Unequal Transition: The Making of China's Wealth Gap*

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Abstract

This paper studies the evolution of wealth inequality during China's rapid economic growth since its market-oriented reforms in the early 1990s. We first document the evolution and composition of China's wealth distribution and summarize stylized facts on aspects of the growth and reform process that are key to understanding wealth accumulation. Then we develop a heterogeneous-agent dynamic general equilibrium model that incorporates two sectors, the rural agricultural sector and the urban manufacturing sector, with endogenous migration, occupation, and durable consumption (housing) choices subject to frictions. In particular, the persistent financial market friction that entrepreneurs face plays a key role, as it ensures that the wealth brought by rapid capital accumulation is accrued predominantly to entrepreneurs. Our quantitative exercise decomposes the rising wealth inequality in China into different contributing factors.

JEL classification: E21, O11, O16, O18

Keywords: Wealth inequality, The Chinese economic reform, Financial frictions, Entrepreneurship, Housing, Migration

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1 Introduction

China's income and wealth inequality has risen dramatically during its economic transition since the start of its market-oriented reforms in 1978. While the growth process has delivered income growth to the vast majority of the population, with the average real income of the bottom 50 percent multiplied by more than five from 1978 to 2015 (Piketty et al., 2019), the income of the top 10 percent rise much more, resulting in a widening gap between the rich and the poor. Wealth inequality, measured by the fraction of wealth owned by the wealthiest 10% households, has increased from 30% in 1995 to about 56% in 2017 (Figure 1.1). While income and wealth inequality rises in most developed economies over the last few decades (Piketty, 2014), China stands out in two aspects: the speed at which the inequality rises and the concurrence of both widening inequality and rapid economic growth. If, as Deng put it four decades ago, "let some people get rich first" is a pre-condition for growth, has the country gone too far in its prioritization of growth over equity?

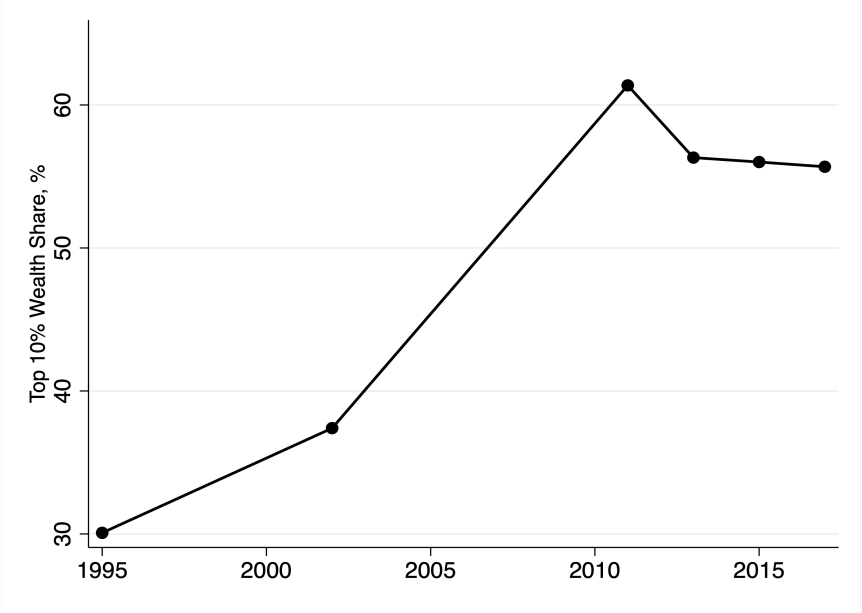


Figure 1.1: Top 10% Wealth Share in China (%)

The goal of this paper is to study the drivers of China's rising wealth inequality over the last three decades in a unified framework that includes all main features of the growth and reform process that are relevant for wealth accumulation. Such a framework will allow us to understand the contribution to inequality of each of the distinct reforms that occurred during the transition and also guide our thinking on how inequality will evolve once growth is slowing down. Combining all publicly available micro datasets with aggregate

statistics, we start by presenting a set of macroeconomic facts on China's economic transition from 1978 to the present day.

The Chinese economic growth process was launched in 1978 with the reform and opening up policy and it accelerated in 1992. The process is characterized by large scale rural-to-urban migration, structural transformation from agriculture to manufacturing, retreat of state-owned enterprises (SOEs) and entry of private firms. In the 1980s and even early 1990s, China was dominantly an agrarian economy with the majority of population living in the rural area. By 2020, however, the primary industry accounts for slightly over 20% of China's total employment, and more than 60% of population resides in the urban areas. The non-agricultural production is reallocated from SOEs to private firms, with SOEs accounting for more than 80% of total urban employment in 1995 and below 10% in 2020.

This transition process generates enormous capital and wealth accumulation. The estimated capital stock in 2017 is about 60 times of that in 1978. Despite rapid capital formation, the rate of return on capital remains at a high level until the 2008 global financial crisis, partly due to the constant labor inflow to cities and the manufacturing sector that keeps wage low. However, the wealth created from fast capital accumulation at a high rate of return is not evenly distributed among the population. Over the past decades, China's financial market remains largely underdeveloped. Private firms face substantial frictions in raising funds and rely largely on self savings for expansion. This also means that an average household only has limited access to financially benefit from the capital accumulation in the more profitable private sector. As a result, entrepreneurial fraction of wealth increases over time and especially at the top.

For the vast middle class, however, the paucity of investment opportunities in financial markets means that more households regard housing as an attractive venue of wealth accumulation, especially given the robust housing price growth across most cities that persists for decades. In 2012, net housing, measured as housing value minus mortgage, accounts for more than 70% of total urban wealth. This ratio is much larger than in even developed economies (Kuhn and Rios-Rull, 2016). The surging demand for housing reflects both a consumption motive from inflows of urban residents as well as an increase in income level generally and an investment motive from riding the price appreciation during the transition.

After sorting out these stylized facts, we build a dynamic general equilibrium incomplete markets model with rural-urban location choice and worker-entrepreneur occupational choice, augmented with frictions to capture the reality of the pre-transition Chinese economy. There are two regions, rural and urban. The agriculture sector is located in the rural area, and manufacturing in the urban area. Each individual chooses whether to live in the rural or urban area, and those who choose to live in an urban area further choose to become a worker or an entrepreneur. Rural residents live in their self-built housing and are hand to mouth. Urban residents need to purchase housing, as a durable good, from the market. In the pre-transition economy, rural-urban migration, entry to entrepreneurship, and housing purchase in the urban area are all subject to frictions. We calibrate the steady state of this frictional economy to empirical moments in 1995, the earliest year for which wealth distribution data is available.

The reform is then modeled as reduction /removal of these frictions and the growth is approximated by an increase the productivity in the urban manufacturing sector, capturing, for example, the impact of opening to international trade on the expansion of the Chinese manufacturing sector. We compare the transitional dynamics towards the new steady state in the model to the economic transition from 1995 onward. With the calibrated model in hand, we perform a number of counterfactual exercises, shutting down individual reform measures, to evaluate the impact of different forces in driving up the wealth inequality from 1995 to 2018.

Related Literature Our paper is most closely related to the literature on income and wealth inequality of China. Combining household survey and Hurun's rich list, Piketty et al. (2019) calculates the evolution of top 10% income share from 1978-2015 and top 10% wealth share from 1995-2015.[1] Built on facts established in the empirical literature, the goal of our paper is to construct a quantitative theory of the evolution of China's wealth inequality and evaluate the contributions of different forces.

The way economic transition is modelled in our paper is related to several papers which examined China's transition from a macroeconomic perspective. Buera and Shin (2013) analyzes the transition dynamics following removal of tax /subsidy distortions in China and other Asian economies. Our paper follows the same approach in the sense that we

[1]The literature on the evolution of income and wealth inequality during the transition has contributions from Khan and Riskin (2005), Benjamin et al. (2008), Piketty and Qian (2009), Santaeulalia-Llopis and Zheng (2018) and Sicular et al., eds (2020), among others.

model the pre-reform economy as a steady state with frictions, and interpret reform-triggered growth as transitional dynamics following complete or partial removal of these frictions. Song et al. (2011) studies the reallocation from SOEs to POEs during the late 1990s and early 2000s. Recently, Garriga et al. (2021) studies the impact of rural urban migration on Chinese housing markets across regions. Tombe and Zhu (2019) studies how the reduction in goods and labor market frictions affects China's aggregate labor productivity during the early 2000s. None of these papers focus on the evolution of wealth inequality.

Last, methodology-wise our paper builds upon and extends Quadrini (2000) and Cagetti and De Nardi (2006), who incorporate entrepreneurship into a heterogeneous agent model á la Aiyagari (1994) to quantitatively match with wealth concentration observed in the US data. We extend their framework to two sectors, a rural (agricultural) sector and a urban (non agricultural) sector, include a durable consumption good of housing, and focus on transitional dynamics, along which both economic growth and wealth concentration are observed.

The rest of paper is organized as follows. Section 2 systematically documents facts related to China's wealth inequality during the economic transition. The model is presented in Section 3. In Section 4, we calibrate the model and perform counterfactual exercises to quantitative evaluate the role of different forces in driving the widening wealth distribution. Concluding remarks follow in Section 5.

2 Facts

In this section, we present the main empirical patterns regarding the Chinese growth process and the evolution of wealth distribution over the past four decades. We make use of all publicly available micro data from China from 1989 to the present times as well as the various Chinese Statistical Yearbooks. The micro datasets we use include the China Household Income Project (CHIP) 1995-2013, China Health and Nutrition Study (CHNS) 1989-2011, China Family Panel Studies (CFPS) 2010-2018, China Household Finance Survey (CHFS) 2011-2017, and Population Census 1990-2020. The economic reforms started in 1978 and accelerated since 1992 especially in urban areas. So wherever possible, we show time series aggregates since 1978, in order to present the full picture of the transition process.

2.1 Wealth Inequality

Following the method in Piketty et al. (2019), we combine CHIP 1995 and 2002, CFPS 2010-2018 with Hurun's Rich List to construct the fraction of wealth owned by the wealthiest 10% of urban population as our baseline measure of wealth inequality. Figure 1.1 plots the evolution of wealth inequality from 1995, which is the earliest year for which household-level wealth information is available in China. The top 10% wealth share in China increases from a level around 30% to close to 60% over a period of 20 years. To put that into perspective, the level of wealth inequality is well below the that of western Europe in mid 1990s and now it's approaching the level of the US at a rate that doubles the rate at which wealth inequality is increasing in the US (Piketty et al., 2019). The speed at which inequality grows in China is startling by any international standard. [2]

Several clarifications are in order. Firstly, different than Piketty et al. (2019), we regard a household rather than an adult as the unit of analysis. This is to recognize that housing purchase is typically a household-level decision and that our analysis is consistent with the setup in the literature on entrepreneurship and wealth inequality (Quadrini, 2000; Cagetti and De Nardi, 2006). We confirm that the differences between these two views are minor.[3] Secondly, we focus on wealth inequality in urban areas, for several reasons.[4] Urban China is where wealth is increasingly concentrated. According to CHIP and CHFS data, the share of urban residents among the national top 10% wealthiest households increases from 30% in 1995 to 86% in 2002 and further to 95% in 2017 (see Table B.1 in Appendix B).[5] This means since early 2000 the main drivers of wealth accumulation for the national top 10% have to do with changes that occur in urban areas. Almost all major economic reforms relevant for the urban sector in China were initiated in mid 1990s after Deng Xiaoping's 1992 southern tour. This means that we can interpret the data from the 1995 survey as coming from a pre-reform steady state with our focus of the urban sector.

[2] A detailed description of the method is provided in Appendix A.

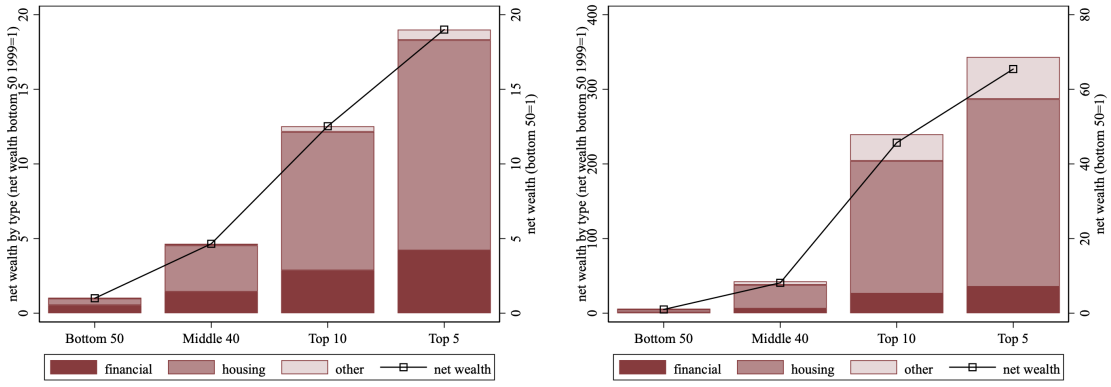
[3] Figure B.1 in Appendix B show the evolution of the top 10% wealth share from 1995-2008 under both assumptions and their levels and trends are very similar. When we use household total wealth measure, we do not adjust for the equivalence scale. This is because we do not observe family size for people in Hurun's Rich List. On the other hand, the average household size of a typical household in the top 10%, middle 40%, and bottom 50% respectively is quite stable over time, so we do not think making that adjustment would change much our results.

[4] We define the urban or rural status as the status of permanent residence (*Changzhu Renkou*) following the National Bureau of Statistics' definition. That is, an urban resident is one who lives in an urban area for more than 6 months in a year.

[5] The low level of 30% in 1995 is mainly due to the fact that while land and housing are always included in rural households' wealth, the majority of urban households in early 1990s lived in state-provided accommodation and by construction had zero housing wealth. As housing market liberalization progressed in late 1990s, by 2000 most urban households become participants in the housing market.

But the same cannot be said for rural China as economic reforms went on throughout 80s and 90s there.[6] In sum, an explanation for the wealth concentration in the urban area goes a long way in accounting for the rise of wealth inequality for the whole nation in the past three decades.

Figure 2.1 shows how average household's net wealth changes over the whole wealth distribution between 1999 and 2017. More specifically, we chart the average net wealth of the bottom 50%, the middle 40%, the top 10% and the top 5% in 1999 and 2017, normalizing the height of total net wealth for the bottom 50% in 1999 to 1. The first thing to notice from the figure is that the absolute level of household wealth increases dramatically from 1999 to 2017. The average net wealth of the bottom 50%, the middle 40% and top 10% in 2017 are roughly 5, 10 and 20 times their 1999 counterparts respectively (dividing the height of the bars in (b) by the counterparts in (a), reading from the left axis). The growth process and reforms seem to have generated wealth for most households in China, albeit to different extents. Though most get wealthier over time, the gap between the rich and the middle class widens: While in 1999 average wealth of the top 10% is roughly 12 times that of the bottom 50%, in 2017 the top 10% is 50 times wealthier than the bottom 50% (reading from right axis). This is consistent with the overall increase in top 10% wealth share we document earlier for urban households.



(a) 1999 (b) 2017

Figure 2.1: Composition of Wealth and Wealth Inequality in Urban China, 1999 vs 2017

Moreover, the composition of wealth for urban households has also undergone significant changes over time. Throughout the wealth distribution and throughout time, the single

[6] In Appendix B, we show in Figure B.2 the wealth inequality in rural China, alongside that of the urban areas and of the nation. We leave the increase of rural wealth inequality to future research.

most important form of wealth for most urban households is housing. The housing-wealth ratio ranges from 45% for the bottom 50% to 74% for the top 5% in 1999, and it declines slightly from 76% to 73% as we climb up the wealth distribution in 2017. However, when we concentrate on the wealth composition of the wealthier households, financial wealth is giving way to non-financial non-housing wealth, which consists mostly of entrepreneurial wealth from private businesses. The accumulation of housing wealth and the accumulation of entrepreneurial wealth therefore are two key mechanisms that we must incorporate in our analysis of the evolution of wealth inequality, which we turn to in the next subsections.

2.2 Housing

China started its housing reform in 1994, which was later extended to the whole country in 1998. Under the pre-reform planned economy system, state firms provided public housing to its employees[7]. After the housing reform, urban households are required to purchase houses from the market.

Table 2.1 shows the average home-ownership rate and housing wealth ratio from 1999-2017. In 1999, the first year after the national rollout of the housing reform, about two-thirds of urban residents own their accommodation. The home ownership rate increases to 79% in 2002, and eventually to 89% in 2017. The housing-wealth ratio in our sample period increases from 66% in 1999 to a relatively stable level of 78% from 2002-2017. The home ownership rate and housing-wealth ratio in urban China is significantly higher than those in developed economies. For example, the home ownership rate in the US is about 65% and the housing-wealth ratio, as reported by Kuhn and Rios-Rull (2016), is 32.4% in 2013 (also see Table B.2 in Appendix B).

A high housing-wealth ratio in China is partly due to the fact that, with an underdeveloped financial market, an average household have limited investment channels. For most Chinese households, housing is used as an important vehicle of wealth accumulation. Rapidly rising housing prices in most cities in the 2000s and 2010s, due to limited land supply and increasing housing demand fueled by rural-urban migration and rising urban income, contribute to the observed high housing-wealth ratio (Garriga et al., 2021). The average housing size per person in urban areas from 6.7 to 32.9 square meters from 1978 to 2012, according to the Urban Household Surveys (see Figure B.4 in Appendix B).

[7] In CHIP 1995, about 40% of urban households owns private housing.

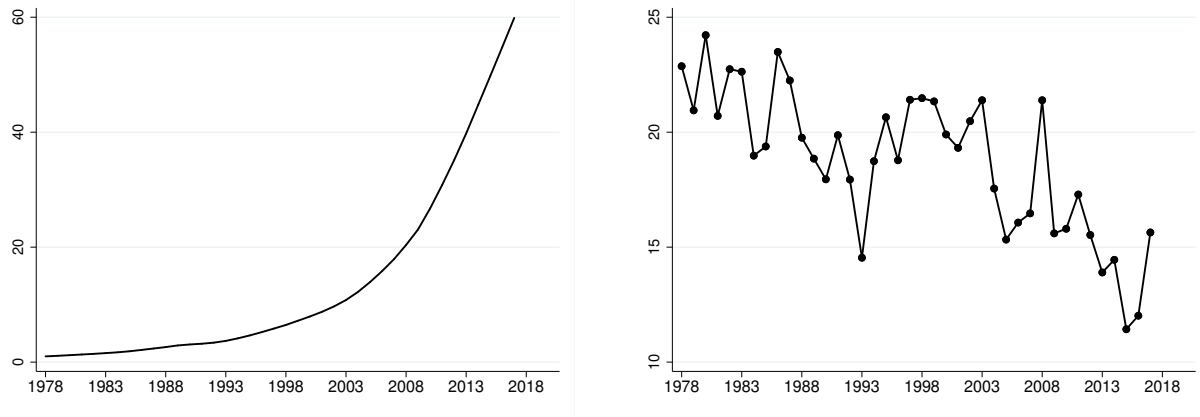
Table 2.1: Housing Wealth Ratio

Year	1999	2002	2011	2017
Home-Ownership Rate	66.14%	78.89%	84.84%	89.01%
Housing-Wealth Ratio	66.48%	78.89%	79.48%	76.50%

Data Source: CHIP 1999, 2002; CHFS 2011, 17.

2.3 Capital Accumulation and the Return of Capital

Rapid capital accumulation has been a critical driver of the Chinese economic growth. We calculate the stock of capital and its rate of return following the method in Bai et al. (2006), extend the series to 2017 and present them in Figure 2.2.[8] From 1978-2017, China's real capital stock grows at an average annual rate of 11.37% (Panel (a)) and the return remains at a high level throughout 1978 to 2008 at above 15%, and shows a declining trend only in the past 10-15 years (Panel (b)).



(a) Real Capital Stock (1978=1)

(b) Return to Capital (%)

Figure 2.2: Real Capital Stock and Return to Capital in China, 1978-2017

A combination of rapid capital accumulation and a stubbornly high rate of return persisting for decades generates growth in income and wealth. However, this gain is unequally distributed among the population largely due to the underdevelopment of the financial

[8] A detailed description of the method is provided in Appendix A.

market. Allen et al. (2005) documents that in 2000, the ratio of bank credit to private firms to GDP in China is 0.24, significantly lower than their sample average, 0.73, among 48 developed and developing countries. There are two implications from these financial frictions: One, entrepreneurs, as owners of firm and capital, largely rely on self saving for capital accumulation, and for those who manage to do that, they enjoy high rates of return for a long period of time. Two, an average worker cannot access this high rate of return from capital accumulation by participating in financial markets, which limits the accumulation of financial wealth for the middle class.

2.4 Entrepreneurship

Entrepreneurs as a rising class of *nouveau riche* emerge from the transition from a SOE-dominant planned system to a market based economy. During the transition, the size of the SOE sector shrinks from employing as high as 85% of urban population in 1995 to 34% in 2002 and further to 13% in 2020 (China Labor Statistical Yearbook, see Figure B.3 in Appendix B). As the SOEs retreat, the entry barriers of private firms are slowly lowered following the promulgation of the *Company Law* in 1994 which formally recognizes private ownership of capital (Jiang et al., 2021). As we will show, this has significantly contributed to the concentration of wealth at the top, in a way similar to other developed economies (Cagetti and De Nardi, 2006).

Table 2.2 presents the fraction of entrepreneurs and their wealth share during the transition. We define a household as entrepreneurial if there is at least one household member who engages in private or individual business operation. In 1988, entrepreneurs only account for 1.07% of total population.[9] This ratio rises slightly to 2.67% in 1995. The wealth share of entrepreneurs was also limited, amounting to only 4.44% in 1995. Both the population and wealth share increase rapidly thereafter. In 2002, entrepreneurs' share in population more than doubles and arrives at 6.23%; their wealth share also increase to 8.18%. The 2002-11 has seen the largest increase in both the population and wealth share. Entrepreneur's population share rises to over 16% in urban population in 2011; together they account for more than 30% of urban wealth, after which both shares stay stable at those levels.

[9]This value is consistent with that from the Urban Household Survey Statistical Yearbook 1989, in which individual and private employers (*geti guzhu yu siyingzhe* accounts for 1.49% of urban employment in 1988. CHIP 1988 does not contain wealth information, we therefore cannot calculate entrepreneurs' wealth share in that year.

Table 2.2: Share of Entrepreneurs in Population and Wealth

Year	1988	1995	2002	2011	2017
Population Share	1.07%	2.67%	6.23%	16.26%	18.01%
Wealth Share		4.44%	8.18%	31.26%	30.23%

Data Source: CHIP 1988, 95 and 2002; CHFS 2011, 18.

The importance of entrepreneurs accounting for the top wealth in China has also increased over time. Table 2.3 lists the population and wealth share of entrepreneurs among the wealthiest 10%, 5% and 1% of urban households. Let's focus on the top 10%. In 1995, the population share of entrepreneurs among wealthiest 10% of households is 4.57%, and the wealth share is slightly larger at 5.32%. Even though these numbers are relatively small, they are both already bigger than entrepreneurs' share in total population. Similar as the trend in the previous table, entrepreneurs' population and wealth shares have a moderate increase from 1995-2002, followed by a substantial jump from 2002-2011 before tapering off. In 2011, entrepreneurs account for more than 24% of the wealthiest 10% of urban households, and their wealth account for around 35%.

Table 2.3: Population and Wealth Share of Entrepreneurs among Wealthiest Households

Year	1995	2002	2011	2017
Among Wealthiest 10% Households				
Pop. Share	4.57%	6.80%	24.74%	28.37%
Wealth Share	5.32%	9.50%	35.98%	34.92%
Among Wealthiest 5% Households				
Pop. Share	4.99%	8.29%	33.32%	31.43%
Wealth Share	5.68%	11.41%	42.93%	38.59%
Among Wealthiest 1% Households				
Pop. Share	6.76%	12.32%	67.68%	45.47%
Wealth Share	6.82%	18.58%	67.10%	55.31%

Data Source: CHIP 1995, 2002; CHFS 2011, 17; Hurun's Rich list

2.5 Urbanization and Structural Transformation

It should be clear from the discussion above that wealth accumulation process during the transition in urban China has much to do with the performance of the housing market as well as the growth in the entrepreneurial private sector. Both mechanisms are further aided by the urbanization which takes place concurrently.

Since 1978, the urban population share increases steadily from near 20% as reported in the 1982 Population Census to over 60% by the 2020 Population Census (Figure 2.3). The non-primary sector employment share evolves in parallel to that of the urban population share, growing from 29.5% in 1978 to 76.4% in 2020 according to the National Bureau of Statistics.[10] These figures suggest that there is substantial labor relocation from the rural agricultural sector to the urban non-agricultural sector.

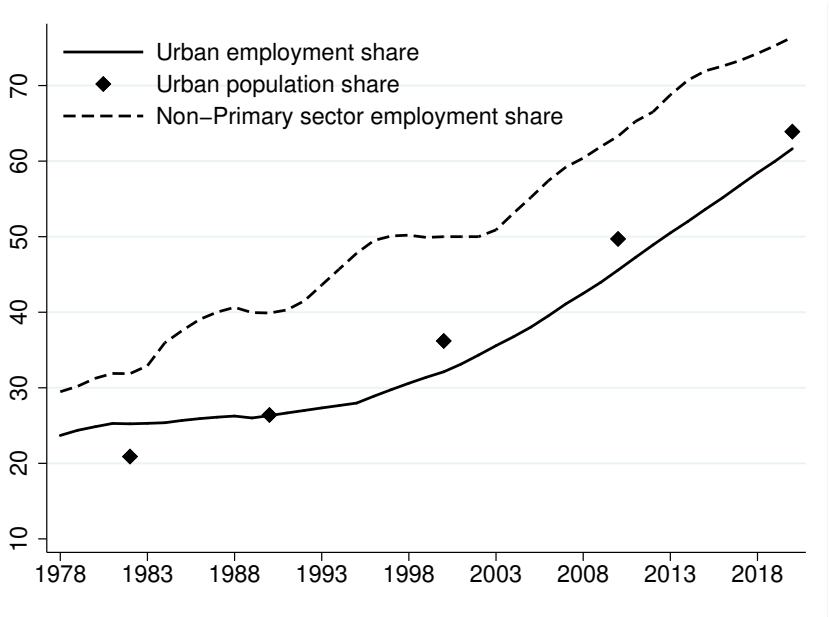


Figure 2.3: Urbanization Rate and Non-Primary Sector Emp. Share in China (%)

Note: Urban and Non-Primary sector employment share is from NBS; Urban population share in 1982, 1990, 2000, 2010, and 2020 is from population census in these years.

This labor movement has two implications on the wealth accumulation process. Firstly, as urban population grows, the demand for urban housing unit which needs to be purchased from the market since late 1990s puts upward pressure on housing prices. Sec-

[10]The fact that non-primary sector employment share increases faster than urban employment share from 1978-1995 is partly due to rural industrialization—rise of township and village enterprises—in the 1980s and early 1990s.

ondly, a constant inflow of rural (relatively unskilled) labor to the non-agricultural sector helps keep the wage rate in that sector low and maintain the relatively high return on capital which we show in Section 2.3.

In sum, the stylized features of the transitional growth of the Chinese economy which are relevant for wealth accumulation can be summarized as follows:

- The liberalization of the housing market accompanied by the soaring house price in cities offers an attractive way of wealth accumulation to the vast majority of urban households
- The SOE reforms which allow entry from private firms to previously state dominated sectors leads to rapid capital accumulation creating a rising affluent class of entrepreneurs
- Financial friction motivates entrepreneurs to save and precludes workers from investing in profitable firms
- Rural-to-urban labor relocation keeps the demand for housing high and the supply of labor abundant, contributing to house price appreciation and persistently high return of capital

In what follows, we present a dynamic general equilibrium incomplete markets model that has all the above mentioned ingredients embedded, which will be the framework of our quantitative assessment.

3 The Model

In this section, we construct a discrete-time general equilibrium incomplete markets model to account for China's wealth distribution and its evolution. The model builds on heterogeneous agent models with occupational choice under financial frictions (Cagetti and De Nardi, 2006; Buera and Shin, 2013). In the model, there are two region-sectors: rural-agriculture and urban-manufacturing. The term "manufacturing" stands for all non-agriculture activities in the urban area. The pre-reform economy is modeled as an economy with low productivity in manufacturing and many frictions in migration, entrepreneurship, and housing market. The economic reform and process is modeled as a gradual improvement in manufacturing productivity as well as reductions in those frictions.

There is a continuum of infinitely lived agents who has the same preference and maximizes the discounted sum of utility as follows

$$\sum_{t=0}^{\infty} \beta^t u(c_{r,t}, c_{m,t}, h_t)$$

where β is the discount factor, and $c_{r,t}$, $c_{m,t}$ and h_t denote consumption of agricultural good, manufacturing good, and housing services respectively. The manufacturing good is the numeraire.

Rural households are modeled in a similar way to Garriga et al. (2021), in that they are hand-to-mouth households who live in self-built houses and have zero wealth. They face a migration decision to move to the urban area and differ in the net migration cost. Upon migration, they will take an ability draw and start from zero wealth in the urban area.

Urban households are characterized by a pair of abilities, (e, z), here $e \in E$ denotes the entrepreneurial ability and $z \in Z$ denotes the working ability. The sets E and Z contains a finite number of values and $(e, z) \in E \times Z$ evolves stochastically according to a Markov process with transition probability from (e, z) this period to (e', z') the next period given by $\Pi((e', z'), (e, z)$. The stochastic process embodies the idiosyncratic labor income shocks workers face and entrepreneurial risk entrepreneurs face. They start a period with financial wealth b (and if they own their housing with housing wealth h), and make occupation, non-durable consumption, and saving (and if possible, housing) decisions. In terms of occupation, more specifically urban households can choose to work as an entrepreneur in the private sector or working as an employed worker in either the private or the state sector.

In what follows, we detail technology, market structure, frictions, and decision in the urban and rural sector separately.

3.1 The Urban Sector

The urban sector consists of SOEs and private firms and urban workers are perfectly mobile between SOEs and private firms.

3.1.1 SOEs

The SOE production is modeled as a representative firm, which have access to the following Cobb-Douglass production function,

$$Y_{s,t} = A_{s,t} K_{s,t}^{\alpha_s} L_{s,t}^{1-\alpha_s}$$

$A_{s,t}$ is the TFP of SOEs in period t; $K_{s,t}$ and $L_{s,t}$ are capital and labor input; α_s is the capital income share in SOE production, and $Y_{s,t}$ is SOE's final output. Given market determined interest rate, r_t, depreciation rate, δ, and urban wage rate, w_t, the representative SOE maximizes the following profit

$$\pi_{s,t} = Y_{s,t} - (r_t \tau_{r,t} + \delta) K_{s,t} - w_t L_{s,t}.$$

where $\tau_{r,t} \leq 1$ is a potentially time-varying interest rate subsidy to SOEs.

3.1.2 Private firms

The manufacturing good can also be produced by private firms, each operated by an entrepreneur. Denote e_i the entrepreneur's ability in private firm i. Firm $i's$ production function is given by

$$y_{i,t} = A_{m,t} e_i \left(k_{i,t}^{\alpha} l_{i,t}^{1-\alpha} \right)^{\nu},$$

where $\alpha < 1$ and $\nu < 1$ governs the decreasing return, and $A_{m,t}$ is the aggregate productivity in the private manufacturing sector in period t. Firm $i's$ production depends on its entrepreneur's ability as well as the aggregate productivity. Because of decreasing return to scale, the entrepreneurs earn positive profits.

Entrepreneurs who operate private firms face the urban wage w and the unsubsidised rental rate of capital $r + \delta$ and are subject to a collateral constraint. Let $\pi(e, z, a)$ denote the profit of an entrepreneur as a function of (e, z, a):

$$\pi(e, z, a) \equiv \max_{k,\, l} \; A_m e \left(k^{\alpha} l^{1-\alpha} \right)^{\nu} - wl - (r + \delta) k$$

subject to the financial friction

$$k \leq \lambda a.$$

The parameter λ captures under-development of the financial market. Given her own wealth a, the entrepreneur can borrow up to λa. The smaller the value of λ, the more

severe of financial frictions. For those who own housing, housing wealth and financial wealth are used as collateral, while for those who cannot trade houses, only financial wealth can be included in a and used as collateral.

3.1.3 Urban Agent's Problem

An urban agent faces the following timeline of decisions. At the beginning of the period, an urban agent first makes an occupational choice – whether to operate a private firm as an entrepreneur or to work as an employed worker. She then receives earnings according to her occupation and the capital income from her savings. An urban agent lives in state provided houses before the housing reform. After the reform, she owns housing and can make adjustment to her housing consumption. She makes non-durable consumption and saving decision before the end of the period.

As the occupational choice affects only static income, it is simply a comparison between two income sources, as in Buera and Shin (2013). The term i denotes earnings of an urban resident, which is either her wage or her income comes from operation profits, whichever is higher.

$$i(e,z,a) = \max\{zw, \pi(e,z,a) - \mu_{e,t}[\pi(e,z,a) - zw] - \tau_{e,t}\}$$

where $\pi(e,z,a)$ is the profit and $\mu_{e,t}$ denotes any time-varying distortions associated with entrepreneurial profit. We intentionally formulate $\mu_{e,t}$ in the form above so that it summarizes distortions that affect the intensive margin of entrepreneurial income, and is separated from the extensive margin of entrepreneurship—entry barriers. The parameter $\tau_{e,t}$ captures firm entry barriers which might change over time.

For urban agents before the housing reform, they are provided with government funded public housing $\bar{h}_u$. Their value function reads

$$V_u^n(e,z,b) = \max_{c_r,c_m,b'} u(c_r,c_m,\bar{h}_u) + \beta \mathbb{E}_{e',z'} V_u^n(e',z',b'), \tag{1}$$

subject to the budget constraint

$$p_r c_r + c_m + b' \leq i(e,z,b) + b(1 + r\tau_r),$$

and a non-borrowing constraint,

$$b' \geq 0.$$

Note that the policy variable τ_r is also imposed on the household sector, which captures the notion that the state-monopolized banking sector suppresses the deposit rate to provide cheap funding to the SOEs.

After the housing reform, urban households need to purchase houses in the market, their value function is

$$V_u(e,z,b,h) = \max_{c_r,\, c_m,\, b',\, h' \in H} u(c_r, c_m, h') + \beta \mathbb{E}_{e',z'} V_u(e', z', b', h'), \tag{2}$$

subject to the budget constraint

$$p_r c_r + c_m + b' + p_h h' \leq i(e,z,b + p_h h(1-\delta_h)) + (1 + r\tau_r)b + p_h h(1-\delta_h),$$

and a non-borrowing constraint,

$$b' \geq 0.$$

Note here we do not allow workers to borrow to purchase houses, which we think is consistent with the fact that in data we calculate net housing—value of houses minus mortgage—in obtaining total household wealth and in China household debt that is not mortgage has been of a small magnitude historically. We assume households can choose housing h' from a finite set $H = \{H_1, H_2, ..., H_N\}$. The reason that we discretize the housing choice, instead of using a continuous variable, is that indivisibility together with an upper bound in housing allows the return to housing to be different than the risk free return, and avoid corner solutions where households hold zero financial wealth. Last, we denote H_t^s the exogenous tradable housing supply at period t. Urban housing price is determined by urban housing market clearing condition which we specify when we discuss equilibrium.

3.2 The Rural Sector

The agriculture sector locates in the rural area, and is run by a "representative" firm that runs a farmland and hires all rural workers. The agriculture sector employs labor as the only input and admits the following constant-return-to-scale technology

$$Y_{r,t} = A_{r,t} L_{r,t},$$

where $L_{r,t}$, $A_{r,t}$ and $Y_{r,t}$ denote agriculture employment, productivity and output.

Rural workers live in self-built houses, which gives them a housing service of $\bar{h}_r$ and is assumed to be hand-to-mouth with zero wealth. Rural households are identical except with regards to the net cost of migration, ϵ. Suppose the net cost of migration is given by $\tau_{m,t}\epsilon$, where the common component $\tau_{m,t}$ is a time-varying policy variable and ϵ is an individual characteristic, which is distributed as $\Phi(\epsilon)$ in the rural population and fixed over time.

The value function for a rural worker with ϵ is then

$$V_r(\epsilon) = \max_{c_r, c_m} \; u(c_r, c_m, \bar{h}_r) + \beta \max\left\{V_r(\epsilon), \mathbb{E}_{e,z} V_u(e, z, \bar{h}_u) - \tau_m - \epsilon\right\} \tag{3}$$

subject to her budget constraint

$$p_r c_r + c_m \leq w_r.$$

Note that the income w_r of a rural household is simply A_r and does not depend on ϵ, the optimal c_r and c_m will be independent of ϵ. Denote them as c_r^r and c_m^r and denote the period utility $u(c_r^r, c_m^r, \bar{h}_r)$ as u^r. Also, the expected value of living in the urban area, $\mathbb{E}_{e,z} V_u(e, z, \bar{h}_z)$, is independent of ϵ, and let's denote it by EV_u. After the housing reform, We further impose the following assumption: an agent does not own houses upon the first period she migrates from rural to urban. Accordingly, in the post-reform era, the expected value of living in the urban area for a rural household is $\mathbb{E}_{e,z} V_u(e, z, 0)$ in the Bellman equation.

In a stationary environment, since The optimal migration/location decision is given by a cutoff in ϵ such that for all households whose $\epsilon \leq \bar{\epsilon}$ defined below, they choose to migrate to urban areas

$$\bar{\epsilon} = EV_u - V_r(\epsilon) - \tau_m. \tag{4}$$

This leads to a simply decision rule that for all $\epsilon > EV_u - u^r/(1-\beta) - \tau_m$, they remain in rural areas and their value becomes $u^r/(1-\beta)$. For those whose $\epsilon \leq EV_u - u^r/(1-\beta) - \tau_m$, they become urban residents and their value is $V(\epsilon) = u^r + \beta(EV_u - \tau_m - \epsilon)$.

Therefore the percentage of population that resides in the urban area in a stationary equilibrium ($\Phi(\bar{\epsilon})$) is an endogenous outcome. And it is conceivable that this proportion

should be larger in an environment where as the urban manufacturing sector is much more productive than the agricultural sector, as this boosts EV_u.

3.3 Stationary Recursive Equilibrium

We define a stationary recursive equilibrium for an economic environment characterized by constant technology parameters (A_r, A_s, A_m) for the agricultural, SOE and private manufacturing sectors respectively and the constant policy parameters $(\tau_r, \tau_e, \mu_e, \tau_m)$ which represent the level of policy interventions in interest rate, housing tradability, entry to private sector, entrepreneurial income, and migration respectively.

Let $\mathbf{x} = (e, z, b, h)$ be an urban household's state vector, while ϵ is the rural household's only state variable. Without loss of generality denote $b \in [0, b_{max}] \equiv B$ and we already have $h \in H$. Therefore, the state space for urban households is $S = E \times Z \times B \times H$. Let the Borel $\sigma-$algebra associated with S be $\mathcal{A}^h$ and the typical subset be $(\mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}) \equiv \mathcal{S} \in \mathcal{A}$. The space $(S, \mathcal{S})$ is a measurable space and for any subset $\mathcal{S}$, let $F(\mathcal{S})$ be the measure of agents in set $\mathcal{S}$. Define the transition function $Q((e, z, b, h), \mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H})$ as the probability that an individual with current state (e, z, b, h) transits to the set $\mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}$ next period, $Q : S \times \mathcal{A} \to [0, 1]$ and

$$\begin{aligned} & Q((e, z, b, h), \mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}) \\ = & \sum_{(e', z') \in \mathcal{E} \times \mathcal{Z}} I\left\{b'(e, z, b, h) \in \mathcal{B} \text{ and } h'(e, z, b, h) \in \mathcal{H}\right\} \Pi((e', z'), (e, z)), \end{aligned}$$

where I is an indicator function.

A stationary recursive equilibrium consists of i) interest rate r, wage rates (w, w_r), and housing price p_h; ii) policy functions agricultural good consumption $c_r(\mathbf{x})$, manufacturing good consumption $c_m(\mathbf{x})$, occupation $o(\mathbf{x})$, savings $b'(\mathbf{x})$, and housing consumption $h'(\mathbf{x^h})$ for urban households, as well as consumption c_r^r and c_m^r for rural households; iii) value functions $V_u(\mathbf{x})$ for urban households specified (2) and $V_r(\epsilon)$ for rural households in (3); iv) exogenously given urban housing supply H^s; v) and invariant probability measures $F(\mathbf{x})$ for urban households, such that

1. Given prices, urban household makes the optimal occupational choice, consumption bundle choice, savings decision (and if before housing reform housing choice) and $V(\mathbf{x})$ is the associated value function;

2. Rural households indeed prefer to live in rural areas that is $\epsilon \leq \bar{\epsilon}$ as in (4) and they make optimal consumption bundle choices and the associated value function is $V_r(\epsilon)$;

3. The representative SOE and private entrepreneurs maximize profits;

4. Factor and goods markets clear:

   - In rural labor market, agricultural labor demand L_r equals supply

   $$L_r = 1 - \Phi(\bar{\epsilon})$$

   - In urban labor market, the sum of SOE labor demand L_s and entrepreneurs' labor demand $l(\mathbf{x})$ equals supply of labor

   $$L_s + \Phi(\bar{\epsilon}) \left(\int I\{o(\mathbf{x}) = e\} l(\mathbf{x}) dF(\mathbf{x}) \right) = \Phi(\bar{\epsilon}) \left(\int I\{o(\mathbf{x}) = w\} dF(\mathbf{x}) \right)$$

   - In capital market, the sum of SOE capital demand and entrepreneurs' capital demand $k(\mathbf{x})$ equals the supply of savings

   $$K_s + \Phi(\bar{\epsilon}) \left(\int I\{o(\mathbf{x}) = e\} k(\mathbf{x}) dF(\mathbf{x}) \right) = \Phi(\bar{\epsilon}) \left(\int b(\mathbf{x}) dF(\mathbf{x}) \right)$$

   - Agriculture good market clears

   $$Y_r = \int_{\bar{\epsilon}} c_r^r(\epsilon) d\Phi(\epsilon) + \Phi(\bar{\epsilon}) \left(\int c_r(\mathbf{x}) dF(\mathbf{x}) n \right)$$

   - Manufacturing good market clears

   $$Y_s + \Phi(\bar{\epsilon}) \left(\int I\{o(\mathbf{x}) = e\} y(\mathbf{x}) dF(\mathbf{x}) \right) = \int_{\bar{\epsilon}} c_m^r(\epsilon) d\Phi(\epsilon) + \Phi(\bar{\epsilon}) \left(\int c_m(\mathbf{x}) dF(\mathbf{x}) \right)$$

   - Urban housing market clears

   $$H^s = \Phi(\bar{\epsilon}) \int h'(\mathbf{x}) dF(\mathbf{x}).$$

5. For all $(\mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}) \in \mathcal{A}$, the invariant probability measure F satisfies

$$F(\mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}) = \int_{E \times Z \times B \times H} Q((e, z, b, h), \mathcal{E} \times \mathcal{Z} \times \mathcal{B} \times \mathcal{H}) dF(e, z, b, h)$$

In the quantitative analysis, we will interpret the stylized facts summarized from before 1995 as a stationary equilibrium in the *pre-reform* economy, with a specific set of parameters $(A_r, A_s, A_m, \tau_r, \tau_e, \mu_e, \tau_m)$ to be calibrated. We will postulate a terminal stationary equilibrium associate with a different set of parameters which we specify either based on the US experience or based on our policy projection. We then compute the entire transition from the pre-reform economy to the terminal economy, allowing $(A_r, A_s, A_m, \tau_r, \tau_e, \mu_e, \tau_m)$ to change in specific ways.

4 Quantitative Analysis

Conceptually, we view the observed 26-year empirical growth process from 1995 to 2021 as a part of a transition from a *pre-reform* steady state to a hypothetical terminal steady state decades into the future. More specifically, we start from the steady state of a model economy without housing market and laden with migration barriers and private sector distortions (entry barrier to the private sector, profit tax on private firms, and interest rate subsidies to SOEs), representing the observable state in 1995. Then, in the first period after the initial steady state we open the housing market and from then on allow all urban households trade houses in the model. We simultaneously vary the frictions and rural and manufacturing productivities over the transition, until the model economy settles down in a hypothetical terminal steady state. The challenge is that we do not yet observe the terminal steady state and for that reason we need to make some assumptions to extrapolate that from the observed reality of 2017-2021. We will conduct robustness checks on these assumptions regarding the terminal steady state to make sure what happens in the first 26 years along the transition, for which we do have the empirical counterpart, is robust to those alternative assumptions. Our strategy of calibrating the entire transition consists of three steps.

In the first step, we calibrate the model shutting down all frictions $(\tau_r, \tau_e, \mu_e, \tau_m)$ to a mixture of moments based on the 2017-2021 Chinese data and on some US data moments to proxy the terminal steady state. In particular, the calibrated parameters characterizing the ability process of entrepreneurs and those characterizing the preference for and the sizes of housing are taken as deep parameters, which we fix throughout the transition path.

In the second step, we work backwards from the terminal productivity level in the private manufacturing sector according to an exogenously calibrated productivity growth process to recover the level of private manufacturing TFP in the initial steady state. We

then calibrate the friction-related parameters in the initial steady state to match the data moments from 1995.

In the third step, once we have calibrated the initial and terminal steady states, we simulate the entire transition under assumptions on how the frictions evolve over time from the initial levels to the terminal levels (zero). By comparing the first years of model simulated moments with those in the data from 1995 to 2021, we can evaluate whether those assumptions on the evolution of frictions are plausible or not.

With the entire transition on hand, we perform a series of counterfactual exercises by shutting down different forces to isolate the impact of various factors in contributing to the evolution of China's wealth inequality over the past decades.

4.1 Calibration

As mentioned, we first calibrate ability-related parameters (e.g. $\bar{e}$, π_e, π_w) in a hypothetical terminal steady state extrapolated from 2017-2020 moments in data. These parameters are assumed to be fixed throughout the transition. The friction-related parameters are set to 0 in this terminal steady state.

Functional forms We simplify the computation by assuming that rural and urban produce the same good. The utility function is specified as

$$u(c,h) = \frac{[c^{1-\eta}(\underline{h}+h)^{\eta}]^{1-\sigma}}{1-\sigma}.$$

We assume that the type distribution in the rural population, $\Phi(\epsilon)$, follows a Pareto distribution,

$$\Phi(\epsilon) = 1 - (\frac{1}{\epsilon})^{\zeta}, \quad \forall\, \epsilon \geq 1.$$

One period in the model corresponds to 1 year in the data. We use a 5-state Markovian process for worker's ability, and use a two-state Markovian process for entrepreneurial ability, and assume the two processes are independent. For worker's ability, we run an AR(1) regression of urban households' wage using the China Family Panel Studies (CFPS) data from 2012-18, and then discretize this AR(1) process to obtain the states and the associated transitional matrix. Entrepreneurial ability is assumed to take two states, 0 and $\bar{e}$, that is, an individual either has the ability to become an entrepreneur or not. Denote π_w and π_e the probability of agents staying at $e = 0$, and $e = \bar{e}$ respectively, for two con-

secutive periods.

We set the discount rate $\beta = 0.96$, relative risk aversion coefficient $\sigma = 2$, physical capital depreciation rate $\delta = 0.06$, and housing depreciation rate $\delta_h = 0.03$. For η, we set $\eta = 0.23$ to target 23% of housing expenditure share in total household expenditure[11]. The housing unit in rural area, $\bar{h}_r$, is normalized as 1 in the terminal steady state.

The productivity of the private manufacturing sector is normalized as 1. We choose parameters in the production function, α_s, α, γ, to match a labor income share of 50%, a value typically adopted in the literature (e.g. Song et al. (2011)). That is, we set $\alpha_s = \alpha/(\alpha + \gamma) = 0.5$. The value $1 - (\alpha + \gamma)$ represents span-of-control, we choose $\alpha + \gamma = 0.85$, a value used in many macroeconomic research (Atkeson and Kehoe, 2007; Restuccia and Rogerson, 2008).

The remaining parameters are calibrated internally. The aggregate productivity at the SOE sector, A_s is calibrated to SOE employment share in 2020, and the agriculture productivity is chosen to match the urban-rural wage gap in 2020. We choose the parameter ζ in the idiosyncratic migration preference distribution to target an urbanization rate of 83%, which is the US level in 2020.

There remain four entrepreneur related parameters. The state transition probabilities π_w and π_e are calibrated to match entrepreneur-to-entrepreneur and work-to-worker transition probabilities observed from 2015-17. The financial friction parameter λ, and entrepreneurial ability, $\bar{e}$, are chosen to match the fraction of entrepreneurs in population, and the average ratio of entrepreneurial household wealth to worker's household wealth.

For the housing choice set $H = \{h_1, h_2, ..., h_N\}$, we pick N=4, and choose $h_1 = 0$, so that households can choose not to purchase a housing. This gives us 5 housing related parameters: the preference parameter $\underline{h}$, the housing sizes h_2, h_3, h_4 ,and the total housing supply H^S. We target the following five moments: the fraction of urban households who own a house; the housing wealth ratio for all households, for the wealthiest 20% households, and for middle 60% households; as well as the overall wealth inequality (the top 10% wealth share). We use the OECD average values, as reported in Causa et al. (2019),

[11]The data source is China Household Survey Statistical Yearbooks. See Table B.3. The number we use is larger than Hao et al. (2020), as we include computed rent from owner occupied housing as part of housing expenditure.

for housing related moments for the extrapolated steady state of China.[12]

In total, we then have 12 moments to internally calibrate the remaining 12 parameters. We choose parameter values to minimize a weighted sum of the distance between model and data moments:

$$\sum_{k=1}^{12} \frac{|\text{model}(k) - \text{data}(k)|}{0.5 * |\text{model}(k)| + 0.5 * |\text{data}(k)|}$$

Table 4.1 summarizes the calibrated parameter values and associated moments in data and model.

Table 4.1: Parameter Values and Moments in Terminal Steady State

Parameter	Value	Moment	Data	Model
A_r	1.33	urban-rural wage gap	1.60	1.60
A_s	0.6	SOE emp share	9.00%	8.57%
$\bar{e}$	2.80	entrep-entrep wealth ratio	2.08	3.02
π_e	0.85	ent-ent trans.	21.00%	14.83%
π_w	0.98	worker-worker trans.	1.99%	4.00%
λ	1.26	% of entrep in pop	13.12%	11.84%
ζ	–	urban pop share	83.00%	83.00%
$\underline{h}$	0.06	home ownership rate	89.01%	91.57%
h_1	0.40	housing-wealth (H-W) ratio	51.00%	52.23%
h_2	1.92	(H-W) ratio for middle 60% HHs.	60.00%	60.36%
h_3	4.37	(H-W) ratio for top 20% HHs.	50.00%	48.09%
H_s	1.07	Top 10% Wealth Share	51.00%	52.07%

Then we fix these deep parameters and calibrate friction related parameters to the pre-reform economy in 1995. Denote T the number of periods it takes to finish the transition. We let A_m to be constant from $t + 41$ to T at its terminal value, 1, and by assuming a constant growth rate of 3% from $t = 0$ to $t = 40$ to back out the value of A_m in the initial steady state. For other parameters in the initial steady state, we calibrate A_s to the SOE employment share in 1995, and A_r to the rural-urban wage gap in 1995. The migration barrier parameter, τ_m, is chosen to match the urban employment share, and τ_e to the

[12]For housing related moments, we do not directly use Chinese data from 2017-2020 for the following reasons: housing price is constant in the model steady state, but increases at about 10% a year annually from 2017-2020 in data. Therefore the average housing wealth ratio of OECD countries, which has finished the industrialization process, is more appropriate targets. In a work in progress, we plan to re-calibrate housing related parameters using data from Taiwan, which has a high home ownership rate comparable to mainland China.

fraction of entrepreneurs in population, both in 1995. We set SOE interest rate subsidy, τ_s, to match capital labor ratio difference between SOE and POE in 1998. Last, we choose μ_e and τ_{hh} to match the average entrepreneur-worker wealth ratio, and top 10% wealth share in 1995.

4.2 Reform-induced Economic Transition

Starting from the initial steady state, we let the urban housing market fully open in the first period and let the productivity and friction-related parameters change in the following way: (i) increase in manufacturing productivity A_m and rural productivity A_r; (ii) change in the SOE sector A_s, which can reflect both TFP and subsidy; (iii) reduction in migration barrier τ_m; (iv) reductions in entry barrier for private firms τ_e and the wedge on entrepreneurial profit μ_e; (v) decrease in financial market distortion parameter, τ_r.

In the data, we have observed 26 years of China's urban transition from 1995-2021. In the model, we assume the shocks last for 40 years, and then stop there. The economy eventually arrive at a long run steady state. We compare the first 26 years of the transition simulated from the model to that in the data. As in Garriga et al. (2021), assumptions on how shocks behave after the 26^{th} year significantly affect the terminal steady state, it, however, has very limited impact on the first 26 years of transition in the model.

From 1995-2017, manufacturing TFP grows at an average rate of 3% per year. Accordingly we set an annual growth rate of 3% of manufacturing TFP which lasts for 40 years in the model. Along the transition, we assume all parameters but SOE subsidy and private firm entry barrier increases from the initial steady state values to their calibrated terminal steady state values in period 40 at a constant growth rate. For private firm entry barrier and SOE subsidy, we assume they decrease to terminal values at a constant rate in 10 years, to be consistent with China's large scale SOE reform in the late 1990s and early 2000s which roughly ends at 2005. Fang et al. (2016) constructed the aggregate housing price indices for three tier of cities in China from 2003-2013. The average annual growth rate across all 120 cities in their sample is about 10%. During the transition in the model, we calibrate the aggregate housing supply such that the housing price grows at an annual rate of 10% for 40 periods.

Figure 4.1 plots the wealth share of top 10% households along the transition generated from the model. This share rises from about 30% in period 1 to more than 60% in period

40, an increase that is comparable to the data. Similarly, the the entrepreneurial wealth share increases from slightly over 0 to more than 40%.

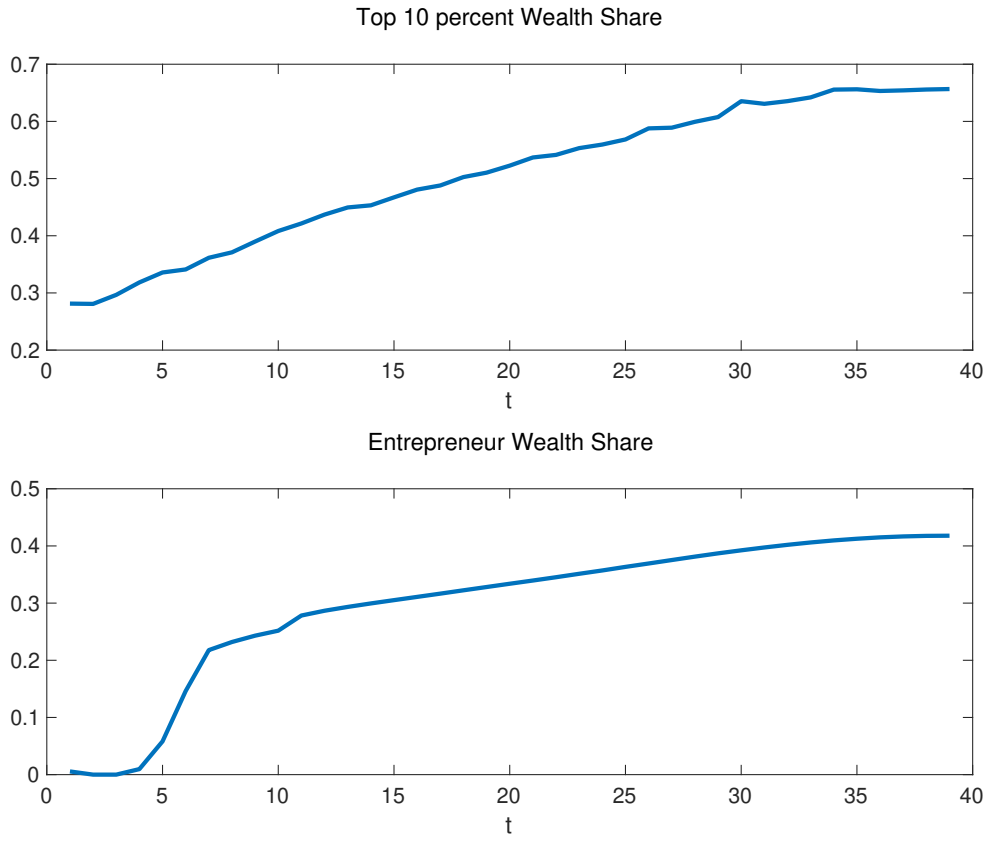


Figure 4.1: Top 10% Wealth Share During Transition in Model

Figure 4.2 presents the allocation of labor during the transitional dynamics. The fraction of entrepreneurs in population increase rapidly especially in the first 10 years after reform. By year 40, the share is about 13%. Accordingly, the employment share of private firms operates by entrepreneurs rises to about 80%, with SOE share decreasing to 20% after 40 years of urban reform. As migration barrier is gradually removed and urban becomes more attractive with a higher income, the agriculture employment share declines steadily from about 70% to less than 20% from year 1-40.

4.3 Counterfactual Exercises

To analyze the impact of different forces on the rising inequality, we have performed a couple of counterfactual exercises. In particular, we let the migration barrier parameter to stay at its initial level along the transition, and counterfactually shut down housing markets ($p_t^h = 0, \ \forall t$) to evaluate one-by-one the impact of each reform measures on China's

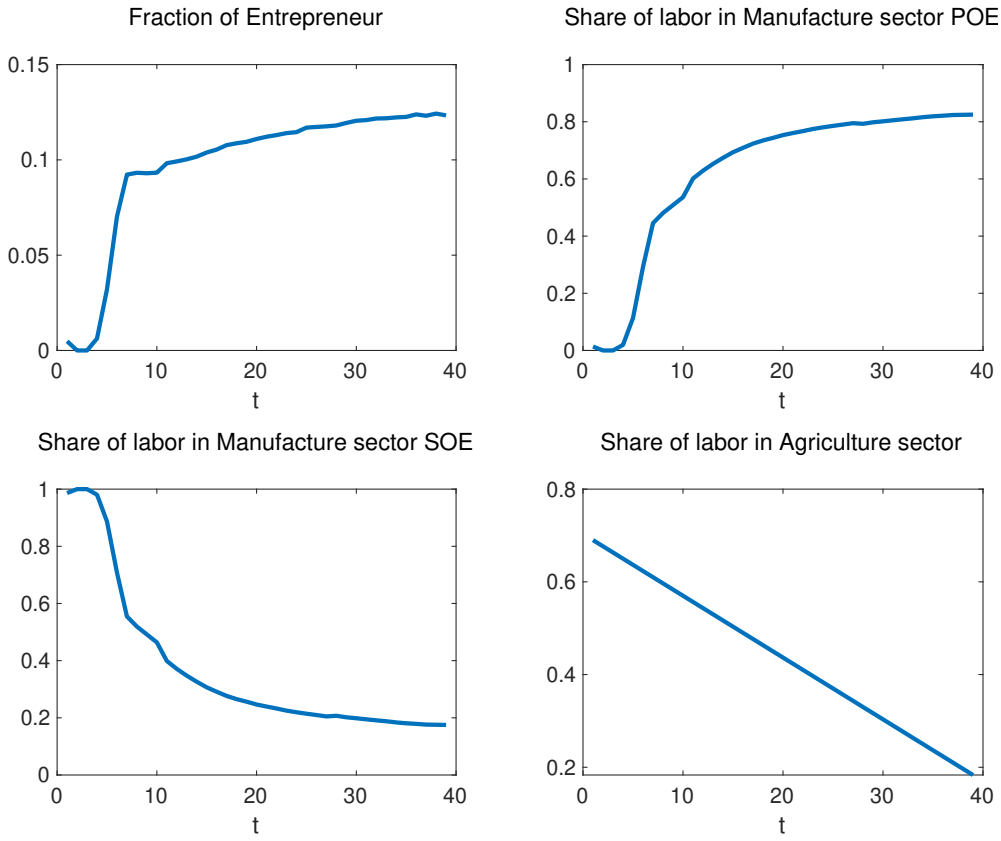


Figure 4.2: Labor Allocation During Transition in Model

rising wealth inequality. Figures B.5 and B.6 present the top 10% wealth share under these two counterfactual exercises. If migration barrier remains at the 1995 level, the inequality would rise more slowly comparing to the baseline. The reason migration might increase inequality is the following: the wealth of rural-to-urban migrants is relatively low, and inflow of migrant labor helps sustain a high return to capital accumulation in the urban.

On the other hand, counterfactually shutting down housing markets increase wealth inequality in our model. Even though wealthier households hold more houses in level, the housing-wealth ratio for this group is relatively low. As most middle class families have only limited access to valuable investment opportunities under severe financial friction, housing acts as a main vehicle of wealth accumulation for most middle class households. Shutting down the housing market therefore increases wealth inequality.

4.4 Discussion and Extensions

We are working on extending the baseline model or discussing the model's implication, in the following dimensions.

Increasing variance of the wage process Wage inequality has increased substantially from 1995 to 2020. To account for this change, we change the variance of the wage process in the model.

Stabilizing Inequality after the 2010s Across various estimates, the wealth inequality flattens after the 2010s. Some researchers date middle to late 2000s as China's Lewis turning point. This coincides with a rapidly rising wage and a declining rate of return to capital. In this section, we explore the impact of such a change through the lens of our model. In particular, we increase the speed of rural productivity from 2010.

Let Some People Get Rich First "Let some people get rich first" is a famous slogan made popular by Deng Xiaoping at the early stage of China's reform. In our model, rising inequality at the early stage of development, driven by profits and expansion of entrepreneurs, is beneficial for economic growth. Our baseline model do not address income tax. In this section, we introduce progressive tax rate, and investigate the impact of changes in this tax rate, especially that for top earners, on inequality as well as on economic growth.

5 Conclusion

In this paper, we study the evolution of wealth inequality in urban China since its market-oriented reforms in the early 1990s. Our research highlights the importance of understanding the rising wealth inequality together with the growth process. Empirically, from 1995-2017, the entrepreneurial share in population and wealth has increased significantly, and housing has been used as a major vehicle for wealth accumulation for most households after the housing reform in the late 1990s. Motivated by the facts, We developed a heterogeneous-agent dynamic general equilibrium model with endogenous migration, occupation, and durable consumption (housing) choices subject to frictions which are to capture China's institutional characteristics, to understand its evolution of wealth distribution in the reform era. Last, we use the model to quantitatively evaluate the impact of different factors /policies on the rising wealth inequality in China from 1995-2020.

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Appendix A Data and Method

A.1 Inequality

The share of wealthiest 10% household in total wealth is obtained by combining micro level household survey, CHIP in 1995 and 2002 and CHFS from 2011-2017, and *Hurun*'s *rich list*. We use the method proposed in Song et al. (2013) to construct weights in CHIP 1995, 2002 and 2007. For Hurun's rich list, we first calculate the percentage of the households in the list to total number of households, and then use the inverse of these percentage as the weight of households in the list. We also revise the original weights in the household survey to take into account the added households from Hurun's list, though this revision is very small given the small percentage of households in the list in total households. All households in Hurun's list are entrepreneurs and we assume they all reside in the urban area. As a robustness check we have also used CFPS from 2010-18.

Adjustment for CHIP-1995 CHIP-1995 is earliest household level data in China that contains wealth information. Urban housing reform in 1994, which later extends to the whole nation in 1998, has nontrivial impact of distribution of urban household wealth in 1995. Urban employees were allowed to purchase housing, say from their working unit, at a discounted price that is significantly lower than the market value. In CHIP-1995, there are 31.03% of urban households with self-purchased private house, as listed in Table A.1. For these households, average purchasing housing price is only 27.8% of their self reported market value of houses[13]. The reported housing market value for most households living in public housing is 0 as they do not own property rights to the house. Simply using the reported housing wealth would greatly overestimate the level of wealth inequality.

To obtain the pre-housing reform steady state wealth distribution, we count the purchase prices of private houses, instead of their estimated market value, as part of household wealth in CHIP-1995. Table A.2 presents the results of wealth inequality in 1995 under different approaches. Also shown is the urban wealth inequality calculated from CHIP in 1999, when the housing reform is finished, majority of households purchased private houses, and we use self reported market value of housing for all households. It is reassuring that the wealth inequality level, after adjusting for housing value, in 1995, matches

[13]CHIP-1995 contains a question on purchasing price for private house owning households. However, the year in which the house was purchase was not asked. Lack of this last piece of information might not cause large bias as China's urban housing reform started in 1994.

Table A.1: Urban house ownership, 1995

	Freq.	Percent
public housing owned by work unit	3,159	45.58%
other public housing	774	11.17%
inherited old private house	298	4.30%
self-built private house	446	6.43%
self-purchased private house	2,151	31.03%
house rented from private owner	56	0.81%
other	47	0.68%
total	6,931	100%

Data source: CHIP

quite well the level in 1999.

Table A.2: Top 10% wealth share in 1995 and 1999

Year	Measure	Value
1995	private housing value = self reported market value	41.61%
1995	private housing value = purchasing price	35.80%
1999	self reported housing market value	34.74%

Data source: CHIP 1995 and 1999

Impact of Alternative Definitions of Entrepreneurs Our baseline definition of entrepreneurs in a given year is households with at least one member who have operated industrial and commercial businesses (*gongshangye jingying*) at that year. As shown in Table 2.2, entrepreneurs account for 19.20% of urban population in 2015. Alternatively we exclude zero employment entrepreneurs, and define entrepreneurs as households that operate private business with at least 1 employment— employment can be either other household members or outside hiring. Table A.3 presents the population and wealth share of entrepreneurs under this alternative definition from CHFS 2015[14]. Using this more narrow definition, the population share of entrepreneurs decreases from 19.20% to 13.12%, and their wealth share declines from 37.27% to 30.25%. The entrepreneur's population share under this alternative definition is more in line with that from CFPS—entrepreneurs

[14]The patterns for other years in CHFS is similar.

account for 12.75% of urban households in 2016 based on the CFPS data.

Table A.3: Share of Entrepreneurs in Population and Wealth, with and without zero-employment entrepreneurs

Year	all	Top10% HHs	Top5% HHs	Top1% HHs
Baseline Definition				
Entrep's Pop Share	19.20%	34.45%	41.14%	63.27%
Entrep's Wealth Share	37.27%	45.45%	51.28%	70.41%
Excluding zero employment cases				
Entrep's Pop Share	13.12%	29.51%	36.76%	58.40%
Entrep's Wealth Share	30.25%	38.14%	43.41%	57.75%

Note: The column for "Top10% HHs" denotes "among wealthiest 10% households". *Data Source:* CHFS 2015.

Adjustment following Piketty et al. (2019) To address the potential inaccuracy of China's household survey data, Piketty et al. (2019) assumes that wealth information in the household survey is accurate up to the 90*th* percentile. Wealth for the top 10% households is scaled up to reflect missing wealth for the wealthiest in household surveys. In particular, they scale top 10% wealth in the survey data by 1.3 in 1995, and 1.5 from 2002 to 2017. Then assume the very top is given by Hurun's rich list, and use Pareto interpolation to obtain the full wealth distribution and calculate the wealth share of top 10% households. We follow the method in Piketty et al. (2019), and plot the time series of inequality in Figure B.1. Though the observation unit is adult and the unit is household in our measures, the time series we obtained are quite similar to Piketty et al. (2019).

Compared to Figure B.1, the inequality level in Figure B.2 is lower. For example, in 1995 the top 10% wealth share is above 40% in Figure B.1, and it is 30% in Figure B.2. In 2017, the top 10% wealth share is about 55% in Figure B.2 and 65% in Figure B.1. Even though the inequality level is lower, the two time series show very similar increasing trend.

A.2 Capital stock

We follow Bai et al. (2006) to calculate capital stock in China and extend the series to 2017. The stock of capital is calculated from the inventory approach

$$K_{t+1} = (1-\delta)K_t + I_t$$

Take 1952 as the initial year. Assume that the economy is initially in an old steady state, so that $I_0 = -(1-\delta)K_0 + K_1 = -(1-\delta)K_0 + K_0\frac{I_1}{I_0}$. It follows that

$$K_0 = \frac{I_0}{\delta + \frac{I_1 - I_0}{I_0}}$$

So we can back-out the value of K_0 using data about I_0, I_1 and δ. Use the average growth rate of investment (*gross fixed capital formation*) from 1953 to 1958 to approximate $\frac{I_1-I_0}{I_0}$. For depreciation rates, take 24% for machinery and equipment, and 8% for structure. We use the inventory approach to first calculate stock for *machinery and equipment* and *structure* separately, and then add them up into an aggregate stock of capital. As in Bai et al. (2006), we adjust for GDP deflators in 1992-1995, while maintaining its overall accumulated growth, to accommodate the vast fluctuation of investment deflators in that period.

The nominal return to capital j is

$$i(t) = \frac{P_Y(t)MPK_j(t)}{P_{K_j}(t)} - \delta_j(t) + \hat{P}_{K_j}(t)$$

where $\hat{P}_{K_j}(t) \equiv \frac{P_{K_j}(t+1)-P_{K_j}(t)}{P_{K_j}(t)}$ is percentage change in price of capital j. Denote $\alpha(t)$ the capital income share, the real return to capital equals to[15]

$$r(t) = i(t) - \hat{P}_Y(t) = \alpha(t)\frac{P_Y(t)Y(t)}{P_K(t)K(t)} + \hat{P}_K(t) - \hat{P}_Y(t) - \delta(t)$$

with $P_K(t)K(t) \equiv \Sigma_j P_{K_j}(t)K_j(t)$, $\delta(t) \equiv \Sigma_j \frac{P_{K_j}(t)K_j(t)}{P_K(t)K(t)}\delta_j(t)$, and $\hat{P}_K(t) \equiv \Sigma_j \frac{P_{K_j}(t)K_j(t)}{P_K(t)K(t)}\hat{P}_{K_j}(t)$. The return to capital can thus be measured according to this formula.

[15] See Appendix for details.

A.3 Estimate Wage Process from CHNS:

Estimate an AR(1) process from the residual adult equivalent household earnings from the urban and rural subsamples of China Health and Nutrition Survey (CHNS) from 1989 to 1997. For urban residents, earnings refer to labor market income and for rural residents, earnings refer to the sum of agricultural income, small business income and labor market income. This reflects the more diverse income sources of the rural households. We first regress log adult equivalent household earnings on dummies of sex, age, education and minority status of the household head and the province in which the household resides by year. Suppose the residual income, y_t follows an AR(1) process:

$$y_t = \rho y_{t-1} + \epsilon_t, \quad \epsilon_t \sim (0, \sigma^2)$$

Denote the different between residual income in year t and in year $t-\tau$ as $\Delta_\tau y_t$. Then,

$$\Delta_\tau y_t = (\rho^\tau - 1) y_{t-\tau} + \rho^{\tau-1}\epsilon_{t-\tau+1} + ... + \rho\epsilon_{t-1} + \epsilon_t.$$

The variance-covariance matrix of these differences contain

$$\begin{aligned} var(\Delta_\tau y_t) &= (\rho^\tau - 1)^2 var(y_{t-\tau}) + \rho^{2(\tau-1)}\sigma^2 + ... + \rho^2\sigma^2 + \sigma^2 \\ cov(\Delta_{\tau_1} y_t, \Delta_{\tau_2} y_{t+\tau_2}) &= (\rho^{\tau_1} - 1)(\rho^{\tau_2} - 1) cov(y_{t-\tau_1}, y_t) + (\rho^{\tau_2} - 1)\sigma. \end{aligned}$$

Note that the LHS moments on differences and the RHS $var(y_{t-\tau})$ and $cov(y_{t-\tau_1}, y_t)$ are all observed from the data. The parameters ρ and σ^2 are estimated using a minimum distance estimator and the standard errors are based on 50 bootstraps (Table A.4)

Table A.4: AR(1) Urban Income Process, CHNS 1989-1997

Para.	Value
ρ	0.751
	(0.031)
σ^2	0.141
	(0.010)
Observations	3712

Appendix B Table and Figures

Table B.1: Urban Proportion among Nation-Wide Wealthiest 10% Households

Year	1995	2002	2011	2013	2015	2017
	30.1%	85.6%	87.5%	93.7%	91.0%	95.1%

Data Source: CHIP 1995, 2002; CHFS 2011-17. This table measures the fraction of the nation's wealthiest 10% households that are from the urban area.

Table B.2: Portfolio composition of the wealth quintiles in the US

	1st	2nd	3th	4th	5th	all
Liquid assets	8.1	17.0	10.2	7.4	5.9	6.4
Retirement accounts	12.0	21.5	19.6	24.3	17.8	18.7
Houses	140.3	207.1	120.5	74.8	21.7	32.4
Vehicles	40.3	73.1	20.3	8.7	1.7	3.7
Mtge+HELOCs	-161.1	-178.4	-71.4	-30.7	-6.0	-12.7
Installment	-129.7	-45.7	-10.9	-3.9	-0.4	-2.3

Data source: Kuhn & Rios-Rull (2016), based on SCF, 2013

Table B.3: Housing consumption share in total urban household consumption, %

	2014	2015	2016	2017	2018	2019	Mean
(1) Rent	1.4	1.2	1.1	1.1	1.4	1.3	1.3
(2) Housing maintenance	2.1	2.3	2.5	2.9	2.6	2.9	2.6
(3) Water, Electricity & fuel	4.7	4.4	4.3	4.2	4.0	3.8	4.2
(4) Imputed rent for self-owned houses	14.2	14.2	14.2	14.6	16.0	16.2	14.9
Total	22.5	22.1	22.2	22.8	24.0	24.2	23.0

Data source: China Household Survey, 2020.

Table B.4: Employment type for urban residents, %

	2014	2015	2016	2017	2018	2019	Mean
(1) Employer	2.4	1.9	1.5	1.2	1.4	1.2	1.6
(2) Public servant	5.7	5.5	5.3	5.2	4.4	4.3	5.1
(3) Personnel in public institutions	11.0	11.2	11.5	11.3	10.7	10.4	11.0
(4) SOE employee	9.6	9.5	9.3	9.0	7.6	7.2	8.7
(5) Other employee	52.6	55.6	55.8	57.0	58.1	59.4	56.4
(6) Agriculture self employed	5.6	5.0	4.8	4.5	6.5	6.1	5.4
(6) Non-agriculture self employed	13.0	11.4	11.8	11.8	11.2	11.3	11.8

Data source: China Household Survey, 2020.

Table B.5: Income and Wage, Yuan per person

	2014	2015	2016	2017	2018	2019
Urban disposable income	28843.9	31194.8	33616.2	36396.2	39250.8	42358.8
Urban wage income	17936.8	19337.1	20665.0	22200.9	23792.2	25564.8
Urban net operating inc	3279.0	3476.1	3770.1	4064.7	4442.6	4840.4
Urban net property inc	2812.1	3041.9	3271.3	3606.9	4027.7	4390.6
Urban net transfer inc	4815.9	5339.7	5909.8	6523.6	6988.3	7653.0
Rural disposable income	10488.9	11421.7	12363.4	13432.4	14617.0	16020.7
$\frac{\text{Urban Wage}}{\text{Rural Income}}$	1.71	1.69	1.67	1.65	1.63	1.60

Data source: China Household Survey, 2020.

Table B.6: Entrepreneur-Worker 2 year transition probability

	Entrepreneur in 2017	Worker in 2017
Entrepreneur in 2015	63.08%	36.92%
Worker in 2015	6.38%	93.62%

Data source: CHFS, 2015-17.

Figure B.1: Wealth Share of Top 10% Households vs Adults in China (%)

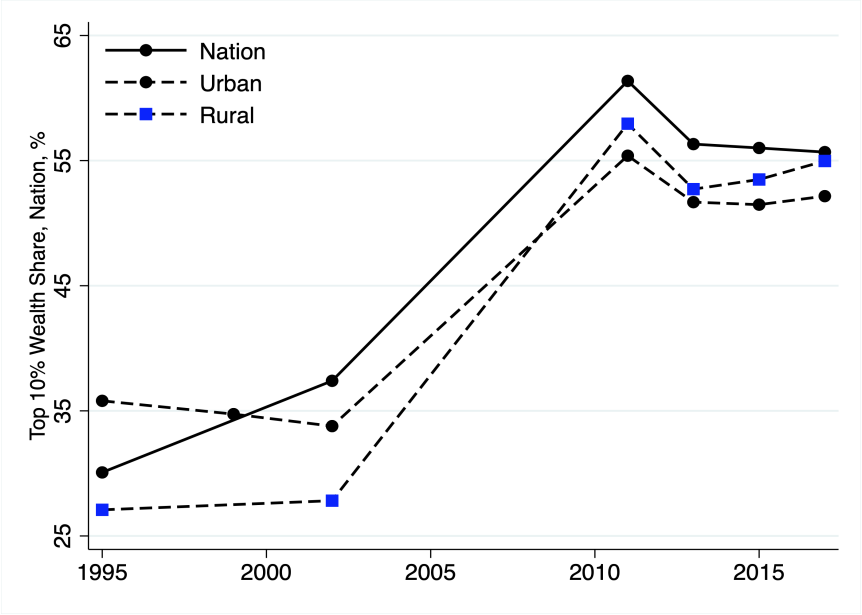

Figure B.2: Top 10% Wealth Share in China, Nation, Urban vs Rural (10%)

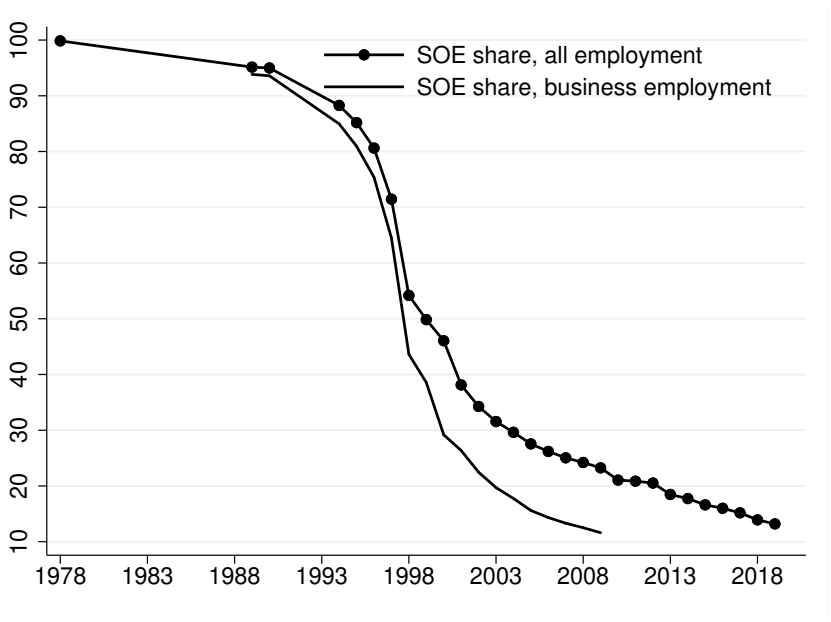

Figure B.3: SOE Share in Urban Employment (%)

Note: For all employment, SOE employment contains state owned enterprises (*qiye*), public institutions (*shiye danwei*), and public agencies and organizations (*jiguan*). For the business sector, SOE employment refers to employment in state owned enterprises. Data source: China Labor Statistical Yearbook, various years.

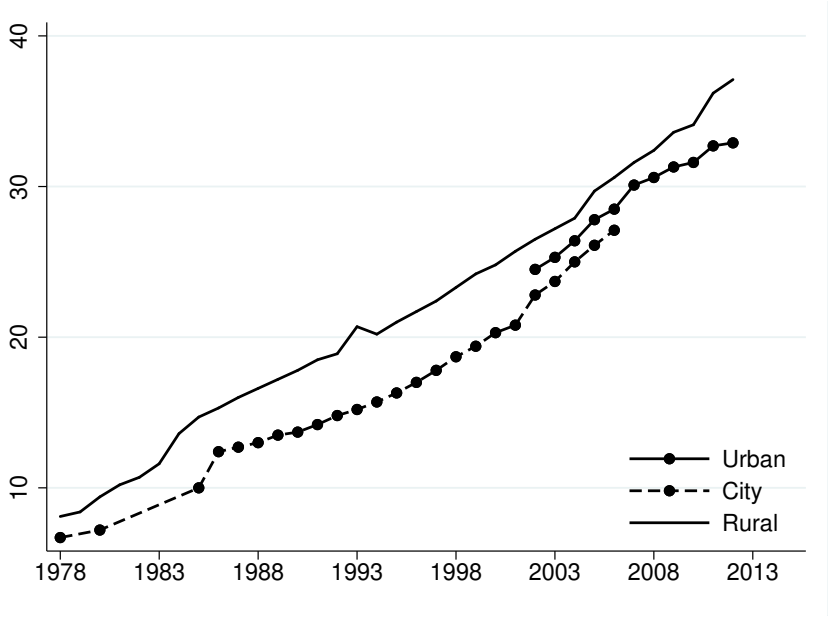

Figure B.4: Average Housing Size per Person (square meter)

Note: Urban (*chengzhen*) contains city (*chengshi*) and township (*zhen*). Data Source: rural and urban household survey, 1978-2013.

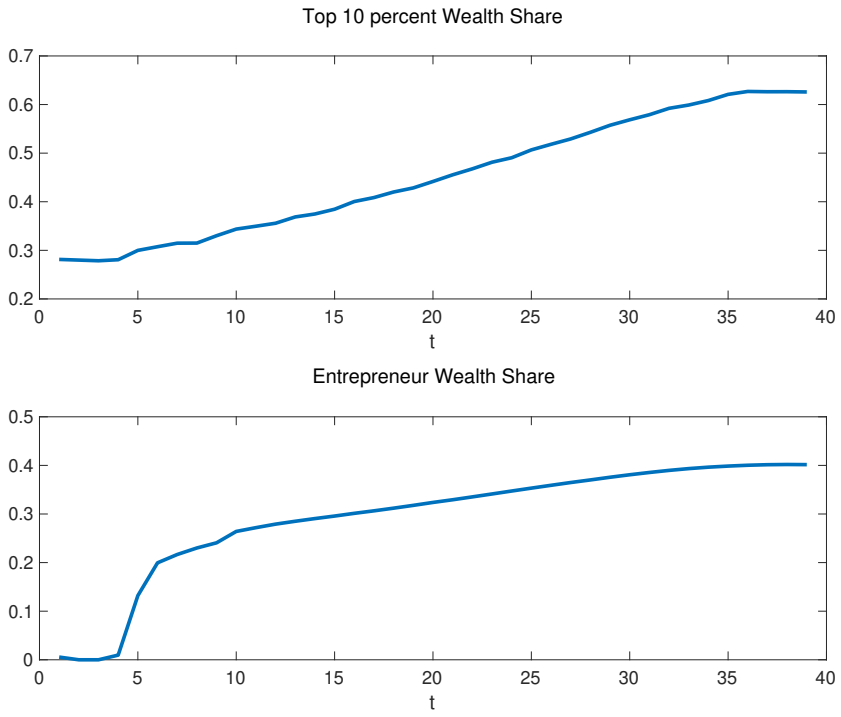

Figure B.5: Counterfactual: Migration Barrier Staying at 1995 level

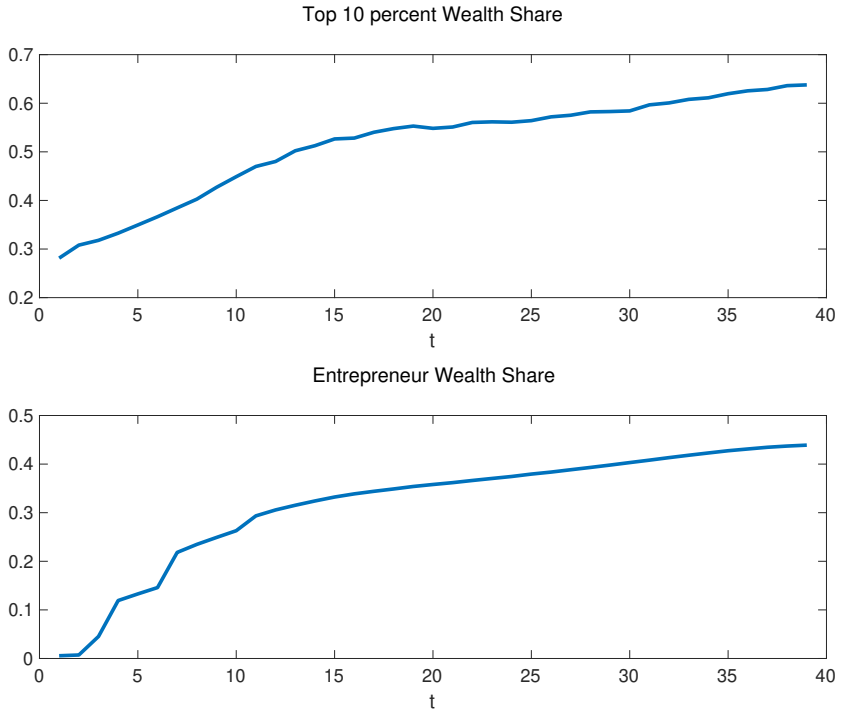

Figure B.6: Counterfactual: Shutting Down Housing Markets

Appendix C Numerical algorithm

C.1 Computing the stationary equilibrium

1. Guess the interest rate in the invariant distribution, r^i (bisection).

2. Then we have the corresponding wage rate w^i in the invariant distribution based on the following equation.
$$w = (1 - \alpha_s) \left(\frac{\alpha_s}{r + \delta} \right)^{\frac{\alpha_S}{1-\alpha_S}}$$

3. Given the guesses on interest rate and wage rate, solve the individuals' optimization problem using value function iteration.

4. Given the optimal decision rule, compute the transition matrix, and then solve the stationary distribution.

5. Then we solve for the aggregate supply of physical capital in the stationary equilibrium. From the following capital market clearing condition we can solve for the capital demand for SOE sector K_s
$$K_s = \int b'(\mathbf{x}) dF(\mathbf{x}) - \int I\{o(\mathbf{x}) = e\} k(\mathbf{x}) dF(\mathbf{x})$$

6. Calculate the demand of labor for SOE
$$L_{SOE} = \frac{(1 - \alpha)(r + \delta)}{\alpha w} K_{SOE}$$
Check the labor market clearing condition in the stationary equilibrium.
$$L_{s,t} + \int I\{o(\mathbf{x}_t) = e\} l_t(\mathbf{x}_t) dF(\mathbf{x}_t) = \int I\{o(\mathbf{x}_t) = w\} dF(\mathbf{x}_t)$$
If there is excess labor demand choose a new interest rate r^{i+1} that is smaller than r^i (which is equivalent to a new wage w^{i+1} that is greater than w^i.) If there is excess labor supply, choose a new interest rate r^{i+1} that is greater than r^i (which is equivalent to a new wage w^{i+1} that is smaller than w^i.)

7. Repeat Step 2-7 until the labor market clear in the stationary equilibrium.

C.2 Value Functions

With migration barrier introduced the way in the text, the value function $V(e,z,a)$ is

$$V(e,z,a) = \nu \log[\exp(V_r(e,z,a))^{1/\nu}) + \exp(V_u(e,z,a) - \tau_m)^{1/\nu})]$$

The probability of a worker live in the urban is

$$\frac{\exp(V_u(e,z,a) - \tau_m)^{1/\nu}}{\exp(V_r(e,z,a))^{1/\nu} + \exp(V_u(e,z,a) - \tau_m)^{1/\nu}}$$

C.3 Computing the transition dynamics

To compute the entire transition dynamics, we have to iterate on the interest rate sequences. Taking the interest rate sequences as given, we solve for the individuals' optimization problem, and then check whether labor and housing markets clear for all periods. We fix T at 130.

1. Guess the interest rate sequence $\{r_t^i\}_{t=0}^T$.
2. Then we have the corresponding wage rate sequence w^i in the transition dynamics based on the following equation.

$$w = (1-\alpha_s)\left(\frac{\alpha_s}{r+\delta}\right)^{\frac{\alpha_s}{1-\alpha_s}}$$

3. Compute the value function of the stationary equilibrium, and let $V_T(\mathbf{x}_t) = V(\mathbf{x}_t)$.
4. By backward induction, taking the wage sequence $\{w^i\}_{t=0}^T$ and the interest rate sequence $\{r_i\}_{t=0}^T$ as given, compute the value function $V_t(\mathbf{x}_t)$ for $t = T-1, ..., 0$.
5. Given the optimal decision rule, calculate transition of distribution function $F_t(\mathbf{x}_t)$
6. Then we solve for the aggregate supply of physical capital in the transition dynamics. From the following capital market clearing condition we can solve for the capital demand for SOE sector $K_{s,t}$

$$K_{s,t} = \int b_{t+1}(\mathbf{x}_t)dF_t(\mathbf{x}_t) - \int I\{o(\mathbf{x}) = e\}k(\mathbf{x}_t)dF_t(\mathbf{x}_t)$$

7. Calculate the demand of labor for SOE

$$L_{s,t} = \frac{(1-\alpha)(r+\delta)}{\alpha w} K_{s,t}$$

Check the labor market clearing condition in the stationary equilibrium. Construct a sequence $\{\bar{r}_t^i\}_{t=0}^T$ that clears the labor market for each period.

$$L_{s,t} + \int I\{o(\mathbf{x}_t) = e\} l_t(\mathbf{x}_t) dF(\mathbf{x}_t) = \int I\{o(\mathbf{x}_t) = w\} dF(\mathbf{x}_t)$$

Update the interest rate sequence: $r_t^{i+1} = \eta r_t^i + (1-\eta)\bar{r}_t^i$

8. Repeat Steps 2–7 until the interest rate sequence also converges.