

Regional Liquidity Shocks, Financial Markets, and Central Bank Policy

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Abstract

Financial markets create trading opportunities affecting the incentives to reveal the private information essential to financial intermediaries in the efficient provision of liquidity insurance. Financial markets and financial intermediaries may be competing mechanisms in the provision of liquidity insurance, and their co-existence impacts risk-sharing, as noted earlier by Jacklin [9], Diamond [5], Allen and Gale [2], among others. The question addressed here is how does the co-existence of local and regional financial markets and intermediation impact the efficiency of equilibrium allocations. Regions are subject to idiosyncratic liquidity demand shocks, specifically randomness in the fraction of early and late households. In the absence of central bank intervention, competitive equilibria with interbank lending and financial intermediation are inefficient relative to the first-best. Competitive equilibria when financial intermediation and financial markets co-exist are also inefficient, resulting in over-investment in the long-term asset. The heterogeneous regions share a common central bank, who can implement welfare-improving policies by acting as a mechanism designer. The optimal policy response by the central bank is a simple portfolio restriction in the form of a liquidity floor, which creates a wedge between the endogenous marginal rate of transformation and the interbank lending rate. Implementation of the optimal policy requires the co-existence of intermediation and financial markets. Even if the central bank could prevent trading demand deposit contracts in financial markets, it is not optimal to do so. The issues studied here might arise, for example, in a banking union with a common central bank or a large country with regional liquidity shocks.

JEL Classification: D81, D82.

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The role of the central bank in facilitating risk-sharing across heterogeneous regions is studied in a model with private information and private trading. Liquidity preference shocks create a demand for liquidity, as in Diamond and Dybvig [4]. Regions are *ex ante* identical and become differentiated by a liquidity shock determining the fraction of early and late consumers. The liquidity preference shock is observed only by the household and the regional liquidity demand shock is known only to the financial intermediary in the region. Financial markets and financial intermediaries may be competing mechanisms in the provision of liquidity insurance, and their co-existence impacts risk-sharing, as noted earlier by Jacklin [13], Diamond [8], Allen and Gale [2], among others. Households can insure against the liquidity preference shock through deposits with the local financial intermediary or else by directly trading in financial markets. The question studied here is what is the optimal central bank policy when there are liquidity preference shocks and regional liquidity demand shocks in the presence of private information and private trading.

Using a Diamond-Dybvig model, Jacklin [13] describes how private trading opportunities can severely limit risk sharing. The implication of private trading opportunities for agents' decisions is studied by Farhi, Golosov and Tsyvinski [9] (FGT) in a Diamond-Dybvig banking model. Optimal central bank policy in a Diamond-Dybvig type model with regional liquidity shocks is studied by Bhattacharya and Gale [4], who derive an incentive efficient allocation. The model here differs from previous work in several ways. FGT use a single region Diamond-Dybvig model in which the return to the long-term investment is exogenous, as is the fraction of patient and impatient agents. Bhattacharya and Gale focus on optimal central bank policy across regions using a model with deterministic returns to investment in which private trading arrangements are prohibited. In contrast, the model presented here has an endogenous return to the long term asset and the focus is on the co-existence of intermediation and financial markets as insurance mechanisms. Regions differ in terms of the fraction of patient and impatient agents and this fraction is random and privately observed.

When the central bank can restrict private trading, it can implement an allocation over time that is incentive efficient, by acting as a mechanism designer. There are two constraints for the allocation to be incentive efficient: (i) the region cannot engage in any further borrowing or lending in an interbank market and (ii) households cannot engage in any private trading arrangements such as a financial market trading negotiable demand deposit contracts. There may be an incentive to engage in additional trading because of differences in the discounted present value of an allocation across regions and differences in the discounted

present value of the allocation provided by a financial intermediary to patient and impatient households. When interbank borrowing and lending cannot be prevented and households do not directly participate in financial markets, the resulting competitive equilibrium is inefficient. The competitive equilibrium is inefficient when financial markets and intermediation co-exist, even if financial intermediaries eliminate arbitrage profit opportunities resulting in an integrated credit markets. The optimal central bank policy is to impose a liquidity floor, as in Farhi, Golosov and Tsyvinski, and to continue to allow trading of deposit contracts in financial markets.

1 Basic Model

There are three periods $t = 0, 1, 2$ and a continuum of locations $\phi \in \Phi \equiv [0, 1]$. There is a representative financial intermediary and a continuum of households at each location.² At $t = 0$, each household at location ϕ is endowed with E units of the consumption good, which is deposited with the financial intermediary at location ϕ . The financial intermediary invests a portion of this deposit $E - k$ in a short-term project, where one unit invested at $t = 0$ yields one unit of consumption at $t = 1$, and the remainder k (capital) is invested in a long-term project yielding output at $t = 2$. The long-term project cannot be interrupted at $t = 1$. An investment of k at $t = 0$ yields θk^α at $t = 2$, where $0 < \alpha < 1$ and $0 < \theta < \infty$. Storage is available between periods 1 and 2, with one unit stored at $t = 1$ yielding one unit at $t = 2$.

Households and Liquidity Demand

A household cares about consumption in periods $t = 1, 2$. At $t = 1$, each household observes a *liquidity preference shock* $v \in \{0, 1\}$. A household has a utility function $U : \mathfrak{R}^+ \rightarrow \mathfrak{R}$ taking the form

$$(1 - v)U(c_1) + v\beta U(c_1 + c_2),$$

where $0 < \beta < 1$. A household realizing $v = 0$ wishes to consume only in the first period (impatient), while households realizing $v = 1$ are indifferent between first and second period consumption (patient). The realization v is private information for the household.

²One may instead assume there is a representative household at each location in which consumption is perfectly divisible across members of the household over time.

Assumption 1 (i) U is continuously twice differentiable, strictly increasing and strictly concave; (ii) U satisfies the Inada conditions; (iii) $U'(c)c$ is non-increasing.

The assumption $U'(c)c$ is non-increasing provides a motive for liquidity insurance, as is standard in the Diamond and Dybvig model. This assumption corresponds to Assumption 1 in Green and Lin [].

A region experiences an idiosyncratic *liquidity demand shock* h , where $h \in H \equiv [\underline{h}, \bar{h}] \subset (0, 1)$ denotes the fraction of impatient agents h (agents realizing $v = 0$) and $1 - h$ is the fraction of patient agents (agents realizing $v = 1$). For each region $\phi \in \Phi$, h is a random variable that is independent and identically distributed across regions. Let $0 < \pi(h) < 1$ denote the probability of realizing h and define the mean

$$h^m \equiv \int_H h\pi(h)dh.$$

A region is said to have a *high demand* for liquidity when $h > h^m$ and a *low demand* when $h < h^m$. The realization h in ϕ is private information observed only by the financial intermediaries in region ϕ .

Denote $c_t : \{0, 1\} \times H \rightarrow R_+$, $t \in \{1, 2\}$ as the period t consumption of a household realizing v residing in a region realizing h . An allocation $(E - k, k, \{c_1(v, h), c_2(v, h)\})$, $v \in \{0, 1\}$ and $h \in H$, is *feasible* if $E \geq k \geq 0$ and

$$E - k = \int_H [hc_1(0, h) + (1 - h)c_1(1, h)] \pi(h)dh, \quad (1)$$

$$\theta k^\alpha = \int_H [hc_2(0, h) + (1 - h)c_2(1, h)] \pi(h)dh. \quad (2)$$

Let Λ denote the set of feasible allocations. At $t = 0$, the representative household at location ϕ has expected utility

$$\mathcal{U} \equiv \int_H \{hU(c_1(0, h)) + (1 - h)\beta U(c_1(1, h) + c_2(1, h))\} \pi(h)dh. \quad (3)$$

The *incentive compatibility constraints* are

$$U(c_1(0, h)) \geq U(c_1(1, h)), \quad (4)$$

$$U(c_1(1, h) + c_2(1, h)) \geq U(c_1(0, h) + c_2(0, h)). \quad (5)$$

If $c_2(0, h) = 0$, impatient agents receive nothing in the second period, and later will be shown to be optimal. In this case, (4) is never binding and the individual incentive compatibility constraints reduce to

$$U(c_1(1, h) + c_2(1, h)) \geq U(c_1(0, h)). \quad (6)$$

Patient agents can always announce they are impatient and store the consumption good, while impatient agents have no incentive to announce they are patient.

Households realizing $v = 0$ prefer to have some insurance against early consumption and this insurance is typically provided by the regional financial intermediary. The random liquidity demand shock h impacts the financial intermediary's ability to provide this liquidity insurance. This shock can be partially smoothed by borrowing and lending when there is an interbank market. The central bank may be able to improve on the risk sharing by acting as a mechanism designer.

Regions and Financial Intermediaries

Financial intermediaries from different regions can borrow or lend in an interbank credit market at rate r , where they act as price takers. Within a region, there is competition among identical financial intermediaries and there may be a *local financial market* with interest rate $r(h)$, where the representative financial intermediary and local households are the only participants in the local market. Only financial intermediaries are able to participate in the interbank credit market.³ An overview of the model is in Figure (1). The three key functions of a financial intermediary are: (i) To facilitate credit flows across regions; (ii) To channel credit to the long-term project; (iii) To provide liquidity insurance for local households.

To facilitate credit flows (i), the financial intermediary borrows or lends in the interbank market to smooth liquidity demand shocks. If the interbank rate r is different from the local financial market rate $r(h)$, then the financial intermediary will arbitrage across markets. Cocco *et al* [5] provide empirical evidence on the importance of the interbank market as a distributor of liquidity. For (ii), financial intermediaries provide loans to producers of the long-term project. There is an extensive literature on the role of a financial intermediary as a delegated monitor in mitigating the impact of asymmetric information; see the discussion in Diamond [7] for example. To simplify the model, I assume the financial intermediary manages the long-term project in lieu of introducing an additional set of agents, although this can be easily modified.⁴ In the provision of liquidity insurance (iii), the financial intermediary offers insurance when it provides higher

³This can be motivated by assuming financial intermediaries have a comparative advantage in inter-regional transactions.

⁴The long term project can be modeled using the costly state verification framework of Gale and Hellwig [10], for example. The financial intermediary has a monitoring technology and it is efficient to delegate monitoring and lending decisions to the intermediary.

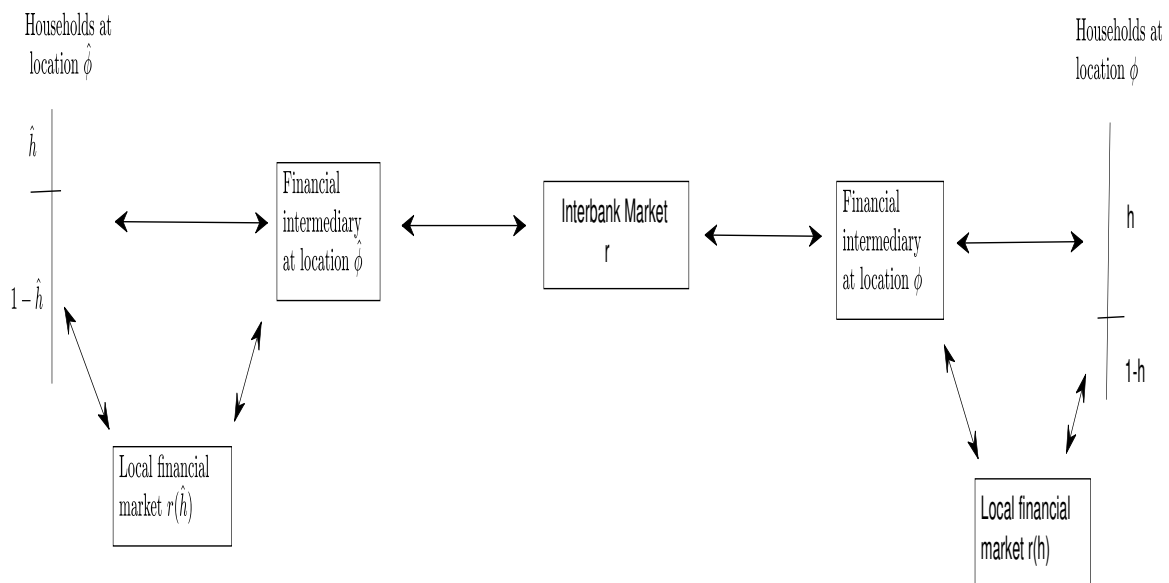


Figure 1: *Overview of the Model - Financial intermediaries can borrow or lend in the interbank market. Households can trade with the local financial intermediary and the local financial market at interest rate $r(h)$. A financial intermediary trades with intermediaries in other regions in the interbank market at interest rate r . Location $\hat{\phi}$ has a fraction \hat{h} of impatient households while location ϕ has a fraction h .*

consumption to a household wishing to consume early than the household could achieve by borrowing against or selling its claims on the long-term project in a financial market.

The following assumption is made to ensure the concept of a region is well-defined.

Assumption 2 (i) Households in region $\phi \in \Phi$ cannot trade directly with households in region $\hat{\phi} \in \Phi$, $\phi \neq \hat{\phi}$; (ii) Households are prevented from participating directly in the interbank market; (iii) Only financial intermediaries in region ϕ can participate in the local credit market in ϕ .

These are extreme assumptions used to make precise the concept of a region. Financial innovation and advances in communication technology have blurred the concept of a regional financial intermediary. There is no difficulty, for example, in allowing the financial intermediaries in different regions to be owned by the same holding company. The assumption only large agents can participate in wholesale financial markets is used in Allen, Gale and Carletti [1], Allen and Gale [2], and succinctly described by Kiyotaki [14].

The financial arrangement between the local representative financial intermediary and local households is assumed to take one of two forms: households trade only with the local financial intermediary, or else there are co-existing local financial markets and financial intermediation. The main issue studied here is how the co-existence of the local and regional financial markets impacts the efficiency of the equilibrium allocations.

2 Observable Liquidity Demand Shocks

The model is first solved under the assumption the realizations (h, v) are publicly observable. This provides a benchmark for comparison with the private information economy. It also explains the impact of assuming an endogenous return to the long-term asset when there is private information, unlike the standard Diamond-Dybvig [6] model where the return to the long-term asset is exogenous. Since $\theta\alpha k^{\alpha-1} < 1$ is a possibility, the social planner may choose to store between periods 1 and 2, where storage in region h is denoted $s(h) = (1 - h)c_1(1, h)$. The social planner maximizes the expected utility of a representative household in a representative region (3) by choosing $\{k, c_1(0, h), c_2(1, h), s(h)\}$, subject to the resource constraint for each period

$$E - k \geq \int_H [hc_1(0, h) + s(h)]\pi(h)dh, \quad (7)$$

$$\theta k^\alpha + \int_H s(h)\pi(h)dh = \int_H [hc_2(0, h) + (1 - h)c_2(1, h)]\pi(h)dh, \quad (8)$$

and the constraint $s(h) \geq 0$ for all h . Since all regions are identical at $t = 0$, there is no incentive to shift resources across regions. After the distribution of the idiosyncratic liquidity demand shock h is realized at $t = 1$, the social planner may choose to store or transfer resources across regions.

Since $c_2(0, h)$ does not enter the objective function, it is optimal to set $c_2(0, h) = 0$. The first-order conditions simplify to a system

$$U'(c_1(0, h)) = \beta\theta\alpha k^{\alpha-1}U'(c_2(1, h)), \quad (9)$$

$$U'(c_1(0, h)) \geq \beta U'(c_2(1, h)), \quad (10)$$

where (10) holds with equality only if $s(h) > 0$.⁵ It follows from the first-order conditions that $(c_1(0, h), c_2(1, h))$ are invariant with respect to h ; let $c_1 = c_1(0, h)$ and $c_2 = c_2(1, h)$. If $U'(c_1) = \beta U'(c_2)$, then for (9) and (10) to hold simultaneously requires $r = \theta\alpha k^{\alpha-1} = 1$, in which case investment in the long-term project satisfies $k_s \equiv (\theta\alpha)^{\frac{1}{1-\alpha}}$. This is the maximum investment in the long-term project and any additional resources transferred from the first to the second period will be made through storage. For convenience, denote total storage as

$$S \equiv \int s(h)\pi(h)dh.$$

For positive storage to be an equilibrium, (9)–(10) imply S solves

$$U' \left(\frac{E - [k_s + S]}{h^m} \right) = \beta U' \left(\frac{\theta k_s^\alpha + S}{1 - h^m} \right).$$

The left side is increasing in S while the right side is decreasing, so a unique solution exists. Since $0 < \beta < 1$

⁵The full problem is stated here. Let λ_t denote the Lagrangian multiplier for the resource constraint at period t . Let $\lambda_s(h)$ denote the multiplier for the non-negativity of storage $s(h)$. The first-order conditions with respect to $\{k, c_1(0, h), c_2(1, h), s(h)\}$ are

$$\lambda_1 = \lambda_2\theta\alpha k^{\alpha-1}, \quad (11)$$

$$\lambda_1 = U'(c_1(0, h)), \quad (12)$$

$$\beta U'(c_2(1, h)) = \lambda_2, \quad (13)$$

$$\lambda_s(h) = \lambda_1 - \lambda_2. \quad (14)$$

The invariance of $c_1(0, h), c_2(1, h)$ with respect to h follows from (12) and (13). Use (11)–(13) to eliminate the multipliers. Use (14) to show $\lambda_1 \geq \lambda_2$. If $\lambda_1 = \lambda_2$, then $s(h) > 0$ for all h . Eliminating the multipliers results in equations (9)–(10).

and U is strictly concave, it follows

$$\left(\frac{E - [(\theta\alpha)^{\frac{1}{1-\alpha}} + S]}{h^m} \right) > \left(\frac{\theta(\theta\alpha)^{\frac{\alpha}{1-\alpha}} + S}{1 - h^m} \right),$$

or equivalently

$$E(1 - h^m) - (\theta\alpha)^{\frac{1}{1-\alpha}} [(1 - h^m) - \alpha^{-1}h^m] > S.$$

Since $S \geq 0$, there will be no storage at $c_1 = c_2$ if

$$(\theta\alpha)^{\frac{1}{1-\alpha}} > \frac{E(1 - h^m)}{1 - h^m + \alpha^{-1}h^m}.$$

The issue is to ensure the production possibility frontier has a marginal rate of transformation exceeding unity at the point where it intersects the 45-degree line, and the following assumption provides that property.

Assumption 3 *Let*

$$(\theta\alpha)^{\frac{1}{1-\alpha}} > \frac{E(1 - h^m)}{1 - h^m + \alpha^{-1}h^m}. \quad (15)$$

The implications of this assumption are explained in Figure (2), which graphs the production possibility frontier for the economy between periods $t = 1$ and $t = 2$. The horizontal axis is consumption of the representative impatient agent and the vertical axis is consumption of the representative patient agent.⁶ There is storage when $\theta\alpha k^{\alpha-1} < 1$, which is indicated by the line segment connecting the point $(\frac{E-k_s}{h^m}, \frac{\theta k_s^\alpha}{1-h^m})$ to the point $(0, \frac{\theta k_s^\alpha + E - k_s}{1-h^m})$.

Substitute the resource constraints into (9) to obtain an equation in the unknown k

$$U' \left(\frac{E - k}{h^m} \right) = \beta U' \left(\frac{\theta k^\alpha}{1 - h^m} \right) \theta \alpha k^{\alpha-1}. \quad (16)$$

The left side is strictly increasing in k while the right side is strictly decreasing so a unique solution exists under the conditions of Assumption 1; let k_f denote this solution. The solution k_f is decreasing in h^m , but otherwise does not depend on the distribution $\pi(\cdot)$ of h . Define

$$\begin{aligned} c_{f1} &= \frac{E - k_f}{h^m}, \\ c_{f2} &= \frac{\theta k_f^\alpha}{1 - h^m}, \end{aligned}$$

⁶The production possibility frontier without storage is

$$E = h^m c_1 + \left(\frac{(1 - h^m)c_2}{\theta} \right)^{\frac{1}{\alpha}}.$$

The slope is $\frac{dc_2}{dc_1} = -\frac{h^m}{1-h^m} \alpha \theta k^{\alpha-1}$.

and $r_f = \theta\alpha k_f^{\alpha-1}$. The portfolio $\{E - k_f, \theta k_f^\alpha\}$ and the allocation $\{c_{f1}, c_{f2}\}$ is the **first-best allocation**. The intertemporal marginal rate of substitution in consumption r_f equals the intertemporal marginal rate of transformation $\alpha\theta k_f^{\alpha-1}$. The Pareto weights do not vary across regions by assumption, so all impatient (patient) households consume c_{f1} (c_{f2}), regardless of location. This allocation results in a distribution of *ex post* utility of the representative household in region h as h varies, while the *ex ante* expected utility is identical across regions.

2.1 Liquidity Preference Shocks are Unobservable

The household's liquidity preference realization v and its consumption are now assumed to be private information known only to the household. The liquidity demand shock h remains publicly observable and there are no regional (local) credit markets. The individual incentive compatibility constraints (6) must hold to induce truth-telling. The first-best solution $\{k_f, c_{f1}, c_{f2}\}$ is derived assuming v is publicly observable, so the incentive-compatibility constraint (6) may not be satisfied. Assumption (2) ensures $\theta\alpha k^{\alpha-1} \geq 1$, but if this holds with equality, then

$$U'(c_1) = \beta U'(c_2),$$

so $c_2 < c_1$, violating (6). This constraint is satisfied if $c_{f2} \geq c_{f1}$, which in turn requires $\beta r_f \geq 1$ in (16). Suppose $\beta r_f = \beta\theta\alpha k^{\alpha-1} = 1$ and define $k_i = (\beta\theta\alpha)^{\frac{1}{1-\alpha}}$, where it follows $k_i < k_s$. When $k = k_i$ the resource constraints imply $c_1 = \frac{E - k_i}{h^m}$ and $c_2 = \frac{\theta k_i^\alpha}{1 - h^m}$. Then

$$c_2 - c_1 = \left[\frac{1}{h^m(1 - h^m)} \right] (\beta\theta\alpha)^{\frac{1}{1-\alpha}} [h^m(\beta\alpha)^{-1} + 1 - h^m]$$

For $c_2 > c_1$, requires

$$(\beta\theta\alpha)^{\frac{1}{1-\alpha}} > \left[\frac{E(1 - h^m)}{1 - h^m + (\beta\alpha)^{-1}h^m} \right].$$

Any $k < k_i$ corresponds to a return $r = \alpha\theta k^{\alpha-1} > \beta^{-1}$. Observe $(\theta\alpha)^{\frac{1}{1-\alpha}} > (\beta\theta\alpha)^{\frac{1}{1-\alpha}}$ and

$$\frac{E(1 - h^m)}{1 - h^m + (\alpha)^{-1}h^m} > \frac{E(1 - h^m)}{1 - h^m + (\beta\alpha)^{-1}h^m}.$$

An incentive-compatible allocation without storage exists under the following assumption.⁷

⁷Under Assumption 4,

$$(\theta\alpha)^{\frac{1}{1-\alpha}} > (\beta\theta\alpha)^{\frac{1}{1-\alpha}} \geq \frac{E(1 - h^m)}{1 - h^m + (\alpha)^{-1}h^m} > \frac{E(1 - h^m)}{1 - h^m + (\beta\alpha)^{-1}h^m}$$

so Assumption 4 is consistent with Assumption 3.

Assumption 4 *Let*

$$(\beta\theta\alpha)^{\frac{1}{1-\alpha}} > \frac{E(1-h^m)}{1-h^m+\alpha^{-1}h^m}.$$

Assumption 4 ensures there is an allocation along the production possibility frontier above the 45 degree line with $\beta r \geq 1$.

The allocation (c_{f1}, c_{f2}) satisfies

$$\frac{c_{f2}}{r_f} \leq c_{f1}. \quad (17)$$

To see this, use the properties $\beta r_f \geq 1$ (or $c_{f2} \geq c_{f1}$) and $U'(c)c$ is decreasing, and the first-order condition to show

$$\frac{c_{f2}}{r_f} - c_{f1} = \left[\frac{\beta c_{f2} U'(c_{f2}) - c_{f1} U'(c_{f1})}{U'(c_{f1})} \right] < \left[\frac{c_{f2} U'(c_{f2}) - c_{f1} U'(c_{f1})}{U'(c_{f1})} \right] < 0.$$

Decentralization

A contingent claims market is briefly described, after which the link with interbank lending is discussed. The liquidity demand shock h is observable, but a household's v and consumption are not. The representative financial intermediary in a region trades in a contingent claims market at $t = 0$ to insure against liquidity demand shocks. One unit of consumption delivered at $t \in \{1, 2\}$ contingent on realizing h can be purchased at price $p_t(h)$. The $t = 0$ budget constraint in the contingent claims market is

$$0 \geq \int_H [p_1(h)\epsilon_1(h) + p_2(h)\epsilon_2(h)] dh. \quad (18)$$

The representative financial intermediary maximizes expected utility (3) subject to the contingent claims constraint (18) and

$$E - k + \epsilon_1(h) = hc_1(0, h) + (1-h)c_1(1, h), \quad (19)$$

$$\theta k^\alpha + \epsilon_2(h) = (1-h)c_2(1, h). \quad (20)$$

Consumption is related to contingent-claims purchases according to

$$hc_{f1} = E - k_f + \epsilon_1(h),$$

$$(1-h)c_{f2} = \theta k_f^\alpha + \epsilon_2(h).$$

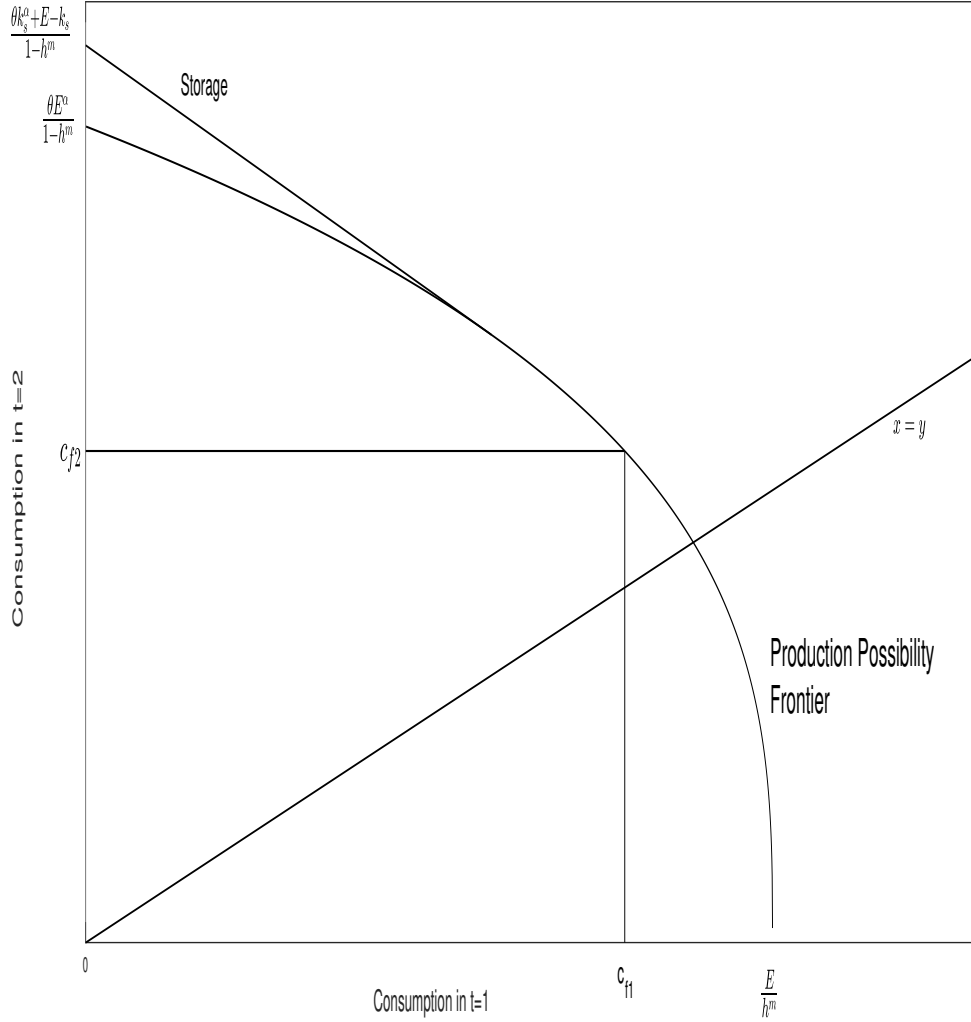


Figure 2: The production possibility frontier has a slope $-\frac{h^m}{1-h^m}\alpha\theta k^{\alpha-1}$. At point A, $1 = \alpha\theta k^{\alpha-1}$. Any resources transferred from $t = 1$ to $t = 2$ will occur through storage. Assumption 3 ensures the slope of the production possibility frontier at point B is greater in absolute value than $\frac{h^m}{1-h^m}$ so $1 < \alpha\theta k^{\alpha-1}$. Assumption 4 ensures the slope at point B is greater than $\frac{h^m}{1-h^m}\frac{1}{\beta}$.

The equilibrium contingent claims prices satisfy

$$r_f \equiv \frac{p_1(h)}{p_2(h)} = \frac{U'(c_{f1})}{\beta U'(c_{f2})}.$$

The contingent claim purchases at $t = 0$ create a sequence of state-contingent transfers across regions over periods $t = 1, 2$. Hence the first best allocation can be supported as a competitive equilibrium with contingent claims when h is observable and v is private information. If an interbank market opens at $t = 1$, there will be no borrowing or lending when the interest rate is r_f .

The contingent claims market is equivalent to deposit insurance, which is defined as a set of mandatory state-contingent transfers $\{(h^m - h)c_{f1}, (h - h^m)c_{f2}\}$ such that

$$0 = \int_H (h - h^m)c_{f1}\pi(h)dh = \int_H (h^m - h)c_{f2}\pi(h)dh.$$

Since, in general,

$$E - k_f + \frac{\theta k_f^\alpha}{r_f} \neq hc_{f1} + (1 - h)\frac{c_{f2}}{r_f},$$

interbank lending and borrowing is not a replacement for contingent claims trading or deposit insurance.

2.1.1 Logarithmic Example

When utility is logarithmic, the optimal investment in the long-term project is $k_f = B(h^m)E$, where $B : H \rightarrow [0, 1]$ is

$$B(h) \equiv \frac{\alpha\beta(1-h)}{\alpha\beta(1-h)+h}. \quad (21)$$

The function $B(\cdot)$ is decreasing and convex.⁸ The equilibrium real rate of interest satisfies $r_f = \theta\alpha k_f^{\alpha-1}$. Consumption satisfies $c_{f2} = \beta r_f c_{f1}$ and equals $h^m c_{f1} = E[1 - B(h^m)]$ and $(1 - h^m)c_{f2} = \theta[B(h^m)E]^\alpha$.

3 Financial Intermediation versus Financial Markets

Each region ϕ realizes a liquidity demand shock h at $t = 1$ that is not observed by any one outside of region ϕ . The household's liquidity preference shock v and consumption are observed only by the household. Since h is private information, a contingent-claims market to insure the liquidity demand shock at $t = 0$ will not

⁸Observe $B'(h) = -\left[\frac{\alpha\beta}{(\alpha\beta(1-h)+h)^2}\right] < 0$ and $B''(h) = 2\left[\frac{\alpha\beta(1-\alpha\beta)}{(\alpha\beta(1-h)+h)^3}\right] > 0$.

exist. In this section, there is no central bank implementing transfers across regions. Two mechanisms for insuring against the liquidity shocks - financial intermediation and financial markets - will be compared. There is an interbank market in both cases and the difference is whether households have access to both financial intermediaries and the local financial market or just financial intermediaries.

3.1 Financial Intermediaries and Interbank Credit

By assumption, there is no local financial market for households, so any liquidity insurance is provided by the local financial intermediary. This implies the local financial markets do not exist in Figure 1. In the standard Diamond-Dybvig model, the demand deposit contract implements the first-best allocation, but also creates a run equilibrium, which is a potential issue here. Although the long-term project cannot be liquidated, it is possible for the financial intermediary to borrow in the interbank market and the debt repayment may induce patient agents to run on the intermediary. Green and Lin [11] eliminate the bank run equilibrium by expanding the set of admissible financial arrangements over those considered by Diamond and Dybvig, demonstrating an incentive-efficient allocation can be implemented in a model without sequential service by linking the allocation to the reported messages of all agents. Although liquidity demand is random in this model, the consumption allocations can be made contingent on the fraction of agents reporting $v = 0$. If the individual incentive compatibility constraint is satisfied, agents will truthfully reveal their types, so a Green and Lin allocation can be implemented. The financial intermediary's problem is first described, after which the Green-Lin mechanism is discussed.

The financial intermediary determines its portfolio $\{E - k, \theta k^\alpha\}$ at time $t = 0$. Since h is unobservable, there is no contingent claims trading. At $t = 1$, it observes h , which is private information. It has access to the interbank market where it can borrow or lend the amount A at interest rate r . Its financial constraints in periods $t = 1, 2$ are

$$E - k = hc_1(0, h) + (1 - h)c_1(1, h) + A, \quad (22)$$

$$\theta k^\alpha + rA = hc_2(0, h) + (1 - h)c_2(1, h). \quad (23)$$

Solve (23) for A and substitute into (22) and rewrite

$$E - k + \frac{\theta k^\alpha}{r} = h \left[c_1(0, h) + \frac{c_2(0, h)}{r} \right] + (1 - h) \left[c_1(1, h) + \frac{c_2(1, h)}{r} \right]. \quad (24)$$

Define

$$W(r, k) \equiv E - k + \frac{\theta k^\alpha}{r}$$

and let $W = W(r, k)$. Given $\{W, r, h\}$, the financial intermediary chooses $\{c_1(v, h), c_2(v, h)\}$ to solve⁹

$$V_I(W, r, h) = \max_{\{c_t(v, h)\}_{v \in \{0, 1\}}} [h(U(c_1(0, h)) + (1 - h)\beta U(c_1(1, h) + c_2(1, h)))] \quad (25)$$

subject to

$$W = h \left[c_1(0, h) + \frac{c_2(0, h)}{r} \right] + (1 - h) \left[c_1(1, h) + \frac{c_2(1, h)}{r} \right]. \quad (26)$$

The solution $(c_1(0, h), c_2(1, h))$ satisfies¹⁰

$$U'(c_1(0, h)) = \beta r U'(c_2(1, h)), \quad (27)$$

$$E - k + \frac{\theta k^\alpha}{r} = h c_1(0, h) + \frac{(1 - h)c_2(1, h)}{r}. \quad (28)$$

If $\beta r \geq 1$, then the individual incentive compatibility constraint (6) is satisfied. To determine if $\beta r \geq 1$ requires construction of the equilibrium, which is addressed below.

In the Green and Lin mechanism, the financial intermediary determines the allocations based on the fraction of households reporting they are impatient; let m denote this fraction. The financial intermediary chooses an allocation $\{c_1(v, m), c_2(v, m)\}$ depending on the report m , where this allocation satisfies

$$U'(c_1(0, m)) = \beta r U'(c_2(1, m)),$$

$$W = m c_1(0, m) + \frac{(1 - m)c_2(1, m)}{r},$$

given (W, r, m) . The solution to (27)-(29) is the Green-Lin allocation mechanism. They demonstrate in Lemma 1 that the solution $(c_1(0, m), c_2(1, m))$ to (27)-(29) has the property the allocations are a non-increasing function of the fraction of households reporting they are impatient m , or equivalently the consumption allocations are rising in the fraction of households reporting they are patient $1 - m$. Green and Lin then demonstrate the solution can be implemented as a truth-telling equilibrium of an allocation mechanism. Under the conditions of Theorem 1, the truth-telling equilibrium is the unique Bayesian Nash equilibrium of

⁹The dual to this problem is studied in Farhi, Golosov and Tsyvinski, and is equivalent to this specification.

¹⁰If $\beta r = 1$, then the patient household is indifferent between receiving its allocation at $t = 1$ or $t = 2$. For convenience, set $c_1(1, h) = 0$.

the mechanism. Assumption 1 corresponds to the Green-Lin Assumption 1. The conditions for Lemma 1 in Green-Lin are satisfied and proven in Proposition 1 below. Hence $\hat{v} = v$ is a dominant strategy, so $m = h$.

At $t = 0$, the financial intermediary solves

$$\max_{\{k\}} \int_H V_I(W(r, k), r, h) \pi(h) dh.$$

Since the function V_I is increasing in W , the problem reduces to

$$W^*(r) = \max_k W(r, k),$$

and the solution is

$$k^*(r) = \left(\frac{\alpha \theta}{r} \right)^{\frac{1}{1-\alpha}}.$$

The steps to define an equilibrium are as follows: The properties of the allocation solving (27)-(28) are described in Proposition 1, assuming $\beta r \geq 1$ where r is given. A competitive equilibrium is then defined. The properties of the competitive equilibrium are contained in Theorem 1 and show an equilibrium interest rate satisfies $\beta r \geq 1$, if it exists.

Define G as the inverse function of U' , specifically if $y = U'(x)$ then $x = G(y)$, and observe $G'(y) < 0$ because U is concave and continuously differentiable. If $U''' > 0$ then $G''(y) > 0$ and $U'''(c) > 0$. The first-order condition (27) is solved for c_2 and substituted into the life-time budget constraint

$$W^*(r) = hc_1 + \frac{1-h}{r} G\left(\frac{U'(c_1)}{r\beta}\right). \quad (29)$$

The right side is strictly increasing in c_1 , hence a unique solution exists; denote the solution $x_1(h, r)$ and define

$$x_2(h, r) = G\left(\frac{U'(x_1(h, r))}{r\beta}\right).$$

The functions $\{x_1(h, r), x_2(h, r)\}$ have the following properties:

Proposition 1 *Let $\beta r \geq 1$. Under Assumption 1,*

- (i) *The functions $x_1(h, r)$ and $x_2(h, r)$ are continuous in (h, r) and nonincreasing in h .*
- (ii) *The fraction of wealth devoted to impatient households $hx_1(h, r)$ is increasing in h and the fraction devoted to patient households $(1-h)x_2(h, r)$ is decreasing in h .*
- (iii) *The impact of an increase in r on $x_1(r, h)$ depends on h , specifically whether the region is a borrower or*

a lender. There exists an $h_b(r)$ such that $x_1(h, r) = \frac{E - k^*(r)}{h}$, so the region is neither a borrower or a lender. There exists an $h_c \in H$ such that, if $h \geq h_c$, $x_1(h, r)$ is nonincreasing in r .

The proof is in the appendix. Hence, when h is high, consumption falls in both periods and a greater fraction of wealth is devoted to early consumers.

The idiosyncratic risk in the liquidity demand shock cannot be diversified away. Figure (2) illustrates the problem solved at $t = 1$ by a region with portfolio $\{E - k^*(r), \theta(k^*(r))^\alpha\}$ and liquidity demand realization $h = 0.2$. The point of tangency of the budget set and the indifference curve is the optimal consumption allocation. The slope of the budget constraint is $-\frac{hr}{1-h}$ and the slope of the indifference curve at the point of tangency is $-\frac{hU'(x_1)}{(1-h)\beta U'(x_2)}$, so for all regions $r = \frac{U'(x_1)}{\beta U'(x_2)}$. As h increases, the budget set rotates clockwise through the production point $\{\frac{E - k^*(r)}{h}, \frac{\theta(k^*(r))^\alpha}{1-h}\}$. For regions with a very low h , the budget set is quite flat, becoming steeper as h rises. The rotation of the budget constraint as h varies illustrates the idiosyncratic risk that cannot be diversified away. Borrowing and lending in the interbank market allows the region to locate anywhere along the budget set, but the inability to insure against the liquidity shock implies the region cannot change the budget set.

The market-clearing conditions are

$$E - k^*(r) = \int_H hx_1(h, r)\pi(h)dh, \quad (30)$$

$$\theta(k^*(r))^\alpha = \int_H (1-h)x_2(h, r)\pi(h)dh. \quad (31)$$

Definition 1 *An interbank competitive equilibrium with private information and no regional financial markets is an allocation $\{x_1(h, r), x_2(h, r)\}$, investment portfolio $\{E - k, k\}$, and interest rate r such that (i) the financial intermediary maximizes (3) subject to the budget constraint (26) and individual incentive compatibility constraint (6) by choosing $k^*(r)$ and the Green-Lin consumption allocation (27)-(28), for all $h \in H$; (iii) the interbank market clears, so (30)-(31) are satisfied at r_b .*

The equilibrium interest rate r_b is the solution to

$$E - k^*(r) = \int_{\underline{h}}^{h_c(r)} hx_1(h, r)\pi(h)dh + \int_{h_c(r)}^{h_b(r)} hx_1(h, r)\pi(h)dh + \int_{h_b(r)}^{\bar{h}} hx_1(h, r)\pi(h)dh. \quad (32)$$

The left side is strictly increasing in r . The second and third terms on the right side are decreasing in r . If

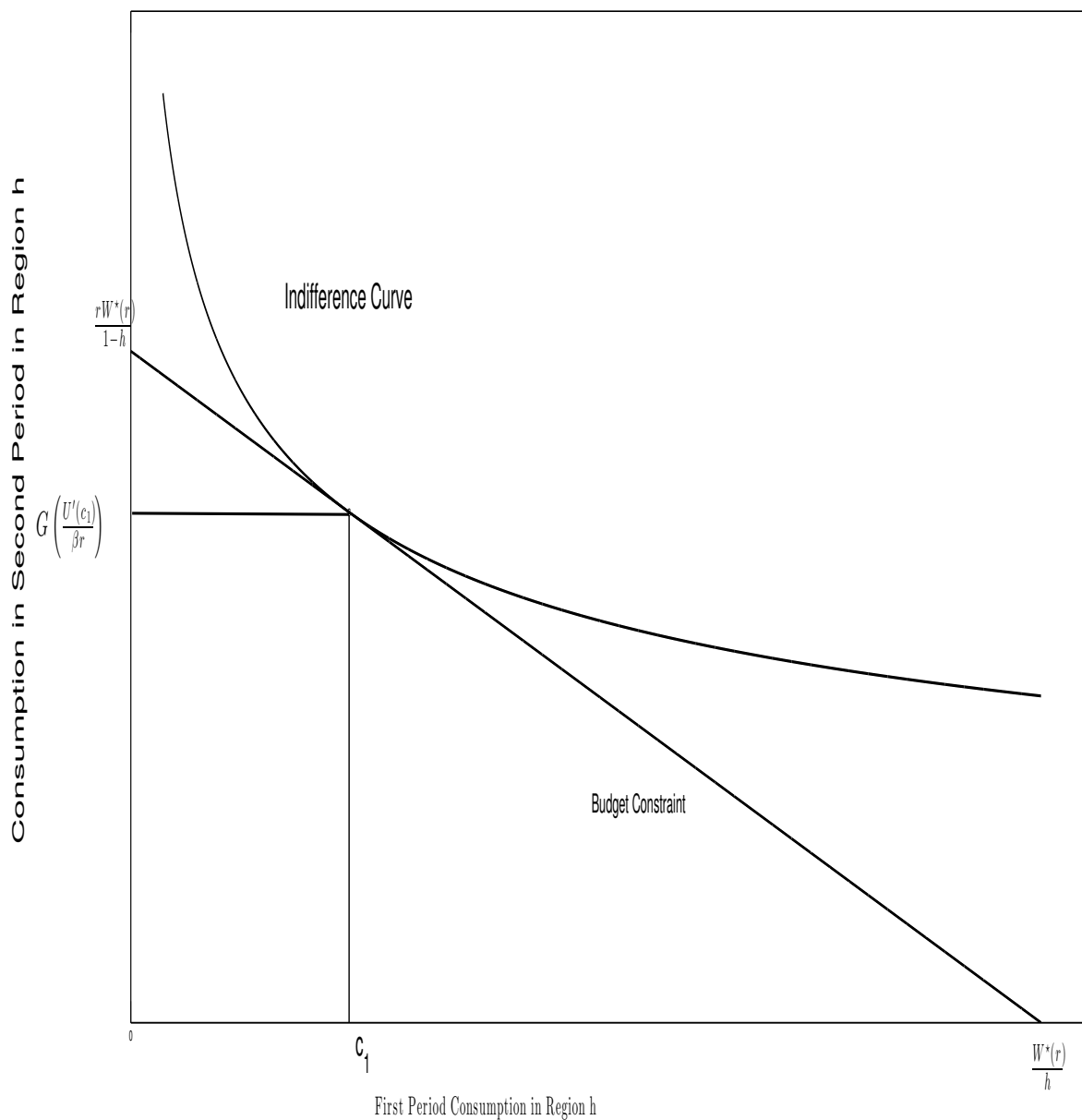


Figure 3: Budget constraint and indifference curve in a region with $h = 0.2$. The budget constraint varies with the liquidity demand shock h . Interbank borrowing allows a region to move along the budget constraint but doesn't alter the constraint, unlike contingent claims trading.

$h_c(r)$ is small, specifically near \underline{h} , the right side is decreasing in r so an equilibrium exists.¹¹ For logarithmic utility, $h_c(r) = \underline{h}$.

Theorem 1 *Let $(r_b, k^*(r_b), \{x_1(h, r_b), x_2(h, r_b)\})$ be an interbank competitive equilibrium (Definition 1). The interbank competitive equilibrium does not support the first-best consumption allocation $\{c_{f1}, c_{2f}\}$. The equilibrium interest rate satisfies $\frac{1}{\beta} < r_b$.*

PROOF.

Observe $x_1(h^m, r_f) = c_{f1}$ because c_{f1} solves $W^*(r_f) = h^m c_{f1} + \frac{(1-h^m)}{r_f} G\left(\frac{U'(c_{f1})}{\beta r_f}\right)$. For any $h \neq h^m$, $x_1(h, r_f) \neq c_{f1}$ because $x_1(h, r)$ is strictly decreasing in h . To show $\beta^{-1} < r_b$, assume $r_b = \beta^{-1}$. Then $k^*(\beta^{-1}) = k_i = (\beta\theta\alpha)^{\frac{1}{1-\alpha}}$. If $r_b = \beta^{-1}$ then (27) implies $x_1(h, \beta^{-1}) = x_2(h, \beta^{-1})$ because U is strictly concave. Using (28), it follows

$$E - k_i + \beta\theta k_i^\alpha = x_1\left(h, \frac{1}{\beta}\right)$$

If $r_b = \beta^{-1}$ is an equilibrium, then $x_1 = x_2$ and

$$E - k_i = \int_H h x_1\left(h, \frac{1}{\beta}\right) \pi(h) dh = h^m [E - k_i + \beta\theta k_i^\alpha]$$

or

$$(\beta\theta\alpha)^{\frac{1}{1-\alpha}} = \frac{E(1-h^m)}{1-h^m + \alpha^{-1}h^m},$$

which violates Assumption 3. ■

If an equilibrium exists, then it has the property $\beta r \geq 1$, so the incentive compatibility constraint (6) is satisfied. An equilibrium exists if the relative curvature of the utility function is not too large or if there is sufficient demand for liquidity insurance, which is a property of the distribution of the liquidity demand shocks. The interest rate in the private information economy is different from the first-best rate, creating a pecuniary externality of the type described by Bhattacharya and Gale [4]. While the financial intermediary provides liquidity insurance, it relies on the interbank market to smooth liquidity demand shocks and the resulting competitive equilibrium is inefficient.

¹¹This is a joint restriction on \underline{h} and the distribution function π . An economy in which $\underline{h} = 0$ and a concentration of probability mass near low values of h may not have an equilibrium because there is insufficient demand for liquidity. In that case, the interest rate would be β^{-1} and there would be storage. This is partly a restriction on the probability density function π . The intuition is there must be enough of a demand for liquidity in the first period for an equilibrium to exist.

3.1.1 Logarithmic Utility

Define $\tau : H \rightarrow [0, 1]$ by

$$\tau(h) \equiv \frac{\beta(1-h)}{\beta(1-h)+h},$$

and observe $\tau(h)$ is decreasing and convex in h .¹² Equilibrium consumption satisfies $hx_1(h, r) = [1 - \tau(h)]W^*(r)$ and $(1-h)x_2(h, r) = \tau(h)rW^*(r)$. The equilibrium interest rate is

$$r_b = (\theta\alpha) \left[\frac{E \int_H \tau(h)\pi(h)dh}{\int_H [\tau(h) + (1-\tau(h))\alpha^{-1}]\pi(h)dh} \right]^{\alpha-1}$$

and investment in the long-term project is $k^*(r_b)$. It can be directly verified that $r_f - r_b > 0$.

3.2 Interbank and Financial Markets Co-Exist

By assumption, financial intermediaries and local financial markets co-exist. In Figure 1, households now trade in local financial markets as well as placing deposits with the local financial intermediary. At $t = 0$ the household deposits E with the local financial intermediary offering the allocation $\mathcal{C} = \{c_1(v, h), c_2(v, h)\}$ providing the highest *ex ante* expected utility. At $t = 1$, each household observes v and decides its announcement \hat{v} , which determines its consumption stream $\{c_1(\hat{v}, h), c_2(\hat{v}, h)\}$. The liabilities of the financial intermediary can be used either as collateral in borrowing or directly traded (as negotiable securities) in the local financial market at interest rate $r(h)$. This opportunity to engage in trading in local financial markets impacts a household's incentive to reveal information.

Since v is private information and consumption is unobservable, a household realizing v may announce $\hat{v} \neq v$. At $t = 1$, the household solves

$$V_H(\mathcal{C}, r(h), v) = \max_{\{\hat{v}, x_1, x_2\}} [(1-v)U(x_1) + v\beta U(x_2)] \quad (33)$$

subject to

$$c_1(\hat{v}, h) + \frac{c_2(\hat{v}, h)}{r(h)} \geq x_1 + \frac{x_2}{r(h)}, \quad (34)$$

where the announcement $\hat{v} \in \{0, 1\}$ determines the discounted present value of the payments on deposits, the left side of (34). Let $\{\hat{x}_1(v, \mathcal{C}, r(h)), \hat{x}_2(v, \mathcal{C}, r(h)), \hat{v}(v, \mathcal{C}, r(h))\}$ denote the solution to (33) subject to

¹²Observe $\tau'(h) = -\left[\frac{\beta}{(\beta(1-h)+h)^2}\right] < 0$ and $\tau''(h) = 2\left[\frac{\beta(1-\beta)}{(\beta(1-h)+h)^3}\right] > 0$.

(34). An impatient household announcing \hat{v} will consume

$$\hat{x}_1(0, \mathcal{C}, r(h)) = c_1(\hat{v}, h) + \frac{c_2(\hat{v}, h)}{r(h)}$$

at $t = 1$, while a patient household announcing \hat{v} will consume

$$\hat{x}_2(1, \mathcal{C}, r(h)) = r(h)c_1(\hat{v}, h) + c_2(\hat{v}, h)$$

at $t = 2$. Since the indirect utility function $V_H(\cdot)$ is increasing in wealth, where wealth is defined as the left side of (34), a household has an incentive to announce the \hat{v} maximizing wealth regardless of his realization v . Specifically, if

$$c_1(0, h) + \frac{c_2(0, h)}{r(h)} < c_1(1, h) + \frac{c_2(1, h)}{r(h)},$$

then all households in the region will announce $v = 1$, while the converse holds if the inequality is reversed. Farhi, Golosov and Tsyvinski [9] demonstrate *an incentive compatible allocation with private trading* will have the property

$$W_h(r, h) \equiv c_1(0, h) + \frac{c_2(0, h)}{r} = c_1(1, h) + \frac{c_2(1, h)}{r}, \quad (35)$$

for each $h \in H$, so the discounted present value of any incentive-compatible allocation is equal for all v , conditional on h and given r .

The financial intermediary can borrow or lend the amount b in the local financial market at interest rate $r(h)$, in addition to trading in the interbank market. Its budget constraints are

$$E - k = hc_1(0, h) + (1 - h)c_1(1, h) + A + b, \quad (36)$$

$$\theta k^\alpha + rA + r(h)b = hc_2(0, h) + (1 - h)c_2(1, h). \quad (37)$$

Solve (37) for A and substitute into (36) and rewrite

$$W(r, k) = b \left[1 - \frac{r(h)}{r} \right] + \left[hc_1(0, h) + (1 - h)c_1(1, h) + \frac{hc_2(0, h) + (1 - h)c_2(1, h)}{r} \right]. \quad (38)$$

The financial intermediary maximizes the expected utility of the representative agent (3) subject to the constraint (38) and the *incentive compatibility constraint with private trading*

$$hU(c_1(0, h)) + (1 - h)\beta U(c_1(1, h) + c_2(1, h)) \geq hV_H(\{c_1(v, h), c_2(v, h)\}, r(h), 0) + (1 - h)V_H(\{c_1(v, h), c_2(v, h)\}, r(h), 1) \quad (39)$$

This constraint replaces the individual incentive compatibility constraint (6).

The financial intermediary will arbitrage between the local financial market and the interbank market: if $r(h) > r$ ($r(h) < r$) then it borrows (lends) the amount A in the interbank market and lends (borrows) the amount b in the local financial market until the *no arbitrage condition*

$$r(h) = r \tag{40}$$

is satisfied.

The financial intermediary chooses a portfolio allocation maximizing $W(k, r)$, where the solution is $W^*(r)$. The elimination of arbitrage profits and competition among local financial intermediaries in a region will result in zero profits so the left side of (38) can be expressed as

$$W^*(r) = E - k^*(r) + \frac{\theta(k^*(r))^\alpha}{r} = W(r, k).$$

Assuming (40) holds and profits are zero, it follows from (38) that

$$W^*(r) = W_h(r, h),$$

where W_h is defined in (35).

The solution to the financial intermediary's problem is (i) a portfolio allocation $\{E - k^*(r), \theta(k^*(r))^\alpha\}$, (ii) interbank position $A(h, r)$, and (iii) local credit market position $b(h)$ such that the no-arbitrage condition is satisfied, and a consumption allocation $c_1(v, r), c_2(v, r)$ satisfying the incentive compatibility constraint with private trading (??) such that, given the interest rate r and consumption allocation, households truthfully reveal their types $\hat{v} = v$.

The assets of the financial intermediary at $t = 1$ equal $W^*(r)$ plus lending in the interbank market (if $A(h) > 0$) or lending in the local financial market ($b > 0$). The liabilities equal the present value of the claims on the intermediary, which include borrowing in either the interbank market or the local financial market in addition to the present value of claims $W_H(h, r)$. The maturity structure between liabilities at $t = 1$ and $t = 2$ is indeterminate. For convenience, set $c_1(v, h) = E - k^*(r)$ and $c_2(v, h) = \theta(k^*(r))^\alpha$. This arrangement satisfies the incentive compatibility constraint with private trading. The financial arrangement between households and the local financial intermediary is discussed briefly below.

Equilibrium in Financial Markets

In a region realizing h , a fraction h of the regional households wish to borrow (or sell) their claims against the long term asset. A fraction $1 - h$ are willing to lend. When the position taken by the financial intermediary is $b = 0$, the equilibrium interest rate $r(h)$ in the local credit market is

$$r(h) = \left[\frac{h}{1-h} \right] \left[\frac{\theta k^\alpha}{E-k} \right], \quad (41)$$

creating a distribution of interest rates across regions.¹³ The local financial intermediary will arbitrage across the interbank market and local financial market, setting A as

$$A(h, r) = (1-h)(E-k) - h \frac{\theta k^\alpha}{r} = -b(h, r).$$

If $r(h) > r$, then $b > 0$ and $A < 0$ because the financial intermediary borrows in the interbank market where interest rates are lower. When $r(h) < r$ the financial intermediary borrows in local markets and lends in the interbank market.

Impatient households consume $W^*(r)$ and patient households consume $rW^*(r)$. The equilibrium condition in the interbank market is an interest rate $r_m(k)$ solving

$$\begin{aligned} 0 &= \int_H A(h, r) \pi(h) dh \\ &= \int_H [(1-h)(E-k)r - h\theta k^\alpha] \pi(h) dh, \end{aligned} \quad (42)$$

given k . The equilibrium interbank interest rate is

$$r_m(k) = \left[\frac{h^m}{1-h^m} \right] \left[\frac{\theta k^\alpha}{E-k} \right]. \quad (43)$$

given k .

Definition 2 *An financial market competitive equilibrium is a consumption allocation $\{c_{m1}, c_{m2}\}$, a portfolio $\{E - k_m, k_m\}$, and interest rates r and $r(h)$ for $h \in H$ such that (i) Households solve (33) subject to*

¹³If households buy and sell claims on the intermediary, the price $q(h)$ in equilibrium satisfies

$$hq(h)\theta k^\alpha = (1-h)(E-k)$$

or $q(h) = (r(h))^{-1}$.

(34) given $r(h)$; (ii) The financial intermediary solves (3) subject to (38) and (39) given $r, r(h)$; (iii) The no arbitrage condition (40) is satisfied; (iv) The interbank equilibrium condition (42) is satisfied at r_m (v) The local financial market is in equilibrium (satisfies (41)).

The financial intermediary anticipates this rate of interest at $t = 0$ when making its portfolio allocation $(E - k, k)$. Substitute $r = \theta\alpha k^{\alpha-1}$ into (43) and solve for k

$$k_m = \frac{(1 - h^m)\alpha E}{h^m + \alpha(1 - h^m)}. \quad (44)$$

This is the optimal investment in the long-term asset when financial and interbank markets co-exist. The choice of k and the market interest rate $r_m = r_m(k_m)$ do not depend on the curvature of the utility function or β . In equilibrium,

$$c_{m1} = E - k_m + \frac{\theta k_m^\alpha}{r_m}, \quad (45)$$

$$c_{m2} = r_m[E - k_m] + \theta k_m^\alpha. \quad (46)$$

The financial intermediary borrows

$$b^*(r_m, h) = h\theta k_m^\alpha - (1 - h)(E - k_m)r_m = \theta k_m^\alpha \left[\frac{h - h^m}{1 - h^m} \right],$$

where the interest rate r_m is substituted.

Proposition 2 Under Assumptions (1)-(4), $\beta r_m \geq 1$ and

$$r_m < r_f,$$

from which it follows $k_m > k_f$.

PROOF.

To show $\beta r_m \geq 1$, substitute for k_m using (44) in the definition of $r_m(k)$ in (43)

$$\beta r_m = \beta \left[\frac{h^m}{1 - h^m} \right] \left[\frac{\theta \left(\frac{(1 - h^m)\alpha E}{h^m + \alpha(1 - h^m)} \right)^\alpha}{E - \left(\frac{(1 - h^m)\alpha E}{h^m + \alpha(1 - h^m)} \right)} \right].$$

Rewriting, it follows

$$\beta r_m = \beta\theta\alpha \left(\frac{\alpha E(1 - h^m)}{h^m + \alpha(1 - h^m)} \right)^{\alpha-1} \geq 1,$$

where the last inequality uses Assumption 4.

To prove $r_m(k_m) < r_f$, define the intertemporal marginal rate of substitution as

$$R(k) \equiv \frac{U' \left(\frac{E-k}{h^m} \right)}{\beta U' \left(\frac{\theta k^\alpha}{1-h^m} \right)}$$

and observe $R(k)$ is increasing in k , as is r_m . For a given k , observe

$$\begin{aligned} r_m(k) - R(k) &= \left[\frac{h^m}{1-h^m} \right] \left[\frac{\theta k^\alpha}{E-k} \right] - \left[\frac{U' \left(\frac{E-k}{h^m} \right)}{\beta U' \left(\frac{\theta k^\alpha}{1-h^m} \right)} \right] \\ &= \left[\frac{h^m}{(E-k)\beta U' \left(\frac{\theta k^\alpha}{1-h^m} \right)} \right] \left[\frac{\theta k^\alpha}{1-h^m} \beta U' \left(\frac{\theta k^\alpha}{1-h^m} \right) - \left(\frac{E-k}{h^m} \right) U' \left(\frac{E-k}{h^m} \right) \right]. \end{aligned}$$

By assumption $U'(c)c$ is decreasing and the allocation satisfies the individual incentive compatibility constraint $\frac{\theta k^\alpha}{1-h^m} \geq \frac{E-k}{h^m}$ because $\beta r_m(k) \geq 1$, so the second term in brackets on the right side is negative. Hence $r_m(k) < R(k)$. Since k_m is the solution to

$$\theta \alpha k_m^{\alpha-1} = r_m(k_m),$$

and k_f solves

$$\theta \alpha k_f^{\alpha-1} = R(k_f),$$

it follows $\theta \alpha k_m^{\alpha-1} < \theta \alpha k_f^{\alpha-1}$ or $k_m > k_f$ because $\theta \alpha k^{\alpha-1}$ is strictly decreasing in k . ■

Hence financial markets result in lower interest rates with a higher fraction of the endowment invested in the long-term asset, or equivalently a lower investment in the short-term asset, relative to the first-best allocation. An important property of this allocation is

$$U'(c_{m1}) \neq \beta r_m U'(c_{m2})$$

and $c_{m1} = \frac{c_{m2}}{r_m}$. Hence no liquidity insurance is provided when agents directly trade in financial markets, a point recognized by Jacklin [13] among others. Since the financial intermediary eliminates arbitrage profits, $r = r(h)$ and the household's consumption is unaffected by the realization h .

3.2.1 Financial Arrangement Between Households and Intermediaries

The deposit contract offered by the financial intermediary $\{c_1(v, h), c_2(v, h)\}$ is assumed to be tradable in the local credit market. It is also assumed the household deposits its entire endowment with the financial intermediary because storage between periods 0 and 1 is ruled out. Maintaining this assumption doesn't preclude other financial arrangements between households and the financial intermediary, as long as the $t = 1$ present value of the allocation to each household, regardless of its announcement of type, is equal to $W^*(r)$ and the no arbitrage condition is satisfied.

As an example following Jacklin, the financial intermediary can be organized as a mutual fund with complete financing by equity capital. At $t = 0$, each household deposits its endowment with the financial intermediary in exchange for an equity share, which is a claim to a two-period dividend stream $(E - k, \theta k^\alpha)$ when the mutual fund invests k in the long-term asset. At $t = 1$, each household receives the dividend $E - k$ and observes its v . Impatient households wish to sell their ex-dividend share while patient households wish to buy shares. There is an active secondary market trading ex-dividend shares. Households selling shares at $t = 1$ receive $(1 - h)(E - k)$ at $t = 1$ in exchange for $h\theta k^\alpha$ at $t = 2$. In the event equity shares are traded only within a region, the share price would be $q(h) = \frac{\theta k^\alpha}{r_m(h)}$ and there would be an arbitrage opportunity across regions. When the equity markets are integrated across regions, the ex-dividend share price in the secondary market is $q = \frac{\theta k^\alpha}{r_m}$. All shares trade at the same price because the dividend stream is identical across regions. Trading equity shares in an integrated market will result in an allocation such that $\frac{c_2}{r} = c_1$.

4 Central Bank Policy

In the absence of intervention by the central bank, the two trading mechanisms - financial intermediation only versus co-existence with financial markets - result in different equilibrium allocations, and neither results in the first-best allocation. The central bank may be able to implement welfare-improving policies by acting as a mechanism designer. Bhattacharya and Gale derive the second-best (incentive-efficient) allocation in a framework similar to the one studied here. In their model, the central bank is a mechanism designer implementing incentive-compatible transfers across regions. The key difference between their approach and mine is the lack of financial markets in Bhattacharya and Gale, whereas the impact of subsequent trading opportunities for both financial intermediaries and households is the focus here.

The potential to trade impacts the social planning problem in several ways. The central bank chooses a feasible allocation $\{E - k, k\}$ at $t = 0$ and consumption allocation $\mathcal{S} \equiv \{c_1(v, h), c_2(v, h)\}_{h \in H, v \in \{0,1\}}$. If the consumption allocation requires transfers of the consumption good across regions, incentives of a region to reveal information are typically affected. A region announcing it is a type \hat{h} will receive $\hat{h}c_1(0, \hat{h}) + (1 - \hat{h})c_1(1, \hat{h})$ at $t = 1$ and $\hat{h}c_2(0, \hat{h}) + (1 - \hat{h})c_2(1, \hat{h})$ at $t = 2$ from the central bank. By assumption, the central bank cannot prevent borrowing and lending in the interbank market, nor can it make the transfers contingent on subsequent activity in the interbank market. A regional financial intermediary realizing h and announcing \hat{h} to the central bank will choose an allocation $\{z_1(v, h), z_2(v, h)\}$, $v \in \{0, 1\}$ subject to its budget constraints

$$\hat{h}c_1(0, \hat{h}) + (1 - \hat{h})c_1(1, \hat{h}) \geq h z_1(0, h) + (1 - h)z_1(1, h) + A + b, \quad (47)$$

$$\hat{h}c_2(0, \hat{h}) + (1 - \hat{h})c_2(1, \hat{h}) + rA + r(h)b \geq h z_2(0, h) + (1 - h)z_2(1, h), \quad (48)$$

where borrowing in the interbank and local financial markets is incorporated. Solve (48) for A , substitute into (47), and rewrite to obtain

$$\begin{aligned} & \left[\hat{h}c_1(0, \hat{h}) + (1 - \hat{h})c_1(1, \hat{h}) + \frac{\hat{h}c_2(0, \hat{h}) + (1 - \hat{h})c_2(1, \hat{h})}{r} \right] + b \left[1 - \frac{r(h)}{r} \right] \\ & = \left[h z_1(0, h) + (1 - h)z_1(1, h) + \frac{h z_2(0, h) + (1 - h)z_2(1, h)}{r} \right]. \end{aligned} \quad (49)$$

The financial intermediary chooses $\{\hat{h}, z_1(v, h), z_2(v, h), b, d_1, d_2\}$, $v \in \{0, 1\}$ to solve

$$V_F(\mathcal{S}, r, h, r(h)) = \max_{\{k, z_1(v, h), z_2(v, h)\}} \int_H [hU(z_1(0, h)) + (1 - h)\beta U(z_1(1, h) + z_2(1, h))] \pi(h) dh \quad (50)$$

subject to (49) and the *individual incentive compatibility constraint with trading*

$$hU(z_1(0, h)) + (1 - h)\beta U(z_1(1, h) + z_2(1, h)) \geq hV_H(\{z_1(v, h), z_2(v, h)\}, r(h), 0) + (1 - h)V_H(\{z_1(v, h), z_2(v, h)\}, r(h), 1), \quad (51)$$

where V_H is defined as the indirect utility of a household, or the solution to

$$V_H(\{z_1(v, h), z_2(v, h)\}, r(h), v) = \max_{\{\hat{v}, x_1, x_2\}} [hU(x_1) + (1 - h)\beta U(x_2)] \quad (52)$$

subject to

$$z_1(\hat{v}, h) + \frac{z_2(\hat{v}, h)}{r(h)} \geq x_1 + \frac{x_2}{r(h)}.$$

This value function $V_F(\mathcal{S}, r, h, r(h))$ is increasing in the present value of the transfers from the social planner, equal to the first term in brackets on the left side of (49). The second term on the left side is arbitrage profits, which equal zero in equilibrium.

Although the model differs in several key ways, the arguments developed by FGT can be applied here. The financial intermediary will announce the \hat{h} maximizing the present value of the transfers from the central bank, or

$$W_S(r) \equiv \max_{\{h\}} \left[hc_1(0, h) + (1 - h)c_1(1, h) + \frac{hc_2(0, h) + (1 - h)c_2(1, h)}{r} \right].$$

The *regional incentive compatibility constraint with interbank markets* is

$$hU(c_1(0, h)) + (1 - h)\beta U(c_2(1, h)) \geq V_F(\{hc_1(h), (1 - h)c_2(h)\}, r, h, r(h)) \quad (53)$$

must hold for each h , where $c_1(1, h) = c_2(0, h) = 0$ for convenience.

Households in a region realizing h take the allocation offered by the financial intermediary as given and trade in the local financial markets at interest rate $r(h)$. Financial intermediaries take the allocation offered by the central bank as given and engage in trading in the interbank market at interest rate r . The central bank takes the existence of the subsequent trading opportunities into account in solving its maximization problem. The social planning problem with interbank and local financial markets is stated next.

(SP3) The central bank chooses a portfolio $(E - k, k)$ and a feasible consumption allocation $\{c_1(v, h), c_2(v, h)\}$, $h \in H$ and $v \in \{0, 1\}$ to maximize the expected utility of a representative household (3) subject to the resource constraints (1) and (2), and the regional incentive compatibility constraint (53), where the function V_F incorporates the individual incentive compatibility constraint and the incentive compatibility constraint with private trading, and $(r, r(h))$ are the equilibrium interest rates.

Definition 3 *The constrained efficient allocation with interbank and local financial markets is a feasible allocation $\{E - k, k, \{c_1(v, h), c_2(v, h)\}_{h \in H, v \in \{0, 1\}}\}$ solving the central bank's problem (SP3), where the no-arbitrage condition holds, and the interbank market and local financial markets are in equilibrium.*

FGT discuss the solution in a model with an exogenous return to the long-term project and without liquidity demand shocks (so $h = h^m$). They demonstrate the social planner can achieve an allocation superior to the one achieved by competitive markets, even in the presence of private-trading opportunities for households

and financial intermediaries. FGT demonstrate the constrained efficient allocation with private trading coincides with the full information allocation. The allocation can be implemented through a *liquidity floor*, which takes the form of a simple portfolio restriction.

Although there is an additional source of uncertainty and trading (interbank market) in this model, the mathematical structure here is very similar to their structure. Instead of formally proving the existence of a solution and determining the properties of this solution, as FGT do, I examine whether the first-best allocation can be implemented using a simple policy, specifically a liquidity floor, as in Allen and Gale [2] and FGT. By construction, if the first-best allocation satisfies all of the constraints when the liquidity floor is in place, then it is the constrained efficient solution when there is private information and co-existence of local and interbank financial markets. A formal proof can be constructed based on the proof of Proposition 1 and Theorem 1 in FGT.

The steps are to examine if the two incentive compatibility constraints and resource constraints are satisfied under the conjecture there is a liquidity floor. The first step is to examine the impact of local financial markets on the incentive compatibility constraint faced by the financial intermediary, specifically the $t = 1$ discounted present value of claims on the financial intermediary must be equal for any $v \in \{0, 1\}$. This property and equilibrium in the local market, implies

$$c_{1m}(h) = \frac{c_{2,m}(h)}{r(h)}$$

and

$$U'(c_{1m}(h)) \neq \beta r(h) U'(c_{2m}(h))$$

Second, the menu of feasible allocations $\mathcal{C} = \{c_1(v, h), c_2(v, h)\}$ chosen by the social planner will have the property the $t = 1$ discounted present value of transfers to a financial intermediary will be equal across for all $h \in H$,

$$W_s(\mathcal{C}, r) = h \left[c_1(0, h) + \frac{hc_2(0, h)}{r} \right] = (1 - h) \left[c_1(1, h) + \frac{c_2(1, h)}{r} \right]$$

to satisfy the regional incentive compatibility constraint with trading ((??)).

The question is whether there is an intervention in the form of a liquidity floor implementing the constrained efficient allocation with interbank trading and financial trading. Suppose the central bank imposes a liquidity floor on financial intermediaries, specifically the portfolio allocation to short term assets must

satisfy

$$E - k \geq M \equiv E - k_f. \quad (54)$$

This is a restriction on the composition of assets on the intermediary's balance sheet. If the constraint is binding, then $k = E - M$. For the constraint to be binding, the interest rate in the interbank market must satisfy $r < r_f$, otherwise $k^*(r) > E - M$. The social planner offers an allocation $\{E - k_f, \theta k_f^\alpha\}$ to each region regardless of its announcement \hat{h} , which satisfies the regional incentive compatibility constraint with interbank markets (53). At $t = 0$, the financial intermediary chooses k subject to the liquidity constraint, so $k = k_f$. In this case, the equilibrium interest rate in the interbank market is

$$r_m(k_f) = \left[\frac{h^m}{1 - h^m} \right] \left[\frac{\theta k_f^\alpha}{E - k_f} \right].$$

As derived earlier, consumption of impatient households will satisfy

$$c_{f1} = E - k_f + \frac{\theta k_f^\alpha}{r_m(k_f)}$$

while consumption of patient households will satisfy

$$c_{f2} = r_m(k_f)(E - k_f) + \theta k_f^\alpha.$$

Denote

$$MRT(k) = \theta \alpha k_f^{\alpha-1}$$

The liquidity floor creates a wedge between the marginal rate of transformation $MRT(k)$ and the marginal rate of substitution in the interbank market $R(k_f) = r_m(k_f)$. Without the liquidity floor, competitive local credit markets and the interbank market would not support the first-best allocation.

These results are summarized in the following theorem.

Theorem 2 *Solutions to the constrained efficient problem with interbank trading and local financial markets (SP3) and the informationally unconstrained problem coincide. The competitive equilibrium with interbank and local financial markets specified in Definition (2) coincides with the constrained efficient allocation with interbank and local financial markets when the additional constraint (54) - a liquidity floor - is imposed.*

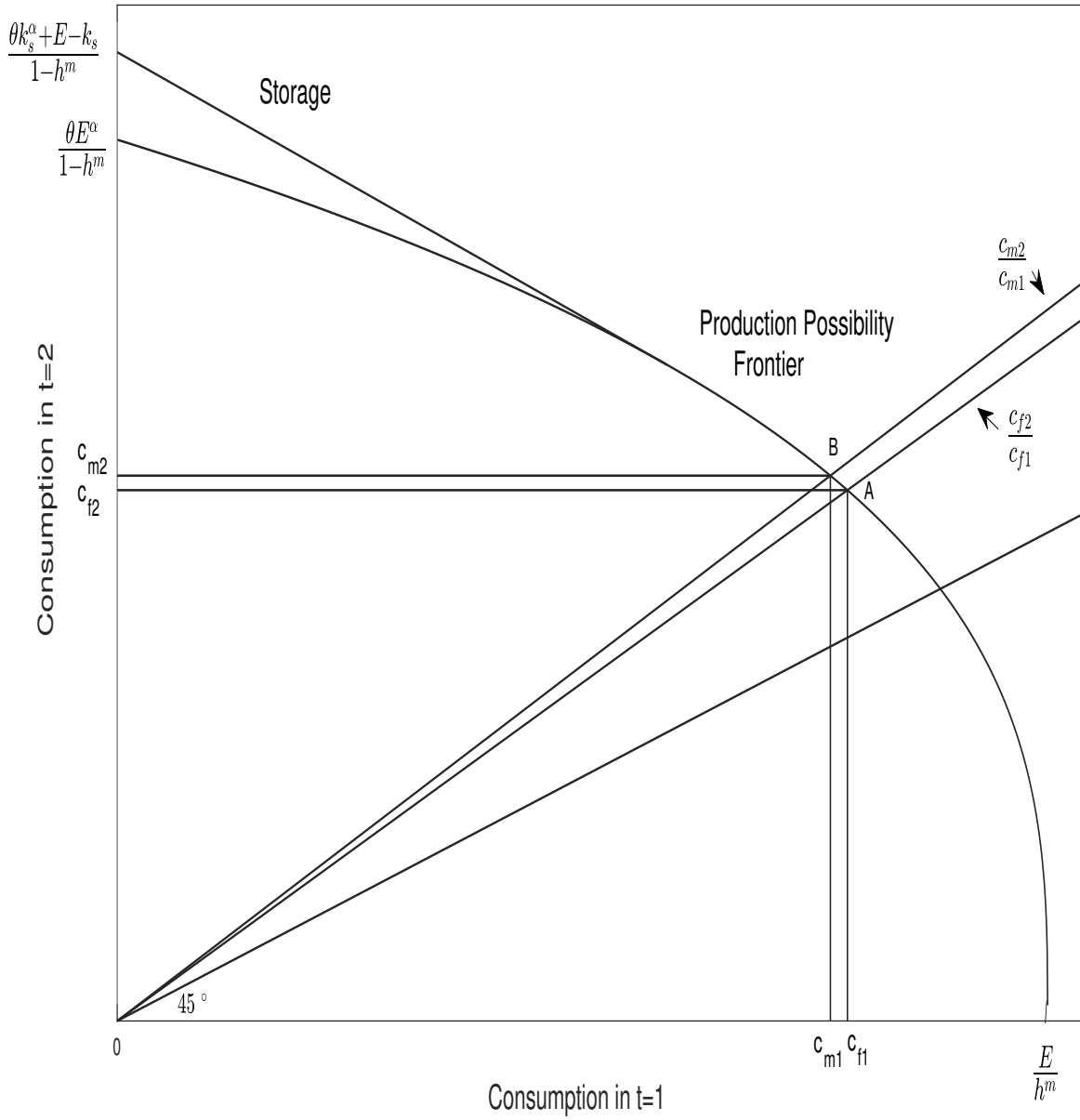


Figure 4: The central bank imposes a liquidity floor M satisfying $c_{f1} = \frac{M}{h^m} = \frac{E - k_f}{h^m}$. The slope at point A is $-\frac{h^m}{1 - h^m} r_f$. The interbank rate with the liquidity floor is $-\frac{c_{f2}}{c_{f1}}$.

5 Conclusion

Private information plays an important role in financial arrangements and pricing. The assumption contractual arrangements cannot be made exclusive leads to private trading, altering the incentives to reveal information. The question addressed here is how private information and private trading interact to impact a regional bank's investment portfolio, securitization of that portfolio, and the ability to smooth liquidity shocks in the interbank market.

A regional bank observes a random variable determining the distribution of its risky investments. It also observes a random variable determining the fraction of households in its region wishing for early consumption. The interaction of private information and private trading creates an incentive for all regions to act as if they are low risk in the securitization process, resulting in an over-investment in the risky asset by high risk regions and an over-investment in the safe asset by low risk regions. This distortion in the composition of assets impacts the region's ability to smooth liquidity shocks. As in FGT, the private-trading constrained efficient allocation can be implemented by imposing a liquidity floor, a restriction on the composition of the investment portfolio, which impacts the equilibrium interest rate in the interbank market.

6 Appendix

Proposition 1

The arguments of $x_1(h, r)$ are suppressed for convenience.

PROOF.

For part (i), continuity follows from Assumption 1. Let

$$D(h, r) \equiv h + \frac{(1-h)}{\beta r^2} G' \left(\frac{U'(x_1)}{\beta r} \right) U''(x_1).$$

Then

$$\frac{\partial x_1(h, r)}{\partial h} = [D(h, r)]^{-1} \left[G \left(\frac{U'(x_1(h, r))}{\beta r} \right) \frac{1}{r} - x_1 \right].$$

Since $\beta r \geq 1$, $U'(x_2) \leq U'(x_1)$ so $x_2 \geq x_1$. Using the first-order condition $r^{-1} = \frac{\beta U'(x_2)}{U'(x_1)}$,

$$\frac{x_2}{r} - x_1 = [\beta x_2 U'(x_2) - x_1 U'(x_1)] [U'(x_1)]^{-1} < [x_2 U'(x_2) - x_1 U'(x_1)] [U'(x_1)]^{-1} < 0,$$

where the inequality follows because $U'(c)$ is decreasing. Hence $x_1(h, r)$ is decreasing in h . The function $x_2(h, r)$ is non-increasing in h because

$$\frac{\partial x_2(h, r)}{\partial h} = \frac{U''(x_1)}{\beta r U''(x_2)} \frac{\partial x_1}{\partial h} \leq 0.$$

Part (ii) follows because

$$\begin{aligned} \frac{\partial h x_1}{\partial h} &= x_1 + h \frac{\partial x_1}{\partial h} \\ &= x_1 \frac{(1-h)}{r} \frac{U''(x_1)}{\beta r U''(x_2)} + \frac{h x_2}{r} > 0 \end{aligned}$$

and

$$\frac{\partial(1-h)x_2}{\partial h} = -\frac{1}{D} \left[h x_2 + (1-h) \frac{U''(x_1)}{\beta r U''(x_2)} x_1 \right] < 0.$$

Property (iii) is established by totally differentiating

$$W^*(r) = h x_1(r, h) + \frac{1-h}{r} G \left(\frac{U'(x_1(r, h))}{\beta r} \right)$$

with respect to r and rewriting to obtain

$$\left[\frac{(1-h)x_2}{r^2} - \frac{\theta k^\alpha}{r^2} \right] + \left[\frac{(1-h)}{r^2} \frac{G' U'(x_1)}{\beta r} \right] = \frac{\partial x_1}{\partial r} [D(h, x_1(h, r), r)], \quad (55)$$

where the derivatives of $W^*(r)$ and $k^*(r)$ have been substituted in. $D(h, x_1(h, r), r)$ is positive. Given r , define $h_b(r)$ as the value of h solving

$$U' \left(\frac{E - k^*(r)}{h} \right) = \beta r U' \left(\frac{\theta (k^*(r))^\alpha}{1 - h} \right),$$

so the region neither borrows or lends. Such a solution exists because the left side is increasing in h while the right side is decreasing. At $h_b(r)$,

$$\frac{\partial x_1(h_b(r), r)}{\partial r} < 0,$$

because the first term on the left side of (55) is zero and the second term is negative. To determine the range of liquidity shocks such that $\frac{\partial x_1(h, r)}{\partial r} \geq 0$, solve (55) for $\frac{\partial x_1}{\partial r}$ and define $\hat{h}_c(r)$ as the value of h such that the left side of (55) equals 0, or $\frac{\partial x_1}{\partial r} = 0$, or

$$h_c(r) = \max \left[\hat{h}, \frac{x_2 \left(1 + \frac{U'(x_2)}{U''(x_2)x_2} \right) - \theta k^\alpha}{x_2 \left(1 + \frac{U'(x_2)}{U''(x_2)x_2} \right)} \right]$$

which determines the liquidity demand shock at which a lending region increases its first-period consumption in response to an increase in r . Observe $h_c(r) < h_b(r)$. For regions with $h < h_c(r)$, $x_1(h, r)$ is increasing in r . For regions with $h \geq h_c$, $x_1(h, r)$ is decreasing in r . ■

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