

# Contracting Frictions with Managers, Financial Frictions, and Misallocation\*

(Preliminary and Incomplete)

Chaoran Chen  
National University of Singapore

Ashique Habib  
International Monetary Fund

Xiaodong Zhu  
University of Toronto

February 16, 2018

---

## ABSTRACT

---

Weak contract enforcement prevents productive firms from hiring outside managers and expanding production in developing countries. We develop a model where firms can increase their span of control by hiring outside managers, but weak contract enforcement distorts this delegation decision since managers can then steal the firm's output unless properly incentivized. Firms overcome this friction by hiring fewer managers and increasing the compensation to managers, which manifests as output wedges at the firm level. We show that these features are consistent with cross-country evidence from the IPUMS-International data: The employment share and the wage premium of managers relative to workers are correlated with a country's level of development. In our model, the distortionary effects are increasing in firm productivity, a necessary property any mechanism needs to generate large losses due to misallocation. We then further introduce financial frictions into our model and estimate this model using the firm-level data from China. We find that our model can account for the fact that larger firms have a higher marginal product than smaller firms, a feature of the data that standard models of financial frictions alone cannot generate. We further show that the contracting frictions generate larger productivity loss than financial frictions.

---

Keywords: Contract Enforcement, Delegation, Financial Frictions, Firm Dynamics, Misallocation, Aggregate Productivity.

JEL classification: E13, L16, L26, O41.

---

\*We thank Diego Restuccia, Daniel Yi Xu, as well as seminar participants at Midwest Macro conference at Pittsburgh and Fudan University for helpful comments. All errors are our own. The views expressed here are those of the authors and should not be attributed to the International Monetary Fund, its Executive Board, or its management. Contact: Chen: Department of Economics, National University of Singapore, 1 Arts Link, AS2 #04-38, Singapore 117568, [ecsc@nus.edu.sg](mailto:ecsc@nus.edu.sg). Habib: International Monetary Fund, 700 19th Street NW, Washington DC 20431, [ahabib@imf.org](mailto:ahabib@imf.org). Zhu: Department of Economics, University of Toronto, 150 St. George Street, Toronto, Ontario, Canada, M5S 3G7, [xzhu@chass.utoronto.ca](mailto:xzhu@chass.utoronto.ca). This draft is preliminary and please do not circulate without permission.

# 1 Introduction

Cross-country income differences are largely explained by differences in total factor productivity (TFP) (Caselli, 2005). A recent literature argues that lower TFP in poor countries can be accounted for by severe resource misallocation, especially financial constraints on firms. A common feature of this literature is that financial frictions, often modelled as collateral constraints à la Buera et al. (2011) and Moll (2014), mainly affect small firms, while large firms are less affected. This feature is inconsistent with firm-level data from developing countries that large firms may potentially face more severe distortions.<sup>1</sup> In this paper, we propose that contracting frictions between firms and outside managers causes significant misallocation by distorting the delegation decisions of large firms. We document evidence consistent with the idea that large firms are particularly distorted, and that contracting frictions with outside managers are particularly worse in developing countries. We then develop a model with both financial and managerial frictions, and quantitatively show how managerial frictions are important for both generating large productivity losses and for matching the documented relationship between firm size and marginal productivity.

Larger firms have higher marginal product of capital, as documented in Hsieh and Olken (2014). However, models with just financial frictions generate larger distortions for smaller firms. We first decompose the dispersion of marginal product of capital into the dispersions of capital labor ratio and marginal product of labor. The dispersion in marginal product of capital indicates frictions in the capital market, while the latter indicate some other frictions since financial frictions do not distort the choice of labor. We argue that higher marginal product of labor of larger firms can be explained by contracting frictions, which results in lacks of delegation to outside managers. We further use household survey data from the *Integrated Public Use Microdata Series* (IPUMS) to document two stylized facts. The fraction of population working as managers increases with a country's GDP per capita, and managers are paid a relatively higher wage premium (efficiency wage) in poor countries with weak contracting enforcement after controlling for observed characteristics. These two stylized facts serve as direct evidence for our modelling of contracting frictions.

---

<sup>1</sup>For example, Hsieh and Olken (2014) find that large firms have higher marginal products of capital and labor than small firms using micro data from India, Mexico, and Indonesia.

We build our model on [Garicano and Rossi-Hansberg \(2006\)](#), [Grobovšek \(2017\)](#) and [Buera et al. \(2011\)](#). In our framework, firms face endogenous collateral constraints as well as contracting frictions with managers. Entrepreneurs can increase their span of control by hiring outside managers and selecting the number of managerial layers. However, weak contract enforcement distorts these delegation decisions. Managers can steal firm’s output when contract enforcement is weak. Firms overcome this friction by hiring fewer managers and increasing the compensation to managers, consistent with our two stylized facts. Larger firms benefit more from outside managers than smaller firms, which is why larger firms are more severely affected by the contracting friction. These larger firms face higher output wedges which generates a positive relationship between marginal product and size, consistent with the data.

We calibrate our benchmark economy without financial frictions to U.S. data. Then we vary the level of both frictions to study the impact on aggregate productivity. Productivity falls monotonically with both frictions. Whereas financial frictions disproportionately affect smaller firms, managerial frictions mainly affect large firms. TFP falls by 24% when managerial frictions completely shut down delegation. Contracting frictions have larger impacts on TFP than financial frictions because contracting frictions distort the larger firms who produce most of the output in the economy.

Our paper mainly contributes to the macro literature on the importance of firm management.<sup>2</sup> We build on the framework of [Grobovšek \(2017\)](#), showing how managerial frictions can help account for observed patterns of factor misallocation.

Our paper is also related to the recent macroeconomic literature on the firm size distribution and firm dynamics, and their relationship to economic development.<sup>3</sup> The closest related paper is [Akcigit et al. \(2017\)](#), who also argue that the lack of delegation explains why firms in poor countries have lower productivity and do not grow over their life cycles. Whereas their paper highlights the importance of selection, we completely shut down this channel in our model. We highlight the importance of contracting frictions with managers

---

<sup>2</sup>See, for example, [Garicano and Rossi-Hansberg \(2006\)](#), [Bloom and Reenen \(2007\)](#), [Bloom and Reenen \(2010\)](#), and [Grobovšek \(2017\)](#), among others.

<sup>3</sup>See, for example, [Haltiwanger et al. \(2013\)](#), [Roys and Seshadri \(2014\)](#), [Hsieh and Klenow \(2014\)](#), and [Akcigit et al. \(2017\)](#), among others.

generate productivity losses by misallocating factors of production, something they abstract from in their paper.

Our paper also contributes to the misallocation literature by building a framework to quantify the importance of contracting frictions and lack of delegation.<sup>4</sup> The contracting frictions with managers leads to endogenous output wedges that are positively correlated with firm productivity. Our work is also related to the literature studying the role of weak institutions as key obstacles of economic development. Weak contract enforcement is an often-cited result of weak institutions in poor countries.<sup>5</sup>

The paper proceeds as follows. Section 2 documents stylized facts both across firms in China and India and across countries. Section 3 describes our model. We quantify the impact of the two types of frictions in Section 4. Section 5 concludes the paper.

## 2 Evidence

In this section we document evidence on contracting frictions both across countries and across firms.

### 2.1 Firm-Level Evidence from China and India

#### 2.1.1 A Simple Accounting Framework

We start by documenting firm-level evidence using data from the manufacturing sector of China and India. To guide our analysis, let us first consider a simple accounting framework.

The profit maximization problem of firm  $i$  in industry  $j$  is given by

$$\pi_{ij} = \tau_{ij}^y p_i y_{ij} - \tau_{ij}^k r k_{ij} - w l_{ij}.$$

$\pi_{ij}$  denotes for the profit of this firm.  $y_{ij}$ ,  $k_{ij}$ , and  $l_{ij}$  are the firm's output, capital input, and labor input, respectively.  $p_i$  is the price of output, which is constant for industry  $i$ .  $\tau_{i,j}^y$

---

<sup>4</sup>For the misallocation literature, see, for example, Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Buera et al. (2011), Moll (2014), and Bento and Restuccia (2017), among others.

<sup>5</sup>For the literature on institutions and economic growth, see Alchian and Demsetz (1973) and Acemoglu et al. (2005) among others.

and  $\tau_{ij}^k$  are output and capital wedges.<sup>6</sup> We further assume that the production function is Cobb-Douglas:  $y_{ij} = z_{ij}^{1-\gamma} (k_{ij}^\alpha l_{ij}^{1-\alpha})^\gamma$ . Then the first order conditions are

$$\alpha\gamma\tau_{ij}^y \frac{p_i y_{ij}}{k_{ij}} = \tau_{ij}^k r \quad \text{and} \quad (1-\alpha)\gamma\tau_{ij}^y \frac{p_i y_{ij}}{l_{ij}} = w. \quad (1)$$

Define  $\frac{p_i y_{ij}}{k_{ij}}$  and  $\frac{p_i y_{ij}}{l_{ij}}$  as the revenue productivity of capital and labor ( $\text{APK}_{ij}$  and  $\text{APL}_{ij}$ ).

We can rewrite Equation 1 as

$$\text{APK}_{ij} \propto \frac{\tau_{ij}^k}{\tau_{ij}^y} \quad \text{and} \quad \text{APL}_{ij} \propto \frac{1}{\tau_{ij}^y}. \quad (2)$$

The capital-labor ratio ( $\text{KOL}_{ij}$ ) of the firm can be written as

$$\frac{k_{ij}}{l_{ij}} = \frac{\alpha}{1-\alpha} \frac{w}{\tau_{ij}^k r} \propto 1/\tau_{ij}^y.$$

We can then show that  $\text{APK}_{ij}$  can be decomposed into  $\text{APL}_{ij}$  and the capital-labor ratio ( $\text{KOL}_{ij}$ ):

$$\text{APK}_{ij} = \text{APL}_{ij} \cdot \frac{1}{\text{KOL}_{ij}}. \quad (3)$$

Intuitively, Equation (3) simply states that the average product of capital can be decomposed into two terms: the average product of labor which only depends on the output wedge but not the capital wedge, and the capital-labor ratio which only depends on the capital wedge but not the output wedge. This decomposition helps us to identify the capital wedges relative to the output wedges in our firm level data.

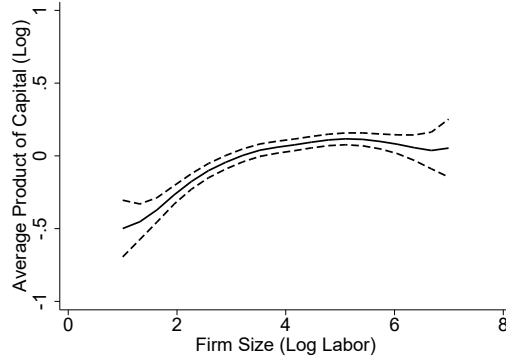
### 2.1.2 Firm-Level Data of China and India

We now document stylized facts in the firm-level data of China and India, where the data cleaning process follows closely [Hsieh and Klenow \(2009\)](#). Literature typically focuses on the differences in average product of capital (APK) across firms as evidence for capital misallocation. Figure 1 shows the APK across firms in the Indian data. We plot firms by their size (measured as the labor input) on the horizontal axis and plot their APK on the

---

<sup>6</sup>Note that it is equivalent to model the capital wedge and labor wedge without the output wedge, but we cannot have all three wedges in this framework.

Figure 1: Average Product of Capital of Indian Firms

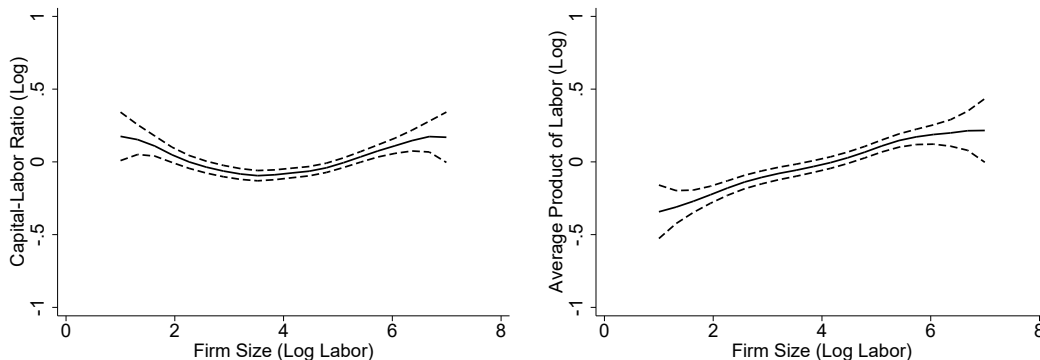


Note: The figure shows the average product of capital across firms of the Indian data in the year 1998. We control on the firm’s age, ownership, location, and 2-digit industry and then use the residuals of APK to obtain the results in the figure. The data cleaning process is detailed in Appendix A. The dashed lines show the 95 percent confidence interval.

vertical axis. In general, larger firms have higher APK, consistent with [Hsieh and Olken \(2014\)](#). In fact suggests that collateral constraints are now likely to explain the dispersion of APK across firms, since standard models of borrowing constraints typically predict that small firms are constraint due to a lack of collateral and therefore have higher APK.

In order to study why large firms have higher APK than small firms, we decompose the APK into the average product of labor (APL) and the capital-labor ratio (KOL) following Equation (3). The results are in Figure 2. The left panel of Figure 2 shows the KOL across firms, where the dispersion arises from the capital wedges only. It is clear that there are severe capital misallocation across firms: larger firms do have higher capital-labor ratio than smaller firms, consistent with the story of collateral constraint, except for those very small firms. The right panel of Figure 2 shows the APL across firms, where the dispersion arises from the output wedges only. It is clear that larger firms have higher APL, consistent with [Hsieh and Olken \(2014\)](#). This indicates that larger firms face higher output wedges than smaller firms, and we need to find explanations different from financial frictions to explain this pattern that APL increases with firm size. As we will show later, our model predicts that the contracting frictions between entrepreneurs and managers exactly work as output wedges across firms, and our model also predicts that APL should increases with firm size when the contract enforcement is weak, consistent with these firm-level data.

Figure 2: Decomposition of Wedges of Indian Firms

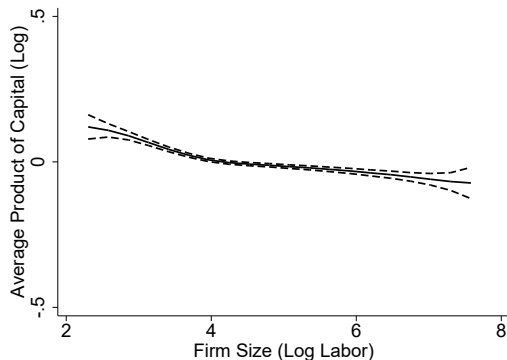


Note: The left figure shows the capital-labor ratio across firms of the Indian data in the year 1998. The right figure shows the average product of labor across firms. We control on the firm’s age, ownership, location, and 2-digit industry and then use the residuals to obtain the results in the figure. The data cleaning process is detailed in Appendix A. The dashed lines show the 95 percent confidence interval.

We also present our results using the firm-level data from China in the year of 2004. The China data are different from the Indian data since the Chinese manufacturing survey only records firms whose gross output in the previous year is greater than five million RMB (roughly 750,000 U.S. dollar in the year 2004). This criterion generates selection effect that biases upward the estimated productivity of small firms: a small firm with only ten workers will have to be extremely productive to produce five million RMB and to be included in the survey, while a large firm with 100 workers is able to produce five million RMB and to be included in the survey even if it is not very productive. Therefore, we mainly discuss on the pattern among large firms for the China data.

Figure 3 shows the APK across firms in China. Larger firms have lower APK than smaller firms. Again this pattern depends on both capital wedges and output wedges, therefore we apply Equation (3) and decompose the pattern into KOL and APL across firms in Figure 4. If we focus only on large firms, we can see that the KOL increases with firm size, consistent with the idea that smaller firms face collateral constraints and have lower capital intensity. Therefore, there are also substantial capital misallocation in China, similar to what we find in the Indian data. The right panel of Figure 4 shows that the APL increases with firm size, in a pattern similar to the Indian data as well. Larger firms in China also face higher output wedges that do not arise from capital frictions. This fact is also consistent with our model

Figure 3: Average Product of Capital of Chinese Firms



Note: The figure shows the average product of capital across firms of the China data in the year 2004. We control on the firm's age, ownership (especially whether this firm is a state-owned enterprise), location, and 2-digit industry and then use the residuals of APK to obtain the results in the figure. The data cleaning process is detailed in Appendix A. The dashed lines show the 95 percent confidence interval.

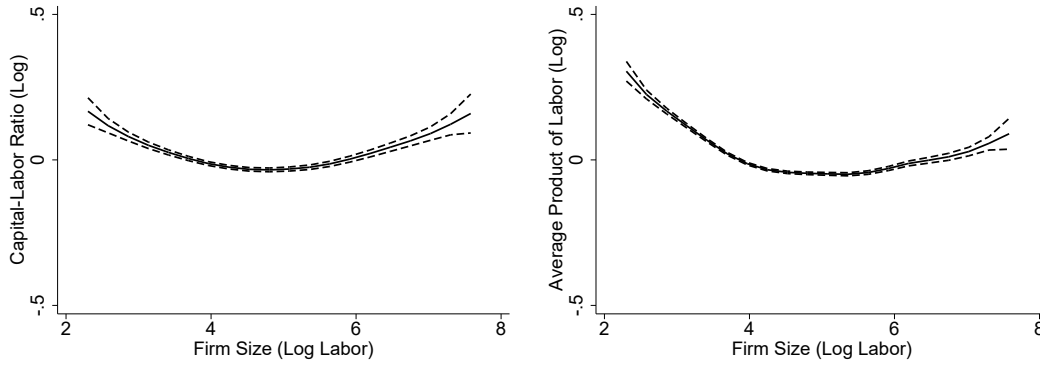
that larger firms are constraint by a lack of delegation to outside managers and therefore have higher output wedges.

It is important to highlight the issue of selection in the China data. As is mentioned before, the criterion for a firm to be in the survey is that its gross output of the previous year is greater than five million RMB. A way to avoid the selection bias is to define firm size in gross output instead of labor. Figure 5 shows the same decomposition when firm size is measured by sales (which are closely related to gross output). We can see that both the capital-labor ratio and the labor productivity are (almost) monotonically increasing in firm size, confirming our findings in the Indian data that smaller firms face higher capital wedges, while larger firms face higher output wedges. We further note that we do observe labor quality differences across firms: the data report the composition of workers with different education levels. The increasing pattern of APL is robust to controlling for these labor quality differences.

To conclude, firm-level data from both China and India suggest that large firms face substantially higher output wedges, which do not arise from capital frictions. The increasing pattern of APL is consistent with our conjecture that large firms are constraint by lacking delegation to outside managers due to weak contract enforcement. The following section uses cross-country evidence to show that the lack of delegation is indeed a problem for many

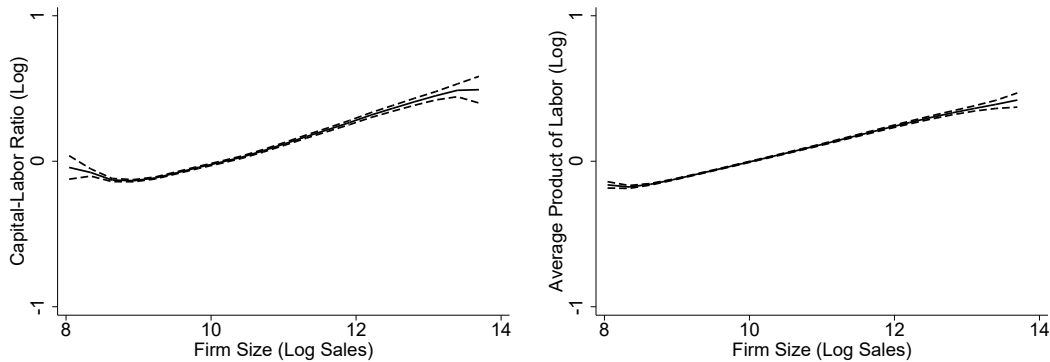


Figure 4: Decomposition of Wedges of Chinese Firms



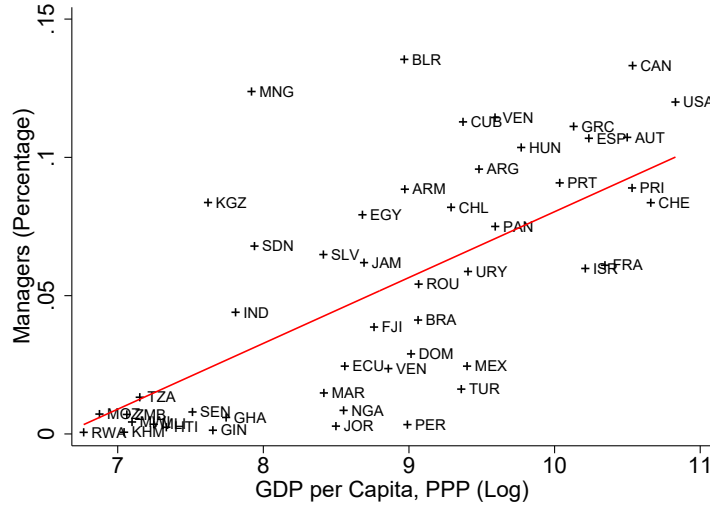
Note: The left figure shows the capital-labor ratio across firms of the China data in the year 2004. The right figure shows the average product of labor across firms. We control on the firm's age, ownership (especially whether this firm is a state-owned enterprise), location, and 2-digit industry and then use the residuals of APK to obtain the results in the figure. The data cleaning process is detailed in Appendix A. The dashed lines show the 95 percent confidence interval.

Figure 5: Decomposition of Wedges of Chinese Firms: Alternative Size Measure



Note: The left figure shows the capital-labor ratio across firms of the China data in the year 2004. The right figure shows the average product of labor across firms. We control on the firm's age, ownership (especially whether this firm is a state-owned enterprise), location, and 2-digit industry and then use the residuals of APK to obtain the results in the figure. The data cleaning process is detailed in Appendix A. The dashed lines show the 95 percent confidence interval. These two figures differ from Figure 4 since firm size is measured by sales instead of labor input.

Figure 6: Fraction of Individuals Working as Managers



Note: The figure shows the correlation between the percentages of individuals working as managers and the country’s GDP per capita (PPP adjusted). There are 49 observations in the figure. Poor countries in general have lower portions of individuals working as managers. The regression coefficient is 0.026, significant at the one percent level, and  $R^2$  is 0.47.

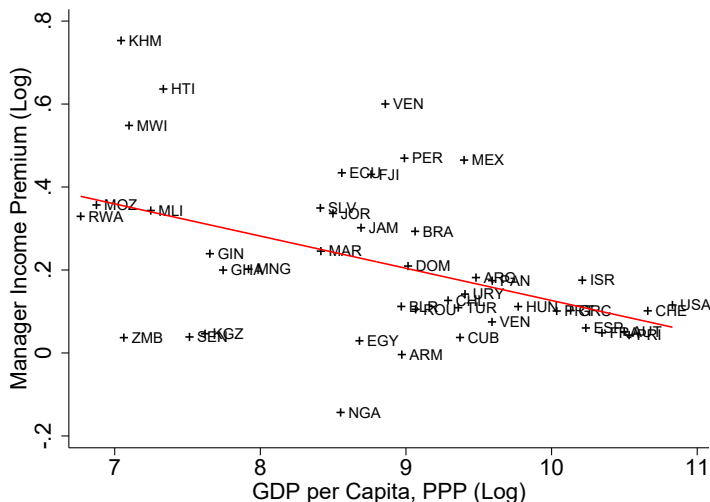
poor countries.

## 2.2 Cross-Country Differences in Contracting Frictions

We document two stylized facts on cross-country differences in contracting frictions. The first stylized fact is that the fraction of individuals working as managers increases in a country’s GDP per capita. Figure 6 plots this stylized facts, with log GDP per capita (PPP adjusted) on the horizontal axis and the fraction of individuals working as managers on the vertical axis. The data are from the *Integrated Public Use Microdata Series, International: Version 6.5* (IPUMS), and see Appendix A for details. Rich countries, such as the United States and Canada, have around 10 percent of individuals working as managers. In contrast, in poor countries very few individuals work as managers. The slope is statistically significant at the one percent level. This stylized fact is consistent with Akcigit et al. (2017), who also find that firms in poor countries tend to delegate less frequently and this lack of delegation explains that why firms in poor countries grow slower over time (Hsieh and Klenow, 2014).

The second stylized fact is that managers tend to earn higher income than individuals of

Figure 7: Income Premium of Managers



Note: The figure shows the income differences between managers and individuals of other occupations versus the country’s GDP per capita (PPP adjusted). There are 45 observations in the figure. The income differences of managers are larger in poor countries. The regression coefficient is  $-0.078$ , significant at the one percent level, and  $R^2$  is  $0.21$ .

other occupations, and the difference is larger in poor countries than in rich countries. In the IPUMS-International data, we observe individual’s income for only a few countries, but we do have information on individual’s consumption, such as the number of rooms, for a large set of countries. We therefore use the number of rooms of individuals as an approximation of income. Before we compare income among individuals of different occupations, we first run the standard Mincer regression to control for observed characteristics, such as education level, age, and gender. Given the fact that the number of rooms may be affected by the number of people living in a same home, we further control for the family size in the regression. We then obtain the residuals from the regression and use them to calculate the income premium of managers, defined as the room number differences between managers and individuals of other occupations, for each country. We then correlate this income premium of managers versus a country’s log GDP per capita. The results are shown in Figure 7. Among 45 countries that we have data, poor countries in general have larger manager income premiums than rich countries. The slope is statistically significant at the one percent level.

Note that we have controlled for observed heterogeneity among individuals in the Mincer

regression, such as age, gender, and education, and only use the residuals to calculate this income premium of managers. Therefore, higher manager income premium in poor countries should not arise from these observed heterogeneity, such as education gaps between managers and individuals of other occupations. Rather, it may reflect contracting frictions between entrepreneurs and managers, which result in efficiency wages paid to managers in poor countries.

To conclude, we find that poor countries have smaller portions of individuals working as managers, but the income premium of managers are higher in poor countries than in rich countries. This is consistent with our conjecture that weak contract enforcement in poor countries result in lacking delegation to outside managers and higher compensation to managers (as efficiency wages). In the next section, we formally describe our model and show how our model with contracting frictions generates these two patterns.

### 3 Model

This model builds on [Garicano and Rossi-Hansberg \(2006\)](#) and [Grobovšek \(2017\)](#). As in [Grobovšek \(2017\)](#), entrepreneurs can increase their span of control by hiring outside managers, potentially employed in multiple layers. If contract enforcement is not perfect, outside managers can get away with stealing a fraction of the output that passes through their hands, which reduces the gains from delegation. We also introduce financial frictions à la [Moll \(2014\)](#) to this framework in a stylized way, which is potentially important in generating the observed pattern of marginal products across firms. To integrate these two frameworks, we make several simplification assumptions to keep our model tractable. We abstract from entry into entrepreneurship and therefore we focus on the misallocation along intensive margin rather than selection. We further restrict managers and entrepreneurs to short-term contracts.

#### 3.1 Households

The economy consists of two types of infinitely-lived households: *worker* households and *entrepreneurial* households.

**Workers.** There are a measure  $N_w$  of worker households. Each of these households consist of a measure 1 of members. In each period, each member has one unit of time that they can supply directly to firms as unskilled labor, or supply to firms as skilled labor after paying a training cost  $\kappa$ . The wage for skilled and unskilled labor are  $w_s$  and  $w_u$ , respectively. Households pool their members' incomes and allocate consumption across members to maximize the household's utility.

Denote  $N_s$  and  $N_u$  as the measure of skilled and unskilled labor. The worker household's problem is therefore to maximize  $N_s(w_s - \kappa) + N_u w_u$ , subject to  $N_s + N_u = N_w$ . In the equilibrium, it has to be that  $w_s = w_u + \kappa$  in order to have positive measures of both types of labor.

**Entrepreneurs.** There are a measure  $N_e$  of infinitely lived entrepreneurs. Each entrepreneur has a production technology that only they can operate. Entrepreneurs differ in their productivity  $z$ , which has a cumulative distribution  $F_z : \mathbb{R}_+ \mapsto [0, 1]$ .

Entrepreneurs also differ in their exogenous borrowing limit  $\bar{k}$ , which has a cumulative distribution  $F_k : \mathbb{R}_+ \mapsto [0, 1]$ . We model financial frictions in this reduced form and abstract from the endogenous borrowing limit arising from collateral constraints, since our work focus more on the contracting frictions between managers and entrepreneurs. Note that this assumption does not affect our quantitative results on misallocation since we calibrate the exogenous distribution of the borrowing limit to firm-level data.

An entrepreneur who begins the period with exogenous borrowing limit  $\bar{k}$  and productivity  $z$  earns profit  $\pi(\bar{k}, z)$  from operating his firm. We will describe shortly how profit  $\pi(\bar{k}, z)$  is determined. Entrepreneurs then consume their profit income.

## 3.2 Production technology

The production technology takes in labor and capital as variable inputs to produce output. There are five distinct types of labor inputs: the entrepreneur's, skilled/unskilled production workers', and skilled/unskilled managerial workers'. Production workers and managerial workers need to be supervised, while the entrepreneur uses his one unit of time to supervise

either production or managerial workers. Managerial workers can supervise other workers.<sup>7</sup>

**Two-layer firm.** The simplest firm consists of an entrepreneur (in layer 2) supervising capital and production workers (in layer 1). The efficiency of the inputs in layer 1 depends on the amount of time the entrepreneur spends supervising them. If a firm employs  $k(1)$  units of capital,  $m_s(1)$  units of skilled production workers, and  $m_u(1)$  units of unskilled production workers, then the efficiency per unit of composite input is:

$$\left( \frac{1}{k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha}} \right)^\theta, \quad \alpha, \mu, \theta \in (0, 1). \quad (4)$$

The parameter  $\theta$  captures the idea that the entrepreneur's marginal efficiency is decreasing in the amount of inputs he manages,  $\alpha$  governs the capital share, and  $\mu$  determines the relative importance between skilled and unskilled workers. The effective units of inputs are

$$\left( \frac{1}{k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha}} \right)^\theta \times \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right) = \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right)^{1-\theta}.$$

If the entrepreneur has productivity  $z$ , then his total output is;

$$z \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right)^{1-\theta}.$$

Notice that this is a standard decreasing returns to scale technology as [Lucas \(1978\)](#), with an entrepreneurial profit share of  $\theta$ .

**Three-layer firm.** The entrepreneur can increase the amount of supervision received by the production workers by adding a managerial layer and staffing it appropriately. Managerial inputs also need supervision. Suppose the entrepreneur employs  $(k(2), m_s(2), m_u(2))$  units of capital, skilled labor, and unskilled labor in the managerial layer and  $(k(1), m_s(1), m_u(1))$  units of capital, skilled labor, and unskilled labor in the production layer. The effective units

---

<sup>7</sup>[Grobovšek \(2017\)](#) considers a model with only labor inputs, where managers use only their time to supervise. Since we introduce capital as a factor of input, we assume managers use both their time and capital to supervise the layer below them. This assumption keeps the capital-labor ratio constant across firms in our model.

of managerial inputs is

$$\left( \frac{1}{k(2)^\alpha (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha}} \right)^\theta \times \left( (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha} \right) = \left( (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha} \right)^{1-\theta}.$$

The parameter  $\gamma < 1$  indicates that the managerial layer is more skilled-labor intensive than the production layer.

The effective units of managerial inputs are used to supervise the inputs in the production layer. The effective units of production inputs is

$$\begin{aligned} & \left( \frac{\left( k(2)^\alpha (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha} \right)^{1-\theta}}{k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha}} \right)^\theta \times \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right) \\ & = \left( k(2)^\alpha (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha} \right)^{\theta(1-\theta)} \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right)^{1-\theta}. \end{aligned}$$

The output of this firm is

$$z \left( k(2)^\alpha (m_s(2)^{1-\mu\gamma} m_u(2)^{\mu\gamma})^{1-\alpha} \right)^{\theta(1-\theta)} \left( k(1)^\alpha (m_s(1)^{1-\mu} m_u(1)^\mu)^{1-\alpha} \right)^{1-\theta}.$$

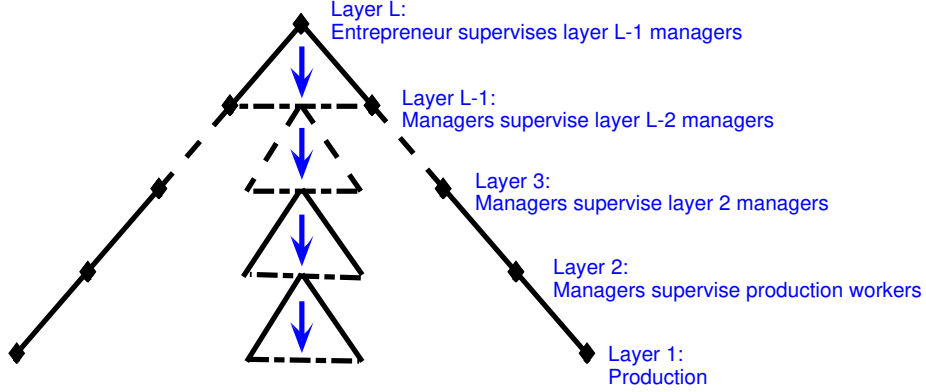
The entrepreneur may potentially gain from adding the additional layer of managers because it increases his span of control by  $\theta(1 - \theta)$ .

***L*-layer firm.** An entrepreneur may choose to add additional layers of managers to further increase his span of control. Suppose a firm has  $L$  layers: the entrepreneur at the top ( $L^{\text{th}}$  layer), managerial resources in layers 2 to  $L - 1$ , and production inputs at the bottom layer.

The structure of this firm can be illustrated in Figure 8. This firm's output is given by

$$o^L = z \prod_{l=1}^{L-1} \left( k(l)^\alpha \left( m_s(l)^{1-\mu\gamma^{l-1}} m_u(l)^{\mu\gamma^{l-1}} \right)^{1-\alpha} \right)^{(1-\theta)\theta^{l-1}}, \quad \forall L \geq 2. \quad (5)$$

Figure 8: Illustration of an  $L$ -layer firm



Define the following elasticities for capital, skilled labor, and unskilled labor:

$$\begin{aligned}\varepsilon_k(l) &\equiv \alpha\theta^{l-1}(1 - \theta), \\ \varepsilon_s(l) &\equiv (1 - \mu\gamma^{l-1})(1 - \alpha)\theta^{l-1}(1 - \theta), \\ \varepsilon_u(l) &\equiv (\mu\gamma^{l-1})(1 - \alpha)\theta^{l-1}(1 - \theta).\end{aligned}$$

We can rewrite this firm's output as

$$o^L = z \prod_{l=1}^{L-1} k(l)^{\varepsilon_k(l)} m_s(l)^{\varepsilon_s(l)} m_u(l)^{\varepsilon_u(l)}, \quad \forall L \geq 2. \quad (6)$$

### 3.3 Entrepreneur's Problem

#### 3.3.1 Managerial Frictions and Output Wedge

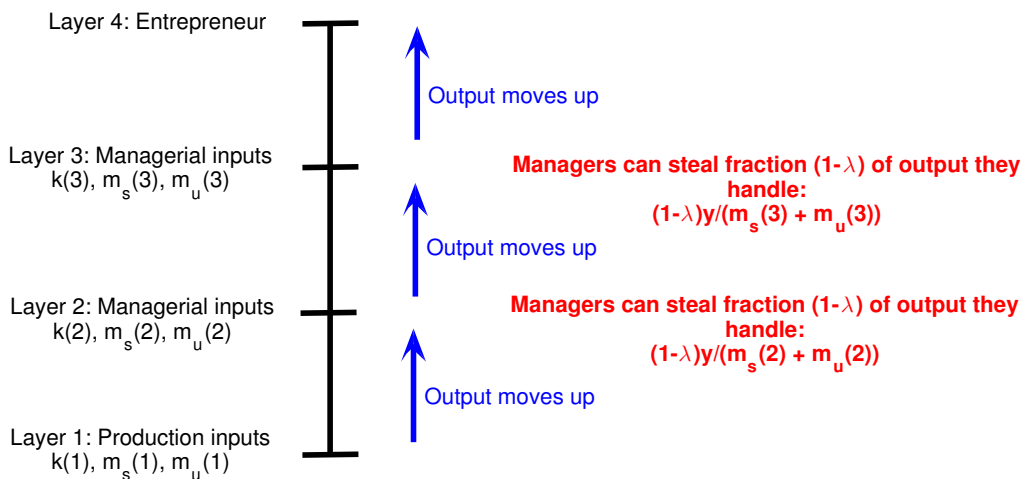
After production takes place, the firm's output moves up the layers to the entrepreneur. There are  $m_s(l) + m_u(l)$  outside managers in the  $l^{\text{th}}$  layer, hence each outside manager in that layer handles a fraction  $o_L / (m_s(l) + m_u(l))$  of the total output. She can steal a fraction  $1 - \lambda$  of this output that she handles, which is given by

$$(1 - \lambda) \left( \frac{o_L}{m_s(l) + m_u(l)} \right).$$

Figure 9 illustrates this problem in a four-layer firm.



Figure 9: Illustration of Managerial Frictions



The entrepreneur overcome this stealing problem by paying an efficient wage  $b(l)$  to each outside manager in layer  $l$  is she does not steal.<sup>8</sup> The total compensation of a skilled (unskilled manager) in the  $l^{th}$  layer is this efficiency wage plus  $w_s$  ( $w_u$ ), which is the market wage rate of skilled/unskilled labor. A stealing manager is detected after  $w_s/w_u$  is paid but before  $b_l$  is paid. The entrepreneur imposes the maximum possible punishment on stealing managers, which is to not pay  $b_l$ .

It is optimal for the manager to maximize his static income. Therefore, he will not steal if and only if

$$w_i + b(l) \geq w_i + (1 - \lambda) \left( \frac{o_L}{m_s(l) + m_u(l)} \right).$$

It follows immediately that the optimal efficiency wage should be exactly when this incentive compatibility constraint binds, which means

$$b(l) = (1 - \lambda) \left( \frac{o_L}{m_s(l) + m_u(l)} \right), \quad \forall l \geq 2. \quad (7)$$

There is no efficiency payment to production workers in the bottom layer, so we have  $b(1) = 0$ .

We can now define the operating income of a firm with  $L$  layers, productivity  $z$  and

---

<sup>8</sup>In our static setup, the relationships between the entrepreneur and all workers last one period, hence the entrepreneur cannot use the threat of losing future employment to keep the manager from stealing.

rented capital  $K$ .

**Definition 1.** *The operating income (revenue minus labor compensation) for an entrepreneur with rented capital  $K$  and productivity  $z$  is:*

$$\tilde{\pi}_L(K, z) = \max_{k(l), m_s(l), m_u(l)} \left\{ z \prod_{l=1}^{L-1} k(l)^{\varepsilon_k(l)} m_s(l)^{\varepsilon_s(l)} m_u(l)^{\varepsilon_u(l)} - \sum_{l=1}^{L-1} (w_s + b(l)) m_s(l) - \sum_{l=1}^{L-1} (w_u + b(l)) m_u(l) : \sum_{l=1}^{L-1} k(l) \leq K \right\}.$$

Using the characterization of the excess compensation  $b(l)$  in Equation (7), we can rewrite the operating income as

$$\tilde{\pi}_L(K, z) = \max_{k(l), m_s(l), m_u(l)} \left\{ (1 - \Lambda(L)) z \prod_{l=1}^{L-1} k(l)^{\varepsilon_k(l)} m_s(l)^{\varepsilon_s(l)} m_u(l)^{\varepsilon_u(l)} - \sum_{l=1}^{L-1} w_s m_s(l) - \sum_{l=1}^{L-1} w_u m_u(l) : \sum_{l=1}^{L-1} k(l) \leq K \right\}, \quad (8)$$

where  $\Lambda(L) \equiv (1 - \lambda)(L - 2)$  is the output wedge arising from the efficiency wage payments.

Equation (8) highlights the feature that the contracting frictions between entrepreneurs and managers can be reduced to the output wedge  $\Lambda(L)$ : the problem of a firm with contracting frictions is equivalent to a firm without such frictions but faces the output wedge  $\Lambda(L)$  which depends on the layer of the firm. Note that in a economy with contracting frictions ( $\lambda < 1$ ), the wedge  $\Lambda(L) \geq 1$  and is increasing in  $L$  with  $\Lambda(L) = 1$  if and only if  $L = 2$ . Intuitively, A two-layer firm is not affected by the contracting frictions since there is no managerial layer within that firm. A firm with more layers of managers faces a higher output wedge, since there are more managers who can steal and a larger amount of efficiency compensation has to be paid. Absent from contracting frictions ( $\lambda = 1$ ), the output wedge  $\Lambda(L)$  is equal to zero for all firms.

### 3.3.2 Optimal Input Choice

Lemma 1 characterizes the optimal allocation of workers and capital to layers as a function of total number of workers hired and capital rented.

The optimal allocation of capital and labor for a firm with capital  $K$ , skilled labor  $M_s$ , and unskilled labor  $M_u$  solves the following output maximization problem:

$$(1 - \Lambda(L)) \tilde{y}_L(K, M_s, M_u, z) = \max_{k(l), m_s(l), m_u(l)} \left\{ (1 - \Lambda(L)) z \prod_{l=1}^{L-1} k(l)^{\varepsilon_k(l)} m_s(l)^{\varepsilon_s(l)} m_u(l)^{\varepsilon_u(l)}, \right. \\ \left. \sum_{l=1}^{L-1} k(l) \leq K, \quad \sum_{l=1}^{L-1} m_s(l) \leq M_s, \quad \sum_{l=1}^{L-1} m_u(l) \leq M_u. \right.$$

**Lemma 1** (Optimal allocation across layers). *The optimal choices of capital and labor at each layer are:*

$$k(l) = \frac{\varepsilon_k(l)}{\bar{\varepsilon}_k(L)} K, \quad m_s(l) = \frac{\varepsilon_s(l)}{\bar{\varepsilon}_s(L)} M_s, \quad m_u(l) = \frac{\varepsilon_u(l)}{\bar{\varepsilon}_u(L)} M_u,$$

where

$$\bar{\varepsilon}_i(L) = \sum_{l=1}^{L-1} \varepsilon_i(l), \quad \forall i \in \{k, s, u\}.$$

It is important to note that the contracting frictions do not distort the share of workers of each type and capital allocated to each layer.

The maximized output of a firm with capital  $K$ , skilled labor  $M_s$ , and unskilled labor  $M_u$  is

$$\tilde{y}_L(K, M_s, M_u, z) = z \hat{\theta}(L) K^{\bar{\varepsilon}_k(L)} M_s^{\bar{\varepsilon}_s(L)} M_u^{\bar{\varepsilon}_u(L)},$$

where

$$\hat{\theta}(L) \equiv \frac{\prod_{l=1}^{L-1} \varepsilon_k(l)^{\varepsilon_k(l)} \varepsilon_s(l)^{\varepsilon_s(l)} \varepsilon_u(l)^{\varepsilon_u(l)}}{\bar{\varepsilon}_k(L)^{\bar{\varepsilon}_k(L)} \bar{\varepsilon}_s(L)^{\bar{\varepsilon}_s(L)} \bar{\varepsilon}_u(L)^{\bar{\varepsilon}_u(L)}}.$$

To provide some intuition, let us consider a special case when  $\gamma = 1$ . Output can be simplified to

$$\tilde{y}_L(K, M_s, M_u, z) = z \hat{\theta}(L) \left( K^\alpha (M_s^{1-\mu} M_u^\mu)^{1-\alpha} \right)^{1-\theta^{L-1}}.$$

The span of control is governed by the term  $1 - \theta^{L-1}$ . This means that if a firm adds more hierarchy ( $L$ ), then the span of control is increased. In the limit, if  $L$  is sufficiently large, the production function of this firm approaches constant return to scale. In the general case, one can show that the span of control ( $\bar{\varepsilon}_k(L) + \bar{\varepsilon}_s(L) + \bar{\varepsilon}_u(L)$ ) also increases with  $L$ .

**Lemma 2** (Operating income, redefined). *The operating income of a firm with productivity  $z$  and employing  $K$  units of capital is*

$$\begin{aligned}\tilde{\pi}_L(z, K) &= \max_{M_s, M_u} \{ \Lambda(L) \tilde{y}_L(z, M_s, M_u, K) - w_s M_s - w_u M_u \} \\ &= \max_{M_s, M_u} \left\{ \Lambda(L) z \hat{\theta}(L) K^{\bar{\varepsilon}_k(L)} M_s^{\bar{\varepsilon}_s(L)} M_u^{\bar{\varepsilon}_u(L)} - w_s M_s - w_u M_u \right\}.\end{aligned}$$

### 3.3.3 Financial Frictions

We assume an entrepreneur faces an exogenous borrowing limit that  $K \leq \bar{k}$ . The optimal capital choice  $(K_L^d(\bar{k}, z))$  for an  $L$ -layered firm solves

$$\pi_L(a, z) = \max_K \{ \tilde{\pi}_L(z, K) - rK : \text{subject to } K \leq \bar{k} \},$$

where  $r$  is the rental rate of capital.

### 3.3.4 Optimal number of layers

The fraction  $\Lambda(L) = (1 - \lambda)(L - 2)$  is the fraction of the output that the entrepreneur has to hand over to managers as excess compensation. Define the maximum layer  $\bar{L}(\lambda)$  as

$$\bar{L}(\lambda) = 2 + \frac{1}{1 - \lambda}. \quad (9)$$

If a firm has more layers than  $\bar{L}$ , then the fraction  $1 - \Lambda(L)$  is greater than one and the entrepreneur's share of output is negative. Therefore, they will choose fewer layers than  $\bar{L}(\lambda)$ .

For example, if  $\lambda = 0$  and managers can steal everything, then  $\bar{L} = 3$ . If a firm hires up to three layers, then they will have to give all their output to their managers. Therefore, firms will choose to employ two layers, i.e. firms without outside managers. If  $\lambda$  is sufficiently small, i.e., managers can barely steal anything, then the maximum layer  $\bar{L}$  can be sufficiently large such that potentially the technology of a firm can be close to constant return to scale.

An entrepreneur with productivity  $z$  and borrowing limit  $\bar{k}$  chooses the number of layers

to maximize profit given by

$$\pi(\bar{k}, z) = \max_{L < \bar{L}(\lambda)} \{ \pi_L(\bar{k}, z) \}. \quad (10)$$

Let  $L^*(\bar{k}, z)$  be the solution to the above optimization problem.<sup>9</sup>

### 3.4 Aggregation and Equilibrium

Let  $G(\bar{k}, z)$  be the joint distribution of entrepreneurial households over the exogenous borrowing limit and productivity. Aggregate capital demand, skilled labor demand, and unskilled labor demand are

$$\mathcal{K}^d = N_e \int_{\bar{k}, z} K^d(\bar{k}, z) G(d\bar{k}, dz), \quad (11)$$

$$\mathcal{M}_s^d = N_e \int_{\bar{k}, z} M_s^d(\bar{k}, z) G(d\bar{k}, dz), \quad (12)$$

$$\mathcal{M}_u^d = N_e \int_{\bar{k}, z} M_u^d(\bar{k}, z) G(d\bar{k}, dz), \quad (13)$$

where  $K^d(\bar{k}, z)$ ,  $M_s^d(\bar{k}, z)$ , and  $M_u^d(\bar{k}, z)$  are the demand of capital, skilled labor, and unskilled labor of an individual firm with borrowing limit  $\bar{k}$  and productivity  $z$  whose problem is solved previously.

We now define the stationary equilibrium as follows:

**Definition 2** (Competitive Equilibrium). *A competitive equilibrium for this economy consists of prices  $r$  and  $w$ , optimal input and layer choices  $K^d(\bar{k}, z)$ ,  $M_s^d(\bar{k}, z)$ ,  $M_u^d(\bar{k}, z)$ , and  $L^*(\bar{k}, z)$ , optimal labor supply  $N_s(w_s, w_u)$  and  $N_u(w_s, w_u)$ , that satisfy the following conditions:*

- i) Given prices and borrowing limits,  $K^d(\bar{k}, z)$ ,  $M_s^d(\bar{k}, z)$ ,  $M_u^d(\bar{k}, z)$ , and  $L^*(\bar{k}, z)$  solves the firm's problem.*
- ii) Given prices, the worker households choose the amount of skilled and unskilled labor that are supplied to the market subject to  $N_s(w_s, w_e) + N_u(w_s, w_e) \leq N_e$ .*

---

<sup>9</sup>Note that we are not able to show that there is an unique solution, though in practice there has always been an unique solution. If there are multiple solutions, we will take the smaller one.

iii) The interest rate  $r$  clears the capital market:  $\mathcal{K}^s = \mathcal{K}^d$ , where  $\mathcal{K}^s$  is the endowment of capital in this economy.

iv) The wage  $w_s$  and  $w_u$  clear the labor market:  $N_s = \mathcal{M}_s^d$  and  $N_u = \mathcal{M}_u^d$ .

### 3.5 Model properties

We now highlight some properties of the model relevant to our analysis.

**Proposition 1.** *The optimal number of layers  $L^*(a, z)$  is weakly increasing in entrepreneurial productivity ( $z$ ) and borrowing limit ( $\bar{k}$ ).*

The optimal number of layers is increasing in the firm's productivity because firm productivity  $z$  and input are complements. Therefore, a more productive firm gains more than a less productive firm from increasing the number of layers.

Lemma 3, which follows closely from proposition 1, shows that the support of the productivity distribution conditional on the borrowing limit can be partitioned into intervals within which all firms choose the same number of layers.

**Lemma 3.** *There exists an increasing sequence of threshold productivities  $\{\bar{z}(L, \bar{k})\}$  such that for  $z \in [\bar{z}(L, \bar{k}), \bar{z}(L + 1, \bar{k}))$ , the optimal number of layers  $L^*(z, \bar{k})$  is  $L$ .*

**Proposition 2.** *In the economy without financial frictions ( $\bar{k} = +\infty$ ), capital and labor demand are strictly increasing in the firm's productivity  $z$ .*

**Proposition 3** (Declining APK without managerial friction). *In an economy without frictions ( $\lambda = 1$  and  $\bar{k} = +\infty$ ), the average product of capital is declining in firm productivity  $z$  and firm size.*

The average product of capital is falling with productivity because the number of layers, and therefore the span of control, is increasing. The marginal product is, however, constant and equals  $r$ .

### 3.5.1 Economies with Contracting Frictions with Managers ( $\lambda < 1$ )

The contracting friction with outside managers disproportionately affects more productive firms. First, they are forced to reduce the number of layers in their organizational structure. Second, if they continue to hire outside managers, they must adjust their scale downward. Lemma 4 shows that this manifests as a higher marginal product of capital.

**Lemma 4.** *In an economy with  $\lambda \in (0, 1)$  where some firms still hire outside managers, the marginal product of capital is increasing in firm productivity  $z$ :*

$$MPK(z) = \frac{r}{\Lambda(L^*(z))}.$$

Lemma 5 characterizes the wage premia earned by managers, as a function of the layer they are employed in and their employer's productivity  $z$ .

**Lemma 5.** *The wage premium earned by an outside manager working at the  $l^{\text{th}}$  layer at a firm with productivity  $z$  is*

$$b(l, z) = \frac{1 - \lambda}{\theta^l} \left[ \frac{1}{\Lambda(L^*(z))} \left( \frac{W}{(1 - \theta)} \right) \right]^{\frac{1}{\nu(L^*(z))}}$$

*The wage premium is increasing in the layer the manager works at ( $l$ ) and the firm's productivity  $z$ .*

The wage premium is increasing with the manager's layer not because higher higher-level managers are more productive, but because there are fewer of them. Since more output passes through each manager's hands, they need to be compensated more to prevent stealing. The higher productivity of the firm increases the wage premium only if the firm employs more layers. Although a higher productivity also directly increases output, it also increases the number of managers at each level and these effects cancel out.

## 4 Quantitative Analysis

We calibrate our model with perfect contract enforcement between entrepreneurs and outside managers ( $\lambda = 1$ ) and financial frictions to moments from US data. By doing this, we are

able to determine the value of parameters governing technologies and ability distribution. We then recalibrate the parameters governing both frictions, as well as some productivity parameters, to firm-level and household-level data of China, and study how both frictions affect aggregate productivity.

## 4.1 Calibration Strategy

We first describe how we calibrate the model without managerial frictions to the U.S. data. There are in total seven parameters of this model: four on technology,  $\{\alpha, \theta, \mu, \gamma\}$ , one on skill accumulation  $\{\kappa\}$ , and two on endowments  $\{N_e, N_w\}$ . In addition, we need to determine the joint distribution of entrepreneur’s ability  $z$  and the financial frictions  $\bar{k}$ .

**Technology:**  $\{\alpha, \theta, \mu, \gamma\}$ . The capital share  $\alpha$  is set to 0.33 following [Gollin \(2002\)](#). The parameter  $\theta$  determines the efficiency of supervision. We choose  $\theta$  to match the moment that 12.0% of household labor works as managers in the *Integrated Public Use Microdata Series (IPUMS)* USA 2005 census data ([Ruggles et al., 2017](#)).  $\mu$  and  $\gamma$  govern the relative importance of skilled labor versus unskilled among workers and different layers of managerial resources. We choose these two parameters to match the following two moments: 1) the portion of skilled workers, which is approximated by the portion of college graduates, is 53.0% from the IPUMS-USA data; 2) the elasticity between firm size and average wage of a firm is 0.047 ([Troske, 1999](#)). The second moment is informative as larger firms hire on average more skilled workers in the managerial layers, and therefore the average labor productivity can be higher even without frictions.

**Skill Accumulation:**  $\{\kappa\}$ .  $\kappa$  determines the schooling cost of a skilled worker. It determines the wage premium of skilled labor relative to unskilled labor. Since we use college graduates to approximate skilled labor, we choose  $\kappa$  to match a college wage premium of 47.6% as we observed in the IPUMS-USA data.

**Endowments:**  $\{N_e, N_w\}$ .  $N_e$  and  $N_w$  determine the relative measure of entrepreneurs versus household labor (which is the sum of measures of workers and managers, or the sum



of measures of skilled and unskilled labor). IPUMS-USA data set has no information on whether an individual works as a manager. [Cagetti and Nardi \(2006\)](#) use the *Survey of Consumer Finances (SCF)* and report that 7.55% of the population work as entrepreneurs in the U.S. We therefore normalize  $N_w = 1$ , and choose  $N_e = 8.17\%$  such that the share of entrepreneurs is  $N_e/(N_e + N_w) = 8.17\%/(1 + 8.17\%) = 7.55\%$ .

**The Distribution of Ability and Financial Frictions  $\{F(z, \bar{k})\}$ .** We assume that entrepreneurial productivity  $z$  follows a log-Normal distribution with a mean of zero and a standard deviation of  $\sigma$ . We discipline  $\sigma$  by targeting the moment that 69% of workers are employed by firms in the top 10% of the size distribution. Although the log-normal distribution typically cannot match the employment shares in the tail of size distribution, we are able to do so because these large firms increase their span of control by hiring outside managers.

For the financial frictions, we follow [Bento and Restuccia \(2017\)](#) and assume that the exogenous borrowing limit takes the following functional form:

$$\log \bar{k}_i = \beta_0 + \beta_1 \log z_i + \varepsilon_i, \quad (14)$$

where  $\beta_1$  determines the correlation between the distortion and ability, while  $\varepsilon_i$  is a random variable following normal distribution with standard deviation of  $\sigma_\varepsilon$ . These two parameters govern the correlation between sales-to-asset ratio and firm size, and the variation of sales-to-asset ratio among firms of the same size. [Crouzet and Mehrotra \(2018\)](#) argue that the *US Census Bureau's Quarterly Financial Report (QFR)* provide a representative sample of the population of U.S. manufacturing firms, while the sample of Compustat is not representative. Therefore, we take two moments from [Crouzet and Mehrotra \(2018\)](#): 1) the quarterly sales-to-asset ratio is 0.60 among firms within the size distribution from zero to 90<sup>th</sup> percentile, while it is 0.39 among firms within the size distribution from 90<sup>th</sup> to 99<sup>th</sup> percentile, meaning that the ratio is around 54% higher for smaller firms; 2) among firms within the size distribution from zero to 99<sup>th</sup> percentile, the average leverage ratio is 0.35, while those firms with leverages in the upper quartile ( $> p75$ ) has an average leverage ratio of 0.47, the ratio of which provides the dispersion of leverage among firms. Note that in

our model, we do not have the separation of assets and debts, but we use the dispersion of the the leverage ratio to approximate that of the capital-to-sales ratio, as in our model, the dispersion only arises from financial frictions.

**Re-Calibration to Chinese Data:**  $\{N_e, N_w, \kappa, \lambda, F(z, \bar{k})\}$ . Previously we calibrate our model to the U.S. data to pin down the parameters governing technology and ability distribution. Our goal is to study how the frictions can explain the observed pattern in the Chinese data, so we further re-calibrate some parameters to reflect the salient features in the Chinese data. First we choose  $N_w = 1$  and  $N_e = 0.042$  such that 3.99% of population works as entrepreneurs in the 2005 Chinese Household Census. We choose  $\kappa$  such that the wage of skilled labor working as production workers is around 47.5% higher than that of unskilled labor working as production workers, after controlling for observables such as age, gender, health status, and marriage status. We choose  $\lambda$  to match the moment that, the wage of skilled labor working as managers is around 49.3% higher than that of skilled labor working as workers, after the same set of controls.

We re-calibrate  $\beta_1$  and  $\sigma_\varepsilon$  in Equation (14) to match two moments from 2004 Chinese Manufacturing Enterprise Census: 1) the elasticity between capital-to-output ratio and firm size (measured by capital) is 0.49, after controlling for firm age, industry, region, and ownership; 2) the standard deviation of the residual of log capital-to-output ratio is 1.025, after controlling for firm size, firm age, industry, region, and ownership. Note that we avoid using labor to measure firm size as we described before that there is selection among small firms and the sample is not representative.

## 4.2 Contracting frictions and aggregate productivity

In our first quantitative exercise, we investigate the effect of increasing the fraction outside managers can steal (i.e. reducing  $\lambda$ ), while maintaining the assumption of no financial frictions (i.e.  $\phi = 1$ ). Figure 10 presents the impact on aggregate productivity.

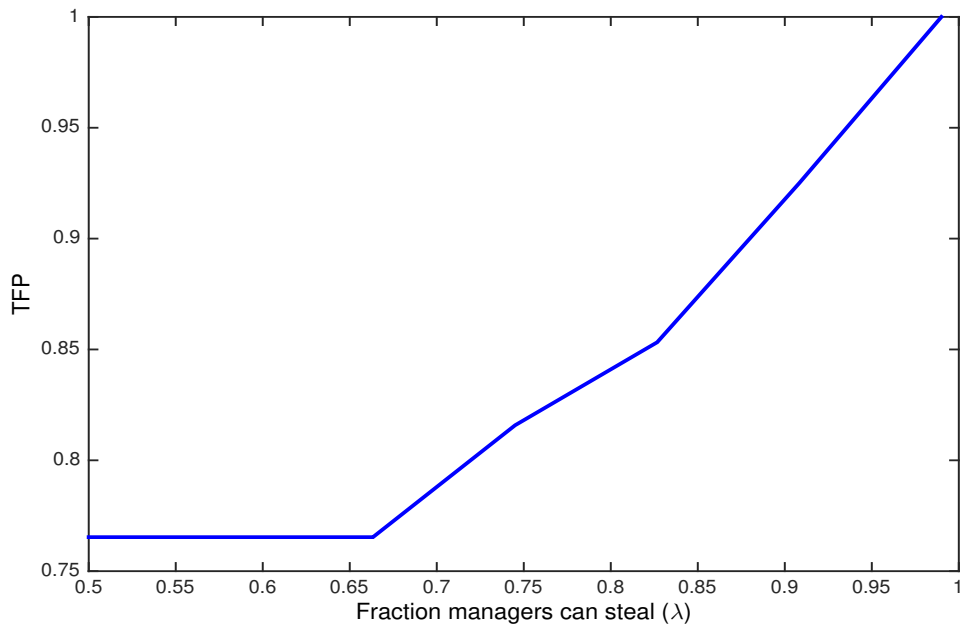


Figure 10: TFP and contracting frictions with managers

In figure 10, we normalize TFP by the level in the frictionless economy.<sup>10</sup> Aggregate productivity falls monotonically to 76% as the contracting friction with managers worsen. When the friction gets sufficiently worse, entrepreneurs stop hiring outside managers.

Figure 11 shows how the fraction of workers employed as outside managers changes as the contracting friction worsens. Entrepreneurs respond to the contracting frictions by hiring fewer outside managers.

---

<sup>10</sup>TFP equals 1 when  $\lambda = 1$ .

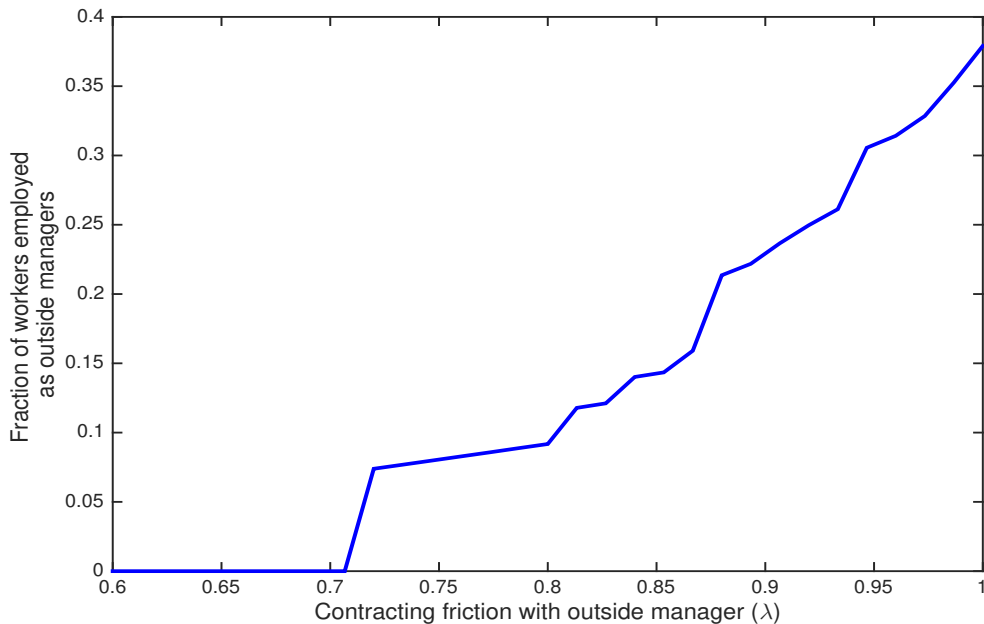


Figure 11: Fraction employed as outside managers

We next study the impact of financial frictions (12), by varying the fraction  $\phi$  lenders can recoup from defaulting entrepreneurs. We do this both for an economy without contracting frictions between entrepreneurs and managers (blue line), and severe contracting frictions between entrepreneurs and managers (red line). We again normalize productivity by that obtained in the frictionless economy.

Productivity falls monotonically as financial frictions worsen, but the total fall is about 5%. The productivity drop is at the lower end of those reported in the literature because we have shut down occupational choice. Distorted selection is often the main driver of productivity losses from financial frictions (as in Buera et al. (2011)).<sup>11</sup>

Figure 12 suggests that there is little interaction between the two frictions. It is interesting to note that in the economy with severe managerial frictions, the impact of financial frictions is smaller. This is because managerial frictions, by effectively compressing the underlying distribution of firm productivity, reduce the scope for financial frictions to further lower aggregate productivity.

<sup>11</sup>We abstract from occupational choice both for technical reasons (see discussion in 3) and because our objective is mainly to asset factor misallocation across operating firms.

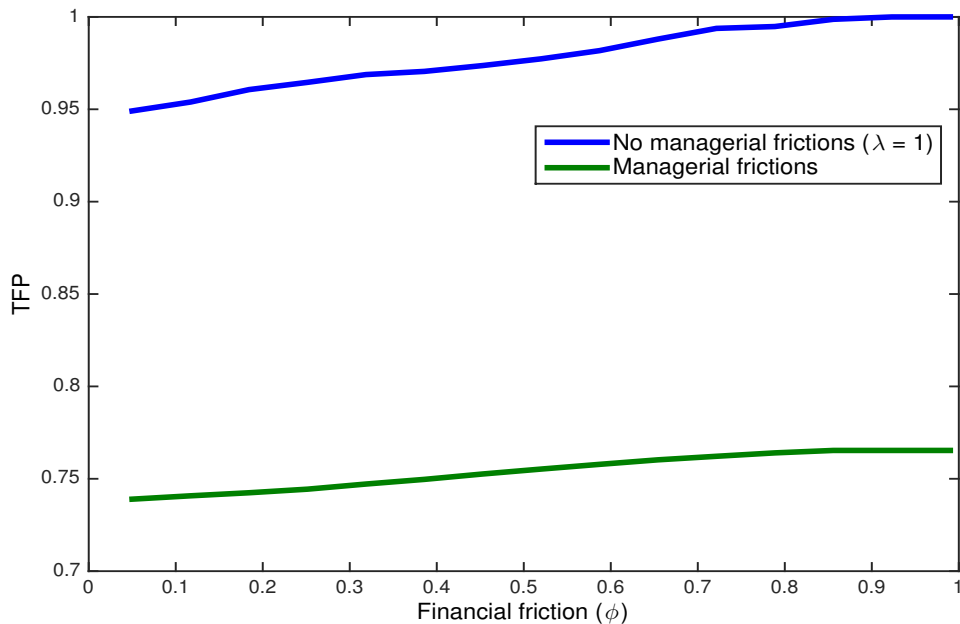


Figure 12: TFP and financial frictions

### 4.3 Cross-sectional properties

In figure 13, we present some preliminary analysis of how the managerial contracting friction affects delegation decisions and wedges. In figure 13, we plot the number of layers against firm productivity. Consistent with our analytical results, the number of layers is (weakly) increasing in firm productivity. As we tighten contracting frictions, the number of managerial layers decline. As low-productivity firms do not employ outside managers in the absence of the friction, they are not affected at all as the friction worsens. However, highly-productive firms do want to employ outside managers and therefore disproportionately affected by worsening managerial frictions.

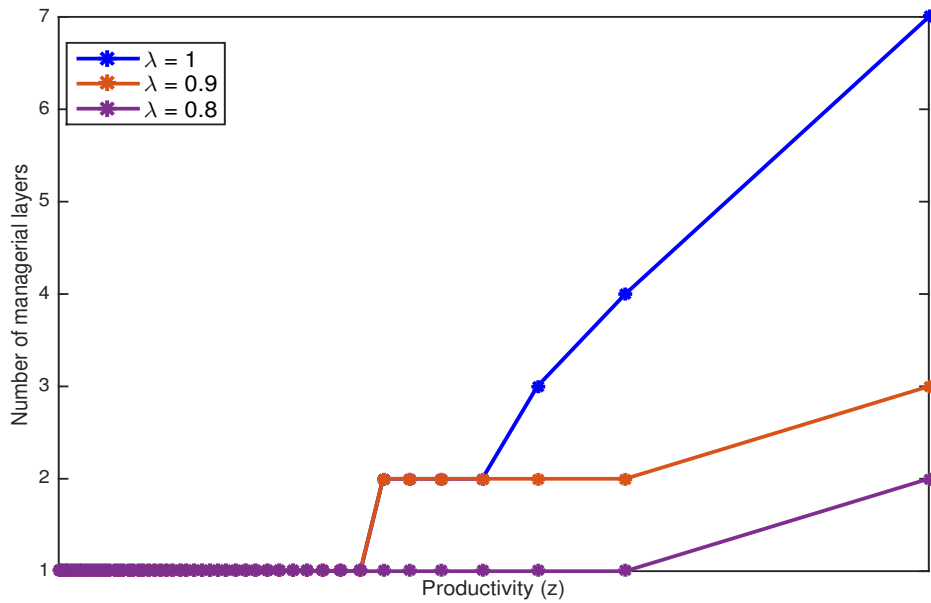


Figure 13: Number of managerial layers

Next we illustrate how our model is able to generate larger distortions for larger firms. We consider two economies: One economy has managerial frictions and the other does not.<sup>12</sup> In the first economy, all firms equate marginal product of capital to the rental rate. In the economy with managerial frictions, large firm still find it optimal to hire outside managers. However, one way they reduce the incentive to steal is by reducing their scale of operation which raises the marginal product of capital above the rental rate.

<sup>12</sup>Both do not have financial frictions

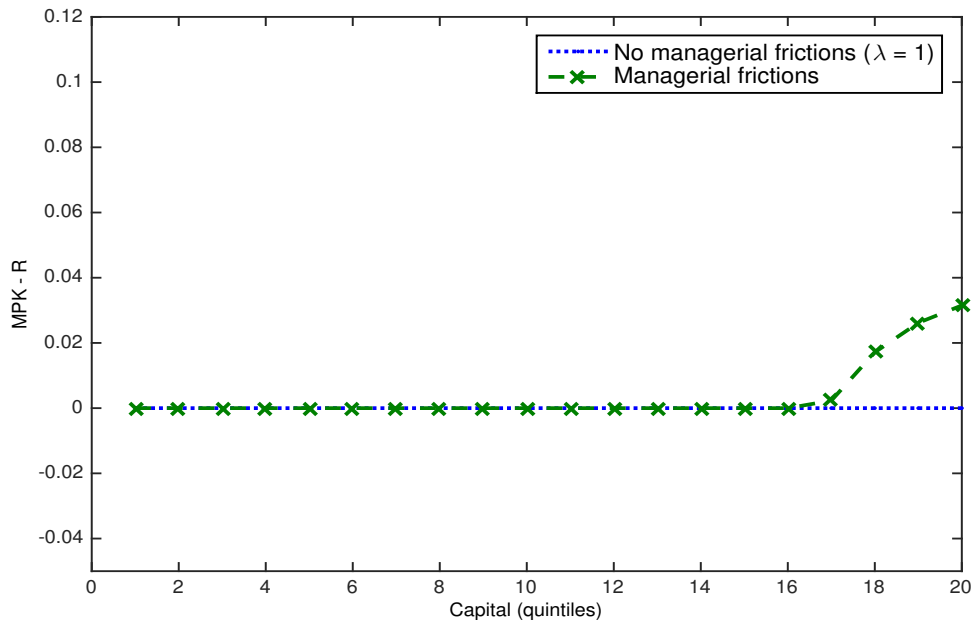


Figure 14: Marginal product of capital and firm size

## 5 Conclusion

In this paper, propose that weak contract enforcement between firms and outside managers is an important source of factor misallocation and productivity losses. We document that marginal products of capital and labor are increasing in firm size and that managers in developing countries receive higher compensation. We develop a model where productive firms hire outside managers to increase their span of control. These firms respond to contracting frictions with managers by paying efficiency wages (which result in output wedges), and by reducing their scale of operation. Our quantitative exercises show how this friction can lead to much larger productivity losses than financial frictions, and is consistent with the relationship between firm size and marginal products we document in the data.

## References

- Acemoglu, D., Johnson, S., and Robinson, J. A. (2005). Institutions as a fundamental cause of long-run growth. *Handbook of Economic Growth*, 1(A):385–472.
- Akcigit, U., Alp, H., and Peters, M. (2017). Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries. *Working Paper*.
- Alchian, A. A. and Demsetz, H. (1973). The Property Right Paradigm. *Journal of Economic History*, 33(1):16–27.
- Bento, P. and Restuccia, D. (2017). Misallocation, Establishment Size, and Productivity. *American Economic Journal: Macroeconomics*, 9(3):267–303.
- Bloom, N. and Reenen, J. V. (2007). Measuring and Explaining Management Practices Across Firms and Countries. *The Quarterly Journal of Economics*, 122(4):1351–1408.
- Bloom, N. and Reenen, J. V. (2010). Why Do Management Practices Differ across Firms and Countries? *Journal of Economic Perspectives*, 24(1):203–224.
- Brandt, L. and Zhu, X. (2010). Accounting for China’s Growth. *Working Paper*.
- Buera, F. J., Kaboski, J. P., and Shin, Y. (2011). Finance and Development: A Tale of Two Sectors. *American Economic Review*, 101(5):1964–2002.
- Cagetti, M. and Nardi, M. D. (2006). Entrepreneurship, frictions and wealth. *Journal of Political Economy*, 114(5):835–870.
- Caselli, F. (2005). Accounting for cross-country income differences. *Handbook of Economic Growth*, 1:679 – 741.
- Crouzet, N. and Mehrotra, N. R. (2018). Small and Large Firms Over the Business Cycle. *Working Paper*.
- Garicano, L. and Rossi-Hansberg, E. (2006). Organization and Inequality in a Knowledge Economy. *Quarterly Journal of Economics*, 121(4):1383–1435.
- Gollin, D. (2002). Getting Income Shares Right. *Journal of Political Economy*, 110(2):458–474.
- Grobovšek, J. (2017). Managerial Delegation and Aggregate Productivity. *Working Paper*.



- Haltiwanger, J., Jarmin, R. S., and Miranda, J. (2013). Who Creates Jobs? Small versus Large versus Young. *Review of Economics and Statistics*, 95(2):347–361.
- Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and Manufacturing TFP in China and India. *Quarterly Journal of Economics*, 124(4):1403–1448.
- Hsieh, C.-T. and Klenow, P. J. (2014). The Life Cycle of Plants in India and Mexico. *Quarterly Journal of Economics*, 129(3):1035–1084.
- Hsieh, C.-T. and Olken, B. A. (2014). The Missing “Missing Middle”. *Journal of Economic Perspectives*, 28(3):89–108.
- Lucas, Jr., R. E. (1978). On the size distribution of business firms. *Bell Journal of Economics*, 9(2):508–523.
- Moll, B. (2014). Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation? *American Economic Review*, 104(10):3186–3221.
- Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous establishments. *Review of Economic Dynamics*, 11(4):707–720.
- Roys, N. and Seshadri, A. (2014). Economic Development and the Organization of Production. *Working Paper*.
- Ruggles, S., Genadek, K., Goeken, R., Grover, J., and Sobek, M. (2017). Integrated Public Use Microdata Series: Version 7.0 [Dataset]. *Minneapolis: University of Minnesota*.
- Troske, K. (1999). Evidence on the employer size-wage premium from worker-establishment matched data. *Review of Economics and Statistics*, 81(1):15–26.