Cycles of Credit Expansion and Misallocation: The Good, The Bad and The Ugly[*]

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Abstract

Burgeoning empirical evidence suggests that credit booms may not only increase misallocation, but also sow seeds for financial crises. However, there is relative under-supply of theory accounting for these facts. To bridge the gap, we develop a unified general equilibrium banking model with heterogeneous firms to analyze the misallocation consequences of credit expansion policy both within and across sectors. The main insight is the trade-off between credit quantity and quality. On the one hand, a moderate credit expansion has a non-monotonic impact on the aggregate output. It raises credit potentially available for production at the cost of more severe productivity distortion. On the other hand, a sufficiently large credit expansion may trigger an interbank market crisis, generating discontinuous effect. The resulting economic recession is exacerbated by the firm-level productivity misallocations. By extending the static model to a dynamic environment, we show that an expansionary credit policy can generate endogenous boom-bust business cycles despite the absence of adverse shocks.

Keywords: Credit Expansion, Credit Misallocations, Financial Risk Capacity, Financial Crisis, Credit Cycles

1 Introduction

Credit expansion policies are among the most important measure by the central banks to cope with the outburst of the recent financial crises. Using China as a laboratory, Bai, Hsieh, and Song (2016) show China's 2009-2010 huge credit expansions, in particular the four-trillion fiscal stimulus program, worsen the overall efficiency of capital allocation by significantly enlarging the dispersion of marginal product of capital across privately owned firms. Complementary to Bai, Hsieh, and Song (2016), Cong et al. (2017) find that the new credit under large-scale fiscal stimulus was allocated more towards low-productivity state-controlled firms. Moreover, Gopinath et al. (2017) show that capital inflow into Italy and Portugal lowers interest rate, which in turn delivers a significant decline in sectoral TFP by mis-allocating credit toward firms that are not necessarily more productive; also see Reis (2013) for similar findings.

The main message that comes across the aforementioned empirical analysis is that credit expansion may worsen the firm-level resource misallocation. However, the credit-driven misallocation is puzzling through the lens of the financial accelerator theories, which imply that a credit boom will relax the borrowing constraint, alleviate credit misallocation, and stimulate the real economy; see Bernanke and Gertler (1989), Bernanke, Gertler, and Gilchrist (1999), and Kiyotaki and Moore (1997) for the pioneering work and Jermann and Quadrini (2012) among others for the recent development. Moreover, the recent empirical studies have highlighted that the financial crises follow credit intensive booms, i.e., "credit booms gone wrong". Based on the information of credit spread, Krishnamurthy and Muir (2017) find that the credit supply expansions are a precursor to crises. Boissay, Collard, and Smets (2016) show that only financial recessions are associated with credit booms and one country's recent path of credit growth helps predict a financial recession. Based on cross-country dataset, Gorton and Ordoñez (2016) reveal that 34 out of 87 credit booms end up with recessions, and those recessions led by credit booms account for 70% of all financial crises.[1]

Motivated by those empirical facts, this paper aims to provide a unified framework to address the consequences of credit supply policy on the financial market as well as the real economy. We highlight the interaction between the credit expansions and the firm-level productivity misallocations, the channel that has not been adequately studied by the recent research. In particular, we ask the following questions: (i) why sometimes credit expansion generate crisis while others do not? (ii) why credit expansion may generate credit misallocations, and does it matter to the aggregate economy? (iii) how to reconcile financial accelerator theory and the empirical facts?

One of the key insights from the literature on financial accelerator is that agency frictions distort investment decisions in economic slump but not in booms. This is because agency costs are negatively related to firms' net worth, which tends to be procyclical. Therefore the financial-accelerator models generally imply that credit booms are "good" times which relax borrowing constraint and alleviate

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[1]Similar patterns are also documented in Schularick and Taylor (2012), Mendoza and Terrones (2008), etc.

misallocations.

To study the potential adverse impacts of a credit expansion policy, the recent theoretical works emphasize the sector-level credit misallocations between the efficient and inefficient sectors. The asymmetric financial structure of two sectors in Bleck and Liu (2017) leads to a non-monotonic credit demand scheme. As a result, a credit expansion that shifts the credit supply curve may bring asymmetric effects on the two sectors. The inefficient one may absorb more capitals from the market, preventing the resources from being efficiently allocated to the productive sector. Credit booms then go wrong. In our paper, besides the cross-sector distortionary effect of the credit expansion, we particularly study the within-sector misallocations. We show that this channel largely amplifies the adverse impacts of credit expansions, making the economy more vulnerable to the exogenous disturbances (e.g., negative TFP shocks) and the recession more severe.

Our analytical framework is constructed by introducing firm-level productivity heterogeneity into a financial-crisis model developed by Boissay, Collard, and Smets (2016). A typical bank meets two types of firms (investment projects) that are respectively from high productivity sector (H sector) and low productivity sector (L sector). The firms in H sector are heterogenous in terms of productivity, and those in L sector are homogeneous. The firms in H sector are on average more productive than those in L sector. If the bank decides to undertake an investment project (or offer corporate loans to the firm) in H sector with the productivity drawn from a publicly observable distribution, it can utilize the internal funds (its own capitals) with a fixed fraction controlled by the credit policy and obtain external finance from other banks through the interbank market.

We assume that the projects in L sector (akin to the storage technology in Boissay, Collard, and Smets (2016)) is not traceable and cannot be seized by the lenders. So the moral hazard problem implies that the borrowing banks always have incentive to divert their interbank loans and invest in the L sector. As a consequence, the lending banks tend to limit the quantity of debts that the borrowers can obtain such that the incentive compatibility condition is satisfied. This financial friction prevents the economy from achieving the first-best equilibrium.

When the credit policy is tight (the quantity of banks' internal funds is small), the economy reaches a second-best equilibrium where the interbank market can be supported. In this equilibrium, those banks that meet firms in H sector with relatively high productivities would undertake investments and become net borrowers. The remaining banks choose to provide interbank loans to the borrowing banks and earn interests. We label this scenario as the "normal regime". We analytically show that the property of the equilibrium critically depends on the magnitude of the credit policy. When the credit policy is very tight (below an endogenous threshold), the economy has unique equilibrium, we label it as the **Good**. When the credit policy becomes moderate, multiple equilibria emerge. Besides the **Good**, a less efficient equilibrium emerges, which is labelled as the **Bad**. Since the **Bad** is strictly Pareto dominated by the **Good**, our analysis mainly focuses on the **Good** one. Furthermore, for the **Good** an expansionary credit policy has non-monotonic effects on the aggregate output. It on

the one hand raises credit supply in the market, resulting in a positive effect of intensive margin; but it on the other hand induces a larger amount of inefficient projects to be financed, leading to a negative effect of extensive margin. The extensive margin caused by the firm-level (within-sector) misallocations is the novel part in our analysis.

When the central bank implements an intensive credit expansion policy that greatly boosts the quantity of capitals available to the banks, the market cannot effectively absorb the oversupplied credit and the interbank market may eventually collapse, leading to a financial crisis as documented in Boissay, Collard, and Smets (2016). We show this crisis equilibrium is unique and label it as the **Ugly**. We want to highlight that the within-sector misallocations make the financial recessions more prolong. Our analysis suggests that when the economy trapped into the **Ugly**, the quantity of inefficient projects that get financed becomes even larger, implying that the resource misallocations among firms in H sector becomes more severe. This channel amplifies the adverse impact of credit expansions on the aggregate economy in the financial crisis.

To further document the dynamic effect of the credit expansion, we extend the baseline model to a dynamic environment. The characteristics of the steady state equilibrium crucially rely on the magnitude of the credit expansion. For a tight credit policy, the steady state turns out to be the **Good**. A further expansion (above a certain threshold) may make the steady state be the **Bad**, and a sufficiently large expansion eventually induces the **Ugly** steady state.

The aforementioned productivity misallocation channel also brings rich dynamics in our dynamic model. Under a very tight credit condition, the steady state is the **Good** one, increasing the credit supply boosts the economy through the financial accelerator mechanism. For a moderate credit condition where the steady state remains the **Good**, a credit expansion may impede the real economy because the negative effects of the within-sector productivity misallocations dominate. The model dynamics show that the economy may experience a large recession in the short run because the economy temporally hits the crisis regime and a mild decline in the long run. While, if the credit condition becomes even looser, the steady state will switch to the **Bad**, and a further credit expansion may cause endogenous boom-bust credit cycles, because the economy periodically switches between the **Good** and the **Ugly**. For a sufficiently loose credit policy, the interbank market may freeze, the steady state becomes the **Ugly** equilibrium.

Although we focus on the impact of credit policy, our model can also generate financial crises caused by unfavorable technology shocks or the slowdown of TFP growth. This prediction conforms to those empirical evidences documented in Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2016). In particular, we show that when the technology progress in the efficient sector relative to inefficient sector declines, the economy may experience severe crisis and endogenous boom-bust credit cycles.

The rest of the paper proceeds as follows. Section 2 introduces a stylized static model in which the bank's own capital stock is fixed. Section 3 characterizes the equilibria under different scenarios

and illustrates the key mechanism of the credit expansion. Section 4 embeds the static analysis of the baseline model to an infinite-horizon dynamic general framework to study under what conditions endogenous cycles of credit misallocation emerge. We offer an literature review in section 5, and then conclude in section 6. All the proofs are put in the appendix.

2 A Static Model

We start with a stylized static model to convey the basic idea. The economy has unit measure of banks. Each bank is endowed with K units of capital. We assume that the quantity of capitals that the bank can convert to the loans potentially available to production sectors or to other banks in the interbank market is ξK. The parameter ξ reflects the tightness of the central bank's credit policy. For instance, ξ could be treated as reserve requirement ratio so it is less than 1.[2] Also, it could be the loan-to-deposit ratio. In the latter case, ξ is the central bank's macro-prudential policy instrument. More broadly speaking, we treat ξ as a measurement of credit expansion.

2.1 Production Sectors

There are two production sectors in the economy. We label the sector with high productivity as h and the one with low productivity as l. Later we will present a formal assumption regarding the sectoral productivities. Each individual bank meets one individual firm in H sector and in L sector. We assume that the firms in H sector are heterogeneous in the sense that each firm receives an idiosyncratic productivity shock z with independent and identical CDF $\mathbf{F}(z)$ with $\mathbf{E}(z) = 1$. The firm's physical capital is fully financed from the bank if the bank decides to invest (i.e., provide loans). Following Gertler and Karadi (2011), we assume that there is no asymmetric information and agency problem between the individual bank and the firm, so the bank takes all the capital income from the firm as the payment to the loans. Note that the banks in our model are indeed heterogeneous because of the productivity heterogeneity of the firms. This setup generates the interbank market (Boissay, Collard, and Smets, 2016).

A typical firm with idiosyncratic productivity z in H sector uses capital k_h and labor n_h to produce goods according to the Cobb-Douglas production technology $y_h = A_h (zk_h)^\alpha n_h^{1-\alpha}$, $\alpha \in (0,1)$, where A_h denotes the sectoral productivity in H sector. The physical capital k_h is fully financed from the bank loan. Note that k_h is not necessarily equal to the individual bank's endowed capital available, ξK, because of the presence of interbank markets. The optimal labor decision is the solution of the static problem $\Pi_h(z) = \max_{n_h} A_h (zk_h)^\alpha n_h^{1-\alpha} - W n_h$, where W is the wage rate. The first order

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[2]For simplicity, we normalize the interest rate paid to the reserves $(1-\xi)K$ to zero.

condition implies that the labor demand satisfies

$$n_h(z) = \left[\frac{(1-\alpha) A_h}{W} \right]^{\frac{1}{\alpha}} z k_h. \tag{1}$$

Due to the constant return to scale technology, the capital income $\Pi_h(z)$ can be expressed as $\Pi_h(z) = \pi_h z k_h$, where the sectoral average marginal rate of return π_h satisfies

$$\pi_h = \alpha A_h^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{W} \right)^{\frac{1-\alpha}{\alpha}}. \tag{2}$$

As we described earlier, the bank's marginal return of capital by investing the firm with productivity z in H sector is simply $\pi_h z$.

For the firms in inefficient L sector, we assume that they are homogeneous and produce the final goods by only using capitals. The production technology follows a linear form: $y_l = A_l k_l$, where A_l is the sectoral productivity. So the bank's marginal return of capital by investing the firm in L sector is simply A_l.

We assume that the households inelastically provide one unit of labor. So the equilibrium labor is simply 1. To characterize the first-best scenario, we make the following assumption.

Assumption 1 $\alpha A_h z_{\max}^\alpha K^{\alpha-1} > A_l$.

The assumption says that the marginal product of capital for the firm with highest productivity at the capital stock K is greater than that in the inefficient sector. Therefore, a social optimal allocation implies that all the capitals should be allocated to the most productive firms in H sector, and with complete credit expansion ($\xi = 1$). We will go back to this assumption later after the characterization of the equilibrium.

2.2 Banks and Interbank Market

There is an interbank market from which the individual bank can supply or obtain loans. We denote R^f as the competitive interest rate prevalent in the interbank market. Given the ξK units of capital available, an individual bank that meets a firm in H sector with idiosyncratic productivity z can choose to (i) lend to other banks in the interbank market with the interest rate R^f, or (ii) borrow from the interbank market with R^f and invest (or provide loans to) the firm in H sector with the rate of return $\pi_h z$.

Let λ denote the ratio of interbank loan to bank's endowed capital. So $\lambda \xi K$ is the quantity of loans the bank borrows from the interbank market. If the bank decides to invest in H sector, the overall capitals available are $(1 + \lambda) \xi K$. The net rate of return is $\pi_h z (1 + \lambda) - R^f \lambda$. While, if the bank chooses to lend all the deposits to other banks, the rate of return is simply R^f.

Notice that as we assume each bank only meets one firm in H sector with productivity z, the banks are essentially heterogeneous in terms of their potential investment projects. In the first-best scenario, as suggested by the Assumption 1, all banks with $z < z_{\max}$ should lend their capitals (ξK) to the banks with $z = z_{\max}$. However, the presence of financial frictions, such as moral hazard problem, may discourage credit trade in the interbank market. In particular, following Boissay, Collard, and Smets (2016), we assume that the borrowers may divert $\theta \in (0, 1)$ proportion of the interbank loans, combining all their resources (endowed capitals plus interbank loans) together and resort to the inefficient L sector. So the total amount of capital that the bank can invest in L sector is $\xi K + \theta \lambda \xi K$. Due to the linear production function, the marginal rate of return per unit of bank's own capital for producing in L sector is $A_l (1 + \theta \lambda)$. [3]

In sum, the rate of return of the bank with z under the aforementioned options is given by

$$R(z) = \max \left\{ R^f, \quad \pi_h z (1 + \lambda) - R^f \lambda, \quad A_l (1 + \theta \lambda) \right\}. \tag{3}$$

Due to the moral hazard problem, the lending banks want to deter the borrowing banks from diverting the interbank loans. To do so, they can limit the quantity of loans that the marginal borrowers (those are indifferent with the first and the second options) can borrow such that they have no interest in diverting:

$$A_l (1 + \theta \lambda) \leq R^f. \tag{4}$$

It can be shown that the above Incentive-Compatibility (IC) condition holds with equality at the optimum.[4] So the market funding ratio in the interbank market can be expressed as

$$\lambda = \frac{R^f - A_l}{\theta A_l}. \tag{5}$$

The market funding ratio λ increases with the market interest rate R^f, decreases with the rate of return in low productivity sector, A_l, and the severity of moral hazard problem, θ. As discussed in Boissay, Collard, and Smets (2016), the positive relationship between λ and R^f reflects the positive selection effect of interbank market rate on the borrowers. That is, when R^f rises, only those banks with efficient projects (z is high) intend to borrow, which in turn mitigates the moral hazard problem and therefore induces a higher market funding ratio λ.

Given the IC condition (4) is satisfied, we now discuss the bank's optimal borrowing/lending strategies. It is straightforward to show that a bank intends to borrow from the interbank market if

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[3]In the current setup, the severity of moral hazard problem is captured by the parameter θ. We could also endogenize θ by considering risk-taking behaviors when the bank chooses to take inefficient project in L sector. We show that in this case θ is decreasing in the interbank market rate R^f. The endogenous severity of moral hazard problem serves as an additional amplifier for the credit expansion policy. The detailed proof is available upon request.

[4]To be rigorous, the optimization problem regarding λ is given by $\max_\lambda \pi_h z (1 + \lambda) - R^f \lambda$, subject to the IC constraint. The first order condition implies IC condition always binds at the optimum, i.e., the borrowers would always achieve the borrowing limit.

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and only if the productivity z is above a threshold z^* that satisfies

$$z^* \equiv \frac{R^f}{\pi_h}, \tag{6}$$

where the sectoral average marginal return to capital in H sector, π_h, is given by (2). If $z < z^*$, investing in H sector is less profitable than lending to the interbank market. As a result, the bank would strictly prefer to the latter option. Otherwise, for the case of $z > z^*$, the bank would choose the former option.

3   Characterizing Equilibrium

Before the discussion of capital market equilibrium, we first specify the aggregate labor N and aggregate output in H sector, Y_h. The aggregate output in L sector will be defined later. In particular, from the individual labor demand (1), the aggregate labor N is given by

$$N = \int_{z \geq z^*} n_h(z) \, d\mathbf{F}(z) = \left[\frac{(1-\alpha) A_h}{W} \right]^{\frac{1}{\alpha}} \tilde{K}_h. \tag{7}$$

Here, $\tilde{K}_h \equiv K_h (1 + \lambda) \int_{z \geq z^*} z d\mathbf{F}(z)$ is the effective capital used in H sector, which depends on the quantity $K_h(1 + \lambda)$ and the quality $\int_{z \geq z^*} z d\mathbf{F}(z)$. For the aggregate output in H sector, we have $Y_h = \int_{z \geq z^*} y_h(z) \, d\mathbf{F}(z) = \frac{W}{1-\alpha} N$. The second equality is due to the optimal condition of labor demand. Combining last two equations leads to the aggregate production function in H sector

$$Y_h = A_h \tilde{K}_h^\alpha N^{1-\alpha}. \tag{8}$$

With the inelastic labor supply (i.e., $N = 1$), the marginal rate of return to capital in H sector is obtained by substituting (7) into (2)

$$\pi_h = \alpha A_h \tilde{K}_h^{\alpha-1}. \tag{9}$$

To sharpen the analysis, we assume for the rest of the paper that individual productivity z conforms to a Pareto distribution with CDF $\mathbf{F}(z) = 1 - \left(\frac{z}{z_{\min}} \right)^{-\eta}$ and $\eta > 2$.[5] We set $z_{\min} = \frac{\eta-1}{\eta}$ so that $\mathbf{E}(z) = 1$.

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[5]Since $z_{\max} = \infty$ under Pareto distribution, Assumption 1 is automatically satisfied. Besides, we have done numerical analysis for other widely used distributions, e.g., Log-normal, uniform, etc, and the results in our main analysis remain unchanged. All the results are qualitatively preserved. The numerical analysis is available upon request.

3.1 Interbank Market Equilibrium: The Good and The Bad

We start with the case where the interbank market is not collapsed. In this case, we must have $R^f > A_l$, and all capitals in the banking sector, ξK, will be allocated to the H sector. Let K_h and K_l denote the aggregate capitals in H sector and l, respectively. Then we have $K_h = \xi K$ and $K_l = 0$.

The interbank capital market clearing condition implies the demand of loans, $\int_{z \geq z^*} \lambda \xi K d\mathbf{F}(z)$, equates the supply of loans, $\int_{z < z^*} \xi K d\mathbf{F}(z)$. This equilibrium condition can be further expressed as

$$[1 - \mathbf{F}(z^*)] \lambda = \mathbf{F}(z^*), \tag{10}$$

where the market funding ratio λ is given by (5). The LHS of above equation indicates that the supply of loans only depends on the extensive margin $\mathbf{F}(z^*)$, which monotonically increases with the cutoff value z^*. While, the RHS of the equation shows that the aggregate demand of loans consists of the extensive margin $1 - \mathbf{F}(z^*)$ and the intensive margin λ. It is straightforward to show that the extensive margin declines with the cutoff z^*. For the intensive margin λ, it is increasing in z^*. To see this, under the market clearing condition, the aggregate effective capital can be expressed as $\tilde{K}_h = \xi K \mathbf{E}(z|z \geq z^*)$, which is the product of the quantity of capital available to the bank and the average production efficiency.[6] From (6) and (9), the equilibrium interest rate satisfies

$$R^f = \pi_h z^* = \alpha z^* \mathbf{E}^{\alpha-1}(z|z \geq z^*) A_h (\xi K)^{\alpha-1}. \tag{11}$$

Notice that with Pareto distribution, we have $\mathbf{E}(z|z \geq z^*) = \frac{z^*}{z_{\min}}$, and thus the equilibrium interest rate R^f strictly increases with z^*. The positive relationship between R^f and z^* reflects the positive selection of the market rate on the production efficiency. From the definition of λ in (5), the market funding ratio increases with R^f, implying the leverage increases with z^* as well. Therefore, the relationship between the aggregate demand of loans $[1 - \mathbf{F}(z^*)] \lambda$ and the cutoff value z^* could be non-monotonic. A rise in the cutoff z^* would raise the borrowing capacity of banks—intensive margin, it also reduces the number of firms that choose to borrow and produce—extensive margin.

Under the first best scenario where the market frictions are absent, the social optimal allocation implies that all the capital (with a complete credit expansion) should be allocated to the firm in H sector with $z = z_{max}$. The equilibrium condition (11) indicates that the market rate equates the marginal product of capital, i.e., $R^f = \alpha z_{max}^\alpha (K)^{\alpha-1} > A_l$, where the second inequality comes from the Assumption 1.

To give an intuitive illustration, Figure 1 plots the demand and supply schemes of loans against the cutoff value z^*. The demand curve (the red lines) presents an inverted-U shape.[7] The intersection

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[6]From the definition of $\tilde{K}_h$, we have $\tilde{K}_h = \xi K (1 + \lambda) \int_{z \geq z^*} z d\mathbf{F}(z) = \xi K \mathbf{E}(z|z \geq z^*)(1 + \lambda)[1 - \mathbf{F}(z^*)]$. From the market clearing condition (10), we immediately have $(1 + \lambda)[1 - \mathbf{F}(z^*)] = 1$.

[7]This inverted-U shape is intuitive. Consider a extreme case of $z^* \to z_{\min}$, the extensive margin becomes 1, so the demand curve fully relies on the intensive margin λ which increases with z^*. Meanwhile, for $z^* \to z_{\max}$, no firm would

point of the demand and supply curves corresponds to the equilibrium cutoff z^*. Note that the equilibrium condition (10) indicates that the credit policy ξ only affects the loan demand. It is straightforward to show that a rise in ξ shifts the demand curve downwardly. [8] As the supply side $\mathbf{F}(z^*)$ does not depend on ξ, the magnitude of the credit policy essentially determines the properties of the equilibria. As shown in Figure 1, when the credit policy is tight (ξ is small), the demand and the supply only cross once, so the interbank market equilibrium is unique; while, if the credit is further expanded, the demand and the supply may cross twice, implying a situation of multiple equilibria; when the credit policy is sufficiently loose, the demand may not intersect with the supply, so the interbank market equilibrium does not exist. We will show later that in the last scenario there exists a unique interbank market collapse equilibrium.

To give a rigorous analysis, we express the equilibrium condition (10) more explicitly as a function of the cutoff z^* by employing the market funding ratio equation (5), the cutoff value condition (6) and π_h in (9). Then we have

$$\frac{\alpha \left(\xi K\right)^{\alpha-1} A_h}{A_l} = \Gamma\left(z^*\right).\tag{12}$$

where $\Gamma\left(z^*\right) \equiv \frac{\theta \frac{\mathbf{F}(z^*)}{1-\mathbf{F}(z^*)}+1}{[\mathbf{E}(z|z\geq z^*)]^{\alpha-1}z^*}$ with $\lim_{z^*\to z_{\min}} \Gamma\left(z^*\right) = \frac{1}{z_{\min}}$ and $\lim_{z^*\to z_{\max}} \Gamma\left(z^*\right) = \infty$. [9]

One advantage of the above expression of market clearing condition is that we can easily analyze the impact of credit policy ξ on the equilibrium cutoff z^*, since the LHS of (12) does not rely on z^*. So the property of equilibrium (unique or multiple) fully depends on the shape of $\Gamma\left(z^*\right)$. We now characterize the property of $\Gamma\left(z^*\right)$.

Assumption 2 *The capital share α, the shape parameter for the Pareto distribution η and the severity of moral hazard problem θ satisfy the condition $1 + \alpha < \eta < \frac{\alpha}{\theta}$.*

Lemma A.1 in the appendix provides a full characterization of the property of $\Gamma\left(z^*\right)$. In particular, $\Gamma\left(z^*\right)$ is strictly convex in z^* because of the condition $\eta > 1 + \alpha$. Moreover, $\Gamma\left(z^*\right)$ achieves its minimum at $\hat{z} = \left(1 + \frac{\alpha/\theta-\eta}{\eta-\alpha}\right)^{\frac{1}{\eta}} z_{\min}$. The condition $\eta < \frac{\alpha}{\theta}$ guarantees that the minimum is interior, i.e., $\hat{z} > z_{min}$.

As discussed previously, the properties of equilibria, given $\Gamma\left(z^*\right)$, mainly relies on the value of credit policy. The following proposition gives a full characterization.

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produce and borrow, so the demand shrinks to zero, which means the demand curve must decline for a sufficient large z^*.

[8]Given the cutoff z^* fixed, (11) implies a larger ξ leads to a lower R^f. Since the market funding ratio λ decreases with R^f, an expansionary policy may shift the demand curve downwardly.

[9]Note that the LHS of (12) after multiplying z_{max}^α is the wedge between the first best marginal product of capital (MPK) and that in L sector. So, $\Gamma\left(z^*\right)$ is proportional to this wedge. It can be further decomposed into two components: the gap between the first best MPK and the second best MPK, and the gap between the second best MPK and that in L sector. Here the second best MPK is defined as $\frac{\partial Y_h}{\partial K_h}$. The first component decreases with the cutoff z^* and the second one increases with the cutoff z^*. The intuition is simple. A higher cutoff z^* means a less severe misallocation among firms, which narrows the gap of MPK between the first best and the second best, but enlarges the gap of MPK between the second best and that in L sector.
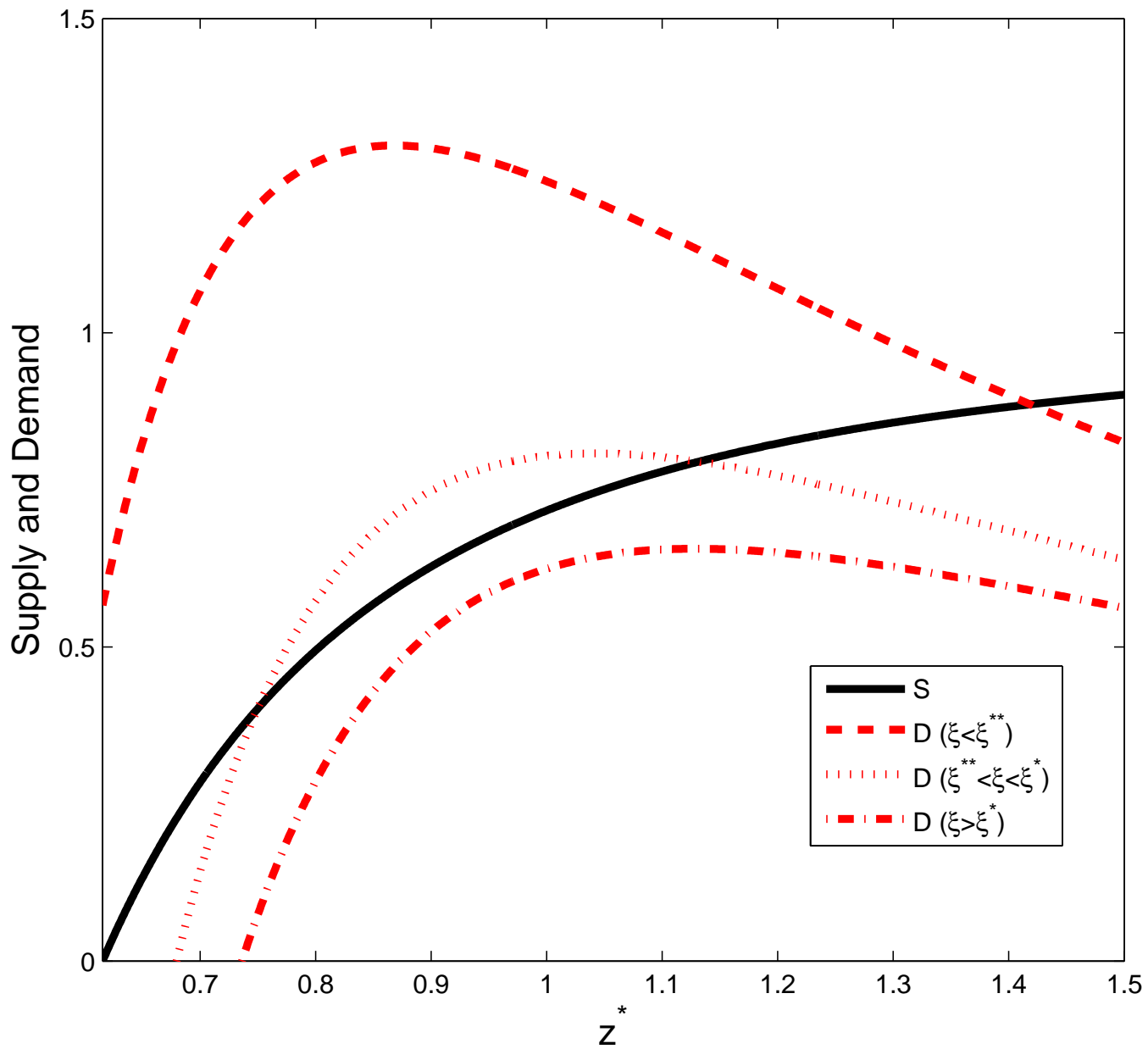
Figure 1: Demand and supply in the interbank market

Proposition 1 *There exist two thresholds of credit policy* $\xi^{**} = \left[\frac{\alpha K^{\alpha-1} \frac{A_h}{A_l}}{\Gamma(z_{min})} \right]^{\frac{1}{1-\alpha}}$ *and* $\xi^* = \left[\frac{\alpha K^{\alpha-1} \frac{A_h}{A_l}}{\Gamma(\hat{z})} \right]^{\frac{1}{1-\alpha}}$
with $\xi^* > \xi^{**} > 0$ *such that: (i) if* $\xi < \xi^{**}$ *the interbank market equilibrium is unique;(ii) if* $\xi^{**} < \xi < \xi^*$ *there may exist multiple interbank market equilibria with* $z^* < \hat{z}$ *and* $z^* > \hat{z}$; *(iii) if* $\xi > \xi^*$ *the interbank equilibrium cannot be supported.*

The part (iii) in Proposition 1 shows that once the total supply of credit in the whole economy exceeds $\xi^* K$, the interbank market equilibrium cannot be supported. In other words, $\xi^* K$ reflects the limit of capacity that the market can absorb and efficiently allocate. This implies that a large credit expansion may trigger a banking crisis due to the discontinuity between different equilibria.[10] We label the regime for the existence of interbank market equilibrium (i.e., $\xi < \xi^*$) as Regime 1 and the regime for the interbank market collapse equilibrium (i.e., $\xi > \xi^*$) as Regime 2.

The parts (i) and (ii) in Proposition 1 state the conditions for the credit policy under which the economy may have unique or multiple equilibria. If the credit policy is sufficiently tight (ξ is below ξ^{**}), the interbank market equilibrium is unique. While if the credit policy is moderate, i.e., $\xi \in (\xi^{**}, \xi^*)$, multiple equilibria emerge. In this scenario, the equilibrium cutoff of productivity z^* is high in one equilibrium and low in the other. Since the value of z^* reflects the efficiency of the credit allocation, we label the equilibrium with high z^* as the **Good** and the equilibrium with low z^* as the **Bad**. Figure 2 gives a graphic illustration for the Proposition 1.

The non-monotonicity of the credit demand regarding z^* (the LHS in (10)) is crucial for the existence of multiple equilibria. As the credit supply (the RHS in (10)) strictly increases with z^*, to support an equilibrium with high cutoff z^* (the **Good**), the demand side must be large as well. This can always be supported by a large market funding ratio λ. To sustain an equilibrium with low cutoff z^* (the **Bad**), the demand side must be low in order to match a low credit supply, which can happen only if the market funding ratio λ is small. However, if the credit policy is sufficiently tight (less than ξ^{**}) such that the market funding ratio cannot be small,[11] the **Bad** equilibrium may not be supported. In this scenario, the **Good** is the unique equilibrium, which gives the part (i) in Proposition 1.

Risk Capacity The thresholds ξ^* and ξ^{**} reflect the different types of risk capacity for the financial market. In particular, a large ξ^* indicates that the interbank market is vulnerable for the crisis. While, a large ξ^{**} implies that the interbank market is vulnerable for generating multiple equilibria. Corollary A.1 in the appendix shows that two risk capacities ξ^* and ξ^{**} increase with the dispersion of sectoral TFP $\frac{A_h}{A_l}$, and decreases with the volatility of idiosyncratic productivity shock z. Moreover, the threshold ξ^* decreases with the severity of moral hazard problem θ.

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[10]A credit boom may induce a banking crisis has been comprehensively documented in Boissay, Collard, and Smets (2016). One major difference between our paper and theirs is that we focus on the credit policy instead of technology growth led credit boom.

[11]According to the equilibrium condition for the market rate R^f, (11), given z^* a small ξ implies a large market rate R^f, which further induces a large market funding ratio λ.
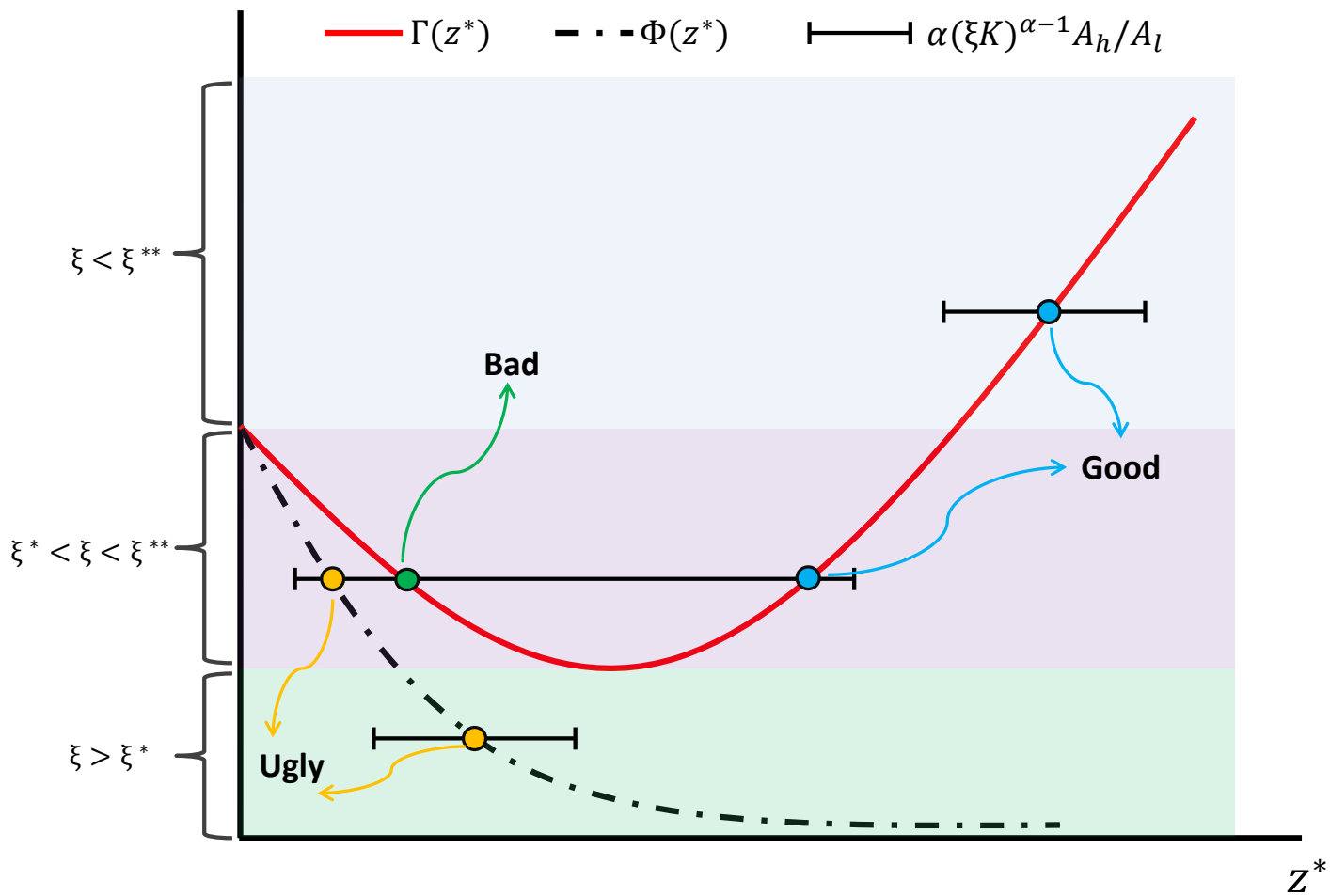
Figure 2: Equilibria Cutoff under Different Regimes

The properties of risk capacity ξ^* have several important implications. Firstly, it suggests that a small TFP shock to the H sector that changes the cross-sector dispersion $\frac{A_h}{A_l}$ may generate asymmetric effect to the aggregate economy. For instance, when the H sector suffers a negative TFP shock (i.e., $\frac{A_h}{A_l}$ decreases), the threshold ξ^* declines. Given the value of ξ, this may lead the economy to the collapsed equilibrium. The above channel is consistent with the analysis in Boissay, Collard, and Smets (2016). One novel thing in our model is that the firm-level misallocation largely amplify the adverse impact caused by the bank crisis. We will go back to this point in the later analysis. Secondly, it implies that a rise in microeconomic uncertainty (variance of z) may trigger an interbank market crisis by reducing the threshold value of regime switch ξ^*.[12] The above two arguments can also be applied to the risk capacity ξ^{**} for the multiple equilibria. That is, a lower sectoral TFP ratio $\frac{A_h}{A_l}$ or a higher volatility of z may make the economy more likely to enter the multiple equilibria regime where the **Bad** equilibrium may emerge. Finally, the property of ξ^* implies that a more severe moral hazard problem (θ is larger) may make the economy more vulnerable to the financial crisis. Intuitively, if a bank can divert a larger fraction of loans from the interbank market, it has stronger incentive to invest in L sector. So more likely the interbank market cannot be sustained.

Allocation Efficiency For the Regime 1 ($\xi < \xi^*$) where the interbank market equilibrium exists, all the credit are allocated to H sector, so there is no cross-sector misallocations. In this regime, the cutoff z^* is the sufficient statistics of the allocation efficiency. A larger z^* indicates more efficient allocations of the market credit. It is straightforward to show that for the **Good** equilibrium, a credit expansion (ξ increases) reduces the cutoff value (see Figure 2), therefore exacerbates the within-sector misallocations. The model-implied adverse impact of credit expansion on the firm-level allocation efficiency conform to the empirical findings in the recent literature, e.g., Bai, Hsieh and Song (2016), Cong and Ponticelli (2016), Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-Sanchez (2017). The within-sector misallocation is the novel channel in our model that departs from the recent banking crisis theories.

Equilibrium Output Since all the capitals are allocated to the H sector, we have $K_h = \xi K$, $K_l = 0$, and $Y_l = 0$. The aggregate output Y is

$$Y = Y_h = A_h \left[\mathbf{E} \left(z | z \geq z^* \right) \xi K \right]^\alpha. \tag{13}$$

We first discuss aggregate output in the **Good** equilibrium (the blue points in Figure 2). In this equilibrium, as the equation (13) shows, the credit expansion has two offsetting effects on the output. On the one hand, it directly raises the market credit that is used for the production, i.e., ξK increases. On the other hand, it induces the banks to finance more low efficient projects thus reduces the average productivity, i.e., $\mathbf{E} \left(z | z \geq z^* \right)$ declines. As a result, given the aggregate TFP A_h

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[12]The fact that ξ^* strictly decreases with microeconomic uncertainty holds not only for Pareto distribution but is also true for other frequently used distributions, including uniform distribution and log-normal distribution. We use Pareto mainly for analytical concern. The numerical results for the robust analysis is available upon request.

and the capital stock K, the output non-monotonically responds to the change of credit policy.

Proposition 2 *Under the Regime 1 where the interbank market equilibrium is supported (i.e., $\xi < \xi^*$), the aggregate output in the **Good** equilibrium is strictly concave in the credit policy ξ and achieves its optimal level at $\xi = \tilde{\xi}$, where $\tilde{\xi} = \left[\frac{\alpha K^{\alpha-1} \frac{A_h}{A_l}}{\Gamma(\tilde{z})} \right]^{\frac{1}{1-\alpha}} < \xi^*$ and $\tilde{z} = \left[\frac{1-\theta}{(\eta-1)\theta} \right]^{\frac{1}{\eta}} z_{\min}$.*

Proposition 2 indicates that in the **Good** equilibrium, there may exist an optimal credit policy so that the output achieves the optimum. When the credit supply below this optimal level (i.e., $\tilde{\xi} K$), the credit expansion would stimulate the output. When the credit supply exceeds the optimum, a further expansion may impede the aggregate economy. If the credit expands further and pass the absorption capacity (i.e., $\xi^* K$), the interbank market will freeze, inducing a banking crisis. The former negative impact of credit expansion is due to the within-sector misallocation, and the latter one is mainly due to the cross-sector misallocation. We will discuss this scenario shortly.

We now briefly discuss the **Bad** equilibrium (the green point in Figure 2). The output in this case also takes the form of (13). The only difference is the equilibrium cutoff z^*. It is straightforward to show that a credit expansion (ξ increases) in this case raises the cutoff and thus the production efficiency (see Figure 2). Meanwhile, a larger ξ also expands the market credit available for the banks. Therefore, for the **Bad** equilibrium, the credit expansion unambiguously raises the aggregate output.

Note that given the credit policy the overall efficiency in the **Bad** equilibrium is below that in the **Good** equilibrium. Corollary A.2 in the appendix compares the aggregate output in both equilibria. It shows that in the multiple equilibria regime where $\xi \in (\xi^{**}, \xi^*)$, the output in the **Good** is strictly higher than that in the **Bad**. Since the cutoff z^* in two equilibria converges to $\hat{z} \equiv \arg\min \Gamma(z^*)$ when $\xi = \xi^*$, the output in two equilibria also converges at the risk capacity for the banking crisis ξ^*, i.e., $Y_{\textbf{Good}}(\xi^*) = Y_{\textbf{Bad}}(\xi^*)$. Note that for the **Bad** equilibrium, if the credit policy is at the risk capacity for the multiple equilibria, $\xi = \xi^{**}$, the equilibrium output is simply $Y = A_h \left(\alpha \frac{A_h}{A_l} \right)^{\frac{\alpha}{1-\alpha}} z_{\min}^{\frac{\alpha}{1-\alpha}}$. In this case, the moral hazard parameter θ no longer matters for the aggregate economy.[13]

3.2 Financial Crisis Equilibrium: The Ugly

We now consider Regime 2 where the interbank market equilibrium cannot be supported, i.e., $\xi > \xi^*$. We label this financial autarky (or crisis) equilibrium as the **Ugly**. Because of the collapse of interbank market, the banks cannot finance from outside, so $\lambda = 0$. The bank's investment options are reduced to two: investing either in the H sector or l. Remember that the marginal rates of return for these two options are $\pi_h z$ and A_l, respectively. The bank's investment decision rule follows trigger

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[13]This is because in this **Bad** equilibrium, even the bank with lowest productivity z_{min} has incentive to borrow from the interbank market, in other words, the supply of credit $\mathbf{F}(z^*)$ is zero. So the moral hazard problem is not essential because of the market freeze.

strategy: invest in H sector if $z > z^*$ and invest in L sector otherwise. The cutoff z^* equates two marginal rates of return, i.e.,

$$\pi_h z^* = A_l. \tag{14}$$

So the aggregate capitals allocated to the H sector and l are given by $K_h = \xi K \left[1 - \mathbf{F}\left(z^*\right) \right]$ and $K_l = \xi K \mathbf{F}\left(z^*\right)$, respectively. Comparing to the interbank market equilibrium where all the capitals are allocated to the efficient H sector, in the **Ugly** equilibrium the cross-sector misallocation emerges.

To determine the equilibrium cutoff z^*, from (14) and the definition of π_h, we have $\alpha A_h \tilde{K}_h^{\alpha-1} z^* = A_l$, where the effective capital satisfies $\tilde{K}_h = \xi K \left[1 - \mathbf{F}\left(z^*\right) \right] \mathbf{E}\left(z | z \geq z^*\right)$. Analog to the previous analysis for Regime 1, we rearrange the terms and obtain

$$\alpha \frac{A_h}{A_l} \left(\xi K\right)^{\alpha-1} = \Phi\left(z^*\right), \tag{15}$$

where $\Phi\left(z^*\right) \equiv \frac{[1-\mathbf{F}(z^*)]^{1-\alpha}}{[\mathbf{E}(z|z\geq z^*)]^{\alpha-1}z^*}$, and $\lim_{z^* \to z_{\min}} \Phi\left(z^*\right) = \frac{1}{z_{\min}}$ and $\lim_{z^* \to z_{\max}} \Phi\left(z^*\right) = 0$. Again, the LHS in last equilibrium condition is a constant, and the RHS only depends on the cutoff z^*. So the property of the **Ugly** equilibrium relies on the shape of the function $\Phi\left(z^*\right)$. Lemma A.2 in the appendix shows that under the Assumption 2, the function $\Phi\left(z^*\right)$ strictly decreases with the cutoff z^*. Therefore, there may exist a unique **Ugly** equilibrium when the credit policy is sufficiently loose, i.e., $\xi > \xi^*$.

Figure 2 provides a graphical illustration for the above analysis. Since $\xi < \xi^*$, the line $\alpha \frac{A_h}{A_l} \left(\xi K\right)^{\alpha-1}$ is below the curve $\Phi\left(z^*\right)$ and only crosses it once (see the yellow point in Figure 2). The intersection point is the unique **Ugly** equilibrium. Under the Pareto distribution, the equilibrium condition (15) admits an analytical solution to z^* such that $z^* = \left[\frac{A_l \xi^{1-\alpha}}{\alpha A_h K^{\alpha-1} z_{\min}} \right]^{\frac{1}{\alpha+\eta-\alpha\eta}} z_{\min}$. Obviously, the cutoff z^* in the **Ugly** equilibrium strictly increases with the credit policy ξ. In Figure 2, the positive relationship between ξ and z^* can be easily verified by shifting the line $\alpha \frac{A_h}{A_l} \left(\xi K\right)^{\alpha-1}$ downwardly. So a credit expansion improves the firm-level efficiency in the H sector.

Regarding the aggregate output, the impact of ξ on the aggregate output now consists of three effects. Firstly, a higher ξ reduces the number of banks that invest in H sector ($\mathbf{F}(z^*)$ declines), so it exacerbates the cross-sector capital misallocation. Secondly, a higher ξ improves the within-sector misallocation because of a larger cutoff z^*. Finally, a higher ξ raises the endowed capital stock that the bank can use. It turns out that the latter two effects dominates the first one, therefore in the **Ugly** equilibrium, the aggregate output strictly increases with credit policy ξ. It is worth noting that as $\Phi\left(z_{min}\right) = \Gamma\left(z_{min}\right) = \frac{1}{z_{min}}$, the **Ugly** equilibrium is coincident with the **Bad** equilibrium at $\xi = \xi^{**}$. As a result, we must have $Y_{\mathbf{Bad}}(\xi^{**}) = Y_{\mathbf{Ugly}}(\xi^{**})$.

Discontinuity So far we have analyzed the impact of credit policy ξ on the aggregate output for different equilibria. To give a complete and rigorous characterization of the relationship between the output and the credit policy, we need to discuss the discontinuity of the aggregate output around

the risk capacity ξ^*.

Proposition 3 *The equilibrium cutoff z^* and the aggregate output are discontinuous for Regime 1 and Regime 2 at $\xi = \xi^*$. In particular, $z^*_{\textbf{Good}}(\xi = \xi^*) = z^*_{\textbf{Bad}}(\xi = \xi^*) > z^*_{\textbf{Ugly}}(\xi = \xi^*)$ and $Y_{\textbf{Good}}(\xi = \xi^*) = Y_{\textbf{Bad}}(\xi = \xi^*) > Y_{\textbf{Ugly}}(\xi = \xi^*)$.*

The proposition indicates that a credit expansion policy that makes ξ exceed the risk capacity ξ^* will induce a discontinuous drop in the allocation efficiency z^* and in the aggregate output. With the discontinuity and together with the previous analysis, we can fully characterize the relationship between the aggregate output and the credit policy. Figure 3 gives a graphic description.

The two thresholds ξ^{**} and ξ^* divide the space into three areas. The blue area corresponds to the unique **Good** equilibrium. The green area corresponds to the unique **Ugly** equilibrium. For the purple area in the middle, there coexist three equilibria. According to the previous discussions, in the Regime 1 the output is concave in the credit policy ξ. It increases when ξ is less than the threshold $\tilde{\xi}$ and decreases when $\xi \in (\tilde{\xi}, \xi^*)$. For the **Bad** and the **Ugly** equilibria, the output is monotonically increasing in the credit policy ξ. In the area of multiple equilibria where $\xi \in (\xi^{**}, \xi^*)$, the output in different equilibria ranks as follows: $Y_{\textbf{Good}} > Y_{\textbf{Bad}} > Y_{\textbf{Ugly}}$.[14] In addition, Proposition 3 suggests that the output presents a drop at $\xi = \xi^*$ between Regime 1 and Regime 2.

For the within-sector allocation efficiency, the relationship between the equilibrium cutoff z^* and the credit policy ξ generally presents a similar pattern to that for the output. Figure 4 gives a graphic description. It shows that after a credit expansion (ξ increases), the efficiency for the **Good** equilibrium is deteriorated (instead of the non-monotonic relationship for the case of output); while for the **Bad** and the **Ugly** equilibria, the efficiency is improved. In the area of multiple equilibria, the cutoff z^* ranks $z^*_{\textbf{Good}} > z^*_{\textbf{Bad}} > z^*_{\textbf{Ugly}}$. Moreover, Proposition 3 implies that when ξ exceeds the risk capacity ξ^*, the efficiency experiences a sharp drop.

3.3 From The Good to The Ugly

In the area of multiple equilibria where $\xi \in (\xi^{**}, \xi^*)$, the ranking for the output and the allocation efficiency shows that the **Good** Pareto dominates the **Bad** and the **Ugly**. So following Boissay, Collard, and Smets (2016), throughout the rest of the paper we exclude the **Bad** and the **Ugly** and only focus the **Good** equilibrium whenever multiple equilibria emerge.

Proposition 3 also conveys an important message that when the economy switches from Regime 1 (normal time) to Regime 2 (crisis time), the output would undertake a sharp reduction. In this sense, our model provides a novel channel through which a small credit expansion policy may lead to excessive aggregate volatility. For instance, given risk capacity ξ^*, if a small credit expansion makes

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[14]The part of $Y_{\textbf{Good}} > Y_{\textbf{Bad}}$ comes directly from Corollary A.2 in the appendix. The part of $Y_{\textbf{Bad}} > Y_{\textbf{Ugly}}$ is due to the facts that (i) Y strictly increases with ξ for both the **Bad** and the **Ugly**; (ii) $Y_{\textbf{Bad}}(\xi^{**}) = Y_{\textbf{Ugly}}(\xi^{**})$ and $Y_{\textbf{Bad}}(\xi^*) > Y_{\textbf{Ugly}}(\xi^*)$.
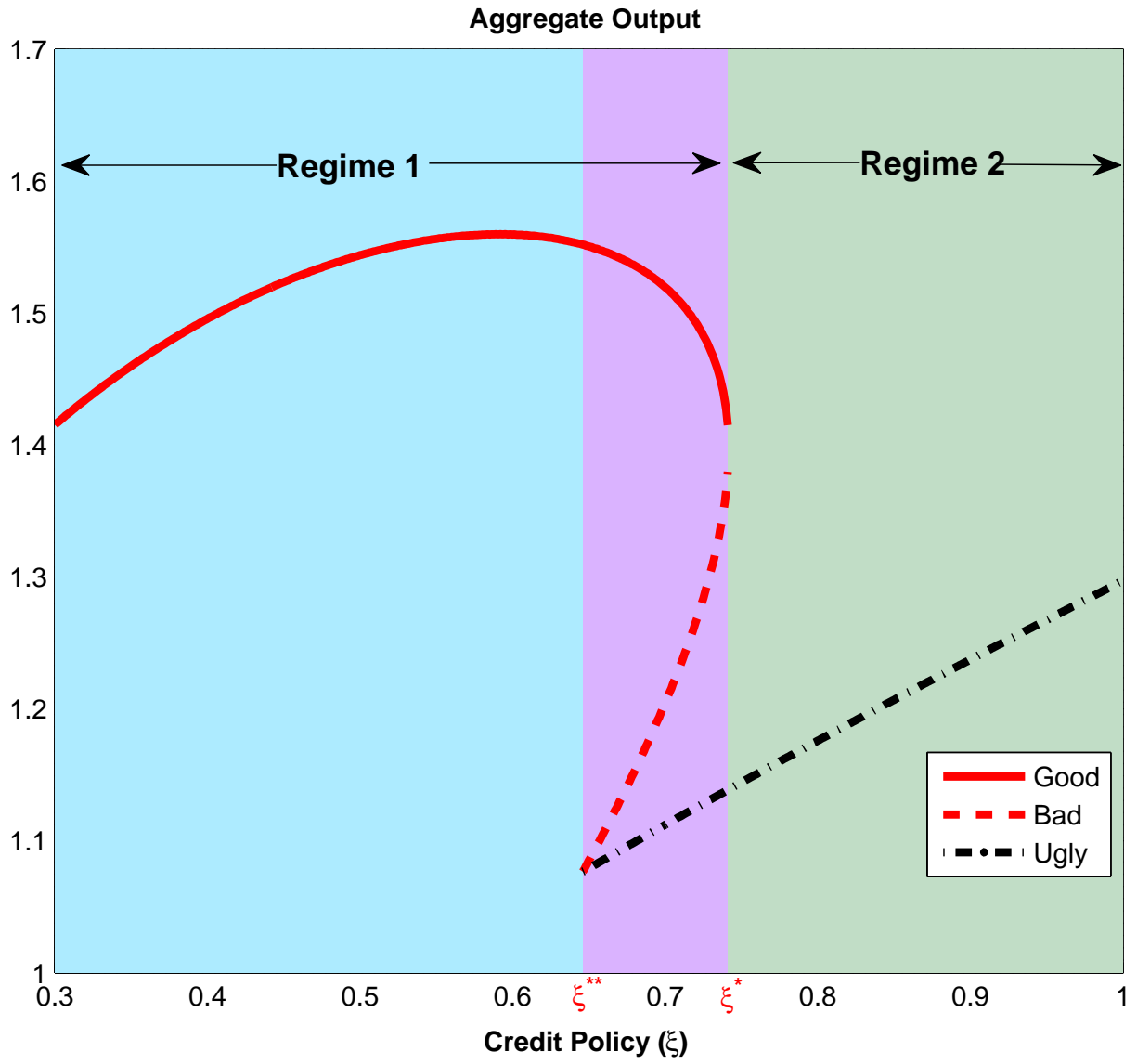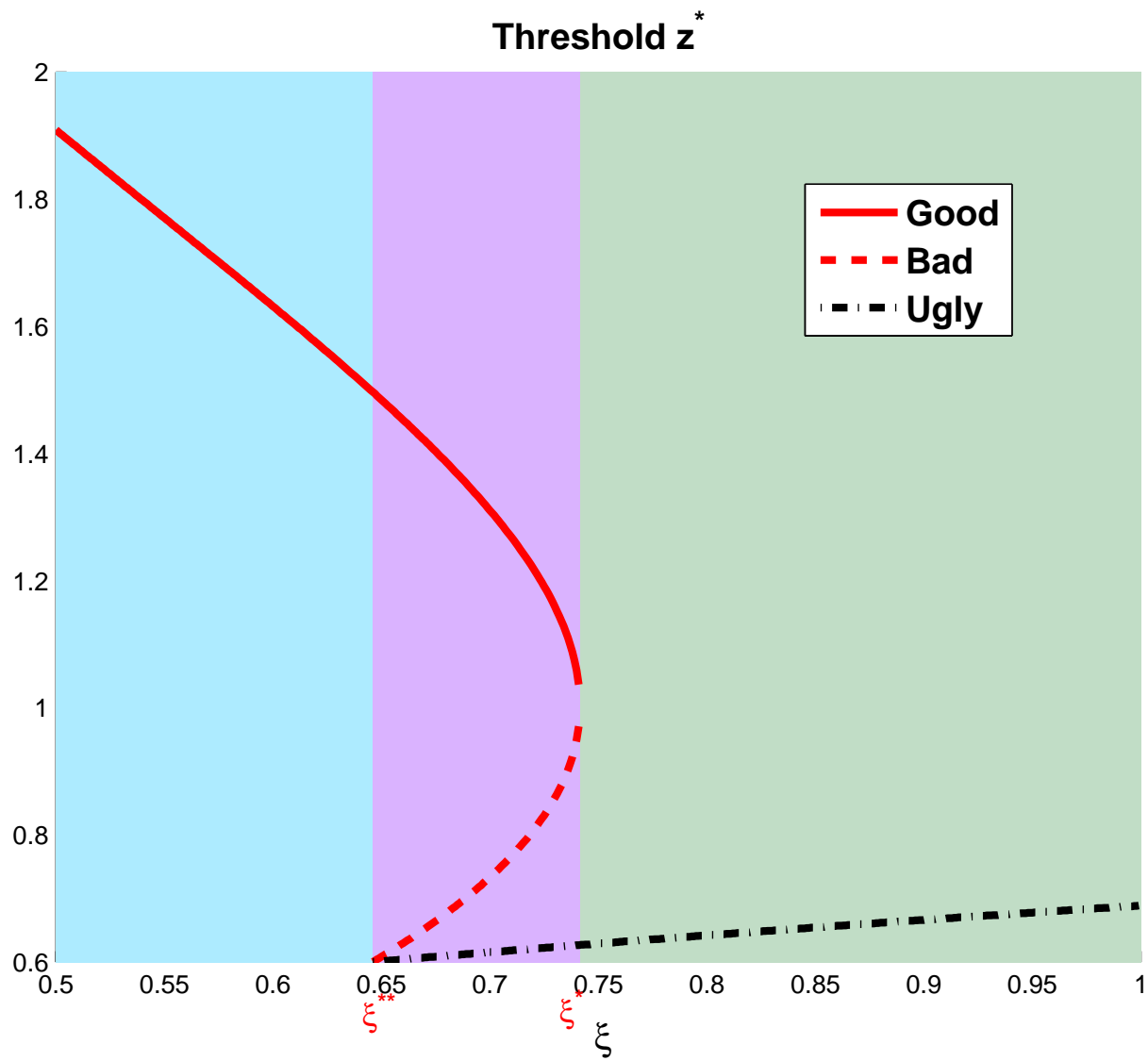
Figure 3: Aggregate Output in Different Equilibria

Figure 4: Efficiency in Different Equilibria

the ξ that is originally less than but close to ξ^* exceed ξ^*, the aggregate output will experience a large drop instead of a continuous reduction due to the regime switch between the **Good** and the **Ugly**.

In addition, as we discussed earlier the risk capacity ξ^* depends on various fundamental factors. Thus, a small change of economic fundamental may trigger a large fluctuation in real economy. For instance, Corollary A.1 in the appendix implies that ξ^* strictly decreases with the sectoral TFP dispersion $\frac{A_h}{A_l}$. So, a small negative TFP shock in H sector (A_h) may cause a large recession due to the collapse of interbank market. This prediction is in line with the technology-led credit boom-bust cycles comprehensively documented in Boissay, Collard, and Smets (2016). One difference between our model and theirs is that the magnitude of the recession is amplified by the within-sector misallocation. Similarly, as ξ^* strictly decreases with the dispersion of idiosyncratic productivity z, a small uncertainty shock that raises the variance of z would cause a large recession in aggregate output. Therefore, our model also provides a new channel to transmit the uncertainty shocks that have been identified to be an important source of aggregate fluctuations, e.g., Bloom et al. (2016).

3.4   The Role of Within-Sector Misallocation

It is worth noting that the crisis due to the interbank market freeze would be exacerbated by the within-sector productivity misallocation. To highlight this channel, we compare the relationship between the output and the credit policy ξ in the baseline model and that in the model without TFP misallocation. In particular, for the latter setup, we follow Boissay, Collard, and Smets (2016) assuming that the firms are identical, i.e., the production function for the firms in h sector is $y_h = A_h k_h^\alpha n_h^{1-\alpha}$, and the banks are heterogeneous in terms of their rate of return. So the bank's problem remains the same as that in the baseline model. However, the aggregate output is different since the aggregate production becomes

$$Y = \begin{cases} A_h \left(\xi K\right)^\alpha & \textbf{Good} \\ A_h \left\{[1 - \mathbf{F}\left(z^*\right)] \xi K\right\}^\alpha + A_l \xi K \mathbf{F}\left(z^*\right) & \textbf{Ugly} \end{cases}. \tag{16}$$

In the baseline model, when the credit expansion triggers the interbank market freeze, the cutoff of productivity z^* falls sharply due to the discontinuity of z^* around ξ^*, which deteriorates the allocation efficiency. This adverse impact on the output is reflected by the term $\mathbf{E}\left(z|z \geq z^*\right)$ in production function (13). While in the model with homogeneous productivity, the within-sector misallocation channel is absent.

Figure 5 compares the relationship between the aggregate output and the credit policy under these two models. To make different models comparable, we set the same values for the common parameters in both models. The figure shows that the recession caused by the interbank market freeze is much more severe in the baseline model. Moreover, the with-sector misallocation also lead to a
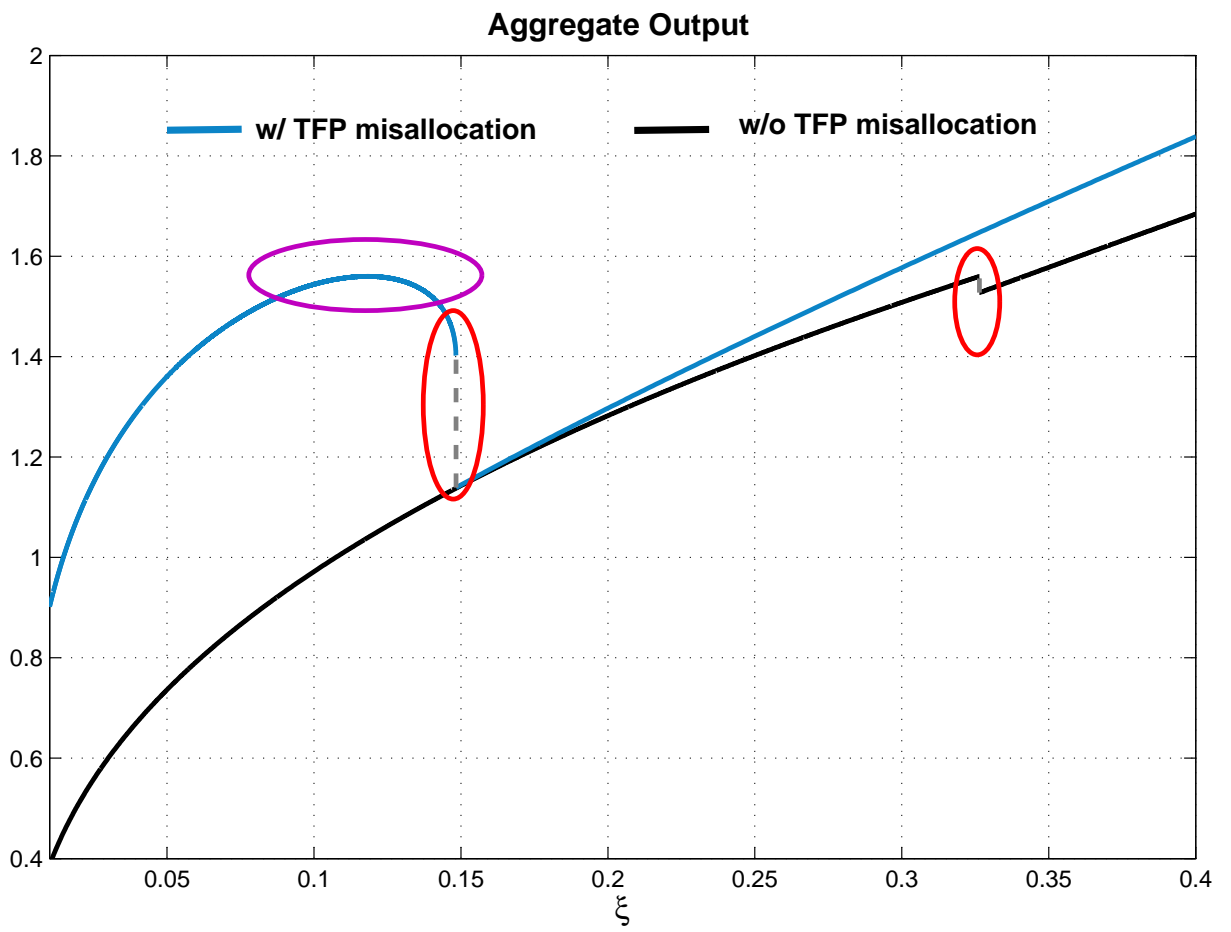
Figure 5: Credit Expansion and Aggregate Output: Model Comparison

non-monotonic impact of credit expansion on the output in the normal time (the **Good** equilibrium). While, in the model without within-sector misallocation the credit expansion monotonically affects the aggregate output.

4   Dynamics

The previous analysis is based on a static model in which the aggregate capital stock K is exogenously given. We now extend the static model to a dynamic one and document the dynamics under the credit policy. The banking and production sectors in the dynamic model are essentially the same as those in the static model. To introduce the dynamics, we assume that the banker (corresponding to the bank in the static model) accumulates capitals. The workers again inelastically provide labors. To simplify the analysis, we assume that the workers do not have access to the capital market and are hand-to-mouth. They consume all of their labor income each period. The idiosyncratic productivity z_t is assumed to follow Pareto distribution and i.i.d. across individuals and over time.

The banker that meets a firm in H sector with idiosyncratic productivity z_t chooses consumption c_t and capital stock in next period k_{t+1} to solve the optimization problem $\max \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \log c_t$, where $\beta \in (0,1)$ is the discount rate. The banker's budget constraint is

$$c_t + k_{t+1} - (1 - \delta) k_t = R_t (z_t) \xi_t k_t, \tag{17}$$

where $R_t (z_t)$ is the rate of return to the capitals that is defined in (3); $\xi_t \in [0,1]$ is a time varying credit policy instrument which is exogenous to the banker. Because of the log utility, the optimal capital decision satisfies

$$k_{t+1} (z_t, k_t) = \beta [R_t (z_t) \xi_t + (1 - \delta)] k_t. \tag{18}$$

Let $\mu_t (k)$ denote the distribution of the capital k in the economy. The aggregate capital satisfies $K_{t+1} = \int \int k_{t+1} (z_t, k_t) \, d\mathbf{F} (z_t) \, d\mu_t (k_t)$. Since z_t is i.i.d., the aggregate capital K_{t+1} can be expressed as

$$K_{t+1} = \beta \left[\xi_t \int R_t (z_t) \, d\mathbf{F} (z_t) + (1 - \delta) \right] K_t = \beta [\alpha Y_{h,t} + Y_{l,t} + (1 - \delta) K_t]. \tag{19}$$

The second equality holds because of the fact that the aggregate capital wealth consists of the income from the sectors h and l and the value after depreciation. According to the analysis in the static model, the sectoral outputs are given by

$$Y_{h,t} = A_h \left[\mathbf{E} (z | z \geq z_t^*) K_{h,t} \right]^{\alpha}, \tag{20}$$

$$Y_{l,t} = A_l K_{l,t}. \tag{21}$$

Recall that the cutoff values of z_t^* for the different regimes are determined by

$$\alpha \frac{A_h}{A_l} (\xi_t K_t)^{\alpha-1} = \begin{cases} \Gamma(z_t^*), & \text{if Regime 1} \\ \Phi(z_t^*), & \text{if Regime 2} \end{cases}. \tag{22}$$

Unlike the static model, given the credit policy ξ_t, the risk capacity ξ^* now is also determined by the aggregate capital stocks. In particular, let K_t^* denote the critical value of capital stock for the switch between Regime 1 and Regime 2, which satisfies

$$\alpha (\xi_t K_t^*)^{\alpha-1} \frac{A_h}{A_l} = \Gamma(\hat{z}_t), \tag{23}$$

where $\hat{z}_t = \arg\min_{z \in (z_{\min}, z_{\max})} \Gamma(z)$. Analog to the static model, we must have

$$K_{h,t} = \begin{cases} \xi_t K_t, & K_t < K_t^*, \text{ (Regime 1)} \\ \xi_t K_t [1 - \mathbf{F}(z_t^*)], & K_t > K_t^*, \text{ (Regime 2)} \end{cases}. \tag{24}$$

The capital allocated to the L sector is given by

$$K_{l,t} = \xi_t K_t - K_{h,t}. \tag{25}$$

To completely describe the dynamics, we need to define the threshold for the capital stock for the existence of multiple equilibria. In particular, let K_t^{**} satisfies

$$\alpha \frac{A_h}{A_l} (\xi_t K_t^{**})^{\alpha-1} = \Gamma(z_{\min}). \tag{26}$$

It is straightforward to see that if $K_t \in [K_t^{**}, K_t^*]$, the **Good**, the **Bad** and the **Ugly** equilibria coexist. In the end, the full dynamic system is described by (19) to (26).

Substituting (22) into (19) reveals that the RHS of (19) is a function of K_t under either Regime 1 or 2, which is denoted as $g(K_t)$. Similar to proofs for the static model, we can prove that $g^{\mathbf{Good}}(K_t)$ is strictly concave in K_t, and both $g^{\mathbf{Bad}}(K_t)$ and $g^{\mathbf{Ugly}}(K_t)$ strictly increases with K_t with $g^{\mathbf{Good}}(K_t^*) = g^{\mathbf{Bad}}(K_t^*)$ and $g^{\mathbf{Bad}}(K_t^{**}) = g^{\mathbf{Bad}}(K_t^{**})$, where (K_t^*, K_t^{**}) are defined respectively in (23) and (26). Figure 6 provides a numerical example of the law of motion of capital stock K_t.

4.1 The Steady States

We now discuss the steady state of the dynamic system. From the law of motion of aggregate capital, in the steady state we must have $rK = \alpha Y_h + Y_l$, where $r = \frac{1}{\beta} - 1 + \delta$. Notice that the steady-state capital is the intersection between the policy function $g(K_t)$ and the 45 degree line. It turns out that the steady-state capital depends on the credit policy ξ. Following proposition provides a summary.
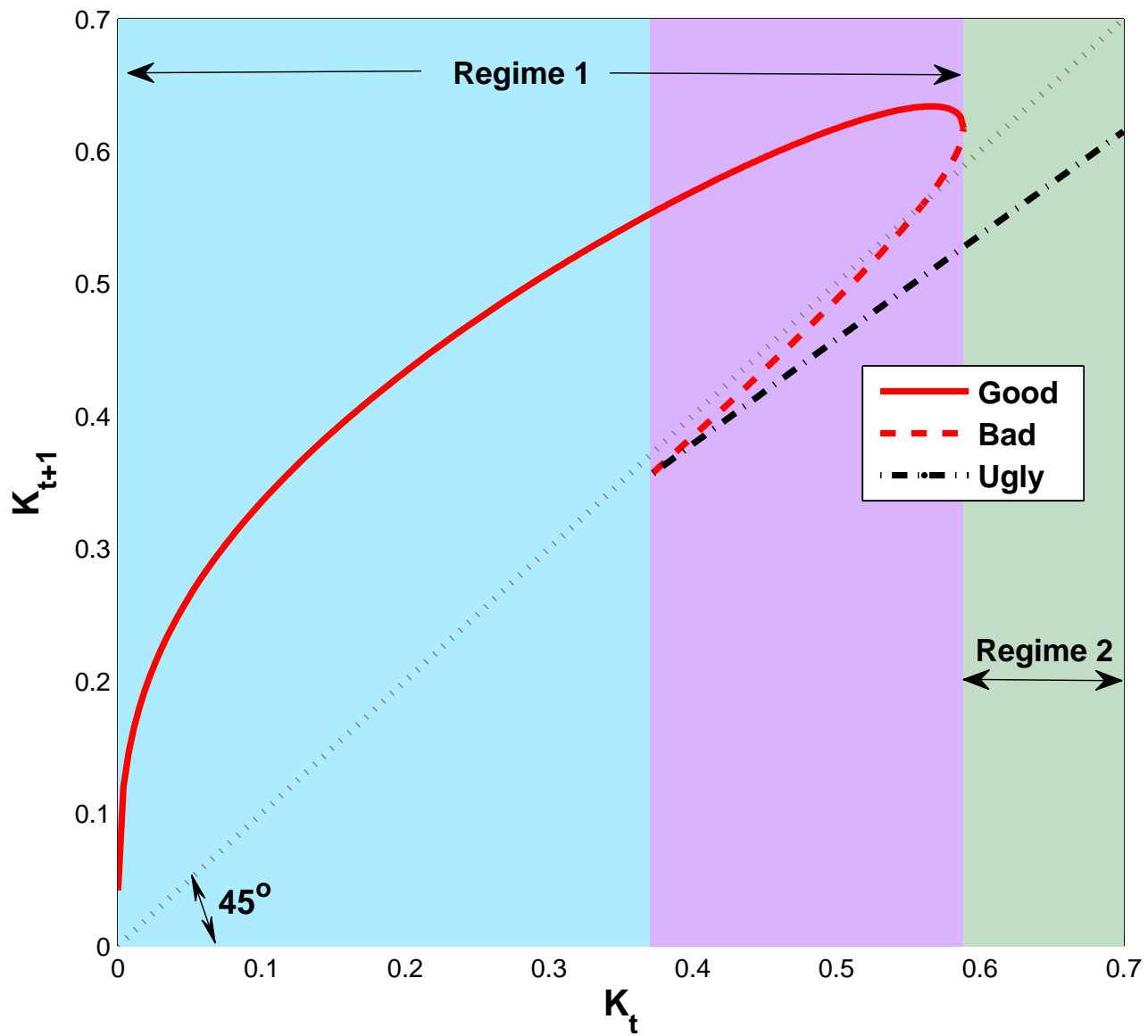
23

Figure 6: Phase Diagram for Aggregate Capital

Proposition 4 *Under Assumption [2] and $\xi < \xi_X \equiv \frac{r}{A_l}$, the dynamic economy has a unique steady state. If the credit policy satisfies $\xi < \xi_L \equiv \frac{\eta-\alpha}{\eta(1-\theta)}\frac{rz_{\min}}{A_l}$, the unique steady state is the **Good** equilibrium; if $\xi_L < \xi < \xi_H \equiv \frac{rz_{\min}}{A_l}$, the steady state is the **Bad** equilibrium; if $\xi_H < \xi < \xi_X$, the steady state is the **Ugly** equilibrium.*

Figure [7] gives graphical illustrations of the above proposition. A credit expansion (ξ increases) reduces two threshold values of capital stock K^* and K^{**}, and shifts the policy function of aggregate capital $g(K_t)$ towards the left. If the credit policy ξ equals ξ_L, the steady-state capital is exactly K^* (the upper-left panel in Figure [7]) therefore for any $\xi < \xi_L$, the steady state capital would be the **Good** equilibrium. If the credit policy equals ξ_H, the unique intersection between the policy function $g(K_t)$ and the 45 degree line is at K^{**} (the upper-right panel in Figure [7]) . So for any $\xi \in (\xi_L, \xi_H)$, the unique steady state is the **Bad** equilibrium. For $\xi = \xi_X$, the policy function $g(K_t)$ does not intersect with the 45 degree line (the bottom-left panel in the Figure), therefore for any $\xi > \xi_X$, the steady state does not exist.

4.2   Dynamics under Credit Expansions

In this section, we aim to discuss the dynamic effect under the expansionary credit policy. The policy function of capital $g(K_t)$ shows that when $K_t \in [K_t^{**}, K_t^*]$, the **Good**, the **Bad** and the **Ugly** equilibria coexist. Since these three equilibria can be Pareto-ranked, following Boissay, Collard, and Smets (2016), we exclude the coordination failure problem, and assume that the most efficient equilibrium (the **Good**) is always selected for the regime of multiple equilibria.

Assume that the economy initially stays at the steady state with the **Good** equilibrium, i.e., the initial level of credit policy satisfies $\xi_0 < \xi_L$. Then in the first period, the credit policy ξ_t undertakes a permanent increase. According to the previous analysis, the dynamic impact of this credit policy may depend on the magnitude of the expansion as well as the initial level of ξ_t. We now discuss the different cases based on the quantitative analysis.

First we do the following parameterizations. The capital share in the production function α is set to 0.4. The shape parameter in the Pareto distribution η is set to 2.5. To make the Assumption [2] hold, the moral hazard parameter θ is set to 0.01, implying the individual borrower can divert 1% of the loan. We normalize the sectoral TFP in L sector to 1, and set the TFP in H sector to 1.2. The discount rate β and the depreciation rate δ are set to 0.9 and 0.1, respectively.

We start with the case where the initial credit policy is tight (i.e., ξ_0 is far below ξ_L). In particular, we set ξ_0 to $0.25 \times \xi_L$. We assume that in the first period ξ_t permanently increases by 1%, 2% and 3%, respectively. Figure [8] presents the transition dynamics of the economy. It shows that when the level of ξ_0 is low, a credit expansion would unambiguously raise the output in H sector. Since in this case ξ_t is sufficiently below the threshold ξ_L, the economy would not hit the Regime 2 along the transition. As a result, the L sector is degenerated (i.e., $Y_l = 0$), and all the capitals are allocated
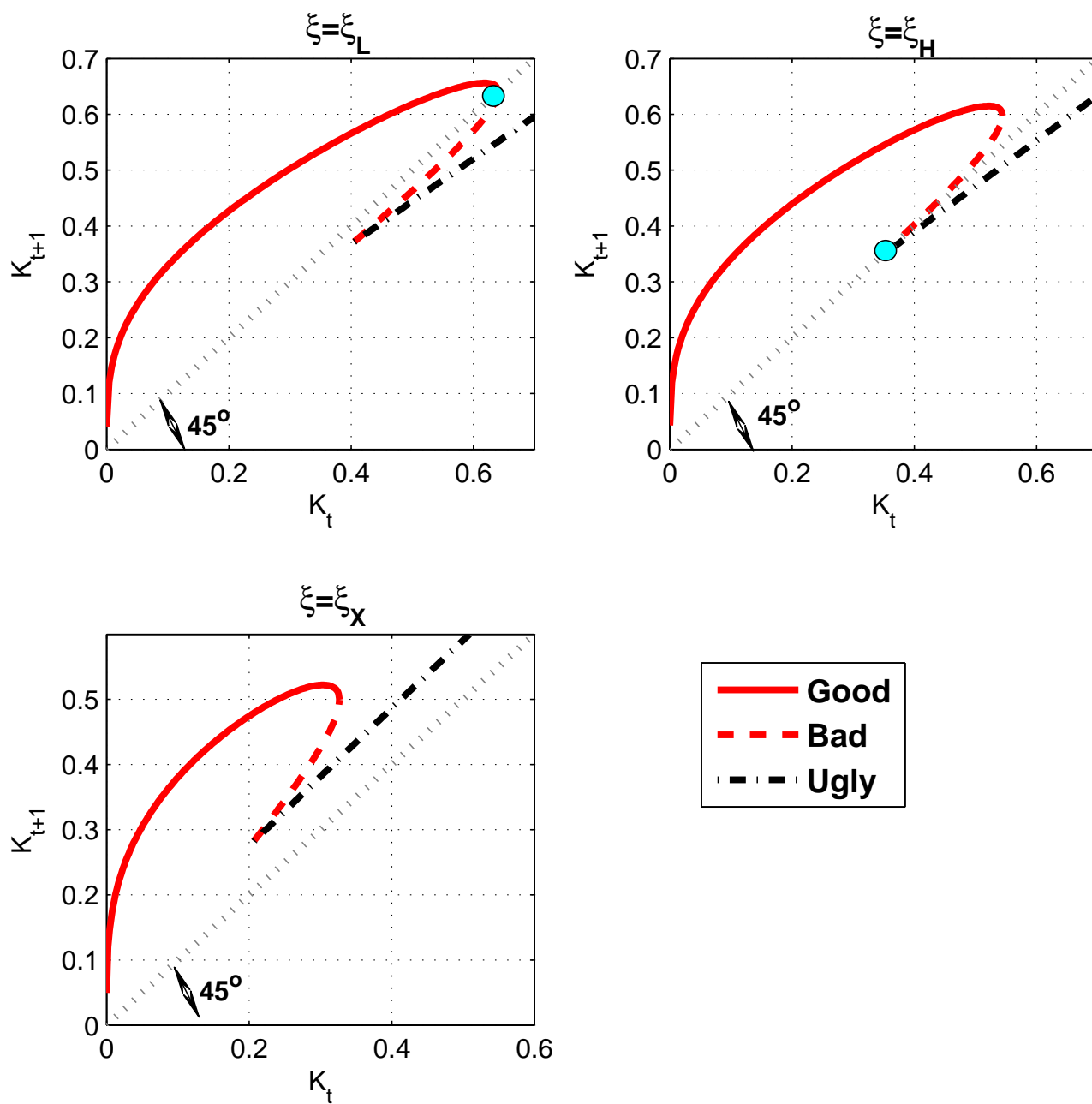
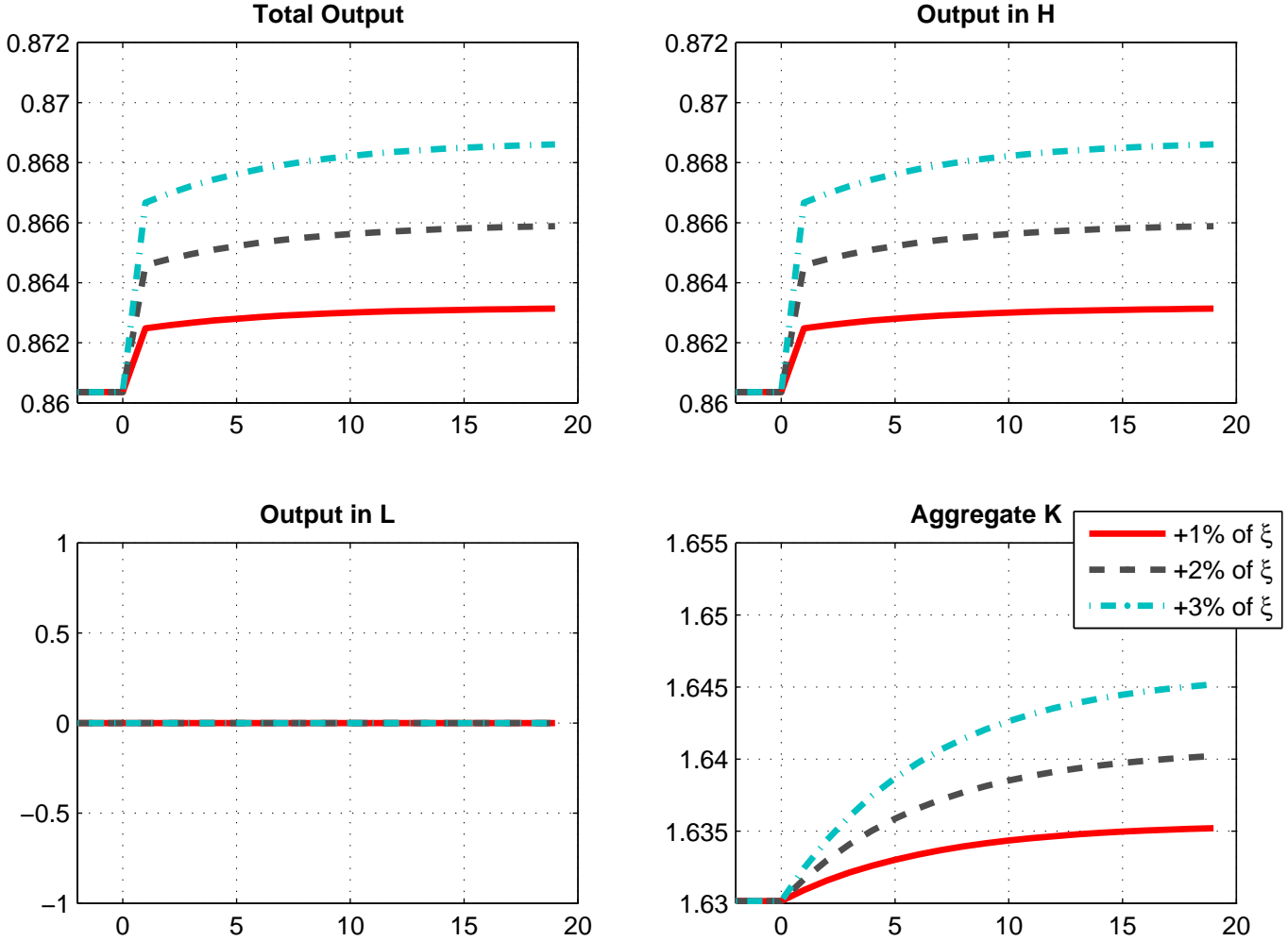Figure 7: Steady States under Different ξ

Figure 8: Transition Dynamics under Credit Expansions: $\xi_0 = 0.25 \times \xi_L$

to H sector. As a result, the cross-sector misallocation is absent. In addition, the magnitude of the responses for the output and capital are proportional to the level of credit expansion.

Now consider a case where the initial credit policy ξ_0 is close to but below the critical value ξ_L. In particular, we specify $\xi_0 = 0.92 \times \xi_L$. We then conduct the same exercise as the previous case. Figure 9 presents the transition dynamics. From the figure, it can be seen that unlike the previous case, a credit expansion has adverse effect on the aggregate economy. Moreover, for a relative small credit expansion with the increment of 1% or 2%, the reduction of aggregate variables are proportional to the change of credit policy. While, for a further large expansion (ξ_t is still less than ξ_L), namely increasing ξ_t by 3%, the economy may hit the Regime 2 during the transition where the interbank market is temporally collapsed. As a result, the output and the capital stock experience a sharp decline in the short run (see the blue lines in Figure 9). Since the steady state remains at the **Good** equilibrium, the transition path eventually converges.

Figure 10 provides a graphical illustration of the dynamics of capital stock through the phase dia-
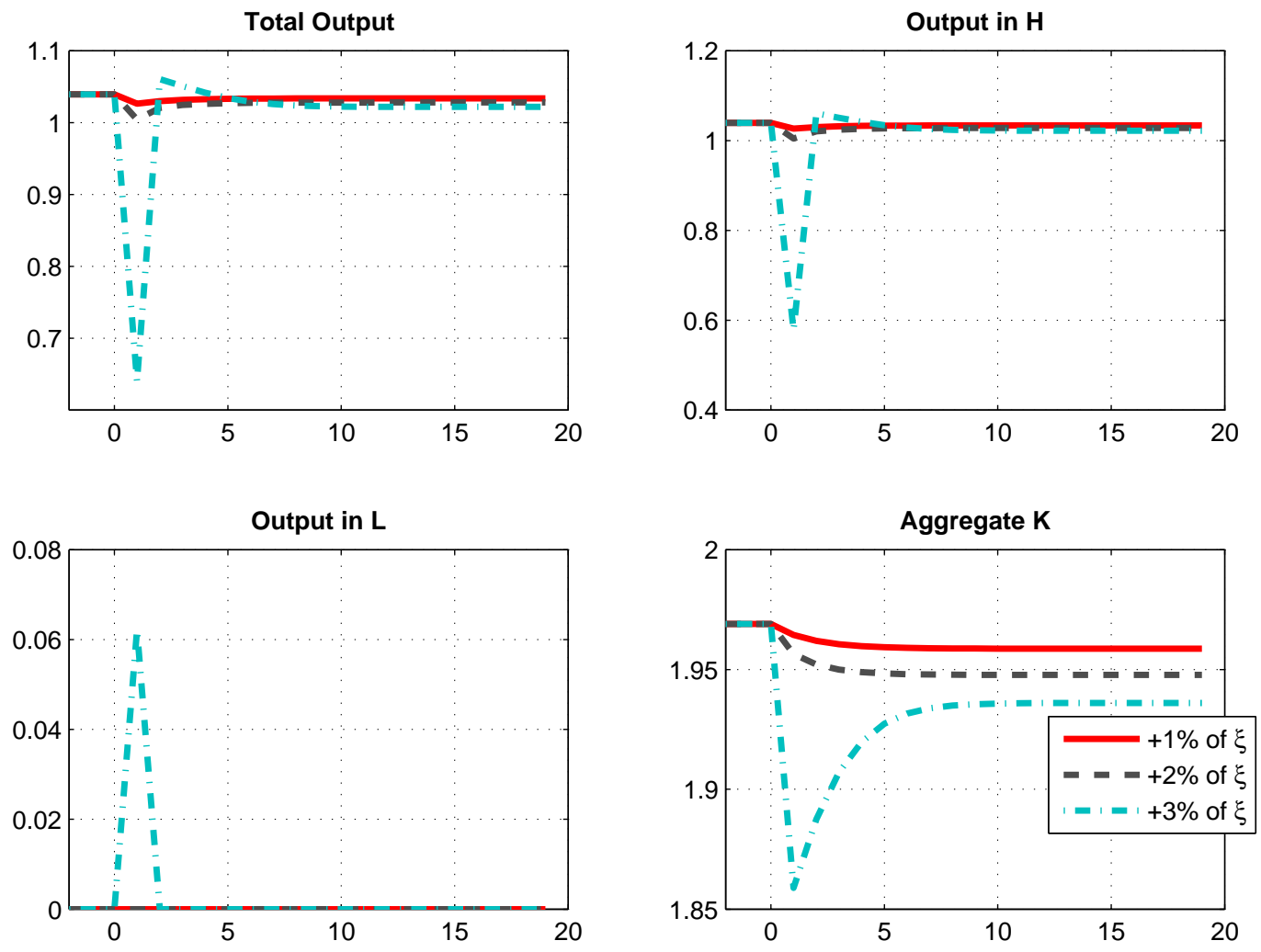
Figure 9: Transition Dynamics under Credit Expansions: $\xi_0 = 0.92 \times \xi_L$
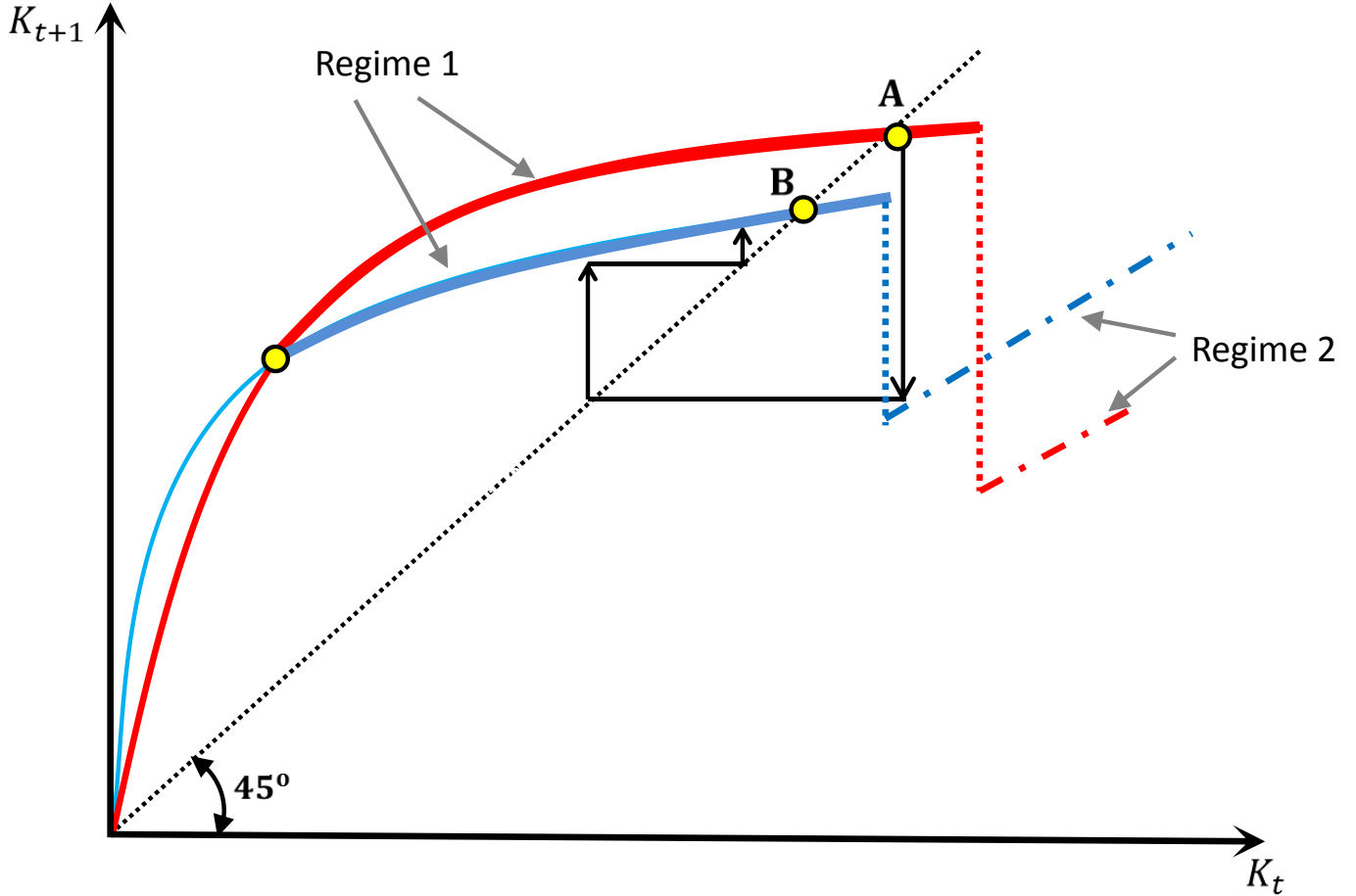
Figure 10: Phase Diagram: $\xi_0 = 0.92 \times \xi_L$

gram. The initial steady state is at point A. A credit expansion in this case reduces the threshold K_t^* and shifts the second half part of the policy function $g(K_t)$ in Regime 1 (the thick line) downwardly and the Regime 2 part (the dot-dashed line) upwardly. The intuition is that for the Regime 1 the credit expansion has non-monotonic impact on the capital. On the one hand, it raises the capitals that the bank can use (intensive margin); On the other hand it induces more banks to invest less efficient firms (extensive margin), exacerbating the resource misallocation. When the capital is close to K_t^* the adverse impact dominates, which explains the downward shift of $g(K_t)$ in the Regime 1. For the Regime 2 where the interbank market is collapsed, a credit expansion unambiguously increases the capitals (see the discussion of the output in the **Ugly** equilibrium), which leads to a upward shift of $g(K_t)$. As a result, during the transition the economy may hit the Regime 2, leading to the collapse of the interbank market. This explains the large drop of capital and output during the transition after a credit expansion.

In the previous two cases, the credit expansion does not alter the property of the steady state. We now consider a case where the credit expansion results in regime switch between different steady

states (from the **Good** to the **Ugly**) and generates endogenous business cycles. In particular, we specify the initial credit policy is close to ξ_L, namely $\xi_0 = 0.96 \times \xi_L$. The steady state in the initial period is the **Good** equilibrium. We again consider various small credit expansions, i.e., ξ_t will permanently increases by 1%, 2% and 3%, respectively. It turns out that under the credit expansion of 3%, ξ_t exceeds the threshold value ξ_L (but is still less than ξ_H). In light of the previous steady-state analysis, when $\xi_t \in (\xi_L, \xi_H)$ the economy has multiple equilibria and the steady state is the **Bad** equilibrium (see Figure 7). Since for the case of multiple equilibria only the **Good** equilibrium is selected, the steady state in this case does not exist. The credit expansion eventually leads to endogenous fluctuations. To see this, Figure 11 plots the transition dynamics for the expansionary credit policies with increments of 1%, 2% and 3% respectively. It shows that when a credit expansion does not make ξ_t exceeds ξ_L, the economy first experiences a sharp drop due to the temporal collapse of the interbank market, and then converges to the steady state—the **Good** (see the red and the black lines). However, if the credit expansion induces ξ_t to pass the threshold ξ_L, the transition dynamics will present oscillations as shown by the blue lines.

The intuition can be seen from the phase diagram Figure 12. A sufficiently large credit expansion shifts the Regime 1 part of policy function $g(K_t)$ (the thick red line) downwardly and the Regime 2 part upwardly. Because of the discontinuity, the new policy function does not intersect with the 45 degree line. In this case, the steady state is not well defined. As a result, after the credit expansion the capital stock periodically switches between the **Good** and the **Ugly** equilibria, resulting in oscillation dynamics.

In sum, the credit expansion has asymmetric impact on the aggregate economy and leads to rich dynamics for the transitions. More specifically, a rise in ξ_t may increase or reduce the aggregate output and capital depending on the initial level of credit policy. It may also cause endogenous fluctuations because of the regime switches between different equilibria.

4.3 Dynamics under Technology Shocks

We now document the dynamic impacts of TFP shocks on the aggregate economy. In particular, we will discuss a shock to TFP in H sector and l, separately. In light of the previous steady state analysis, the change of TFP in H sector does not influence the critical values ξ_L and ξ_H (see the Proposition 4). Therefore, given a credit policy ξ, a reduction in A_h may not change the steady state regime. That is, if the the economy is initially at the **Good** steady state, the new steady state after a permanent reduction of A_h remains the **Good** regardless the magnitude of the adverse shock. So, a negative shock to A_h does not lead to oscillation dynamics. We now consider a concrete quantitative example. We assume that in the period 0 the economy is at the steady state. We specify $\xi_t = 0.9 \times \xi_L$, so the initial steady state is the **Good** equilibrium. In the period 1, the TFP in H sector declines permanently by 1%, 2% and 3%, respectively. Figure 13 presents the corresponding
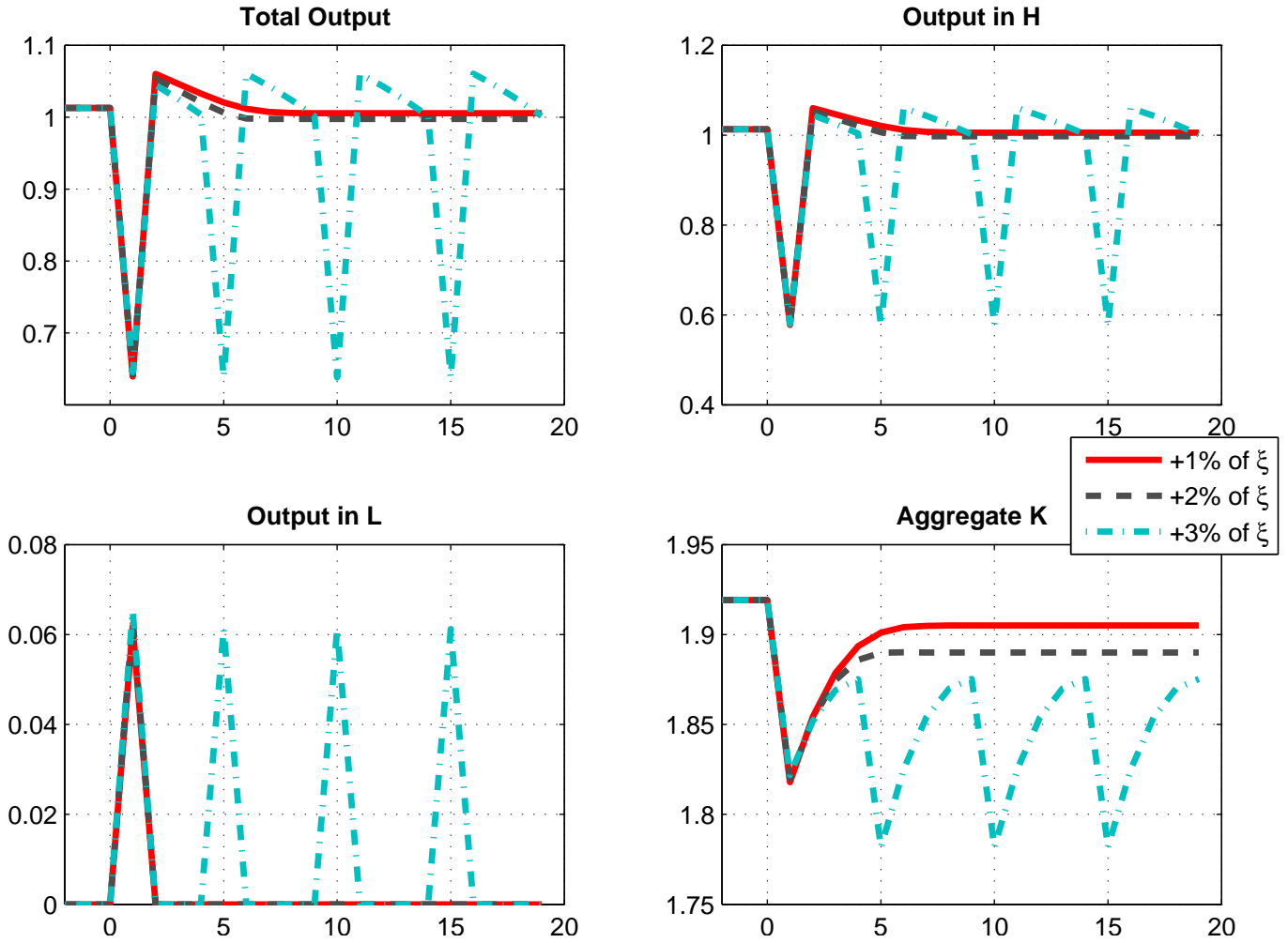
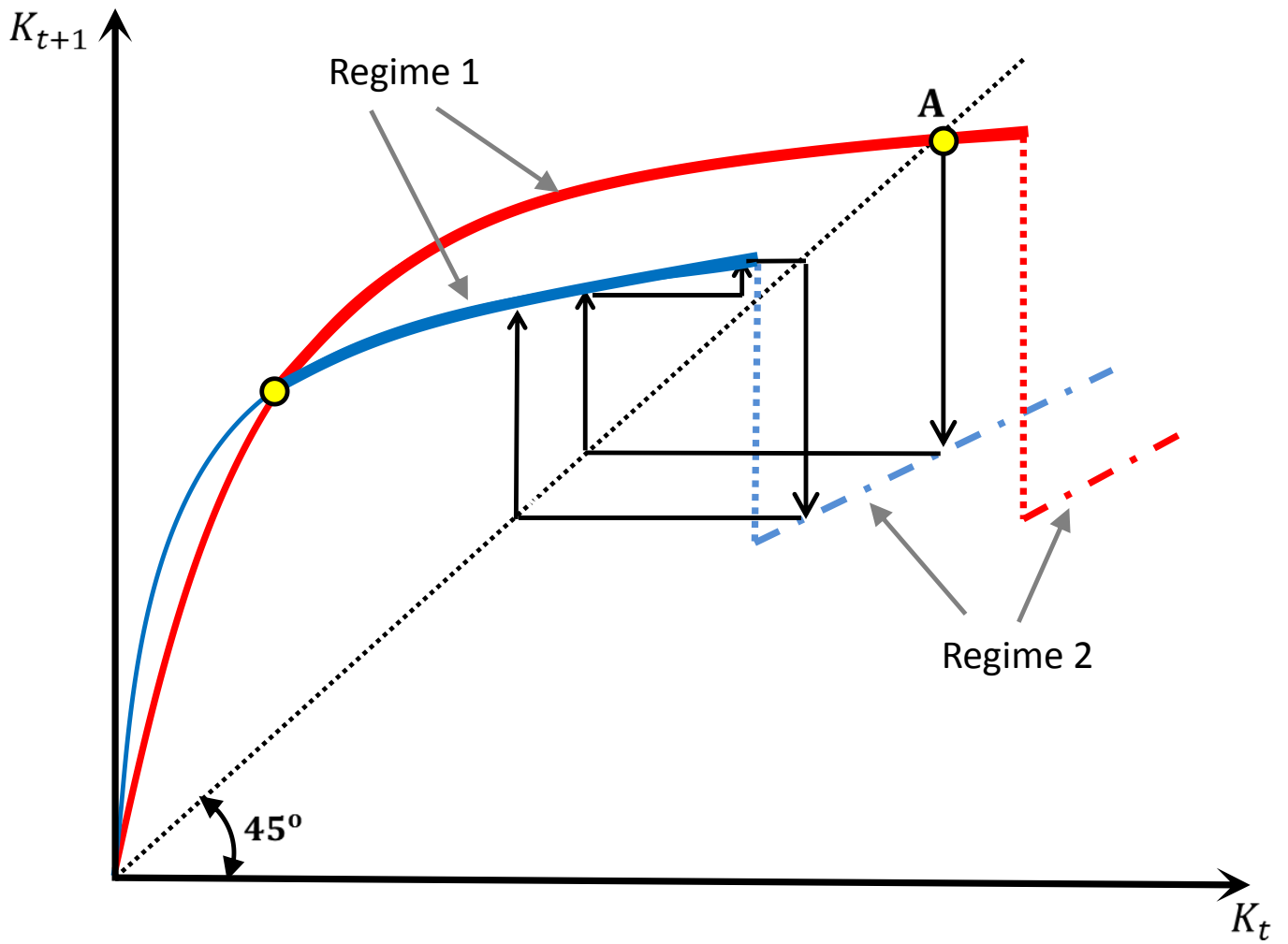Figure 11: Transition Dynamics under Credit Expansions: $\xi_0 = 0.96 \times \xi_L$

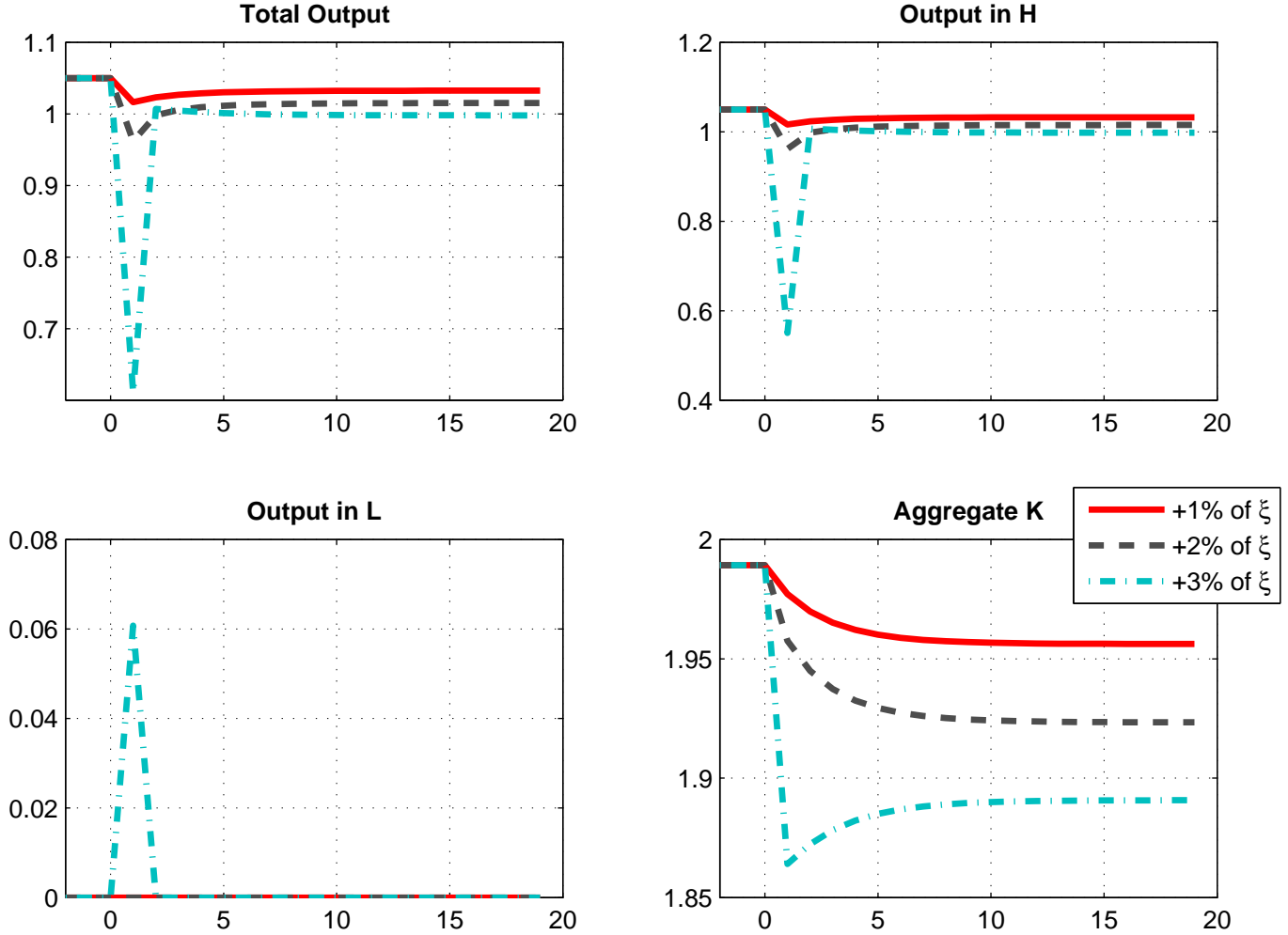Figure 12: Phase Diagram: $\xi_0 = 0.96 \times \xi_L$ and $\xi_1 > \xi_L$

Figure 13: Transition Dynamics under Negative A_h shock: $\xi = 0.9 \times \xi_L$

transition dynamics. It shows that an adverse TFP shock in efficient H sector depresses the aggregate economy. For a relative small shock (namely 1% and 2%), the depression is proportional to the decline in TFP. While, a larger shock (namely 3%) triggers a large-scale decline in the aggregate output and capital in the short run because of the temporal collapse of the interbank market. This pattern looks very similar to those in the credit expansion where $\xi_t < \xi_L$.

We now turn to the TFP shocks to L sector. Since a rise of A_l reduces the dispersion of TFPs in two sectors, to document the negative effect of A_l shocks, we consider an increase of A_l. As has been shown in the steady-state analysis, unlike the A_h shock the A_l shock matters for the threshold values ξ_L and ξ_H, therefore a sufficiently large A_l shock may alter the regime of steady states, resulting in oscillation dynamics. To see this, we consider a quantitative example. Akin to the previous analysis, we assume the economy initially stays at the **Good** steady state, where $\xi = 0.9 \times \xi_L$. In the period 1, the TFP in L sector increases by 1%, 5% and 10% respectively. Figure 14 reports the transition dynamics. For a small increase of A_l (namely, 1%), the aggregate output and capital experience
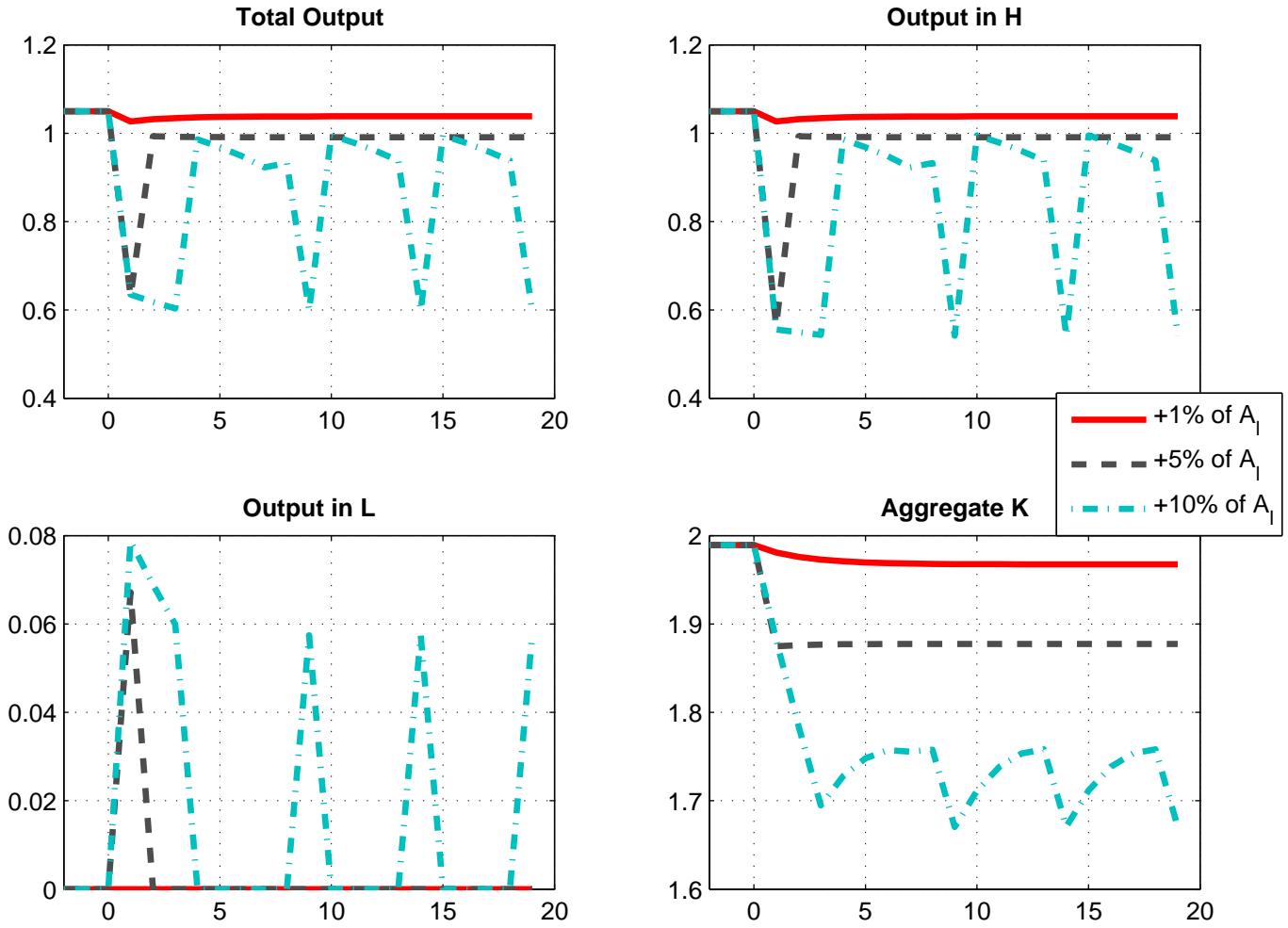
Figure 14: Transition Dynamics under Positive A_l Shock: $\xi = 0.9 \times \xi_L$

moderate declines. If A_l increases further (namely, 5%) the economy may temporally hit the crisis regime causing a large drop in output and capital in the short run. Since the new steady state now is still a **Good** equilibrium, the transition path eventually converges. However, if the increase of A_l is considerably large (e.g., 10%), the steady state may not be well defined (the policy function $g(K_t)$ does not cross the 45 degree line), consequently the transition paths present oscillation dynamics (the blue lines in Figure 14). The above analysis indicates that sectoral TFP shocks may have distinct effects on the real economy. Unlike the standard business cycle theories, the TFP shocks to the inefficient sector may lead to endogenous fluctuations.

5 Related Literature

The current paper is generally related to an extensive volume of literature which we do not attempt to go through here. Instead, we only highlight papers that are most closely related. First, one of the purpose of this paper is to develop a model in which agency frictions in the interbank markets influence investment in both economic downturns and economic booms. Our paper is definitely not the first paper to use agency problems in financial markets to generate endogenous instability and fluctuations. The idea that the separation of creditors and investors may generate macroeconomic (in)stability dates back at least to Keynes (1936) and Harrod (1939). Theoretical progress made on endogenous credit cycles due to agency frictions include Aghion, Banerjee, and Piketty (1999), Suarez and Sussman (1997), Gu et al. (2013), Matsuyama (2013) and recently by Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2016).

Moreover, our model predicts asymmetric effects of shocks to the economy, which is in line with empirical findings. In particular, Ordoñez (2013) empirically shows the asymmetric effects of financial frictions: quickly during crises but slowly during recoveries. I show that this asymmetry is stronger in countries with less developed financial systems and greater financial frictions. Early works that study the asymmetry of financial and real variables include Veldkamp (2005), Jovanovic (2006) and Van Nieuwerburgh and Veldkamp (2006), etc.

More importantly, our paper contributes the literature on the misallocation consequences of credit expansion (liquidity injection) policy, especially after the recent global financial crisis.[15] Using Portugal as a case in point, Reis (2013) shows that the capital inflow can encourage the expansion of relatively unproductive firms in the nonchargeable sector at the expense of more productive tradable-sector firms. Gopinath et al. (2017) show that capital inflow into Italy and Portugal lowers interest rate, which in turn implies a significant decline in sectoral TFP by misallocating credit toward firms that are not necessarily more productive. Bai, Hsieh, and Song (2016) find that the declining trend of

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[15]Our paper contributes to the literature on misallocations. See Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) for general discussion and Banerjee and Moll (2010), Jermann and Quadrini (2012), Moll (2014), Buera, Kaboski, and Shin (2011), Brandt and Zhu (2000) for misallocation driven by financial frictions.

the dispersion of firm's marginal product of capital has been persistently reverted after the 4-trillion fiscal stimulus by China in 2009 and 2010. Cong et al. (2017) find that in contrast to the pre-stimulus years, credit stimulus was allocated relatively more towards state-owned, low-productivity firms than to privately-owned, high-productivity firms. Our theoretical predictions are consistent with all of aforementioned empirical findings.

Our paper is also related to the literature that studies the credit boom led financial crisis. The most relevant research is Boissay, Collard, and Smets (2016). Their empirical analysis indicates that banking crisis is more likely to burst after intensive credit expansion, and in turn tends to generate deep and long-last recessions. Similarly, Gorton and Ordoñez (2016) shows some credit boom end in a crisis while others do not. Moreover, they find that credit booms start with an increase in productivity growth, which subsequently falls faster during bad booms. Additionally, Bleck and Liu (2017) develop a model of credit expansion and credit misallocation across sectors. The main difference between our paper and the above three papers is that we emphasize the importance of the interaction between within-sector misallocation and cross-sector misallocation. Instead, there is only across-sector misallocation in Boissay, Collard, and Smets (2016) and Gorton and Ordoñez (2016). As illustrated by Proposition 2 in Section 3.1, the within-sector misallocation in our paper not only implies an inverted U-shape for the relationship between credit expansion and output, but also generates a quantitatively larger effect at the regime switch point. Boissay, Collard, and Smets (2016) also generates a non-monotonic demand curve because of the moral hazard problem. In their paper, a large credit boom may trigger a interbank market freeze equilibrium. Gorton and Ordoñez (2016) provides an alternative mechanism, in which a recession after a credit boom is due to the regime switch between the information insensitive and sensitive equilibria.[16] Note that in these papers, the credit booms are the consequences of the productivity grows, so they do not particularly study the credit expansion policy. Moreover, the financial risk capacity due to within-sector misallocation is smaller than that in the literature, including Boissay, Collard, and Smets (2016).[17]

6   Conclusion

Increasing empirical works reveal that a large portion of financial crises follow credit expansions, the facts that have been explained by the recent theoretical work through the sectoral level misallocation channel. Although the firm level misallocations have been proved to be important for the real economy (Hsieh and Klenow, 2009), its consequences on the credit expansion policy remains unclear.

To fill the gap, we introduce the firm level misallocations into a banking crisis model to evaluate both the aggregate and disaggregate consequences of credit expansion policy. In our model, banks

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[16]Also see Dong, Miao, and Wang (2016) who develop a dynamic model with adverse selection in the financial market to study the interaction between funding liquidity and market liquidity and the implications of credit shock.

[17]See Bigio (2015), among others, for the discussion of financial risk capacity of banking sector under adverse selection in financial markets, which is used to explain the slow recovery of bank capital and economic activity.

can offer loans to the firms with idiosyncratic productivities. The quantity of credits that the banks can provide is controlled by the government through the credit policy. The heterogeneity of bank's investment return gives rise to an interbank market. The moral hazard and asymmetric information leads to different regimes of the market equilibrium. A credit expansion on the one hand raises the credit supply that may stimulate the production side, on the other hand it leads more inefficient projects to be financed. In a stylized static model, we analytically show that the credit condition determines the properties of the equilibrium. A unique **Good** equilibrium where the interbank market is well functioned can be achieved when the credit policy is sufficiently tight. Multiple equilibria (the **Good** and the **Bad**) may emerge when the credit policy becomes looser. In the **Bad** equilibrium, the within-sector misallocation is relatively acute though the interbank market still works. If the credit expansion policy is sufficiently large, the unique **Ugly** equilibrium emerges where the interbank market collapses. So an intensive credit expansion may cause regime switch between the **Good** equilibrium to the banking crisis equilibrium (the **Ugly**). The contraction effect of crisis is exacerbated by the within-sector misallocations due to the credit expansion at firm level. In a dynamic setup, the firm level misallocations provide a novel channel through which a large credit expansion may cause interbank market collapse and endogenous boom-bust cycles for the real economy.

To make the transmission mechanism as transparent as possible, the setup of our work is intentionally stylized, and therefore we cannot do full justice to reality. To make the model setup more realistic, we could introduce the labor input into the inefficient sector, which may make this sector coexists with the efficient sector when the equilibrium is the **Good**. We believe this extension will make the model dynamics be richer and fit the data better. Our model can also be flexibly extended to a small open economy by taking the inefficient sector as an international capital market. The extended model can be used to discuss the financial integration or foreign interest rate shocks on the domestic capital market. It may offer new insights on capital control for the international capital flows. We can embed our model into a growth model to address the connections between cycles and trends as the discussions in Gorton and Ordoñez (2016). We leave all these potential topics for the future research.

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Appendix

A Proofs

Lemma A.1 *Under Assumption 2, $\Gamma(z^*)$ is strictly convex with z^* over $(z_{\min}, z_{\max})$, and achieves its minimum at $\hat{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}} z_{\min} > z_{\min}$.*

Proof of Lemma A.1: When z follows Pareto distribution such that $\mathbf{F}(z) = 1 - (z/z_{\min})^{-\eta}$ with $\eta > 1$ and $\mathbf{E}(z) = \frac{\eta}{\eta-1} z_{\min} = 1$, we can easily prove that $\mathbf{E}(z|z \geq z^*) = \frac{\eta}{\eta-1} z^* = \frac{z^*}{z_{\min}}$. Then $\Gamma(z^*)$ can be simplified as

$$\Gamma(z^*) = z_{\min}^{\alpha-1} \left[\frac{\theta}{z_{\min}^{\eta}} (z^*)^{\eta-\alpha} + (1-\theta)(z^*)^{-\alpha} \right]. \tag{A.1}$$

Since $\eta > 1 > \alpha$, we must have $\eta - \alpha > 0$. Moreover, we can derive

$$\Gamma'(z) = z_{\min}^{-1+\alpha} \left[\frac{\theta}{(z_{\min})^{\eta}} (\eta - \alpha) z^{\eta-\alpha-1} - \alpha(1-\theta) z^{-\alpha-1} \right],$$

$$\Gamma''(z) = z_{\min}^{-1+\alpha} \left[\frac{\theta}{(z_{\min})^{\eta}} (\eta - \alpha)(\eta - \alpha - 1) z^{\eta-\alpha-2} + \alpha(\alpha+1)(1-\theta) z^{-\alpha-2} \right].$$

Under the first part of Assumption 2, i.e., $\eta > 1 + \alpha$, we know that $\Gamma(z^*)$ is strictly convex in z^*, i.e., $\Gamma''(z) > 0$. So the minimum of $\Gamma(z^*)$ is achieved under the first order condition $\Gamma'(z^*) = 0$, i.e.,

$$z^* = \hat{z} \equiv \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}} z_{\min}. \tag{A.2}$$

We can obtain interior solution of $\Gamma'(z^*) = 0$ (i.e., $\hat{z} > z_{\min}$) if and only if $\eta < \frac{\alpha}{\theta}$, i.e., the second half of Assumption 2.

Notice that the solution of the equilibrium equation (12) guarantees the condition $R^f > A_l$. To see this, from (6) we have $R^f = \pi_h z^*$. So the condition $R^f > A_l$ is equivalent to $\alpha(\xi K)^{\alpha-1} \frac{A_h}{A_l} > \frac{1}{[\mathbf{E}(z|z \geq z^*)]^{\alpha-1} z^*}$. Since the solution of (12) implies

$$\alpha(\xi K)^{\alpha-1} \frac{A_h}{A_l} = \frac{\theta \frac{\mathbf{F}(z^*)}{1 - \mathbf{F}(z^*)} + 1}{[\mathbf{E}(z|z \geq z^*)]^{\alpha-1} z^*} > \frac{1}{[\mathbf{E}(z|z \geq z^*)]^{\alpha-1} z^*},$$

and thus the non-collapse condition $R^f > A_l$ is satisfied. **Q.E.D.**

Proof of Proposition 1: When $\xi > \xi^*$, the condition

$$\alpha(\xi K)^{\alpha-1} \frac{A_h}{A_l} \geq \Gamma(\hat{z}) \tag{A.3}$$

is violated. So the equation (12) does not have a solution, i.e., the interbank market equilibrium cannot be supported. Therefore, the interbank market equilibrium exists if and only if $\xi \leq \xi^*$.

From the definition of two thresholds,

$$\xi^{**} = \left[\frac{\alpha K^{\alpha-1} \frac{A_h}{A_l}}{\Gamma(z_{\min})} \right]^{\frac{1}{1-\alpha}}, \tag{A.4}$$

$$\xi^* = \left[\frac{\alpha K^{\alpha-1} \frac{A_h}{A_l}}{\Gamma(\hat{z})} \right]^{\frac{1}{1-\alpha}}, \tag{A.5}$$

we immediately know that $\xi^{**} < \xi^*$. The property of $\Gamma(z)$ further implies that under $\xi < \xi^{**}$, the equilibrium condition (12) has unique solution. While if $\xi^{**} < \xi < \xi^*$, there exist two equilibria: one is less efficient and the other is Pareto improving. We label the equilibrium for $z^* > \hat{z}$ as the **Good** equilibrium and the one for $z^* < \hat{z}$ as the **Bad** equilibrium. **Q.E.D.**

Corollary A.1 *Under Assumption 2, the risk capacity ξ^* and ξ^{**} increase with the sectoral TFP dispersion A_h/A_l and decreases with the volatility of the idiosyncratic productivities. Moreover, ξ^* increases with the severity of moral hazard problem θ.*

Proof of Corollary A.1: The definition of ξ^{**} in (A.4) immediately suggests that ξ^{**} increases with A_h/A_l, and is independent of θ. Furthermore, with the Pareto distribution, ξ^{**} can be further expressed as

$$\xi^{**} = \left[\alpha K^{\alpha-1} \frac{A_h}{A_l} \frac{\eta-1}{\eta} \right]^{\frac{1}{1-\alpha}},$$

which implies ξ^{**} increases with η. Since $\mathbf{Var}(z)$ decreases with η, we know that ξ^{**} decreases with $\mathbf{Var}(z)$.

The comparative statics of ξ^* with respect to $\frac{A_h}{A_l}$ is obvious. As shown in the proof of Lemma A.1, we have

$$\hat{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha} \right)^{\frac{1}{\eta}} z_{\min},$$

and therefore we have

$$\Gamma(\hat{z}) = \left(\frac{\eta}{\eta-1} \right) \left(\frac{\eta}{\eta-\alpha} \right) \left(\frac{\eta-\alpha}{\alpha} \right)^{\frac{\alpha}{\eta}} \theta^{\frac{\alpha}{\eta}} (1-\theta)^{1-\frac{\alpha}{\eta}}.$$

We can easily verify that $\Gamma(\hat{z})$ strictly increases with the severity of moral hazard θ under the condition $\theta < \frac{\alpha}{\eta}$, which is satisfied under the Assumption 2. Note that the definition of ξ^* implies that ξ^* strictly decreases with θ.

42

Finally, the variance of z under Pareto distribution is given by

$$\mathbf{Var}\,(z) = \frac{\left[\mathbf{E}\,(z)\right]^2}{\eta\,(\eta - 2)} = \frac{1}{\eta\,(\eta - 2)},$$

which is well defined and strictly decreases with η provided $\eta > 2$. Moreover, we can verify that $\Gamma\,(\hat{z})$ strictly increases with η. Therefore ξ^* decreases with $\mathbf{Var}\,(z)$. **Q.E.D.**

Proof of Proposition 2: Under the Pareto distribution, (12) and (A.1) implies

$$\xi = \left(\alpha \frac{A_h}{A_l}\right)^{\frac{1}{1-\alpha}} \frac{1}{K} \Gamma\,(z^*)^{-\frac{1}{1-\alpha}}. \tag{A.6}$$

Put last equation into (13) and recall that $\mathbf{E}\,(z|z \geq z^*) = \frac{z^*}{z_{\min}}$, we can express the output as

$$Y = A_h \left(\alpha \frac{A_h}{A_l}\right)^{\frac{\alpha}{1-\alpha}} \Omega\,(z^*)^{-\frac{\alpha}{1-\alpha}}, \tag{A.7}$$

where

$$\Omega\,(z^*) = \frac{\theta}{z_{\min}^{\eta}}\,(z^*)^{\eta - 1} + (1 - \theta)\,(z^*)^{-1}.$$

We can easily show that $\Omega''\,(z^*) > 0$, i.e., $\Omega\,(z^*)$ is convex in z^*, and reaches its minimum at $\tilde{z} = \left(\frac{1-\theta}{(\eta-1)\theta}\right)^{\frac{1}{\eta}} z_{\min}$. According to Assumption 2, $\theta < \frac{\alpha}{\eta} < \frac{1}{\eta}$, and therefore $\frac{1-\theta}{(\eta-1)\theta} > 1$. Then immediately we know that $\tilde{z} > z_{\min}$. Since ξ is strictly decreasing in z^*, we must have that the aggregate output achieves its optimal level when $\xi = \tilde{\xi} \equiv \left[\frac{\alpha K^{\alpha-1}\frac{A_h}{A_l}}{\Gamma(\tilde{z})}\right]^{\frac{1}{1-\alpha}}$. Recall that the threshold value of the regime switch ξ^* satisfies $\xi^* = \left[\frac{\alpha K^{\alpha-1}\frac{A_h}{A_l}}{\Gamma(\hat{z})}\right]^{\frac{1}{1-\alpha}}$, where $\hat{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}} z_{\min}$. Since $\hat{z} < \tilde{z}$, we must have $\tilde{\xi} < \xi^*$.

It remains for us to prove Y is concave in ξ. First, since $\Omega\,(z^*)$ is convex in z^*, equation (A.7) implies that Y is concave in z^*. Second, equation (12) implies that

$$\xi = \left[\frac{\Gamma\,(z^*)}{\alpha K A_h/A_l}\right]^{-\frac{1}{1-\alpha}}.$$

We can easily prove that $\Gamma\,(z^*)$ is convex with z^*. Meanwhile, $x^{-\frac{1}{1-\alpha}}$ can be easily verified as a convex function. Therefore, ξ is convex in z^*, or equivalently, z^* is concave in ξ. Consequently, the concavity of both $Y\,(z^*)$ and $z^*\,(\xi)$ implies that $Y = Y\,(\xi) = Y\,(z^*\,(\xi))$ is concave with ξ.

Since

$$\Gamma\,(\tilde{z}) = \frac{\theta \frac{\mathbf{F}(\tilde{z})}{1 - \mathbf{F}(\tilde{z})} + 1}{[\mathbf{E}\,(z|z \geq \tilde{z})]^{\alpha - 1}\,\tilde{z}} = \left(\frac{\eta}{\eta - 1}\right)^2\,(\eta - 1)^{\frac{\alpha}{\eta}} \cdot \theta^{\frac{\alpha}{\eta}}\,(1 - \theta)^{1 - \frac{\alpha}{\eta}},$$

43

we can easily prove that $\tilde{\xi}$ increases with cross-sector TFP dispersion $\frac{A_h}{A_l}$, decreases with aggregate capital stock K and decreases with the severity of moral hazard problem θ, similar to comparative statics of ξ^* in Proposition A.1. **Q.E.D.**

Corollary A.2 *Under the Regime 1 for the **Bad** equilibrium, i.e., $\xi \in [\xi^{**}, \xi^*]$, the aggregate output Y is strictly increasing in ξ, and $Y^{Bad}(\xi^*) = Y^{Good}(\xi^*)$.*

Proof of Corollary A.2: From the definition of $\tilde{\xi}$, we know that $\tilde{\xi}$ satisfies $\alpha \left(\tilde{\xi} K\right)^{\alpha-1} \frac{A_h}{A_l} = \Gamma(\tilde{z})$, where $\tilde{z} = \left[\frac{1-\theta}{(\eta-1)\theta}\right]^{\frac{1}{\eta}} z_{\min}$. Under the Assumption 2, we have $\tilde{z} > \hat{z} > z_{\min}$. As the function is $\Omega(z^*)$ convex, i.e., $\Omega''(z^*) > 0$, we must have $\frac{\partial Y}{\partial z^*} > 0$ for any $z^* \in [z_{\min}, \tilde{z}]$. From the previous analysis, we already have $\frac{\partial z^*}{\partial \xi} > 0$, therefore we eventually obtain $\frac{\partial Y}{\partial \xi} = \frac{\partial Y}{\partial z^*} \frac{\partial z^*}{\partial \xi} > 0$. Moreover, when $\xi = \xi^*$, the **Bad** equilibrium converges to the **Good** one since $\Gamma(z)$ now reaches minimum at $\hat{z}$. Therefore, we must have $Y^{Bad}(\xi^*) = Y^{Good}(\xi^*)$. **Q.E.D.**

Lemma A.2 *Under the Assumption 2, there exists a unique equilibrium (the **Ugly**) in the Regime 2 where $\xi > \xi^*$.*

Proof of Lemma A.2: Under the Pareto distribution, we have

$$\Phi(z^*) = z_{\min}^{-(1-\alpha)(1-\eta)} (z^*)^{(1-\alpha)(1-\eta)-1},$$

with the properties $\lim_{z^* \to z_{\min}} \Phi(z^*) = \lim_{z^* \to z_{\min}} \Gamma(z^*) = \frac{1}{z_{\min}}$ and $\lim_{z^* \to +\infty} \Phi(z^*) = 0$. Since $\Phi(z^*)$ is monotonically decreasing, to show the existence of unique solution of (15) is equivalent to show $\alpha \frac{A_h}{A_l}(\xi K)^{\alpha-1} < \Phi(z_{\min})$. This is indeed the case. Remember that under the Assumption 2, i.e., $\eta < \frac{\alpha}{\theta}$, we have $\Gamma(z_{\min}) > \Gamma(\hat{z})$ because $\hat{z}$ solve the problem $\min_z \Gamma(z)$. Moreover, as $\Phi(z_{\min}) = \Gamma(z_{\min})$, we immediately have $\Phi(z_{\min}) > \Gamma(\hat{z})$. Since in the Regime 2, the credit policy ξ must satisfy $\xi > \xi^*$ or equivalently $\alpha \frac{A_h}{A_l}(\xi K)^{\alpha-1} < \Gamma(\hat{z})$. Then, we must have

$$\alpha \frac{A_h}{A_l}(\xi K)^{\alpha-1} < \Gamma(\hat{z}) < \Phi(z_{\min})$$

Consequently, equation (15) has a unique solution. **Q.E.D.**

Lemma A.3 *The sectoral outputs Y_h and Y_l and the aggregate output Y increase with the credit policy ξ.*

Proof of Lemma A.3: Once the equilibrium z^* is solved from (15), it is straightforward to obtain the sectoral outputs. In particular, we have

$$Y_h = A_h \{\xi K [1 - \mathbf{F}(z^*)] \mathbf{E}(z|z \geq z^*)\}^\alpha = \frac{A_l \xi K}{\alpha} \int_{z > z^*} \frac{z}{z^*} d\mathbf{F}(z), \tag{A.8}$$

where the second equality is obtained by using (15),and $Y_l = A_l \xi K \mathbf{F}(z^*)$.

Meanwhile, substituting (15) into (A.8), and we know that Y_h strictly increases with ξ. On the other hand, (15) implies that z^* strictly increases with ξ, and so does Y_l. Consequently, the aggregate output $Y = Y_h + Y_l$ also monotonically increases with ξ. **Q.E.D.**

Proof of Proposition 3: According to the definition, the ξ^* satisfies $\alpha (\xi^* K)^{\alpha-1} \frac{A_h}{A_l} = \Gamma(z_1^*)$, where $z_1^* = \arg\min_z \Gamma(z) = \hat{z}$. Let z_2^* denote the equilibrium cutoff in Regime 2 when $\xi = \xi^*$, i.e., $\alpha (\xi^* K)^{\alpha-1} \frac{A_h}{A_l} = \Phi(z_2^*)$. Since as discussed earlier $\Phi(z) < \Gamma(z)$ for any $z > z_{\min}$ and $\Phi(z)$ is monotonically decreasing in z, we must have $z_2^* < z_1^*$. So the equilibrium cutoff z^* is discontinuous at $\xi = \xi^*$.

Based on the above Lemma, we can further show that the aggregate output is discontinuous at the $\xi = \xi^*$. In particular, the aggregate output in Regime $j \in \{1, 2\}$ can be written as

$$Y_j = Y_{j,h} + Y_{j,l} = A_h \left[K_{j,h} \mathbf{E}\left(z | z \geq z_j^*\right) \right]^\alpha + A_l \left(\xi K - K_{j,h} \right),$$

where $K_{j,h}$ is the capitals used in H sector in Regime j satisfying

$$K_{1,h} = \xi K,$$
$$K_{2,h} = \xi K \left[1 - \mathbf{F}(z_2^*) \right].$$

Notice that since $K_{1,h} = \xi K$ is independent with z_1^*, we have

$$\begin{aligned}
\frac{\partial Y_1}{\partial K_{h,1}} &= A_h \alpha K_{1,h}^{\alpha-1} \left[\mathbf{E}\left(z | z \geq z_1^*\right) \right]^\alpha - A_l \\
&= \pi_{1,h} \mathbf{E}\left(z | z \geq z_1^*\right) - A_l \\
&> \pi_{1,h} \hat{z}_1 - A_l.
\end{aligned}$$

The second line is due to the definition of $\pi_{1,h}$ (see equation 9). It worth noting that the equilibrium in the Regime 1 implies the interbank market does not collapse, i.e., $\pi_{1,h} z_1^* = R^f > A_l$. Thus, we must have $\frac{\partial Y_1}{\partial K_{h,1}} > 0$. With this monotonicity property, we further have

$$\begin{aligned}
Y_1 &= A_h \left[K_{1,h} \mathbf{E}\left(z | z \geq z_1^*\right) \right]^\alpha + A_l \left(\xi K - K_{1,h} \right) \\
&> A_h \left[K_{2,h} \mathbf{E}\left(z | z \geq z_1^*\right) \right]^\alpha + A_l \left(\xi K - K_{2,h} \right) \\
&> A_h \left[K_{2,h} \mathbf{E}\left(z | z \geq z_2^*\right) \right]^\alpha + A_l \left(\xi K - K_{2,h} \right) \\
&= Y_2.
\end{aligned}$$

The second line is due to the fact $K_{1,h} > K_{2,h}$ and $\frac{\partial Y_1}{\partial K_{h,1}} > 0$, while the third line just applies $z_1^* > z_2^*$.

Finally, it remains for us to pin down the condition under which $Y_1(\xi = \xi^*) > Y_2(\xi = 1)$, i.e.,

the output in Regime 2 is always below the level at threshold value ξ^* in Regime 1. First, note that according the definition of output, we have

$$
\begin{aligned}
Y_1\left(\xi=\xi^*\right) &= A_h\left[\xi^* K \mathbf{E}\left(z \mid z \geq z_1^*\right)\right]^\alpha, \\
Y_2\left(\xi=1\right) &= A_h\left[K_{2,h} \mathbf{E}\left(z \mid z \geq z_2^*\right)\right]^\alpha + A_l\left(\xi K - K_{2,h}\right),
\end{aligned}
$$

where we have already proved that

$$
\begin{aligned}
\xi^* &= \left(\frac{\alpha \frac{A_h}{A_l}}{\Gamma(\hat{z})}\right)^{\frac{1}{1-\alpha}} \frac{1}{K}, \\
z_1^* &= \arg\min_z \Gamma(z) = \hat{z} = \left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}} z_{\min}, \\
K_{2,h} &= \xi K\left[1 - \mathbf{F}\left(z^*\right)\right]|_{\xi=1} = K\left[1 - \mathbf{F}\left(z^*\right)\right] \\
z_2^* &= \left[\frac{A_l}{\alpha A_h K^{\alpha-1} z_{\min}} \xi^{1-\alpha}\right]^{\frac{1}{\alpha+\eta-\alpha\eta}} z_{\min}|_{\xi=1} = \left[\frac{A_l}{\alpha A_h K^{\alpha-1} z_{\min}}\right]^{\frac{1}{\alpha+\eta-\alpha\eta}} z_{\min}.
\end{aligned}
$$

We further have

$$
Y_1\left(\xi=\xi^*\right) = A_h\left[\left(\frac{\alpha \frac{A_h}{A_l}}{\Gamma(\hat{z})}\right)^{\frac{1}{1-\alpha}}\left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{1}{\eta}}\right]^\alpha,
$$

and

$$
\begin{aligned}
Y_2\left(\xi=1\right) &= A_h\left[K\left(1 - \mathbf{F}\left(z_2^*\right)\right) \mathbf{E}\left(z \mid z \geq z_2^*\right)\right]^\alpha + A_l K \mathbf{F}\left(z_2^*\right) \\
&= A_h\left(K \int_{z_2^*}^{z_{\max}} z \, dF\right)^\alpha + A_l K \mathbf{F}\left(z_2^*\right) \\
&= A_h\left(\alpha \frac{A_h}{A_l} z_2^*\right)^{\frac{\alpha}{1-\alpha}} + A_l K\left[1 - \left(\frac{z_2^*}{z_{\min}}\right)^{-\eta}\right] \\
&= \left(\frac{\alpha z_{\min} A_h}{A_l}\right)^{\frac{\alpha(\eta-1)}{\alpha+\eta-\alpha\eta}}\left(1 - \alpha z_{\min}\right) A_h K^{\frac{\alpha}{\alpha+\eta-\alpha\eta}} + A_l K.
\end{aligned}
$$

Therefore, $Y_1\left(\xi=\xi^*\right) > Y_2\left(\xi=1\right)$ if and only if

$$
\left(\frac{A_h}{A_l}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\Gamma(\hat{z})}\right)^{\frac{\alpha}{1-\alpha}}\left(1 + \frac{\alpha/\theta - \eta}{\eta - \alpha}\right)^{\frac{\alpha}{\eta}} > \left(\alpha z_{\min}\right)^{\frac{\alpha(\eta-1)}{\alpha+\eta-\alpha\eta}}\left(1 - \alpha z_{\min}\right)\left(\frac{A_h}{A_l}\right)^{\frac{\eta}{\alpha+\eta-\alpha\eta}} K^{\frac{\alpha}{\alpha+\eta-\alpha\eta}} + K.
$$

First, note that $\alpha < 1$, and $z_{\min} < 1$. Therefore $1 - \alpha z_{\min} > 0$. Then we know that RHS of the above inequality strictly increases with K. Second, note that $\frac{1}{1-\alpha} > \frac{\eta}{\alpha+\eta-\alpha\eta}$. Consequently, we know that $Y_1\left(\xi=\xi^*\right) > Y_2\left(\xi=1\right)$ if and only if (1) K is small enough, or (2) $\frac{A_h}{A_l}$ is large enough. **Q.E.D.**

Proof of Proposition 4: We analyze the steady state in Regime 1 and that in Regime 2 sequentially.

Regime 1: i.e., $R^f > A_l$, only the H sector prevails. So $Y_l = 0$, and $Y = Y_h = A_h [\mathbf{E}(z|z \geq z^*) \xi K]^\alpha$. Then (K, z^*) are jointly determined by

$$
\begin{aligned}
\alpha \frac{A_h}{A_l} (\xi K)^{\alpha-1} &= \Gamma(z^*), \\
rK &= \alpha A_h [\mathbf{E}(z|z \geq z^*) \xi K]^\alpha.
\end{aligned}
$$

Combining last two equations yields

$$
\frac{r}{A_l \xi} = \frac{1}{z^*} \left[\theta \frac{\mathbf{F}(z^*)}{1 - \mathbf{F}(z^*)} + 1 \right] \mathbf{E}(z|z \geq z^*),
$$

From which we can further solve the cutoff value of productivity z^* as $z^* = \left(\frac{\frac{r z_{\min}}{A_l \xi} - (1-\theta)}{\theta} \right)^{\frac{1}{\eta}} z_{\min}$. Therefore, the Regime 1 can be supported if and only if $\xi < \xi_H = \frac{r z_{\min}}{A_l}$. Notice that the value of z^* varies with the credit policy ξ. So given that $\xi < \frac{r z_{\min}}{A_l}$, we need to further analyze whether it is a **Good** or **Bad** equilibrium in Regime 1.

In particular, if $z^* > \hat{z}$, where $\hat{z} = \arg\min_z \Gamma(z)$, then it is the **Good** equilibrium; if $z^* \in (z_{\min}, \hat{z})$, then it is the **Bad** equilibrium. Notice that under the Assumption 2 that $\frac{\eta(1-\theta)}{\eta-\alpha} > 1$, the **Good** equilibrium can be supported if and only if $\xi < \xi_L \equiv \frac{\eta-\alpha}{\eta(1-\theta)} \xi_H$. In turn, the **Bad** one can be supported if $\xi \in (\xi_L, \xi_H)$.

Regime 2: i.e., $R^f = A_l$, then the H sector and l coexist. We have $Y_l = A_l \xi K \mathbf{F}(z^*)$ and $Y_h = A_h [\mathbf{E}(z|z \geq z^*) \xi K]^\alpha$. Then (K, z^*) are jointly determined by

$$
\begin{aligned}
\alpha \frac{A_h}{A_l} (\xi K)^{\alpha-1} &= \Phi(z^*), \\
rK &= \alpha Y_h + Y_l.
\end{aligned}
$$

The last equation implies

$$
\mathbf{F}(z^*) = \frac{\frac{r}{A_l \xi} - \frac{1}{z_{\min}}}{1 - \frac{1}{z_{\min}}}.
$$

To guarantee the equilibrium exist, we must have $0 < \frac{\frac{r}{A_l \xi} - \frac{1}{z_{\min}}}{1 - \frac{1}{z_{\min}}} < 1$. Since $\xi > \xi_H$ implies that $\frac{\frac{r}{A_l \xi} - \frac{1}{z_{\min}}}{1 - \frac{1}{z_{\min}}} > 0$ always holds, the above condition implies ξ must satisfy $\xi < \xi_X \equiv \frac{r}{A_l}$. Since ξ_X is greater than ξ_H, for the case of Regime 2, the ξ must satisfies $\xi \in (\xi_H, \xi_X)$. **Q.E.D.**