suggest such policy uncertainty is undesirable. The current paper takes a data-driven approach.

The main contributions of this paper are twofold: (i) First, I use a simple model to illustrate the propagation mechanism of LFFU. The analytical calculations make clear how LFFU twists households' both short- and long-run expectations. LFFU introduces additional non-linear state variables—households' beliefs (i.e., perceived fiscal targets)—into the model. Non-linear model dynamics are preserved by solving the model around households' perceived steady states, which are time-varying. A parallel (i.e., equation (14) and (15)) is established between FIRE and IIAU solutions to emphasize why not knowing fiscal targets introduces prevalent uncertainties. Impulse responses to a tax cut could be *more* (see Figure 2) or *less* (see Figure 3) expansionary in the IIAU model than in the FIRE model and crucially depend on the underlying fiscal financing scheme. While tax cuts are always expansionary in FIRE across all horizons, in IIAU the same tax cuts could be contractionary in longer terms (see Figure 3). Conditional on different households' beliefs, purely expectation-driven fluctuations can cause the IIAU economy to either *over-* or *under-*perform

¹[Hollmayr and Matthes \(2015\)](#) considers a one-time discrete policy change and let households learn every fiscal details and finds learning about fiscal policy introduces more volatility to the economy. [Richter and Throckmorton \(2015\)](#) calibrates the debt targets to an exogenous discrete-time Markov chain and let households learn both the state and transition matrix of the Markov chain. They find unknown debt targets lead to welfare losses and may reduce the stimulative effect of the ARRA and the Bush tax cuts.

its FIRE counterpart (see Figure 7). (ii) More importantly, I quantify effects of LFFU by fully estimating the RBC model. Structural parameters, including those which govern the evolution of households' beliefs, are estimated by utilizing the conditional linearity of the model structure and a time-varying Kalman filter. Data strongly prefer an IIAU specification over FIRE with a large *log* Bayes factor close to 100, and suggest increasing transfers history is the main driving force of changing perceived targets. Beyond better empirical performance, the estimated IIAU model implies much smaller real frictions and a lot weaker debt responses of fiscal instruments than the FIRE model. Depending on households' beliefs on fiscal targets, LFFU introduces state-dependent and time-varying fiscal multipliers. The IIAU model can generate larger than unity government spending multipliers that usually cannot be generated under FIRE in the RBC framework. Filtered historic beliefs show recent perceived long-run transfers levels and capital tax rates are high, while perceived long-run government spending levels and labor tax rates deviate little from historical means. These filtered beliefs correspond to a type of fiscal pessimism defined later in the paper.

This paper is related to a burgeoning literature on belief-driven business cycle fluctuations. A partial list of some prominent work includes [Lorenzoni \(2009\)](#), [Barsky and Sims \(2012\)](#), [Schmitt-Grohé and Uribe \(2012\)](#), and [Blanchard et al. \(2013\)](#). While much of the literature emphasizes roles of news and/or noise of TFP shocks, this work focuses exclusively on households' beliefs regarding long-run fiscal policy, and highlights subsequent state-dependent fiscal impacts. This work is therefore also connected to a large empirical literature (e.g., [Auerbach and Gorodnichenko \(2012\)](#), [Bachmann and Sims \(2012\)](#), [Caggiano et al. \(2015\)](#), [Fazzari et al. \(2015\)](#)) that provides evidence of state-dependent and time-varying fiscal multipliers. While most of the empirical literature use business cycles (i.e., economic booms and recessions)¹ as states, this paper extends states to households' perceived long-run fiscal levels.

2 A Simple Model

I use a neoclassical growth model and simple tax rules to illustrate the underlying mechanism of low-frequency fiscal uncertainty. Consider an economy with a unit measure of identical households who choose a sequence of consumption and investment $\{C_t, I_t\}_{t=0}^{\infty}$ to maximize expected constant relative risk aversion (CRRA) utility $E_0^{\mathcal{I}} \sum_{t=0}^{\infty} \beta^t \frac{(C_t)^{1-\varphi}}{1-\varphi}$ where $\beta \in (0, 1)$ is the subjective discount factor and φ is the degree of relative risk aversion. $E_t^{\mathcal{I}}$ is an expectation operator conditional on the representative household's information set \mathcal{I}_t (defined in Section 2.1). A perfectly competitive firm produces output subject to a Cobb-Douglas technology $Y_t = A_t K_{t-1}^{\alpha} L_t^{1-\alpha}$ with capital share $\alpha \in (0, 1)$. Households pay income taxes and receive lump-sum transfers T from the government. Households' choices are constrained by $C_t + I_t \leq [1 - \exp(\tau_t)](W_t L_t + R_t K_{t-1}) + T_t$ where $\exp(\tau_t)$ is the income tax rate and W_t, R_t are wage and capital rental rate. Since leisure is not valued in

¹Other states examined include monetary policy (e.g., [Davig and Leeper \(2011\)](#)), government indebtedness (e.g., [Ilzetzki et al. \(2013\)](#), [Nickel and Tudyka \(2014\)](#), [Bi et al. \(2016\)](#)), downward nominal wage rigidity (e.g., [Shen and Yang \(2018\)](#)) and the binding zero lower bound on nominal interest rate (e.g., [Christiano et al. \(2011\)](#) and [Erceg and Lindé \(2014\)](#)).

utility, households provide labor inelastically and $L_t \equiv 1$. Capital depreciates at a constant rate δ and follows the law of motion $K_t = (1 - \delta)K_{t-1} + I_t$. Define $a_t = \log(A_t)$.¹ Technological progress (in logs) follows a stationary AR(1) process

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim N(0, 1) \quad (1)$$

with $\rho_a \in [0, 1)$. Government runs a balanced budget each period, i.e. $G_t + T_t = \exp(\tau_t)Y_t$, where G denotes government spending. For simplicity, I assume the fiscal authority always spends a constant fraction of their tax revenue $w_G \in [0, 1]$ on government spending, i.e., $G_t = w_G \exp(\tau_t)Y_t$.² Tax rates (in logs) are exogenous and follow

$$\tau_t = \tau^* + u_t \quad (2)$$

where u_t is modeled as a stationary and persistent AR(1) process

$$u_t = \rho_u u_{t-1} + \sigma_u \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \quad (3)$$

with $\rho_u \in [0, 1)$ and $\rho_u \approx 1$. The persistent component u_t generates slow-moving tax rates as observed in data. All shocks $\varepsilon_{a,t}, \varepsilon_t$ are mutually orthogonal at all leads and lags. Tax rules (2) and (3) imply the long-run average tax rate (in logs) is given by τ^* and a deterministic steady-state tax rate level $\exp(\tau^*)$. In contrast to the 2% inflation target that is well-known among the public, in reality many fiscal variables do not have explicit targeted values, not to mention the private sector's awareness of (or lack thereof) a country's long-run fiscal stance. To raise such awareness, I call τ^* the tax target. Whether households know τ^* or not gives rise to a type of policy uncertainty that is advocated in this paper.

I make the following timing assumptions: First, the economy enters time period t with K_{t-1}, A_{t-1} and τ_{t-1} determined. Second, shocks $\varepsilon_{a,t}, \varepsilon_t$ hit the economy and determine current A_t and tax rate τ_t . Households update beliefs on τ^* if there is low-frequency fiscal uncertainty. Conditional on updated beliefs, households form expectations, provide labor supply and make consumption/saving decisions which determine output and therefore tax revenue. Government spending and transfers adjust to satisfy the balanced-budget constraint. All things considered in the second step happen *simultaneously*.

The equilibrium conditions of the model consist of the inter-temporal Euler equation (4) and aggregate resource constraint (5)

$$C_t^{-\varphi} = \beta E_t^{\mathcal{I}} C_{t+1}^{-\varphi} [1 - \delta + (1 - \exp(\tau_{t+1}))\alpha A_{t+1} K_t^{\alpha-1}], \quad (4)$$

$$C_t + K_t = (1 - w_G \exp(\tau_t)) A_t K_{t-1}^{\alpha} + (1 - \delta) K_{t-1}. \quad (5)$$

¹Throughout the paper, $\log(x)$ refers to the natural logarithm of x , i.e., $\log_e(x)$.

²This assumption allows me to focus only on one fiscal targets, i.e., τ^* , instead of both tax and government spending targets. In the following RBC section such constant fiscal financing scheme assumption will be relaxed.

I consider two information sets: a standard *full information rational expectation* (FIRE) case and an *incomplete information* case where households must learn about the unknown tax target τ^* . A small misspecification prevents households from learning τ^* and they stick to the anticipated utility approach to make decisions on C_t, K_t . This approach is pioneered by [Kreps \(1998\)](#), introduced into a macro setting by [Cogley and Sargent \(2008\)](#), and is common in the learning literature. I call the *incomplete information anticipated utility* case IIAU.

2.1 Households' information sets \mathcal{I}_t

I first illustrate the commonalities between FIRE and IIAU. In both cases households completely understand the non-policy model structure. Before choosing C_t and K_t they have access to the entire history of observables $\{C^{t-1}, K^{t-1}, \tau^t, a^t, Y^{t-1}, T^{t-1}, G^{t-1}\}$.¹ They also know non-policy parameters $\{\beta, \varphi, \alpha, \delta, \rho_a, \sigma_a\}$ perfectly.² Besides acting as price takers and making consumption and saving decisions, households are also aware of the balanced government budget constraint and the constant government spending share w_G .

In FIRE households also understand the structure of fiscal policy: They know the tax rules (2), (3) and have learned $\{\tau^*, \rho_u, \sigma_\varepsilon\}$. Beliefs on the tax target thus have been firmly and correctly anchored at τ^* .

In IIAU households' beliefs on the tax target are not anchored: They do not know the actual tax target τ^* and are not convinced it is a constant.³ Observing a highly persistent and slow-moving tax series, households suspect tax targets are time-varying. Their perceived tax rules are given by

$$\tau_t = \tau_t^* + u_t \quad (6)$$

$$\tau_t^* = \tau_{t-1}^* + \sigma_\eta \eta_t, \quad \eta_t \sim N(0, 1) \quad (7)$$

$$u_t = \rho_u u_{t-1} + \sigma_\varepsilon \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \quad (8)$$

where the perceived law of motion of the tax target (7) is a unit root process, a standard assumption in the literature on time-varying coefficient models in empirical macroeconomics (see [Cogley and Sargent \(2005\)](#) and [Primiceri \(2005\)](#)). Households perceive η_t, ε_t shocks as mutually *i.i.d.*

Comparing the actual tax rules (2)-(3) and the perceived rules (6)-(8) implies households misperceive part of the highly persistent yet stationary u_t as evidence of a changing tax target τ_t^* . In the FIRE model households' long run expectations of average tax rates are anchored at τ^* . Households are convinced effects of ε_t will die out eventually and that the tax rate τ_t will converge to τ^* . However, in the IIAU model, when $\sigma_\eta > 0$ households do not know whether tax rates τ_t will converge, and which level they will converge to in the long run. The non-stationary unit root process (7) allows beliefs on τ_t^* to drift over time.

¹I use $X^t = \{X_t, X_{t-1}, \dots\}$ to denote the history of variable X up to time t . The persistent component u^t is not directly observable in both models, though it can be easily learned by FIRE households.

²Since a^t is observable and households know technology follows an AR(1), knowing ρ_a, σ_a implies households can figure out the history of technology shocks ε_a^t perfectly in both FIRE and IIAU.

³The households' initial beliefs on tax target is fixed at τ^* .

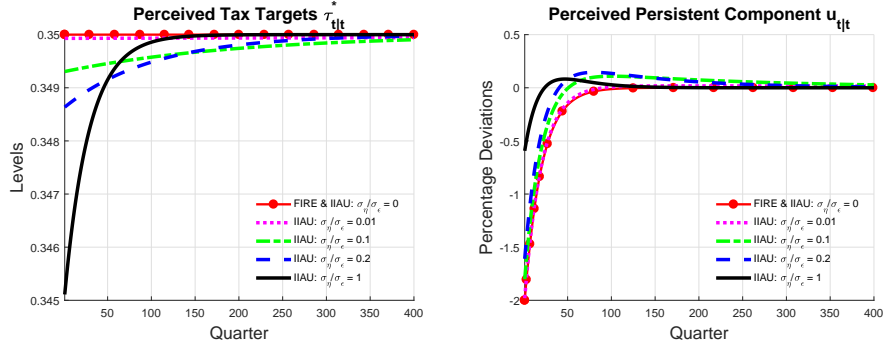


Figure 1: Impulse response functions of perceived tax target $\tau_{t|t}^*$ (Left Panel) and perceived persistent component $u_{t|t}$ (Right Panel) to a $1-\sigma_\varepsilon$ negative shock ε_t for various $w = \sigma_\eta/\sigma_\varepsilon$; The single shock hits the economy at $t = 1$. The initial perceived tax target $\tau_{1|0}^* = \tau^*$. Parameter values are fixed at: $\tau^* = 0.35$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$.

The perceived tax rules (6)-(8) create a signal extraction problem as households need to disentangle τ_t^* and u_t when only history of τ_t is available. Since (6)-(8) is linear, I assume households use the optimal learning algorithm for linear systems—the Kalman filter—to update beliefs in IIAU. The signal extraction problem (6)-(8) highlights multiple roles played by τ_t in the IIAU model: Besides acting as realizations of the fiscal instrument which finances government spending, lump-sum transfers and distorts the real economy, τ_t 's also serve as “signals” and shape household’s opinions on long-run fiscal target.

Figure 1 plots impulse response functions of perceived tax target $\tau_{t|t}^*$ and perceived persistent component $u_{t|t}$ to a $1-\sigma_\varepsilon$ negative shock ε_t for various $\sigma_\eta/\sigma_\varepsilon$. I use estimated values from Section 3.1 when calibrating τ^* , ρ_u and σ_ε . The ratio of standard deviations $\sigma_\eta/\sigma_\varepsilon$ is chosen to be bounded above by one as it would be unrealistic to speculate large drift of households beliefs within one period given the actual slow-moving tax process (2) and no structural breaks. I consider $\sigma_\eta/\sigma_\varepsilon \in [0, 0.01, 0.1, 0.2, 1]$. The initial $\tau_{1|0}^*$ is fixed at the actual tax target τ^* . Observing a tax cut starting at $t = 1$, for all $\sigma_\eta/\sigma_\varepsilon > 0$ households’ perceived targets $\tau_{1|1}^*$ decrease: The larger $\sigma_\eta/\sigma_\varepsilon$ is, the bigger drop in $\tau_{1|1}^*$. Without subsequent tax shocks, households’ perceived targets $\tau_{t|t}^*$ slowly converge back to τ^* . Such slow convergence of beliefs is apparently due to the large persistence of tax rates τ_t . The larger $\sigma_\eta/\sigma_\varepsilon$ is, the faster the convergence speed is. Intuitively, as $\sigma_\eta/\sigma_\varepsilon$ approaches 0, households are more “stubborn” to change their beliefs and once $\tau_{t|t}^*$ deviates from τ^* , it takes a long time to converge back to τ^* .¹

The right panel of Figure 1 plots $u_{t|t}$. For all $\sigma_\eta/\sigma_\varepsilon > 0$ households’ perceived persistent components are above the actual persistent component u_t . Since the sum of $u_{t|t}$ and $\tau_{t|t}^*$ is equal to the observed tax rate τ_t by (6), $u_{t|t} > u_t$ as long as $\tau_{t|t}^* < \tau^*$: Under-estimation of the tax target must imply over-estimation of the persistent component u_t . It is important to recognize the perceived persistent component $u_{t|t}$, returned by the Kalman recursion, does not follow an AR(1) process. Interestingly, Figure 1 shows although for all $\sigma_\eta/\sigma_\varepsilon$ households are initially correct regarding the sign of u_t , over time they “overshoot” and misperceive a negative u_t (i.e., fiscal expansion) as a

¹Under the current parameterization and $\sigma_\eta/\sigma_\varepsilon = 0.01$, $\tau^* - \tau_{t|t}^* > 10^{-6}$ even after 1000 years.

positive $u_{t|t}$ (i.e., fiscal contraction). As we will see in Section 2.2, while $\tau_{t|t}^*$ anchors households' expected tax rate in the long run, $u_{t|t}$ anchors households' expected tax deviation (from $\tau_{t|t}^*$) in the short run. Different combinations of $\tau_{t|t}^*$ and $u_{t|t}$'s impact households' expectation formation and decision making across all horizons.

I now solve the model for both information sets and explain how households form expectations in the IIAU model.

2.2 Expectation Formation and Model Solution

It suffices to focus on the capital level K_t to solve the model. Substituting C_t in (4) using (5) gives

$$1 = \beta E_t^I \left\{ \left[\frac{(1 - w_G \exp(\tau_t)) A_t K_{t-1}^\alpha + (1 - \delta) K_{t-1} - K_t}{(1 - w_G \exp(\tau_{t+1})) A_{t+1} K_t^\alpha + (1 - \delta) K_t - K_{t+1}} \right]^\varphi [1 - \delta + (1 - \exp(\tau_{t+1})) \alpha A_{t+1} K_t^{\alpha-1}] \right\} \quad (9)$$

I solve (9) by log-linearizing it around the actual steady state in the FIRE model and around the perceived steady state in the IIAU model. As pointed out by [Hollmayr and Matthes \(2015\)](#), solving the IIAU model around the perceived steady state has two nice interpretations. First, private agents (i.e., households) in the model need to solve their perceived model and they log-linearize around their perceived steady state. Second, solving the IIAU model around the perceived steady state can better capture the behavior of the non-linear equation (9) when the economy is far away from the actual steady state.

Denote the time- t updated tax target $\tau_{t|t}^*$. While the actual steady-state capital stock

$$K^* = \left[\frac{\beta^{-1} + \delta - 1}{\alpha (1 - \exp(\tau^*))} \right]^{1/(\alpha-1)} \quad (10)$$

is time-invariant in the FIRE model, in IIAU the perceived steady-state capital

$$K_{t|t}^* = \left[\frac{\beta^{-1} + \delta - 1}{\alpha (1 - \exp(\tau_{t|t}^*))} \right]^{1/(\alpha-1)} \quad (11)$$

is time-varying due to changes in $\tau_{t|t}^*$. The elasticity of $K_{t|t}^*/K^*$ with respect to $(1 - \exp(\tau_{t|t}^*))/(1 - \exp(\tau^*))$ is $1/(1 - \alpha) > 1$. Given a capital share $\alpha = 1/3$, if $(1 - \exp(\tau_{t-1}^*))$ is 1% lower than $(1 - \exp(\tau^*))$ ¹, then $K_{t|t}^*$ will be 1.5% lower than K^* .

Denote $k_t = \log(K_t)$, the actual steady-state capital(in logs) $k^* = \log(K^*)$ and the perceived steady state $k_{t|t}^* = \log(K_{t|t}^*)$. Log-linearizing (9) produces a second-order difference equation in

¹For instance, if $\exp(\tau^*) = 0.35$ and $\exp(\tau_{t-1}^*) = 1 - (1 - 0.35)/1.01 \approx 0.3564$, then $(1 - \exp(\tau_{t-1}^*))$ is 1% lower than $(1 - \exp(\tau^*))$.

capital,

$$\begin{aligned} \text{FIRE: } & \Phi_1(\tau^*)E_t^{FIRE} [k_{t+1} - k^*] + \Phi_2(\tau^*) [k_t - k^*] + \Phi_3(\tau^*) [k_{t-1} - k^*] \\ & = \Phi_4(\tau^*)a_t + \Phi_5(\tau^*)E_t^{FIRE} (\tau_{t+1} - \tau^*) + \Phi_6(\tau^*) (\tau_t - \tau^*) \end{aligned} \quad (12)$$

$$\begin{aligned} \text{IIAU: } & \Phi_1(\tau_{t|t}^*)E_t^{IIAU} [k_{t+1} - k_{t|t}^*] + \Phi_2(\tau_{t|t}^*) [k_t - k_{t|t}^*] + \Phi_3(\tau_{t|t}^*) [k_{t-1} - k_{t|t}^*] \\ & = \Phi_4(\tau_{t|t}^*)a_t + \Phi_5(\tau_{t|t}^*)E_t^{IIAU} (\tau_{t+1} - \tau_{t|t}^*) + \Phi_6(\tau_{t|t}^*) (\tau_t - \tau_{t|t}^*) \end{aligned} \quad (13)$$

where $\Phi_i(\cdot)$'s are coefficients of the linearized equation written as functions of either τ^* or $\tau_{t|t}^*$. Comparing (13) to (12) highlights the differences between rational expectation and anticipated utility: k^* changes to $k_{t|t}^*$ when moving from FIRE to IIAU, and all τ^* in (12) have been replaced by $\tau_{t|t}^*$ in (13). This is due to the underlying assumption of how households form expectations in the anticipated utility approach: Households update their beliefs on the tax target using Bayes' law in their signal extraction problem (6)-(8) but optimize myopically in their expectation formation embedded in (9).¹ This property allows us to treat $\tau_{t|t}^*, k_{t|t}^*$ as constant when linearizing (9).

It's worthwhile to emphasize how households form $E_t^I \tau_{t+1}$. In FIRE households can observe the persistent component u_t directly. From (2), (3) it follows

$$E_t^{FIRE}(\tau_{t+1} - \tau^*) = E_t^{FIRE} u_{t+1} = \rho_u u_t.$$

In IIAU given perceived tax rules (6)-(8)

$$E_t^{IIAU}(\tau_{t+1} - \tau_{t|t}^*) = E_t^{IIAU}(\tau_{t+1}^* + u_{t+1} - \tau_{t|t}^*) = E_t^{IIAU}(\underbrace{\tau_{t+1}^* - \tau_t^*}_{\text{unpredictable}} + u_{t+1}) = u_{t+1|t} = \rho_u u_{t|t},$$

where $u_{t|t}$ are households' time- t perceived persistent component and $u_{t+1|t}$ is its one-step ahead prediction, both obtained by households' Kalman recursion.

Appendix A shows solutions to (12) and (13) are of the form

$$\text{FIRE : } k_t = \phi_0(\tau^*) + \phi_1(\tau^*)k_{t-1} + \phi_2(\tau^*)a_t + \phi_3(\tau^*)\tau_t \quad (14)$$

$$\text{IIAU : } k_t = \phi_0(\tau_{t|t}^*) + \phi_1(\tau_{t|t}^*)k_{t-1} + \phi_2(\tau_{t|t}^*)a_t + \phi_3(\tau_{t|t}^*)\tau_t \quad (15)$$

While in the FIRE model τ^* is simply a fixed parameter, in IIAU $\tau_{t|t}^*$ are used to update the steady-state tax rate and the steady state of the economy, and therefore enter the system as an *additional non-linear* state variable. Its effects are widespread: First, it impacts *all* response coefficients of k_t to linear state variables 1, k_{t-1} , a_t , τ_t . Second, although I only focus on the capital level K_t , it is easy to see all other endogenous variables (i.e., C_t , Y_t , G_t and T_t) will be influenced by households' beliefs $\tau_{t|t}^*$ as well.

¹The anticipated utility approach encodes a form of bounded rationality that abstracts from the private agents' precautionary motive driven by uncertainty to future beliefs. The anticipated utility approach is commonly used in the literature on adaptive learning. Cogley and Sargent (2008) shows that the loss of using anticipated utility decision making is not large compared to Bayesian decision making.

The widespread impact of $\tau_{t|t}^*$ is generated by complicated general equilibrium effects, both in the short/long run and on the supply/demand side: Interpreting $\tau_{t|t}^*$ as households' long run expectations of the tax target, a different $\tau_{t|t}^*$ changes the expected relative price of investment in the distant future, thus introducing a different substitution effect. It also changes the income effect by altering the expected future steady-state lump-sum transfers payment. At the same time, an incorrect $\tau_{t|t}^*$ must correspond to a misperceived persistent component, i.e., $u_{t|t} \neq u_t$. The perceived component $u_{t|t}$ governs households' short-term expectations of next period's tax deviation, thus modifying income and substitution effects in the short run. The perceived target $\tau_{t|t}^*$ also impacts both the supply and demand sides: while a higher $\tau_{t|t}^*$ tends to depress investment and causes households to supply less capital to firms, the constant share w_G embedded in the government budget constraint implies demand to capital becomes larger. All things combined, introducing LFFU impacts households' expectations and decision making across all horizons on both supply and demand sides.

2.3 Impulse Responses of Capital

2.3.1 Correct initial belief: $\tau_{1|0}^* = \tau^*$

I now consider impulse responses of capital to a $1-\sigma_\varepsilon$ negative shock ε_t (i.e., a tax cut) for various $\sigma_\eta/\sigma_\varepsilon$ and w_G when the economy starts at its deterministic steady state and a correct initial perceived tax target $\tau_{1|0}^* = \tau^*$. To focus exclusively on fiscal policy, technology shocks have been shut down and $a_t \equiv 0$. Other parameter $\beta, \alpha, \varphi, \delta, \rho_a, \sigma_a$ values are typical for a quarterly model. Along with $\sigma_\eta/\sigma_\varepsilon$ values considered in Figure 1, I consider $w_G = [0, 0.3, 0.4, 0.6, 1]$. When $w_G = 0$ ($w_G = 1$) government spends 100% of its tax revenue on lump-sum transfers (government spending).

Figure 2 plots the impulse responses when the government spends all of its tax revenues on lump-sum transfers, i.e., $w_G = 0$. Effects of the tax cut are expansionary for all $\sigma_\eta/\sigma_\varepsilon$'s values across all horizons. The stimulus effect reaches its peak around the 24th quarter—6 years after the shock hits the economy. When $\sigma_\eta/\sigma_\varepsilon = 0$ and $\tau_{1|0}^* = \tau^*$, the FIRE and IIAU models share the same impulse responses. As $\sigma_\eta/\sigma_\varepsilon$ increases the same tax cut becomes more expansionary in IIAU models. The differences between FIRE and IIAU start small in the short run and gradually increase over the medium run. In the long run, capital's impulse responses slowly converge to its FIRE counterpart. As in Figure 1, the smaller $\sigma_\eta/\sigma_\varepsilon$ is, the slower the convergence rate will be.

Figure 3 plots the impulse responses of capital when government spends all of its tax revenues on government spending, i.e., $w_G = 1$. In sharp contrast to Figure 2, as $\sigma_\eta/\sigma_\varepsilon$ increases the IIAU impulse responses are uniformly less expansionary than its FIRE counterpart. More importantly, while the tax cut is stimulus across all horizons under FIRE, under IIAU it becomes contractionary in the long run. Such contractionary effects are mainly due to misperceived persistent components (i.e., $u_{t|t} > 0$ while $u_t < 0$) seen in Figure 1: Long-run impacts of tax cuts may be perverse, but

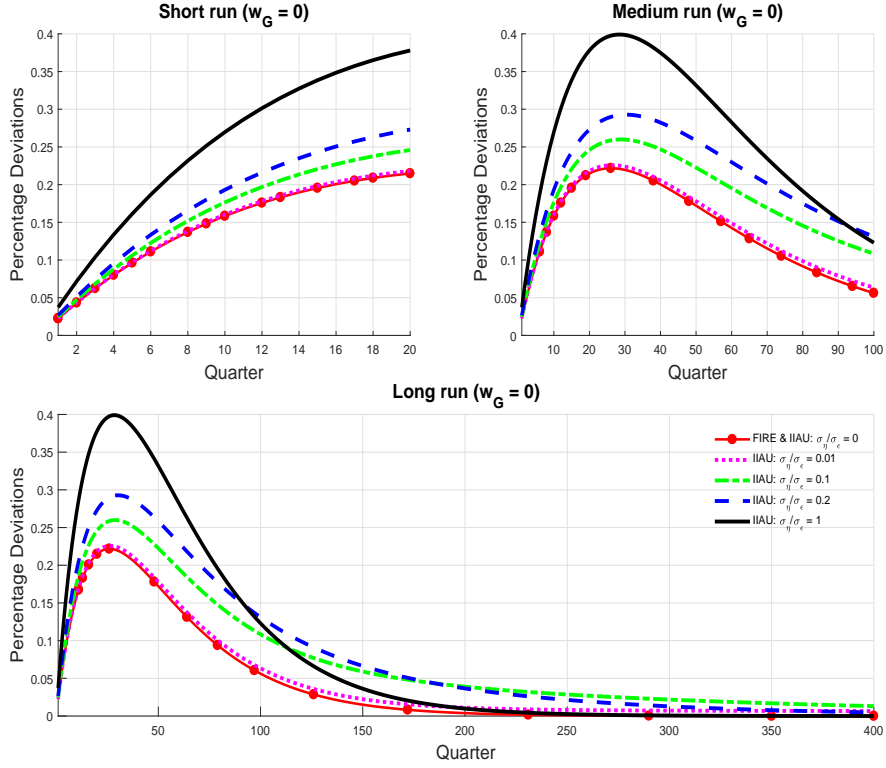


Figure 2: Impulse response functions of capital to a $1\text{-}\sigma_\varepsilon$ negative shock ε_t across different time horizons for various $\sigma_\eta/\sigma_\varepsilon$ and $w_G = 0$. Impulse responses are represented as percentage deviations from the deterministic steady-state capital. The single shock hits the economy at $t = 1$. The initial capital level $k_0 = k^*$ and perceived tax target $\tau_{1|0}^* = \tau^*$. Technology shocks have been shut down and $a_t \equiv 0$. Other parameter values are fixed at: $\beta = 0.99$; $\alpha = 1/3$; $\varphi = 2$; $\delta = 0.025$; $\tau^* = 0.35$; $\rho_a = 0.9$; $\sigma_a = 0.0062$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$.

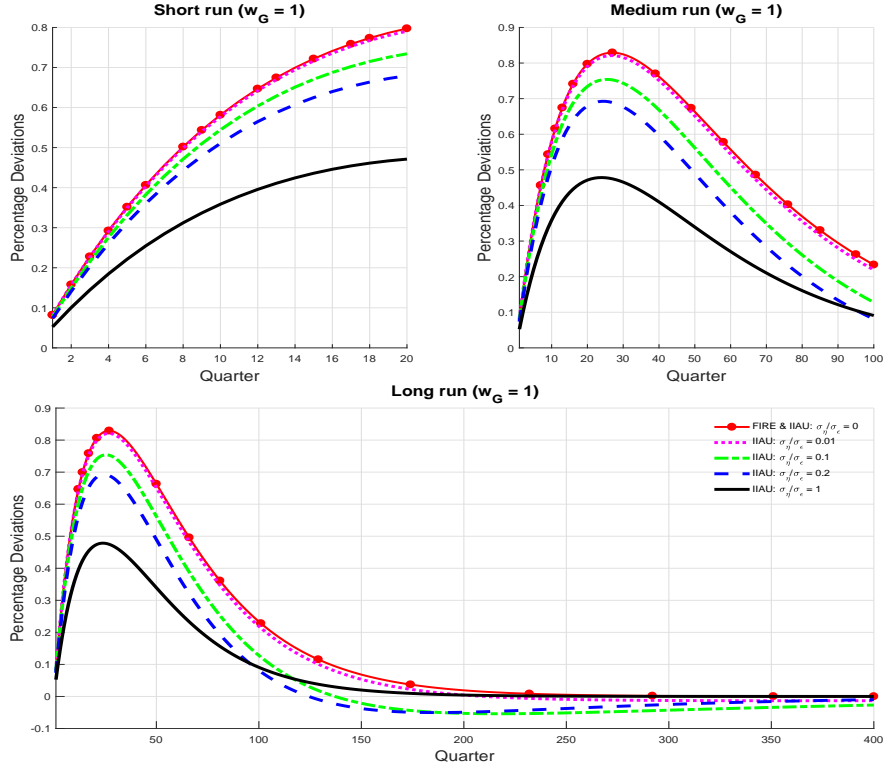


Figure 3: Impulse response functions of capital to a $1\text{-}\sigma_\varepsilon$ negative shock ε_t across different time horizons for various $w = \sigma_\eta/\sigma_\varepsilon$ and $w_G = 1$. Impulse responses are represented as percentage deviations from the deterministic steady-state capital. The single shock hits the economy at $t = 1$. The initial capital level $k_0 = k^*$ and perceived tax target $\tau_{1|0}^* = \tau^*$. Technology shocks have been shut down and $a_t \equiv 0$. Other parameter values are fixed at: $\beta = 0.99$; $\alpha = 1/3$; $\varphi = 2$; $\delta = 0.025$; $\tau^* = 0.35$; $\rho_a = 0.9$; $\sigma_a = 0.0062$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$.

not for fiscal financing reasons.¹ They arise due to misspecified beliefs (i.e., $\tau_{t|t}^*$ and $u_{t|t}$).

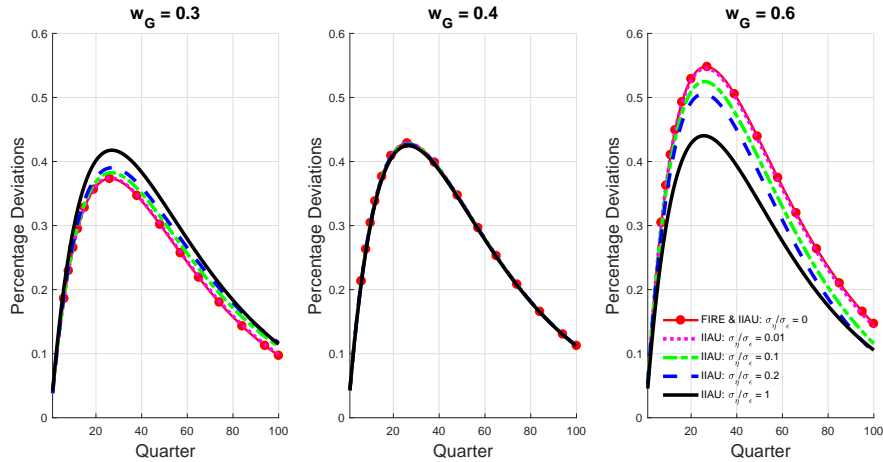


Figure 4: Impulse response functions of capital to a $1\text{-}\sigma_\varepsilon$ negative shock ε_t for various $\sigma_\eta/\sigma_\varepsilon$ and $w_G \in [0.3, 0.4, 0.6]$. Impulse responses are represented as percentage deviations from the deterministic steady-state capital. The single shock hits the economy at $t = 1$. The initial capital level $k_0 = k^*$ and perceived tax target $\tau_{1|0}^* = \tau^*$. Technology shocks have been shut down and $a_t \equiv 0$. Other parameter values are fixed at: $\beta = 0.99$; $\alpha = 1/3$; $\varphi = 2$; $\delta = 0.025$; $\tau^* = 0.35$; $\rho_a = 0.9$; $\sigma_a = 0.0062$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$.

Given the IIAU impulse responses are initially close to their FIRE counterparts, can either undershoot or overshoot it over the medium run and will eventually converge to the FIRE counterpart in the long run, continuity of the model solution with respect to w_G implies there must be a w_G such that the FIRE and IIAU models share similar impulse responses. Figure 4 shows this is indeed the case when w_G is close to 0.4. Since Figures 2, 3 and 4 are generated under the same $\tau_{t|t}^*$ and $u_{t|t}$ (see Figure 1) given a $\sigma_\eta/\sigma_\varepsilon$, they highlight the fact that besides the belief parameter σ_η , impacts of LFFU also crucially depend on the underlying fiscal financing scheme.²

So far we have assumed $\tau_{1|0}^* = \tau^*$. Figure 1 shows with a negative $1\text{-}\sigma_\varepsilon$ shock, the perceived tax target will decline between 0 and 50 basis points. Since the economy is subject to tax shocks all the time, what happens if $\tau_{1|0}^*$ has already deviated away from τ^* ?

2.3.2 Incorrect initial belief: $\tau_{1|0}^* \neq \tau^*$

I now analyze the effects of LFFU at various initial perceived tax target $\tau_{1|0}^*$. Since the underlying tax series impacts households' beliefs and capital level jointly, I follow Bi et al. (2016) and simulate the model to pin down the joint distribution of $\tau_{1|0}^*$, capital k_0 and tax rate τ_0 . The simulation starts at $t = -200$ where the economy lands at its deterministic steady state. The initial capital level k_{-201} and tax rate τ_{-201} are fixed at k^* and τ^* , respectively. The initial perceived tax target $\tau_{-200|-201}$ is fixed at the actual target τ^* . The economy is subject to both technology and tax shocks $\varepsilon_{a,t}, \varepsilon_t$ from $t = -200$ to $t = 0$. A total of 50000 simulations are performed and for each

¹The fiscal financing parameter is fixed at $w_G = 1$.

²While in this simple model the fiscal financing parameter w_G is an exogenous constant, in the following RBC model the composition of fiscal financing will be endogenous determined and time-varying.

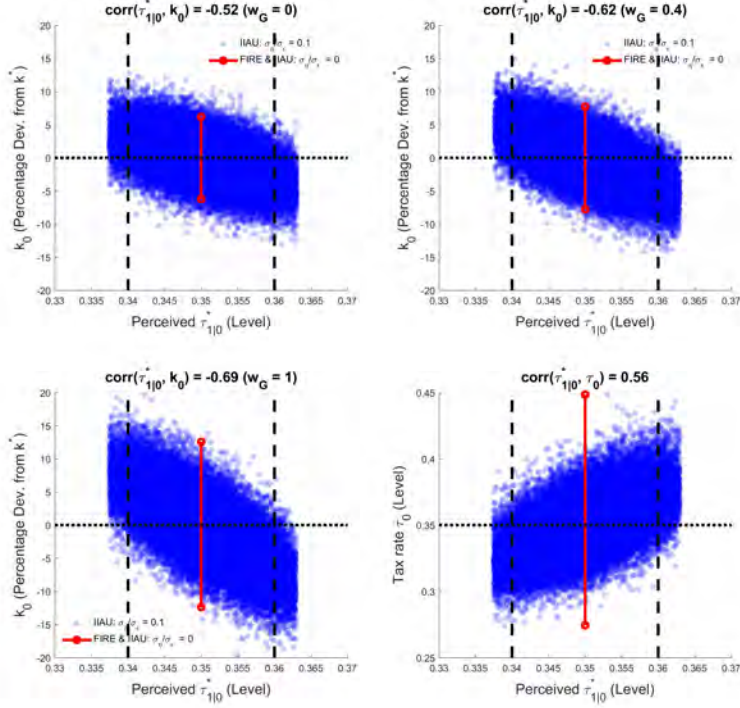


Figure 5: Simulated joint distribution of $(\tau_{1|0}^*, k_{-1})$ and $(\tau_{1|0}^*, \tau_{-1})$ at $t = 0$ for $\sigma_\eta/\sigma_\varepsilon = 0.1$ and $w_G = [0, 0.4, 1]$. The economy is simulated 50000 times. Each simulation is 200 quarters. The initial capital k_{-200} , tax rate τ_{-200} and perceived tax target $\tau_{-200|-201}$ are fixed at k^* , τ^* , and τ^* , respectively. Other parameter values are fixed at: $\beta = 0.99$; $\alpha = 1/3$; $\varphi = 2$; $\delta = 0.025$; $\tau^* = 0.35$; $\rho_a = 0.9$; $\sigma_a = 0.0062$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$.

distribution of interest, a 90-percent interval is retained to remove the effects of outliers.

Figure 5 presents the scatter plot of the joint state distributions of $(\tau_{1|0}^*, k_0)$ and $(\tau_{1|0}^*, \tau_0)$ for various government spending shares $w_G \in [0, 0.4, 1]$. Two groups are considered: the FIRE model (or equivalently, the IIAU model with $\sigma_\eta/\sigma_\varepsilon = 0$) and the IIAU model with $\sigma_\eta/\sigma_\varepsilon = 0.1$. The joint distribution shows that $\tau_{1|0}^*$ and k_0 are negatively correlated. In the FIRE model the fluctuations of capital are relatively small while in the IIAU models depending on $\tau_{1|0}^*$, capital k_0 can deviate further away from k^* . Across simulations the correlation coefficient between $\tau_{1|0}^*$ and k_0 is between -0.52 and -0.69 . Not surprisingly, $\tau_{1|0}^*$ and τ_0 are positively correlated, with a correlation coefficient of 0.56 . While in the FIRE model movements of tax rates cannot shift the perceived tax target, in the IIAU model the perceived tax target depends on current tax rate (and its history). For instance, when $\tau_{1|0}^* = 0.36$, simulations in Figure 5 show the current tax rate τ_0 must be higher than 30%.

These joint distributions indicate it is safe to choose

$$\exp(\tau^{*,low}) = 0.34, \quad \exp(\tau^{*,high}) = 0.36$$

as $\tau_{1|0}^*$ while keeping (k^*, τ^*) as the initial capital level k_0 and the initial tax rate τ_0 for various w_G . With $\tau_0 = \tau^*$, $\tau_{1|0}^*$ imposes a restriction on the initial perceived persistent component

$$u_{1|0} = \rho_u u_{0|0} = \rho_u (\tau_0 - \tau_{0|0}^*) = \rho_u (\tau^* - \tau_{1|0}^*).$$

Figure 6 plots the impulse responses of $\tau_{t|t}^*$ and $u_{t|t}$ at different $\tau_{1|0}^* = [\tau^{*,low}, \tau^*, \tau^{*,high}]$ and $\tau_0 = \tau^*$. It is worth mentioning from $t = 1$ we shut down both technology and tax shocks at all time. Without subsequent tax shocks, the tax series τ_t will simply be at the constant level τ^* . From Figure 6 we see $\tau_{t|t}^*$ and $u_{t|t}$ gradually converge back to their FIRE counterparts, but fairly slowly. The perceived tax target is still 10 basis points greater(or lower) than the actual tax target after one century.

With a fixed $\sigma_\eta/\sigma_\varepsilon = 0.1$, Figure 7 plots the impulse responses of capital for different $\tau_{1|0}^*$ and w_G . With $\tau_0 = \tau^*$, $k_0 = k^*$ and no incoming shocks from $t = 1$, these fluctuations are purely driven by changing beliefs (i.e., $\tau_{t|t}^*$ and $u_{t|t}$): While in the FIRE model the economy stays put since it reaches its steady state, in the IIAU model since $\tau_0 = \tau^* \neq \tau_{1|0}^*$, households believe the current economy is not at its perceived steady state, i.e., $k_0 = k^* \neq k_{1|0}^*$. Furthermore, there are two offsetting forces. If households under-estimate the tax target, i.e., $\tau_{1|0}^* < \tau^*$, then from (10), (11) we know $k_{1|0}^* > k^*$. To converge to its perceived steady state, the economy's capital level tends to grow from k^* to $k_{1|0}^*$. On the other hand, under-estimation of τ^* must imply over-estimation of u_0 . Given $u_0 = 0$, we have $u_{1|0} > 0$ and households perceived current tax persistent component is contractionary. It follows the economy's capital level needs to shrink in response to such perceived contractionary fiscal policy. Which force dominates depends on the fiscal financing parameter w_G . Figure 7 shows the impulse responses are hump-shaped and LFFU can cause the IIAU economy to chronically over- or under-perform its FIRE counterpart. Alternatively, one can think of the impulse responses as the cost or benefit of a credible announcement of the actual tax τ^* at $t = 0$.¹ The effects crucially depend on the initial beliefs (i.e., $\tau_{1|0}^*$) and the composition of fiscal financing (i.e., w_G).

2.3.3 Match US quarterly data moments

The simple neoclassical growth model with technology shocks has a renowned history for describing stylized facts of US aggregate activity well(See King and Rebelo (1999)). Does adding LFFU impede its ability to match quarterly data moments? As a diagnosis, I simulate the FIRE and IIAU models to test their ability to generate consumption, investment, and output's relative standard deviations, autocorrelations and cross correlations under different combinations of $\sigma_\eta/\sigma_\varepsilon$ and w_G . For each parameterization 50000 simulations are performed. The simulation starts at $t = -200$ where the economy lands at its deterministic steady state and $\tau_{-200|-201} = \tau^*$, $\tau_{-201} = \tau^*$ and $k_{-201} = k^*$. Each simulation is of length 500 quarters. The first 200 quarter data are discarded to allow different initial beliefs on the tax target and the persistent component u_t . Table 1 reports the moments of the data and Table 2 reports the simulated moments. Adding an persistent AR(1)

¹With anchored tax target τ^* , the IIAU and FIRE economies are identical.

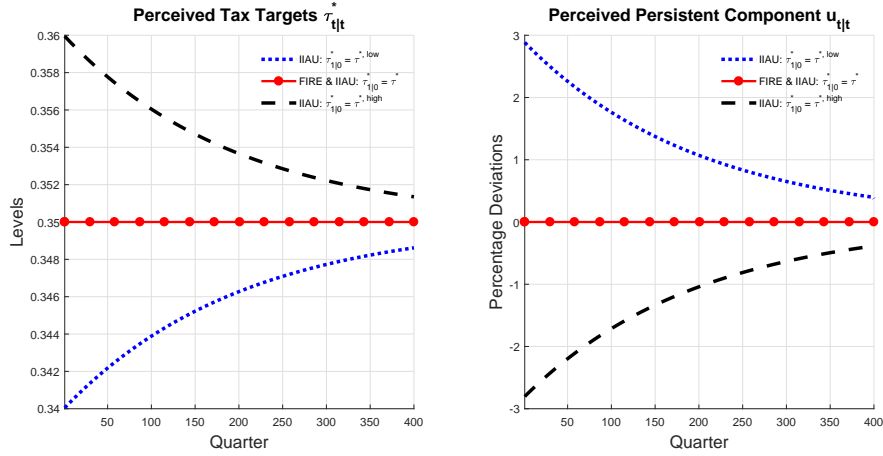


Figure 6: Impulse response functions of the perceived tax target $\tau_{t|t}^*$ (Left Panel) and the perceived persistent component $u_{t|t}$ (Right Panel) for various τ_{10}^* . The initial tax rate $\tau_0 = \tau^*$. There are no subsequent tax shocks. Parameter values are fixed at: $\tau^* = 0.35$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$; $\sigma_\eta/\sigma_\varepsilon = 0.1$.

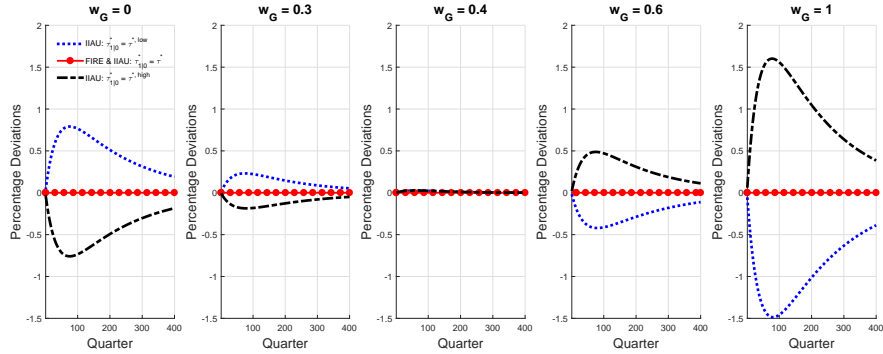


Figure 7: Impulse response functions of capital for various τ_{10}^* and $w_G \in [0, 0.3, 0.4, 0.6, 1]$. The initial capital level $k_0 = k^*$ and tax rate $\tau_0 = \tau^*$. There are no subsequent tax shocks. Parameter values are fixed at: $\beta = 0.99$; $\alpha = 1/3$; $\varphi = 2$; $\delta = 0.025$; $\tau^* = 0.35$; $\rho_a = 0.9$; $\sigma_a = 0.0062$; $\rho_u = 0.95$; $\sigma_\varepsilon = 0.02$; $\sigma_\eta/\sigma_\varepsilon = 0.1$.

component u_t pushes more low-frequency movements into both FIRE and IIAU models, causing Y_t , C_t and I_t to be slightly more auto-correlated than in actual data. Other moments are matched fairly well by both models. The IIAU models perform as well as its FIRE counterpart: The differences of simulated moments between FIRE and IIAU models across different $\sigma_\eta/\sigma_\varepsilon$ and w_G are small and insignificant.

Data			
	SD	AR(1)	Cor(\cdot, y_t)
Y_t	1	0.87	1
C_t	0.82	0.89	0.88
I_t	4.56	0.85	0.91

Notes: Data for the period 1967Q1-2017Q3 are taken from St. Louis Fed's FRED database(GDPC1 for output Y_t , PCECC96 for consumption C_t , GDPIC1 for investment I_t . To account growth, Y_t, I_t, C_t are in logs, HP-filtered, and multiply by 100 to express them in percentage deviations from trend. Results are robust to the bandpass filter(Christiano and Fitzgerald (2003)).

Table 1: Selected moments of output Y_t , investment I_t and consumption C_t in the data.

	FIRE or IIAU: $\sigma_\eta/\sigma_\varepsilon = 0$			IIAU: $\sigma_\eta/\sigma_\varepsilon = 0.1$			IIAU: $\sigma_\eta/\sigma_\varepsilon = 0.2$			IIAU: $\sigma_\eta/\sigma_\varepsilon = 1$		
	SD	AR(1)	Cor(\cdot, y_t)	SD	AR(1)	Cor(\cdot, y_t)	SD	AR(1)	Cor(\cdot, y_t)	SD	AR(1)	Cor(\cdot, y_t)
w_G = 0												
Y_t	1	0.93	1	1	0.93	1	1	0.93	1	1	0.94	1
	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-
C_t	0.69	0.97	0.86	0.70	0.97	0.85	0.71	0.97	0.84	0.76	0.96	0.79
	(0.06)	(0.01)	(0.05)	(0.06)	(0.01)	(0.05)	(0.07)	(0.01)	(0.06)	(0.07)	(0.02)	(0.08)
I_t	3.76	0.89	0.86	3.80	0.89	0.85	3.85	0.89	0.84	4.10	0.90	0.78
	(0.41)	(0.03)	(0.04)	(0.43)	(0.03)	(0.04)	(0.45)	(0.03)	(0.05)	(0.54)	(0.03)	(0.06)
w_G = 0.4												
Y_t	1	0.94	1	1	0.94	1	1	0.94	1	1	0.94	1
	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-
C_t	0.77	0.98	0.87	0.77	0.99	0.87	0.78	0.98	0.87	0.78	0.98	0.87
	(0.11)	(0.01)	(0.06)	(0.11)	(0.01)	(0.06)	(0.11)	(0.01)	(0.06)	(0.18)	(0.01)	(0.11)
I_t	4.05	0.90	0.75	4.04	0.90	0.75	4.05	0.90	0.75	4.06	0.90	0.75
	(0.54)	(0.03)	(0.07)	(0.54)	(0.03)	(0.07)	(0.54)	(0.03)	(0.07)	(0.56)	(0.03)	(0.08)
w_G = 0.6												
Y_t	1	0.94	1	1	0.94	1	1	0.94	1	1	0.94	1
	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-
C_t	0.92	0.99	0.82	0.93	0.99	0.81	0.94	0.98	0.80	0.97	0.98	0.77
	(0.15)	(0.01)	(0.06)	(0.16)	(0.01)	(0.09)	(0.17)	(0.01)	(0.10)	(0.11)	(0.01)	(0.06)
I_t	4.30	0.91	0.69	4.27	0.91	0.69	4.28	0.91	0.70	4.07	0.90	0.73
	(0.62)	(0.03)	(0.09)	(0.62)	(0.03)	(0.09)	(0.61)	(0.03)	(0.09)	(0.55)	(0.03)	(0.07)
w_G = 1												
Y_t	1	0.95	1	1	0.95	1	1	0.95	1	1	0.93	1
	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-	-	(0.02)	-
C_t	1.30	0.98	0.77	1.40	0.98	0.74	1.50	0.98	0.71	1.80	0.96	0.55
	(0.22)	(0.01)	(0.12)	(0.25)	(0.01)	(0.13)	(0.28)	(0.01)	(0.14)	(0.39)	(0.01)	(0.19)
I_t	4.88	0.92	0.58	4.81	0.92	0.58	4.74	0.92	0.59	4.11	0.91	0.69
	(0.80)	(0.02)	(0.11)	(0.78)	(0.02)	(0.11)	(0.75)	(0.02)	(0.1)	(0.61)	(0.03)	(0.09)

Notes: Y_t, C_t, I_t 's are in logs and multiply by 100. Means and standard deviations(in parentheses) of simulated moments are reported.

Table 2: Simulated moments of output Y_t , investment I_t and consumption C_t in the models.

The simple model sheds light on the importance of the belief parameter σ_η , the composition of fiscal spending w_G and the impact of different beliefs $\tau_{t|t-1}^*$. However, Table 2 suggests matching aggregate moments may not be sufficient to identify the sources and impacts of LFFU. In the following I quantify the effects of LFFU by employing full likelihood Bayesian estimation to a more elaborated real business cycle (RBC) model with balanced growth.

3 The RBC model

The RBC model is extended to include both inter- and intra-temporal preference shocks, external habit formation, investment adjustment costs and variable capacity utilization. A detailed fiscal sector with distortionary taxes, government spending, lump-sum transfers, and one-period government bonds is added on top of the model. The economy consists of a representative household, competitive firms, and a central government (the fiscal authority). The model structure \mathcal{M} includes both the non-policy section \mathcal{M}^{NP} and the fiscal policy section \mathcal{M}^{FP} .

Because of its central role in the current analysis, I describe \mathcal{M}^{FP} first.

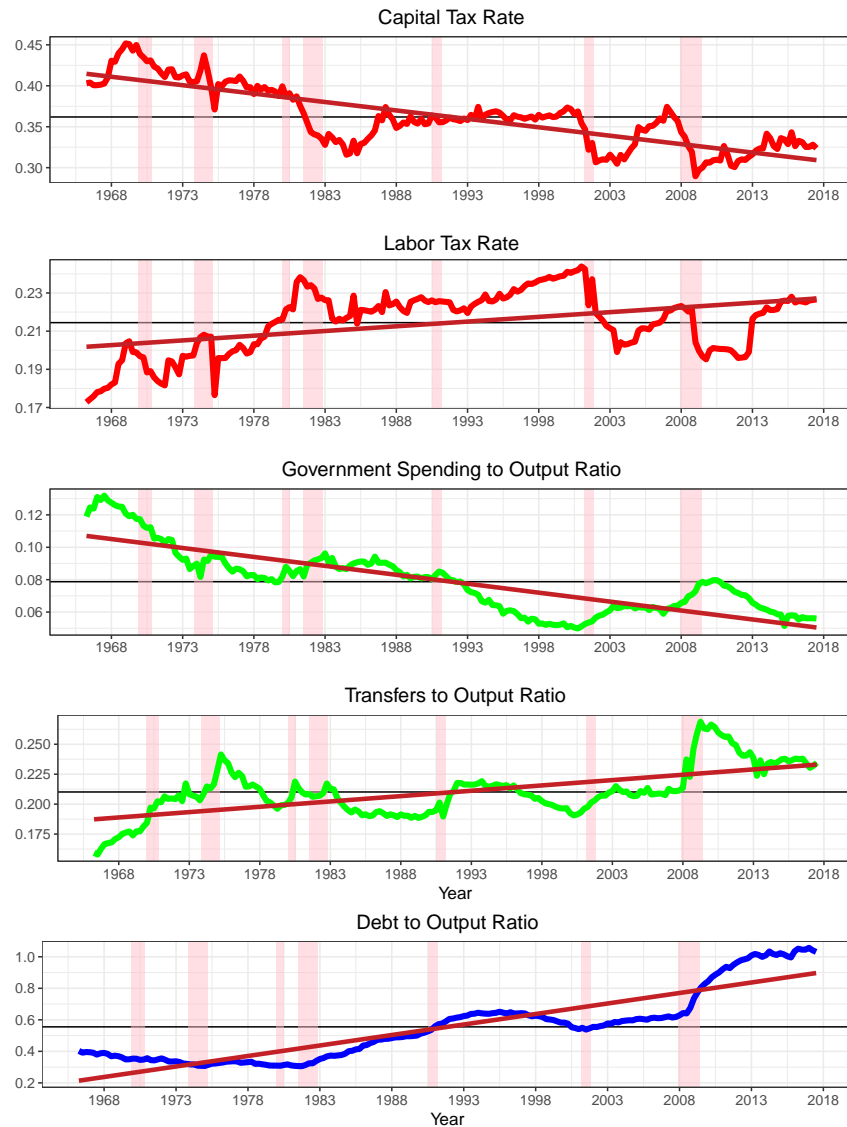


Figure 8: Historical U.S. fiscal series: 1966Q2 - 2017Q3. The horizontal lines are means of each fiscal instrument. The dark red lines run linear regressions for each fiscal instrument with time. The shaded regions are NBER dated recessions.

3.1 The model structure: Fiscal policy \mathcal{M}^{FP}

Fiscal policies are implemented in terms of simple rules as in [Leeper et al. \(2010\)](#). Given a constant, observable consumption tax rate $\tau_C^* = 0.023^1$, four rules govern the evolution of the following fiscal instruments: government spending as a share of output G_t/Y_t , transfers as a share of output Z_t/Y_t^2 , labor tax rate $\tau_{L,t}$, and capital tax rate $\tau_{K,t}$. Three components are included in each fiscal rule: The first component represents “automatic stabilizers” fiscal policies usually designed to offset fluctuations in the macro economy. This is modeled as responses of fiscal instruments to detrended log output(i.e., the output gap) \tilde{y}_t .³ The second component allows fiscal instruments to adjust to the indebtedness of the government and is modeled as fiscal instruments’ responses to lagged log debt-to-GDP gap, $\log(B_{t-1}/Y_{t-1}) - \log(BY^*)$, where BY^* is the debt target. This feature aims to ensure the real debt is on a stable path. The last component describes the exogenous AR(1) innovation processes.

I follow the standard exercise in the literature and assume the actual fiscal targets of G_t/Y_t , Z_t/Y_t , $\tau_{K,t}$ and $\tau_{L,t}$ are all constant. Denote the corresponding targets GY^* , ZY^* , τ_K^* and τ_L^* . The fiscal rules are:

$$\log(\tau_{K,t}) - \log(\tau_K^*) = \varphi_{\tau_K} \tilde{y}_t + \gamma_{\tau_K} [\log(B_{t-1}/Y_{t-1}) - \log(BY^*)] + u_t^{\tau_K} + \phi_{\tau_K}^{\tau_L} u_t^{\tau_L} \quad (16)$$

$$\log(\tau_{L,t}) - \log(\tau_L^*) = \varphi_{\tau_L} \tilde{y}_t + \gamma_{\tau_L} [\log(B_{t-1}/Y_{t-1}) - \log(BY^*)] + u_t^{\tau_L} + \phi_{\tau_K}^{\tau_L} u_t^{\tau_K} \quad (17)$$

$$\log(G_t/Y_t) - \log(GY^*) = -\varphi_G \tilde{y}_t - \gamma_G [\log(B_{t-1}/Y_{t-1}) - \log(BY^*)] + u_t^G \quad (18)$$

$$\log(Z_t/Y_t) - \log(ZY^*) = -\varphi_Z \tilde{y}_t - \gamma_Z [\log(B_{t-1}/Y_{t-1}) - \log(BY^*)] + u_t^Z \quad (19)$$

where for $X = \{\tau_K, \tau_L, G, Z\}$, the persistent component u_t^X is modeled as an AR(1) process

$$u_t^X = \rho_u^X u_{t-1}^X + \sigma_\varepsilon^X \varepsilon_t^X \quad (20)$$

In rules (16) - (19), φ_X are the output response parameters and γ_X are debt response parameters. Signs in front of φ_X , γ_X have been adjusted so that $\varphi_X, \gamma_X \geq 0$ for $X = \{\tau_K, \tau_L, G, Z\}$ from an *ex ante* perspective: Presumably, the fiscal authority should conduct expansionary fiscal policy when the economy is in a recession and raise taxes and/or contract spending when debt-to-GDP ratios are high. Since policy makers often change tax rates jointly, following [Leeper et al. \(2010\)](#), a co-movement parameter $\phi_{\tau_K}^{\tau_L}$ is introduced in tax rules (16), (17) to capture the contemporaneous changes of two tax rates due to either a $\varepsilon_{u,t}^{\tau_K}$ or a $\varepsilon_{u,t}^{\tau_L}$ shock. Each of the ε_t^X 's is distributed *i.i.d.* $N(0, 1)$.

¹This value follows [Leeper et al. \(2017\)](#) and is the sample mean of the consumption tax rate series. Consumption taxes are usually assumed to be exogenous and there are rarely unpredictable permanent changes to consumption tax rates. Estimation results in [Leeper et al. \(2010\)](#) reveal little interaction between consumption taxes and other fiscal instruments. Since households know τ_C^* , fiscal uncertainties are thus concentrated on other fiscal variables. Introducing time-varying consumption tax rates with known τ_C^* will not change main conclusions of the paper. In [Leeper et al. \(2010\)](#) τ_C^* is calibrated to 0.028.

²Government spending and transfers are scaled by output to account for the impact of growth.

³The detrended log output \tilde{y}_t is defined as $\log(Y_t) - \log(Y_t^*)$ where Y_t^* is the steady-state output level along the balanced growth path.

True Fiscal target \mathcal{FT}^*	Value
τ_K^*	0.3619
τ_L^*	0.2145
GY^*	0.0787
BY^*	0.5553
Historical mean	Value
\overline{ZY}	0.2102

Notes: Since households know τ_C^* in both FIRE and IIAU models, the constant consumption tax rate τ_C^* is treated as a parameter and is not part of the fiscal targets. The transfers target ZY^* depends on estimated parameters in the RBC model and is endogenously determined by the model structure to balance the government budget constraint in the long run.

Table 3: Calibrated Time-invariant Fiscal Targets and Historical mean of Z_t/Y_t

Figure 8 plots the historical U.S. fiscal series constructed from the national income and product accounts (NIPA).¹ Appendix B provides more details on data construction. To preserve low-frequency fiscal movement and the underlying composition of fiscal financing, all fiscal series are neither demeaned nor detrended. A glance of Figure 8 immediately reveals fiscal series are highly persistent: They move slowly over decades and appear to have their own trends. Such trends, however, cannot last forever as tax rates and government spending-to-GDP ratios are naturally bounded between zero and one and transfers- and debt-to-GDP ratios, in general, cannot go to negative territories and explode to extremely high values. Figure 8 also suggests estimating the time-invariant fiscal targets τ_K^* , τ_L^* , GY^* , ZY^* , BY^* directly from data is unlikely yield promising results as data are not informative in revealing these fiscal series’s long-run values. I follow the common exercise in the literature and calibrate τ_K^* , τ_L^* , GY^* , BY^* by historical means. The transfers target, ZY^* , is determined endogenously by the RBC model structure to balance the government budget constraint in the long run.² Table 3 presents the calibrated fiscal targets. The remaining fiscal parameters are estimated along with other structural parameters using Bayesian methods.

I now introduce the non-policy model structure \mathcal{M}^{NP} , which is a standard RBC model that allows balanced growth.

3.2 The model structure: Non-policy sections \mathcal{M}^{NP}

3.2.1 Households

There is a large number of infinitely lived, identical households. Households derive utility from private consumption, C_t , relative to a habit stock hC_{t-1} where $h \in [0, 1]$, and disutility from labor

¹It is noteworthy that the tax series plotted in Figure 8 and used in the following model estimation are *average* tax rates, rather than *marginal* tax rates. While Barro and Redlick (2011) reports a marginal income tax rate history, separate marginal capital and labor tax rates are not readily available. The exercise of using average tax rates follows Jones (2002) and Fernández-Villaverde et al. (2015).

²The model implied transfers target is around 20% and is slightly lower than its historical mean, which is 21.02%.

supply, L_t . The representative household maximizes the intertemporal utility function

$$E_0^{\mathcal{I}} \sum_{t=0}^{\infty} \beta^t \exp(u_t^\beta) \left[\frac{(C_t - hC_{t-1})^{1-\gamma}}{1-\gamma} - \omega A_t^{1-\gamma} \exp(u_t^\ell) \frac{L_t^{1+\kappa}}{1+\kappa} \right] \quad (21)$$

where $\beta \in (0, 1)$ is the time discount factor, γ denotes constant relative risk aversion, ω is a preference weight on disutility of labor, and $1/\kappa$ is the Frisch elasticity of labor supply. $E_t^{\mathcal{I}}$ is an expectation operator conditional on the household's information set \mathcal{I}_t (defined in Section 3.3). Preferences are subject to both an intertemporal discount factor shock,

$$u_t^\beta = \rho_\beta u_{t-1}^\beta + \sigma_\beta \varepsilon_t^\beta, \quad \varepsilon_t^\beta \sim N(0, 1)$$

and an intratemporal labor supply shock

$$u_t^\ell = \rho_\ell u_{t-1}^\ell + \sigma_\ell \varepsilon_t^\ell, \quad \varepsilon_t^\ell \sim N(0, 1)$$

A_t is the level of labor augmenting technology. The term $A_t^{1-\gamma}$ is introduced in (21) to ensure a balanced growth path. During each period household receive after-tax wage and capital rental income, lump-sum transfers Z_t from the government and spend income on consumption, investment I_t and government bonds B_t . The household's budget constraint is given by

$$(1 + \tau_C^*)C_t + B_t + I_t = (1 - \tau_{L,t})W_t L_t + (1 - \tau_{K,t})R_t^K v_t K_{t-1} + R_{t-1}B_{t-1} + Z_t \quad (22)$$

where τ_C^* , $\tau_{L,t}$, $\tau_{K,t}$ are tax rates on consumption C_t , labor income $W_t L_t$ and capital rental income $R_t^K v_t K_{t-1}$. The effective capital level supplied to firms is given by $v_t K_{t-1}$, where v_t measures capacity utilization in period t . R_t is the gross interest rate on the one-period, risk-free government bond B_t .

The law of motion for capital is

$$K_t = (1 - \delta(v_t))K_{t-1} + \exp(u_t^I) [1 - s(I_t/I_{t-1})] I_t \quad (23)$$

where $s(I_t/I_{t-1})I_t$ is the investment adjustment cost that satisfies $s(\cdot) = s'(\cdot) = 0$ and $s''(\cdot) > 0$ along the balanced growth path as in [Christiano et al. \(2005\)](#). Capital accumulation (23) is also subject to an investment-specific efficiency shock u_t^I where

$$u_t^I = \rho_I u_{t-1}^I + \sigma_I \varepsilon_t^I, \quad \varepsilon_t^I \sim N(0, 1).$$

Following [Schmitt-Grohé and Uribe \(2012\)](#), the capital utilization function $\delta(v_t)$ is¹

$$\delta(v_t) = \delta_0 + \delta_1(v_t - 1) + \frac{\delta_2}{2}(v_t - 1)^2. \quad (24)$$

¹In (24) $\delta_1 = \exp(g_A)^\gamma/\beta - (1 - \delta_0)$ so that the capacity utilization v_t equals 1 in the steady state.

The household maximizes utility (21), subject to the budget constraint (22), the law of motion for capital (23) and the quadratic variable utilization function (24).

3.2.2 Firms

There is a continuum of identical, competitive firms with Cobb-Douglas production functions

$$Y_t = (v_t K_{t-1})^\alpha (A_t L_t)^{1-\alpha}. \quad (25)$$

where Y_t denotes the output produced with effective capital level $v_t K_{t-1}$, labor L_t and labor augmenting technology A_t . Capital share $\alpha \in (0, 1)$. The logarithm of A_t 's growth rate, $u_t = \log(A_t) - \log(A_{t-1})$, follows a stationary AR(1) process

$$u_t^A = (1 - \rho_A)g_A + \rho_A u_{t-1}^A + \sigma_A \varepsilon_t^A, \quad \varepsilon_t^A \sim N(0, 1) \quad (26)$$

It follows $\exp(g_A)$ is the quarterly growth rate of the economy along the balanced growth path.

The representative firm rents capital and labor from the household to maximize profit

$$Y_t - R_t^K v_t K_{t-1} - W_t L_t$$

subject to (25). Competitive production market indicates wages W_t and capital rental rate R_t^K are paid at their marginal product

$$W_t = \frac{(1 - \alpha)Y_t}{L_t}, \quad R_t^K = \frac{\alpha Y_t}{v_t K_{t-1}}.$$

3.2.3 Government Budget Constraint

Let G_t denote government spending. The intertemporal government budget constraint is

$$B_t + \tau_t^K R_t^K v_t K_{t-1} + \tau_t^L W_t L_t + \tau_C^* C_t = R_{t-1} B_{t-1} + G_t + Z_t \quad (27)$$

3.2.4 Equilibrium conditions

In equilibrium households and firms are optimizing and the capital rental, labor and bond markets all clear. Debt and capital accumulation also satisfy the transversality conditions. The final goods market is in equilibrium if the aggregate demand by the household and government equals to the aggregate production:

$$Y_t = C_t + I_t + G_t \quad (28)$$

Appendix C provides the complete equilibrium conditions of the RBC model.

Sections 3.2.1 - 3.2.4 conclude the non-policy model structure \mathcal{M}^{NP} .

3.3 Household's information set \mathcal{I}_t

In line with the simple model section, I consider FIRE and several IIAU models. Let \mathcal{O}^t denote the complete history of variables related to the households' signal extraction problem¹ and \mathcal{P} be the entire parameter list appeared in the RBC model. Depending on whether households' beliefs on $\{\tau_K^*, \tau_L^*, GY^*\}$ are anchored (or not), I consider two IIAU model specifications: IIAU anchored and IIAU unanchored. Table 4 summarizes the alternative information sets.

	Full Information		Incomplete Information	
	FIRE	IIAU anchored	IIAU unanchored	
Non-policy structure \mathcal{M}^{NP}	✓	✓	✓	✓
Observables \mathcal{O}^t	✓	✓	✓	✓
Parameters $\mathcal{P} \setminus \{\tau_K^*, \tau_L^*, GY^*, ZY^*, BY^*\}^a$	✓	✓	✓	✓
Fiscal policy structure \mathcal{M}^{FP}	✓	✓ except [b]	✓ except [a], [b]	
[a] Beliefs on $\{\tau_K^*, \tau_L^*, GY^*\}$	✓	✓	unanchored	
[b] Beliefs on $\{ZY^*, BY^*\}$	✓	unanchored	unanchored	

Table 4: Alternative information sets: FIRE *vs.* IIAU

^aThe backslash \setminus denotes set subtraction.

Since households' signal extraction problems in the IIAU models are analogous to their counterpart in the simple model, I briefly describe it here. In the IIAU anchored model, households' beliefs on long-run government spending, capital and labor taxes are correctly anchored: Only households' perceived transfers target ZY_t^* and debt targets BY_t^* can drift. Motivated by the recent concerns on the ever-increasing larger demand on social security, Medicare and Medicaid, I assume households' perceived transfers-to-GDP target is time-varying and follows a unit root process

$$\log(ZY_t^*) = \log(ZY_{t-1}^*) + \sigma_{ZY}^* \varepsilon_{ZY,t}^* \quad (29)$$

Since unit root processes are non-stationary, (29) violates rational expectation and anticipated utility plays its role as households making decisions conditional on their current beliefs ZY_t^* and BY_t^* . I don't model the $\varepsilon_{ZY,t}^*$ -shocks explicitly. Rather, they can be interpreted as demographic and/or political shocks that shift the perceived transfer-to-GDP target. As populations age in many advanced economies, instable transfers are likely to become more common. The $\varepsilon_{ZY,t}^*$ -shock therefore is a shortcut for capturing both the slow-moving demographic changes and the complex political process that leads to changes of households' beliefs.² Since beliefs $\{\tau_K^*, \tau_L^*, GY^*\}$ are anchored, due to a changing transfers target ZY_t^* , the debt-to-GDP target BY_t^* is also time-

¹More explicitly, $\mathcal{O}^t = \{\tau_K^t, \tau_L^t, G^t, Z^t, B^t, Y^t, A^t\}$. Households need the history of technology, A^t , to figure out the perceived output gap.

²Motivated by the concept of fiscal foresight(See [Leeper et al. \(2013\)](#)), [Fève and Pietrunti \(2016\)](#) emphasizes the effects of noisy fiscal policy by introducing noisy signals about future shocks to government spending. While there are in general foresight with regarding future tax rates changes and government spending, this is not the case to transfers. As of now we don't know any potential reforms, if there'll be any, to social security, Medicare and Medicaid. To emphasize the impacts of low-frequency fiscal uncertainty here I don't introduce any foresight and noise signals on fiscal shocks.

varying and is endogenously determined by the model structure to balance the government budget constraint in the long run. When $\sigma_{ZY}^* = 0$ and the initial belief $ZY_{1|0}^*$ matches the actual target ZY^* , the IIAU anchored model becomes the FIRE model.

In the IIAU unanchored model, none of the fiscal targets' beliefs are anchored and the households' learning problem is more complicated. To put it simple, households misperceive each AR(1) fiscal shocks in (16)-(19) as a summation of a unit root and an AR(1) process, exactly the same way as they did in perceived law of motions (6)-(8) in the simple model section. Denote the unit root processes

$$\Omega_t^X = \Omega_{t-1}^X + \sigma_\eta^X \eta_t^X, \quad \eta_t^X \sim N(0, 1) \quad (30)$$

for $X = \{\tau_K, \tau_L, G, Z\}$. The perceived shocks η_t^X 's are assumed to be mutually *i.i.d.* Appendix C establishes the 4×1 vector $\Omega_t = [\Omega_{\tau_K,t}, \Omega_{\tau_L,t}, \Omega_{G,t}, \Omega_{Z,t}]$ that summarizes the five time- t perceived fiscal targets¹

$$\mathcal{FT}_t^* = \{\tau_{K,t}^*, \tau_{L,t}^*, GY_t^*, ZY_t^*, BY_t^*\}$$

The perceived standard deviations (i.e., σ_η^X 's) govern how large households' beliefs on fiscal targets can drift within one period. Compared with the IIAU model with anchored $\{\tau_K^*, \tau_L^*, GY^*\}$, this IIAU specification allows all perceived fiscal targets to drift over time. Concerns on instable transfers thus could cause beliefs to change on long-run government spending and tax liabilities. Conversely, persistent changes in government spending could also change households' perception of long-run transfer levels. When taking (30) to data, the initial beliefs $\Omega_{X,1|0}$'s are fixed at values such that households' initial perceived fiscal targets match the true targets, i.e., $\mathcal{FT}_{1|0}^* = \mathcal{FT}^*$. When $\sigma_{\eta,X} = 0$ for all $X = \{\tau_K, \tau_L, G, Z\}$, the IIAU unanchored model becomes the FIRE model. More details on establishing households' perceived fiscal policy and their signal extraction problem can be found in Appendix C and D.

3.4 Model solution

The FIRE model is solved by linearized methods around the unique, deterministic steady state of the economy Ω . Let \mathbf{y}_t denote the vector of endogenous state variables and \mathbf{s}_t the actual exogenous state vector. The FIRE model can be described in terms of

$$\mathbf{F}(\Omega)E_t^{FIRE}\mathbf{y}_{t+1} + \mathbf{G}(\Omega)\mathbf{y}_t + \mathbf{H}(\Omega)\mathbf{y}_{t-1} + \mathbf{M}(\Omega)\mathbf{s}_t + \mathbf{N}(\Omega)E_t^{FIRE}\mathbf{s}_{t+1} = 0 \quad (31)$$

where $\mathbf{F}, \mathbf{G}, \mathbf{H}, \mathbf{M}, \mathbf{N}$ are matrices that depend on Ω . The stochastic difference equation (31) can be solved by Uhlig (1995) to yield the FIRE law of motion

$$\mathbf{y}_t = \mathbf{P}(\Omega)\mathbf{y}_{t-1} + \mathbf{Q}(\Omega)\mathbf{s}_t \quad (32)$$

¹We lose one degree of freedom from \mathcal{FT}_t^* to Ω_t due to the long-run government budget constraint.

LFFU causes households' perceived fiscal targets to drift over time and thus generates time-varying perceived steady state of the economy. Let $\mathcal{FT}_{t|t}^*$ denote households' time- t perceived fiscal targets and $\Omega_{t|t}$ be the resulting perceived steady state. The IIAU models are linearized around $\Omega_{t|t}$. Let $\mathbf{s}_{t|t}$ denote households' perceived exogenous state vector. The stochastic difference equation for the IIAU model is

$$\mathbf{F}(\Omega_{t|t})E_t^{IIAU}\mathbf{y}_{t+1} + \mathbf{G}(\Omega_{t|t})\mathbf{y}_t + \mathbf{H}(\Omega_{t|t})\mathbf{y}_{t-1} + \mathbf{M}(\Omega_{t|t})\mathbf{s}_{t|t} + \mathbf{N}(\Omega_{t|t})E_t^{IIAU}\mathbf{s}_{t+1} = 0 \quad (33)$$

Anticipated utility let households treat $\Omega_{t|t}$ as constant when forming time- t expectations and allows us to solve (33) the same way as (31), giving the IIAU law of motion

$$\mathbf{y}_t = \mathbf{P}(\Omega_{t|t})\mathbf{y}_{t-1} + \mathbf{Q}(\Omega_{t|t})\mathbf{s}_{t|t} \quad (34)$$

The perceived steady state $\Omega_{t|t}$ and perceived exogenous state vector $\mathbf{s}_{t|t}$ are byproducts of households signal extraction problem and can be derived from the Kalman recursion.

3.5 Estimation

I use Bayesian methods to estimate structural parameters of both FIRE and IIAU models using quarterly US data. There are eight observables: log differences of aggregate consumption, investment and hours worked; capital and labor tax rates; government spending-, transfers- and debt-to-GDP ratios. Data are neither detrended nor demeaned. The sample period is 1966Q2 to 2017Q3. Details of the data construction appears in Appendix E.1.

The FIRE model can be estimated using the standard Kalman filter. In the IIAU model with anchored $\{\tau_K^*, \tau_L^*, GY^*\}$, the perceived steady state of the economy other than transfers and debt is still at the unique, deterministic steady state Ω . Intuitively, as GDP is defined as the sum of consumption, investment and government spending in a closed economy, transfers and debt are *not* part of GDP and don't impact the allocations of other real variables in steady state. Solving the model around the deterministic steady state Ω by the algorithm of Blanchard et al. (2013) implies the standard Kalman filter is still applicable to the IIAU model with anchored $\{\tau_K^*, \tau_L^*, GY^*\}$.

The Kalman filter is not directly applicable to the unanchored IIAU model as (34) shows $\Omega_{t|t}$, determined by $\mathcal{FT}_{t|t}^*$, enters the system as additional non-linear state variables. In general the appearance of non-linear state variables requires us to use sequential Monte Carlo methods(i.e., particle filters) to estimate the model¹. Direct parameter estimation of the non-linear state-space model by sequential Monte Carlo methods is computationally intensive.² More importantly, sequential Monte Carlo methods, which are simulation-based, imply the model likelihood can only be

¹For applications of using particle filters on estimating DSGE models, see Herbst and Schorfheide (2016).

²It should be noticed that conditional on $\Omega_{t|t}$, or equivalently $\mathcal{FT}_{t|t}^*$, the law of motion (34) is linear. To utilize such conditional linearity, a Rao-Blackwellized particle filter(RBPF) can be combined with MCMC algorithms to estimate the model parameters. A RBPF that is applicable to models considered here is Schon et al. (2005). Early experiment shows while the filter performs well in extracting non-linear state variables given a set of parameters, it is running considerably slower than the time-varying Kalman filter in estimation even for a small number of particles.

approximated as we tracking particles to “mimic” the distributions of the non-linear state variables. On the other hand, model likelihood can be calculated analytically in linear state space models by Kalman filters. Since the model’s empirical performance (i.e., model fit comparisons) is essential in evaluating the importance of LFFU, I approximate the IIAU equilibrium conditions (33) around $\Omega_{t|t-1}$ so that a time-varying Kalman filter can be applied to the IIAU model estimation. Denote households’ perceived fiscal targets at the beginning of time- t $\mathcal{FT}_{t|t-1}^*$ and let $\Omega_{t|t-1}$ be the corresponding perceived steady state of the economy.¹ Log-linearizing the IIAU equilibrium conditions around $\Omega_{t|t-1}$ gives

$$\mathbf{F}(\Omega_{t|t-1})E_t^{IIAU}\mathbf{y}_{t+1} + \mathbf{G}(\Omega_{t|t-1})\mathbf{y}_t + \mathbf{H}(\Omega_{t|t-1})\mathbf{y}_{t-1} + \mathbf{M}(\Omega_{t|t-1})\mathbf{s}_t + \mathbf{N}(\Omega_{t|t-1})E_t^{IIAU}\mathbf{s}_{t+1} = 0 \quad (35)$$

Following the algorithm in Blanchard et al. (2013), solving (35) gives a law of motion

$$\mathbf{y}_t = \mathbf{P}(\Omega_{t|t-1})\mathbf{y}_{t-1} + \mathbf{Q}(\Omega_{t|t-1})\mathbf{s}_t + \mathbf{R}(\Omega_{t|t-1})\mathbf{s}_{t|t} \quad (36)$$

Alternatively, (36) can be interpreted as the model solution to an IIAU environment in which conditional on their last period’s beliefs $\mathcal{FT}_{t|t-1}^*$, households form expectations *in the first step* and solve for economic outcomes and update beliefs to $\mathcal{FT}_{t|t}^*$ simultaneously *in the second step*. The updated belief $\mathcal{FT}_{t|t}^*$ will be used as $\mathcal{FT}_{t+1|t}^*$ next period to form expectations. Intuitively, since in general households don’t change perceived fiscal targets drastically within the time period from $\mathcal{FT}_{t|t-1}^*$ to $\mathcal{FT}_{t|t}^*$ ², the recursive timing assumption (36) suggested won’t cause much differences than the simultaneous timing convention embedded in (34). Indeed, prior predictive analysis show (36) is a very good approximation of (34).³ Appendix A.1 illustrates the algorithm by reformulating the simple growth model’s equilibrium condition (9) in terms of (35) and solving it in terms of (36).

More importantly, (36) implies conditional on time- $(t-1)$ non-linear state variables $\Omega_{t|t-1}$, the model dynamics is linear. It follows a time-varying Kalman filter is applicable to (36) for parameter estimation⁴ and model likelihood can be calculated exactly. Section 4.1 and 4.2 report the log-marginal data density and posteriors of the unanchored IIAU model based on (36).

3.5.1 Prior distributions

Besides the calibrated time-invariant fiscal targets in Table 3, a few parameters are fixed. Table 5 summarizes the calibration⁵, which is standard for quarterly models. Table 6 lists prior distributions for non-policy structural parameters, mostly taken from Leeper et al. (2017).⁶ These priors cover a broad range of parameters values and are widely used in a large literature on Bayesian

¹Due to the random walk nature of the law of motion for the fiscal targets, $\mathcal{FT}_{t|t-1}^* = \mathcal{FT}_{t-1|t-1}^*$. Within the time period households update $\mathcal{FT}_{t|t-1}^*$ to $\mathcal{FT}_{t|t}^*$.

²Or equivalently, period-by-period as $\mathcal{FT}_{t|t-1}^* = \mathcal{FT}_{t-1|t-1}^*$.

³Results are available upon request.

⁴See Section 13.8 of Hamilton (1994) for the implementation of time-varying Kalman filters.

⁵The effective discount factor $q = \beta / \exp(g_A \gamma)$.

⁶Priors of the CRRA parameter γ and the quadratic capacity utilization parameter δ_2 are not available in Leeper et al. (2017) and are taken from Leeper et al. (2010).

	Value	Description	Target
q	$1.02^{-0.25}$	Effective discount factor	2% annual real interest rate
δ_0	0.025	Capital depreciation(quarterly)	10% average annual depreciation rate
α	1/3	Capital income share	2/3 labor share of income
L^*	1/3	Steady-state labor supply	Average labor supply is 1/3

Table 5: Calibrated Structural Parameters

	Description	Prior	Mean	SD	90 percent int.
$100g_A$	Quarterly balanced growth rate	Normal	0.5	0.05	[0.42, 0.58]
h	Habit formation coefficient	Beta	0.5	0.2	[0.17, 0.83]
γ	Constant relative risk aversion	Gamma	1.75	0.5	[1.02, 2.65]
κ	Inverse of Frisch elasticity	Gamma	2	0.5	[1.18, 2.80]
s''	Invest. adjust. cost coefficient	Normal	6	1.5	[3.54, 8.47]
δ_2	Capacity util. parameter(quadratic)	Gamma	0.7	0.5	[0.12, 1.67]
ρ_A	AR(1) for technology growth rate	Beta	0.5	0.2	[0.17, 0.83]
ρ_β	AR(1) for beta preference shocks	Beta	0.5	0.2	[0.17, 0.83]
ρ_ℓ	AR(1) for labor preference shocks	Beta	0.5	0.2	[0.17, 0.83]
ρ_I	AR(1) for invest. adjust. cost shocks	Beta	0.5	0.2	[0.17, 0.83]
$100\sigma_A$	Std. dev. of technology shocks	Inv. Gamma	0.1	1	[0.02, 0.28]
$100\sigma_\beta$	Std. dev. of beta preference shocks	Inv. Gamma	0.1	1	[0.02, 0.28]
$100\sigma_\ell$	Std. dev. of labor preference shocks	Inv. Gamma	0.1	1	[0.02, 0.28]
$100\sigma_I$	Std. dev. of invest. adjust. cost shocks	Inv. Gamma	0.1	1	[0.02, 0.28]

Table 6: Priors of Non-policy Structural Parameters

estimation of DSGE models.¹

I now specify prior distributions for fiscal parameters in \mathcal{M}^{FP} . Importance of these priors is evident: First, it disciplines the fiscal behaviour when taking the reduced-form rules to the structural model. Second, it validates the signal extraction problem and highlights why and how LFFU arises. Third, evolution of households' beliefs in the signal extraction problem is also disciplined by estimated \mathcal{M}^{FP} . For these reasons I use fairly diffuse priors on all fiscal parameters. Table 7 presents the prior distributions. The prior for output response coefficient φ_X 's is uniform on $[-5, 5]$, where the lower and upper bounds of the flat distribution are chosen to be not restrictive. We thus allow both pro- and counter-cyclical responses of fiscal instruments to output gap before confronting rules to data and the model. Following [Leeper et al. \(2010\)](#), debt response coefficients γ_X 's are assumed to have a Gamma distribution with a mean of 0.4 and standard deviation of 0.2. For the tax co-movement parameter $\phi_{\tau_K}^{\tau L}$ the prior is uniform on $[-1, 1]$. Due to the symmetry of $\phi_{\tau_K}^{\tau L}$ in (16) and (17), the assumption $|\phi_{\tau_K}^{\tau L}| < 1$ is not restrictive.² For AR(1) coefficients and standard deviations of fiscal shocks I use the same Beta and Inverse Gamma distributions as non-fiscal

¹See [Leeper et al. \(2017\)](#) for a long list of literature who adopted these priors.

²This is because $u_t^{\tau K} + \phi_{\tau_K}^{\tau L} u_t^{\tau L}$, $u_t^{\tau L} + \phi_{\tau_K}^{\tau L} u_t^{\tau K}$ in (16), (17) can be redefined as $v_t^{\tau K} + \phi_{\tau_K}^{\tau L} v_t^{\tau L}$, $v_t^{\tau L} + \phi_{\tau_K}^{\tau L} v_t^{\tau K}$ with $\varphi_{\tau_K}^{\tau L} = 1/\phi_{\tau_K}^{\tau L}$.

	Description	Prior	Lower Bd.	Upper Bd.
φ_X 's	Output Response Parameters	Uniform	-5	5
$\phi_{\tau_K}^{\tau_L}$	Tax Co-movement Parameter	Uniform	-1	1
	Description	Prior	Mean	SD
γ_X 's	Debt Response Parameters	Gamma	0.4	0.2
ρ_u^X 's	AR(1) of fiscal shocks	Beta	0.5	0.2
$100\sigma_{X,\varepsilon}$'s	Std. dev. of fiscal shocks	Inv. Gamma	0.1	1

Table 7: Priors of Fiscal Policy Parameters

Model	Parameter	Prior	Mean	SD
IIAU anchored	$\sigma_{ZY}^*/\sigma_\varepsilon^Z$	Beta	0.2	0.1
For $X = \{\tau_K, \tau_L, G, Z\}$				
IIAU unanchored	$\sigma_\eta^X/\sigma_\varepsilon^X$	Beta	0.2	0.1

Table 8: Priors of Belief Parameters

shocks.

Belief parameters(i.e., σ_{ZY}^* and σ_η^X 's for $X = \{\tau_K, \tau_L, G, Z\}$) are reparameterized as $\sigma_{ZY}^*/\sigma_\varepsilon^Z$ and $\sigma_\eta^X/\sigma_\varepsilon^X$ respectively. Since we expect households do not adjust beliefs on $\log(ZY_t^*)$ or Ω_t^X 's drastically on a quarterly basis, I choose the Beta distribution that guarantees it assumes values on $[0, 1]$. A larger upper bound of the prior distribution seems implausible. All belief parameters listed in Table 8 has prior mean 0.2 and prior standard deviation 0.1. This prior distribution puts a substantial weight on small values near zero¹, thus forcing data to be the deciding factor to positive belief parameters. As a robustness check, I also use a Uniform prior distribution on the interval $[0, 1]$. Appendix E.3 reports model fit comparisons under this more diffuse, flat prior distribution. Appendix E.3 also reports model fit comparisons using only Great Moderation data starting in the mid-1980s. Our main conclusions are still valid under these alternative specifications.

4 Estimation Results

4.1 Model fit comparisons

I first evaluate the models' empirical performance² by calculating the log-marginal data densities.³Table 9 reports the data densities and Bayes factors for several FIRE and IIAU models. The

¹Its [5%, 95%]-quantile interval ranges approximately between 0.05 and 0.34.

²For each model specification, a sample of 1 million posterior draws was created with the first 500,000 draws discarded as the burn-in process. Every 200th draws was kept to lower correlations of the Metropolis-Hastings(MH) draws. The posterior mode and the inverse Hessian at the posterior mode resulting from the optimization procedure were used to define the transition matrix in the MH algorithm. The step size in the MH algorithm is chosen to yield acceptance rates between 20% and 40% in all models. The sample size of each model's posterior equals to 2500. I use trace plots, ACF plots and Geweke's separated partial means tests to diagnose the chain convergence.

³Log-marginal data densities are calculated using Geweke (1999) modified harmonic mean estimator with a truncation parameter of 0.5.

Bayes factor is the ratio of the probabilities from having observed the data given each model. It is a measure of the evidence provided by the data in favor of one model over another. Based on [Jeffreys \(1961\)](#) criterion, a log relative Bayes factor larger than 4.6¹ indicates data strongly prefer the current model specification than the alternative. An alternative criterion, provided by [Kass and Raftery \(1995\)](#), requires a log Bayes factor larger than 10. The log Bayes factor of the IIAU model with anchored $\{\tau_K^*, \tau_L^*, GY^*\}$ relative to the FIRE model is -0.5 , thus there's no strong evidence in favor of the IIAU anchored model. Since the IIAU anchored model nests FIRE, the negative log Bayes factor suggests data discriminate against the IIAU anchored model, which is a more complex model with an additional parameter σ_{ZY}^* .² On the other hand, the log relative Bayes factor of the IIAU unanchored model is around 100, suggesting data strongly prefer this model specification, even though it has four additional parameters $\{\sigma_\eta^G, \sigma_\eta^Z, \sigma_\eta^{\tau_K}, \sigma_\eta^{\tau_L}\}$. Since the differences between FIRE and IIAU unanchored models are only due to LFFU, the large Bayes factor indicates impacts of LFFU is both significant and large.

Table 9 also reports the log-marginal data density of a FIRE model with linearly detrended fiscal observables. With different observables, the relative Bayes factor of the two FIRE models is not defined. Along with calibration of steady-state fiscal variables, detrending fiscal series is also a common exercise in the literature. Such two exercises are inconsistent with each other. Not detrending fiscal series not only respects the long-run government budget constraint fiscal targets inherently need to satisfy, but also preserves the low-frequency movements, which motivate the LFFU in the first place.

Model	Log-marginal data density	Log relative Bayes factor
FIRE	4200.3	0
FIRE(<i>linearly detrended fiscal obs.</i>)	4225.1	NA
IIAU anchored	4199.8	-0.5
IIAU unanchored	4309.6	109.3
IIAU unanchored with only $\sigma_\eta^Z > 0$	4304.9	104.6

Table 9: Model fit comparisons: FIRE *vs.* IIAU; Full sample period: 1966Q2-2017Q3

4.2 Posterior distributions

Before looking into IIAU unanchored models' posteriors, I first compare priors and posteriors of the two FIRE models and the IIAU anchored model.

The IIAU model with anchored $\{\tau_K^*, \tau_L^*, GY^*\}$ yields almost identical posterior distributions as the FIRE model without detrending. Differences between the two FIRE models' posteriors considered in Table 9 are noticeable: Linear detrending fiscal observables decreases the AR(1) coefficients $\{\rho_u^X : X = \tau_K, \tau_L, G, Z\}$'s posterior mode from 0.95-0.99 to 0.93-0.97: Fiscal variables are still quite persistent, even after linear detrending. Figure E.1 and E.2 in Appendix E.2 plot

¹Given a log relative Bayes factor 4.6, the relative Bayes factor is $\exp(4.6) \approx 100$.

²In fact, as can be seen in Figure E.2, data cannot identify σ_{ZY}^* in the IIAU anchored model.

	Mean and [5%, 95%] of posterior distribution					
	FIRE		FIRE ^a		Leeper et al. (2010)	
h	0.68	[0.61, 0.74]	0.70	[0.64, 0.77]	0.5	[0.4, 0.6]
κ	1.56	[0.97, 2.11]	1.79	[1.19, 2.44]	1.9	[1.4, 2.6]
s''	11.1	[9.16, 12.8]	10.8	[9.02, 12.5]	5.5	[5.1, 5.9]
δ_2	0.49	[0.28, 0.66]	0.42	[0.25, 0.59]	0.29	[0.20, 0.42]
ρ_I	0.75	[0.67, 0.84]	0.66	[0.57, 0.74]	0.55	[0.47, 0.64]
$100\sigma_\beta$	7.51	[6.70, 8.27]	7.35	[6.58, 8.10]	7.0	[6.4, 7.7]
$100\sigma_I$	16.8	[13.7, 19.8]	18.3	[15.1, 21.5]	6.4	[5.7, 7.2]
γ_{τ_K}	0.16	[0.06, 0.26]	0.25	[0.14, 0.37]	0.39	[0.28, 0.51]
γ_{τ_L}	0.24	[0.14, 0.33]	0.14	[0.06, 0.23]	0.049	[0.019, 0.09]
γ_G	0.35	[0.16, 0.52]	0.23	[0.09, 0.36]	0.23	[0.15, 0, 0.31]
γ_Z	0.10	[0.03, 0.17]	0.13	[0.06, 0.23]	0.5	[0.41, 0, 0.59]

Table 10: Selected Posteriors of Structural Parameters: FIRE vs. FIRE(*linearly detrended fiscal obs.*) vs. Leeper et al. (2010)

^awith linearly detrended fiscal observables;

the prior and posterior distributions of the three models. Estimation results of the non-policy parameters are comparable to Leeper et al. (2010), with a few exceptions. With balanced growth and non-detrended fiscal observables, more prominent low-frequency movements force the model to prefer larger real frictions. In particular, habit formation h , the Frisch elasticity $1/\kappa$, quadratic capital utilization cost δ_2 , persistence of the investment specific shock ρ_I , and the standard deviation of discount factor shocks σ_β are slightly larger than Leeper et al. (2010)'s results. Detrending fiscal observables yields smaller estimated real frictions. The FIRE models, both detrended and without detrending, imply much larger investment adjustment coefficient s'' and investment specific shocks σ_I . See Table 10.

Turning to fiscal policy parameters, the differences between my results and Leeper et al. (2010)'s are mostly concentrated on output response coefficients φ_X 's and debt response coefficients γ_X 's. These differences are mainly due to different tax observables¹ and slightly different fiscal rules.² Leeper et al. (2010) also detrended fiscal variables linearly. As can be seen in Table 10 and the second column of Figure E.2, linear detrending fiscal observables in the current setting brings all debt response estimates closer to Leeper et al. (2010)'s results.

Speaking of posteriors of IIAU anchored and unanchored models, the belief parameters are of first-order importance. Data cannot identify $\sigma_{ZY}^*/\sigma_\varepsilon^Z$ and penalize the IIAU anchored model's log-marginal data density. On the other hand, all $\sigma_\eta^X/\sigma_\varepsilon^X$'s in the IIAU unanchored model are fairly well-identified. In particular, while estimated $\sigma_\eta^{\tau_K}$, $\sigma_\eta^{\tau_L}$ and σ_η^G are concentrated on small values near

¹Leeper et al. (2010) use capital and labor tax *revenues* as observables. To visualize tax series as seen in Figure 8, I follow Fernández-Villaverde et al. (2015) and use constructed tax *rates* as observables.

²To account for balanced growth, fiscal variables respond to lagged debt-to-GDP ratio in the current setup, instead of lagged debt. Balanced growth also requires to consider government spending- and transfers- to-GDP ratios as fiscal instruments, instead of government spending and transfers themselves. Along with different tax observables, the output response coefficients γ_X 's thus are not directly comparable.

zero, there's strong evidence that $\sigma_\eta^Z > 0$. Figure 9 compares the posterior distributions of σ_{ZY}^* and σ_η^Z . Reestimating the IIAU unanchored model with only $\sigma_\eta^Z > 0$ shows most of the improvement in model fit of the IIAU unanchored model comes from a positive σ_η^Z (See Table 9). Appendix E.3 provides further supporting evidence of the IIAU unanchored model with only $\sigma_\eta^Z > 0$ when using a flat prior on all belief parameters and when only using Great Moderation data(1985Q1-2017Q3). Given these results, the following discussions mainly focus on the IIAU unanchored model with only $\sigma_\eta^Z > 0$ while suppressing $\sigma_\eta^{\tau K} = \sigma_\eta^{\tau L} = \sigma_\eta^G = 0$. When there's no confusion, the IIAU unanchored model with only $\sigma_\eta^Z > 0$ is called IIAU for simplicity.

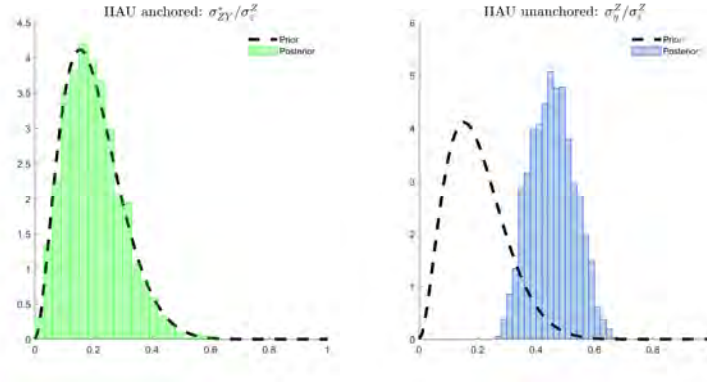


Figure 9: Prior versus posterior distributions of $\sigma_{ZY}^*/\sigma_\epsilon^Z$ (Left Panel) in the IIAU anchored model and $\sigma_\eta^Z/\sigma_\epsilon^Z$ (Right Panel) in the IIAU unanchored model with only $\sigma_\eta^Z > 0$;

Description		Mean and [5%, 95%] of posterior distribution			
		FIRE		IIAU	
$100g_A$	Quarterly balanced growth rate	0.42	[0.34, 0.48]	0.52	[0.49, 0.55]
h	Habit formation coefficient	0.68	[0.61, 0.74]	0.66	[0.60, 0.73]
γ	Constant relative risk aversion	1.43	[1.00, 1.84]	1.74	[1.21, 2.21]
κ	Inverse of Frisch elasticity	1.56	[0.97, 2.11]	1.43	[0.96, 1.92]
s''	Invest. adjust. cost coefficient	11.1	[9.16, 12.8]	6.45	[4.76, 8.23]
δ_2	Capacity util. parameter(quadratic)	0.49	[0.28, 0.66]	0.40	[0.20, 0.63]
ρ_A	AR(1) for technology growth rate	0.17	[0.07, 0.26]	0.17	[0.08, 0.27]
ρ_β	AR(1) for beta preference shocks	0.64	[0.60, 0.69]	0.30	[0.20, 0.41]
ρ_ℓ	AR(1) for labor preference shocks	0.98	[0.97, 0.99]	0.99	[0.98, 0.99]
ρ_I	AR(1) for invest. adjust. cost shocks	0.75	[0.67, 0.84]	0.64	[0.52, 0.75]
$100\sigma_A$	Std. dev. of technology shocks	0.90	[0.82, 0.98]	0.86	[0.79, 0.93]
$100\sigma_\beta$	Std. dev. of beta preference shocks	7.51	[6.70, 8.27]	2.51	[2.13, 2.92]
$100\sigma_\ell$	Std. dev. of labor preference shocks	2.44	[1.79, 3.10]	2.66	[2.16, 3.22]
$100\sigma_I$	Std. dev. of invest. adjust. cost shocks	16.8	[13.7, 19.8]	11.3	[8.83, 13.7]

Table 11: Posteriors of Non-policy Structural Parameters: FIRE vs. IIAU

Figure 10, 11 compare posteriors of the FIRE and IIAU models¹ and Table 11, 12 reports the means and [5%, 95%]-quantiles. Adding LFFU changes several structural parameters' estimates drastically. Perhaps most strikingly, the required intertemporal discount factor shocks(i.e., u_t^β 's) and investment adjustment cost shocks(i.e., u_t^I 's) are much smaller in the IIAU unanchored model: LFFU decreases' the AR(1) coefficient(i.e., ρ_β)'s posterior mean from 0.64 in FIRE to 0.30, and the standard deviations(i.e., $100\sigma_\beta$)'s posterior mean from 7.51 to 2.51.² The estimated $100\sigma_I$'s posterior mean decreases from 16.8 in the FIRE model to 11.3 in the IIAU model. The estimated ρ_I is also smaller in the IIAU model. The IIAU unanchored model also indicates a smaller investment adjustment cost coefficient s'' , 6.45 compared to 11.1 in FIRE, and slightly smaller capacity utilization parameter. It seems like the FIRE model underestimates the balanced growth rate.³The FIRE model's 90% posterior interval of $100g_A$ is [0.34, 0.48], leading to an annual balanced growth rate between 1.37% and 1.93%. The estimated $100g_A$ in the IIAU unanchored model is [0.49, 0.55], yielding annual balanced growth rates between 1.97% and 2.22%, which are values closer to the calibrated 2% annual balanced growth rate commonly used in the literature.

Speaking of fiscal parameters' estimates, the main differences between FIRE and IIAU models are concentrated on output and debt response coefficients. An interesting pattern arises: While the FIRE model suggests strong countercyclical tax rates responses to output gap(i.e., large $\varphi_{\tau_K}, \varphi_{\tau_L}$), the IIAU model says capital taxes don't respond to output gap that strongly and no response of labor tax rates(i.e., $\varphi_L \approx 0$). On the other hand, the IIAU model implies slightly larger countercyclical spending coefficients φ_G and φ_Z . At the same time, the IIAU model suggests much weaker debt responses, i.e., smaller γ_X 's for all $X = \{\tau_K, \tau_L, G, Z\}$ compared to the FIRE model. Ignoring LFFU thus could lead to biased opinion toward how fiscal policy stabilizes debt.

¹For common parameters, the differences of posteriors between the IIAU unanchored and the IIAU unanchored with only $\sigma_\eta^Z > 0$ are negligible.

²In Leeper et al. (2010) the 90% posterior credible intervals for ρ_β , $100\sigma_\beta$ are [0.62, 0.69] and [6.4, 7.7].

³Such an underestimation can also be seen in Leeper et al. (2017). Their posterior means of $100g_A$ are between 0.25 and 0.35 across different model specifications and can be found in the Online appendix.

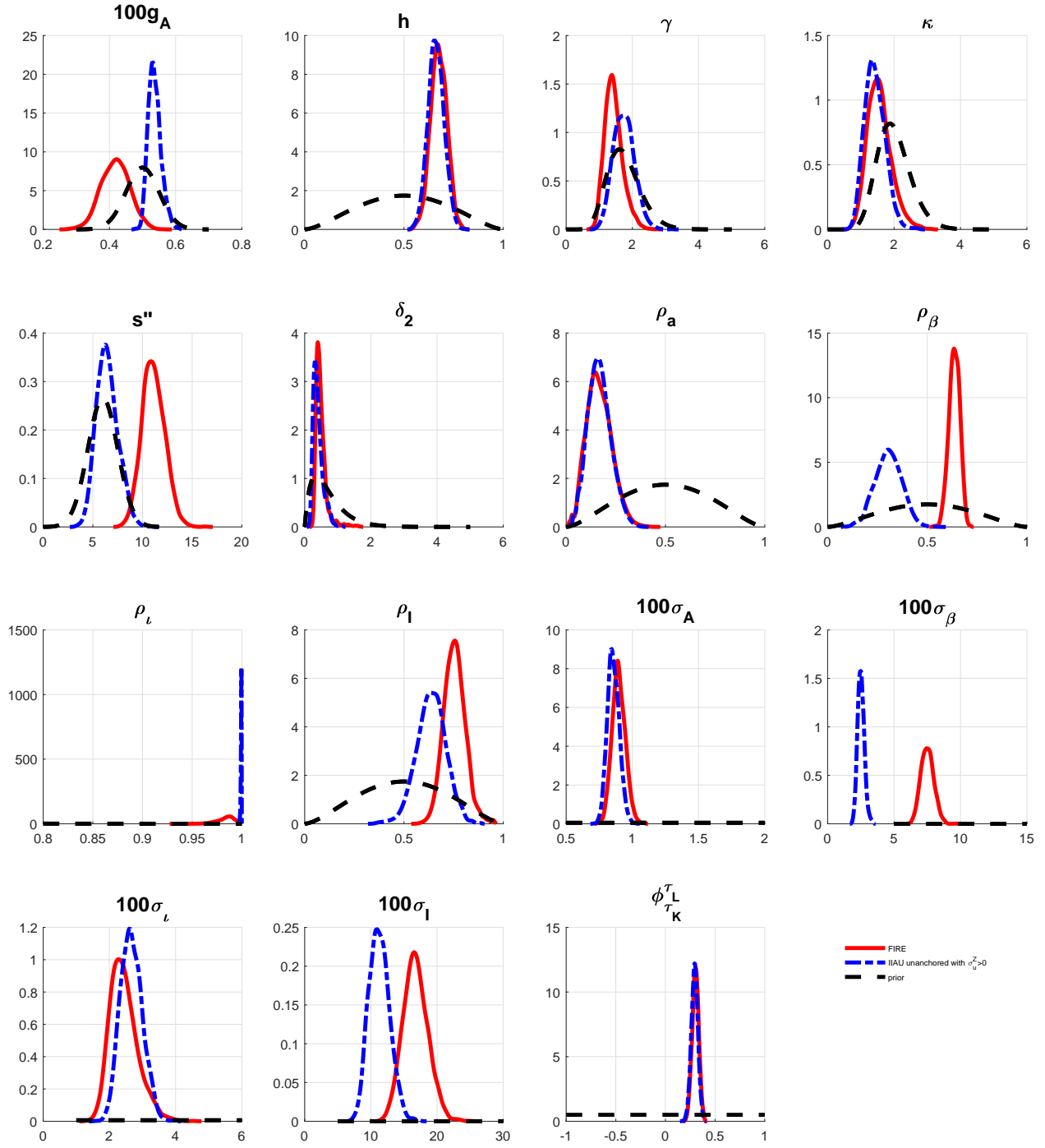


Figure 10: Prior versus posterior distributions of the FIRE model and the IIAU unanchored model with only $\sigma_\eta^Z > 0$: Non-policy parameters and the tax comovement parameter $\phi_{\tau_K}^{\tau_L}$; Parameters are estimated using full sample period from 1966Q2 to 2017Q3.

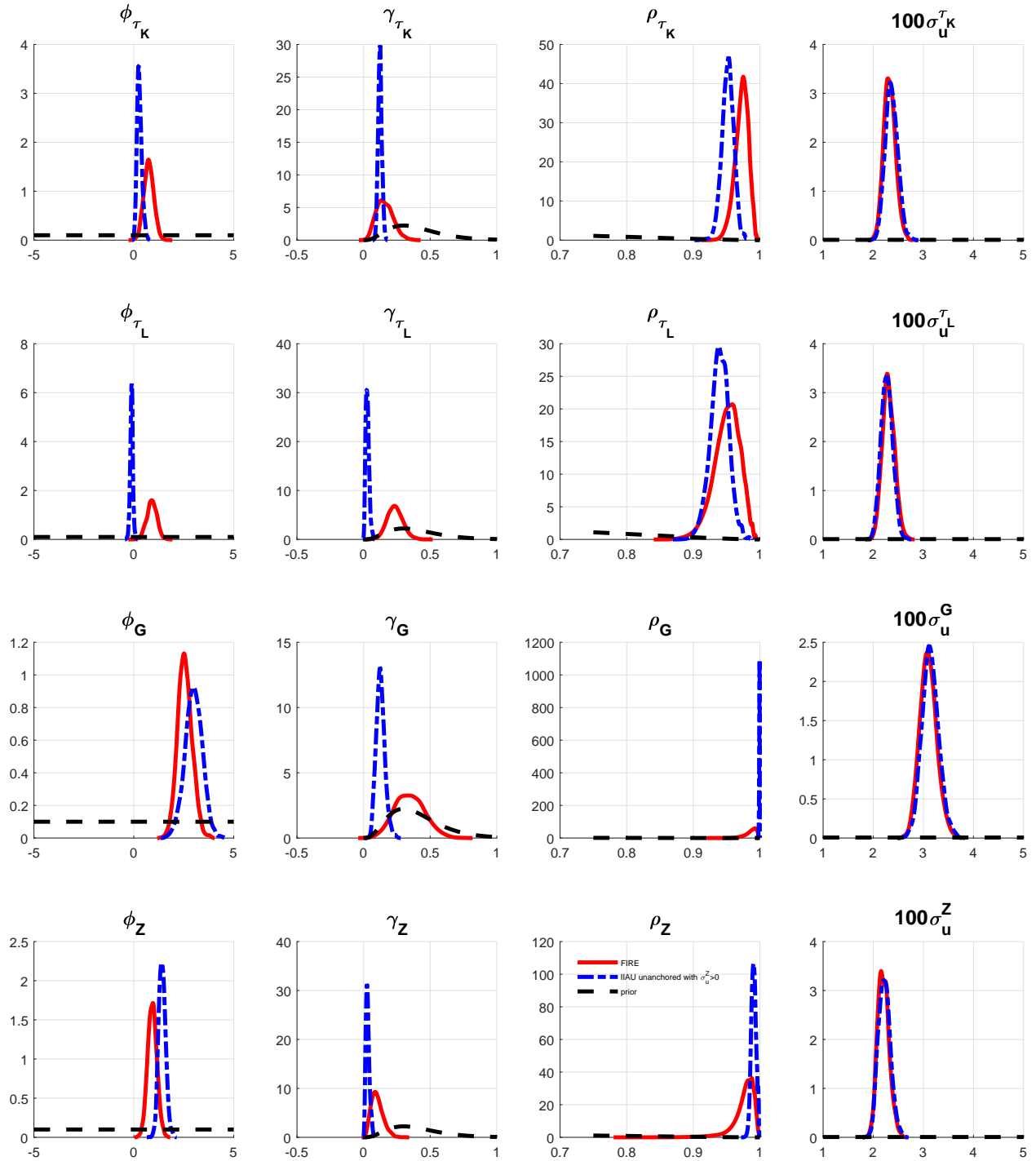


Figure 11: Prior versus posterior distributions of the FIRE model and the IIAU unanchored model with only $\sigma_\eta^Z > 0$: Fiscal policy parameters; The first column plots output response coefficients, the second column plots debt response coefficients. The third and fourth columns plot AR(1) coefficients and standard deviations of fiscal shocks; Parameters are estimated using full sample period from 1966Q2 to 2017Q3.

Mean and [5%, 95%] of posterior distribution				
	FIRE		IIAU	
Tax Co-movement Parameter				
ϕ_{TK}^{TL}	0.30	[0.25, 0.36]	0.26	[0.21, 0.30]
Output Response Parameters				
φ_{TK}	0.75	[0.34, 1.15]	0.28	[0.08, 0.47]
φ_{TL}	0.90	[0.51, 1.30]	-0.12	[-0.21, 0.01]
φ_G	2.56	[1.99, 3.15]	2.99	[2.29, 3.69]
φ_Z	0.92	[0.56, 1.28]	1.42	[1.15, 1.68]
Debt Response Parameters				
γ_{TK}	0.16	[0.06, 0.26]	0.12	[0.10, 0.15]
γ_{TL}	0.24	[0.14, 0.33]	0.03	[0.01, 0.05]
γ_G	0.35	[0.16, 0.52]	0.13	[0.07, 0.17]
γ_Z	0.10	[0.03, 0.17]	0.03	[0.01, 0.05]
AR(1) Coeff. of Persistent Components				
ρ_u^{TK}	0.97	[0.96, 0.99]	0.95	[0.94, 0.97]
ρ_u^{TL}	0.95	[0.92, 0.98]	0.94	[0.91, 0.96]
ρ_u^G	0.99	[0.97, 0.99]	0.99	[0.99, 0.99]
ρ_u^Z	0.98	[0.96, 0.99]	0.99	[0.98, 0.99]
Std. Dev. of Fiscal Shocks				
$100\sigma_\varepsilon^{TK}$	2.33	[2.14, 2.52]	2.37	[2.17, 2.57]
$100\sigma_\varepsilon^{TL}$	2.30	[2.10, 2.49]	2.27	[2.08, 2.46]
$100\sigma_\varepsilon^G$	3.10	[2.83, 3.38]	3.14	[2.87, 3.41]
$100\sigma_\varepsilon^Z$	2.20	[2.03, 2.39]	2.22	[2.04, 2.41]
Belief Parameters				
$\sigma_\eta^Z/\sigma_\varepsilon^Z$	NA	NA	0.45	[0.33, 0.58]

Table 12: Posteriors of Fiscal Policy Parameters: FIRE *vs.* IIAU

4.3 Impulse responses to fiscal shocks

This section compares impulse responses of output to fiscal shocks between FIRE and IIAU models conditional on different perceived fiscal targets $\mathcal{FT}_{1|0}^*$.

4.3.1 Correct initial belief: $\mathcal{FT}_{1|0}^* = \mathcal{FT}^*$

Figure 12 plots both FIRE and IIAU models' impulse responses following a temporary one standard deviation exogenous increase in each fiscal instrument. Households' initial beliefs in the IIAU model, i.e., the perceived fiscal targets $\mathcal{FT}_{1|0}^*$, are fixed at the true fiscal targets \mathcal{FT}^* . When initial beliefs are correct, Figure 12 suggests both models imply qualitatively similar but quantitatively different short-run (between 0 and 5 years) output dynamics. From short run to medium run, the differences are larger. While in the FIRE model tax increases (both capital and labor) remain contractionary in the medium run, in the IIAU model the posterior intervals indicate they could become expansionary. In contrast, the IIAU model suggests deeper contractionary

effects of increases in government spending and transfers in the medium run.

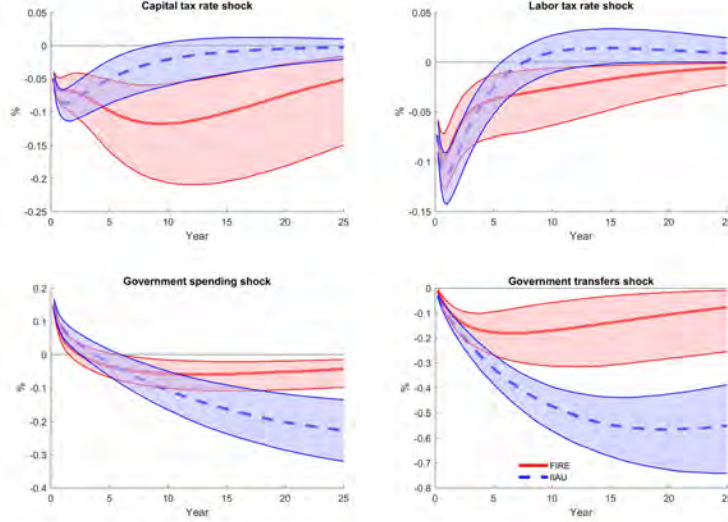


Figure 12: Impulse response functions of output to a $1-\sigma_\varepsilon^X$ positive shock ε_t^X for $X = \{G, Z, \tau_K, \tau_L\}$; The single shock hits the economy at quarter 1. The initial perceived fiscal targets \mathcal{FT}_{10}^* in the IIAU model are fixed at the true fiscal targets \mathcal{FT}^* . The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior quantiles. The x -axis measures years.

The different output dynamics in Figure 12 come from two sources: various structural parameters implied by different posteriors and the underlying transmission mechanism of LFFU, that is, the interplay between fiscal shocks and perceived fiscal targets and the subsequent different expectation formation processes.¹ To see their roles more clearly, recall in the IIAU unanchored model with only $\sigma_\eta^Z > 0$ we have imposed $\sigma_\eta^{\tau_K} = \sigma_\eta^{\tau_L} = \sigma_\eta^G = 0$: When there are no current and subsequent transfers shocks, households won't adjust beliefs on transfers target. This eliminates households' motivations to change beliefs on other fiscal targets as they are convinced all capital tax, labor tax, and government spending shocks are transitory. Consequently, households won't adjust perceived debt target. It follows the first three panels of Figure 12 purely reflect the effects of different posteriors: If we discard the belief parameter σ_η^Z and use the remaining IIAU posteriors to simulate the FIRE economy, we will get identical IIAU impulse responses.

With $\sigma_\eta^Z > 0$ and a temporary ε_t^Z -shock, households will misinterpret part of transitory movements in government transfers as permanent. This leads to adjustments of their perceived u_t^Z and fiscal targets, which govern households' expectations of fiscal financing in the short run and in the long run. The last panel of Figure 12 shows the combined effects of different posteriors and the transmission mechanism. To separate the effects, Figure 13 plots output's impulse responses to a one standard deviation transfers shock ε_t^Z of the FIRE economy using the IIAU posteriors. With common IIAU posteriors, Figure 13 shows the FIRE and IIAU models implies almost identical im-

¹Eventually, the different posteriors are also originated from LFFU. Without adding LFFU, all IIAU models degenerate to FIRE and we won't be able to recover the IIAU posteriors.

pulse responses in the short run. Over time, the persistent transfers shock is more contractionary in the IIAU model.

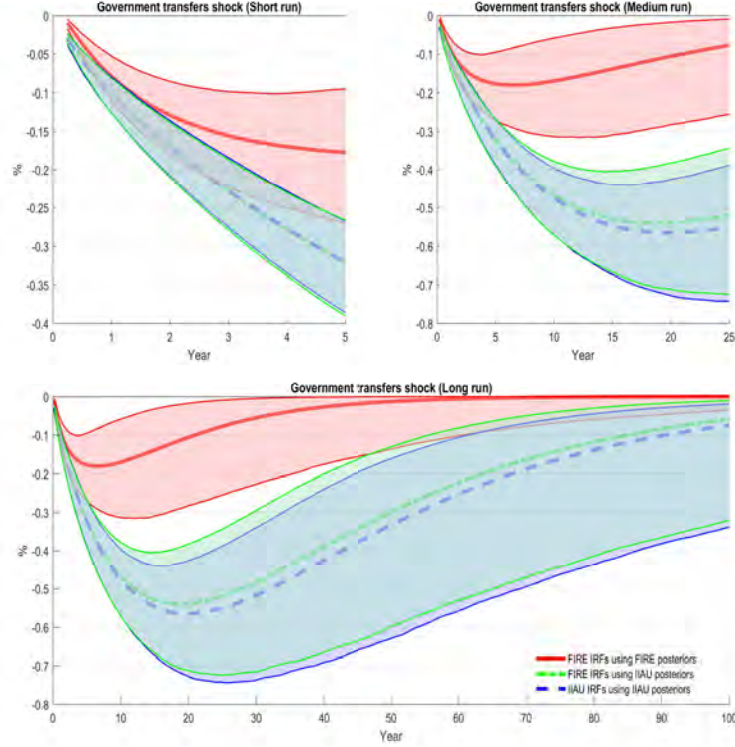


Figure 13: Impulse response functions of output to a one standard deviation government transfers shock; The single shock hits the economy at quarter 1. The initial perceived fiscal targets $\mathcal{FT}_{1|0}^*$ in the IIAU model are fixed at the true fiscal targets \mathcal{FT}^* . The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior intervals. The x -axis measures years.

The differences in FIRE and IIAU impulses responses using the common IIAU posteriors are due to both misspecified short-run and long-run expectations. Figure 14 plots the impulse responses of perceived transfer-to-GDP target $ZY_{t|t}^*$ and AR(1) component $u_{t|t}^Z$ in the IIAU model, along with actual realized ZY_t and u_t^Z to a $1-\sigma_\varepsilon^Z$ transfers shock. The left panel of Figure 14 also plots actual transfer-to-GDP target, ZY^* . Since ZY^* is not calibrated in Table 3 and depends on the underlying parameters, ZY^* in Figure 14 is represented as a band, not a line. The narrow width of the band indicates ZY^* is not very sensitive to estimated structural parameters. Given a positive ε^Z -shock, the left panel of Figure 14 shows households initially overestimate ZY^* by 0.2%-0.4%. Along with ZY_t , the perceived transfer-to-GDP target $ZY_{t|t}^*$ converges to ZY^* , but rather slowly. The rate of convergence of $ZY_{t|t}^*$ is smaller than ZY_t 's. Households' short-run expectation of the current AR(1) transfer shock is governed by $u_{t|t}^Z$. The right panel of Figure 14 suggests households correctly learn the positive sign of the transfer innovation but always underestimate the AR(1) transfers shocks u_t^Z . As time passes by, they misinterpret positive u_t^Z 's as negative $u_{t|t}^Z$'s, a pattern we have seen in Figure 1 in the simple model section. In contrast to the simple model, an underestimation of $u_{t|t}^Z$ doesn't imply a one-to-one overestimation of $\log(ZY_{t|t}^*)$. Rather, it implies a one-to-one

overestimation of $\Omega_{t|t}^Z$, which can be written explicitly as $\log(ZY_{t|t}^*) + \varphi_Z y_{t|t}^* + \gamma_Z \log(BY_{t|t}^*)$.¹

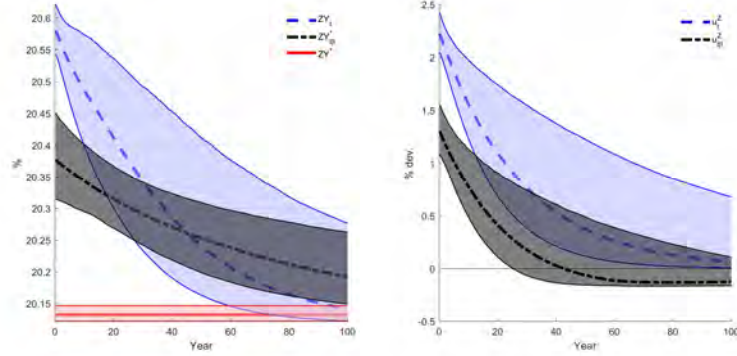


Figure 14: Impulse responses of actual realized transfers-to-GDP ratios *vs.* perceived transfers-to-GDP targets and actual AR(1) transfers shocks u_t^Z *vs.* perceived transfers shocks $u_{t|t}^Z$ to a one standard deviation government transfers shock; The single shock hits the economy at quarter 1. The initial perceived fiscal targets $\mathcal{FT}_{1|0}^*$ in the IIAU model are fixed at the true fiscal targets \mathcal{FT}^* . The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior quantiles. The x -axis measures years.

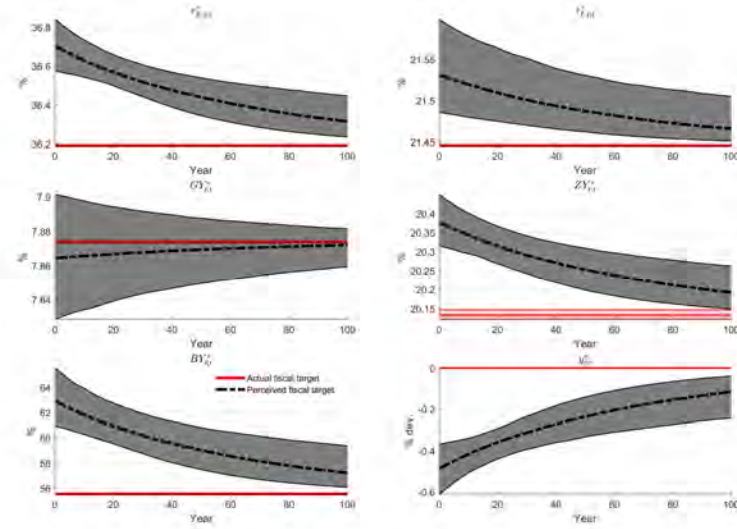


Figure 15: Impulse responses of perceived fiscal targets $\mathcal{FT}_{t|t}^*$ and perceived steady-state output (detrended) $y_{t|t}^*$ to a one standard deviation government transfers shock; The single shock hits the economy at quarter 1. The initial perceived fiscal targets $\mathcal{FT}_{1|0}^*$ in the IIAU model are fixed at the true fiscal targets \mathcal{FT}^* . The medium lines are means and the boundary lines are the 5% and 95% posterior quantiles. The red lines are actual fiscal targets. The x -axis measures years.

What happened to $y_{t|t}^*$, $BY_{t|t}^*$, and other perceived fiscal targets in $\mathcal{FT}_{t|t}^*$? Figure 15 plots the impulse responses of $\mathcal{FT}_{t|t}^*$ and $y_{t|t}^*$ ² to a $1-\sigma_\varepsilon^Z$ transfers shock. A persistent yet transitory,

¹ $y_{t|t}^*$ is households' time- t perceived detrended steady-state log output that is consistent with households' time- t perceived fiscal targets $\mathcal{FT}_{t|t}^*$.

² Due to the random walk property of Ω_t^X 's implied by equation (30), $\mathcal{FT}_{t+1|t}^* = \mathcal{FT}_{t|t}^*$ and $y_{t+1|t}^* = y_{t|t}^*$.

positive transfers shock increases perceived transfers-to-GDP target(level) between 0.2% and 0.4%, slightly increases(and decreases) perceived labor tax target(and government spending target). The perceived capital tax target jumps about 0.5% on impact. The perceived debt-to-GDP ratio is most volatile: The 90% confidence intervals show perceived debt-to-GDP target could rise from calibrated 55.5% to values between 61% and 66%. These perceived fiscal targets imply the detrended steady-state log output decrease by 0.3%-0.6%.

The initial large drop in the perceived steady-state output $y_{t|t}^*$ may seem surprising, given the actual realized output y_t doesn't decline that much until 4-5 years later(See Figure 13). This is because $y_{t|t}^*$ measures households' belief of detrended output in the distant future. The forward-looking nature of the model structure allows $y_{t|t}^*$ to display larger variations than y_t .

Figure 15 shows perceived targets converge to the true targets over time. The relationships between perceived targets and realized fiscal series are more intertwined. While households learn perceived targets from the observed fiscal series, the perceived targets also impact households' expectation formation and in consequence, their decision making. It follows the equilibrium outcomes, i.e. output and debt-to-GDP ratios, depend on households' perceived targets. Since endogenous fiscal rules respond to output and debt-to-GDP ratios, the realized fiscal series also depend on the perceived targets. Without subsequent shocks, the perceived targets and realized fiscal series share similar long-run behaviour and converge back to the actual target. However, the rate of convergence of the perceived targets is always smaller than the realized fiscal series. The left panel of Figure 14 compares $ZY_{t|t}^*$ and ZY_t . Figure 16 re-illustrates the point by comparing the perceived targets and realized series for other fiscal variables.

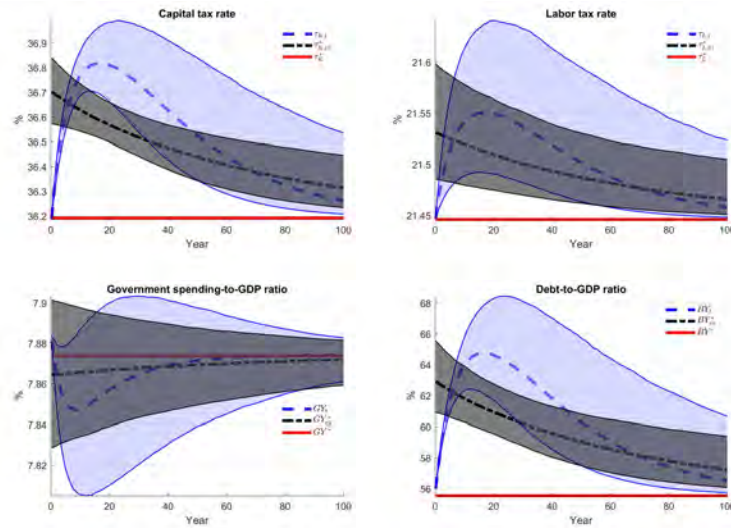


Figure 16: Impulse responses of perceived fiscal targets $\mathcal{FT}_{t|t}^*$ and realized fiscal series to a one standard deviation government transfers shock; The single shock hits the economy at quarter 1. The initial perceived fiscal targets $\mathcal{FT}_{1|0}^*$ in the IIAU model are fixed at the true fiscal targets \mathcal{FT}^* . The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior quantiles. The red lines are actual fiscal targets. The x -axis measures years.

4.3.2 Incorrect initial belief: $\mathcal{FT}_{1|0}^* \neq \mathcal{FT}^*$

So far we have assumed the initial belief $\mathcal{FT}_{1|0}^*$ matches \mathcal{FT}^* . What if $\mathcal{FT}_{1|0}^*$ has already deviated away from \mathcal{FT}^* ? This section focuses on impulse responses of output to fiscal shocks conditional on incorrect initial beliefs $\mathcal{FT}_{1|0}^* \neq \mathcal{FT}^*$. Non-belief variables' initial values are still fixed at their deterministic steady states.

The model structure imposes restrictions on what valid $\mathcal{FT}_{1|0}^*$'s we could consider. In particular, since in the IIAU unanchored model data prefers only $\sigma_\eta^Z > 0$, we only need to consider two belief state variables, $\Omega_{t+1|t}^Z$ and $u_{t|t}^Z$.¹ Rewriting households' perceived transfer rule as²

$$\Omega_Z^* + u_t^Z = \Omega_{t|t}^Z + u_{t|t}^Z = \Omega_{t+1|t}^Z + u_{t|t}^Z \quad (37)$$

gives a restriction belief variables have to satisfy at any time. Figure 17 plots the joint distribution of $(u_0^Z, \Omega_{1|0}^Z - \Omega_Z^*)$ after simulating the IIAU economy a large number of times. The correlation of u_0^Z and $\Omega_{1|0}^Z - \Omega_Z^*$ is 0.98. Given the economy is at its deterministic steady state at $t = 0$, we impose $u_0^Z = 0$. The 90% quantile of $\Omega_{1|0}^Z - \Omega_Z^*$ conditional on $u_0^Z = 0$ is $[-0.047, 0.055]$. I choose $\Omega_{1|0}^Z - \Omega_Z^* = \pm 0.02$ to illustrate the impacts of incorrect beliefs $\mathcal{FT}_{1|0}^*$.

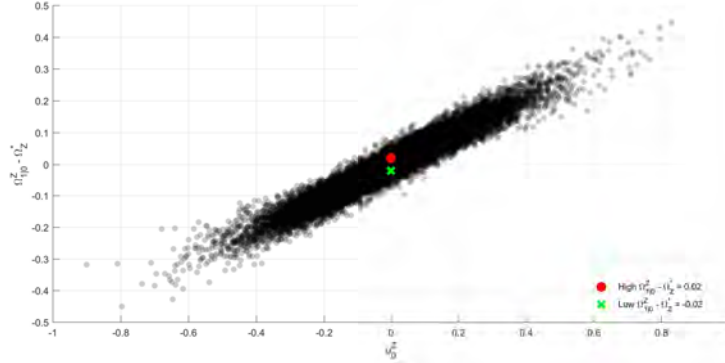


Figure 17: Simulated joint distribution of $(u_0^Z, \Omega_{1|0}^Z - \Omega_Z^*)$ at $t = 0$. The IIAU economy is simulated 1 million times. Each simulation randomly draws parameters from posteriors and is of length quarters. The initial state of the economy is at its deterministic steady state at $t = -200$. The initial perceived fiscal target \mathcal{FT}_{-200}^* is fixed at true target \mathcal{FT}^* .

Given parameters households know³, there is a one-to-one mapping between $\Omega_{1|0}^Z$ and $\mathcal{FT}_{1|0}^*$. Figure 18 plots the simulated distributions of $\mathcal{FT}_{1|0}^*$ along with perceived detrended (log) steady-state output $y_{1|0}^*$, conditional on either high or low $\Omega_{1|0}^Z - \Omega_Z^*$ values. A positive $\Omega_{1|0}^Z - \Omega_Z^*$ implies relatively high (mis-)perceived capital and labor tax rates, transfers, debt targets and a low government spending target. It follows the implied perceived detrended steady-state output is relatively low. A negative $\Omega_{1|0}^Z - \Omega_Z^*$, on the other hand, implies a relatively high steady-state output. Since

¹Given $\sigma_\eta^{\tau K} = \sigma_\eta^{\tau L} = \sigma_\eta^G = 0$, it follows other belief state variables $\Omega_{t|t}^{\tau K} \equiv \Omega_{\tau K}^*$, $\Omega_{t|t}^{\tau L} \equiv \Omega_{\tau L}^*$, $\Omega_{t|t}^G \equiv \Omega_G^*$ are constant and perceived AR(1) components match actual AR(1) components, i.e., $u_{t|t}^{\tau K} = u_t^{\tau K}$, $u_{t|t}^{\tau L} = u_t^{\tau L}$, $u_{t|t}^G = u_t^G$.

²The constant Ω_Z^* is similarly defined as $\Omega_{t|t}^Z$ with perceived targets replaced by actual targets.

³That is, $\mathcal{P} \setminus \{\tau_K^*, \tau_L^*, GY^*, ZY^*, BY^*\}$. See Table 4.

$y_{1|0}^*$ describes households' perceived potential output (i.e., natural level of output), we loosely define households are “optimistic” when $y_{1|0}^* > y^*$ and “pessimistic” when $y_{1|0}^* < y^*$.

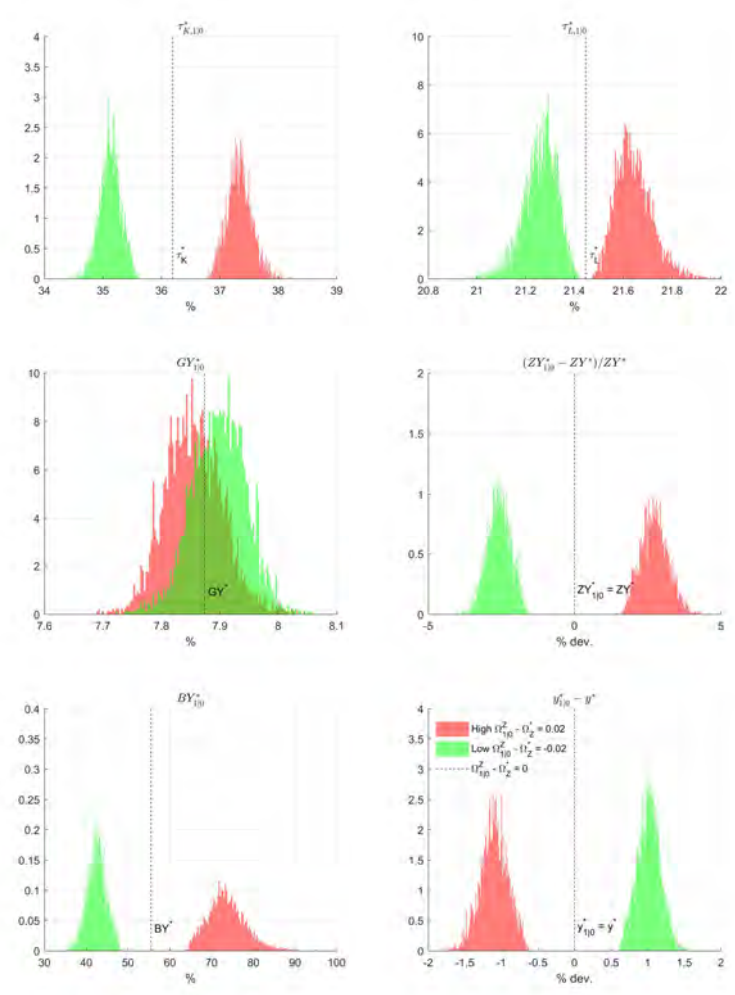


Figure 18: Simulated distributions of $\mathcal{FT}_{1|0}^*$ and $y_{1|0}^*$ conditional on $\Omega_{1|0}^Z - \Omega_Z^* = \pm 0.02$. Each simulation randomly draws parameters from posteriors and the perceived steady states are simulated 10 million times in total.

Figure 19 plots impulse responses of output to different fiscal shocks conditional on both high and low $\Omega_{1|0}^Z - \Omega_Z^*$'s. To isolate the interaction between transfers shocks and perceived targets, I focus on the other three fiscal instruments: capital taxes, labor taxes, and government spending. Both expansionary and contractionary fiscal policies are considered. Non-zero $\Omega_{1|0}^Z - \Omega_Z^*$'s introduce state-dependent fiscal effects and a common pattern emerges across all three fiscal instruments: A fiscal shock, regardless of its sign, is always more expansionary if households are optimistic, and is always more contractionary if households are pessimistic. The implied fiscal impacts are

asymmetric: If effects of fiscal shocks were symmetric, then the lower panel of Figure 19 should be the mirror image of the upper panel.

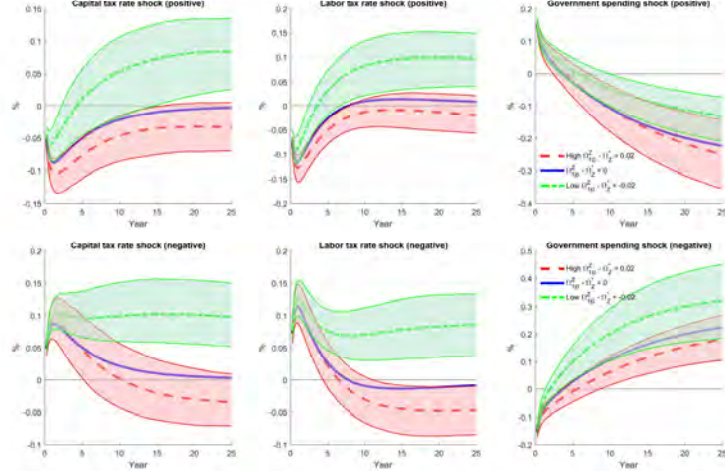


Figure 19: Impulse responses of output to a $1\text{-}\sigma_{\varepsilon}^X$ positive (Upper Panel) and negative (Lower Panel) shock ε_t^X for $X = \{\tau_K, \tau_L, G\}$ conditional on different $\Omega_{1|0}^Z - \Omega_Z^*$'s; The single shock hits the economy at quarter 1. The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior quantiles. The blue solid lines are the median impulse responses when $\Omega_{1|0}^Z - \Omega_Z^* = 0$. The x -axis measures years.

To further highlight the state dependency, Figure 20 plots present-value multipliers for output of each fiscal shock. Following Mountford and Uhlig (2009), the present value of additional output over a k -period horizon generated by a change in the present value of government spending can be calculated as

$$\text{Present-Value Multiplier}(k) = \frac{\sum_{j=0}^k \left(\prod_{i=0}^j R_{t+i}^{-1} \right) \Delta Y_{t+j}}{\sum_{j=0}^k \left(\prod_{i=0}^j R_{t+i}^{-1} \right) \Delta G_{t+j}} \quad (38)$$

Other fiscal multipliers are defined similarly.¹ If effects of expansionary and contractionary fiscal shocks were symmetric, then calculations of fiscal multipliers would be independent of the sign of these shocks. Quite contrary, Figure 20 shows over the medium to long term, signs of tax multipliers reversed once switching from a tax raise to a tax cut. Furthermore, tax multipliers vary significantly across states (i.e., different $\Omega_{1|0}^Z - \Omega_Z^*$'s). This is consistent with Sims and Wolff (2018)'s findings, who also generates a wide range of tax multipliers but conditional on different states of output.

¹The tax multipliers are defined as the the present value of additional output generated by a change in the present value of corresponding tax revenues.

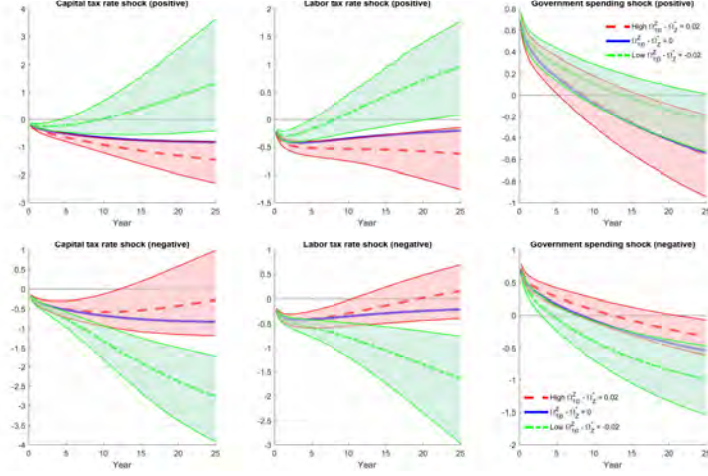


Figure 20: Fiscal multipliers of output to a $1-\sigma_\varepsilon^X$ positive (Upper Panel) and negative (Lower Panel) shock ε_t^X for $X = \{\tau_K, \tau_L, G\}$ conditional on different $\Omega_{1|0}^Z - \Omega_Z^*$'s; The single shock hits the economy at quarter 1. The x -axis measures years.

What causes more expansionary fiscal impacts under optimism regardless of the sign of the shock? Conditional on different $\Omega_{1|0}^Z - \Omega_Z^*$'s, Figure 21 tracks movements of $\Omega_{t|t}^Z - \Omega_Z^*$, $u_{t|t}^Z$ and output when no shocks hit the economy. When initial belief of fiscal targets is correct (i.e., $\Omega_{1|0}^Z - \Omega_Z^* = 0$) and there are no shocks, the impulse responses are trivially zero as the economy stays at its deterministic steady state at $t = 0$. Nonzero $\Omega_{1|0}^Z - \Omega_Z^* = 0$'s, however, suggest households don't believe the economy reaches its potential output. Similar to Figure 7 in Section 2, there are two offsetting forces: While $\Omega_{1|0}^Z - \Omega_Z^* < 0$ implies a optimistic $y_{1|0}^* > y^*$ so that output tends to increase from y^* to $y_{1|0}^*$, the restriction (37) implies $u_{t|t}^Z > 0$ and must be the inverse of $\Omega_{1|0}^Z - \Omega_Z^*$'s given $u_t^Z \equiv 0$. Given higher transfers policy is contractionary in the RBC model, output will decrease given positive $u_{t|t}^Z$'s. While in Section 2 the net effect depends on the fiscal financing parameter w_G , here the estimated parameters determine the fiscal financing scheme and imply the expansionary effect dominates the contractionary effect, generating more expansionary fiscal effects as we have seen in the last panel of Figure 21. Over time $\Omega_{t|t}^Z - \Omega_Z^*$ and $u_{t|t}^Z$ gradually converge back to zero. Due to the extremely high persistence of u_t^Z ($\rho_u^Z \approx 0.99$), the convergence takes more than a century.

A well-known result in the literature is that the RBC model and standard New Keynesian DSGE models cannot generate government spending multipliers larger than unity. A prominent example is Leeper et al. (2017), who uses prior predictive analysis and shows the probability of $PV(\Delta Y/\Delta G) > 1$ is 0 across all horizons in a similar RBC model and several New Keynesian models with active monetary policy and passive fiscal policy.¹ More recently, Ramey (2019) summarizes a large body of both empirical and theoretical studies and concludes government purchases multipliers are likely to be between 0.6 and 1 over horizons between 0 and 20 quarters.² The last column of Figure 21 suggests similar results. However, adding LFFU can help generate large multipliers.

¹See Leeper (1991) for how active/passive monetary and fiscal policies are defined.

²See Table 1 of Ramey (2019).

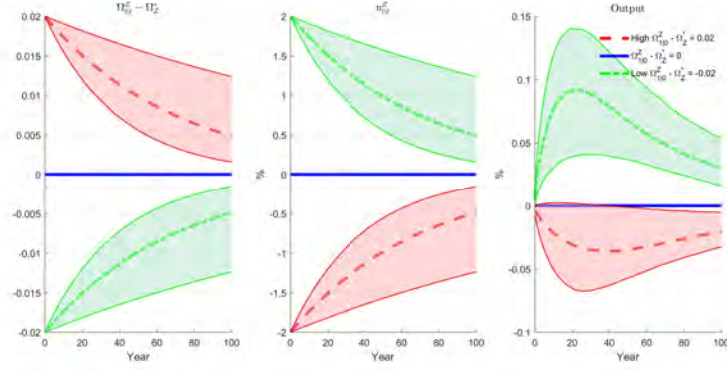


Figure 21: Impulse responses of $\Omega_{t|t}^Z - \Omega_Z^*$, $u_{t|t}^Z$ and output conditional on different $\Omega_{1|0}^Z - \Omega_Z^*$'s when the economy is subject to no shocks; The middle lines are median impulse responses; the boundary lines are the 5% and 95% posterior quantiles. The blue solid lines are the median impulse responses when $\Omega_{1|0}^Z - \Omega_Z^* = 0$. The x -axis measures years.

Figure 22 plots both 2-year and 10-year multipliers of output to either a positive or negative government spending shock of different sizes. When size of the shock is small, a positive ε_1^G -shock under optimism (i.e., $\Omega_{1|0}^Z - \Omega_Z^* < 0$), or a negative ε_1^G -shock under pessimism (i.e., $\Omega_{1|0}^Z - \Omega_Z^* > 0$) can generate multipliers larger than unity at both 2-year and 10-year horizons. In contrast, when $\Omega_{1|0}^Z - \Omega_Z^* = 0$ and there are no subsequent transfers shocks so that the IIAU model becomes FIRE, multipliers are strictly less than one.

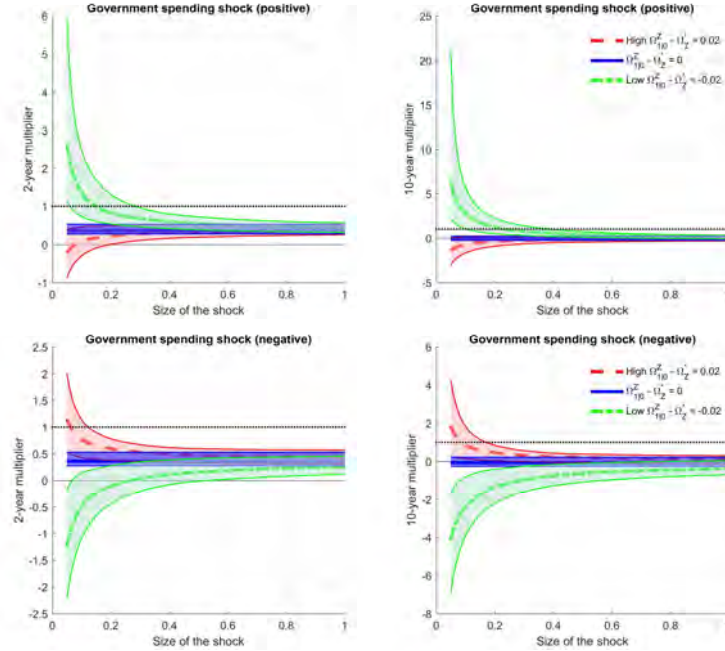


Figure 22: Government spending multipliers of output to a $k\sigma_\varepsilon^G$ positive (Upper Panel) and negative (Lower Panel) shock ε_1^G conditional on different $\Omega_{1|0}^Z - \Omega_Z^*$'s; The x -axis measures size of the shock k and the y -axis measures either 2-year (Left) or 10-year (Right) multipliers.

A closer look at the present-value multiplier definition (38) helps explain why large multipliers are possible under LFFU when ε^G -shocks are small. When $\Delta G_t \equiv 0$ for all t , Figure 21 shows $\Delta Y_t > 0$ under optimism and $\Delta Y_t < 0$ under pessimism. These are fluctuations purely driven by changing beliefs(i.e., $\Omega_{t|t}^Z - \Omega_Z^*$'s and $u_{t|t}$'s), rather than any materialized government purchases. It follows multipliers defined by (38) can either approach $+\infty$ or $-\infty$ in the limit when government spending shocks are small and ΔG_t approaches 0. Indeed, Figure 22 shows as size of the shock goes to 0¹, multipliers can vary significantly. This points to how to identify exogenous variations(i.e., size and sign) in fiscal policy, a long-standing issue pertinent to empirical fiscal studies.

So far we have focused on output impulse responses and fiscal multipliers to a single shock and highlighted the state-dependency of fiscal calculations due to LFFU. The economy, however, is subject to subsequent shocks. This raises caveats to medium- and long-run fiscal evaluations as impacts of fiscal policy now depend on households' beliefs at each time point. What do data tell us about these beliefs? The next section looks at filtered perceived fiscal targets, i.e. $\mathcal{FT}_{t|t}^*$, using the IIAU model's posteriors.

4.4 Filtered beliefs

Before getting started, it is worth mentioning the filtered series will depend on the assumptions we have made to the true fiscal targets \mathcal{FT}^* . In particular, while households inside the economy confront unknown fiscal targets directly, I assume econometricians who estimate models don't face LFFU by allowing them to calibrate actual fiscal targets to historical means \mathcal{FT}^* . This approach probably underestimates low-frequency fiscal uncertainty.² At the same time, without survey data that directly measure households' beliefs, it is also hard to check the validity of the filtered beliefs.

With these challenges in mind, Figure 23 plots filtered $\{\tau_{K,t|t}^*, \tau_{L,t|t}^*, GY_{t|t}^*, ZY_{t|t}^*\}$ since the mid-80s(the so called Great Moderation). I consider two sets of posteriors, the posterior for the entire sample(1966Q2-2017Q3) and the posterior for a subsample of 1985Q1-2017Q3. For each posterior, the initial perceived fiscal target, either $\mathcal{FT}_{1966Q2|1966Q1}^*$ or $\mathcal{FT}_{1985Q1|1984Q4}^*$, is characterized by a mean and an error covariance matrix. Means of both initial perceived targets are chosen to be the actual target \mathcal{FT}^* . The error covariance matrix, which characterizes households' confidence in their initial belief, is chosen to have large diagonal values, representing great uncertainty about the true value of initial beliefs.³

¹The lower bound of the shock size in Figure 22 is 0.05.

²Alternatives are to model actual fiscal targets as Markov switching processes(See Richter and Throckmorton (2015)) or to consider structural breaks(See Hollmayr and Matthes (2015)). While these alternatives may seem more realistic, they bring additional nonlinearities into the model structure and complicate the estimation substantially. Adding these nonlinearities will not change the main theme of LFFU: As long as perceived fiscal targets deviate from actual ones, households' expectation formation and decision making are altered.

³For details on how to calculate the error covariance matrix, see Section 13.2 of Hamilton (1994).

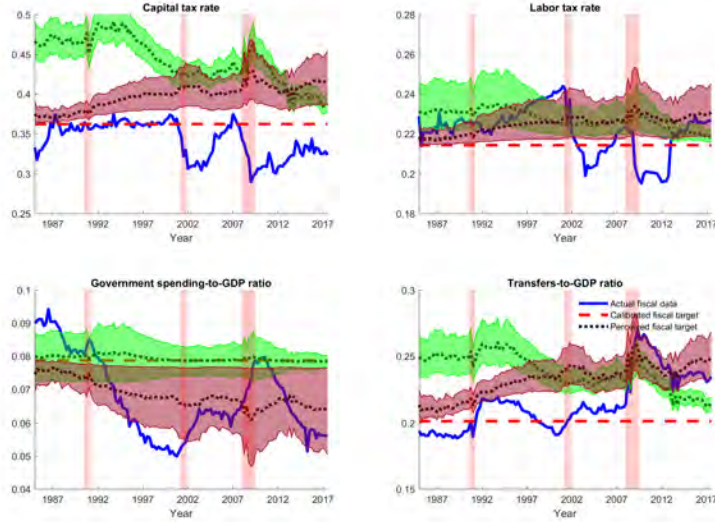


Figure 23: Filtered perceived fiscal targets $\mathcal{FT}_{t|t}^*$ since Great Moderation using either (i). Full sample data(1966Q2-2017Q3) posterior (Light green) or (ii). Subsample data(1985Q1-2017Q3) posterior (Dark red).

Figure 23 shows filtered $\tau_{L,t|t}^*$'s are fairly stable and are slightly higher(around 1%-2%) than the calibrated τ_L^* . While the full sample posterior indicates $GY_{t|t}^*$'s deviate little from GY^* , the subsample posterior suggests $GY_{t|t}^*$ may gradually decrease from calibrated 7.8% to 6.5% over the last three decades. Both the full sample and subsample posteriors indicates $\tau_{K,t|t}^*$'s and $ZY_{t|t}^*$'s are larger than their full information counterparts: The increasing transfers cause households to raise their perceived long-run transfers levels, they also believe it will be funded by higher long-run capital tax rates.

Concerns on relatively high public debt levels are at the center of fiscal discussions during and after the Great Recession. The left panel of Figure 24 shows households beliefs on debt target may rapidly increase during 2008-2009 due to large stimulus fiscal packages but subside from 2007 to 2013. While the full sample posterior suggests a tight perceived debt-to-GDP target interval of [80%, 150%] after 2013, the subsample posterior suggests a wide range of perceived debt targets between 350% and 1100%.

While data cannot pin down the magnitude of perceived debt targets, they do suggest recent perceived fiscal targets imply households' pessimism about the natural level of output. The right panel of Figure 24 plots percentage deviations of households' perceived steady-state output $y_{t|t-1}^*$ from true steady-state output y^* after 2007. While the full sample posterior filtered series suggests households are less pessimistic today than 10 years ago as $y_{t|t-1}^* - y^*$ increases, the Great Moderation subsample suggests this is not a robust results and households are as pessimistic, if not more, as they were one decade ago.

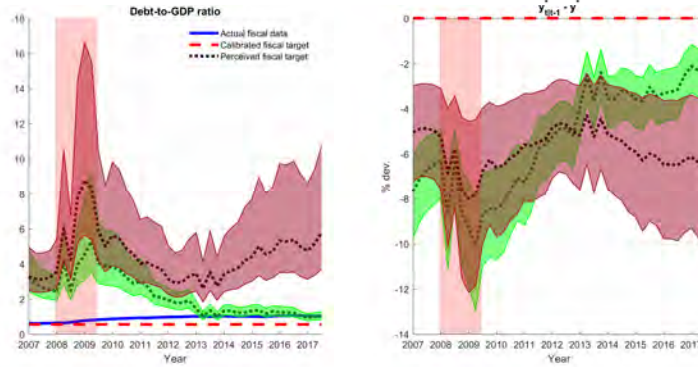


Figure 24: Filtered perceived debt target and $y_{t|t-1}^* - y^*$ (% deviation) from 2007Q1 to 2017Q3 using either (i). Full sample data period from 1966Q2 to 2017Q3 (Light green) or (ii). Subsample data period from 1985Q1 to 2017Q3 (Dark red).

5 Conclusion

This paper explores impacts of low-frequency fiscal uncertainty caused by unknown long-run fiscal levels. With population aging, no foreseeable reform plans of the social welfare programs and relatively high public debt levels, this type of uncertainty seems particularly relevant in today's policy discussions. Within a standard RBC framework, adding LFFU significantly improves a benchmark FIRE model's fit and implies much smaller real frictions. Depending on households' optimism and pessimism¹ regarding government finance in the long run, LFFU introduces state-dependent fiscal impacts and generate time-varying fiscal multipliers. Data point to the increasing transfer payments as the main driving force of low-frequency fiscal uncertainty. This paper isolates belief-driven fluctuations only caused by LFFU. How households' perceived fiscal targets interact with other business-cycle beliefs, for example, beliefs about permanent and transitory productivity shocks and/or monetary policy stance, is another interesting topic worth exploring. I focus on aggregate effects of fiscal policy and ignore intergenerational welfare implications of LFFU. Given both Social Security and Medicare are largely pay-as-you-go plans, a natural extension is to consider a overlapping generation model and quantify welfare benefits/costs of LFFU. Finally, I exclude monetary policy and ignore monetary-fiscal policy interactions. Government bond considered here thus are real assets and debt levels are independent of the recent nominal interest rate normalization. I leave these issues for future research.

¹Sims (2014) defines a similar Fiscal pessimism within the Fiscal Theory of Price Level and focuses on its implications for short-run inflation dynamics.

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Appendix A Solving the simple model

The FIRE and IIAU models (12), (13) are solved by method of undetermined coefficients by forming different sets of state variables. With a slight abuse of notation, define

$$A = \Phi_1(\tau^*), \quad B = \Phi_2(\tau^*), \quad C = \Phi_3(\tau^*), \quad D = \Phi_4(\tau^*), \quad E = \Phi_5(\tau^*), \quad F = \Phi_6(\tau^*)$$

in the FIRE model and

$$A = \Phi_1(\tau_{t|t}^*), \quad B = \Phi_2(\tau_{t|t}^*), \quad C = \Phi_3(\tau_{t|t}^*), \quad D = \Phi_4(\tau_{t|t}^*), \quad E = \Phi_5(\tau_{t|t}^*), \quad F = \Phi_6(\tau_{t|t}^*)$$

in the IIAU model where all τ^* have been replaced by $\tau_{t|t}^*$. In FIRE k_t solves

$$AE_t^{FIRE}[k_{t+1} - k^*] + B[k_t - k^*] + C[k_{t-1} - k^*] = Da_t + (E\rho_u + F)u_t \quad (\text{A.1})$$

while in IIAU k_t solves

$$AE_t^{IIAU}[k_{t+1} - k_{t|t}^*] + B[k_t - k_{t|t}^*] + C[k_{t-1} - k_{t|t}^*] = Da_t + (E\rho_u + F)u_{t|t} \quad (\text{A.2})$$

We guess the FIRE solution is given by

$$k_t = f_0 + f_1 k_{t-1} + f_2 a_t + f_3 u_t$$

It follows

$$\begin{aligned} E_t^{FIRE} k_{t+1} &= E_t^{FIRE} [f_0 + f_1 k_t + f_2 a_{t+1} + f_3 u_{t+1}] \\ &= f_0 + f_1 (f_0 + f_1 k_{t-1} + f_2 a_t + f_3 u_t) + f_2 \rho_a a_t + f_3 \rho_u u_t \end{aligned} \quad (\text{A.3})$$

Plugging (A.3) into (A.1) and matching the coefficients in front of 1, k_{t-1} , a_t and u_t give

$$A(f_0 + f_1 f_0 - k^*) + B(f_0 - k^*) - Ck^* = 0 \quad (\text{A.4})$$

$$A f_1^2 + B f_1 + C = 0 \quad (\text{A.5})$$

$$A(f_1 f_2 + f_2 \rho_a) + B f_2 = D \quad (\text{A.6})$$

$$A(f_1 f_3 + f_3 \rho_u) + B f_3 = E\rho_u + F \quad (\text{A.7})$$

Stationarity of k_t requires the root to the quadratic equation (A.5), f_1 , has a module less than 1. The other coefficients f_0, f_2, f_3 can be pinned down by solving the remaining linear equations.

We guess the IIAU solution is of the form

$$k_t = g_0 + g_1 k_{t-1} + g_2 a_t + g_3 u_{t|t}$$

It follows

$$\begin{aligned} E_t^{IIAU} k_{t+1} &= E_t^{IIAU} [g_0 + g_1 k_t + g_2 a_{t+1} + g_3 u_{t+1|t+1}] = g_0 + g_1 k_t + g_2 \rho_a a_t + g_3 u_{t+1|t} \\ &= g_0 + g_1 [g_0 + g_1 k_{t-1} + g_2 a_t + g_3 u_{t|t}] + g_2 \rho_a a_t + g_3 \rho_u \tilde{u}_{t|t} \end{aligned} \quad (\text{A.8})$$

Plugging (A.8) into (A.2) and matching the coefficients in front of 1, k_{t-1} , a_t and $u_{t|t}$ yield

$$A(g_0 + g_1 g_0 - k_{t|t}^*) + B(g_0 - k_{t|t}^*) - C k_{t|t}^* = 0 \quad (\text{A.9})$$

$$A g_1^2 + B g_1 + C = 0 \quad (\text{A.10})$$

$$A(g_1 g_2 + g_2 \rho_a) + B g_2 = D \quad (\text{A.11})$$

$$A(g_1 g_3 + g_3 \rho_u) + B g_3 = E \rho_u + F \quad (\text{A.12})$$

Comparing (A.9)-(A.12) to (A.4)-(A.7) immediately gives

$$g_0 = f_0(\tau_{t|t}^*), \quad g_1 = f_1(\tau_{t|t}^*), \quad g_2 = f_2(\tau_{t|t}^*), \quad g_3 = f_3(\tau_{t|t}^*)$$

Re-defining

$$\begin{aligned} \phi_0(\tau^*) &= f_0(\tau^*) - f_3(\tau^*), & \phi_1(\tau^*) &= f_1(\tau^*), & \phi_2(\tau^*) &= f_2(\tau^*) \\ \phi_0(\tau_{t|t}^*) &= g_0(\tau_{t|t}^*) - g_3(\tau_{t|t}^*), & \phi_1(\tau_{t|t}^*) &= g_1(\tau_{t|t}^*), & \phi_2(\tau_{t|t}^*) &= g_2(\tau_{t|t}^*) \end{aligned}$$

gives (14), (15) as desired.

A.1 Solving the simple model using (35) and (36)

To illustrate the algorithm used in the IIAU model estimation, I now solve the simple growth model (9) using the formulation of (35). Denote $\tau_{t|t-1}^*$ time- t perceived tax target updated at the end of time- $(t-1)$. Log-linearizing (9) around $\tau_{t|t-1}^*$ gives

$$\begin{aligned} &\Phi_1(\tau_{t|t-1}^*) E_t^{IIAU} [k_{t+1} - k_{t|t-1}^*] + \Phi_2(\tau_{t|t-1}^*) [k_t - k_{t|t-1}^*] + \Phi_3(\tau_{t|t-1}^*) [k_{t-1} - k_{t|t-1}^*] \\ &= \Phi_4(\tau_{t|t-1}^*) a_t + \Phi_5(\tau_{t|t-1}^*) E_t^{IIAU} (\tau_{t+1} - \tau_{t|t-1}^*) + \Phi_6(\tau_{t|t-1}^*) (\tau_t - \tau_{t|t-1}^*) \end{aligned} \quad (\text{A.13})$$

where all $\tau_{t|t}^*, k_{t|t}^*$'s in the original IIAU model equilibrium condition (13) have been replaced by $\tau_{t|t-1}^*, k_{t|t-1}^*$. We guess the solution to (A.13) is of the form

$$k_t = g_0 + g_1 k_{t-1} + g_2 a_t + g_3 (\tau_t - \tau_{t|t-1}^*) + g_4 u_{t|t}$$

It follows

$$\begin{aligned}
E_t^{IIAU} k_{t+1} &= E_t^{IIAU} [g_0 + g_1 k_t + g_2 a_{t+1} + g_3(\tau_{t+1} - \tau_{t+1|t}^*) + g_4 u_{t+1|t+1}] \\
&= g_0 + g_1 k_t + g_2 \rho_a a_t + g_3 E_t^{IIAU} (\tau_{t+1} - \tau_{t+1|t}^*) + g_4 u_{t+1|t} \\
&= g_0 + g_1 \left[g_0 + g_1 k_{t-1} + g_2 a_t + g_3(\tau_t - \tau_{t|t-1}^*) + g_4 u_{t|t} \right] + g_2 \rho_a a_t + (g_3 + g_4) \rho_u u_{t|t} \tag{A.14}
\end{aligned}$$

and

$$E_t^{IIAU} (\tau_{t+1} - \tau_{t|t-1}^*) = E_t^{IIAU} (\tau_{t+1}^* + u_{t+1} - \tau_{t|t-1}^*) = \tau_{t|t}^* - \tau_{t|t-1}^* + \rho_u u_{t|t} = (\tau_t - \tau_{t|t-1}^*) + (\rho_u - 1) u_{t|t} \tag{A.15}$$

Plugging (A.14), (A.15) into (A.13) and matching the coefficients in front of $1, k_{t-1}, a_t, \tau_t - \tau_{t|t-1}^*$ and $u_{t|t}$ yield

$$A(g_0 + g_1 g_0 - k_{t|t-1}^*) + B(g_0 - k_{t|t-1}^*) - C k_{t|t-1}^* = 0 \tag{A.16}$$

$$A g_1^2 + B g_1 + C = 0 \tag{A.17}$$

$$A(g_1 g_2 + g_2 \rho_a) + B g_2 = D \tag{A.18}$$

$$A g_1 g_3 + B g_3 = E + F \tag{A.19}$$

$$A(g_1 g_4 + (g_3 + g_4) \rho_u) + B g_4 = E(\rho_u - 1) \tag{A.20}$$

where A, B, C, D, E, F 's are defined as

$$A = \Phi_1(\tau_{t|t-1}^*), B = \Phi_2(\tau_{t|t-1}^*), C = \Phi_3(\tau_{t|t-1}^*), D = \Phi_4(\tau_{t|t-1}^*), E = \Phi_5(\tau_{t|t-1}^*), F = \Phi_6(\tau_{t|t-1}^*)$$

Comparing (A.16), (A.17), (A.18) to (A.9), (A.10), (A.11) immediately gives

$$g_0 = f_0(\tau_{t|t-1}^*), \quad g_1 = f_1(\tau_{t|t-1}^*), \quad g_2 = f_2(\tau_{t|t-1}^*)$$

Adding up (A.19), (A.20) and comparing it to (A.12) yield

$$g_3 + g_4 = f_3(\tau_{t|t-1}^*)$$

where g_3 can be found by solving (A.19). It follows $g_4 = f_3(\tau_{t|t-1}^*) - g_3$.

Appendix B Fiscal data

This section describes how fiscal data are constructed. I follow Jones (2002), Fernández-Villaverde et al. (2015) and Leeper et al. (2010) closely and use National Income and Product Accounts(NIPA) to construct aggregate effective tax rates τ_K, τ_L , government spending G , transfers Z and government debt B .

B.1 Labor income taxes

The average personal income tax is

$$\tau_p = \frac{\text{PIT}}{\text{WSA} + \text{PRI}/2 + \text{CI}} \quad (\text{B.21})$$

where PIT stands for federal, state, and local taxes on personal income (NIPA Table 3.2, line 3 plus NIPA Table 3.3, line 4), WSA is the wage and salary accruals (NIPA Table 1.12, line 3), PRI is proprietor's income (NIPA Table 1.12, line 9) and CI is capital income. CI is defined as $CI = \text{PRI}/2 + \text{RI} + \text{CP} + \text{NI}$ where RI is rental income (NIPA Table 1.12, line 12), CP is corporate profits (NIPA Table 1.12, line 13), and NI is the interest income (NIPA Table 1.12, line 18).

Labor income taxes τ_L is defined as

$$\tau_L = \frac{\tau_p(\text{WSA} + \text{PRI}/2) + \text{CSI}}{\text{CEM} + \text{PRI}/2} \quad (\text{B.22})$$

where CSI is taxes paid on personal income plus contributions to Social Security (NIPA Table 3.1, line 7) and CEM is compensation of employees (NIPA Table 1.12, line 2).

B.2 Capital taxes

The average capital tax rate τ_K is defined as

$$\tau_K = \frac{\tau_p \text{CI} + \text{CT} + \text{PRT}}{\text{CI} + \text{PRT}} \quad (\text{B.23})$$

where CT is taxes on capital income, taxes on corporate income (NIPA Table 3.1, line 5) and PRT is property taxes (NIPA Table 3.3, line 8).

B.3 Government spending

Government expenditure G is defined as

$$G = \text{GCE} + \text{GI} + \text{NPA} - \text{GFC} \quad (\text{B.24})$$

where GCE is government consumption expenditure (NIPA Table 3.2, line 24), GI is gross government investment (NIPA Table 3.2, line 44), NPA is net purchases of non-produced assets (NIPA Table 3.2, line 46), and GFC is government consumption of fixed capital (NIPA Table 3.2, line 47).

The government spending to output ratio is given by G/Y where the output is defined by $Y = C + I + G$. Consumption, C , is defined as the sum of personal consumption expenditure on nondurable goods (NIPA Table 1.1.5, line 5) and on services (NIPA Table 1.1.5 line 6). Investment, I , is defined as personal consumption expenditure on durable goods (NIPA Table 1.1.5, line 4) and gross private domestic investment (NIPA Table 1.1.5, line 7).

B.4 Government transfers

Lumpsum transfers Z is defined as the summation of net current transfers CT, net capital transfers CAT and subsidies (NIPA Table 3.2, line 35) SUB, minus the tax residue TRE.

$$Z = CT + CAT + SUB - TRE \quad (\text{B.25})$$

Net current transfers CT are defined as current transfer payments CTP (NIPA Table 3.2, line 25) minus current transfer receipts CTR (NIPA Table 3.2, line 18),

$$CT = CTP - CTR \quad (\text{B.26})$$

Net capital transfers CAT are defined as capital transfer payments CATP (NIPA Table 3.2, line 45) minus capital transfer receipts CATR (NIPA Table 3.2, line 41),

$$CAT = CATP - CATR \quad (\text{B.27})$$

The tax residue TRE is defined as the sum of current tax receipts CTR (NIPA Table 3.2, line 2), contributions for government social insurance GSI (NIPA Table 3.2, line 11), income receipts on assets IRA (NIPA Table 3.2, line 14) and the current surplus of government enterprises CSGE (NIPA Table 3.2, line 22), minus total tax revenue T .

$$TRE = CTR + GSI + IRA + CSGE - T \quad (\text{B.28})$$

T is the summation of consumption, labor and capital tax revenues,

$$T = T_C + T_L + T_K = (TPI - PRT) + [\tau_p(WSA + PRI/2) + CSI] + (\tau_p CI + CT + PRT)$$

Since households both produce Y and receive transfers Z , transfers to output ratio is defined as Z/Y and doesn't need to be adjusted by population growth.

B.5 Government debt

Government debt to output ratio B/Y is the Total Public Debt as Percent of Gross Domestic Product (GFDEGDQ188S), obtained from the Federal Reserve Bank of St. Louis's FRED dataset.

Appendix C Equilibrium conditions of the RBC model

For the FIRE model, the following equations along with the four (actual) fiscal rules (16) - (19) and the four AR(1) processes described by (20) characterize the equilibrium conditions of the RBC

model.

$$\exp(u_t^\beta)(C_t - hC_{t-1})^{-\gamma} = E_t^{\mathcal{I}} \beta R_t \exp(u_{t+1}^\beta)(C_{t+1} - hC_t)^{-\gamma} \quad (\text{C.29})$$

$$\omega A_t^{1-\gamma} \exp(u_t^\ell) L_t^{1+\kappa} (1 + \tau_C^*) = (C_t - hC_{t-1})^{-\gamma} (1 - \tau_t^L) (1 - \alpha) Y_t \quad (\text{C.30})$$

$$q_t = \beta E_t^{\mathcal{I}} \frac{\exp(u_{t+1}^\beta)(C_{t+1} - hC_t)^{-\gamma}}{\exp(u_t^\beta)(C_t - hC_{t-1})^{-\gamma}} \left\{ (1 - \tau_{t+1}^K) \frac{\alpha Y_{t+1}}{K_t} + q_{t+1} [1 - \delta(v_{t+1})] \right\} \quad (\text{C.31})$$

$$\frac{\alpha Y_t (1 - \tau_t^K)}{v_t K_{t-1}} = q_t [\delta_1 + \delta_2 (v_t - 1)] \quad (\text{C.32})$$

$$1 = q_t \exp(u_t^I) \left\{ [1 - s_t(\cdot)] - s'(\cdot) \frac{I_t}{I_{t-1}} \right\} + \beta E_t^{\mathcal{I}} \left\{ \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \exp(u_{t+1}^I) s'_{t+1}(\cdot) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\} \quad (\text{C.33})$$

$$W_t = \frac{(1 - \alpha) Y_t}{L_t} \quad (\text{C.34})$$

$$R_t^K = \frac{\alpha Y_t}{v_t K_{t-1}} \quad (\text{C.35})$$

$$T_t^K = \tau_t^K \alpha Y_t \quad (\text{C.36})$$

$$T_t^L = \tau_t^L (1 - \alpha) Y_t \quad (\text{C.37})$$

$$Y_t = (v_t K_{t-1})^\alpha (A_t L_t)^{1-\alpha} \quad (\text{C.38})$$

$$K_t = [1 - \delta(v_t)] K_{t-1} + \exp(u_t^I) \left[1 - s\left(\frac{I_t}{I_{t-1}}\right) \right] I_t \quad (\text{C.39})$$

$$B_t + T_t^K + T_t^L + \tau_C^* C_t = R_{t-1} B_{t-1} + G_t + Z_t \quad (\text{C.40})$$

$$Y_t = C_t + I_t + G_t \quad (\text{C.41})$$

$$u_t^A = \log(A_t) - \log(A_{t-1}) \quad (\text{C.42})$$

$$u_t^A = (1 - \rho_A) g_A + \rho_A u_{t-1}^A + \sigma_A \varepsilon_t^A \quad (\text{C.43})$$

$$u_t^\beta = \rho_\beta u_{t-1}^\beta + \sigma_\beta \varepsilon_t^\beta \quad (\text{C.44})$$

$$u_t^\ell = \rho_\ell u_{t-1}^\ell + \sigma_\ell \varepsilon_t^\ell \quad (\text{C.45})$$

$$u_t^I = \rho_I u_{t-1}^I + \sigma_I \varepsilon_t^I \quad (\text{C.46})$$

For the IIAU model, denote time- t households' perceived fiscal targets

$$\mathcal{FT}_t^* = \{\tau_{K,t}^*, \tau_{L,t}^*, GY_t^*, ZY_t^*, BY_t^*\}$$

Households' perceived fiscal policy is given by

$$\log(\tau_{K,t}) - \log(\tau_{K,t}^*) = \varphi_{\tau_K} \hat{y}_t + \gamma_{\tau_K} [\log(B_{t-1}/Y_{t-1}) - \log(BY_t^*)] + \tilde{u}_t^{\tau_K} + \phi_{\tau_K}^{\tau_L} \tilde{u}_t^{\tau_L} \quad (\text{C.47})$$

$$\log(\tau_{L,t}) - \log(\tau_{L,t}^*) = \varphi_{\tau_L} \hat{y}_t + \gamma_{\tau_L} [\log(B_{t-1}/Y_{t-1}) - \log(BY_t^*)] + \tilde{u}_t^{\tau_L} + \phi_{\tau_K}^{\tau_L} \tilde{u}_t^{\tau_K} \quad (\text{C.48})$$

$$\log(G_t/Y_t) - \log(GY_t^*) = -\varphi_G \hat{y}_t - \gamma_G [\log(B_{t-1}/Y_{t-1}) - \log(BY_t^*)] + \tilde{u}_t^G \quad (\text{C.49})$$

$$\log(Z_t/Y_t) - \log(ZY_t^*) = -\varphi_Z \hat{y}_t - \gamma_Z [\log(B_{t-1}/Y_{t-1}) - \log(BY_t^*)] + \tilde{u}_t^Z \quad (\text{C.50})$$

where I use \tilde{u}_t^X to denote households' perceived persistent components. The perceived law of motion for \tilde{u}_t^X is

$$\tilde{u}_t^X = \rho_X \tilde{u}_{t-1}^X + \sigma_\varepsilon^X \tilde{\varepsilon}_t^X, \quad \tilde{\varepsilon}_t^X \sim N(0, 1) \quad (\text{C.51})$$

for $X = \{\tau_K, \tau_L, G, Z\}$. In (C.47)-(C.50) the \hat{y}_t is the log output gap that's consistent with households' perceived targets \mathcal{FT}_t^* . That is,

$$\hat{y}_t = \log(Y_t) - \log(\mathcal{Y}_t^*) \quad (\text{C.52})$$

where \mathcal{Y}_t^* is the perceived steady-state output level along the balanced growth path based on \mathcal{FT}_t^* . Let y_t^* be the corresponding detrended perceived steady-state log output. Define the time-varying terms which involve fiscal targets and y_t^* in (C.47)-(C.50) as

$$\Gamma_t^{\tau_K} = \log(\tau_{K,t}^*) - \varphi_{\tau_K} y_t^* - \gamma_{\tau_K} \log(BY_t^*) \quad (\text{C.53})$$

$$\Gamma_t^{\tau_L} = \log(\tau_{L,t}^*) - \varphi_{\tau_L} y_t^* - \gamma_{\tau_L} \log(BY_t^*) \quad (\text{C.54})$$

$$\Gamma_t^G = \log(GY_t^*) + \varphi_G y_t^* + \gamma_G \log(BY_t^*) \quad (\text{C.55})$$

$$\Gamma_t^Z = \log(ZY_t^*) + \varphi_Z y_t^* + \gamma_Z \log(BY_t^*) \quad (\text{C.56})$$

and let Ω_t^X solves the following linear system

$$\begin{bmatrix} \Gamma_t^{\tau_K} \\ \Gamma_t^{\tau_L} \\ \Gamma_t^G \\ \Gamma_t^Z \end{bmatrix} = \begin{bmatrix} 1 & \phi_{\tau_K}^{\tau_L} & 0 & 0 \\ \phi_{\tau_K}^{\tau_L} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_t^{\tau_K} \\ \Omega_t^{\tau_L} \\ \Omega_t^G \\ \Omega_t^Z \end{bmatrix} \quad (\text{C.57})$$

The perceived law of motion of Ω_t^X 's for $X = \{\tau_K, \tau_L, G, Z\}$ are unit root processes, given by

$$\Omega_t^X = \Omega_{t-1}^X + \sigma_\eta^X \eta_t^X, \quad \eta_t^X \sim N(0, 1) \quad (\text{C.58})$$

I assume all perceived shocks $\tilde{\varepsilon}_t^X, \eta_t^X$'s are mutually *i.i.d.* Since Ω_t^X 's embed information on perceived fiscal targets, they will jointly determine the perceived steady state of the economy.

For the IIAU model, equilibrium conditions consist of the above equations (C.29) - (C.46), the actual fiscal policy rules (16) - (20), along with households' perceived fiscal policy (C.47) - (C.58).

Appendix D Solving the RBC models

The FIRE model is solved by log-linearizing the equilibrium conditions around the deterministic steady state using Uhlig (1995). Following Hollmayr and Matthes (2015), the IIAU model is solved by log-linearizing the equilibrium conditions around households' perceived steady states to better capture the nonlinear dynamics of the economy when it is far away from deterministic steady state

due to time-varying perceived fiscal targets. Comparing (32) and (34) implies we need to establish households' signal extraction problem to derive $\Omega_{t|t}$ and $\mathbf{s}_{t|t}$. Since the mapping between $\Omega_{t|t}$ and $\mathcal{FT}_{t|t}^*$ and the mapping between $\mathcal{FT}_{t|t}^*$ and $\Omega_{t|t}^X$'s defined in Appendix C are both one-to-one, it suffices to derive the law of motion for $\Omega_{t|t}^X$ and $\mathbf{s}_{t|t}$. Since all non-policy shocks $u_t^A, u_t^\beta, u_t^\ell, u_t^I$ are all perfectly observable in the current setup, in the following signal extraction problem I only include actual and perceived fiscal shocks in the exogenous state vector \mathbf{s}_t .

Define $\mathbf{s}_t = [\Omega_t^{\tau K}; \Omega_t^{\tau L}; \Omega_t^G; \Omega_t^Z; u_t^{\tau K}; u_t^{\tau L}; u_t^G; u_t^Z]_{8 \times 1} = [\Omega_t^X; u_t^X]'$. The state transition equation for \mathbf{s}_t is given by

$$\begin{bmatrix} \Omega_t^X \\ u_t^X \end{bmatrix} = \begin{bmatrix} I_{4 \times 4} & 0 \\ 0 & \text{diag}(\rho_X)_{4 \times 4} \end{bmatrix} \begin{bmatrix} \Omega_{t-1}^X \\ u_{t-1}^X \end{bmatrix} + \begin{bmatrix} \text{diag}(\sigma_\eta^X)_{4 \times 4} & 0 \\ 0 & \text{diag}(\sigma_\varepsilon^X)_{4 \times 4} \end{bmatrix} \begin{bmatrix} \eta_t^X \\ \varepsilon_t^X \end{bmatrix} \quad (\text{D.59})$$

The observation equation is given by

$$\begin{bmatrix} \log(\tau_{K,t}) - \varphi_{\tau K} y_t - \gamma_{\tau K} \log(B_{t-1}/Y_{t-1}) \\ \log(\tau_{L,t}) - \varphi_{\tau L} y_t - \gamma_{\tau L} \log(B_{t-1}/Y_{t-1}) \\ \log(G_t/Y_t) + \varphi_G y_t + \gamma_G \log(B_{t-1}/Y_{t-1}) \\ \log(Z_t/Y_t) + \varphi_Z y_t + \gamma_Z \log(B_{t-1}/Y_{t-1}) \end{bmatrix} = \begin{bmatrix} 1 & \phi_{\tau K}^{\tau L} & 0 & 0 \\ \phi_{\tau K}^{\tau L} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Omega_t^{\tau K} + u_t^{\tau K} \\ \Omega_t^{\tau L} + u_t^{\tau L} \\ \Omega_t^G + u_t^G \\ \Omega_t^Z + u_t^Z \end{bmatrix} \quad (\text{D.60})$$

where $y_t = \log(Y_t/A_t)$ is the detrended log output. Equation (D.59) and (D.60) form household's signal extraction problem. The famous Kalman recursion gives the law of motion of households' beliefs $\mathbf{s}_{t|t}$, which contains $\Omega_{t|t}^X$.

Appendix E Bayesian estimation of the RBC model

E.1 Data description

The RBC models are estimated using eight observables: Consumption per capita C_t , Investment per capita I_t , Hour worked L_t , Capital taxes $\tau_{K,t}$, Labor taxes $\tau_{L,t}$, Government spending to GDP ratio G_t/Y_t , Transfers to GDP ratio Z_t/Y_t and Debt to GDP ratio B_t/Y_t . Data ranges from 1966Q2-2017Q3. Fiscal series $\tau_{K,t}, \tau_{L,t}, G_t/Y_t, Z_t/Y_t, B_t/Y_t$ are constructed using NIPA data and details can be found in Appendix B. I follow [Leeper et al. \(2017\)](#) to construct C_t, I_t and L_t . Nominal consumption is defined as total personal consumption expenditures on nondurables and services (NIPA Table 1.1.5, lines 5 and 6). Nominal investment is defined as gross private domestic investment (Table 1.1.5, line 7) and personal consumption expenditures on durables (NIPA Table 1.1.5, line 4). Nominal variables are converted to real values by dividing the GDP deflator (NIPA Table 1.1.4, line 1). Let *Popindex* be the index of population, constructed such that 2009Q3 = 100. Population data is civilian noninstitutional population in thousands, ages 16 years and over, not seasonally adjusted (CNP16OV, available on FRED). C_t and I_t are defined by the ratio of their

nominal values and *Popindex*. Hour worked L_t are defined as

$$L_t = \frac{H * Emp}{100 * Popindex}$$

where H is the index for average weekly hours, non-farm business sector, constructed such that 2009Q3 = 100 (PRS85006023, available on FRED) and Emp is the civilian employment for sixteen years and over, measured in thousands, seasonally adjusted (CE16OV, available on FRED).

Finally, C_t , I_t and L_t are converted into growth rates (by taking log differences) and fiscal series $\tau_{K,t}$, $\tau_{L,t}$, G_t/Y_t , Z_t/Y_t , B_t/Y_t are transformed to logs.

E.2 Prior versus posterior distributions: FIRE vs. IIAU anchored

See Figure E.1 and E.2. The former compares priors and posteriors of the non-policy parameters and the latter compares priors and posteriors of fiscal policy parameters.

E.3 Model fit comparisons under alternative specifications

Table E.1 provides model fit comparisons when all belief parameters' prior distributions are uniformly distributed on $[0, 1]$ using full sample period. Table E.2 compares model fit using only Great Moderation data (i.e., 1985Q1-2017Q3). Under both alternative specifications, the IIAU anchored model underperforms the FIRE model while the IIAU unanchored model significantly improves log-marginal data density by around 100. Posterior estimates change slightly. For illustration, Figure E.3 and E.4 compares posteriors of the FIRE model and the IIAU unanchored model¹ using subsample data from 1985Q1 to 2017Q3. Our main conclusion still stands.

Model	Log-marginal data density	Log relative factor
FIRE	4200.3	0
FIRE(<i>linearly detrended fiscal obs.</i>)	4225.1	NA
IIAU anchored	4199.8	-0.5
IIAU unanchored	4314.3	114.3
IIAU unanchored with only $\sigma_\eta^Z > 0$	4309.9	109.6

Table E.1: Model fit comparisons under flat belief priors: FIRE vs. IIAU; Full sample period: 1966Q2-2017Q3

¹With only $\sigma_\eta^Z > 0$;

Model	Log-marginal data density	Log relative Bayes factor
FIRE	2745.9	0
FIRE(<i>linearly detrended fiscal obs.</i>)	2777.9	NA
IIAU anchored	2745.5	-0.4
IIAU unanchored	2832.0	86.1
IIAU unanchored with only $\sigma_{\eta}^Z > 0$	2833.8	87.9

Table E.2: Model fit comparisons: FIRE *vs.* IIAU; Subsample period: 1985Q1-2017Q3

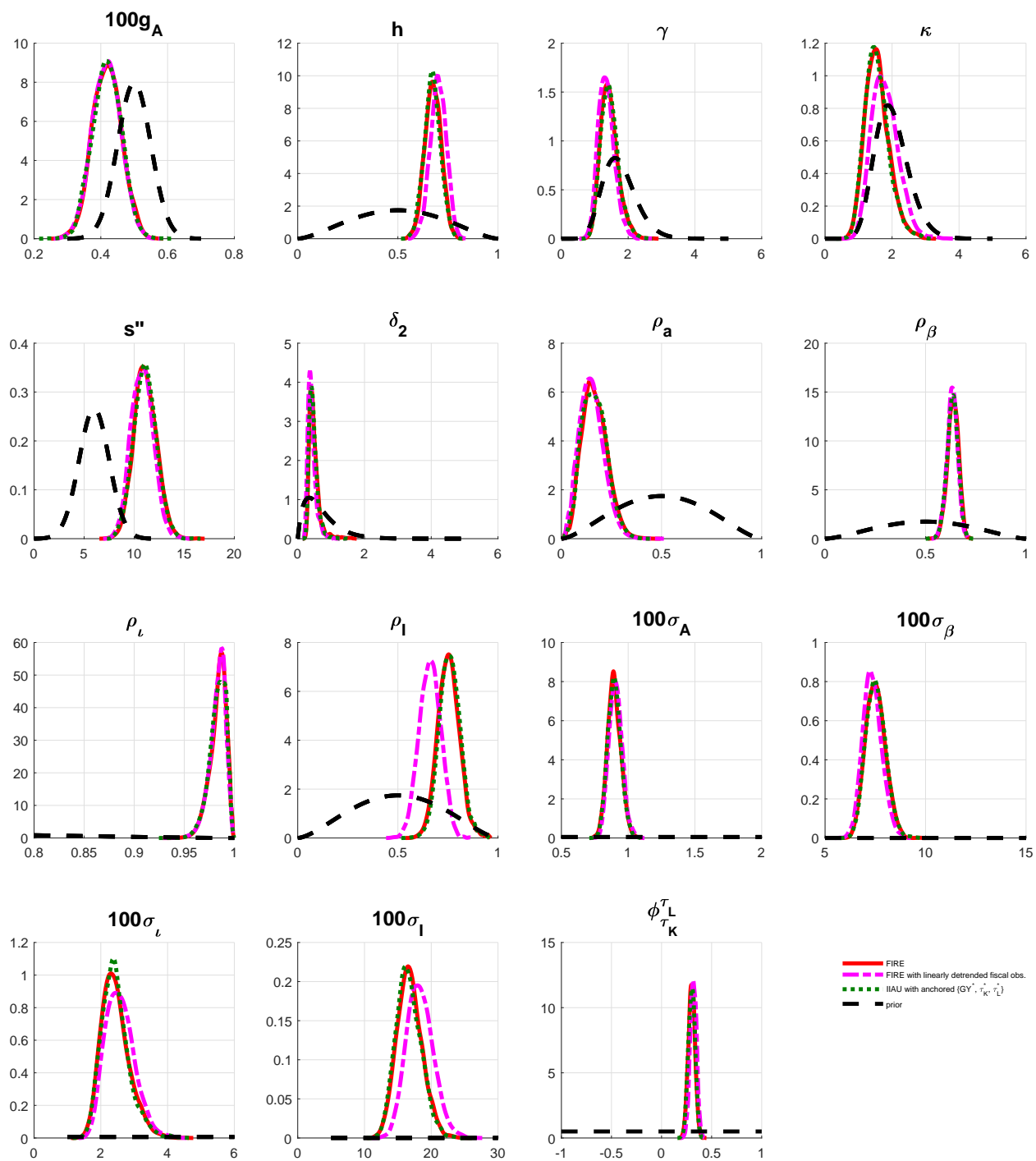


Figure E.1: Prior versus posterior distributions of the FIRE model, the IIAU model with anchored $\{GY^*, \tau_K^*, \tau_L^*\}$ and the FIRE model with linearly detrended fiscal observables: Non-policy parameters and the tax comovement parameter $\phi_{\tau_K}^{\tau_L}$; Parameters are estimated using full sample period from 1966Q2 to 2017Q3.

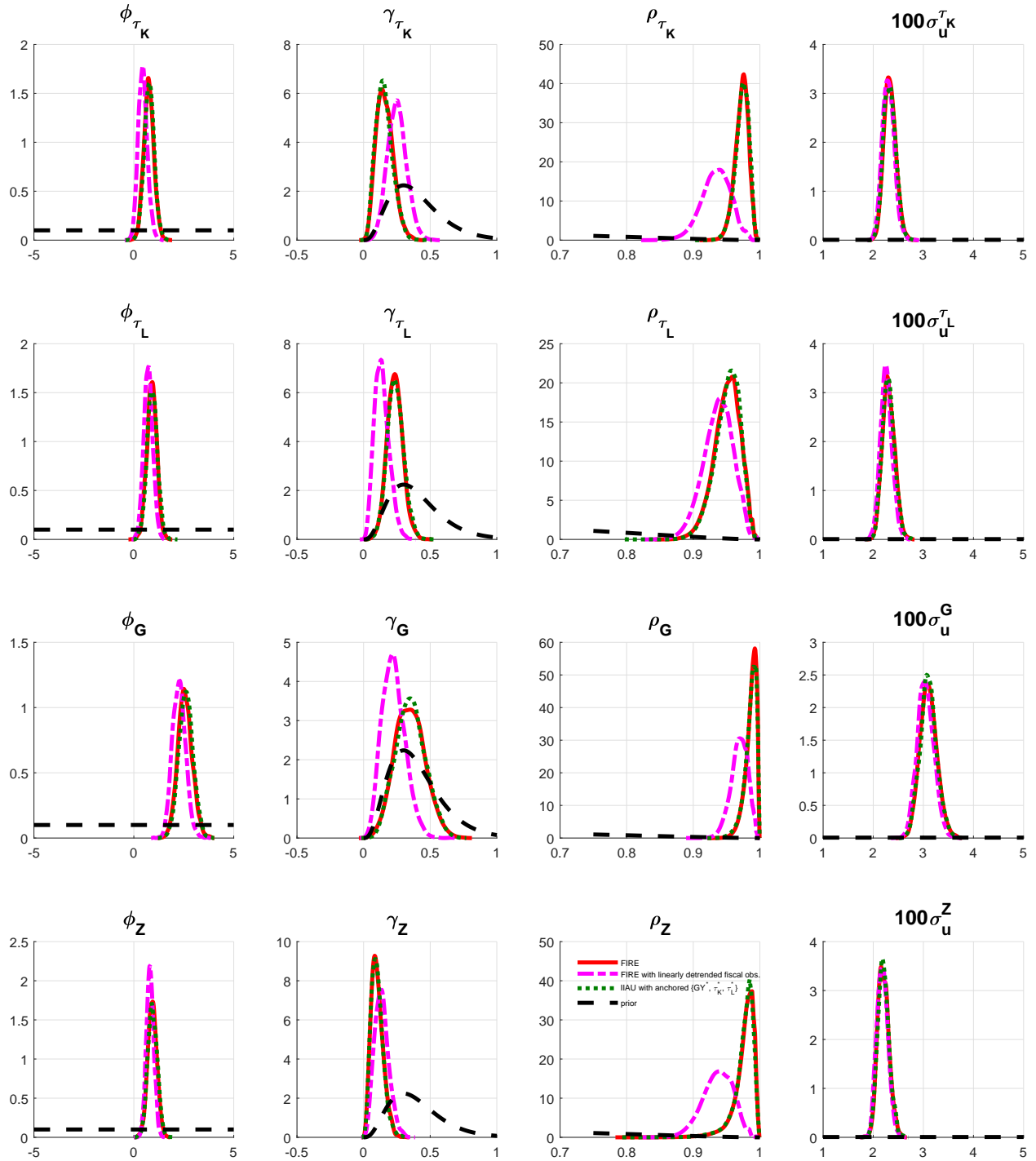


Figure E.2: Prior versus posterior distributions of the FIRE model, the IIAU model with anchored $\{GY^*, \tau_K^*, \tau_L^*\}$ and the FIRE model with linearly detrended fiscal observables: Fiscal policy parameters. The first column plots output response coefficients, the second column plots debt response coefficients. The third and fourth columns plot AR(1) coefficients and standard deviations of fiscal shocks; Parameters are estimated using full sample period from 1966Q2 to 2017Q3.

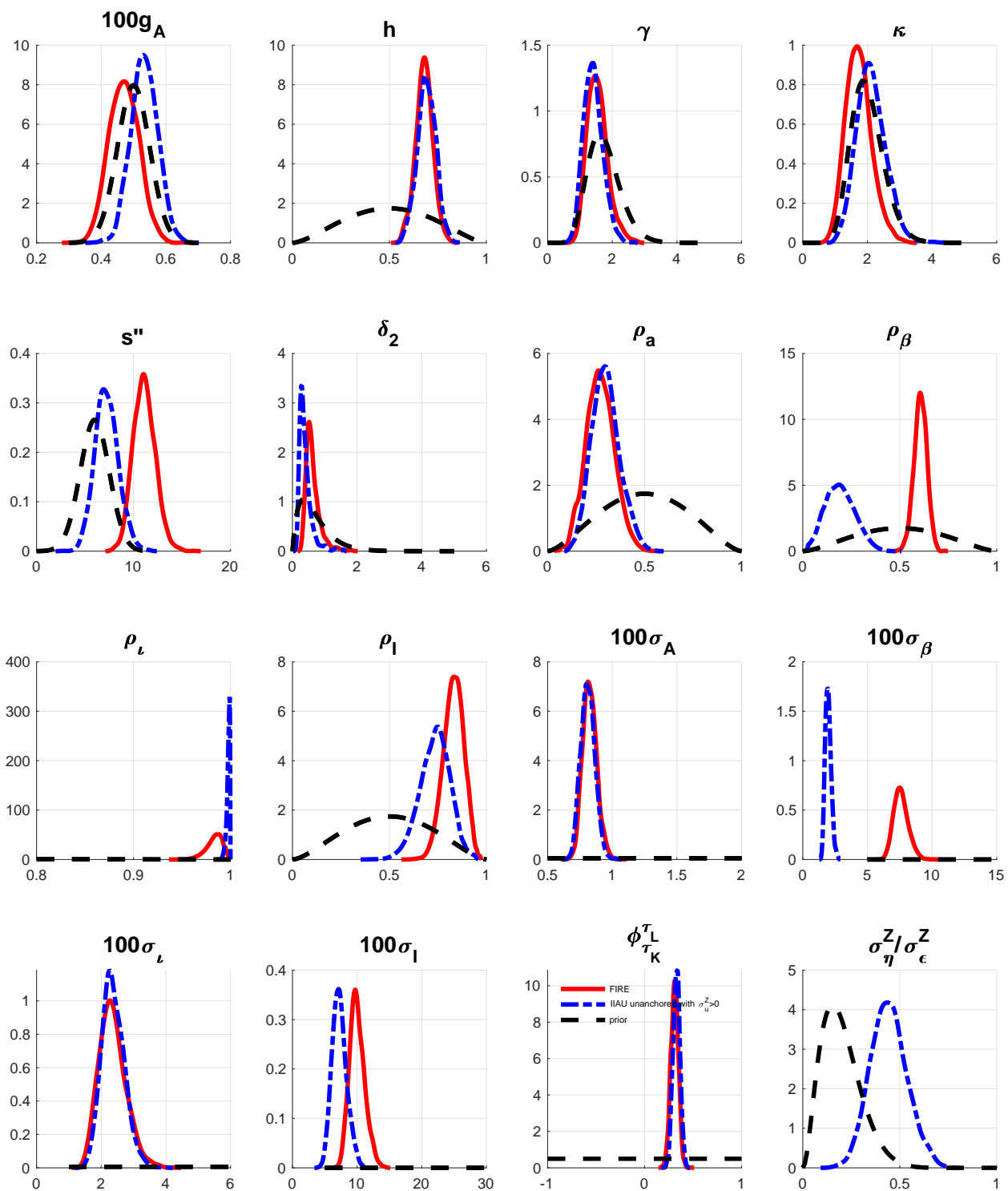


Figure E.3: Prior versus posterior distributions of the FIRE model and the IIAU unanchored model with only $\sigma_\eta^Z > 0$: Non-policy parameters and the tax comovement parameter ϕ_{TK}^{TL} ; The last panel compares prior and posterior of the belief parameter σ_η^Z in the IIAU unanchored model. Parameters are estimated using subsample period from 1985Q1 to 2017Q3.

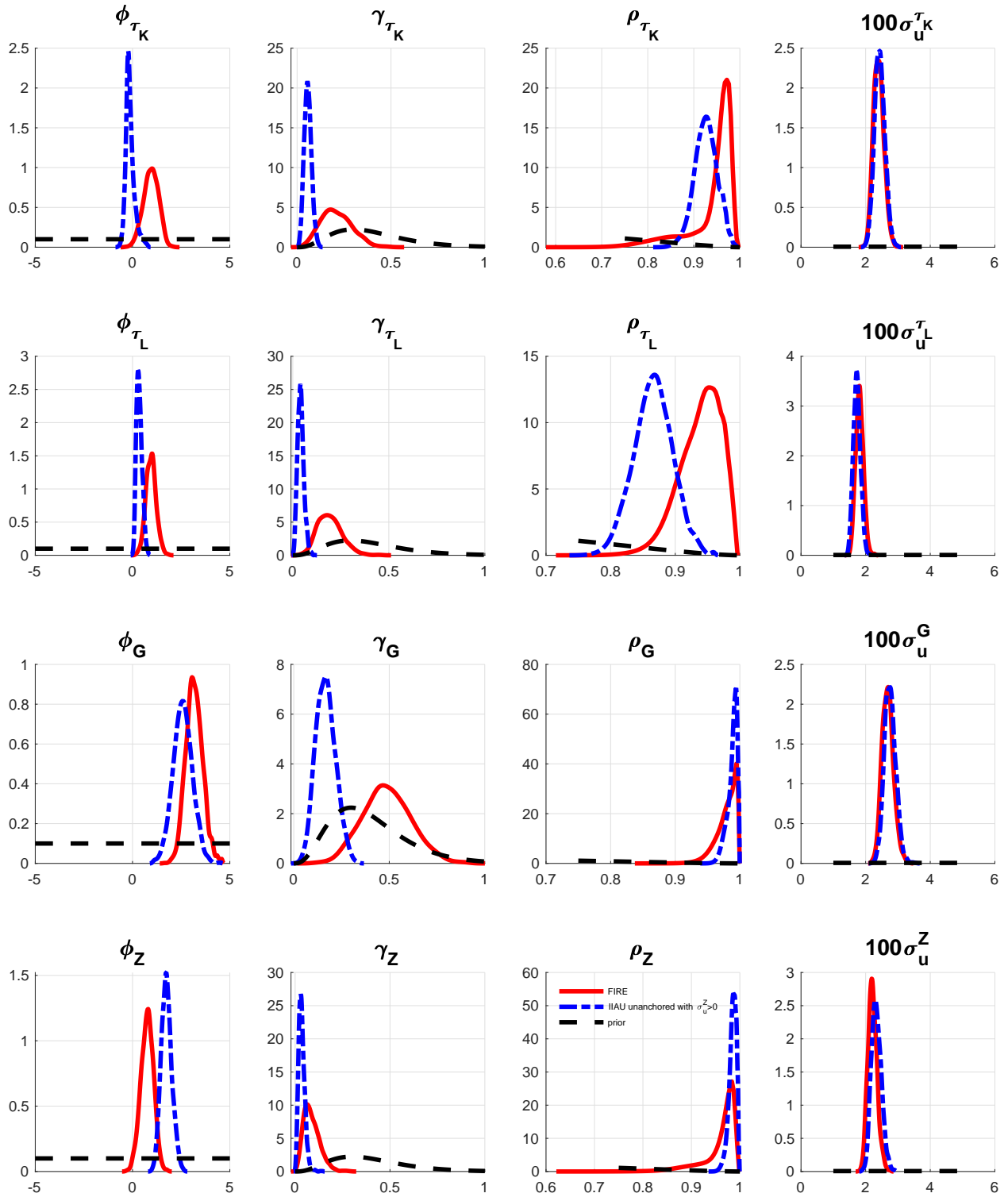


Figure E.4: Prior versus posterior distributions of the FIRE model and the IIAU unanchored model with only $\sigma_\eta^Z > 0$: Fiscal policy parameters; The first column plots output response coefficients, the second column plots debt response coefficients. The third and fourth columns plot AR(1) coefficients and standard deviations of fiscal shocks; Parameters are estimated using subsample period from 1985Q1 to 2017Q3.