On Targeting Frameworks and Optimal Monetary Policy

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Abstract

Speed limit policy, a monetary policy strategy that focuses on stabilizing inflation and the change in the output gap, consistently delivers better welfare outcomes than flexible inflation targeting or flexible price level targeting in empirical New Keynesian models when policymakers lack the ability to commit to future policies. Even if the policymaker can commit under an inflation targeting strategy, the discretionary speed limit policy performs better for most empirically plausible model parameterizations from a normative perspective.

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1 Introduction

The optimal delegation problem in monetary policy studies how a central bank can best serve the interests of society when the optimal state-contingent plan derived under the true social objective function is time-inconsistent. Starting with Rogoff (1985), several authors have shown that assigning the central bank an objective that differs from the true social objective can lead to better normative outcomes under discretionary policymaking than otherwise.¹

One such central bank objective is the speed limit policy under which, according to Walsh (2003), the policymaker focuses on stabilizing inflation and the change in the output gap. We show that in the discretionary Markov equilibrium, the speed limit policy framework consistently outperforms flexible inflation targeting and often performs better than flexible price level targeting in a set of New Keynesian models (NKM) ranging from the purely forward-looking textbook version of the NKM and its extensions to the medium-scale DSGE model in Christiano, Eichenbaum, and Evans (2005) as implemented and estimated in Smets and Wouters (2007) (CEE/SW model).²

The speed limit policy performs strongly in the discretionary Markov equilibrium as it captures a robust feature of the optimal monetary policy under commitment (henceforth, optimal commitment policy) in NKMs: The policymaker promises to keep future monetary policy tight in response to shocks that drive up inflation, such as a positive price markup shock, as evidenced by a slow closing of the negative output gap under the optimal commitment policy. The persistent rise in the policy interest rate deters excessive price and wage adjustments by the private sector in the impact period and reduces overall movements in inflation under the optimal commitment policy. Importantly, the price level is not necessarily stationary under the optimal commitment policy. The speed with and the extent to which nominally rigid prices and wages return to their long-run trend paths depend on the degree of price and wage indexation to past inflation.

As the speed limit policy interprets the idea of stabilizing the real economy as preventing large *changes* in the output gap as opposed to *deviations* of the output gap from zero, the policymaker prefers delaying the closing of the negative output gap after the inflationary shock by construction and keeps future monetary policy tight regardless of the policymaker's ability to commit. If the private sector understands this behavior of the central bank, the rise in inflation is kept small while the price level rises permanently by a small amount. The price level targeting framework also incorporates the idea of keeping monetary policy tight after an inflationary shock albeit through a different mechanism. By assumption, the policymaker is determined to drive the price level back to its trend path under this framework and keeps the interest rate elevated to undo earlier changes induced by the shock. Anticipating such a policy move, households and firms feel deterred from implementing large changes in prices and wages in the first place.

By contrast, the inflation targeting framework lacks a built-in mechanism that facilitates implementing

¹Important contributions include King (1997), Svensson (1997), Svensson (1999), Clarida, Gali, and Gertler (1999), Walsh (2003), Woodford (2003b), Nessen and Vestin (2005), Vestin (2006), and Bilbiie (2014).

² Consistent with the literature, we define that under a flexible targeting framework the central bank minimizes the discounted infinite sum of a period loss function that reflects the central bank's preferences over stabilizing prices and the real economy subject to its model of the economy. Under inflation targeting, the loss function places weight on the squared deviations of inflation from its long-run target and of the output gap from zero as in Svensson (2010). The price level (in deviation from a deterministic trend) takes the place of inflation in the loss function under price level targeting; in addition the loss function places weight on the squared deviations of the output gap from zero. Finally as in Walsh (2003), the central bank's loss function features an aversion to squared deviations of inflation from its target and of the growth rate of the output gap under the speed limit policy.

tight monetary policy after an inflationary shock in the discretionary Markov equilibrium. As the policymaker intends to stabilize inflation and the level of the output gap, the policymaker will not be expected to drive prices back to their trend level or to delay the closing of the output gap under the inflation targeting objective. In line with the "weight-conservative" central banker of Rogoff (1985), placing a high weight on stabilizing inflation helps improving the performance of the inflation targeting framework, but is generally too crude to make inflation targeting attractive relative to the speed limit policy under discretionary policymaking. Only in the simplest NKMs with a high degree of indexation to past inflation can inflation targeting perform best, since in this case the desirability of returning the price level to its previous trend vanishes under the optimal commitment policy. In more complex models featuring habit persistence in consumption or sticky nominal wages (unless highly indexed to inflation as well) or the empirical CEE/SW model inflation targeting is undesirable irrespective of the degree of price indexation when policymakers cannot commit.³

Although, we view the case of discretionary policymaking as more realistic, we also report findings for the case that the central bank can commit to future actions.⁴ Under commitment, the inflation targeting central bank does drive prices and wages back towards their long-run trends if so desired under the optimal commitment policy and performs reliably best across models from the textbook NKM to the CEE/SW model with the speed limit policy a close second. Since under price level targeting the central bank will never allow for permanent changes in prices and wages, this framework performs worst when prices and wages are highly indexed to past inflation.⁵

Several experiments in the CEE/SW model lend further support to the speed limit policy framework when policymakers can only act under discretion. Beyond parameterizing the model at the mode of the posterior distribution reported in Smets and Wouters (2007), we consider alternative parameter choices drawn from the Laplace approximation to the posterior distribution. When the objective functions are parameterized optimally for each parameter draw, the speed limit policy dominates for almost all 30,000 empirically plausible draws when policymakers act under discretion. Surprisingly, the speed limit policy under discretion outperforms the inflation targeting framework under commitment for the majority of draws (including our benchmark parameterization). When we compare the targeting frameworks for selected specifications of the objective functions that do not vary across the 30,000 parameterizations of the CEE/SW model, the speed limit policy almost always dominates regardless of the central bank's ability to commit.

Our findings prevail in a version of the CEE/SW model that is estimated with euro area data instead of US data or a version that reduces the importance of wage markup shocks relative to labor supply shocks to address concerns about identification raised in Chari, Kehoe, and McGrattan (2009) and Justiniano, Primiceri, and Tambalotti (2013). Finally, we also account for the limitations of conventional monetary policy imposed by the zero lower bound constraint on the nominal interest rate. Unless long-lasting and frequent zero-bound episodes cannot be eliminated by raising the long-run inflation target, our results go

³ Under habit persistence smoothing a quasi-difference of the output gap enters in the true social loss functions as a motive which is well captured by the speed limit policy objective; under sticky nominal wages with a moderate degree or no inflation indexation, the optimal commitment policy pushes the levels of prices and wages back towards their deterministic trends even if prices are highly indexed.

 $^{^4}$ See Bernanke and Mishkin (1997) and King (2004) for further elaborations on this issue.

 $^{^{5}}$ In the case of commitment, adopting a simple objective function for the central bank can be justified on the grounds of improving transparency, accountability and the pursuit of the central bank's legal mandate.

unchallenged.

In terms of scope and focus, our paper is closest to Walsh (2003). In a simple NKM with sticky prices and backward-looking elements in the form of lagged inflation and lagged output gap Walsh (2003) illustrates the potential advantages of the speed limit policy. However, the model in Walsh (2003) is not fully micro-founded and social welfare is measured by an ad hoc loss function that is not derived from the preferences of the representative household. Furthermore, the underlying model is calibrated rather than estimated and lacks many of the features found to be of empirical relevance in works such as Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2005). In contrast to Walsh (2003), we find that the speed limit policy outperforms inflation and price level targeting under discretion regardless of the degree of backward-looking inflation dynamics in the CEE/SW model. In Walsh (2003) and in simple NKMs, this conclusion applies only for the case of an intermediate degree of backward-looking behavior.

Restricting attention to the case of a fully committed policymaker Debortoli, Kim, Lindé, and Nunes (2015) report strong support in favor of inflation targeting using the CEE/SW model, a result we confirm and extend to a range of other empirically relevant parameterizations of the CEE/SW model. However, as the optimal inflation targeting under commitment is dominated by the optimal speed limit policy under discretion for many empirically plausible parameterizations, our results appear more general.

The remainder of the paper proceeds as follows. In Section 2, we analyze inflation targeting, price level targeting, and speed limit policy in a sequence of simple NKMs. We consider a wide range of parameterizations and variations of the CEE/SW model in Section 3. Concluding remarks are offered in Section 4. A technical appendix provides information on our methodology, details on the models, and additional results.

2 Baseline New Keynesian Model

Throughout this paper, we refer to the NKM presented in Woodford (2003a), Gali (2008) or Walsh (2010) as the textbook NKM. This model features sticky nominal prices as in Calvo (1983) and a production technology that requires only labor as input. Sales subsidies offset the distortions arising from monopolistic competition in the steady state. Finally, the economy experiences technology and markup shocks. One at a time, we consider the role of features commonly present in empirical DSGE models: (i) intrinsic inflation inertia, (ii) steady state distortions, (iii) consumption habits, and (iv) sticky wages. Appendix A offers details on our computational approach. The models are described in Appendix B.

2.1 Simple objective functions and targeting frameworks

Broadly speaking, analysis of monetary policy distinguishes between targeting frameworks and instrument rules. Under a targeting framework, the central bank optimizes an objective function. An inflation targeting central bank, for example, is instructed to keep a selected inflation measure in the neighborhood of a specific target value. The central bank is granted some flexibility in pursuing this goal and can deviate from its target in the short run to buffer the impact of shocks (flexible inflation targeting).⁶ Given a specific model

 $^{^{6}}$ In practice, a targeting framework fullfils a list of formal criteria. State of the art inflation targeting, for example, is commonly characterized as featuring the following elements, see Hammond (2012): (1) price stability as the main goal of monetary policy, (2) public announcement of a quantitative target for inflation, (3) policy based on inflation forecast, (4) mechanisms for transparency and

of the economy, the policymaker derives a set of optimality conditions for the targeting variables to fulfill under the targeting framework. By contrast, an instrument rule as in Taylor (1993) is a is a formula that specifies directly the functional relationship between the central bank's instrument and a set of variables.

For model-based policy analysis, the central bank's objective function under a targeting framework specifies the variables that characterize the long-run goal(s) of the central bank and the weights assigned to each of these variables as argued in Svensson (2010). In line with the literature, we represent loss functions associated with the targeting frameworks of interest as:

1. inflation targeting (IT)

$$L_t^{IT} = \pi_{p,t}^2 + \lambda_x^{IT} \left(x_t^{gap} \right)^2 \tag{1}$$

2. price level targeting (PLT)

$$L_t^{PLT} = \hat{p}_t^2 + \lambda_x^{PLT} \left(x_t^{gap} \right)^2 \tag{2}$$

3. speed limit policy (SLP)

$$L_t^{SLP} = \pi_{p,t}^2 + \lambda_x^{SLP} \left((x_t^{gap}) - (x_{t-1}^{gap}) \right)^2$$
(3)

where $\pi_{p,t}$ denotes deviations of the inflation measure from its value along the balanced growth path (henceforth the long-run target), \hat{p}_t is the log-deviation of the price level from its value along the balanced growth path (henceforth the long-run trend), and x_t^{gap} measures the (model-specific) output gap. We refer to λ_x^{TF} as the weight on the activity measure under framework TF.

Each objective function implies a long-run commitment to price stability expressed in terms of a longrun inflation target, or equivalently, a deterministic trend in the price level to provide a nominal anchor. The central bank minimizes the discounted sum of losses subject to the equations that describe the behavior of the economy. We consider both the case that in doing so the policymaker can commit to future policy actions and the case that such a commitment is not feasible (discretion). A targeting framework is referred to as optimal, when the objective function associated with this framework is parameterized to minimize the expected welfare loss under this objective relative to the social optimum. The social optimum is defined by the economic outcomes under the optimal commitment policy when the policymaker's preferences are consistent with the true social loss function. Following Woodford (1999), we adopt the concept of "optimality from a timeless perspective" to derive commitment policies throughout this paper.

2.2 Targeting frameworks in the textbook NKM

We start our discussion of targeting frameworks using the textbook NKM. At the core of the linear version of this model lies the New Keynesian Phillips Curve (NKPC) which links inflation, $\pi_{p,t}$, to the (welfarerelevant) output gap, x_t ,

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p (\sigma_L + \sigma_C) x_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t}.$$
(4)

accountability. Suitably adapted, these elements would also be present in other targeting frameworks. By contrast, our discussion of targeting frameworks treats monetary policy as the solution to an optimal control problem under a specific objective function for each framework. Given our broader perspective, the analysis in this paper is also of relevance for central banks that do not adopt a formal targeting framework, but rather search for monetary policy strategies that achieve the central bank's mandate as in the case of the U.S. Federal Reserve.

Here and subsequently, all variables are expressed in deviation from their steady state values (relative if carrying a "hat", absolute otherwise). The markup shock, $u_{p,t}$, follows a known stochastic process. The composite parameter $\kappa_p(\sigma_L + \sigma_C)$ measures the slope of the NKPC and the parameter ι_p represents the degree of indexation to past inflation as in Christiano, Eichenbaum, and Evans (2005). The aggregate demand curve

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_C} \left(i_t - E_t \pi_{p,t+1} - g_{mu,t}^* \right)$$
(5)

provides the connection between the output gap, inflation, the nominal interest rate, i_t , and the natural rate of interest, $g_{mu,t}^* = \sigma_C \left[E_t \hat{y}_{t+1}^* - \hat{y}_t^* \right]$. The natural level of output in this model

$$\hat{y}_t^* = \frac{1 + \sigma_L}{\sigma_L + \sigma_C} \hat{\xi}_{A,t} \tag{6}$$

is obtained from a counterfactual economy without nominal rigidities and without markup shocks. The natural level of output responds to changes in technology, $\hat{\xi}_{A,t}$; other shocks that could move the natural level of output and thus the natural rate of interest, but from which we abstract for now, are shocks to household preferences or government spending. The output gap is defined as the difference between actual output and the natural level of output, $x_t = \hat{y}_t - \hat{y}_t^*$. As in Woodford (2003a), the preferences of the representative household (or equivalently the social welfare function in this context) are approximated to the second-order as

$$E_{t_0}\left(\frac{1}{2}\sum_{t=t_0}^{\infty}\beta^{t-t_0}L_t\right) \tag{7}$$

with the true (approximate) social loss function L_t satisfying

$$L_t = (\sigma_L + \sigma_C) (x_t)^2 + \frac{1 + \theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2$$
(8)

with σ_L , σ_C , θ_p being known parameters.

To fix ideas, we consider first the performance of each targeting framework in the fully forward-looking NKM, i.e., $\iota_p = 0$. The policies associated with each framework are obtained by replacing the true social loss function L_t in equation (7) with the loss functions in (1)-(3). Each framework is evaluated for a range of weights on the activity measure, λ_x^{IT} , λ_x^{PLT} , and λ_x^{SLP} , respectively, both under commitment and discretion with $x_t^{gap} = x_t$. Table 1 provides the parameterization of the model (and of all its extensions). For each targeting framework we consider and for the optimal commitment policy, shocks that transmit through the natural real interest rate, such as the technology shock, have no welfare consequences as adjustments in the nominal interest rate prevent movements of inflation and the output gap so as to prevent any welfare consequences. Blanchard and Gali (2007) refer to this feature of the textbook NKM as divine coincidence.⁷ In the following, we restrict attention to markup shocks which by contrast cannot be neutralized.

Figure 1 plots the unconditional welfare loss for each framework relative to the optimal commitment

 $^{^{7}}$ This feature of the model requires that shocks are sufficiently small in order for policy not to be constrained by the zero lower bound on the nominal interest rate.

policy expressed as consumption equivalent variation (CEV). The weight on the activity measure for which the welfare loss is minimized under a targeting framework is indicated by "o" for price level targeting (PLT), "*" for speed limit policy (SLP), and "o" for inflation targeting (IT). The optimal weights on the activity measure are low relative to the weights on the inflation measure (which is normalized to 1) and the welfare losses of not implementing the optimal commitment policy are small both under commitment and discretion for each framework.

Figure 1 reproduces some well-known results. Under inflation targeting, a central bank acting under commitment can replicate the optimal commitment policy; the solid line in the top panel assumes the value of zero for the optimal choice of the weight λ_x^{IT} in the objective function. In the textbook NKM without indexation, the true social loss function (8) is written solely in terms of contemporaneous inflation and the welfare-relevant output gap. The central bank's preferences over inflation and the output gap under inflation targeting coincide with the true social loss function, if $\lambda_x^{IT} = \lambda_x \equiv (\sigma_C + \sigma_L)\kappa_p \frac{\theta_p}{1+\theta_p}$. Thus, the welfare loss under optimal inflation targeting relative to the optimal commitment policy must be zero. Given the modifications in the objective functions for price level targeting (\hat{p}_t instead of $\pi_{p,t}$) and speed limit policy ($x_t^{gap} - x_{t-1}^{gap}$) relative to the true social loss function the outcomes under these two targeting frameworks are suboptimal by construction.

The equivalence between inflation targeting and the optimal commitment policy breaks down for any change in the model environment, most notably if the central bank lacks commitment. For example, in response to a *transitory* markup shock, the optimal commitment policy manages to reduce deviations of inflation and the output gap from their target values in the impact period by allowing these variables to deviate from their target values also in future periods after the shock has ceased. A central bank acting under discretion with the objective in (8) for $\iota_p = 0$, however, will find it optimal to eliminate these deviations from target in future periods to fully stabilize the economy earlier (stabilization bias). As households and firms correctly anticipate this behavior, the discretionary central bank will not be able to reap the benefits of the optimal commitment policy in the impact period thereby causing larger movements in inflation and the output gap.⁸

Borrowing the idea of a "(weight-) conservative central banker" from Rogoff (1985), Clarida, Gali, and Gertler (1999) show that the optimal inflation targeting central bank puts lower weight on the activity measure than society does, i.e., $\lambda_x^{IT} < \lambda_x$, which mitigates, but does not eliminate, the negative welfare consequences of the stabilization bias. Thus, the CEV in Figure 1 is positive for optimal inflation targeting under discretion.⁹ Changes to the functional form of the policymaker's objective function can induce further welfare improvements: The welfare loss under optimal price level targeting is close to zero and is marginally higher under the optimal speed limit policy in Figure 1.

To understand the strong performance of price level targeting and speed limit policy in the textbook NKM when the policymaker acts under discretion, we revisit the effects of a markup shock under the optimal commitment policy. Let the shock lead initially to an unexpected rise in inflation and a drop in

 $^{^{8}}$ In the case of the textbook NKM with an efficient steady state and the central bank's preferences coinciding with those of the representative household the true social loss function is given by equation (8) regardless of the central bank's ability to commit.

 $^{^{9}}$ Rogoff (1985) formulates the idea of a conservative central bank to overcome the inflation bias that arises under policy discretion in a model with product or labor market distortions akin to Barro and Gordon (1983). A subsidy to offset market distortions also eliminates the inflation bias under discretionary policy in this setting. Yet, in the textbook NKM, even with an efficient steady state due to such subsidies, the optimal commitment policy continues to be time-inconsistent as discussed in the text.

the output gap. Over time the optimal commitment policy drives the price level back to its long-run trend by pushing inflation temporarily below its long-run target. The explanation for the optimality of price level stability (relative to its long-run trend) recognizes the link between price dispersion and inflation: the cross-sectional variation of prices is proportional to the squared value of inflation as shown in Woodford (2003a) and Appendix B.2. By assumption, firms that do not adjust prices optimally in the current period adjust prices by the value of the long-run inflation target instead. Suppose, that the central bank does not plan to return the price level to its long-run trend. Firms that have not adjusted optimally for some time will be far off the new price level and thus contribute to increased dispersion of prices. When such firms are finally called upon to adjust optimally, a sizable price adjustment will contribute to higher inflation. If the central bank does return the price level to its long-run trend, firms that have not adjusted optimally for some time will find their prices to be close to the expected long-run price level; hence prices adjust little when these firms are called upon to do so. In addition, firms that happen to adjust optimally closer in time to the impact of the shock will be deterred from raising prices: if the price level will return to its long-run trend over time, larger price adjustments early on bear the risk of the firms' prices to be far off the price level over time absent future optimal adjustments. As price level targeting under discretion will drive the price level back to its long-run trend by construction, whereas inflation targeting considers past deviations of inflation from its target bygones, the former outperforms the latter.¹⁰

An equivalent description of the optimal commitment policy focuses on the dynamics of the output gap after an inflationary markup shock: an increase of inflation above its target is subsequently countered by tighter monetary policy resulting in a negative output gap. Anticipating such a policy, forward-looking firms restrain their price response in the first place. Rewriting equation (5), we express the output gap as the sum of current and future real interest rates using

$$x_t = -\frac{1}{\sigma_C} \left(i_t - \pi_{p,t+1} \right) - \frac{1}{\sigma_C} E_t \left[\sum_{j=1}^{\infty} \left(i_{t+j} - \pi_{p,t+1+j} \right) \right], \tag{9}$$

where we have set $g_{mu,t+j}^* = 0$ for all j, and we express inflation as the discounted sum of output gaps

$$\pi_{p,t} = \kappa_p(\sigma_L + \sigma_C)x_t + \kappa_p(\sigma_L + \sigma_C)E_t \left[\sum_{j=1}^{\infty} \beta^j x_{t+j}\right] + E_t \left[\sum_{j=0}^{\infty} \beta^j u_{p,t+j}\right].$$
(10)

Following equation (9), tight future monetary policy in terms of higher future real interest rates affects negatively the contemporaneous and expected future values of the output gap. In turn, expectations of a slowly closing output gap reduce the trade off between contemporaneous inflation and the output gap in equation (10) for a given markup shock. As the speed limit policy assigns dislike to changes in the output gap, $x_t^{gap} - x_{t-1}^{gap}$, it replicates the slow closing of the output gap under the optimal commitment policy.

Yet, the speed limit policy cannot replicate the optimal commitment policy as it fails to drive the price level back to its long-run trend. As under inflation targeting the price level changes permanently

 $^{^{10}}$ Following Vestin (2006), we prove in Appendix B.3 that for purely *transitory* markup shocks, as opposed to the ARMA(1,1) shock underlying Figure 1, optimal price level targeting under discretion replicates the optimal commitment policy. Even when the markup shock is *persistent*, the response of the economy under the optimal price level targeting and speed limit policy are close to optimal. Bibbie (2014) shows how to construct a loss function for the central bank that replicates under discretion the optimal commitment policy regardless of the persistence of the markup push shock.

under the speed limit policy. However, the built-in mechanism of closing the output gap slowly by running tighter monetary policy after an inflationary shock reduces the initial increase in the price level under the discretionary speed limit policy compared to inflation targeting. The problem with inflation targeting is not that deviations of inflation from target are considered bygones, but the lack of a mechanism to commit to tight future monetary policy after an inflationary shock.

The superior performance of price level targeting should not be mistaken as a general result. The speed with and the extent to which the price level returns to its long-run trend under the optimal commitment policy is sensitive to a range of model features, but the need to promise keeping monetary policy tight after inflationary shocks for longer is a general feature of the optimal commitment policy. Whether price level targeting or speed limit policy strikes a better balance between the path of the price level and other policy considerations when the policymaker lacks commitment is the quantitative question explored in this paper.

2.3 Extensions of the textbook NKM

The welfare ordering of the targeting frameworks in the textbook NKM is robust to the addition of other features. Inflation targeting is the preferred framework under commitment; price level targeting and speed limit policy outperform inflation targeting under discretion. Figure 2 explores the performance of the speed limit policy and price level targeting relative to inflation targeting as a function of the degree of price indexation, ι_p , for (i) the textbook NKM, (ii) the textbook NKM with a distorted steady state, (iii) a model with external consumption habit, (iv) and a model with sticky nominal wages. With the inflation targeting framework set to be the point of reference, a negative CEV indicates that the framework under investigation is inferior to inflation targeting and superior otherwise. We turn to a detailed discussion of each model variation.

2.3.1 The role of price indexation in the textbook NKM

The textbook NKM with price indexation is given by equations (4)-(8) with $0 < \iota_p \leq 1$. The lagged inflation rate enters equation (4) through the behavior of those firms that are not selected to reset prices optimally in the current period. Following the literature, we assume that these non-selected firms adjust prices by the geometric average of the steady state inflation rate and the inflation rate that prevailed in the previous period.

The weight ι_p governs the social desirability of undoing earlier changes in the price level. If non-selected firms adjust prices by the steady state inflation rate ($\iota_p = 0$), prices of these firms grow along the long-run trend of the price level. The optimal commitment policy limits welfare-costly price dispersion by promising to drive the price level back to its long-run trend over the medium run.

By contrast, when inflation is fully indexed ($\iota_p = 1$), the prices of non-selected firms reflect the deviations of the price level from its previous trend. The optimal commitment policy contains price dispersion, which is proportional to $(\pi_{p,t} - \pi_{p,t-1})^2$ for $\iota_p = 1$, by considering past deviations of inflation from its long-run target bygones and by allowing the price level to change permanently. If monetary policies attempted to revert the price level to its previous trend, it would cause unnecessary price dispersion in future periods. In analogy to the case without indexation, the optimal commitment policy under full indexation promises to return inflation (rather than prices) back to its long-run trend while it is the change in inflation (rather than the change in prices) that enters the true social loss function. This promise of the central bank deters firms that adjust prices optimally in a given period from choosing a price that is far off the price under the automatic indexation scheme for non-selected firms.

If the degree of price indexation falls strictly between 0 and 1, the price level is stationary under the optimal commitment policy, but the horizon over which the price level returns to its long-run trend lengthens with the degree of indexation. As in the case of the textbook NKM without indexation, a shock that calls for monetary tightening in the current period under the optimal commitment policy also calls for tighter policy in future periods as evidenced by a slow closing of the output gap.¹¹

Turning to the evaluation of targeting frameworks, note that in the presence of indexation to past price inflation, the inflation targeting objective cannot be parameterized to match the true social loss function in equation (8). Nevertheless, as shown in the first row of panels in Figure 2, optimal inflation targeting outperforms price level targeting and speed limit policy under commitment for any degree of price indexation, ι_p , owing to the fact that the objective functions for price level targeting and speed limit policy depart even more from the true social loss function. The dominance of inflation targeting is most striking when indexation is high and the price level returns to its long-run trend very slowly, if at all, under the optimal commitment policy. In particular, price level targeting performs poorly in this case given its tendency to force the price level back to trend too quickly.

Under the optimal commitment policy, the monetary authority relates acceptable deviations of inflation from target to the change in the output gap and past inflation:

$$\pi_{p,t} = -\frac{\theta_p}{1+\theta_p} \left(x_t - x_{t-1} \right) + \iota_p \pi_{p,t-1}.$$
(11)

An inflation targeting policymaker also aspires to set inflation in accordance with the change in the output gap. But such a policymaker responds to expected future changes in the output gap and discards the role of past inflation:

$$\pi_{p,t} = -\frac{\lambda_x^{IT}}{\lambda_x} \frac{\theta_p}{1+\theta_p} \left((x_t - x_{t-1}) - \beta \iota_p E_t \left(x_{t+1} - x_t \right) \right).$$
(12)

For a markup shock with a strong transitory component as under our parameterization, the optimal commitment policy allows inflation to rise and the output gap to turn negative initially followed by a period of below-target inflation and a gradual closing of the output gap. Under commitment, inflation targeting induces dynamics similar to those under the optimal commitment policy, when the central bank places a higher weight on stabilizing the output gap, $\lambda_x^{IT} > \lambda_x = (\sigma_L + \sigma_C) \frac{\theta_p \kappa_p}{1+\theta_p}$. The higher weight on the activity measure compensates for the fact that the expected (positive) output gap growth term in equation (12) operates in the opposite direction of the lagged inflation term in equation (11). Finally, inflation targeting under commitment performs strongly although it fails to drive the price level back fully to its original trend.

Under discretion, price level targeting and speed limit policy deliver better outcomes than inflation

¹¹ Stationarity of the price level (or the lack thereof) under the optimal commitment policy can be shown by writing the first order conditions as $-\frac{\theta_p}{1+\theta_p}x_t = \hat{p}_t - \iota_p\hat{p}_{t-1}$. For $\iota_p < 1$, the price level must return to its long-run trend for the output gap to be closed and inflation to be at its long-run target. For $\iota_p = 1$, the output gap is closed if and only if $\hat{p}_t - \hat{p}_{t-1} = \pi_{p,t} = 0$.

targeting for low and moderate degrees of price indexation ($\iota_p < 0.8$), but not for a high degree as inflation becomes increasingly persistent irrespective of policy. High inflation persistence feeds into higher expected inflation after an inflationary shock; an inflation targeting central bank will thus be expected to keep interest rates high to curb inflation. This feature of the textbook NKM with (high) indexation allows the discretionary central bank to indirectly commit to running tight future monetary policy and to preventing the output gap from closing too quickly thereby containing the initial response of inflation. The higher the degree of indexation, the more powerful is the fact that the inflation targeting objective replaces the quasi-difference in inflation in the true social loss function with inflation. In the limiting case of $\iota_p = 1$, optimal inflation targeting under discretion can even implement the optimal commitment policy under suitable assumptions for the nature of the underlying stochastic shocks—just as price level targeting can implement the optimal commitment policy for the case of $\iota_p = 0$.

More formally, provided that shocks are sufficiently small to prevent the zero lower bound constraint from binding, note that in the model without indexation, $\iota_p = 0$, the price level targeting central bank adopts the objective function $L_t^{PLT} = \hat{p}_t^2 + \lambda_x^{PLT} (x_t)^2$ and faces the NKPC of the form

$$(\hat{p}_t - \hat{p}_{t-1}) = \kappa_p (\sigma_L + \sigma_C) x_t + \beta E_t (\hat{p}_{t+1} - \hat{p}_t) + u_{p,t}.$$
(13)

In the case of full indexation, $\iota_p = 1$, the inflation targeting central bank adopts the objective function $L_t^{IT} = \pi_{p,t}^2 + \lambda_x^{IT} (x_t)^2$ and faces the NKPC of the form

$$(\pi_{p,t} - \pi_{p,t-1}) = \kappa_p(\sigma_L + \sigma_C)x_t + \beta E_t (\pi_{p,t+1} - \pi_{p,t}) + u_{p,t}.$$
(14)

Substituting $\pi_{p,t}$ with \hat{p}_t reveals that inflation targeting under discretion in the model with $\iota_p = 1$ is isomorphic with price level targeting under discretion in the model with $\iota_p = 0$. As the optimal commitment policy stabilizes the price level absent indexation, but stabilizes the inflation rate under full indexation, inflation targeting performs close to optimal when $\iota_p = 1$ by analogy. Price level targeting and speed limit policy impose too tight monetary policy in future periods when prices are fully indexed.¹²

Finally, this discussion shows that for a high degree of indexation optimal inflation targeting under discretion can outperform inflation targeting under commitment. This observation raises the question under what conditions it is desirable to assign the central bank a (simple) loss function that departs from the true social loss function when policymakers can fully commit to future actions.

2.3.2 Inefficient steady state

Theoretical works building on the New Keynesian paradigm often assume that the steady state of the model is efficient as subsidies/taxes offset the distortions from monopolistic competition. By contrast, works on empirical DSGE models—including the seminal contributions of Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007)—tend to abstract from such subsidies and taxes. The (in-)efficiency of the steady state affects the welfare ranking of policies through the definition of the output gap.

¹² For an intermediate degree of indexation, $0 < \iota_p < 1$, hybrid price level targeting with the objective function $L_t^{hPLT} = (\hat{p}_t - \iota_p \hat{p}_{t-1})^2 + \lambda_{hPLT} (x_t^{gap})^2$ can be shown to perform at least as well as inflation or price level targeting. See Roisland (2005) and Gaspar, Smets, and Vestin (2007) for additional discussion.

Following Benigno and Woodford (2005) the true social loss function in the model with an inefficient steady state satisfies

$$L_t = (\sigma_L + \sigma_C) \left(\tilde{x}_t\right)^2 + \frac{1 + \theta_p}{\theta_p \kappa_p} \left(\pi_{p,t} - \iota_p \pi_{p,t-1}\right)^2$$
(15)

where \tilde{x}_t denotes the welfare-relevant output gap. The structural equations are given by

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p (\sigma_L + \sigma_C) \tilde{x}_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + \frac{\sigma_L + \sigma_C}{\sigma_L + \sigma_C + (\Phi - 1)(1 + \sigma_L)} u_{p,t}$$
(16)

$$\tilde{x}_{t} = E_{t}\tilde{x}_{t+1} - \frac{1}{\sigma_{C}}\left(i_{t} - E_{t}\pi_{p,t+1} - \tilde{g}_{mu,t}\right)$$
(17)

$$\tilde{y}_t = \frac{1+\sigma_L}{\sigma_L+\sigma_C} \hat{\xi}_{A,t} - \frac{(\Phi-1)\frac{1+\sigma_L}{\sigma_L+\sigma_C}}{\sigma_L+\sigma_C+(\Phi-1)(1+\sigma_L)} u_{p,t}$$
(18)

with $\tilde{g}_{mu,t} = \sigma_C [E_t \tilde{y}_{t+1} - \tilde{y}_t]$. At first glance, it appears that we have merely replaced the output gap term " x_t " with " \tilde{x}_t " and rescaled the impact of the markup shock. However, the two definitions of the output gap respond differently to the markup shock. Under the definition $x_t \equiv \hat{y}_t - \hat{y}_t^*$, the target output level \hat{y}_t^* defined in equation (6) does not respond to the markup shock; all else equal under the definition $\tilde{x}_t \equiv \hat{y}_t - \tilde{y}_t$, the output gap will respond by less to a markup shock since the relevant output level \tilde{y}_t defined in equation (18) moves in the same direction as actual output. Absent steady state distortions, i.e., $\Phi = 1$, the two definitions of the output gap coincide. Furthermore, in response to a technology shock, the divine coincidence continues to apply under the optimal commitment policy regardless of steady state distortions.

Applying this change in the definition of the relevant output gap to the three targeting frameworks, i.e. $x_t^{gap} = \tilde{x}_t$, the second row of panels in Figure 2 plots the results for the case of a distorted steady state with the sales subsidy set equal to zero. Both under commitment and discretion, price level targeting and speed limit policy appear closer to inflation targeting than in the case of an efficient steady state. The reason for this finding is the reduced impact of the markup shock in the model with an inefficient steady state ($\Phi > 1$): in the NKPC the markup shock is scaled by a term smaller than unity and movements in the output gap are curtailed by the adjustments in \tilde{y}_t . With the effective magnitude of the markup shock reduced the welfare losses under each targeting framework relative to the optimal commitment policy shrink.

The behavior of the output gap, and thus the ranking of targeting frameworks, is sensitive to the definition of potential output. If $x_t^{gap} = x_t$ despite the distorted steady state the measured output gap is larger after a markup shock all else equal, and calls for a larger adjustment in policy than under the output gap definition of \tilde{x}_t . When using x_t as the output gap measure despite the presence of steady state distortions, inflation targeting improves its performance and dominates price level targeting and speed limit policy already for the moderate degree of price indexation of $\iota_p = 0.4$.

2.3.3 Habit persistence

When the household's utility function depends on a quasi-difference in consumption (habit persistence), the implied output gap enters with its quasi-difference into the (approximate) true social loss function. Under external consumption habits as in Smets and Wouters (2007), the linear-quadratic form of the model is

given by the loss function

$$L_t = \sigma_L (x_t)^2 + \frac{\sigma_C}{(1-h)(1-h\beta)} (x_t - hx_{t-1})^2 + \frac{1+\theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2$$
(19)

and the structural equations

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p \widehat{mc}_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t}$$
(20)

$$\widehat{mc}_t = \sigma_L x_t + \frac{\sigma_C}{1-h} \left(x_t - h x_{t-1} \right) + \frac{h\beta}{1-h\beta} g^*_{mu,t}$$
(21)

$$(x_t - hx_{t-1}) = E_t (x_{t+1} - hx_t) - \frac{1 - h}{\sigma_C} (i_t - E_t \pi_{p,t+1} - g_{mu,t}^*)$$
(22)

where $g_{mu,t}^*$ is defined as $g_{mu,t}^* = \frac{\sigma_C}{1-h} \left[E_t \left(\hat{y}_{t+1}^* - h \hat{y}_t^* \right) - \left(\hat{y}_t^* - h \hat{y}_{t-1}^* \right) \right]$. The efficient output level satisfies the difference equation

$$\sigma_L \hat{y}_t^* + \frac{\sigma_C}{(1-h)(1-h\beta)} \left(\hat{y}_t^* - h \hat{y}_{t-1}^* \right) - h\beta \frac{\sigma_C}{(1-h)(1-h\beta)} E_t \left(\hat{y}_{t+1}^* - h \hat{y}_t^* \right) = (1+\sigma_L) \hat{\xi}_{A,t}.$$
 (23)

The degree of habit persistence is measured by the parameter $h \in [0, 1)$. The model with habit persistence features endogenous persistence, since the lagged value of the output gap enters into the NKPC and the aggregate demand curve, which in turn affects the dynamics of inflation.¹³ The presence of the lagged output gap term in the true social loss function (19) strengthens the motive for smoothing the evolution of the output gap under the optimal commitment policy.

As shown in the third row of panels in Figure 2, the speed limit policy can outperform inflation targeting under commitment for a moderate degree of habit persistence (h = 0.7) and low inflation inertia due to little or no price indexation. Abstracting from price indexation, the true social loss function resembles the objective function of the speed limit policy framework: A reasonably high degree of habit persistence implies that most of the weight is placed on the term $(x_t - hx_{t-1})^2$ in the true social loss function and the optimal speed limit policy under commitment mimics the optimal commitment policy. Overall, under commitment, the differences between speed limit policy and inflation targeting are much reduced for any degree of price indexation. Price level targeting performs relatively poorly under commitment for a high degree of price indexation as in the previous two model variations.

When policy is conducted under discretion, inflation targeting never outperforms the other two frameworks regardless of the degree of inflation indexation. Compared to the textbook NKM the differences between frameworks are of much larger magnitude. The advantage of speed limit policy and price level targeting over inflation targeting narrows considerably as the degree of price indexation ι_p approaches 1. However, the isomorphism of inflation targeting for $\iota_p = 1$ with price level targeting for $\iota_p = 0$ under discretion no longer applies in the presence of consumption habits. Higher inflation persistence as a result of indexation allows the discretionary inflation targeting central bank to commit indirectly to tighter

¹³ When habits are external, the decisions taken by the household members are not efficient under flexible prices even if a sales subsidy removes the distortions from monopolistic competition in the goods market. To render the steady state of the model efficient, we introduce a consumption tax; yet, the dynamics remain inefficient even for technology shocks. With the term $\frac{h\beta}{1-h\beta}g_{mu,t}^*$ entering equation (20) through the definition of the marginal cost term, \widehat{mc}_t , the central bank is unable to perfectly stabilize inflation and the welfare-relevant output gap in response to technology shocks. As discussed in Leith, Moldovan, and Rossi (2012) and Woodford (2003a), consumption habits have to be specified as internal in order for the divine coincidence to re-emerge.

monetary policy in the future after an inflationary shock. Yet, the expected future policy under inflation targeting is not tight enough. When consumption experiences habit persistence, the optimal commitment policy engages in more smoothing of the output gap which strengthens the motive of keeping monetary policy tight after an inflationary shock. The inflation targeting objective does not capture this additional motive and provides less stabilization of the economy.

2.3.4 Sticky wages

Sticky nominal wages as in Erceg, Henderson, and Levin (2000) are the final feature that we consider in isolation. In detail, the loss function can be shown to satisfy

$$L_t = (\sigma_L + \sigma_C) (x_t)^2 + \frac{1 + \theta_p}{\theta_p \kappa_p} (\pi_{p,t} - \iota_p \pi_{p,t-1})^2 + \frac{1 + \theta_w}{\theta_w \kappa_w} (\pi_{w,t} - \iota_w \pi_{p,t-1})^2$$
(24)

while the structural equations are summarized by

$$(\pi_{p,t} - \iota_p \pi_{p,t-1}) = \kappa_p \widehat{mc}_t + \beta E_t (\pi_{p,t+1} - \iota_p \pi_{p,t}) + u_{p,t}$$
(25)

$$\widehat{mc}_t = \hat{\omega}_t - \hat{\xi}_{A,t} \tag{26}$$

$$(\pi_{w,t} - \iota_w \pi_{p,t-1}) = \kappa_w \left(\widehat{mrs}_t - \hat{\omega}_t \right) + \beta E_t \left(\pi_{w,t+1} - \iota_w \pi_{p,t} \right) + u_{w,t}$$
(27)

$$\widehat{mrs}_t - \hat{\omega}_t = (\sigma_L + \sigma_C) x_t - (\hat{\omega}_t - \hat{\omega}_t^*)$$
(28)

$$(\hat{\omega}_t - \hat{\omega}_t^*) = (\hat{\omega}_{t-1} - \hat{\omega}_{t-1}^*) + \pi_{w,t} - \pi_{p,t} - (\hat{\omega}_t^* - \hat{\omega}_{t-1}^*)$$
(29)

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_C} \left(i_t - E_t \pi_{p,t+1} - g_{mu,t}^* \right).$$
(30)

The NKPC for wages, equation (27), links wage inflation, $\pi_{w,t}$, to the gap between the marginal rate of substitution (between consumption and leisure), \widehat{mrs}_t , and the real wage, $\hat{\omega}_t$. The policymaker places weight on stabilizing price and wage inflation with the weights inversely related to the slopes of the respective NKPCs. To maintain comparability with the previous models we focus on price markup shocks.

As in the model with flexible wages, a policy that promises to be tight in the future—summarized by the discounted sum of future (negative) output gaps in equation (31)—acts towards stabilizing the output gap, and (a weighted average of price and wage) inflation in the impact period:

$$\pi_{p,t} + \frac{\kappa_p}{\kappa_w} \pi_{w,t} = \kappa_p (\sigma_L + \sigma_C) x_t + \kappa_p (\sigma_L + \sigma_C) E_t \left[\sum_{j=1}^{\infty} \beta^j x_{t+j} \right] + E_t \left[\sum_{j=0}^{\infty} \beta^j u_{p,t+j} \right].$$
(31)

The optimal split between movements in wage and price inflation depends on the relative stickiness between prices and wages as captured by the slope coefficients κ_p and κ_w and the evolution of the real wage. According to equation (25),

$$\pi_{p,t} = \kappa_p E_t \left[\sum_{j=0}^{\infty} \beta^j \omega_{t+j} \right] + E_t \left[\sum_{j=0}^{\infty} \beta^j u_{p,t+j} \right].$$
(32)

If the central bank allows the real wage to fall persistently, it can lean against the initial rise in inflation. However, a decline in the future real wage also requires that prices rise faster than wages. If the policymaker places a high weight on stabilizing price inflation, the adjustment process has to operate more through wage inflation. Under the optimal commitment policy, tight monetary policy in the periods following an inflationary shock undoes almost all of the earlier changes in the price and wage level, but prices and wages are not stationary unless there is no inflation indexation, i.e., $\iota_p = \iota_w = 0$. The speed with which price and wage changes are undone depends on the degree of indexation. Unless both prices and wages are highly indexed, this process is rather fast. When prices and wages are fully indexed ($\iota_p = \iota_w = 1$), there is no partial undoing of earlier changes in prices and wages at all.

With these features of the optimal commitment policy in mind, we return to Figure 2. The fourth row of the figure shows that inflation targeting outperforms the other frameworks, when the policymaker can commit. To induce outcomes that are close to the optimal commitment policy, inflation targeting under commitment must place a sufficiently low weight on price inflation to prevent wages from carrying too much of the burden of the real wage adjustment. Overall, when the central bank implements its objective under commitment, the welfare differences across targeting frameworks are small and comparable to those in the previous models.

When the targeting frameworks are implemented under discretion, speed limit policy and price level targeting dominate inflation targeting—and for the case of no wage indexation depicted in Figure 2—this finding does not depend on the degree of price indexation. Given the features of the optimal commitment policy, price level targeting is best suited to stabilize the economy although it pushes prices and wages back to their long-run trends. Discretionary inflation targeting views all changes to prices and wages as permanent; promising to revert price inflation to its long-run target is not a sufficient deterrent against changes in prices and wages. Finally, the speed limit policy keeps the initial response of prices and wages in check as the private sector expects changes in the output gap to be smooth reflecting once again the idea to keeping future monetary policy tight after an inflationary shock. Overall, the welfare outcomes under the speed limit policy are close to those under price level targeting.

In contrast to the previous models, the relative performance of discretionary inflation targeting worsens when prices are increasingly indexed while keeping the degree of wage indexation unchanged. More price indexation implies more persistent price inflation after a markup shock, which leads the inflation targeting central bank wanting to stabilize price inflation more aggressively and thereby to put more burden on wage inflation in the adjustment process. The performance of inflation targeting improves for a higher degree of price indexation, when wage indexation is also high—in this case the optimal commitment policy ends up stabilizing inflation rates and does little to push prices and wages back towards their previous trends.¹⁴

Finally, if we keep the degree of price indexation constant and low, a higher degree of wage indexation implies a better relative performance of the optimal inflation targeting under discretion. An increase in wage indexation has little impact on the persistence of price inflation and on the optimal parameterization of the inflation targeting objective. Furthermore, changes in prices and wages are quickly pushed back under the optimal commitment policy. However, the welfare losses under each framework relative to the optimal commitment policy shrink since wage dispersion, measured by $\pi_{w,t} - \iota_w \pi_{p,t}$, drops for higher values of ι_w . While the welfare differences become smaller, the ranking of targeting frameworks is preserved.

 $^{^{14}}$ If wage indexation is kept fixed at a high value, the advantage of speed limit policy and price level targeting over inflation targeting first increases as the degree of price indexation rises from 0 before eventually falling (and possibly turning negative) as the degree of price indexation approaches 1.

2.3.5 Comparison with Walsh (2003)

Walsh (2003) concludes that a high degree of price indexation is necessary in order for inflation targeting to outperform speed limit policy and price level targeting when policymakers cannot commit to future policy paths.¹⁵ Our analysis generalizes this insight to the case of sticky wages: all prices and wages that experience nominal rigidities must be highly indexed for inflation targeting to perform strongly under discretion. Furthermore, our findings point to the role of consumption habits as increasing the central bank's motive for keeping future monetary policy tight after an inflationary shock to curb the dispersion of prices and wages. This feature is not captured in Walsh (2003) who assumes a model-invariant social loss function of the form $\pi_{p,t}^2 + \lambda (x_t^{gap})^2$ in departure from the linear-quadratic approximation of the preferences of the representative household.

3 Empirical models of the business cycle

Moving beyond the textbook NKM, we extend our analysis to the medium scale CEE/SW model which features sticky nominal prices and wages both with partial indexation to past inflation, physical capital and investment with capital utilization and investment adjustment costs, habit persistence in consumption, a variable elasticity of substitution between intermediate goods as in Kimball (1995) and the same for labor types, a distorted steady state, and shocks to technology, the risk premium, government spending, investment, price and wage markups, and monetary policy as detailed in Appendix D.

An important step in extending our analysis is to obtain a second-order accurate approximation to the preferences of the representative household. We follow a numerical approach. Let the $N \times 1$ vector of endogenous variables in the CEE/SW model be denoted by x_t , with the partition $x_t = (\tilde{x}'_t, i_t)'$. The variable i_t is the policy instrument of the central bank. The vector ζ_t refers to the set of exogenous variables. Given the central bank's choice of the policy instrument for all periods $t \ge t_0$, $\{i_t\}_{t=t_0}^{\infty}$, the remaining N-1 endogenous variables satisfy the N-1 structural model equations

$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0 \tag{33}$$

in equilibrium.

With the intertemporal preferences of society given by $\mathcal{U} = E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t)$, the optimal commitment policy is derived from the maximization program

$$\max_{\{x_t\}_{t=t_0}^{\infty}} E_0 \sum_{t=t_0}^{\infty} \beta^{t-t_0} U(x_{t-1}, x_t, \zeta_t)$$

s.t.
$$E_t g(x_{t-1}, x_t, x_{t+1}, \zeta_t) = 0$$

$$g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}.$$
 (34)

The constraint $g(x_{t_0-2}, x_{t_0-1}, x_{t_0}, \zeta_{t_0-1}) = \bar{g}_{t_0}$ captures the policymaker's ability to pre-commit before the

 $^{^{15}}$ See Appendix C for model details. Figure ?? replicates our analysis for the model in Walsh (2003) for both the case of discretion and commitment with the latter one not being included in Walsh (2003).

beginning of time in $t = t_0$ to embed the idea of optimality from a timeless perspective as in Woodford (2003a).¹⁶

Using the toolbox developed in Bodenstein, Guerrieri, and LaBriola (2014), the first-order conditions associated with the program in (34) can be used to obtain the purely quadratic approximation to the intertemporal preferences of society. The true social loss function

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\frac{1}{2} \hat{x}'_t A(L) \hat{x}_t + \hat{x}'_t B(L) \zeta_{t+1} \right] + \beta^{-1} \hat{\varphi}^{*\prime}_{t_0-1} C(0) \hat{x}_{t_0}$$
(35)

correctly ranks (the first-order accurate) outcomes $\{\hat{x}_t\}_{t=t_0}^{\infty}$ obtained under any monetary policy from the perspective of the optimal commitment policy (from a timeless perspective). The matrices A(L) and B(L)represent the approximation of the preferences with "L" denoting the lag-operator. As discussed in Benigno and Woodford (2012), the term $\beta^{-1}\hat{\varphi}_{t_0-1}^{*'}C(0)\hat{x}_{t_0}$ punishes violations of the pre-commitment constraint under the assessed policy in the case of discretion.¹⁷ Appendix A provides the details of obtaining and evaluating the welfare criterion (35) and of solving for the decision rules under discretion and commitment.

As in Section 2, we compare the welfare implications under inflation targeting, speed limit policy, and price level targeting both under commitment and discretion. At times, we also report results from two nominal income targeting frameworks included in Walsh (2003):

1. nominal income targeting 1 (NIT)

$$L_t^{NIT} = \pi_{p,t}^2 + \lambda_x^{NIT} \left(\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1} \right)^2$$
(36)

2. nominal income targeting 2 (NIT-II)

$$L_t^{NIT-\text{II}} = (x_t^{gap})^2 + \lambda_x^{NIT-\text{II}} (\pi_{p,t} + \hat{y}_t - \hat{y}_{t-1})^2.$$
(37)

The optimal parameterization of a targeting framework, i.e., the optimal choice of λ_x^{TF} , minimizes the welfare distance between the targeting framework and the optimal commitment policy as measured by the welfare criterion in equation (35). In this section, we follow Smets and Wouters (2007) in measuring the output gap as the difference between actual output and the potential output defined as the output level that would have prevailed absent nominal rigidities and inefficient markup shocks to prices and wages.

Our analysis of targeting frameworks in the CEE/SW model proceeds as follows. First, we fix the parameters of the model at their posterior mode estimated in Smets and Wouters (2007). We then explore alternative parameterization of the CEE/SW model obtained by drawning from the Laplace approximation to the posterior distribution in Smets and Wouters (2007).

We close by assessing robustness of our findings along three dimensions. First, we compute optimal targeting frameworks for the CEE/SW model when the model is estimated with data for the euro area

 $^{^{16}}$ Benigno and Woodford (2012) and Debortoli and Nunes (2006) show that assuming policy to be conducted under suitable precommitments is generally needed to obtain a purely quadratic approximation to the preferences of the representative household. For the models in Section 2, the assumption of the timeless perspective is key for deriving the true social loss function when the steady state is not efficient; see also Appendix B.

 $^{^{17}}$ In practice, the correction term tends to be small. Although we did not emphasize this term in Section 2, we did include it in our computations when needed.

instead of the United States. Second, we investigate how our findings are affected by the difficulties of distinguishing between wage markup shocks and preference shocks that shift the marginal utility of labor. And finally, we explore the implications resulting from the zero lower bound on nominal interest rates.

3.1 Targeting frameworks in the CEE/SW model

Figure 3 summarizes our findings for the CEE/SW model. As before, we consider variations in the degrees of price and wage indexation. The top row of panels shows how the degree of price indexation ι_p impacts the relative ordering of the five targeting frameworks in the CEE/SW model. A vertical line marks the posterior mode of $\iota_p = 0.22$. The results nicely relate to our earlier findings. With consumption habits at 0.71 and sticky nominal wages, the optimal speed limit policy is a close second to inflation targeting when the policymaker acts under commitment. As price level targeting places too much importance on price stability and disregards the need to smooth the evolution of the output gap, the welfare outcomes are somewhat inferior. The two nominal income targeting frameworks are strictly outperformed by the speed limit policy and the price level targeting framework. The overall magnitude of the welfare differences is significantly larger in the CEE/SW model than in the simple NKMs, reflecting the presence of additional model features and shocks that introduce welfare-relevant policy trade-offs.

Under discretion, the speed limit policy framework strictly outperforms all other frameworks irrespective of the degree of price indexation. At the posterior mode parameterization of the model, the optimal speed limit policy exceeds welfare under inflation targeting by more than 0.30% of steady state consumption, whereas the advantage of the price level targeting framework over inflation targeting is a bit smaller with 0.25%. As in the textbook NKM with sticky wages, the advantage of the optimal speed limit policy over inflation targeting is larger when the degree of price indexation is higher while keeping the degree of wage indexation constant. Even the two nominal income targeting frameworks strongly outperform inflation targeting in the discretionary Markov equilibrium.

As shown in Figure 4, discretionary speed limit policy and price level targeting capture key features of the optimal commitment policy in the CEE/SW model in response to price markup and wage markup shocks. Given the estimated moderate degree of indexation ($\iota_p = 0.22$ and $\iota_w = 0.59$), price and wage dispersion are closely related to price and wage inflation, which are kept low by the promise of tight future monetary policy after an inflationary shock under the optimal commitment policy. As a result, the price and wage levels return slowly towards their pre-shock trends, although not completely. Noticeably, the speed limit policy considers deviations of price and wage inflation from their long-run target values bygones. However, given the built-in promise of keeping future policy tight after an inflationary shock this policy reduces overall inflation and the rise in the price and wage levels. Price level targeting as a monetary policy strategy signals tight monetary policy in response to inflationary shocks through explicitly promising to return prices and wages to their earlier trends. For a moderate degree of indexation, the resulting stabilization of price and wage inflation is close to optimal. By contrast, the inflation targeting objective does not include built-in features that would allow the central bank to promise tight future monetary policy in an environment with low to moderate inflation indexation under discretion. Thus, the inflation targeting central bank is less effective at stabilizing the economy: Inflation is persistently higher and the output gap drops by more on impact compared to the optimal commitment policy and the other targeting frameworks in Figure 4.

The CEE/SW model abstracts from taxes/subsidies that could correct the distortions associated with monopolistic competition in the production of intermediate goods and the labor market. The second row of panels in Figure 3 reveals that if these distortions are removed, inflation targeting improves its relative performance slightly.

As for the textbook NKM with sticky wages, we vary the degree of wage indexation in the bottom row of panels. Varying the degree of wage indexation away from its posterior mode of $\iota_w = 0.59$ while keeping the degree of price indexation at its posterior mode of $\iota_p = 0.22$ reveals that a lower degree of wage indexation goes along with a relatively poorer performance of inflation targeting under discretion as in the previous section. Under commitment, changing the degree of wage indexation impacts the relative performance of the frameworks in a manner similar to changes in price indexation.

3.2 Deconstructing the results

While the outcomes in the CEE/SW model resemble those in Section 2, we also consider one of the many sequences of expanding the textbook NKM step-by-step to the CEE/SW model. We present results for the case of discretion. Figure 5 plots the CEV values for each framework relative to the inflation targeting framework under discretion. Starting from the textbook NKM with preferences being specified as in Smets and Wouters (2007)—titled SW–Woodford—and using the parameters estimated by Smets and Wouters (2007) where applicable we introduce the following changes step-by-step:

- remove taxes/subsidies for intermediate goods,
- government spending, physical capital and investment, including capital utilization and investment adjustment costs, and related shocks,
- sticky wages (with a wage subsidy to offset distortions in the labor market and no wage markup shock),
- a wage markup shock,
- remove the wage subsidy,
- habit persistence,
- a higher degree of nominal rigidities measured by the probabilities of not adjusting prices or wages optimally from $\xi_p = 0.65$ and $\xi_w = 0.73$ to $\xi_p = 0.85$ and $\xi_w = 0.88$, respectively, in order to match the slopes of the NKPC between a model with and without a variable elasticity of substitution (Kimball aggregator),
- a variable elasticity of substitution as in Kimball (1995).

The figure confirms the importance of indexation, sticky wages, and habit persistence in determining the ranking of targeting frameworks under discretion. Absent sticky wages, a higher degree of price indexation plays out in favor of inflation targeting under discretion. In the presence of sticky nominal wages this finding is overturned. Furthermore, the magnitude of welfare differences increases with sticky wages and the associated wage markup shocks. Habit persistence in consumption raises the overall welfare costs of not implementing the optimal commitment policy and thus the advantage of speed limit policy and price level targeting over inflation targeting. With the true social loss function featuring an explicit motive for

smoothing the quasi-difference in the output gap, the speed limit policy gets even closer to the price level targeting framework. The role of capital accumulation and investment adjustment costs on the quantitative differences between targeting frameworks is relatively minor.

In addition to the features discussed in Section 2, the variable elasticity of substitution is the other feature of quantitative importance as it increases the strategic complementarity in price setting. The Kimball aggregator impacts our outcomes mostly through changing the slope of the NKPCs. Moving from the bottom left panel in the figure (constant elasticity of substitution and $\xi_p = 0.65$ and $\xi_w = 0.73$) to the bottom right panel (variable elasticity of substitution and $\xi_p = 0.65$ and $\xi_w = 0.73$) directly, the welfare differences between price level targeting (or speed limit policy) and inflation targeting triple. Yet, considering the intermediate step of the middle panel (constant elasticity of substitution and $\xi_p = 0.88$) and $\xi_w = 0.88$) reveals that this increase could also be obtained by raising the degree of nominal rigidities while keeping the slopes of the NKPCs the same between the last two panels. Similar conclusions regarding the importance of the various model features emerge when we change the sequence of introducing them or when policymakers act under commitment.

3.3 Robustness to alternative parameterizations

To explore the sensitivity of our findings to alternative, yet empirically plausible, parameter choices. We draw 30000 parameter specifications from the Laplace approximation to the posterior distribution Smets and Wouters (2007) and we

- 1. compute the optimal weights on the activity measure in the objective functions, λ_x^{TF} , associated with inflation targeting, speed limit policy, and price level targeting for each parameter draw and compare welfare for each parameter draw under these optimal weights,
- 2. compare welfare across targeting frameworks for each parameter draw when the weights on the activity measure in the objective function are fixed at specific values.

We exclude the NIT and NIT-II framework from this exercise as they were strictly dominated by price level targeting and speed limit policy.

The first experiment, referred to as the "optimal weights case," confirms that the ordering of targeting frameworks is robust to alternative empirically plausible parameterizations of the CEE/SW model. Figure 6 plots the distribution of welfare losses relative to the optimal commitment policy (expressed in CEV) for each draw of parameters and targeting framework. Under commitment (the top row of panels), the distribution of welfare losses is similar across targeting frameworks, although the losses tend to be slightly smaller under inflation targeting. The median loss under inflation targeting is -0.0288, whereas it reaches -0.0538 under price level targeting and -0.0454 under the speed limit policy. Large losses are rare for all frameworks. Table 2 Panel (a) reports the frequency with which each of the frameworks performs better than the remaining two. The optimal inflation targeting framework emerges as the winner for 97% of the parameter draws. Table 2 Panel (d) is designed to shed light on the magnitude of the welfare differences. For each draw of parameters we compute the welfare difference between a given targeting framework and the best performing framework of the remaining two and report the percentiles of the resulting distribution of welfare differences in increasing order. Since inflation targeting almost always performs best, when

policymakers can commit, the differences reported in columns 3 and 4 basically coincide with the differences between price level targeting and inflation targeting and between the speed limit policy and inflation targeting, respectively. Only for 5% of the parameter draws does the difference between the price level targeting and the inflation targeting framework exceed -0.0493; for the speed limit policy framework, the value is even smaller with -0.0280. For the inflation targeting framework, the advantage over the next best targeting framework is smaller than 0.0280 for about 95% of the draws. The values at the *n*th percentile for column 2 (IT) and the (100 - n)th percentile for column 4 (SLP) indicate that the speed limit policy framework is the second-best performing framework for most parameter draws.

Under discretion, the distributions of welfare losses induced by the three targeting frameworks look much less alike. In Figure 6 (the middle row of panel), the distribution of welfare losses relative to the optimal commitment policy is noticeably more dispersed for price level targeting and, in particular, for inflation targeting than under commitment. By contrast, the distribution under the speed limit policy under discretion may deliver better welfare outcomes (1) than the optimal speed limit policy under commitment, and (2) than optimal inflation targeting under commitment. The first claim is true for any parameterization we consider; the second claim is true for more than 50% of the parameter draws and in particular it is true when the parameters in the CEE/SW model are fixed at their posterior mode. Table 2, Panel (a) further reveals the superiority of speed limit policy under discretion. It is found to perform better than inflation targeting and price level targeting for most parameter draws (around 98%). As shown in Panel (d), the advantage of the speed limit policy framework over the inflation targeting framework can be sizeable (column 5). Although price level targeting performs consistently better than inflation targeting under discretion, it rarely performs best (column 6).

The final row of Figure 6 plots the cumulative distribution functions of the optimal weights on the activity measure. For each framework, the optimal weights tend to be larger and the distributions of weights are more dispersed under commitment than under discretion. For example, the median weight under the speed limit policy framework is $\lambda_x^{SLP} = 11.86$ for commitment, but only $\lambda_x^{SLP} = 3.39$ for discretion.

The robust performance of the speed limit policy framework across commitment and discretion not only applies to a wide range of empirically plausible parameterizations of the CEE/SW model when the weights on the activity measure are set optimally for each draw and framework. Our second set of experiments finds that the performance of the speed limit policy framework is also less sensitive to the exact choice of the weight on the activity measure: (1) We fix the weight on the activity measure for each targeting framework at the value found to be optimal when the parameters in the CEE/SW model are fixed at their posterior mode (under commitment and discretion, respectively) and compute the welfare losses relative to the optimal commitment policy for each of the 30000 parameter draws. (2) We fix the weight on the activity measure for each targeting framework at the value found to be optimal framework at the value found to be optimal framework at the value found to be optimal under commitment (discretion) when the parameters in the CEE/SW model are fixed at their posterior mode and compute the welfare losses relative to the optimal commitment policy for each of the 30000 parameter draws. (2) We fix the weight on the activity measure for each targeting framework at the value found to be optimal under commitment (discretion) when the parameters in the CEE/SW model are fixed at their posterior mode and compute the welfare losses relative to the optimal commitment policy for each of the 30000 parameter draws, but solve the model under the assumption that the policymaker acts under discretion (commitment). Subsequently, we refer to (1) as the "fixed weights case" and to (2) as the "exchanged weights case."

As reported in Table 2, Panel (b), in the fixed weights case, the speed limit policy performs best for 16.5% of the parameter draws under commitment—up from 2.7% in the original experiment—and it maintains its superior performance under discretion by outperforming the other frameworks for 98% of the draws. Figure 7 plots the distribution of welfare losses under the fixed weights case relative to the optimal weights case. The welfare losses that are caused by the policymaker using the optimal weights for a given parameter draw are small under commitment across regimes, but are often sizeable under discretion for both price level targeting and, in particular, inflation targeting.

The exchanged weights case explores the sensitivity of the targeting frameworks to both parameter uncertainty and uncertainty about the ability of the policymaker to commit. As shown in Table 2, Panel (c) when policy is conducted under commitment, but the policymaker uses the weights found to be optimal under discretion for the posterior mode parameterization of the CEE/SW model, the speed limit policy framework performs best for 99% of the parameter draws. Under discretion, the speed limit policy framework performs best for 98% of the draws. Figure 8 also plots the distribution of welfare losses under the exchanged weights case relative to the optimal weights case for each framework. The inflation targeting framework is very sensitive to getting the weight on the activity measure right as evidenced by the high share of large welfare losses exceeding 1% (measured as CEV) for more than 50% of the parameter draws. Under the speed limit policy framework such large losses are never observed.

The speed limit policy framework emerges as the most desirable setting in our analysis of the CEE/SW model. Across parameterizations, the optimal speed limit policy consistently outperforms the inflation targeting and the price level targeting framework under discretion; under commitment the speed limit policy framework is a very close second to the inflation targeting framework; the optimal speed limit policy framework implemented under discretion delivers higher social welfare than optimal inflation targeting under commitment. Finally, the performance of the economy under a speed limit policy is much less sensitive to the exact parameterization of the objective function which is of relevance if the policymaker faces uncertainty about the correct specification of the economy.

3.4 Additional robustness checks

We conclude our analysis with robustness checks regarding the data used to estimate the model, the role of the relative importance of labor supply shocks versus wage markup shocks, and the limitations of monetary policy imposed by the zero lower bound constraint on the nominal interest rate.

3.4.1 Robustness to alternative data

Smets and Wouters (2007) estimated the CEE/SW model using U.S. data. Figure 9 compares the performance of all five targeting frameworks when the CEE/SW model is estimated using data for the euro area instead.¹⁸

¹⁸ Smets and Wouters (2005) estimate a medium-scale DSGE model for the euro area, but the details of the model differ from those in Smets and Wouters (2007). To maintain comparability of results, we estimate the model specified in Smets and Wouters (2007) using data for the euro area from the Area Wide Model database (see Fagan, Henry, and Mestre (2005)). Data on consumption, investment, GDP, hours and wages are expressed in 100 times the log. Inflation is the first difference of the log GDP deflator. The interest rate is the short-term interest in the AWM database. As stated in Smets and Wouters (2005), total employment data is used in place of hours worked due to the absence of hours worked data for the euro area.

Qualitatively, the results for the euro area are similar to those derived from U.S. data. From a quantitative perspective, the case for price level targeting and speed limit policy is even stronger. Their advantage over inflation targeting measured in terms of steady state consumption doubles under discretion. Under commitment the inflation targeting framework maintains a small advantage over speed limit policy and price level targeting.

3.4.2 Shocks to labor supply and wage markups

Chari, Kehoe, and McGrattan (2009) point to an identification problem in the CEE/SW model that preference shocks shifting the marginal disutility of labor cannot be easily distinguished from wage markup shocks. Gali, Smets, and Wouters (2011) and Justiniano, Primiceri, and Tambalotti (2013) impose assumptions to overcome this identification problem.¹⁹ While in comparison to the CEE/SW model wage markup shocks play a less important role in both these papers, wage markup shocks continue to contribute significantly to the fluctuations in inflation in Gali, Smets, and Wouters (2011). Given the different welfare implications of the inefficient wage markup shocks, which creates a monetary policy trade off, and the efficient labor supply shocks, the relative importance of these two shocks may influence the ranking of targeting frameworks.

Figure 10 provides a preliminary inquiry into the importance of the issues raised by Chari, Kehoe, and McGrattan (2009) for the ranking of frameworks. We compute the welfare differences between targeting frameworks by changing the relative importance of wage markup and labor supply shocks. Following Gali, Smets, and Wouters (2011) and Justiniano, Primiceri, and Tambalotti (2013), we model the labor supply shock as a shock to the marginal disutility of labor. The labor supply shock is specified to match the unconditional variance of the wage markup shock and to induce responses similar in magnitude to those induced by the wage markup shock. The relative weight on the labor supply shock depicted along the horizontal axis governs the relative importance of the two shocks.

Both for the commitment and the discretion case, the ranking of targeting frameworks is independent of the relative importance of wage markup and labor supply shocks with the exception of the NIT and the NIT-II framework for the case of discretion and a high importance of the labor supply shock. As the importance of the inefficient wage markup shock is reduced, the welfare differences between targeting frameworks shrink by construction. Monetary policy can mostly offset the welfare consequences of the labor supply shock; when wage markup shocks are absent from the model, price markup shocks are the only remaining source of inefficient fluctuations.

As long as one believes wage markup shocks to play some role in driving business cycle fluctuation as in Gali, Smets, and Wouters (2011), the speed limit policy framework under discretion strongly outperforms all other frameworks under discretion (as well as the inflation targeting framework under commitment). But even for the assessment in Justiniano, Primiceri, and Tambalotti (2013), which assigns little importance to wage markup shocks, the speed limit policy framework performs well. Absent certainty about the true data-generating process, adopting the speed limit policy framework may turn out to be a prudent choice.

¹⁹ Gali, Smets, and Wouters (2011) obtain identification by embedding a theory of unemployment and by including data on unemployment. Justiniano, Primiceri, and Tambalotti (2013) do not exploit the connection between unemployment and wage markups and assume instead a particular stochastic structure for the latter (white noise) to obtain identification.

3.4.3 Zero lower bound on nominal interest rates

Following earlier work on optimal policy design, we have abstracted from the implications for monetary policy imposed by the zero lower bound on the nominal interest rate. This way of preceding allows us to include larger models and to consider aspects of parameter uncertainty. Furthermore, the probability of the policy rate reaching zero (and staying at zero for several periods) is low in the CEE/SW model. As long as the time that the economy spends at the zero bound is short, economic outcomes when the zero bound is enforced barely differ from the outcomes when the policy rate is allowed to violate the zero bound. Thus, the optimal parameterization of each targeting framework is expected to change by little if we were to impose the zero bound in our analysis. Nevertheless, we want to touch on the challenges for monetary policy design presented by the zero bound at least in closing.

Figure 11 plots the impulse responses of selected variables to a combination of contractionary demand shocks under the optimal commitment policy. The figure also plots the responses under inflation targeting, price level targeting, and the speed limit policy: the policymaker acts under discretion, the model parameters are fixed at the posterior mode, and the objective functions are parameterized as found to be optimal absent the zero bound constraint.²⁰ In response to the shock, the optimal commitment policy lowers the short-term interest rate to zero, although not for long, and allows for mild deflation of prices and wages. The output gap turns negative and closes slowly. Further out, the optimal commitment policy allows for only very minor overshooting of price and wage inflation above their long-run target values and the output gap hardly rises above zero.

Although operated under discretion, all three targeting frameworks perform closely to the optimal commitment policy. The inflation and price level targeting central banks are more aggressive at stabilizing price and wage inflation and the output gap. As the shock pushes price and wage inflation, and the output gap in the same direction, the high relative weight on price inflation in the objective function of the inflation targeting central bank to mimic the behavior of the price level targeting central bank.²¹

The optimal speed limit policy computed in Section 3.1 allows for larger deviations of inflation and the output gap than the optimal commitment policy. Under the speed limit policy, the policymaker seeks to adjust the output gap gradually. While such gradualism is of advantage in response to price and wage markup shocks—keeping the output gap negative after an inflationary shocks signals tight future monetary policy and reduces the initial rise in inflation—it is of potential disadvantage after large demand shocks that push the policy interest rate to zero. The slow closing of the output gap under the speed limit policy prevents price and wage inflation from a fast return to their long-run targets. Shocks that are more contractionary than the ones underlying Figure 11 can exacerbate this feature of the speed limit policy.

 $^{^{20}}$ Initially, the economy is assumed to be growing along the balanced growth path. In period 1 the economy experiences a negative one-standard deviation risk-premium shock together with a negative 10-standard deviation shock to government spending. In addition, we lowered the value of the nominal interest rate along the balanced growth path to 4%. The problem is solved using the piece-wise linear approach in Eggertsson and Woodford (2003), Coibion, Gorodnichenko, and Wieland (2012), and Guerrieri and Iacoviello (2015); we abstract from modifications of the social loss function that could result from the zero bound constraint.

 $^{^{21}}$ This result is not at odds with Adam and Billi (2007) or Bodenstein, Hebden, and Nunes (2012) who point out the importance of commitment at the zero lower bound when the central bank maximizes the discounted utility of the representative household. In our application, the discretionary central bank places a higher weight on stabilizing price inflation than under the true social loss function and is therefore much better positioned to stabilize the economy through accommodative monetary policy than in those papers for the case of discretion.

This potential drawback of the speed limit policy can be ameliorated by reducing the weight on the activity measure in the objective function. To convey this idea, Figure 11 also plots the impulse responses under a speed limit policy with a reduced weight on the output gap under the label Alt. SLP (that is one tenth of the weight found to be optimal in Section 3.1). Under the reduced weight, the speed limit policy closely resembles the optimal commitment policy. While the dramatic reduction in weight on the activity measure worsens the performance of the speed limit policy to price and wage markup shocks in particular, this specific parameterization of the speed limit policy still outperforms the optimal inflation and the optimal price level targeting framework under discretion computed in Section 3.1 for the posterior mode parameterization of the model.²²

The optimal parameterization let alone the ranking of targeting regimes in the CEE/SW model may hardly be affected if we enforced the zero bound constraint on nominal interest rates. If shocks that call for lowering the policy interest rates to zero are more frequent than in the CEE/SW model, price level targeting might be preferred to the speed limit policy under discretionary policymaking given a low value of the long-run inflation target. However, raising the long-run inflation target may constitute a viable alternative: the monetary authority can adopt a speed limit policy which is effective in ameliorating the time inconsistency problem associated with price and wage markup shocks while significantly reducing the likelihood of zero bound events. Whether these benefits outweigh the costs of achieving a long-run inflation target is an empirical question beyond the scope of this section.²³

4 Conclusion

The debate on targeting frameworks has often focused on the differences between inflation and price level targeting. In models that follow the New Keynesian paradigm, the optimal commitment policy tends to undo most, if not all, changes of price and wage inflation from their long-run targets over time to realign prices and wages with their long-run trends. Given this insight, price level targeting appears to be a natural contender to inflation targeting when policymakers act under discretion.

However, we argue that speed limit policy is a clear alternative to both the inflation targeting and the price level targeting framework. The objective function underlying the speed limit policy framework with its long-run commitment to stable inflation and its short-run focus on inflation and smooth changes in the output gap leads to better outcomes than all other frameworks when policymakers act under discretion in many circumstances. When policymakers act under commitment, the differences between the three targeting frameworks are negligible. Most importantly, the speed limit policy under discretion outperforms inflation targeting under commitment in numerous cases. We show the relative superiority of the speed limit policy framework in a sequence of simple NK models, that introduce inflation indexation, habit persistence in consumption, and sticky wages, and in the CEE/SW model. The optimal speed limit policy is more robust to empirically-relevant alternative parameterizations of the CEE/SW model and to unclarity about

 $^{^{22}}$ Abstracting from the zero lower bound, the optimal parameterization of each framework is primarily determined by the price and wage markup shocks. Ironically, the optimal weight on the activity measure under the speed limit policy is higher when these markup shocks are more important which in turn impedes the central bank's ability to stabilize the economy in the face of large negative demand shocks and zero interest rates.

 $^{^{23}}$ Pursuing higher inflation targets has captured the imagination of policymakers in the aftermath of the Great Recession, see Williams (2016). Coibion, Gorodnichenko, and Wieland (2012) compute the optimal inflation target for a discretionary central bank to fall just below 3%; Billi (2011) reports significantly higher numbers.

the ability of the central bank to commit. Unless the economy can experience large and persistent negative (demand) shocks and the costs of raising the long-run inflation target are high, the speed limit policy will also outperform inflation and price level targeting under discretion when the zero lower bound constraint on nominal interest rates is enforced in the model.

Since speed limit policies have not yet been as thoroughly examined as inflation and price level targeting, a range of open questions remain to be addressed. How would a speed limit policy perform under model settings that included informational rigidities, or financial frictions? How does a central bank's ability to measure the output gap accurately in real-time—an issue explored in Orphanides (2003)— influence the relative performance of targeting frameworks? What about central bank communication of current and future policy goals? Given the promising performance of speed limit policies shown in this paper, it appears worth to continue exploring the implications of this policy and find answers to the preceding questions.

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	Parameters governing												
			sticky prices			sticky wages			other				
Model	description	ξ_p	ι_p	θ_p	${ar au}_p$	ξ_w	ι_w	θ_w	$\bar{\tau}_w$	h	σ_C	σ_L	β
1	textbook model	0.8	0	0.61	0.61	_	_	_	_	0	1.39	1.92	0.9984
2	price indexation	0.8	[0;1]	0.61	0.61	_	_	_	_	0	1.39	1.92	0.9984
3	inefficient steady state	0.8	[0;1]	0.61	0	_	_	_	_	0	1.39	1.92	0.9984
4	consumption habits	0.8	[0;1]	0.61	0.61	_	_	_	_	0.7	1.39	1.92	0.9984
5	sticky wages	0.8	[0;1]	0.61	0.61	0.8	0	0.5	0.5	0	1.39	1.92	0.9984

Table 1: Parameter values for the textbook NKM and its extensions

Note: The table documents the parameter values of the textbook NKM and its extensions underlying Figures 1 and 2. Model 1 is the textbook NKM without indexation. In Model 2 we augment the textbook NKM to allow for price indexation. Model 3 features distortions in the steady state. Habit persistence in consumption is introduced in Model 4. Finally, Model 5 allows for sticky nominal prices and wage. In all models, an ARMA(1,1) price markup shock is the sole source of fluctuations with the autocorrelation coefficient $\rho_u = 0.9$, the moving average coefficient $\rho_{u\epsilon} = 0.74$, and the standard deviation for innovations $\sigma_u = 0.0014$.

a: Frequency of being the best framework: optimal weights case						
	IT	PLT	SLP			
Commitment	0.9723	0.0004	0.0273			
Discretion	0.0000	0.0167	0.9833			

Table 2: Performance of targeting frameworks under parameter uncertainty

b: Frequency of being the best framework: fixed weights case

	IT	PLT	SLP
Commitment	0.8281	0.0073	0.1646
Discretion	0.0000	0.0168	0.9832

c: Frequency of being the best framework: exchanged weights case

	IT	PLT	SLP
Commitment	0.0036	0.0056	0.9908
Discretion	0.0000	0.0162	0.9838

d: Percentiles of CEV differences

Discretion

Commitment

Quantile	IT	PLT	SLP	IT	PLT	SLP
5%	0.0022	-0.0493	-0.0280	-0.7717	-0.2169	0.0114
10%	0.0055	-0.0413	-0.0244	-0.6086	-0.1621	0.0193
15%	0.0081	-0.0368	-0.0222	-0.5291	-0.1350	0.0258
20%	0.0098	-0.0338	-0.0207	-0.4772	-0.1172	0.0310
25%	0.0110	-0.0316	-0.0194	-0.4349	-0.1035	0.0362
30%	0.0119	-0.0297	-0.0184	-0.4013	-0.0927	0.0409
35%	0.0128	-0.0280	-0.0174	-0.3728	-0.0837	0.0458
40%	0.0135	-0.0264	-0.0166	-0.3475	-0.0757	0.0510
45%	0.0142	-0.0250	-0.0158	-0.3253	-0.0685	0.0565
50%	0.0150	-0.0237	-0.0150	-0.3053	-0.0624	0.0624
55%	0.0158	-0.0226	-0.0143	-0.2856	-0.0565	0.0685
60%	0.0165	-0.0213	-0.0136	-0.2674	-0.0510	0.0757
65%	0.0174	-0.0202	-0.0128	-0.2483	-0.0458	0.0837
70%	0.0183	-0.0191	-0.0120	-0.2294	-0.0409	0.0927
75%	0.0193	-0.0178	-0.0110	-0.2105	-0.0362	0.1035
80%	0.0206	-0.0165	-0.0099	-0.1910	-0.0310	0.1172
85%	0.0222	-0.0149	-0.0082	-0.1675	-0.0258	0.1350
90%	0.0243	-0.0130	-0.0056	-0.1388	-0.0193	0.1621
95%	0.0279	-0.0094	-0.0023	-0.0970	-0.0114	0.2169

Note: The table summarizes the performance of inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the parameters of the CEE/SW model are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). Panel (a) states the frequency of each targeting regime being the best performing one for both the case of commitment and discretion. The weight on the activity measure λ_x^{TF} is chosen optimally for each framework and each parameter draw. In Panel (b) the weight on the activity measure λ_x^{TF} is fixed for each framework at the value that is optimal when the model is parameterized at the posterior mode. All other parameters are drawn from the Laplace approximation to the posterior distribution. In Panel (c) when policy is conducted under commitment (discretion) the weight on the activity measure λ_x^{TF} is fixed for each framework at the value that is optimal under discretion (commitment) for the posterior mode parameterization of the model. All other parameters are drawn from the Laplace approximation to the cell. All other parameters are drawn from the Laplace approximation to the model. All other parameters are drawn from the Laplace approximation to the posterior distribution. In Panel (d), we first compute the CEV difference between the best performing and the second best performing framework for each parameterization; we then rank the differences by size for each framework and compute percentiles.





Note: The figure plots the welfare loss for each targeting framework against the optimal commitment policy under different values for λ_x^{TF} . The only source of fluctuations is an ARMA(1,1) markup shock. Welfare is reported in terms of consumption equivalent variation multiplied by 100. The weight λ_x^{TF} for which the welfare loss is minimized is indicated by "o" under price level targeting (PLT), "*" under speed limit policy (SLP), and "o" under inflation targeting (IT), respectively.



Figure 2: Welfare evaluation of targeting frameworks in extensions of the textbook NKM

COMMITMENT

DISCRETION



Changing Price Indexation

Note: Welfare performance of price level targeting and speed limit policy relative to inflation targeting in the textbook NKM and its extensions with varying degree of price indexation ι_p under commitment and discretion. The only source of fluctuations is an ARMA(1,1) markup shock. Welfare is reported in terms of consumption equivalent variation multiplied by 100. The top row depicts the results in the textbook NKM with an efficient steady state and price indexation. Each of the following rows differs from the textbook NKM by a single feature: distorted steady state (second row), external consumption habits (third row), and sticky nominal wages (last row).



Figure 3: Welfare evaluation of targeting frameworks in the CEE/SW model

Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting (IT) in the CEE/SW model under commitment and discretion. Parameters are set at the mode of the posterior distribution reported in Smets and Wouters (2007). Welfare is measured in terms of consumption equivalent variation multiplied by 100. In the first two rows of panels, we vary the degree of price indexation. The second row deviates from Smets and Wouters (2007) by correcting steady state inefficiencies due to external habits and monopolistic competition. The third row considers variations in the degree of wage indexation.

wage indexation ι_{w}

wage indexation ι_w



Note: The figure compares the impulse responses to a price and wage markup shock under the optimal commitment policy, inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP). The two markup shocks follow ARMA(1,1) processes. See also Appendix D. 34

Figure 5: Understanding the welfare rankings in the CEE/SW model under discretion: introducing features sequentially



DISCRETION

Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting (IT) in the CEE/SW model under discretion. From top left to bottom right we augment the textbook NKM step-by-step by the features in Smets and Wouters (2007): Goods subsidies are removed to render the steady state inefficient, capital and government spending are added in top right panel. In the second row, sticky wages with a wage subsidy to remove distortions in the labor market are introduced, a wage markup shock is added, and finally, the wage subsidy is removed. In the final row, we introduce external consumption habits, increase the nominal rigidities to obtain the same slopes in the NKPCs in the model without variable elasticity of substitution as in the full CEE/SW model with a Kimball (1995) aggregator in the bottom right panel.





Note: The figure plots the distribution of welfare and the optimized weights λ_x^{TF} for inflation targeting (IT), price level targeting (PLT) and speed limit policy (SLP) under commitment and discretion when the parameters of the CEE/SW model are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The top row shows the density distribution of the consumption equivalent variation (CEV) under commitment, the middle row shows the results under discretion. The bottom row of panels depicts the cumulative distribution function (CDF) of the optimal weights under discretion and commitment for each framework in a single panel.





Note: The figure plots the cumulative welfare distribution under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the weights on the activity measure are fixed at the values that are optimal under the posterior mode parameterization of the CEE/SW model relative to the case when the weights on the activity measure are set optimally for each parameter draw and targeting framework. All other parameters are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The upper panel plots the cumulative distribution function (CDF) under commitment; the bottom panel plots the cumulative distribution function (CDF) under discretion.

0.5

CEV difference

0.6

0.7

0.8

0.9

1

0

0.1

0.2

0.3

0.4





Note: The figure plots the cumulative welfare distribution under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) when the weights on the activity measure under commitment (discretion) are fixed at the values that are optimal under the posterior mode parameterization of the CEE/SW model with discretion (commitment) relative to the case when the weights on the activity measure are set optimally for each parameter draw and targeting framework. All other parameters are drawn from the Laplace approximation to the posterior distribution in Smets and Wouters (2007). We simulate 30000 draws. The upper panel plots the cumulative distribution function (CDF) under commitment; the bottom panel plots the cumulative distribution function.

Figure 9: Welfare evaluation of targeting frameworks in the CEE/SW model estimated with euro area data



Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting in the CEE/SW model estimated with euro area data (1975Q4 to 2008Q3) under commitment and discretion. In the first row of panels the degree of price indexation is varied. The second row considers variations in the degree of wage indexation. The degree of indexation at the posterior mode is indicated with $\iota_p = 0.128$ for prices and $\iota_w = 0.374$ for wages, respectively.

Figure 10: Welfare evaluation of targeting frameworks: relative importance of wage markup shocks and labor supply shocks



Note: Welfare performance of price level targeting (PLT), speed limit policy (SLP), and the two nominal income targeting frameworks (NIT, NIT-II) relative to inflation targeting in the modified CEE/SW model when allowing for labor supply and wage markup shocks. This version of the model features preferences that are separable in consumption and leisure. The relative importance of the two shocks is controlled by the weight parameter indicated on the x-axis. "0" indicates the absence of the labor supply shock and "1" indicates the absence of the wage markup shock. The wage markup shock follows an ARMA(1,1) process as in Smets and Wouters (2007), whereas the labor supply shock is assumed to be an AR(1) process. The labor supply shock is scaled to ensure similar magnitudes of the shock as the ARMA(1,1) wage markup shock when comparing the unconditional variances of the shocks.



Figure 11: Welfare evaluation of targeting frameworks under the zero lower bound constraint

Note: The figure compares the impulse responses to a negative one-standard deviation risk premium shock and a negative 10-standard deviation shock to government spending under inflation targeting (IT), price level targeting (PLT), and speed limit policy (SLP) each under discretion and the under the optimal commitment policy. The shocks are large enough for the policy interest rate to be constrained by the zero lower bound.