Search-Based Endogenous Asset Liquidity
and the Macroeconomy[*]

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Abstract

We develop a search-theory of asset liquidity which gives rise to endogenous financing constraints on investment in an otherwise standard dynamic general equilibrium model. Asset liquidity describes the ease of issuance and resaleability of private financial claims, which is the outcome of a costly search-and-matching process for such claims implemented by financial intermediaries. Limited liquidity of private claims creates a role for liquid assets, such as government bonds, to ease financing constraints. We show that endogenising liquidity is essential to generate positive co-movement between asset (re)saleability and asset prices. When the cost of channelling funds to entrepreneurs rises, investment and output fall while the hedging value of liquid assets increases, driving up liquidity premia. In the U.S., such intermediation cost shocks can account about 2/3 of the variation in output over the past three decades, and more than 90% of the variation in liquidity premia as measured by the convenience yield of government bonds.

Keywords: endogenous asset liquidity; asset search frictions; financing constraints; general equilibrium

classification: E22; E44; G11

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1 Introduction

The 2007-09 financial crisis is associated with a wide-spread liquidity freeze across many financial market segments.[1] A deterioration of asset liquidity along with falling asset prices has since been recognised as a key propagation mechanism of financial distress to the real economy (see Brunnermeier (2009)). Empirical evidence, however, points to variation in asset liquidity not only during financial crises, but also at business cycle frequency.[2] Are such fluctuations in asset liquidity an inherent and important characteristic of the business cycle?

This paper answers the above question affirmatively by showing that a macroeconomic model featuring endogenous variation in asset liquidity arising from costly financial intermediation is able to match salient, but hitherto difficult-to-explain, business cycle features.

To this end, we introduce search in the intermediation of financial assets into an otherwise standard real business cycle model. Financial assets are backed by physical capital and used to finance idiosyncratic investment opportunities. Search frictions limit the liquidity, i.e. saleability, of these assets and give rise to endogenous financing constraints on firms' investment. These frictions also motivate investors' demand for highly liquid and safe assets to hedge liquidity risks. We model such assets as government bonds, which are not subject to search frictions and, hence, provide a liquidity service in addition to being a store of value. Because of their special service, government bonds carry a liquidity premium, such that their price exceeds their fundamental value.[3]

The introduction of intermediation costs for financial assets has important implications for the dynamics of both financial and real variables in our model. Crucially, it allows asset prices and liquidity to move in the same direction. A persistent fall in asset liquidity, for instance, limits the amount of financial claims that can circulate in the economy. This tightens firms' financing constraints and exerts upward pressure on asset prices. At the same time, lower asset liquidity raises the costs of adjusting asset portfolios through the sale of financial assets. Therefore, asset demand falls, pushing down asset prices. In our model, this demand effect may dominate, such that asset prices and liquidity fall together. The co-movement of asset prices and liquidity is a powerful amplification and propagation

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[1]Dick-Nielsen, Feldhütter, and Lando (2012) identify a structural break in the market liquidity of corporate bonds at the onset of the sub-prime crisis as the liquidity risk component of spreads of all but AAA rated bonds increased and turnover rates declined. Similarly, the liquidity of commercial paper declined dramatically as reported by Anderson and Gascon (2009), with money market mutual funds, the main investors in the commercial paper market, shifting to highly liquid and secure government securities.

[2]Studies by Huberman and Halka (2001), Chordia, Roll, and Subrahmanyam (2001), Chordia, Sarkar, and Subrahmanyam (2005), and Naes, Skjeltorp, and Odegaard (2011) assert that market liquidity is procyclical and highly correlated across asset classes such as bonds and stocks in the U.S..

[3]This definition follows Geromichalos, Licari, and Suárez-Lledó (2007) and Nosal and Rocheteau (2011).

mechanism.

A recessionary shock, which pushes both variables down, tightens firms' financing constraints substantially as both lower liquidity and asset prices limit the ability to raise funds for investment. As a result, investment contracts sharply, amplifying also the impact of the recessionary shock on output. This mechanism allows shocks to the cost of financial intermediation to generate volatile and pro-cyclical asset prices, which co-move positively with asset liquidity as observed in the data. Due to the interaction of these financial dynamics with firms' financing constraints, even modest intermediation shocks trigger substantial variation in real variables. This makes them good candidates for explaining observed business cycle fluctuations.

Asset price dynamics in our model contrast with existing studies of the business cycle implications of exogenous variation in asset liquidity. Popular general equilibrium models with liquidity frictions, such as Kiyotaki and Moore (2012) (henceforth KM) and Jermann and Quadrini (2012), introduce *exogenous* shocks to the liquidity of private financial assets. Adverse liquidity shocks tighten firms' financing constraints. But they do not reduce the demand for investment and financial claims to capital. Therefore, adverse exogenous liquidity shocks create excess demand on the asset market, which unambiguously pushes up asset prices at the same time as the economy slides into recession. The counterfactual response of asset prices to financial shocks in these frameworks has been extensively explored and documented by Shi (2015). Since the counter-cyclical asset price response dampens the sensitivity of financing constraints to financial shocks, very large liquidity shocks or additional frictions are needed to generate deep recessions in these frameworks (see, e.g., Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017)). Financial shocks, therefore, appear ill-suited in these models to explain regular business cycles, much in contrast to our framework.

We exploit the dynamic properties of our model to assess the empirical relevance of real versus financial shocks. In particular, we contrast total factor productivity (TFP) shocks with intermediation cost shocks. The latter capture the cost-effectiveness of financial intermediaries in providing financial services and directly affect their profitability.

Both shocks generate procyclical asset liquidity and prices for a wide range of reasonable parameterisations of the model. But only adverse intermediation cost shocks induce portfolio rebalancing towards highly liquid government bonds, manifested in a higher liquidity premium. Negative TFP shocks, for instance, persistently decrease the return to capital, making investment into capital goods less profitable both today and in the future. While asset demand falls, investors have a weak incentive to hedge against future financing constraints associated with less liquid asset markets. This is reflected in a fall of the liquidity premium. By contrast, adverse intermediation cost shocks do not reduce the return

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to capital. Investors strongly value the hedging service provided by government bonds and rebalance their asset portfolios accordingly, resulting in a surging liquidity premium. Our model, thus, predicts that the dynamics of the liquidity premium can serve to discriminate between financial and real shocks. Finally, more active portfolio rebalancing increases asset price volatility.

We confront the model with financial and macroeconomic data, we measure the liquidity premium of government debt by the convenience yield associated with U.S. Treasuries following Krishnamurthy and Vissing-Jorgensen (2012).[4] The convenience yield tends to increase in recessions and correlates negatively with real output, while the value of physical capital contracts in recessions and correlates positively with real output. In addition, we explicitly model the provision of financial intermediation services in order to match stylised facts about the U.S. financial sector. Specifically, intermediaries facilitate the flow of funds from investors to firms by matching supply of and demand for financial assets using capital and labour as inputs. They charge fees to the sellers and buyers of financial assets to cover their costs. We match average working hours related to financial intermediation, which correlate positively with real output. We also broadly match estimates of the total cost of financial intermediation in the U.S. provided by Philippon (2015).

Because of the dynamic properties of the model, we find that intermediation shocks explain the vast majority of variation in the convenience yield and asset prices in the sample period 1982Q1-2017Q2. Their ability to also explain real variables depends on the presence of variable capital utilisation. In particular, intermediation cost shocks substantially affect the cost of capital and, hence, its optimal utilisation rate. Variation in the utilisation rate, in turn, amplifies fluctuations in e.g. investment and consumption. An estimated version of the model with variable capital utilisation, therefore, finds that 68% of output fluctuations are explained by intermediation cost shocks in our sample period, compared to around 32% for TFP shocks.

Relation to the Literature. By studying intermediation cost shocks which affect asset liquidity, we complement the literature on financial shocks as possible drivers of cyclical fluctuations in the spirit of KM,Shi (2015), and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017). Ajello (2016) studies exogenous shocks to intermediation costs instead of asset liquidity. Kurlat (2013) and Bigio (2015) extend KM with endogenous resaleability through adverse selection.[5] However, the latter papers ignore the role of liquid assets.

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[4]Specifically, we measure the convenience yield as the difference between the yield on 20 year AAA-rated corporate bonds and 20 year Treasury bills following Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017). Since both AAA-rated bonds and Treasury bills have negligible default risks, the yield spread of same-maturity bonds reflects differences in liquidity.

[5]Eisfeldt (2004) and Guerrieri and Shimer (2014) are also notable examples, but these studies do not consider the feedback effects of liquidity fluctuations on production.

We add to the literature by linking endogenous asset liquidity to costly search through financial intermediaries and by matching the convenience yield carried by liquid government debt. We show that exogenous intermediation costs, which drive a wedge between the purchase and sale price of financial assets as in Ajello (2016), are necessary, but not sufficient to generate the positive co-movement asset liquidity and prices. Instead, asset liquidity needs to endogenously affect intermediation costs in order to sufficiently change asset demand. Asset search frictions have this property. Our paper further contributes to the empirical business cycle literature by confronting a macroeconomic model with the size and volatility of the financial sector (as measured by working hours in financial intermediaries).[6]

The microfoundations of our framework draw on the pioneering work of Duffie, Gârleanu, and Pedersen (2005), (Lagos and Rocheteau, 2009), and (Weill, 2007), which have used search theory to model asset liquidity in over-the-counter (OTC) markets in a partial equilibrium setting.[7] This literature has also emphasised the ambiguous impact of trading frictions on asset prices. For instance, Gârleanu (2009) show that trading frictions reduce asset demand and supply simultaneously, such that the turnover volume declines. But, as in our framework, the asset price response depends on which side of the market is affected more strongly. Our general equilibrium setting additionally links asset liquidity to financing constraints.

Search frameworks have also been applied to a wide range of financial markets, including those for federal funds, corporate bonds, private equity and asset-backed securities.[8] Rocheteau and Weill (2011) provides an extensive survey on search theory and asset market liquidity.[9] Meanwhile, Den Haan, Ramey, and Watson (2003), Wasmer and Weil (2004), and Petrosky-Nadeau and Wasmer (2013) have emphasised the role of search frictions in credit markets and their impact on aggregate dynamics.

Search-theoretic models of money, such as Shi (1995), Trejos and Wright (1995), Lagos and Wright (2005), Rocheteau and Wright (2005), and Guerrieri and Lorenzoni (2009) have further highlighted the importance of money for transaction purposes on anonymous search markets. The framework has been extended to analyse privately created liquid assets (Lagos and Rocheteau, 2008), trading delays with market makers (Lagos and Zhang, 2016), and

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[6]Jaccard (2013) also allows financial intermediaries to produce liquidity services by renting capital and hiring labour, but the author does not study endogenous asset liquidity that are tied to investment financing constraints. We view the link between asset liquidity and financing constraints as crucial. Gazzani and Vicondoa (2016) provide evidence that liquidity shocks in secondary sovereign debt markets can have potent real effects on firms' financing constraints.

[7]An alternative approach to modelling asset liquidity focuses on information frictions, such as the adverse selection models in Eisfeldt (2004) and Guerrieri and Shimer (2014).

[8]See, e.g., Feldhutter (2011); Ashcraft and Duffie (2007); Wheaton (1990); Duffie, Gârleanu, and Pedersen (2007)

[9]Search theory has been successfully employed to explain a range of empirical micro-features of asset markets, such as trading delays as well as volatile market depth, trading volumes and bid-ask spreads (Bao, Pan, and Wang, 2011; Gavazza, 2011).

bank-deposits (Williamson, 2012). Rocheteau (2011) shows that the trading restrictions from the money-search framework can be derived from tractable microfoundations exploiting the relative information-sensitivity of different financial assets. Lester, Postlewaite, and Wright (2012) feature asset liquidity and pricing with multiple types of assets with informational asymmetries. Assets are imperfect substitutes, similar to our setting.

Our model differs from these studies in that we consider endogenous supply of financial assets backed by capital in a standard business cycle model. These assets are themselves subject to search frictions, similar to Geromichalos and Herrenbrueck (2016) and Mattesini and Nosal (2016); but, crucially, asset liquidity interacts with investment financing constraints. Government debt, an imperfect substitute that is not subject to search frictions, is used to relax the financing constraints. In our quantitative exploration, we then focus on the convenience yield of government bonds and the size and volatility of the financial sector. These aspects distinguish our paper from Rocheteau and Rodriguez-Lopez (2014) and Branch, Petrosky-Nadeau, and Rocheteau (2016) who also study the endogenous supply of (liquid) assets.

Financial assets in our framework in practice capture both stocks and debt securities issued and traded for the purpose of financing physical investment, which requires recourse to costly financial intermediation services.[10] In the case of stocks, such services are used, for instance, in the context of initial public offerings (IPOs) and mergers and acquisitions (M&A). Bonds, in turn, largely trade on OTC markets where dealers offer brokerage and settlement services. Following the above-mentioned literature, search frictions affecting asset transactions are well-suited to capture these features in a generic fashion.[11]

In a macroeconomic context, Yang (2014) and Cao and Shi (2014), also apply search theory to asset or capital markets. In the former study, TFP shocks can generate co-movement between asset liquidity and prices similar to our model. However, we show that TFP shocks generate a pro- rather than countercyclical convenience yield. The latter authors emphasize capital reallocation rather than financial intermediation.

Finally, while sharing similarities, this paper differs along important dimensions from our previous work Cui and Radde (2016). First, the latter introduces directed search and

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[10]Formally, financial assets are modelled as equity stakes. However, since private claims do not carry default risk in our model, they stand for broader funding sources of investment including debt instruments, such as bonds.

[11]While our framework mainly echoes features of market-based financial intermediation, search frictions have also been used to model credit intermediation by banks, both in the finance (see afore-mentioned references) and in the macroeconomic literature (De-Fiore and Uhlig, 2011). Hence, the intermediation process could also be regarded as bank-based, with financial intermediaries being interpreted as banks offering costly screening and monitoring services and channeling funds in the form of loans from depositors to borrowers. In the interest of tractability and to preserve the generic nature of the intermediation process, we refrain, however, from modeling financial intermediaries' balance sheets more explicitly.

intermediation chains on asset markets in contrast to the random search approach used here. The model exhibits equilibrium multiplicity even when both liquid and partially liquid assets circulate, thereby complicating its tractability. Second, the current paper offers both theoretical and empirical insights into the dynamic behaviour of asset liquidity and asset price and the distinct role of financial shocks to explain fluctuation in the convenience yield and real economic activity.

2   The Basic Framework

In this section, we describe a simple partial-equilibrium model featuring liquid government bonds and less liquid privately-issued financial assets. At this stage, we maintain the assumption of exogenous asset saleability as in Shi (2015) and only endogenously model the price of privately-issued assets. We show that asset prices can positively co-move with asset saleability once transactions of privately-issued claims become costly.[12]

2.1   The Environment

Consider a discrete time and infinite-horizon economy with four types of agents: a continuum of households with measure one, goods-producing firms, financial intermediaries, and a government. The consumption good is used as the numeraire. In the following partial equilibrium model, we only describe households and financial intermediaries.

Preferences

At the beginning of each period $t = 0, 1, 2, ...$, all members of a representative household are identical and equally divide the assets of the household. During a period, each member receives a status draw, becoming an entrepreneur with probability $\chi \in [0, 1]$ or a worker, otherwise. The type-draw is independent across members and over time. An entrepreneur has investment projects but no labour endowment, while a worker has a unit of labour endowment but no investment project. Both groups are temporarily separated until the end of the period. Such a large household structure facilitates aggregation because the model features only ex-post heterogeneity among the household members. This structure has been used both in labour (see, e.g., Andolfatto (1996); Shimer (2010)) and macro-finance literature (Atkeson, Eisfeldt, and Weill (2015); Bianchi and Bigio (2014)) to reduce dimensionality.

Let c_t^e be the consumption of an individual entrepreneur, and c_t^w and ℓ_t be the consumption and hours worked of an individual worker, respectively. The household aggregates the

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[12]We use the terms "asset liquidity" and "asset saleability" interchangeably.

utility of consumption and the dis-utility of labour supply from all its members according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{\chi u(c_t^e) + (1-\chi)U(c_t^w, \ell_t)\}, \quad \beta \in (0,1) \tag{1}$$

where β is the household's discount factor and the expectation is taken over exogenous shocks to asset liquidity ϕ_t (more details later). $u(.)$ is a standard strictly increasing and concave utility function of consumption. $U(.,.)$ is also a strictly increasing and concave utility function of consumption and leisure.

Timing and Technologies

Each period t is characterised by four stages.

The Household's Decision Stage. Asset liquidity shocks are realised. Household members' types are still unknown, so the household evenly divides its assets among all its members. Each household holds a portfolio of government bonds (fully liquid assets b_t), physical capital (k_t), and private financial claims (s_t). Capital will be rented out to goods-producing firms to produce consumption goods at a later stage. On every unit of capital, there is a private claim, which is either sold to other households or retained by the issuing household. All claims on capital have the same liquidity and expected return.[13] They can be sold at the same price q_t. The household holds a diversified portfolio of private claims on the capital stock of the economy.

At this stage, the household maximizes (1) by specifying actions to be implemented by its members. Each entrepreneur is instructed to consume c_t^e, invest i_t, and hold a portfolio of private claims and liquid assets (s_{t+1}^e, b_{t+1}^e). Each worker is instructed to consume an amount c_t^w, supply labour ℓ_t, and hold an asset portfolio (s_{t+1}^w, b_{t+1}^w). After receiving these instructions, all members go to the market and remain separated until the end of period t.

The Production Stage. The type shock is realised. Competitive goods-producing firms rent capital stock at a fixed rate r and hire labour hours at a fixed rate w to produce numeraire consumption goods. After production, workers receives wage income, and owners of private claims receive the rental income from capital. Each unit of capital depreciates at rate $\delta \in (0,1)$ to $1-\delta$. Every existing private claim is rescaled by a factor of $1-\delta$.

The Consumption-Investment Stage. All members pay lump-sum taxes τ, consume, and adjust their asset portfolios; entrepreneurs invest. Workers use their savings to buy private

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[13]Private claims have a contingent payoff that varies with state of the economy and are, hence, akin to equity stakes. However, as our model does not feature bankruptcies of capital-producing firms, private claims do not carry default risk. Therefore, we think of them as capturing funding sources for investment more broadly, i.e. including debt instruments, such as corporate bonds and banking assets traded in inter-bank markets. We do not distinguish between these types of assets more formally in order to preserve tractability.

claims and government bonds. A government bond that pays off one unit of consumption goods tomorrow is sold at a fixed price p_b. Entrepreneurs use their savings and seek further external funding to finance scaleable investment projects, which can transform one unit of consumption goods into one unit of capital stock. To that end, they sell private claims to the rental income from their investment projects as well as retained claimed to existing capital in exchange for consumption goods.

The Pooling Stage. Finally, all members return to their respective households, again pooling their assets across all members.

Portfolio Adjustment Frictions

Government bonds are fully saleable and there is no cost of trading them. Household members cannot issue or short-sell them, and so are subject to a portfolio adjustment constraint

$$b_{t+1}^j \geq 0 \quad \forall j \in \{e, w\} \tag{2}$$

Privately created financial assets, however, are only partially convertible into consumption goods in each period, and this conversion entails a cost. We assume that individuals' search for and matching with counterparties is more costly than delegating this process to specialised financial intermediaries. Financial intermediaries can, thus, facilitate the flow of funds from savers to entrepreneurs.

Not all private financial assets for sale can be successfully matched to buyers on account of search frictions, putting a lower bound on entrepreneurs' holdings of private assets. Let $f \in [0, 1)$ be the probability of a buy-side offer being matched to a sell-side offer; conversely, let $\phi_t \in [0, 1)$ be the probability of a sell-side offer being matched to a buy-side offer.

Entrepreneurs first obtain consumption goods after selling claims to capital. These consumption goods are then invested to create new capital. Note that this timing assumption creates a possible diversion problem. We, therefore, assume that financial intermediaries spend resources to monitor the delivery of capital that backs the financial assets. As a result, for every unit of capital there is exactly one unit of private claims, such that the amount of assets *offered* for sale is bounded from above by entrepreneurs' ability to deliver capital.

The amount of private financial assets *retained* by entrepreneurs is, in turn, bounded from below: due to the limited saleability of private financial assets, they must retain $(1 - \phi_t)$ fraction of claims on each new investment project i_t and previously accumulated claims $(1 - \delta)s_t$ (adjusted for depreciation); Therefore, a household member's private asset position must satisfy the portfolio adjustment constraint

$$s_{t+1}^j \geq (1 - \phi_t) [i_t + (1 - \delta)s_t] \quad \forall j \in \{e, w\} \tag{3}$$

ϕ_t is thus interpreted as asset liquidity (saleability) which financial frictions are tied to.

Finally, financial intermediaries charge transaction fees to cover costs related to e.g. screening and monitoring services. To model this, we let financial intermediaries pay κ units of consumption goods to process one buy order and to monitor the delivery of one sell order. Since not all offers are matched, financial intermediaries need to process f^{-1} buy orders and monitor ϕ_t^{-1} sell orders per unit of asset transactions. Transaction fees drive a wedge between the prices at which financial assets are purchased and sold. Specifically, let q_t^w denote the price offered to buyers, and q_t the price offered to sellers. Since the profit of each transaction accruing to an intermediary is the difference between the purchase and sale prices $q_t^w - q_t$, the following zero profit condition holds

$$q_t^w - q_t = \kappa \left(\frac{1}{f} + \frac{1}{\phi_t} \right). \tag{4}$$

In other words, the spread between the purchase and (re-)sale price of private assets covers the intermediation costs.

2.2 The Household's Problem

We switch to recursive notation from this point on for expositional simplicity. That is, x_t is expressed as x, while x_{t+1} is expressed as x_+.

In a typical period t, the household makes consumption, savings, and investment plans (c^e, s_+^e, b_+^e, i) for each entrepreneur as well as labour supply, consumption, and savings plans $(\ell, c^w, s_+^w, b_+^w)$ for each worker. The household faces a resource constraint on each member. All members are endowed with s units of claims and b units of bonds from the household.

An Entrepreneur's Constraints. After paying taxes, entrepreneurs finance new investment $(i > 0)$ and consumption $(c^e > 0)$ with capital rental income rs as well as net receipts from trading government bonds $b - p_b b_+^e$ and private claims $q \left[i + (1-\delta)s - s_+^e \right]$:

$$c^e + i \leq rs + b - p_b b_+^e + q \left[i + (1-\delta)s - s_+^e \right] - \tau \tag{5}$$

$$\text{where } s_+^e \geq (1 - \phi) \left[i + (1-\delta)s \right] \text{ following (3)}$$

To understand the net receipts from trading private claims, notice that after capital depreciation, the entrepreneur owns $(1-\delta)s$ legacy claims and his new investment i. The claims to old and new capital are either sold to other households or retained by the entrepreneur for the household. Since the entrepreneur holds s_+^e at the end of the period, the amount $i + (1-\delta)s - s_+^e$ is sold via financial intermediaries at a price q.

We focus on an equilibrium in this economy where the portfolio adjustment constraints

(2) and (3) bind for entrepreneurs (which happens when $q > 1$, such that it is profitable to issue claims and invest). Being financing-constrained, entrepreneurs will seek to maximise their resources for investment projects by selling as many private claims as possible, such that $s_+^e = (1 - \phi)[i + (1 - \delta)s]$, while dissaving all liquid assets, i.e. $b_+^e = 0$. In this case, the resource constraint (5) simplifies to

$$c^e + (1 - \phi q)\, i \leq [r + (1 - \delta)\phi q]\, s + b - \tau, \tag{6}$$

which we refer to as the *financing constraint*. The financing constraint can be interpreted in the following way: to invest in new capital stock, the entrepreneur's liquid net worth $[r + (1 - \delta)\phi q]\, s + b$, net of consumption c^e and taxes τ, can be leveraged at $(1 - \phi q)^{-1}$. Therefore, the financing constraint (6) effectively implies an upper bound on investment i.

A Worker's Constraints. A worker's resource constraint differs from that of an entrepreneur along two dimensions. First, a worker receives labour income $w\ell$. Second, the worker does not have investment projects (i.e., $i = 0$), but seeks to acquire $s_+^w - (1 - \delta)s$ units of private claims for saving purposes. The expenditure on asset transactions amounts to $q^w [s_+^w - (1 - \delta)s]$ where again $q^w > q$ is the price offered to buyers. The resource constraint is thus

$$c^w + q^w [s_+^w - (1 - \delta)s] \leq w\ell + rs + b - p_b b_+^w - \tau \tag{7}$$

Notice that workers should also respect the portfolio adjustment constraints (2) and (3). However, in equilibrium, these constraints will be slack as workers are buyers of private claims sold by entrepreneurs and also hold government bonds.

The Household's Constraints. Let household-wide aggregates for consumptions as well as current- and next-period holdings of private claims and government bonds be denoted as

$$z \equiv \chi z^e + (1 - \chi)z^w \text{ for } z \in \{c, s, b, s_+, b_+\} \tag{8}$$

Multiplying entrepreneurs' financing constraint (6) by χ and workers' resource constraint (7) by $1 - \chi$, adding them up by using (8), and observing the binding portfolio adjustment constraints $s_+^e = (1 - \phi)[i + (1 - \phi)s]$ and $b_+^e = 0$, we obtain a household-wide resource constraint:

$$c + q^w s_+ + p_b b_+ \leq (1 - \chi)w\ell + rs + b + [q^w - \chi\phi(q^w - q)](1 - \delta)s$$
$$+ [q^w - 1 - \phi(q^w - q)]\chi i - \tau \tag{9}$$

The Household's Problem. Since entrepreneurs' portfolio adjustment constraints bind,

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i.e. $s_+^e = (1-\phi)\,[i + (1-\phi)s]$ and $b_+^e = 0$, the household's choice set can be simplified to $(i,$ $c^e,\, c^w,\, s_+,\, b_+,\, \ell)$. Let $v\,(s, b; \Gamma)$ be the value of a typical household with net private financial claims s, money holdings b, given the aggregate state $\Gamma \equiv \phi$. The value $v(s, b; \Gamma)$ satisfies the following Bellman equation

$$v(s, b; \Gamma) = \max_{\{i, c^e, c^w, s_+, b_+, \ell\}} \left\{ \chi u\,(c^e) + (1 - \chi)\, U\,(c^w, \ell) + \beta \mathbb{E}_\Gamma\,[v(s_+, b_+; \Gamma_+)] \right\} \qquad (10)$$

subject to (6), (9), and non-negativity constraints

$$i \geq 0, \quad c^e \geq 0, \quad c^w \geq 0, \quad s_+ \geq 0, \quad b_+ \geq 0, \text{ and } \ell \in [0, 1]$$

Characterising the Problem. Let $U_c = U_c(c^w, \ell)$ and $U_\ell = U_l(c^w, \ell)$ denote the partial derivatives of U with respect to consumption and hours worked. Let $\rho \chi U_c$ be the Lagrangian multiplier of the resource constraint of the entrepreneur (6). The rescaling χU_c in the multiplier simplifies the optimality conditions in the following. The optimal choice of (ℓ, c^e, i) then satisfies

$$U_\ell = w U_c \qquad (11)$$

$$u_c = (1 + \rho) U_c \qquad (12)$$

$$q^w - 1 - \phi(q^w - q) \leq (1 - \phi q)\,\rho \text{ and } i \geq 0 \qquad (13)$$

where the last first-order condition holds with complementary slackness.

(11) is a standard labour supply condition.

(12) captures the ratio of the marginal utilities of consumption between an entrepreneur and a worker. If $\rho > 0$, entrepreneurs are financing constrained and, therefore, have a higher marginal utility of consumption compared to workers.

(13) is a key equation in the model and characterises the optimal choice of investment, relating its marginal benefit (left-hand side) to its marginal cost (right-hand side). First consider the cost-side. For one unit of investment, the entrepreneur can raise ϕq in external funds (consumption goods) by selling financial claims to the newly produced capital, such that it only has to finance a fraction $(1 - \phi q)$ internally, which can be interpreted as a "down-payment" on investment. The cost is adjusted by ρ, which is the shadow price of an entrepreneur's financing constraint in terms of the household's consumption.

The marginal benefit of investing, on the other hand, reflects the net value of newly created capital to the household. Specifically, by investing one unit of consumption goods, an entrepreneur creates a claim worth q^w units of consumption goods to the household at cost 1. This implies a net gain of $q^w - 1$. However, for the fraction ϕ that is issued the

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household reduces the value of its asset portfolio by $q^w - q$, which decreases the marginal gain. The household invests a positive amount if the marginal cost does not exceed the marginal benefit, and decides not to invest $(i = 0)$ otherwise.

Finally, we show the two Euler equations for bonds $b_+ > 0$ and private claims $s_+ > 0$.

$$1 = \mathbb{E}_\Gamma \left[\frac{\beta U_{c,+}}{U_c} \frac{1}{p_b} \left[1 - \chi + \chi(1 + \rho_+) \right] \right] = \mathbb{E}_\Gamma \left[\frac{\beta U_{c,+}}{U_c} \frac{1}{p_b} \left(1 + \chi \rho_+ \right) \right] \tag{14}$$

$$1 = \mathbb{E}_\Gamma \left[\frac{\beta U_{c,+}}{U_c} \left[\frac{r_+ + (1-\delta)q_+^w}{q^w} + \frac{r_+ + (1-\delta)\phi_+ q_+^w}{q^w} \chi \rho_+ - \frac{(1-\delta)\phi_+ \left(q_+^w - q_+ \right)}{q^w} \chi(1 + \rho_+) \right] \right].$$
$$\tag{15}$$

where $\beta U_{c,+}/U_c$ is the stochastic discount factor of an unconstrained worker.

In (14), $1/p_b$ denotes the standard real return on government bonds. How this return is valued from the point of view of the household depends on whether the marginal government bond purchased by a worker today winds up in the hands of a worker or an entrepreneur next period. If the bond is held by a financially unconstrained worker next period, its marginal utility amounts to the real return $1/p_b$. This happens with probability $1 - \chi$. If, however, the bond winds up in the hands of a financially constrained entrepreneur, the marginal utility of consumption of this household member is raised by a factor $1 + \rho_+$ as the additional liquid resources relax the investment constraint. This happens with probability χ. The expression capturing the future value of liquid assets in equation (14) can be compounded into $1/p_b + \chi \rho_+/p_b$, where the first term is the standard real return on government bonds, while the second term reflects the liquidity premium of government bonds.

In (15), the return from holding private claims consists of three parts. The first part and the second part are similar to the two parts in the payoff from holding bonds: a standard return $\frac{r_+ + (1-\delta)q_+^w}{q^w}$ and a premium associated with the fact that private claims also relax the financing constraint, but only up to the saleable fraction ϕ_+. As government bonds are fully liquid, ϕ_+ appears only in this asset pricing equation for private claims. The third part is an adjustment to account for the fact that private claims are effectively sold at a discounted price q_+ below the purchase price q_+^w.

2.3 Asset Price and Asset Liquidity

Given the wage and rental rates $\{w, r\}$, the intermediation technology κ, and the government bond price p_b, we can solve for the household's optimal choices, and, more importantly, determine the price of private claims as a function of asset liquidity ϕ.

Since the interesting equilibrium is one in which entrepreneurs' financing constraint binds,

investment is profitable and, hence, non-negative ($i > 0$). The optimality condition for investment (13) then holds with equality.

Suppose intermediation is costless, i.e. $\kappa = 0$. The zero profit condition for intermediaries (4) then implies $q^w = q$, such that equation (13) simplifies to

$$q - 1 = (1 - \phi q)\rho \rightarrow q = 1 + \frac{1 - \phi}{\phi + \rho^{-1}} > 1$$

To understand this condition, notice that for any given shadow price of the financing constraint ρ, the marginal benefit of investing (i.e., $q - 1$) is strictly increasing in q, while the down-payment $(1 - \phi q)$ on investment is strictly decreasing in q. When liquidity shocks push down ϕ, the down-payment rises. Therefore, when the falling ϕ tightens the financing constraint and raises the shadow price of the financing constraint ρ, the marginal cost of investment goes up for any price q. The equilibrium asset price q needs to rise to equate the marginal benefit and cost of investment.[14]

Intuitively, a fall in ϕ amounts to a negative supply shock in the asset market, creating excess demand for private financial claims. Hence, the asset price must rise to equilibrate supply and demand. That is, when $\kappa = 0$, (13) implies that $\partial q / \partial \phi < 0$; the asset price q would generally *increase* in response to a negative liquidity shock. This mechanism resembles the result in Shi (2015).

By contrast, when financial intermediation is costly, i.e., $\kappa > 0$, the asset price and asset saleability *can* move in the same direction. In this case, the price of buying exceeds that of selling assets. Both buyers and sellers will then need to consider the costs of asset transactions incurred today and in the future (should they become entrepreneurs and need to sell private claims) when taking portfolio decisions. The impact of transaction costs on asset demand can push down the asset price when current or future asset liquidity ϕ falls.

To understand this effect, we again use equation (13), replacing q^w from the zero profit condition (4):

$$q + \kappa\left(\frac{1}{\phi} + \frac{1}{f}\right) - 1 - \phi\kappa\left(\frac{1}{\phi} + \frac{1}{f}\right) = (1 - \phi q)\rho \rightarrow q = \frac{1 + \rho - \kappa(1 - \phi)(\phi^{-1} + f^{-1})}{1 + \phi\rho}$$

First, with a falling ϕ, a smaller fraction of investment will be financed through the sale of private claims. As households retain a lager fraction of the claims to new capital valued at the purchase price q^w, rather than selling them at the lower (re-)sale price q, their portfolio

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[14]Theoretically, ρ could fall with ϕ if the negative liquidity shock tightened the household's resource constraint (9) more than the financing constraint (6), hence lowering workers' consumption relative to entrepreneurs'. However, this is an unlikely outcome, as liquidity shocks directly reduce entrepreneurs' consumption.

loss from selling private assets mechanically shrinks (the above condition reflects the fact that, in equilibrium, the wedge between the purchase and sale price equals the intermediation cost). This makes investment more attractive at the margin. In addition, a falling ϕ also raises the intermediation cost per successful asset transaction, $\kappa\left(\frac{1}{\phi} + \frac{1}{f}\right)$, as well. Both effects induce households to accumulate a larger fraction of their asset portfolio through entrepreneurs rather than workers. As a result, the demand for acquiring private financial claims through workers falls, exerting downward pressure on the asset price q.

Second, there is a much less obvious dynamic effect on entrepreneurs' optimal amount of investment arising from future expected selling costs. This is because the degree of financing constraints ρ is endogenous and essentially forward-looking. Lower asset liquidity in the future implies that claims created today will be more costly to sell in the future. Anticipating this, households demand less physical investment today, somewhat relaxing entrepreneurs' financing constraints, i.e. pushing ρ down. This effect on ρ reduces the marginal cost of investment for any given q.

If the demand effects dominate the negative supply effect mentioned before, current or future liquidity shocks can increase the marginal benefit of investment more than the marginal cost of investment for any given q; the asset price q would, thus, need fall with ϕ in order to satisfy the optimality condition for investment.

In the steady state, ρ is a constant according to the Euler equation for bonds (14). Then, a permanent reduction in ϕ reduces asset price q, i.e., $\partial q/\partial\phi > 0$, if and only if ϕ is small enough. Intuitively, demand is sensitive to changes in asset liquidity when asset liquidity is sufficiently low, because it then disproportionately affects the effective intermediation cost $\kappa(\phi^{-1} + f^{-1})$.

Proposition 1:

Suppose $\kappa > 0$, $p_b > \beta$, and that both private claims and government bonds are valued. In the steady state,

$$\rho = \rho^* \equiv \frac{\beta^{-1}p_b - 1}{\chi} > 0$$

is a constant determined by (14). Then, $\partial q/\partial\phi > 0$ across steady states if and only if

$$0 \leq \phi < \min\{\phi^*, 1\}$$

where $\phi^* = \dfrac{\kappa + \sqrt{\kappa^2 + \kappa(\rho^*)^{-1}\left[\rho^* + \left(\rho^* - \frac{\kappa}{f}\right)\left(\frac{1}{\rho^*} + 1\right)\right]}}{\rho^* + \left(\rho^* - \frac{\kappa}{f}\right)\left(\frac{1}{\rho^*} + 1\right)}$.

Proof. See Appendix A.1. □

By way of comparison, in Ajello (2016), the overall intermediation cost $\tilde{\kappa} = \kappa(\phi^{-1} + f^{-1})$

is exogenous and independent of ϕ. A temporary, but persistent, reduction in ϕ pushes up asset price q in his simulation.[15] Shocks to intermediation cost $\tilde{\kappa}$, on the other hand, can reduce q for *fixed* asset liquidity ϕ.

If we replace $\kappa(\phi^{-1} + f^{-1})$ by $\tilde{\kappa}$ in our model, the asset price can be expressed as

$$q = \frac{1 + \rho - \tilde{\kappa}(1 - \phi)}{1 + \phi\rho}$$

again, using equation (13). Then, $\partial q/\partial\phi > 0$ holds in the steady state whenever $\tilde{\kappa} > \rho$. However, this parameter restriction implies that $q < 1$, which means that entrepreneurs are not financing constrained. This implies that in an economy with binding investment financing constraints and exogenous intermediation cost shocks, the asset price and liquidity cannot move in the same direction.

In order to generate $\partial q/\partial\phi > 0$, Proposition 1 highlights that there is some specific relationship between asset liquidity and intermediation costs (and the tightness of financing constraint) that needs to be satisfied, at least in the steady state. Then, it is likely that researchers need to exogenously move both κ and ϕ together in order to generate $\partial q/\partial\phi > 0$ off the steady state. This can be avoided by endogenising asset liquidity. Indeed, in our full model developed in the next section, which simultaneously features endogenous variation in intermediation cost κ, asset liquidity ϕ, and asset price q, the asset price and liquidity can move together as we will show in the quantitative exercises.

3 General Equilibrium with Endogenous Liquidity

We now endogenise asset liquidity and embed it into a general equilibrium model.

3.1 The Extended Environment

Consumption Goods Producers

Firms in the consumption goods sector produce output y^g by renting capital k^g and hiring labour ℓ^g from households. Their technology follows

$$y^g = AF^g(k^g, \ell^g)$$

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[15] Ajello (2016) shows that one can only revert this relationship by introducing a number of additional frictions, such as sticky prices and inertial monetary policy rules. That is why his paper focuses on shocks to the intermediation costs instead of shocks to liquidity ϕ.

where F^g is homogeneous function in k^g and l^g with degree one, and A measures exogenous total factor productivity (TFP) in the economy. In view of production technology and frictionless capital and labour markets, the rental rate of capital and the wage rate equal the corresponding marginal products denoted as

$$r = AF_k^g \text{ and } w = AF_\ell^g \tag{16}$$

Financial Intermediaries

Competitive intermediaries provide financial services by producing asset demand and supply offers, after receiving instructions from buyers and sellers. They operate a costly matching function for the demand and supply of private financial claims and determine the transaction price for successful matches, settling trades and monitoring their execution. The supply of financial services in terms of matchable buy and sell orders is measured by $AF^f\left(k^f, \ell^f\right)$, where F^f is homogeneous in capital and labour input, k^f and l^f, with degree one.

Financial intermediaries pay rental and wage incomes to capital owners and workers. In addition, we assume financial intermediaries need to spend an exogenous fraction Δ of these factor payments to settle trades and monitor their execution. Δ measures the cost-effectiveness of financial intermediaries. An increase in Δ amounts to an efficiency loss in the production of financial services, which, ceteris paribus, reduces intermediaries' profits. In equilibrium, it translates into a shock to the intermediation cost for asset transactions faced by workers and entrepreneurs (see below).[16]

Therefore, the cost of matching asset demand and supply orders κ, that is passed to savers and investors, amounts to the relative price of financial services in terms of consumption goods. In the general equilibrium setting, κ is an *endogenous* feature of the intermediation process. To see this, notice that for each unit of successful transactions, the financial intermediaries need to post f^{-1} units of purchase orders and ϕ^{-1} units of sale orders, respectively. Given that in total the intermediaries produce $AF^f(k^f, \ell^f)$ units of buy and sell orders, the total volume of transactions is thus $AF^f(k^f, \ell^f)/(f^{-1} + \phi^{-1})$. Further, since each unit of asset transactions yields a gross profit $q^w - q$, we can write the net profit of the financial intermediaries as

$$\Pi^f = \left[\frac{q^w - q}{\frac{1}{f} + \frac{1}{\phi}}\right] AF^f(k^f, \ell^f) - (1 + \Delta)\left(rk^f + w\ell^f\right).$$

Given frictionless capital and labour markets, intermediaries thus demand capital k^f and

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[16]We will show that this financial shock is able to generate volatile and pro-cyclical asset prices which co-move positively with asset liquidity and find that it is an important driver of U.S. business cycles.

labour hours ℓ^f such that

$$r = \left[\frac{q^w - q}{\frac{1}{f} + \frac{1}{\phi}}\right]\frac{AF_k^f}{1 + \Delta} \text{ and } w = \left[\frac{q^w - q}{\frac{1}{f} + \frac{1}{\phi}}\right]\frac{AF_\ell^f}{1 + \Delta} \tag{17}$$

Now, from the zero profit condition (4), we know that the cost for intermediating one unit of asset transactions, $q^w - q = \kappa\left(\frac{1}{\phi} + \frac{1}{f}\right)$, is endogenous, and the cost of asset orders in terms of consumption goods, κ, is defined as

$$\kappa \equiv \frac{(1 + \Delta)\,r}{AF_k^f} = \frac{(1 + \Delta)\,w}{AF_\ell^f}. \tag{18}$$

The costly matching technology for asset transactions is characterised by a mapping of the tightness of asset markets into matching probabilities f and ϕ for both asset demand and supply. Let asset market tightness θ be the ratio of total purchase orders divided by total sale orders, the same as the total amount of assets on sale. Since the total number of orders is $AF^f(k^f, \ell^f)$ and the total amount of assets on sale is $\chi\left[i + (1 - \delta)s\right]$,[17] we can define the tightness as

$$\theta \equiv \frac{AF^f(k^f, \ell^f)}{\chi\left[i + (1 - \delta)\,s\right]} - 1. \tag{19}$$

The matching technology implies that, on the buy-side, a fraction

$$f = f(\theta) \equiv \mu(\theta) \tag{20}$$

of purchase orders is satisfied on average through successful matches *ex post* for some function $\mu(.)$. The matching function that delivers the above matching probability $f(\theta)$ of the demand-side is assumed to exhibit constant returns to scale and should be increasing in both the measure of assets to be sold and the measure of purchase orders. Therefore, we assume that the function determining the matching probability $\mu\colon \mathbb{R}^+ \to \mathbb{R}^+$ is continuous, non-increasing, and convex function with $\lim_{\theta \to 0} \mu(\theta) = \infty$. On the sell-side, asset saleability now captures the fraction of assets that can be sold *ex post* in a given period, which is also a function of the tightness

$$\phi = \phi(\theta) \equiv \theta\mu(\theta) \tag{21}$$

where $\phi(\theta)$ is a non-decreasing and concave function w.r.t. θ, with $\phi(0) = 0$ and $\phi(\theta) \le 1$ for all θ. In particular, $\lim_{\theta \to \infty} \mu(\theta) = 0$.

Once asset orders are matched, financial intermediaries settle the transaction price on behalf of buyers and sellers through Nash bargaining.

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[17]Each entrepreneur posts $i + (1 - \delta)s$ units of assets on sale, and their population is measured by χ.

Government

Each period the government spends g, sets tax τ collected lump-sum from the household, redeems all matured bonds, and issues an amount B_+ of new real bonds, where (g, τ, B_+) are normalised by the measure of households. If $\tau < 0$, the household receives lump-sum transfers from the government. The government budget constraint can be written as

$$g + B = \tau + p_b B_+ \tag{22}$$

where p_b is again the price of bonds in terms of numeraire consumptions goods. Government policies are kept simple, as they are not the focus of this paper. That is, the quantities (g, B_+) are assumed to be positive constants. The lump-sum τ must then vary to satisfy the government budget constraint.

3.2 Equilibrium Characterisation

Let the aggregate capital stock each period be K. Then, the aggregate state Γ in the household problem becomes $\Gamma \equiv \{K, B, A, \Delta\}$. The financial market variables κ and ϕ, which were treated as exogenous state variables in the partial equilibrium analysis, are now functions of the aggregate state Γ.

We now proceed to characterise the equilibrium. The household's optimality conditions (11) - (15) remain valid. We still need to characterise the asset market in equilibrium.

Shadow Price

There exists a shadow price q^f between intermediaries. To be specific, an intermediary can choose to trade with buyers and/or sellers of private claims. Consider, for instance, an intermediary who acts on behalf of buyers. This intermediary obtains q^w from ultimate buyers, but needs to screen $1/f$ buy orders at unit cost κ for each successful asset transaction.

Therefore, the intermediary's profit from delivering an asset to buyers is $q^w - \kappa/f$. The intermediary can also monitor the delivery of the asset by sellers. This requires screening $1/\phi$ sell orders at unit cost κ, and subsequently pay out q to the selling entrepreneur. The cost of acquiring an asset is, thus, $q + \kappa/\phi$. Alternatively, the intermediary can ask another intermediary to monitor the delivery, paying it the shadow price q^f. As intermediaries are competitive, the equilibrium shadow price will satisfy

$$q^f = q^w - \frac{\kappa}{f} = q + \frac{\kappa}{\phi}. \tag{23}$$

That is, profits are zero on either side of an asset transaction, such that intermediaries

are indifferent between only engaging with buyers, only with sellers or with both buyers and sellers. If condition (23) were violated, intermediaries could earn non-zero profits from taking only one side of a trade, such that the market would break down.

Price Settlement

Now, we can derive the relationship between the shadow price and equilibrium transaction prices between ultimate sellers and buyers. Once a sell and a buy orders have been matched, intermediaries bargain on behalf of sellers (entrepreneurs) and buyers (workers) over the transaction price. Let $\tilde{q}^f$ denote this transaction price offered by intermediaries to either side of a match.

Following a related concept in the labour-search literature, the transaction price is determined by bargaining at the margin, i.e. over an incremental asset transaction in a successful match following (Shimer, 2010). Specifically, we compute the marginal transaction surplus of individual buyers and sellers at an arbitrary price $\tilde{q}^f$ relative to the outside option of not engaging in an additional transaction. Let $v_s^w(\tilde{q}^f)$ and $v_s^e(\tilde{q}^f)$ denote the marginal transaction surpluses of individual workers and entrepreneurs.

A Worker's Marginal Transaction Surplus. Consider a worker who has the opportunity to purchase an incremental amount of private assets $\epsilon > 0$ at an arbitrary price $\tilde{q}^f$ this period. All prices revert to equilibrium prices next period. After modifying (10) accordingly, the value for the worker is

$$\hat{v}^w(\tilde{q}^f, \epsilon) = U\left(c^w, \ell\right) + \beta\mathbb{E}_\Gamma\left[v(s_+ + \epsilon, b_+; \Gamma_+)\right] \text{ s.t.}$$

$$c^w + \tau + \tilde{q}^f\epsilon = w\ell + rs + b - p_b b_+^w + q^w\left[(1 - \delta)s - s_+^w\right]$$

where b_+^w and s_+^w are chosen as per the household's instructions. Differentiating this value function w.r.t. to ϵ and evaluating the derivative at $\epsilon = 0$, we obtain the worker's marginal value of an incremental asset transaction

$$v_s^w(\tilde{q}^f) = -U_c\tilde{q}^f + \beta\mathbb{E}_\Gamma\left[v_s(s_+, b_+; \Gamma_+)\right] \tag{24}$$

An Entrepreneur's Marginal Transaction Surplus. On the other side of the trade, an entrepreneur has an incremental ϵ units of investment, the claims to which he can sell at price $\tilde{q}^f$. He invests ϵ and delivers the ϵ units of claims to capital, retaining $\left(\tilde{q}^f - 1\right)\epsilon$ as profit. After we modify (10), the value for the entrepreneur is

$$\hat{v}^e(\tilde{q}^f, \epsilon) = u\left(c^e\right) + \beta\mathbb{E}_\Gamma\left[v(s_+ + i_\epsilon, b_+; \Gamma_+)\right] \text{ s.t.}$$

$$c^e + \tau + (1 - \phi q)(i + i_\epsilon) = rs + (1 - \delta)s + \frac{b}{P} + (\tilde{q}^f - 1)\epsilon$$

where i_ϵ is the extra investment implemented after obtaining the additional resources from selling the ϵ units of claims. Notice that the entrepreneur again can issue a fraction ϕ of the incremental investment i_ϵ at the equilibrium price q.

Because the entrepreneur is financing constrained, he would not spend the additional resources on liquid assets or consumption, but invest them fully into new capital. Since entrepreneurs can leverage the additional resources, they can fund an incremental investment of size $i_\epsilon = (\tilde{q}^f - 1)\epsilon/(1 - \phi q)$. Appendix A.2 contains a formal proof of this claim. Differentiating $\hat{v}^e(\tilde{q}^f, \epsilon)$ w.r.t. to ϵ and evaluating the derivative at $\epsilon = 0$, we obtain the entrepreneur's surplus of an additional unit of successful transactions

$$v_s^e(\tilde{q}^f) = \frac{\tilde{q}^f - 1}{1 - \phi q} \beta \mathbb{E}_\Gamma [v_s(s_+, b_+; \Gamma_+)] \tag{25}$$

Price Settlement. Assume that there are gains from trade, i.e., there is a price $\tilde{q}^f$ satisfying both $v_s^w(\tilde{q}^f) \geq 0$ and $v_s^e(\tilde{q}^f) \geq 0$. We require that the bargained asset price maximizes the (generalized) Nash product

$$\left[v_s^w \left(\tilde{q}^f \right) \right]^{1-\omega} \left[v_s^e \left(\tilde{q}^f \right) \right]^\omega \tag{26}$$

where $\omega \in (0, 1)$ is the bargaining weight assigned to entrepreneurs. In equilibrium, $\tilde{q}^f = q^f$, because otherwise intermediaries could earn positive profits ($\tilde{q}^f < q^f$) or would run losses ($\tilde{q}^f > q^f$).

Proposition 2:
The solution of the Nash bargaining problem satisfies

$$q^f = (1 - \omega) + \omega q^w$$

Using $q^f = \tilde{q}^f$ and the zero profit conditions (23)*, we have*

$$q = \max\{1, 1 + \kappa \left(\frac{\omega}{1 - \omega} \frac{1}{f} - \frac{1}{\phi} \right)\}. \tag{27}$$

Suppose the economy is in the steady state with $\rho = \rho^*$ as a constant and $q > 1$. Then, $\partial q/\partial \phi > 0$ across steady states if and only if

$$\rho^* + 1 < \frac{\mu(\theta) [1 - \theta\mu(\theta)]^2}{[\omega\theta - (1 - \omega)] [\mu(\theta) + \theta\mu'(\theta)]} \tag{28}$$

Proof. See Appendix A.3. □

Equation 27 is similar to the entry conditions commonly found in the asset search literature (Rocheteau and Weill, 2011; Vayanos and Wang, 2007). If the Euler equation for private assets determines the asset price, then demand and supply conditions as captured by the matching probabilities ϕ and f need to be such that condition (27) is satisfied in order to induce individual agents to participate in the market.

As regards the relationship between the price and liquidity of private assets, the general equilibrium framework confirms the intuition developed in the partial equilibrium setting: both can move in the same direction as long as i) financial intermediation is costly, and ii) the demand for private assets falls relative to their supply. To see the last point, notice that if entrepreneurs are more financing constrained, ρ increases and the marginal cost of investment is higher; then, it is more difficult for condition (28) to be met.

Recursive Equilibrium

A *recursive competitive equilibrium* with private claims and liquid assets consists of a mapping of state variables $(K, B, A, \Delta) \rightarrow (K_{+1}, B_+, A_+, \Delta_+)$ and equilibrium objects that are functions of the state variables: the household endowment with private claims and government bonds $\{s, b\}$, policy functions for consumption, labour, investment, asset demand from the workers, and portfolio choices $\{c^e, c^w, \ell, i, s_+, b_+\}$, the demand for factor inputs $\{k^g, \ell^g, k^f, \ell^f\}$, asset market features $\{\theta, \phi, f\}$, and a collection of prices $\{q, p_b, w, r, \kappa\}$, lump-sum taxes (or transfers) τ, such that given government policy (g, B_+) the following conditions hold:

(1) given prices, the policy functions solve the representative household's decision problem represented by (11), (12), (13), (14), and (15), and the household's constraints (6) and (8);

(2) the optimality conditions for goods and financial service producers in (16), (17), and (18) hold;

(3) given market tightness $\theta \in [0, +\infty)$ defined in (19), the probability of accommodating demand for assets and asset saleability satisfy $f = \mu(\theta)$ and $\phi = \theta\mu(\theta)$, respectively, and the price settlement condition (27) holds;

(4) the government budget constraint (22) holds, and the markets for capital and private claims, labour, and liquid assets, clear, i.e.,

$$\text{private claims /capital:}\quad s = K = k^g + k^f \tag{29}$$

$$\text{labour:}\quad (1 - \chi)\ell = \ell^g + \ell^f \tag{30}$$

$$\text{liquid assets:}\quad b_+ = B_+ \tag{31}$$

(5) the law of motion of the aggregate capital stock is consistent with the aggregation of

individuals' investment $K_{+1} = (1 - \delta) K + \chi i$.

Note that $s = K$ states the fact that there are claims on all capital, and the supply of capital K equals the demand for capital $k^g + k^f$. The supply of labour hours $(1 - \chi) \ell$ equals the demand for labour hours $\ell^g + \ell^f$.

We verify Walras' law by checking the goods market clearing condition

$$c + \chi i + \frac{\Delta}{1 + \Delta} \kappa A F^f(k^f, \ell^f) + g = A F^g(k^g, \ell^g) \tag{32}$$

where $\frac{\Delta}{1+\Delta} \kappa A F^f(k^f, \ell^f)$ is the extra cost of financial intermediation reflecting the value added of the financial sector $\frac{1}{1+\Delta} \kappa A F(k^f, \ell^f)$. Aggregate output, y, is then measured by

$$y \equiv A F^g(k^g, \ell^g) + \frac{\kappa}{1 + \Delta} A F^f(k^f, \ell^f)$$

which sums up the value-added of the consumption goods and the financial sectors. Using this definition, we can add the household's spending on financial intermediation (in order to achieve investment χi), i.e., $\kappa A F^f(k^f, \ell^f)$, on both sides of the goods market clearing condition (32), and obtain the aggregate resources constraint

$$c + \chi i + \kappa A F^f(k^f, \ell^f) + g = y \tag{33}$$

where $\kappa A F^f(k^f, \ell^f) + \chi i$ is gross investment and corresponds to "investment" in a national account.

3.3 Convenience Yield

Before closing the model, we discuss how we translate the liquidity premium into a measurable object.

The idea that government-provided assets are more liquid than private claims is the central feature of our model. As a result of this feature, agents are willing to pay a (liquidity) premium for holding government bonds, such that their price exceeds their fundamental value. The value that investors assign to the liquidity and/or safety attributes offered by government debt is also often referred to as the "convenience yield" as expounded in Krishnamurthy and Vissing-Jorgensen (2012). Similar to Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), this convenience yield arises in our model because the whole fraction of liquid assets can be used to relax agents' future financing constraints.

To map the convenience yield into an observable quantity, it is convenient to express it as a ratio. Consider a one-period bond, which is similar to government bonds (with zero supply in equilibrium), except that only a fraction ϕ is saleable. An entrepreneur owning

this asset needs to retain a fraction $(1 - \phi)$, which is returned to the household.[18] Such a bond would satisfy the Euler equation

$$1 = \mathbb{E}_\Gamma \left[\frac{\beta U_{c,+}}{U_c} \frac{1}{\tilde{p}_b} \left(1 + \chi \phi_{+1} \rho_+ \right) \right].$$ (34)

where $\chi \phi_{+1} \rho_+$ reflects the fact that only a ϕ_+ fraction of bonds can relax the financing constraint should a worker becomes an entrepreneur with probability χ. The ratio between the real return on this bond, which provides a limited liquidity service, and government bonds, which are fully liquid, is defined as the convenience yield

$$CY \equiv \frac{(\tilde{p}_b)^{-1}}{(p_b)^{-1}} = \frac{p_b}{\tilde{p}_b}.$$

To understand the relationship between the convenience yield and entrepreneurs' financing constraint, we consider the steady state. First, the asset pricing equation for government bonds (14), implies that

$$\bar{\rho} = \chi^{-1} \left(\beta^{-1} \bar{p}_b - 1 \right)$$

where $\bar{p}_b$ denotes the steady-state value of bond price. If entrepreneurs' financing constraint binds, we have $\bar{\rho} > 0$ and thus $\bar{p}_b > \beta$. From (34), since $\phi \in (0, 1)$, we know that

$$\tilde{p}_b = \frac{\chi \phi \bar{\rho} + 1}{\chi \bar{\rho} + 1} \bar{p}_b = \bar{p}_b - \frac{\chi(1 - \phi)}{\chi + \bar{\rho}^{-1}} \bar{p}_b < \bar{p}_b$$

In other words, the real interest rate on liquid assets $\bar{p}_b^{-1}$ has to be lower than the rate of time preference β^{-1} in such a constrained economy, reflecting the liquidity premium.[19] By providing a full liquidity service, government debt mitigates financing constraints better than other assets that have limited liquidity service. Therefore, government debt carries a positive convenience yield, $\bar{CY} > 1$.

In the following quantitative analysis, we find that aggregate productivity shocks and financial shocks generate different dynamics of the convenience yield.

4 Quantitative Analysis

Having set up a framework that endogenously links asset prices to asset liquidity, we confront the model with macroeconomic and financial data and assess its dynamic properties. The

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[18] In the model, this bond would arise, for instance, if it payed off after entrepreneurs have to finance investment opportunities and consumption.

[19] Similar results obtain in an economy in Cui (2016). A related paper Cui (2017) shows that having a low interest rate is optimal given distortionary taxation.

quantitative exercise will further highlight the role of financial market frictions in transmitting aggregate shocks.

4.1 Functional Forms

The production function has a standard Cobb-Douglas form

$$F(k^g, \ell^g) = (k^g)^\alpha (\ell^g)^{1-\alpha} \text{ and } F^f(k^f, \ell^f) = A^f \left(k^f\right)^\alpha \left(\ell^f\right)^{1-\alpha}$$

where α and $1 - \alpha$ refer to the capital and labour share and A^f is a scaling parameter. In addition to the baseline specification of the model with constant utilisation and depreciation rates of physical capital, we extend the model with variable capital utilisation in the following quantitative analysis, which improves its fit. The differences in the dynamic properties between the baseline and the extended version are discussed in Section 4.4. In this variant, capital is assumed to depreciate as a function of its variable utilisation rate:

$$\delta(u) = \delta_0 + \delta_1(u - \bar{u}) + 0.5\delta_2(u - \bar{u})^2$$

where $\bar{u}$ and δ_0 are the steady-state utilisation and depreciation rates of capital. In the baseline specification, $u = \bar{u}$ always. Now, there should be an optimal choice of capital utilisation, which we derive in Appendix B.1.

The matching function is specified as

$$\mu(\theta) = \xi\theta^{-\eta}$$

where η is the matching elasticity and ξ is the matching efficiency parameter. Similar to the functional forms typically used in the labour search literature, this specification implies that the number of matches exhibits constant return to scale in the ratio of purchase orders to sale orders.

The entrepreneurs' and workers' utility functions are specified, respectively, as

$$u(c) = \tilde{u}(c, 0) \text{ and } U(c, \ell) = \zeta\tilde{u}(c, \ell)$$

where ζ reflects the weight on the utility from workers relative to entrepreneurs, and $\tilde{u}$ is a common utility function. We choose to focus on the substitution effect on labour supply, because the wealth effect is already affecting portfolio choice. We assume that $\tilde{u}(c, \ell)$ is a

GHH (Greenwood, Hercowitz, and Huffman (1988)) utility function taking the form

$$\tilde{u}(c,\ell) \equiv \frac{\left(c - \frac{\mu \ell^{1+\nu}}{1+\nu}\right)^{1-\sigma} - 1}{1-\sigma}$$

where σ is relative risk-aversion parameter, ν is the labour supply elasticity, and μ is a parameter that governs the steady-state hours worked.[20]

Finally, we consider standard AR(1) processes for aggregate productivity and the efficiency loss (or costs) in financial intermediation, i.e.,

$$\log A_t = \rho_A \log A_{t-1} + (1 - \rho_A) \log \bar{A} + \epsilon_t^A, \quad 0 < \rho_A < 1$$

$$\log(1 + \Delta_t) = \rho_f \log(1 + \Delta_{t-1}) + (1 - \rho_f) \log(1 + \Delta) + \epsilon_t^f, \quad 0 < \rho_f < 1$$

with i.i.d. $\epsilon_t^A \sim N(0, \sigma_A^2)$ and i.i.d. $\epsilon_t^f \sim N\left(0, \sigma_f^2\right)$.

4.2 Parameterisation

Calibration. One period is set to a quarter. We calibrate the steady state of the model to match several characteristics of the U.S. economy in the sample period 1982Q1-2017Q2 (see Table 1).[21]

We set $\sigma = 1$ in order to limit the degree of risk aversion. We also choose $\nu = 1/1.5$, which is common in macroeconomic models as it falls in the range of Frisch labour supply elasticity estimates typically found in empirical studies.

Because of our assumption of an even distribution of household resources among its members, χ captures the share of household wealth accruing to entrepreneurs. We calibrate χ to 0.0280, to target a steady-state purchase price of private assets $q^w = 1.15$. The purchase price q^w captures Tobin's Q (excluding transaction costs), which ranges from 1.1 to 1.21 in the U.S. economy according to COMPUSTAT data. The calibration target for q^w implies a steady-state (re-)sale asset price $q = 1.05 > 1$, so entrepreneurs are indeed financing constrained.[22]

The capital share of output in the goods sector of $\alpha = 0.3062$ is set to target the

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[20]This utility function is common in models featuring variable capital utilisation and facilitates the steady state solution. The main results are robust to a more complex specification. See the discussion in the calculation of steady-state values in the Appendix.

[21]The reason for choosing 1982 as the starting point is that the years before 1982 may still be in the transition of financial liberalisation.

[22]Note that the number χ is smaller than the one used in Shi (2015)(who targets the share of firms that implement investment), but larger than the one used in Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), reflecting modelling differences in our paper. See later for a robustness check with respect to the steady-state value of χ.

Table 1: **Steady state calibration**.

Parameter	Baseline Value	Target/Source
β: Household discount factor	0.9890	Annualized convenience yield 121 basis points
σ: Relative risk aversion	1	Exogenous
ν: Labour supply elasticity	0.6700	Frisch elasticity = 1.5
μ : Utility weight on leisure	4.3519	Hours of work = 25%
χ: Mass of entrepreneurs	0.0280	$q^w = 1.15$
ζ: Entrepreneurs' utility weight	22	Government debt-to-output 64%
δ_0: Depreciation rate of capital	0.0200	Investment-to-capital ratio = 8%
α: Capital share	0.3062	Investment-to-output ratio = 18%
Δ: Efficiency loss in financial intermediation	0.0452	Hours share in financial sector = 2.11%
$\bar{u}$: Steady-state utilization rate	1	Normalisation
$\bar{A}$: Steady-state aggregate productivity	1	Normalisation
ω: Bargaining weight	0.6631	$f = \phi = 0.5$
ξ: Matching efficiency	0.5000	Total cost / intermediated assets = 2.5%
g: government expenditures	0.1154	Government spending share of output = 18%
τ: lump-sum taxes	0.1174	Price of government bonds $p_b = 1/1.005$

investment-to-output ratio, which is about 18% based on quarterly data from 1982Q1 to 2017Q2 from the US National Income and Product Account (NIPA) obtained from Bureau of Economic Analysis (BEA).

To calibrate the steady-state depreciation rate δ_0, we construct the nominal value of capital PqK in the private sector (including both non-financial and financial sectors) from the quarterly flow-of-funds data (see Appendix C for details), which corresponds to the nominal value of all claims to capital in the model. Once we have the time series of the value of capital, we can further obtain the capital value-to-output ratio, which has a sample average of 2.35 (real output measured in terms of annual rates). Since $q = 1.05$ in the steady state , and we thus set the capital-to-output ratio as 2.24. Using both the capital-to-output and investment-to-output ratios, we pin down the annual depreciation rate in the steady-state, which is about 8%, implying $\delta_0 = 0.02$ at quarterly frequency.

To calibrate the efficiency loss in financial intermediation Δ, we target the size of the financial sector relative to the total economy in terms of hours worked. There is no quarterly data of hours worked in financial industry, and we obtain yearly total hours worked (including full-time and part-time hours) and the hours worked in the financial sector (excluding hours in the real estate sector) from BEA (see Appendix C for details). Since 1982, the starting point of our sample, the hour share of the financial sector has been broadly stable at about 2.11% of total hours worked (Figure 1). While this share may seem low, it is again due to the exclusion of hours in the real estate sector. To be specific, the hour share of the combined "Finance, Insurance, and Real Estate" sector is close to 8% in recent years. Note that a higher hour share would make liquidity frictions and financial intermediation quantitatively more important, such that the relatively low hour share is a useful restriction to discipline

Figure 1: **The share of hours worked in the financial sector relative to total hours.**
(1982-2016)
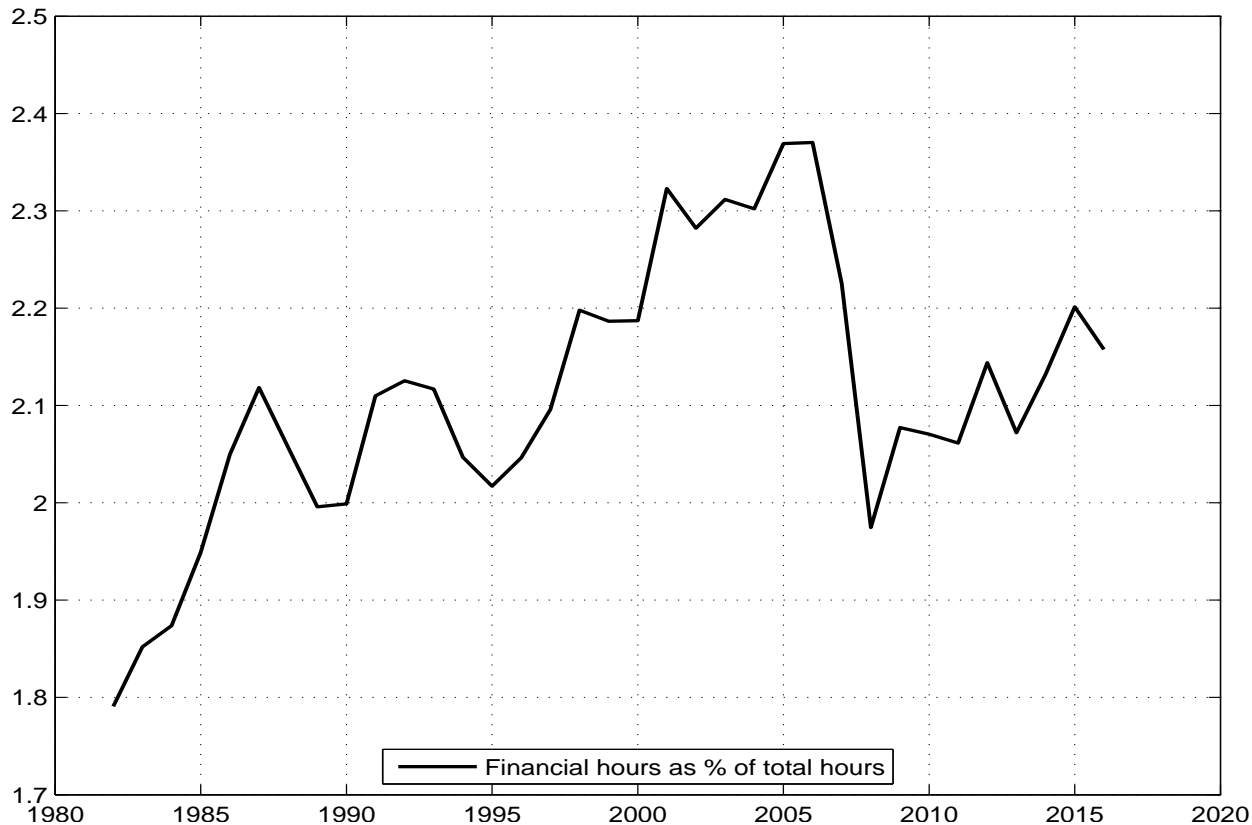
our model.

The parameters $\{\xi, \omega, \eta\}$ govern the matching function and bargaining on the asset market. We choose two targets for these two parameters. First, we tie our hand by setting $\bar{\phi} = \bar{f}$, so that $\theta = \phi/f = 1$ in the steady state. Second, we target the steady-state asset liquidity $\bar{\phi}$. Since $\phi = \xi\theta^{1-\eta}$, we immediately know that $\xi = \bar{\phi}$ and η does not affect the steady state (we will estimate η, since it is important for the model's dynamics). We choose $\phi = 0.50$ so that both buy and sell orders on average take two quarters to be processed. There is one more important reason for this choice. Philippon (2015) finds that the ratio of financial income to total intermediated assets (including privately-issued assets and liquid assets) is 1.5-2%. In our model, $\phi = 0.50$ implies 2.5% for this ratio, which is remarkably close to the estimate by Philippon (2015) given that our model does not feature bank deposits (which

are part of intermediated assets in the data).[23][24] Notice that $\phi = 0.50$ is higher than what previous studies find about turn-over rate of assets in flow-of-funds.[25] This reflects the modeling choice of capturing primary and secondary market together to simplify the algebra. In addition, this calibration again stacks the odds against the quantitative importance of liquidity frictions in our setup. Finally, the restriction $\bar{\phi} = \bar{f}$ enables us to calibrate the bargaining weight ω.

We set the ratio of government spending to total output to 18% as in the data. We also set the inverse of the price of government bonds to $1/p_b = 1.005$ to pin down lump-sum taxes. This implies an annual return of 2%, well in line with the 1.72% return of US government liabilities with one year residual maturity and about 2.2% for 20 year maturity (see Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017).

In addition, government debt in the flow-of funds data corresponds to all liabilities of the Federal Government, that is, Treasury securities net of holdings by the central bank and the budget agency plus reserves, vault cash and currency net of remittances to the Federal government (again following Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017)). This is about 64% of total output since 1982. ζ indicates the importance of entrepreneurs in the household. It determines their consumption (and investment) and thus the need to carry liquid assets. We calibrate ζ such that the ratio of the real value of liquid assets to output, B/Y, is 64% in the steady state.

We set $\beta = 0.989$ by targeting the convenience yield. We follow Krishnamurthy and Vissing-Jorgensen (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) to measure this yield by the difference in annualised yields between 20 year Moody Aaa-rated corporate bonds and 20 year Treasury bonds. As argued by Krishnamurthy and Vissing-Jorgensen (2012), Aaa bonds have very little default risks, such that the yield spread with Treasuries of similar maturity almost entirely reflects liquidity risks. Note that the data

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[23]The total financial income in the model is $\frac{\kappa}{1+\Delta}AF^f(k^f, \ell^f)$, and the total volume of intermediated assets is $\phi\chi[i + (1-\delta)K] + B$. Note that we include liquid assets as part of the intermediated assets to match the data. This is consistent with Philippon (2015), who includes all liquid assets in his calculation. While we assume for ease of exposition that liquid assets do not need to be intermediated, one could equivalently assume that they are intermediated at close to zero costs.

[24]The implied spread for private claims $(q^w - q)/q^w$ is about 7%. This corresponds broadly to the fees incurred in initial public offerings on the primary market (see, e.g., Chen and Ritter (2007-2009)). For merger and acquisition (M&A) fees, there is no published record. But there is a famous "Double Lehman" compensation structure designed by M&A specialists. Double Lehman is a variation on the Lehman Formula to bridge the gap between the small (less than $1 million) and large (greater than $100 million) deals. Under Double Lehman, the M&A specialist fee is structured as follows: 10% of the first million dollars involved in the transaction; 8% of the second million; 6% of the third million; 4% of the fourth million; 2% of everything thereafter.

[25]Nezafat and Slavik (2010) use the US flow-of-funds data for non-financial firms to estimate the stochastic process of ϕ. The long-run average is close to 0.30, which would imply a stronger degree of financial frictions than in our calibration.

on Aaa bonds has the advantage of being widely available and spanning our whole sample period. The historical yield spreads between these two assets are plotted in 2. The sample mean is 121 basis points (annualised).[26]

Finally, the steady state is affected by the scaling parameter for financial sector productivity $A^f = 40.4758$ and the first-order sensitivity of depreciation rate $\delta_1 = 0.0310$. They are however *not* free parameters under the targets we impose. We show how to determine the two parameters in Appendix B.

Estimation. Given the dynamic implications of the model following productivity and intermediation cost shocks, the two key series for its estimation are the cyclical components of the convenience yield and output. We de-trend output and the convenience yield using the one-sided HP filter (with a smoothing coefficient of 1600 as typically used for quarterly data) over our sample period. The convenience yield correlates negatively with output (-0.47) as suggested by its counter-cyclical movement in Figure (2) reflecting portfolio rebalancing towards liquid assets.

Since the model has implication for asset price and hours worked in the financial sector, we add the cyclical components of the capital value and financial hours worked as observations along with two observational errors in order to avoid stochastic singularity issues. Observational errors capture unexplained variation in the data. Specifically, denote

$$q_t K_t - (qK)_t^{observed} = \epsilon_t^q$$

$$\ell_{t-3}^f + \ell_{t-2}^f + \ell_{t-1}^f + \ell_t^f - (\ell^f)_t^{observed} = \epsilon_t^\ell \text{ if } t \text{ corresponds to a 4th quarter}$$

where ϵ_t^q and ϵ_t^ℓ are i.i.d. normal random variables with zero mean and variance σ_q^2 and σ_ℓ^2, respectively. The specification for hours worked in the financial sector is due to the fact that they are observed at annual frequency.

We use the same filter as before to de-trend capital value observed at quarterly frequency. We use the one-sided HP filter with a smoothing coefficient of 6.75 (used for yearly data) to de-trend financial hours worked. Both the capital value and financial hours tend to fall in recessions. The capital value correlates positively with output (0.61).

We log-linearize the model around the deterministic steady state and express it as a Kalman filtering problem. Note that missing observations (financial hours) do not change the standard procedure of the filtering problem. Using maximum likelihood methods, we estimate the parameters associated with the two structural shock processes Since the second-order term of the depreciation function δ_2 and matching sensitivity η cannot be determined

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[26]The data is reported as constant maturity yield with 6 months payments. We transform the data into annualised yields.

Figure 2: **Cyclical components of the convenience yield and asset prices against real GDP.** All series are cyclical components of one-sided HP filtered original series (multiplied by 100). The shaded areas are NBER dated recessions.
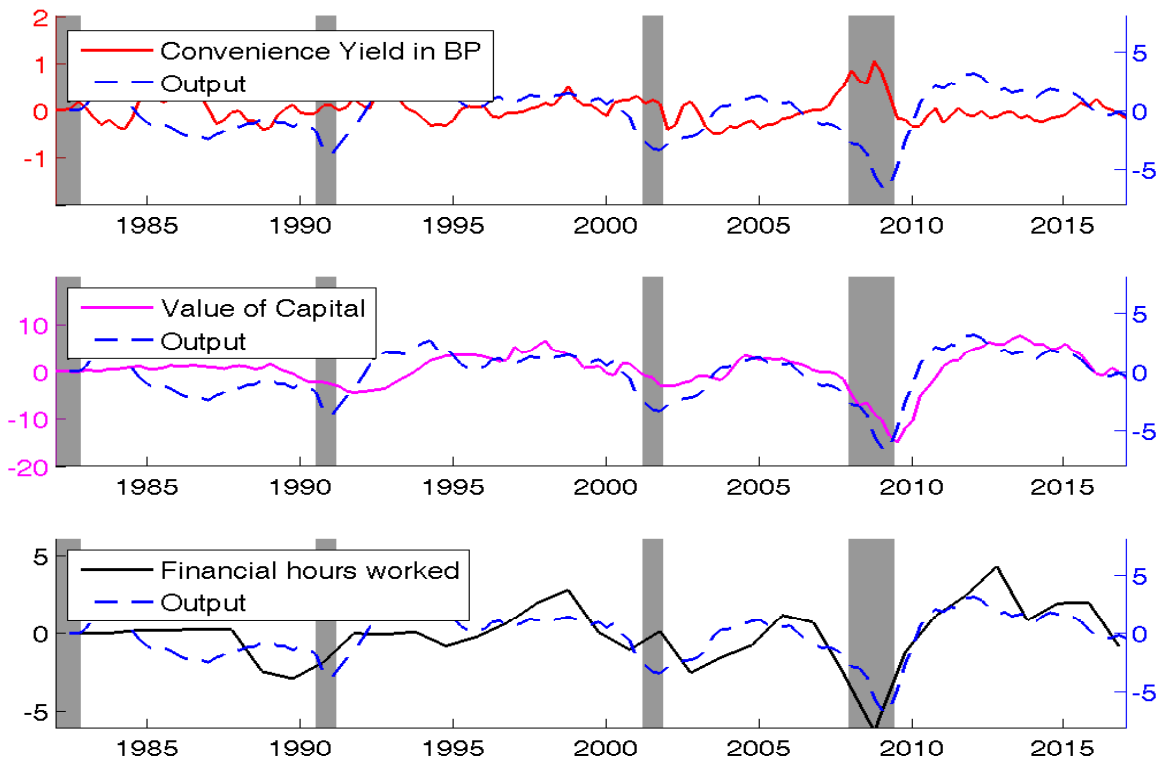
in the deterministic steady state, we also estimate δ_2 and η.

The estimated productivity process is slightly more persistent than the intermediation cost process ($\rho_A > \rho_f$). The value η is close to 1, so that $\phi'(\theta) = (1 - \eta)\xi\theta^{-\eta}$ is small and condition (28) is easily satisfied. That is, asset price is likely to fall with a falling asset liquidity ϕ.

4.3   Dynamic and Empirical Properties

In this section, we discuss the model's dynamics after TFP shocks and shocks to financial intermediation efficiency - or intermediation cost shocks (when parameters are set at their maximum likelihood estimates). Importantly, we find that these two types of shocks generate opposite convenience yields dynamics; TFP shocks also generate less volatile asset price movements than intermediation cost shocks.

Negative TFP Shocks. Suppose an adverse aggregate productivity shock hits the economy at time 0 . This shock depresses the marginal product of capital and its value to the

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Table 2: **Estimated parameters**.

	Parameter	Estimated value	Standard errors
Persistence of TFP shocks	ρ_A	0.9207	0.0282
Persistence of intermediation shocks	ρ_f	0.8585	0.0361
St.d. of TFP shocks	σ_A	0.0024	0.0001
St.d. of intermediation shocks	σ_f	0.4607	0.0321
St.d. of observational errors on capital value	σ_q	0.0278	0.0017
St.d. of observational errors on labour hours	σ_ℓ	0.0150	0.0011
Sensitivity of utilization	δ_2	0.0030	0.0005
Sensitivity of matching	η	0.9200	0.0523

household. Search for investment becomes less attractive and the amount of purchase orders from workers for private financial claims drops. The demand-driven fall is reflected in the endogenous drop in asset market tightness θ and asset liquidity ϕ, which amplifies the initial shock in two ways: (1) it reduces the quantity of assets that entrepreneurs are able to sell; (2) the asset price falls. Both effects tighten entrepreneurs' financing constraints. As a result, investment falls; consumption also falls because fewer resources are produced at the lower level of aggregate productivity.

In principle, government debt's liquidity service becomes more valuable to households when private claims' liquidity declines. However, in the case of a persistent TFP shock, lower expected returns to capital make *future* investment less attractive. This effect weakens the incentive to hedge against asset illiquidity for future investment. The former effect has a positive impact on the convenience yields, while the latter has a negative impact.

In our calibration, the decline in the profitability of investment projects is sufficient to reduce the convenience yield. Therefore, the demand for liquid assets falls, which is reflected in the falling price p_b (recall that government bond supply is inelastic, such that change in demand fully feeds through to the price). To the extent that productivity reverts back to the steady state, while asset liquidity is still subdued, hedging becomes more attractive which explains the relatively fast recovery of the convenience yield.

Note that the fall in asset liquidity ϕ following TFP shocks is much less pronounced than in the case of intermediation shocks (discussed below). Although negative TFP shocks reduce asset liquidity ϕ, the search-and-matching process with Nash bargaining mitigates this effect of productivity shocks as their impact is mostly absorbed by the asset price (see a similar discussion in the context of labour markets in Shimer (2005)).

Negative Financial Shocks. Rather than affecting the production frontier of the economy, shocks to intermediation efficiency only impair the capacity of the financial sector to inter-mediate funds between workers and entrepreneurs. Suppose a rise of intermediation cost hits

Figure 3: **Impulse responses after a one standard deviation shock to TFP or intermediation costs at time 0**. "BP" stands for basis points. "ss" stands for the steady state value.
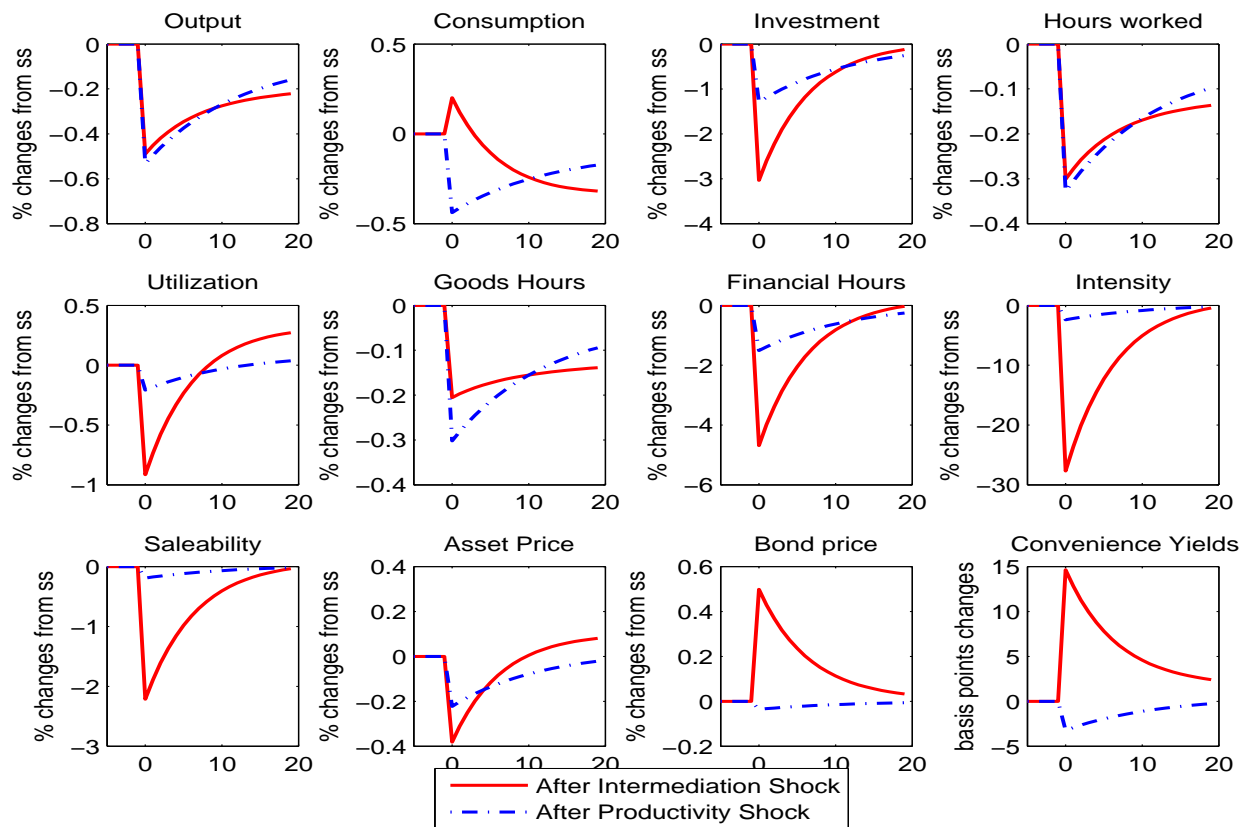
the economy at time 0. Since participation in financial markets is more costly now and later, households seek to reduce their exposure to private claims, such that demand on the asset market falls. On the supply side, financing-constrained entrepreneurs would still like to sell as many assets as possible in order to take full advantage of profitable investment opportunities. Therefore, asset demand on the search market shrinks relative to supply(reflected in the sharp decline in asset market tightness/intensity θ), reducing the likelihood for assets on sale to be matched with buy orders. As a result, asset liquidity falls sharply.

The sharp drop in asset liquidity tightens entrepreneurs' financing constraints substantially. Investment falls, and the marginal product of capital rises, putting upward pressure on the asset price. But the demand effect dominates (again reflected in asset market tightness/intensity θ), such that the re-sale value of capital q_t falls strongly and amplifies the initial shock by depressing entrepreneurs' net worth further. This effect is mirrored in a significant decline of investment activity, the impact response of which is about 6 times stronger than that of output.

As saving via the financial market becomes more expensive with higher intermediation costs, workers reduce their labour supply and consume slightly more after the initial shock. Entrepreneurs, on the other hand, have to cut back on consumption significantly in view of their tightly binding financing constraints. Given the small population share of entrepreneurs, aggregate consumption increases slightly initially, while output falls on impact because of the drop of labour hours and the falling utilisation of capital. However, lower investment into the capital stock soon reduces the marginal product of labour and the wage rate. As labour income of workers falls, consumption persistently drops below the steady state. As a result, output is also persistently compressed.

While the intermediation efficiency shock depresses the demand for and the liquidity of private assets, it substantially increases demand for liquid assets. Future investment remains profitable since the productivity of capital is not affected by the shock. To take advantage of future investment opportunities, households seek to hedge against the persistent illiquidity of private claims by rebalancing their asset holdings towards liquid government bonds. This is reflected in the increasing bond price and convenience yield.

The accumulation of government bonds relaxes future financing constraints: on the one hand, entrepreneurs can finance more out of their stock of liquid assets; on the other, buyers have more liquid resources to purchase private claims. Both effects improve liquidity conditions on the private asset market going forward. As a result, both the asset price and asset liquidity overshoot above the steady-state levels after about 3 years. Together, the sharp decline of the asset price on impact and its subsequent overshooting enable intermediation shocks to generate more asset price volatility than productivity shocks.

Variance Decomposition. Intermediation costs successfully generates counter-cyclical convenience yields, mimicking the deterioration of private assets' liquidity relative to publicly issued liquid assets typically observed in recessions. As a result, the convenience yield can serve as a discriminant between the sources of recessions. In addition, they generate more volatile asset prices than TFP shocks as discussed before. Based on these dynamic properties, we estimate the shock series shown in Figure 4.

Through the lens of our model, the 2007-09 recession stands out as being driven by a combination of exceptionally large intermediation costs and fall of productivity. The sharp fall in intermediation cost shocks in 2009-10 may be related to the asset-purchase programmes implemented by the Federal Reserve in 2009-10. These programmes replaced illiquid private assets, such as mortgage-backed securities, with highly liquid central bank reserves in the hands of banks and private investors, thereby preventing the intermediation capacity of financial markets from collapsing further (see Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017)). Intermediation cost shocks have also contributed considerably to the

Figure 4: **Estimated shocks.** The figure shows the means of the estimated shocks (ε_t^f and ε_t^A). Both shocks are normalised by their respective standard deviations. Shaded areas indicate NBER-dated recessions.
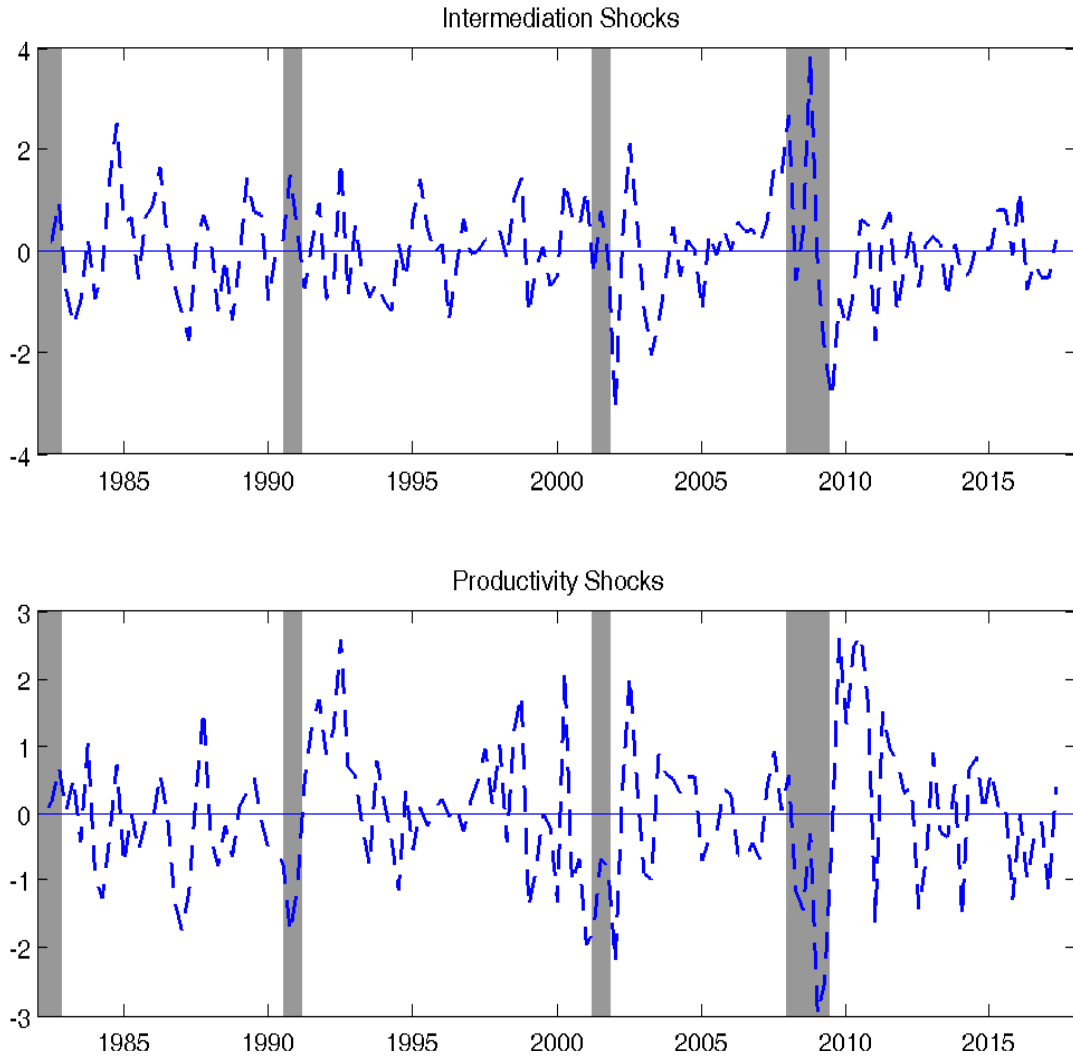
1990-91 recession and to the economic boom in the wake of the bursting of the dotcom bubble in the early 2000s, possibly reflecting the contribution of the financial sector to the emerging housing market boom. Table 3 summarises the share variation in selected macroeconomic and financial variables that can be separately explained by the two shocks during our sample period. As anticipated based on the impulse responses, the dynamics of the convenience yield and investment are mainly explained by intermediation shocks; productivity shocks still explains around 32% of output fluctuations and around 20.5% of the variation in consumption.

The large explanatory power of intermediation shocks for macroeconomic variables owes, to a substantial degree, to variable capital utilisation as discussed below. Specifically, intermediation shocks affect the cost of capital and change utilisation significantly. This effect absorbs the contribution of productivity shocks in a standard real business cycle model with productivity shocks as the only exogenous disturbance.

This notwithstanding, the observational error accounts for around 26% of the variation in the capital value, suggesting that our model omits some important drivers of asset prices, such as risk and uncertainty in goods production or financial intermediation.

Table 3: **Variance Decomposition.** OE stands for observational errors. qK is the value of capital. ℓ^f is financial hours worked. Contributions are shown in percent.

|	Productivity	Intermediation	OE on qK	OE on ℓ^f
Output	32.32	67.68	0	0
Consumption	20.52	79.48	0	0
Investment	23.91	76.09	0	0
Financial hours	14.83	74.72	0	10.45
Convenience yield	5.34	94.66	0	0
Capital value	7.74	76.19	26.07	0

4.4 Discussion

Against the backdrop of these results, we perform a number of robustness checks.

Hedging Value of Liquid Assets

While rising intermediation costs increase the hedging value of government bonds and the convenience yield as shown in Section 4.2, TFP shocks may have an ambiguous effect on the incentive to hold government bonds. There are two effects from TFP shocks. Persistently low productivity diminishes the return on capital and financial claims, such that investing in private claims becomes less profitable and the willingness to hedge idiosyncratic investment risks shrinks. At the same time, low productivity depresses entrepreneurs' net worth, such that financing constraints become more binding. This effect should raise the hedging motive and the willingness to hold money. In the baseline experiment in Figure 3, the first effect dominates the second effect, such that the convenience yield contracts strongly.

We will illustrate in the following that the above result is robust to a wide range of parameters by assessing the sensitivity of the convenience yield and the Lagrangian multiplier $\chi \rho U_c$ associated with the entrepeneurs' financing constraint (6), which measures the importance of financing constraints from the perspective of households. In particular, we perform

Figure 5: **Robustness check 1 (persistence of TFP shocks).** Different impulse response functions after the same innovation in TFP, but with different persistence ρ_A. "BP" stands for basis points. "ss" stands for the steady state value.
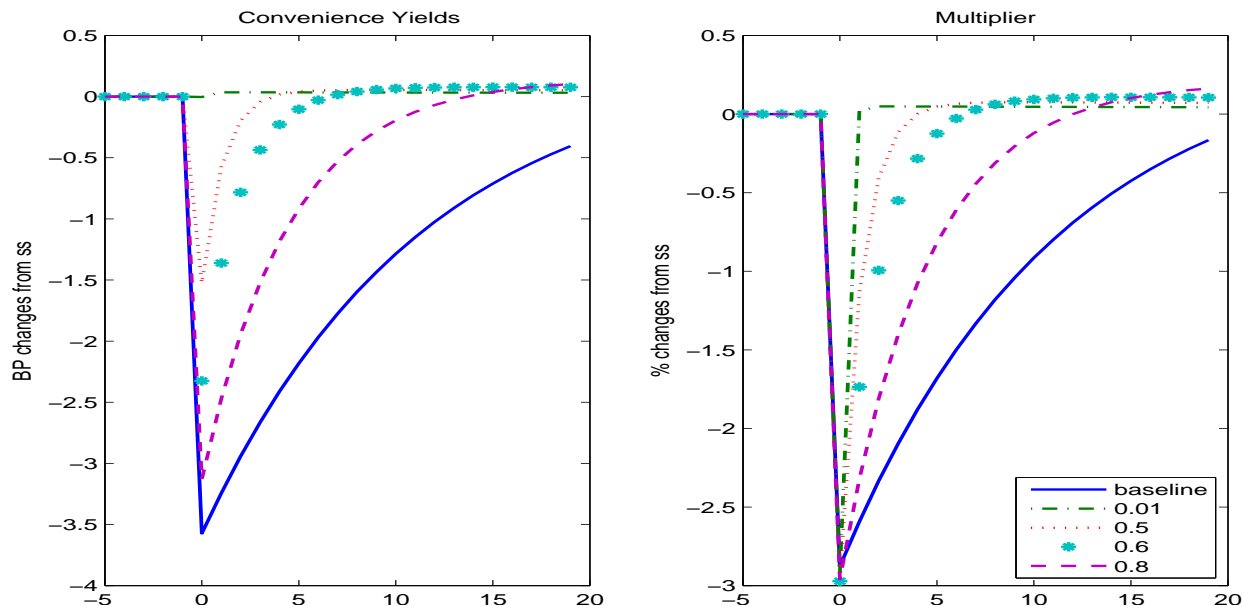

robustness analysis with respect to the persistence of TFP shocks and the wealth share of entrepreneurs χ. Changing the persistence affects the duration of the low productivity spell, which may impact the relative importance of both effects. Varying χ affects the tightness of financing constraints directly.

Figure 5 demonstrates that the convenience yield falls after all but very short-lived TFP shocks, while the persistence of this fall is clearly a function of the persistence of the fall in productivity. With highly persistent TFP shocks the demand for investment will be restored slowly, such that the convenience yield falls more and stays below the steady state level for longer than with less persistent shocks. Once demand for investment reverts back to normal, both the multiplier on the financing constraint and the convenience yield overshoot. This is because the more persistent TFP shocks are, the less wealth the household accumulates during the recovery phase from the negative productivity shock. As a result, financing constraints are more acute once the demand for investment normalises.

As we reduce the mass of entrepreneurs χ, fewer resources are allocated to entrepreneurs with investment opportunities. This implies that financing constraints bind more acutely with falling productivity prima facie increasing the hedging value of government bonds. However, both the convenience yield and the multiplier fall with productivity shocks under all values for χ. This again implies that it is quite implausible for adverse TFP shocks to generate a higher convenience yield.

Figure 6: **Robustness check 2 (lower χ).** Different impulse response functions after the same innovation in TFP, but with different entrepreneur wealth shares. "BP" stands for basis points. "ss" stands for the steady state value.
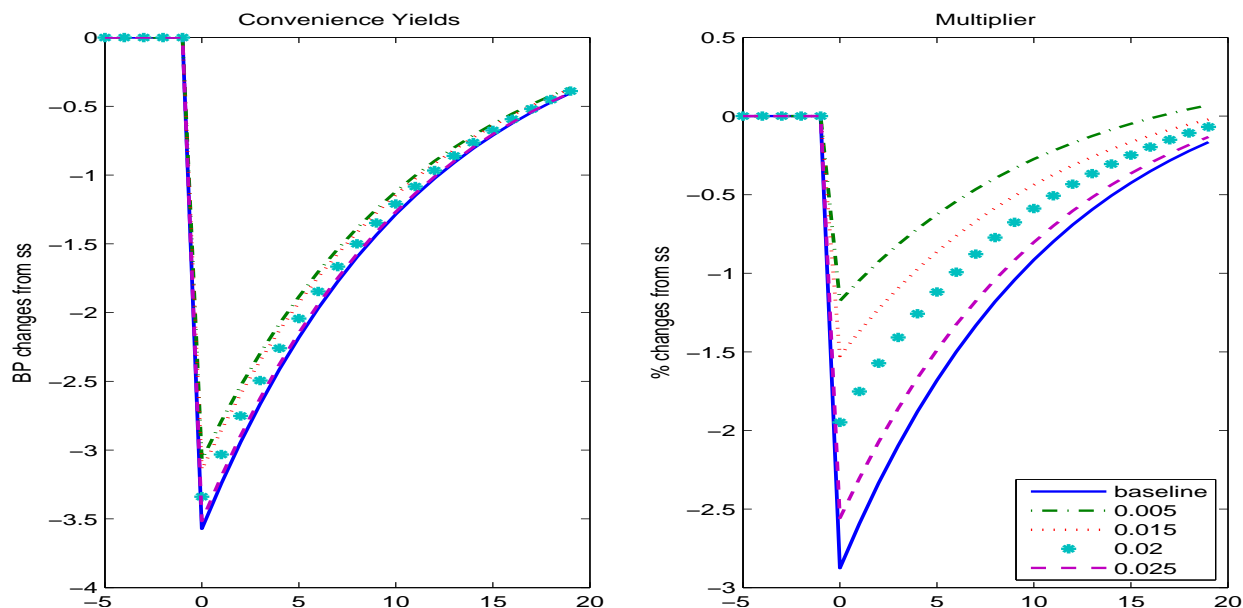
To summarise, we find that the impact of TFP shocks on the return to capital - and hence demand for claims issued against capital - dominates that on entrepreneurs' net worth and the tightness of their financing constraints. In contrast to financial shocks, it is, therefore, difficult for adverse TFP shocks to push up the convenience yield and the tightness of financing constraints.[27]
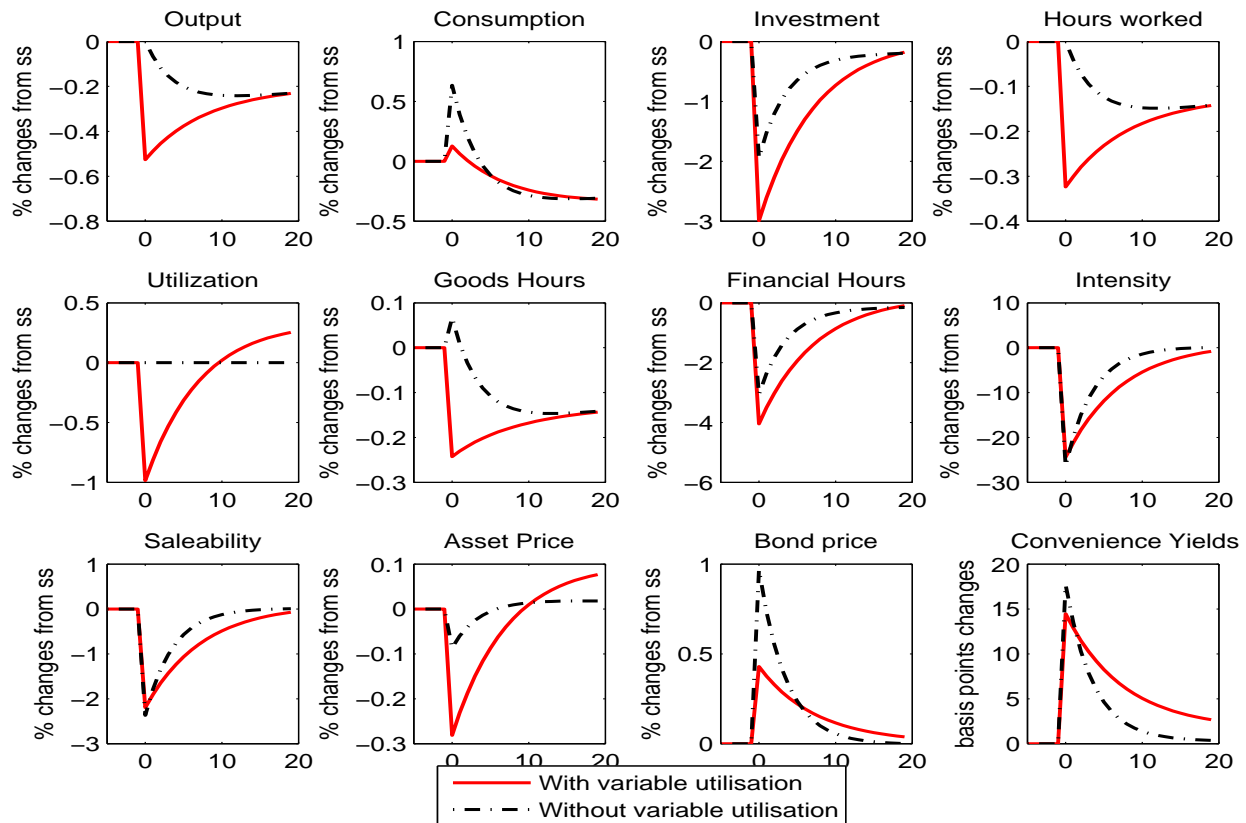
Variable Capital Utilisation

To illustrate the contribution of variable capital utilisation to the dynamic results, we re-estimate the model by fixing the utilisation rate at its steady-state level $u = \bar{u}$ and fixing η as before. We then plot the impulse responses with parameters set at the estimated level ($\rho_A = 0.9237$, $\rho_f = 0.7634$, $\sigma_A = 0.0033$, and $\sigma_f = 0.5341$).

We clearly notice that fixing utilisation dampens the effect of financial shocks on output and the asset price. Since investment falls strongly also with fixed capital utilisation, consumption has to be above the steady state to compensate the weaker contraction of out-

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[27]From this discussion it is clear that the result would change if productivity shocks only affected the net worth of constrained agents (entrepreneurs in our model), but had no effect on the demand for investment. For instance, in the seminal Kiyotaki and Moore (1997) model, financing constraints are tighter if a fall in productivity only affects constrained agents ("farmers" in their model), but not unconstrained agents ("gatherers" in their model). But this immediately raises the question why only constrained agents should be subject to productivity shocks.

Figure 7: **Impulse responses after a standard deviation shock to intermediation costs at time 0**. No variable capital utilisation versus variable capital utilisation.

put. This contrasts with the result under variable utilisation where capital financing becomes more costly after financial shocks and the household decides to utilise capital less intensively. With variable capital utilisation, Figure 7 shows that rising intermediation costs generate a larger output contraction and consumption is smooth and mostly below the steady-state level.

With fixed utilisation rates, the explanatory power of intermediation shocks for real macroeconomic variables also shrinks (Table 4), while the convenience yield is still mostly explained by intermediation cost shocks. For example, with fixed capital utilisation about 45.6% of output fluctuations are explained by financial shocks compared to about 68% with variable capital utilisation, while variation in the convenience yield is predominantly explained by financial shocks in either case (91.47% without and 94.66% with variable capital utilisation).

How can this difference be explained? When financing constraints are tightened by financial shocks, capital becomes expensive to finance. Given the option, it is then optimal to utilise capital less intensely in order to preserve productive capital stock for the future. In view of this interaction between financing constraints and variable capital utilisation, fi-

Table 4: **Variance Decomposition.** OE stands for observational errors. qK is the value of capital. ℓ^f is financial hours worked. Contributions are shown in percent.

	Productivity	Intermediation	OE on qK	OE on ℓ^f
Output	54.37	45.63	0	0
Consumption	42.00	58.00	0	0
Investment	51.27	48.73	0	0
Financial hours	33.43	56.24	0	10.23
Convenience yield	8.53	91.47	0	0
Capital value	20.74	41.71	37.56	0

nancial shocks in our framework are similar to investment-specific technology shocks (e.g., Greenwood, Hercowitz, and Huffman, 1988; Fisher, 2006; Primiceri, Justiniano, and Tambalotti, 2010), whose impact is amplified by their effect on endogenous asset liquidity and prices.[28]

5   Conclusion

We endogenise asset liquidity in a macroeconomic model with search frictions. Assets are claims backed by physical capital. Endogenous variation in asset liquidity is triggered by shocks that affect asset demand and supply either directly (intermediation cost shocks), or indirectly (productivity shocks). By tightening entrepreneurs' financing constraints, these shocks feed into real activity. Agents hedge against endogenous financing constraints arising from illiquid assets by holding liquid government bonds. These portfolio choices hark back to Keynes' speculative motive for cash balances (Keynes, 1936) and Tobin's theory of risk-based liquidity preferences (Tobin, 1958, 1969).

We show that asset prices can positively co-move with asset saleability. The *endogenous* nature of asset liquidity is key to match this positive correlation, as adverse *exogenous* liquidity shocks that tighten financing constraints would induce asset price booms in recessions.

The liquidity service provided by government bonds is reflected in a liquidity premium. This premium rises as financing constraints bind more tightly. Shocks to the cost of financial

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[28]Importantly, this interpretation confirms Shi (2015)'s discussion on potential solutions to generate positive co-movement between asset prices and liquidity. In particular, financial shocks need to trigger a sufficient reduction in the need for investment (the demand effect). In this regard, the intermediation cost shock in our model acts as a negative shock to effective productivity (reducing the need for investment) as well as to liquidity and financing constraints. Although Ajello (2016) also find that rising intermediation costs can generate falling asset prices, our contribution is to show that asset liquidity ϕ can also fall with the asset price q, which is not feasible in Ajello (2016).

intermediation, therefore, increase the hedging value of liquid assets, enabling our model to replicate the pro-cyclical nature of the liquidity premium during recent U.S. business cycles.

For future research, it may be fruitful to link risks/uncertainty to asset liquidity along the line of Lagos (2010), while maintaining the link between financing constraints and asset liquidity. Doing so will deepen our understanding of the links and differences between liquidity and safety and their impact on aggregate dynamics. In a recent paper, Del Negro, Giannone, Giannoni, and Tambalotti (2017) find that both liquidity and safety premia associated with U.S. government debt have increased in recent years.

Regarding government interventions, our framework suggests that, as in KM, open market operations in the form of asset purchase programs can have real effects by easing liquidity frictions. However, government demand may crowd out private demand due to congestion externalities in an endogenous liquidity framework. Therefore, future research could focus on the optimal design of conventional and unconventional monetary as well as fiscal policy measures in the presence of illiquid asset markets.

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Appendices

A Proofs

This section contains all the proofs to the propositions and claims in the main text.

A.1 Proof to Proposition 1

In steady state, $\rho = \rho^* = \chi^{-1}\left[\beta p_b^{-1} - 1\right] > 0$ is a constant according to the Euler equation for bonds (14). From the optimal condition for investment (13), we know that

$$h(\phi, q) \equiv q - 1 + \kappa(1-\phi)\left(\frac{1}{\phi} + \frac{1}{f}\right) - \rho(1 - \phi q) = 0$$

Now, we use the implicit function theorem:

$$\frac{\partial q}{\partial \phi} = -\frac{\partial h/\partial \phi}{\partial h/\partial q} = \frac{\left(\frac{\kappa}{\phi} + \frac{\kappa}{f}\right) + (1-\phi)\frac{\kappa}{\phi^2} - \rho q}{(1-\phi) + \rho\phi}$$

As $(1-\phi) + \rho\phi > 0$, $\partial q/\partial\phi > 0$ iff the numerator is positive. Using the original $h(\phi, q) = 0$ to express $q = 1 + \frac{(1-\phi)\left[\rho - \kappa\left(\frac{1}{f} + \frac{1}{\phi}\right)\right]}{1 + \phi\rho}$, we find (after some simplification) that $\partial q/\partial\phi > 0$ is equivalent to

$$\left[\rho + \left(\rho - \frac{\kappa}{f}\right)\left(\frac{1}{\rho} + 1\right)\right]\phi^2 - 2\kappa\phi - \kappa\rho^{-1} < 0 \tag{35}$$

Notice that the coefficient in front of ϕ^2 is positive if entrepreneurs are financing constrained. To see this, from the condition $q = 1 + \frac{(1-\phi)\left[\rho - \kappa\left(\frac{1}{f} + \frac{1}{\phi}\right)\right]}{1 + \phi\rho} > 1$, the following is true

$$\rho - \frac{\kappa}{f} > \frac{\kappa}{\phi} > 0$$

and therefore $\rho + \left(\rho - \frac{\kappa}{f}\right)\left(\frac{1}{\rho} + 1\right) > 0$. Therefore, (35) is equivalent to

$$0 \le \phi < \min\{\phi^*, 1\}$$

where

$$\phi^* = \frac{\kappa + \sqrt{\kappa^2 + \kappa\left(\rho^*\right)^{-1}\left[\rho^* + \left(\rho^* - \frac{\kappa}{f}\right)\left(\frac{1}{\rho^*} + 1\right)\right]}}{\rho^* + \left(\rho^* - \frac{\kappa}{f}\right)\left(\frac{1}{\rho^*} + 1\right)}$$

as $\phi \in [0, 1)$. $\square$

A.2 Proof to the Claim about Marginal Surplus

The claim states that an entrepreneur spends all the additional resources on investment. We formally show this claim here.

Recall that the value of an additional ϵ units of claims yields the value to the entrepreneur as

$$\hat{v}^e(\tilde{q}^f, \epsilon) = u\left(c^e\right) + \beta\mathbb{E}_\Gamma\left[v(s_+ + i_\epsilon, b_+; \Gamma_+)\right] \text{ s.t.}$$

$$c^e + (1 - \phi q)(i + i_\epsilon) = rs + (1-\delta)s + \frac{b}{P} + (\tilde{q}^f - 1)\epsilon$$

where i_ϵ is the extra investment implemented after obtaining the additional resources from selling the ϵ units of claims. Notice that the entrepreneur again can issue ϕ fraction i_ϵ at the equilibrium price q.

We have already obtained the surplus when the entrepreneur spends all resources on investment. That is, specifying $i_\epsilon = (\tilde{q}^f - 1)\epsilon/(1 - \phi q)$, differentiating $\hat{v}^e(\tilde{q}^f, \epsilon)$ w.r.t. to ϵ, and evaluating the derivative at $\epsilon = 0$, we obtain the entrepreneur's surplus of an additional unit of successful transactions

$$v_s^e(\tilde{q}^f) = \frac{\tilde{q}^f - 1}{1 - \phi q}\beta\mathbb{E}_\Gamma\left[v_s(s_+, b_+; \Gamma_+)\right]$$

If the entrepreneur, however, spends all the resource on consumption, we should set $i_\epsilon = 0$ and have instead

$$u_c\left(\tilde{q}^f - 1\right)$$

We will prove that

$$u_c\left(\tilde{q}^f - 1\right) < \frac{\tilde{q}^f - 1}{1 - \phi q}\beta\mathbb{E}_\Gamma\left[v_s(s_+, b_+; \Gamma_+)\right]$$

so that the entrepreneur does not want to consume the extra resources. The relationship is equivalent to

$$(1 - \phi q)u_c < \beta\mathbb{E}_\Gamma\left[v_s(s_+, b_+; \Gamma_+)\right]$$

Using the envelop condition of the household problem, we have $\beta\mathbb{E}_\Gamma\left[v_s(s_+, b_+; \Gamma_+)\right] = U_c q^w$. Then, the above relationship is equivalent to

$$\frac{(1 - \phi q)u_c}{U_c} < q^w$$

which is true. To see this, by using the first order conditions **??** and 13, we know

$$\frac{(1 - \phi q)u_c}{U_c} = (1 - \phi q)(1 + \rho) = (1 - \phi q)\frac{(1 - \phi)q^w}{1 - \phi q} = (1 - \phi)q^w < q^w \quad \square$$

A.3 Proof to Proposition 2

The bargaining solution simplifies to

$$\frac{1 - \omega}{q^w - q^f} = \frac{\omega}{q^f - 1}$$

after we impose $\tilde{q}^f = q^f$. Therefore, $q^f = (1 - \omega) + \omega q^w$. Together with the zero profit conditions $q^f = q^w - \kappa/f = q + \kappa/\phi$, we obtain

$$q = 1 + \kappa\left(\frac{\omega}{1 - \omega}\frac{1}{f} - \frac{1}{\phi}\right)$$

as in the main text. Now, let us substitute out κ. From (13), we know that

$$q = 1 + \frac{(1 - \phi)\left[\rho - \kappa\left(\frac{1}{f} + \frac{1}{\phi}\right)\right]}{1 + \phi\rho}$$

The above two equations can solve κ, and we express asset price q as a function of ρ, ϕ, and f only:

$$q = 1 + \frac{\rho}{\frac{1 + \phi\rho}{1 - \phi} + \frac{\phi + f}{\tilde{\omega}\phi - f}}$$

where $\tilde{\omega} = \frac{\omega}{1 - \omega}$. Again, in steady state, $\rho = \rho^*$. Therefore, if and only if $\partial\{\frac{1 + \phi\rho}{1 - \phi} + \frac{\phi + f}{\tilde{\omega}\phi - f}\}/\partial\theta < 0$, $\partial q/\partial\theta > 0$ and $\partial q/\partial\phi > 0$ (because $\partial\phi/\partial\theta > 0$). Notice that

$$\frac{\partial\{\frac{1 + \phi\rho}{1 - \phi} + \frac{\phi + f}{\tilde{\omega}\phi - f}\}}{\partial\theta} = \frac{(\rho + 1)\left[\mu(\theta) + \theta\mu'(\theta)\right]}{\left[1 - \theta\mu(\theta)\right]^2} - \frac{\mu(\theta)}{(1 - \omega)(\tilde{\omega}\theta - 1)}$$

after using $f = f(\theta) = \mu(\theta)$ and $\phi(\theta) = \theta\mu\theta$. We thus know that $\partial q/\partial\phi > 0$ is equivalent to

$$\rho + 1 < \frac{\mu(\theta)\left[1 - \theta\mu(\theta)\right]^2}{(1 - \omega)\,(\tilde{\omega}\theta - 1)\left[\mu(\theta) + \theta\mu'(\theta)\right]} = \frac{\mu(\theta)\left[1 - \theta\mu(\theta)\right]^2}{\left[\omega\theta - (1 - \omega)\right]\left[\mu(\theta) + \theta\mu'(\theta)\right]} \qquad \square$$

B Equilibrium Conditions

This section contains the equilibrium conditions we use in the quantitative exercise.

B.1 Optimal Variable Capital Utilisation

With variable utilisation, the household's constraints are modified as

$$c^e + (1 - \phi q)\,i \leq \left[ur + \left[1 - \delta(u)\right]\phi q\right]s + b - \tau$$

$$c + q^w s_+ + p_b b_+ \leq (1 - \chi)w\ell + urs + b + \left[q^w - \chi\phi(q^w - q)\right]\left[1 - \delta(u)\right]s \\ + \left[q^w - 1 - \phi(q^w - q)\right]\chi i - \tau$$

Recall that the Lagrangian multipliers are $\chi U_c\rho$ and U_c for each constraints, we therefore know the optimal utilisation rate satisfies

$$\delta'(u)\left[q^w - \chi\phi\left(q^w - q\right) + \chi\rho\phi q\right] = r(1 + \chi\rho) \tag{36}$$

The Euler equation for private claims also needs to be adjusted, see (39) in the following.

B.2 Recursive Competitive Equilibrium

We list the equilibrium conditions with variable utilisation in the following. We use the functional forms assumed in the quantitative exercise. Given the aggregate state variables $\Gamma = (K, A, \Delta)$, the exogenous laws of motion of (A_+, Δ_+), and government policy $g = \bar{g}$ and $B = B_+ = \bar{B}$, the equilibrium maps Γ to $\Gamma' = (K_+, A_+, \Delta_+)$ such that

$$\left(K_+, i, c^e, c^w, \ell, \ell^g, \ell^f, k^g, k^f, \rho, \theta, \phi, f, q, q^w, r, w, \kappa, p_b, \tau, u\right)$$

satisfies the following equilibrium conditions obtained from the main text:

1. The representative household's optimality conditions:

$$w = \mu\ell^\nu \tag{37}$$

$$u'(c^e) = \rho U'\left(c^w - \frac{\mu\ell^{1+\nu}}{1+\nu}\right) \tag{38}$$

$$1 = \mathbb{E}_\Gamma\left[\frac{\beta U_{c,+}}{U_c}\left[\frac{u_+ r_+ + \left[1 - \delta(u_+)\right]q_+^w}{q^w} + \frac{r_+ + \left[1 - \delta(u_+)\right]\phi_+ q_+^w}{q^w}\chi\rho_+ - \frac{\left[1 - \delta(u_+)\right]\phi_+\left(q_+^w - q_+\right)}{q^w}\chi\left(1 + \rho_+\right)\right]\right] \tag{39}$$

$$p_b = \mathbb{E}_\Gamma\left[\frac{\beta U_{c,+}}{U_c}\left(1 + \chi\rho_+\right)\right] \tag{40}$$

$$q^w - 1 - \phi(q^w - q) = (1 - \phi q)\rho \tag{41}$$

$$i = \frac{\left[ur + \left[1 - \delta(u)\right]\phi q\right]K + B - c^e - \tau}{1 - \phi q} \tag{42}$$

$$\delta(u) = \delta_0 + \delta_1(u - \bar{u}) + 0.5\delta_2(u - \bar{u})^2 \tag{43}$$

$$\left[\delta_1 + \delta_2(u - \bar{u})\right]\left[q^w - \chi\phi\left(q^w - q\right) + \chi\rho\phi q\right] = r(1 + \chi\rho) \tag{44}$$

where (44) follows (36).

2. Firms:

$$r = \alpha A \left(\frac{k^g}{\ell^g} \right)^{\alpha-1} \text{ and } w = (1-\alpha)A \left(\frac{k^g}{\ell^g} \right)^{\alpha} \tag{45}$$

$$(1+\Delta)r = \kappa\gamma A A^f \left(\frac{k^f}{\ell^f} \right)^{\alpha-1} \text{ and } (1+\Delta)w = \kappa(1-\gamma)A A^f \left(\frac{k^f}{\ell^f} \right)^{\alpha} \tag{46}$$

$$q^w - q = \kappa \left(\frac{1}{f} - \frac{1}{\phi} \right) \tag{47}$$

$$q = 1 + \kappa \left(\frac{1}{f} - \frac{1}{\phi} \right) \tag{48}$$

$$\phi = \xi\theta^{1-\eta} \tag{49}$$

$$f = \xi\theta^{-\eta} \tag{50}$$

$$(1+\theta)\chi\left[i + \left[1 - \delta(u)\right]K\right] = AA^f \left(k^f\right)^{\alpha} \left(\ell^f\right)^{1-\alpha} \tag{51}$$

3. Government budget constraint:

$$(1 - p_b)B = \tau \tag{52}$$

4. Capital accumulation:

$$K_{+1} = \left[1 - \delta(u)\right]K + \chi i \tag{53}$$

5. Market clearing:

(a) The consumption goods market

$$\chi c^e + (1-\chi)c^w + \chi i + \frac{\Delta\kappa A A^f \left(k^f\right)^{\alpha} \left(\ell^f\right)^{1-\alpha}}{1+\Delta} = A\left(k^g\right)^{\alpha}\left(\ell^g\right)^{1-\alpha} \tag{54}$$

(b) Factor markets

$$k^f + k^g = uK \tag{55}$$

$$\ell^f + \ell^g = (1-\chi)\ell \tag{56}$$

Notice that in the steady state we calibrated, δ_1 and A^f are not free parameters because the equilibrium restrictions (44) and (46) imply that

$$\delta_1 = \frac{r(1 + \chi\rho)}{\left[q^w - \chi\phi\left(q^w - q\right) + \chi\rho\phi q\right]}$$

$$A^f = \frac{(1+\Delta^f)r}{\kappa\gamma A} \left(\frac{k^f}{\ell^f} \right)^{1-\gamma}$$

and r, χ, ρ, q^w, q, ϕ, Δ^f , κ, and k^f/ℓ^f are known in the steady state.

C Data

This section contains the detail of the data we use for the quantitative exercise.

C.1 Output and Hours of Work

Real consumption, investment, and government expenditures (and thus output) are from standard BEA Table 1.1.6. Hours of work are from Table 6.9 (annual hours worked by full-time and part-time employees

by industry). To the best of our knowledge, there is no quarterly Table. We use the hours under "Finance, Insurance, and Real Estate" and also hours under "Domestic Industries" which is used for total hours worked.

The financial hours in the model should exclude hours worked in real estate. Unfortunately, Table 6.9 combines hours worked in finance and insurance industry and real estate industry. We use the following adjustment. BEA also publishes a Table of value added of each industry.[29] The division there is finer. We look for value added from finance and insurance and value added from real estate separately. We compute the financial hours such that the ratio of financial hours to real estate hours is the same as the ratio of value added in finance and insurance to that in real estate.

C.2   Convenience Yields

The Moody's Aaa index is constructed from a sample of long-maturity (≥ 20 years) industrial and utility bonds (industrial only from 2002 onward). We also use the yield on 20-year maturity Treasury bonds. Both data series are from the FRED database (series AAA and GS20). The convenience yield is then computed by the ratio of the gross return of Aaa bonds and that of Treasury bonds.

Note that bonds are quoted as constant maturity. For example, constant maturity treasury (CMT) yields are read directly from the Treasury's daily yield curve and represent "bond equivalent yields" for securities that pay semiannual interest, which are expressed on a simple annualized basis. This is consistent with market practices for quoting bond yields in the market and makes the CMT yields directly comparable to quotations on other bond market yields. However, these yields are not effective annualized yields or Annualized Percentage Yields (APY), which include the effect of compounding. We perform the transformation before computing the convenience yields.

C.3   Value of Capital

Using flow-of-funds data from the Federal Reserve Board (i.e., Z1 report), We consolidate the balance sheet of non-profit organization (B.100), the non-financial non-corporate sector (B.103), and the non-financial corporate sector (B102), and the financial sector (the balance sheet account in S.6.a Financial Business) to obtain the market value of aggregate capital.

For non-profit organizations, we sum real estate and equipment and software. For the non-corporate sector, we sum real estate, equipment and software, intellectual property products, and inventories. For the corporate sector, we obtain the market value of the capital stock by summing the market value of equity and liabilities net of financial assets. We then subtract from the market value of capital for the private sector the government credit market instruments, TARP, and trade receivables. For the financial sector, we sum structures, equipments, and intellectual property products.

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[29]https://www.bea.gov/iTable/index_industry_gdpIndy.cfm