Pension Plan Systems and Asset Prices

Nuno Coimbra[†] Francisco Gomes[‡] Alexander Michaelides[§] Jialu Shen[¶]

February 1, 2022

Abstract

We show that incorporating defined benefit pension funds in an asset pricing model with incomplete markets improves its ability to jointly match the historical equity premium and riskless rate. We identify a new risk channel arising from fluctuations in the funding position of the pension fund, which increase the volatility of household disposable income. We further emphasize the importance of the pension fund's size and asset demands in determining equilibrium asset prices. We then use our calibrated model to study the implications of a shift from an economy with defined benefit pension schemes to one with defined contribution plans. We find that the new steady-state is characterized by a higher riskless rate and a lower equity premium.

JEL Classification: E21, E44, G11, G12, G50.

Key Words: Equity Premium, Intermediary-Based Asset Pricing, Pension Funds, Defined Contribution and Defined Benefit Pension Plans, Incomplete Risk Sharing, Limited stock market participation.

[*]We thank Harjoat Bhamra, Paymon Khorrami, Howard Kung, Francis Longstaff and seminar participants at the Bank of France for their helpful comments.

[†]Bank of France, nuno.coimbra@banque-france.fr.

[‡]London Business School and CEPR, fgomes@london.edu.

[§]Imperial College Business School and CEPR, a.michaelides@imperial.ac.uk.

[¶]Trulaske College of Business, University of Missouri, jshen@missouri.edu.

1 Introduction

The most important savings motive for most individuals is financing consumption during retirement. In most countries, the majority of those savings are made automatically through defined benefit pension plans. The accumulated wealth is then managed by a pension fund which is responsible for making pension payments during retirement. Therefore, individuals do not choose how much to save (in this particular category), or how to invest those savings. In general equilibrium asset pricing models, it is common practice to abstract from these pension arrangements, mostly for tractability reasons. However, the value of total wealth held by defined benefit pension funds (hereafter DBPF) is quite significant, averaging to about 100% of U.S. GDP between 1996 and 2017.[1] Assuming that this wealth is invested according to household first-order conditions abstracts from the important institutional considerations and restrictions that apply to these pension funds. Furthermore, the return on this endowment of wealth is an important source of funding of pension benefits, and this has important implications for household risk and its correlation with the stock market.

In this paper, we consider an asset pricing model with an explicit defined benefit pension scheme. We first show how the implications of such a model differ from those obtained with the standard formulation and, in particular, improve the ability to match standard asset pricing moments. We then use the model to study the implications of shifting from an economy where retirement savings are managed by defined benefit pension schemes, to one where those schemes have been replaced by defined contribution plans, a trend that we currently observe in several countries. Models that abstract from pension arrangements are silent on the potential asset pricing and macro-economic implications of this seismic shift in the retirement savings landscape.

Our baseline economy is a production economy with incomplete markets and overlapping generations of households. During working life households contribute to a defined benefit pension plan, make social security contributions and can supplement both of these with their own private savings. Once retired households collect both social security benefits and defined benefit pension payments. In addition, they can also use their accumulated private savings to finance consump-

[1]This has implications for the overall development of the financial system across different economies, as emphasized by Scharfstein (2018).

tion. The model includes borrowing constraints and uninsurable labor income risk, to capture precautionary motives (as in Hubbard et al. (1995), Carroll (1997), Gourinchas and Parker (2002) and Cocco et al. (2005), for example), so retirement is not the only savings motive for households. Households face earnings risks with negative skewness over the business cycle, a feature of earnings risk that has been documented by Guvenen et al. (2014).[2]

We consider two types of households with heterogeneous preferences. They both have the same risk aversion coefficient, but one group has a strong preference for savings (high discount factor and high elasticity of intertemporal substitution, EIS), while the other has a low discount factor and a low EIS. The latter represent households who save very little, but we generalize the common "hand-to-mouth" formulation by allowing them to be optimizing agents with some savings. This is particularly important when we consider different comparative statics and counterfactuals, as we want to allow those agents to re-optimize their behavior as well.

In most countries the majority of the population does not invest in equities. In the U.S, where the percentage of stock market participants is comparatively higher, the number is still close to 50% (see Gomes et al. (2020) for a recent review). Therefore, those households are not directly exposed to stock market risk, and have a low rate of return on their savings.[3] Importantly, the incentives to become a stock market participant are likely to change if households are enrolled in a defined contribution pension plan versus a defined benefit pension plan. Therefore, we incorporate limited stock market stock market participation in the model and treat it as an endogenous decision (as in Vissing-Jørgensen (2002), Gomes and Michaelides (2005), and Fagereng et al. (2017)) which is allowed to change both over time for a given pension system, and across pension systems.

We first show that the model is able to match the historical average riskless rate and the market Sharpe ratio, a particularly difficult combination to achieve in the context of production-based asset pricing models (see, for example, Jermann (1998), Storesletten et al. (2007), Gomes and Michaelides (2008), Guvenen (2009), Croce et al. (2012), Favilukis (2013), Gomes et al. (2013), Kung and Schmid (2015), Favilukis et al. (2017)), or Elenev et al. (2021)). In our baseline model,

[2]Catherine (2020) and Shen (2020) study the portfolio choice implications of this earnings process, while Constantinides and Ghosh (2017) considers the asset pricing implications.

[3]The asset pricing implications of limited stock market participation are investigated by Mankiw and Zeldes (1991), Basak and Cuoco (1998), Luttmer (1999), Cao et al. (2005) Gomes and Michaelides (2008), Guvenen (2009) and Favilukis (2013), among others.

the volatility of equity returns is 13.7%, compared with 19.8% in the data. As a result, even though we match the Sharpe ratio, our equity premium is only 4.65%. This is still quite high for a production-based asset pricing model (see previous references).[4] Finally, our economy delivers a stable riskfree rate (volatility of 1.34%) and limited stock market participation consistent with the data (participation rate of 52.3%).

We then compare our results with those obtained in an otherwise identical economy where we set the endowment of the DBPF equal to zero. The DBPF is, therefore, a simple pass-through entity, collecting pension contributions from current workers and paying pension benefits to current retirees.[5] When both models are calibrated to deliver the same riskless rate and stock market participation, our baseline economy has a substantially higher Sharpe ratio (0.34 versus 0.24). When the funding position of the pension fund deteriorates, the pension contributions must increase to cover the gap.[6] Therefore, our model captures an important additional source of exposure to stock returns for households, arising from the investment decisions of the pension fund. Since the endowment of the defined benefit is partially invested in the stock market, this will impact both the volatility of household consumption growth and its correlation with stock returns.

In practice negative shocks to pension fund wealth can be reflected either in increases to the contribution rates of workers or in increases to the contributions made by firms. We explore both of these adjustment margins in the context of our model. These two alternatives have slightly different implications for risk premia and consumption volatility. In the second case the risk is directly absorbed by shareholders, while increases in workers' contribution rates increase the volatility of consumption for all households. However, regardless of the set-up, in equilibrium this additional channel leads to a higher Sharpe ratio and a higher equity premium, relative to an economy that ignores it.

Pension fund contributions (or benefits) are infrequently adjusted, as the financial position of the pension fund reaches a certain threshold. In our model, the contributions (either those

[4]We could increase the volatility of equity returns further, while keeping a low volatility of the riskless rate. However, that would imply a counter-factually high standard deviation of consumption growth.

[5]For brevity, we refer to such a model as "noDBPF". Since the DBPF does not have an endowment with its associated return, the contributions from current workers must be higher.

[6]Alternatively, the pension benefits must fall, or we can have a combination of the two. Economically both potential adjustments represent a source of risk for households.

from workers or those from firms) are adjusted every year to keep the endowment of the DBPF unchanged. This is done mainly for tractability reasons, to avoid having the funding position of the DBPF as an additional state variable.[7] Regardless of the set-up, the crucial economic mechanism remains: household disposable income, and hence consumption is impacted by the returns on the endowment of the DBPF. If we instead captured this with infrequent adjustments, then such risk would manifest itself in a series of infrequent large jumps as opposed to more regular small fluctuations. Arguably the risk premium implications might even be larger than the ones we capture with our current modelling approach.

Having established that our model matches extremely well the historical riskless rate, the Sharpe ratio on equities, the volatility of consumption growth and the stock market participation rate, we proceed to our counter-factual analysis. Given the current funding problems of most defined benefit pension schemes, we observe an important shift towards DC pension plans in several countries. Different dimensions of the costs and benefits associated with this change have been studied by İmrohoroğlu et al. (1998), Conesa and Krueger (1999) and Nishiyama and Smetters (2007), for example, but they have not considered the joint macroeconomic and asset pricing implications that we are studying here. This part of our paper is similar to the analysis in Abel (2001), but he studies the implications of shifting social security investments to the stock market, while we consider a change in the pension system, from Defined Benefit to Defined Contribution. More precisely, we compare the equilibrium in our baseline economy with the one obtained in an otherwise identical economy where the DBPF has been shut down, and households are now saving for retirement fully in their own private pension accounts ("DC-only economy").

In the "DC-only economy", there is a reduction in precautionary savings, since the volatility of income is lower, as households are no longer exposed to fluctuations in DB contribution rates. This is reflected in lower wealth accumulation early in life. Later on, as they start saving for retirement, private wealth accumulation is naturally higher, since they must compensate for the reduction in retirement income. However, households do not have the incentive to fully substitute for the DBPF's wealth, since they could have already saved more before if that was optimal for them. The pension fund is "forcing" households with a low discount factor to save more

[7] In addition, it avoids having to set arbitrary rules for the threshold levels of the fund's position that would trigger an adjustment in the contributions.

for retirement than they would otherwise. So, total retirement wealth is lower in the "DC-only economy" which, combined with a decrease in precautionary savings, implies a reduction in total wealth in the economy. The implications for asset returns are not straightforward though, since the asset demand of households differs from the demand of the DBPF and, furthermore, it has shifted in the new equilibrium. Therefore we again require our quantitative model to understand the impact on equilibrium returns.

We find that the total capital stock is only slightly lower in the DC-only economy. Despite this small difference, the asset pricing moments are very different across the two economies. In the absence of the DBPF, the demand for bonds is reduced, implying a higher equilibrium risk-free rate, while the lower standard deviation of consumption growth leads to a smaller equity premium and Sharpe ratio. The lower equity premium reduces stock market participation among households with a high savings motive, but the percentage of stockholders among those with a low savings motive increases, since they must now save more for retirement. On net, stock market participation is slightly higher in the new steady-state.

Our paper is related to a growing literature studying the importance of financial intermediaries for macroeconomic activity and asset pricing. Previous literature has mostly focused on the role of banks and asset managers such as mutual funds and hedge funds (e.g. Adrian and Shin (2010), He and Krishnamurthy (2013), Adrian and Shin (2014), Adrian et al. (2014), Brunnermeier and Sannikov (2014), Khorrami (2021) and He and Krishnamurthy (2018) for a recent review). Here we consider the importance of asset owners: defined benefit pension funds. In addition to considering the role of a different type of institution, our paper differs from this previous work because we are not just focusing on the role of intermediary constraints and/or frictions. We are also highlighting the importance of taking into account the wealth endowment of these asset owners, and the implications for household risk arising from fluctuations in this endowment. Models that do not include that endowment are missing wealth that was accumulated in the past, when these funds were first created, and therefore missing the indirect link between stock returns and household disposable income that this creates.[8]

[8]Naturally we are not implying that those models are mis-specified. In the absence of the endowment, and its returns, pension payments are fully financed by the contributions of current workers, which are therefore higher than in our baseline economy.

Our paper is also related to the literature on delegated portfolio management where institutional investors operate under certain constraints when making asset allocation decisions. In the area of pension funds, institutional investors must respect complex regulations. For instance, the 1974 Employee Retirement Income Security Act and the 2008 Pension Protection Act (PPA) seek to protect beneficiaries of corporate defined benefit pension plans. These regulations affect the pension asset allocation in equities via a trade-off between risk shifting and risk management (Rauh (2009)), or even through the effects of different disclosure requirements (Chuk (2013)). Corporate pension plans might also be subject to performance constraints faced by other institutional investors (Basak and Pavlova (2013)), or have mandates that determine how to respond to changing asset prices (Gabaix and Koijen (2021)), subject to the legal constraints imposed by the relevant regulations. Public pension plans do not fall under the PPA, and there is an active discussion about incentives to invest in equities based on actuarial assumptions (Lucas and Zeldes (2009)), and evidence that different political constraints affect the asset allocation decision (for example, Andonov et al. (2017)).

Finally, our paper is also part of the literature on asset pricing with incomplete risk sharing, such as the production economy models of Storesletten et al. (2007), Gomes and Michaelides (2008), Favilukis (2013), Gomes et al. (2013), Favilukis et al. (2017) and Elenev et al. (2021), and the exchange economy models of Telmer (1993), Lucas (1994), Constantinides and Duffie (1996), Constantinides et al. (2002) and Heaton and Lucas (1996).

The paper is organized as follows. Section 2 presents the baseline model and its calibration, while in Section 3 we discuss the equilibrium results. In Section 4, we compare those results with the ones obtained when we ignore the defined benefit pension fund. In Section 5, we compare our current equilibrium with one where DB plans have been fully phased out, and we conclude in Section 6.

2 The Model

2.1 Outline

We consider an asset pricing production economy model with heterogeneous households and incomplete markets. Markets are incomplete because households face uninsurable labor income risk with borrowing constraints, and because they have a finite horizon. Households can invest in two assets, a claim to the risky capital stock (equity) and a riskless government bond. Investing in equities requires paying participation costs, both a first-time entry cost and a per-period cost. We consider two groups of households with heterogeneous preferences. This heterogeneity is important for obtaining significant cross-sectional wealth inequality and for generating endogenous limited stock market participation with realistic participation costs (see, for example, Gomes and Michaelides (2005) and Gomes and Michaelides (2008)).

From ages 20 to 65 (working life), households supply labor inelastically and face countercyclical earnings risks as in Guvenen et al. (2014). In the baseline version of the model, during retirement (after age 65), households receive income from both social security and a defined benefit pension.[9] The social security payments are financed by taxes on current workers' wages. The defined benefit pension is financed both by contributions made by current workers and/or firms, and by the return on the accumulated wealth of the pension fund.

Since we are focusing on household risk and savings, the production side of the model is fairly standard. All firms are identical and perfectly competitive. They combine capital and labor, using a constant returns to scale technology, to produce a non-durable consumption good. We close the model by explicitly modelling a government sector. The government issues bonds, which therefore exist in a positive net supply. This is crucial for matching household portfolios and therefore for obtaining a realistic calibration of household-level risk.[10] The interest payments on public debt are financed by (linear) taxes on capital gains, bequests and wages, with government expenditures determined by the government budget constraint.

[9]These can be supplemented by private savings which, in our model, also capture savings into defined contribution pension schemes. Therefore, in our baseline economy, both systems (DB and DC) co-exist as is currently the case in the U.S..

[10]This comes at a cost however, since having a positive supply of the riskless asset makes it harder to match asset pricing moments in this class of models (see Gomes and Michaelides (2008)).

2.2 Firms

Since the focus of the paper is on the role of pension plans, retirement savings and household risk, the production side of the economy is quite standard.

2.2.1 Production technology

Firms produce a single non-durable consumption good using a standard Cobb-Douglas production function, with total output at time t given by:

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where K is the total capital stock in the economy, L is the total labor supply, and Z is a stochastic productivity shock, which follows the process:

$$Z_t = G_t U_t \tag{2}$$

$$G_t = (1+g)^t \tag{3}$$

where g captures aggregate growth, and the productivity shocks (U_t) follow a two-state Markov chain capturing business cycle fluctuations.

Standard frictionless production economies cannot generate sufficient return volatility, since agents can adjust their investment plans to smooth consumption over time (see Jermann (1998) or Boldrin et al. (2001)), but introducing adjustment costs of capital in a model with incomplete markets is conceptually challenging.[11] We address this problem by following a common approach in this literature and modelling the depreciation rate as stochastic:[12]

$$\delta_t = \bar{\delta}(U_t) + \sigma^{\delta}(U_t)\eta_t \tag{4}$$

[11] Adjustment costs would add an intertemporal dimension to the firm's problem, and the solution to such problem is not well defined under incomplete markets (see Grossman and Hart (1979)). Favilukis et al. (2017) offer a practical solution to this problem by using a sensible stochastic discount factor in the firm's optimization problem.

[12] See, for example, Krueger and Kubler (2006), Storesletten et al. (2007), Gomes and Michaelides (2008), Gottardi and Kubler (2011), and Gomes et al. (2013).

where η_t is an i.i.d. standard normal shock. Both the period mean ($\bar{\delta}$) and standard deviation (σ^{δ}) of depreciation are correlated with aggregate productivity shocks (U_t) to match business cycle fluctuations, as discussed in the calibration section. Therefore, δ_t is interpreted a more general measure of economic depreciation, combining physical depreciation, adjustment costs, capital utilization, and investment-specific productivity shocks.[13]

2.2.2 Maximization problem

Firms are perfectly competitive, so they take wages (W_t) and return on capital (R_t^K), as given. They face no frictions (e.g. no adjustment costs of capital) and make their decisions after observing the aggregate shocks. Therefore, they solve a sequence of static maximization problems with no uncertainty:

$$\max_{K_t, L_t} Z_t K_t^{\alpha} L_t^{1-\alpha} - W_t L_t - R_t^K K_t \tag{5}$$

The first-order conditions to this optimization problem are

$$W_t = (1-\alpha) Z_t (K_t/L_t)^{\alpha} \tag{6}$$

$$R_t^K = \alpha Z_t (L_t/K_t)^{1-\alpha} + 1 - \delta_t \tag{7}$$

2.3 The government sector

The government sector issues one-period riskless bonds, which therefore exist in positive net supply allowing us to match the average portfolio allocations in the data.[14] The government's budget constraint is

$$C_t^G + R_t^B B_t = B_{t+1} + T_t \tag{8}$$

where C^G is government consumption, B is public debt, R^B is the gross interest rate on government bonds, and T denotes the tax revenues.

The government collects revenues from proportional taxes on capital income (tax rate τ^K), on

[13] Greenwood et al. (1988) use the same approach to model fluctuations in capital utilization.

[14] If the riskless asset is in zero net supply, then the average risky portfolio share in the model would be 100%, which is inconsistent with the data on household portfolios. Furthermore, since we have limited stock market participation, this would require stockholders to have an equity share above 100%, which would deliver a higher equity premium, but would again be counter-factual.

bond interest payments (tax rate τ^B), wages (tax rate τ^W) and bequests (tax rate τ^E). Government expenditures do not enter the agents' utility functions, and are determined as the residual from Equation (8), given the (exogenous) level of debt, the (exogenous) tax rates, and the (endogenous) interest rate on bonds.

2.4 Households and financial markets

There are two types of households in the model (A and B). They are ex-ante different because type-B households have preferences that imply high wealth accumulation, while type-A households have preferences that lead them to consume most (but not all) of their labor income. As a result, in equilibrium, most type-B households will find it optimal to pay the participation costs and invest in stocks, while most of type-A households will invest only in government bonds.

2.4.1 Life-cycle and preferences

We follow the convention in life-cycle models and let adult age (a) correspond to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 periods (age 100). The probability of being alive at age $(a+1)$, conditional on being alive at age a, is denoted by p_a (with $p_0 = 1$), and at each point in time there is a stationary age distribution of households in the economy, with no population growth.

Households have Epstein-Zin-Weil preferences (Epstein and Zin (1989) and Weil (1990)) defined over consumption of a single non-durable good (C_a^i):

$$V_a^i = \left\{(1-\beta)C_a^{1-1/\psi} + \beta(E_a(p_a V_a^{1-\gamma}))^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}} \tag{9}$$

where β is the discount factor, γ is the coefficient of relative risk aversion and ψ is the elasticity of intertemporal substitution.[15] Both types of households (A and B) have the same degree of risk aversion, so $\gamma^A = \gamma^B$, but they have a different EIS and different discount factors, as discussed in

[15] We do not include a bequest motive in the model because, in general equilibrium with overlapping generations and stochastic mortality, this would require (young) agents to form expectations about the future bequest that they might receive. Instead we assume that accidental bequests in the model are fully taxed by the government. Another alternative would be to assume perfect annuity markets after retirement, but that would eliminate longevity risk from the model.

the calibration section.

2.4.2 Labor income

Before retirement (i.e. until age 65), all households supply labor inelastically, and face individual-specific productivity shocks. Individual labor income (H^i_{at}) is the product of individual productivity (L^i_{at}) and the aggregate wage per unit of productivity (W_t):

$$H^i_{at} = W_t L^i_{at} \tag{10}$$

The aggregate wage is determined in equilibrium by equation (6), while the stochastic process for individual productivity is given by a permanent component P^i_{at} and a transitory shock ε^i_t.:

$$L^i_{at} = P^i_{at}\varepsilon^i_t \tag{11}$$

$$P^i_{at} = \exp(f(a))P^i_{a-1,t-1}\xi^i_t \tag{12}$$

where $f(a)$ is a deterministic function of age, capturing the typical hump-shape profile in life-cycle earnings and ξ^i_t are shocks to the permanent component. We assume that $\ln \varepsilon^i$ is independent and identically distributed with mean $\{-.5 * \sigma^2_\varepsilon\}$, and variances σ^2_ε.

Following the evidence in Guvenen, Ozkan and Song (2014), we assume $\ln \xi^i_t$ is a mixture of normal distributions, so that conditional on the state of the economy U_t the innovation $\ln \xi^i_t$ is drawn from one distribution with probability q_1 and with probability $(1 - q_1)$ from a second distribution:

$$\ln \xi^i_t = \begin{cases} \ln \xi^i_{t,1} \sim N(\mu_{1,U_t}, \sigma^2_{1,U_t}) & \text{with prob } q_1 \\ \ln \xi^i_{t,2} \sim N(\mu_{2,U_t}, \sigma^2_{2,U_t}) & \text{with prob } 1 - q_1 \end{cases} \tag{13}$$

By making both conditional skewness in the innovations to the permanent income shocks (ξ), and the expected growth rates, dependent the aggregate productivity state (U_t), we allow for countercyclical earnings risk as also emphasized by Guvenen et al. (2014).

2.4.3 Financial markets

There are two financial assets in the model, so markets are incomplete. The first asset (stocks/equity) is a claim on the capital stock of firms and has a risky return (R_t^K). The second asset is the one-period riskless bond issued by the government. Since there is no default on government bonds in the model, the rate of return the riskless asset can be written as

$$R_t^B = \frac{1}{P_{t-1}^B} \tag{14}$$

where P^B denotes the bond price.

Before investing in stocks for the first time, households must pay a one-time fixed cost: $F^0 P_{at}^i W_t$. This entry fee captures both explicit pecuniary costs (e.g. transaction cost from opening a brokerage account and/or hiring a financial advisor), and the (opportunity) cost of acquiring information about the stock market. In addition, every period in which they have positive stock-holdings, households must pay a (lower) per-period participation cost, $F^1 P_{at}^i W_t$, which reflects the (opportunity) cost of managing the portfolio and (again) acquiring information about the stock market. The participation costs are scaled by the current value of the permanent component of labor income (P_{at}^i) and by the aggregate wage (W_t), both because it significantly simplifies the solution of the model, and because this is consistent with the opportunity cost interpretation.

Households cannot borrow against their future labor income, and cannot short either asset, so both their bond holdings, B_{at}^i, and their stock holdings, K_{at}^i, must be non-negative:

$$B_{at}^i \geq 0 \tag{15}$$

$$K_{at}^i \geq 0 \ . \tag{16}$$

2.5 Pension system and social security

In the baseline version of the model, retired households receive income in the form of both social security and defined benefit pension payments. Naturally, consumption at retirement is also financed by their private savings, which can include defined contribution pension schemes. The baseline economy should therefore be viewed as an economy where both systems (DB and DC) co-exist as is currently the case in the U.S..[16] Current employees in the U.S. are enrolled in DC plans, DB plans, hybrid DB-DC systems, or neither. In addition, several of those enrolled in DC or Hybrid plans also have a legacy DB plan. We capture this large heterogeneity in a simplified form by considering that our households are all enrolled in a hybrid plan.

The defined benefit pension is managed through a pension fund that collects contributions paid from the total wage bill of current employees and pays the pensions of current retirees. In addition, as in reality, the pension fund has an accumulated stock of wealth (from previous contributions), and therefore the return on its endowment is another source of income that can be used to finance pension payments. Social security is modelled as a fully funded pay-as-you-go system. In the U.S. social security also has a trust fund, but this is being depleted and is projected to disappear in the near future. The potential implications of this are certainly interesting to explore. However, in our paper we are already extending the standard framework by modelling pension funds. Hence, we leave this question for other research.[17]

2.5.1 Retirement income

For both systems, the benefits at retirement are proportional to the product of their permanent income and the aggregate wage. Total pension payments are as follows:

$$\text{pension}_{at}^{i} = (\lambda^{ss} + \lambda^{db}) P_{a^R t^R}^{i} W_t \ , \ a > a^R \tag{17}$$

where λ^{ss} and λ^{db} are the (exogenous) replacement ratios for (respectively) social security and the defined benefits scheme. The notation t^R refers to the year in which the individual has retired (i.e. the year with $a = a^R$).

2.5.2 Social security

Social security benefits are financed by a proportional tax rate on labor income (τ^{ss}), which is determined endogenously by the relative demographic weights of workers and retirees, so that the

[16] Later on we consider an alternative economy where DB systems have been fully phased out.

[17] In addition, studying the implications of the dynamics of the social security trust fund would require solving for a full dynamic transition path along this dimension.

system is balanced at all times.[18]

$$\sum_{a=20}^{65}\int_{i\in I^a}\tau^{ss}L_{at}^{i}w_t di=\sum_{a=66}^{100}\int_{i\in I^a}[\lambda^{ss}\exp(f(a^R))w_t P_{a^R t^R}^{i}]di\ , \tag{18}$$

where the notation I^a refers to the set of individuals with age a. This equation determines the value of the social security tax/contribution (τ^{ss}) for a given value of the social security retirement replacement ratio (λ^{ss}).

2.5.3 Defined benefit pension scheme

Defined pension benefits are financed by a proportional contribution rate out of labor income (τ_t^{db}), and this value is also determined by the relative demographic weights of workers and retirees, so that the pension fund is not running a deficit or a surplus. Given our set-up, this contribution rate captures both the direct salary deductions taken from employee wages, and the top-up contributions made by firms.

In addition, the pension fund also has an endowment of net worth W^P, which represents the accumulated wealth of the contributions of past generations. Suppose the system is suddenly closed such that current generations suddenly stopped paying any more contributions, and there is no additional accrual of retirement benefits (so no additional liabilities for the Pension Fund). In that case, the current net worth (plus its expected return) is defined to be exactly enough to pay off the existing liabilities in expectation. We assume that the pension fund keeps its net worth fixed at this level $\bar{W}^P$, so that it adjusts contribution rates to compensate for realized returns. In reality, when pension funds are running deficits, these can be covered either by higher contributions from the sponsoring companies, or increased contribution rates for current employees. In our paper, we consider both of these adjustment margins.[19]

The portfolio allocation of the pension fund is captured in the following reduced-form equa-

[18] Since the scheme is fully funded we do not need to consider these taxes and payouts in the government budget constraint.

[19] Another option would be a decrease in accrued retirement benefits going forward. In extreme cases, there could also be an actual (partial) default on previously promised retired benefits.

tion:[20]

$$\alpha_t^P = a^P + b^P * P_t^B, \tag{19}$$

where (α^P) is the risky share of the pension fund. This formulation allows the asset allocation of the pension funds to respond to moves in interest rates. In particular, if b^P is negative (as we will consider), this captures a search-for-yield behavior, with the risky share of pension fund increasing when the bond price (riskless rate) rises (falls). Since the Pension Fund is tax-exempt, its return (R_t^P) is then given by:

$$R_t^P = \alpha_t^P R_t^K + (1 - \alpha_t^P) R_t^B. \tag{20}$$

Adjustment through changes in employee contributions

In our first version of the model, we assume that fluctuations in pension wealth are reflected in changes to employee contribution rates. In periods of high returns, contribution rates will fall and vice-versa to stabilize pension fund wealth.[21] At any point in time, τ_t^{db} can therefore be calculated according to the following equation

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau_t^{db} L_{at}^i w_t di + (R_t^P - 1)\bar{W}^P = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R t^R}^i] di. \tag{21}$$

Given Equation (21), we can find $\bar{\tau}_{db}$ as a function of steady-state $\bar{W}^P$, and unconditional expected returns and wages. We then calculate the net worth that corresponds to the fixed point of that expression and the sum of expected net cumulative contributions (capitalized using pension fund returns) from its inception until all age groups are included. Formally, let NC_a be the net contribution of age group a (contributions - benefits). We have that:

$$\bar{W}^P = \sum_{t=1}^{T} E(R^P)^{t-1} \sum_{a=1}^{t} NC_a. \tag{22}$$

Net worth is endogenous, given that it is a function of returns and wages, but will be fixed

[20]We ignore the issue of the optimal design of a pension fund asset allocation (see Dahlquist et al. (2018)).

[21]In principle, we could also adjust the pension payments, but defaults on accrued benefits are rare events, and this would make retirement income counterfactually volatile.

over time for a given calibration due to the contribution rule described in Equation (21).

Adjustment through changes in employer contributions

As an alternative to the previous set-up, we also consider a version of the model where fluctuations in pension wealth are offset by changes in employer contributions to the pension plan.

We implement this by setting τ^{db} equal to the steady-state level of the previous case, and introducing employer contributions which reduce corporate profits and therefore lower the gross returns to capital R_t^K. Let τ^{kdb} represent pension fund contributions as a proportion of gross returns, so that $\tau_t^{kdb}\ R_t^K K_t$ is the total value of employer contributions to the pension fund at time t. Fund returns R_t^P in this case are:[22]

$$R_t^P = \alpha_t^P (1 - \tau_t^{kdb}) R_t^K + (1 - \alpha_t^P) R_t^B \tag{23}$$

The budget constraint of the pension fund under a fixed endowment of wealth (W^P) is now:

$$\sum_{a=20}^{65} \int_{i \in I^a} \tau^{db} L_{at}^i w_t di + r_t^P \bar{W}^P = \sum_{a=66}^{100} \int_{i \in I^a} [\lambda^{db} \exp(f(a^R)) w_t P_{a^R t^R}^i] di + \tau_t^{kdb}\ R_t^K K_t \tag{24}$$

Combining equations (23) and (24) for given returns and wages, we can calculate τ_t^{kdb} such that pension fund wealth remains fixed over time. This will lead to a negative correlation between τ^{kdb} and gross return to capital R_t^K, and will increase the volatility of (net) equity returns

2.6 The individual optimization problem

2.6.1 Household wealth accumulation

At the beginning of each period (t), agents earn after-tax returns on their wealth invested in bonds B_{at}^i and (potentially) also in stocks, K_{at}^i. We define the dummy variable I_E^i as equal to one in the period in which the entry cost is paid, and zero otherwise, and the dummy variable I_S^i as equal to one if the household has a positive holding of stocks, and zero otherwise. We then capture the

[22]Likewise, the equity returns for households are also decreased in the same way.

total participation costs paid by agent i at time t with the notation

$$PC_{at}^{i} = I_{E}^{i} F^{0} P_{at}^{i} W_{t} + I_{S}^{i} F^{1} P_{at}^{i} W_{t} \tag{25}$$

Wealth (cash-on-hand) at time t is then given by:

$$X_{at}^{i} = K_{at}^{i}(1 + (1 - \tau^{K}) r_{t}^{K}) + B_{at}^{i}(1 + (1 - \tau^{B}) r_{t}^{B}) + L_{at}^{i}(1 - \tau^{ss} - \tau^{W} - \tau_{t}^{db}) W_{t} - PC_{at}^{i} \tag{26}$$

before retirement ($a < a^{R}$), and by:

$$X_{at}^{i} = K_{at}^{i}(1 + (1 - \tau^{K}) r_{t}^{K}) + B_{at}^{i}(1 + (1 - \tau^{B}) r_{t}^{B}) + (\lambda^{db} + \lambda^{ss}) P_{a^{R} t^{R}}^{i}(1 - \tau^{W}) W_{t} - PC_{at}^{i} \tag{27}$$

during retirement ($a \geqslant a^{R}$). Naturally if the household chooses not to pay the participation cost then $K_{at}^{i} = 0$ in these equations.

2.6.2 Household expectations

Households maximize utility given their expectations about future asset returns and aggregate wages. Under rational expectations, the latter are given by equations (6) and (7), and are therefore determined by the equilibrium level of the capital stock, which is a function of the distribution of asset holdings in the economy. As standard in the literature we follow the approach proposed by Krusell and Smith (1998) and approximate the full distribution of asset holdings (an infinite-dimensional state space) with a small set of moments. As discussed in the Online Appendix, our model can accurately forecast the capital stock using its lagged mean (last-period's aggregate capital stock, K_t) and the realizations of the two aggregate shocks (productivity, U_t , and stochastic depreciation, η_t):

$$K_{t+1} = \Gamma_{K}(K_{t}, U_{t}, \eta_{t}) \ . \tag{28}$$

Since government bonds are only riskless over one period, households must also forecast future bond prices (P_{t+1}^{B}). The forecasting rule for P_{t+1}^{B} is:

$$P_{t+1}^{B} = \Gamma_{P}(P_{t}^{B}, K_{t}, U_{t}, \eta_{t}) \ . \tag{29}$$

Details are given in the Online Appendix. This procedure introduces four aggregate state variables in the individual's maximization problem (P_t^B, K_t, U_t, and η_t).

2.6.3 The dynamic programming problem

We write the model in a stationary form, by scaling all variables by aggregate productivity growth ($G_t^{\frac{1}{1-\alpha}}$). We further normalize the individual variables by the current level of permanent labor income (P_{at}^i), to reduce the dimensionality of the state vector by one variable. Normalized variables are denoted by lower-case letters.[23]

After the normalizations, the individual maximization problem has seven state variables. Age (a), normalized cash on hand (x_{at}^i), stock market participation status (E_a^i, a zero-one variable indicating whether the entry cost has been paid or not), and the four aggregate variables from the forecasting equations ((28) and (29)). The full optimization problem is written as:

$$V_a(x_{at}^i, E_a^i; k_t, U_t, \eta_t, P_t^B) = \underset{\{k_{a+1,t+1}^i, b_{a+1,t+1}^i\}_{a=1}^A}{Max} \{(1-\beta)(c_{at}^i)^{1-1/\psi} \tag{30}$$

$$+\beta(E_t[(\frac{P_{a+1,t+1}^i}{P_{at}^i}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V_{a+1}^{1-\rho}(x_{a+1,t+1}^i, E_{a+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B)])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}},$$

subject to the constraints:

$$k_{a+1,t+1}^i \geq 0 \quad , \quad b_{a+1,t+1}^i \geq 0 \tag{31}$$

$$c_{at}^i + b_{a+1,t+1}^i + k_{a+1,t+1}^i = x_{at}^i \tag{32}$$

and

$$x_{a+1,t+1}^i = \begin{cases} \frac{\left[k_{a+1,t+1}^i(1+(1-\tau^K)r_{t+1}^K)+b_{a+1,t+1}^i(1+(1-\tau^K)r_{t+1}^B)\right]}{[(P_{a+1,t+1}^i/P_{at}^i)(1+g)^{\frac{1}{1-\alpha}}]} \\ +\varepsilon^i(1-\tau^{ss}-\tau^W-\tau_t^{db})w_{t+1} - I_E^i F^0 w_{t+1} - I_S^i F^1 w_{t+1} \quad a < a^R \\ \frac{\left[k_{a+1,t+1}^i(1+(1-\tau^K)r_{t+1}^K)+b_{a+1,t+1}^i(1+(1-\tau^K)r_{t+1}^B)\right]}{[(P_{a+1,t+1}^i/P_{at}^i)(1+g)^{\frac{1}{1-\alpha}}]} \\ +(\lambda^{db}+\lambda^{ss})w_{t+1} - I_E^i F^0 w_{t+1} - I_S^i F^1 w_{t+1} \quad a > a^R \end{cases}, \tag{33}$$

the stochastic process for individual labor productivity (equations (10) to (13)), and the forecasting

[23]Specifically, household-specific variables are normalized as $x_{at}^i \equiv \frac{X_a^i}{P_{at}^i G_t^{\frac{1}{1-\alpha}}}$, $c_{at}^i \equiv \frac{C_a^i}{P_{at}^i G_t^{\frac{1}{1-\alpha}}}$, $b_{a+1,t+1}^i \equiv \frac{B_{a+1,t+1}^i}{P_{at}^i G_t^{\frac{1}{1-\alpha}}}$, $k_{a+1,t+1}^i \equiv \frac{K_{a+1,t+1}^i}{P_{at}^i G_t^{\frac{1}{1-\alpha}}}$ while aggregate variables are normalized as $k_t \equiv \frac{K_t}{G_t^{\frac{1}{1-\alpha}}}$, and $w_t \equiv \frac{W_t}{G_t^{\frac{1}{1-\alpha}}}$.

equations (28) and (29).

The individual takes as given all aggregate variables, i.e. capital stock, returns, bond price, wages, tax rates and the other government variables.

2.7 Equilibrium

Equilibrium prices and quantities and determined by the following set of conditions:

1. Firms hire capital and labor to maximize profits (equations (6) and (7)).

2. Individuals choose their consumption and asset allocation to maximize their expected lifetime utility, i.e. maximize equation (41) subject to the constraints described in the previous subsection.

3. The social security system is balanced at all times:

$$\int_i \int_{a \in I_W} \tau^{ss} L^i_{at} w_t dadi = \int_i \int_{a \in I_R} [\lambda^{ss} \exp(f(a^R)) w_t P^i_{at^R}] dadi \ , \tag{34}$$

where the left-hand side is integrated over all workers ($a \in I_W$), while the right-hand side is integrated over retirees ($a \in I_R$). This equation determines the value of the social security tax/contribution (τ^{ss}) for a given value of the social security retirement replacement ratio (λ^{ss}).

4. The defined benefit pension fund is in a balanced path with a constant endowment (W^P), and an endogenous contribution rate (τ^{db}_t) as given by Equation (21).

5. The government budget (equation (8)) is balanced every period for a given ratio of government debt to GDP.

6. All markets clear, specifically the markets for capital, bonds and the consumption good:[24]

$$k_t = \int_i \int_a P^i_{a-1,t-1} k^i_{at} dadi \tag{35}$$

$$b_t = \int_i \int_a P^i_{a-1,t-1} b^i_{at} dadi \tag{36}$$

$$U_t k_t^{\alpha} L_t^{1-\alpha} = \frac{C_t^G}{G_t^{\frac{1}{1-\alpha}}} + \ (1+g)^{\frac{1}{1-\alpha}} k_{t+1} - (1-\delta_t) k_t + \int_i \int_a P^i_{at} c^i_{at} dadi \tag{37}$$

By Walras' law, once two of these equations are verified, the third is also automatically satisfied.

[24] The market for labor is trivial since there is no labor-leisure choice.

7. Household expectations for market prices (equations (28) and (29)) are verified in equilibrium.

We describe the numerical solution of the model in the Online Appendix.

2.8 Calibration

In this section, we discuss the calibration of the model. A summary of the parameter values considered in the baseline calibration is reported in Table 1.

[INSERT TABLE 1 HERE]

2.8.1 Aggregate variables

The productivity shock follows a first-order Markov process with two values, corresponding to expansions and recessions. We calibrate the transition matrix to fit NBER data. The probability of remaining in recessions (π_r) is 16/37, and the probability of remaining in expansions (π_e) is 60/81, yielding an average business cycle duration of six years. We also fit the expansion and recession values of productivity to match the conditional growth rate of labour income as in Guvenen et al. (2014), as described in more detail below. This implies a standard deviation of TFP of 2.6%. The capital's share of output (α) is set to 34%, while the average annual depreciation rate (δ) is set to 10%, with a volatility of 10%.

The aggregate supply of bonds is calibrated to deliver an endogenous ratio to GDP of around 40%, based on the average value of U.S. Treasury securities held by the U.S. public taken from the Congressional Budget Office.[25] The tax rate on bond interest payments (τ^B) is set at 20%, while tax rate on stock returns (τ^K) is 40%. This is meant to capture a 20% personal tax rate on both sources of equity income (dividends and capital gains), and a 20% corporate tax rate on firm profits. Bequests are fully taxed ($\tau^E = 100\%$), but total bequests are a very small fraction of total government revenues (both in the model and in the data), so this assumption is only made for simplicity.[26]

[25] We specifically consider U.S. debt held by the U.S. public only, since ours is a closed-economy model.

[26] Otherwise we would have to re-distribute this wealth to the surviving generations, and households would need to form expectations over these transfers and consider those expectations in their optimization problem.

Since the firms in the model are unlevered, the return on capital is a return on unlevered equity. We obtain the implied levered equity return by assuming a leverage ratio of 1/3, from Rajan and Zingales (1995).

2.8.2 Household variables

We calibrate the conditional survival probabilities ($\{p_a\}_{a=1}^{81}$) from the mortality tables of the National Center for Health Statistics. Both types of households (A and B) have a risk aversion coefficient of 6 ($\gamma^A = \gamma^B = 6$). Type-A households have a subjective discount factor (β^A) of 0.876 and an elasticity of intertemporal substitution (EIS, ψ^A) of 0.2, thus delivering low wealth accumulation and endogenous low stock market participation. Households of type B have a subjective discount factor (β^B) of 0.983 and an EIS (ψ^B) of 0.6. These values are also chosen to match the volatility of consumption growth, the level and volatility of the risk-free rate, and the stock market participation rate. We set the entry cost of participation (F^0) to 6% and the per-period cost of participation (F^1) to 2%.[27]

We take the deterministic labor income profile from Cocco et al. (2005). The variance of the transitory shocks (σ_ε) is set to 10%, while for the permanent income shocks, we rely on the values in Guvenen et al. (2014) who estimate a quantitative labor income model using a large data set of administrative income data. The moments of permanent income shocks can be calculated based on these estimates, and we can then calibrate the parameters for the mixture of normal distributions during expansions and recessions. The probability of the mixture normal distribution ($q_1 = 0.49$) is the same as in Guvenen et al. (2014). We estimate the remaining eight moments to match the first four moments during expansions and the first four moments during recessions, yielding similar estimates to Guvenen et al. (2014), and likewise delivering countercyclical expected growth rates: 0.045 during booms and -0.002 during recessions. Considering that each household faces the same expected growth rate, instead of incorporating expect growth rates in permanent income shocks, we treat these as aggregate TFP growth. The variance of the log of labour income during expansions and recessions is 0.1, and its skewness is 0.8 in expansions and -1.02 in recessions. Kurtosis is 5.6 during expansions and 5.2 during recessions.

[27] These values are within the range previously considered in the literature; see Gomes and Michaelides (2008), Favilukis (2013), and Fagereng et al. (2017).

2.8.3 Pension fund

The total replacement ratio of age-65 income ($\lambda^{db} + \lambda^{ss}$) is set to 0.68212, which is also taken from the labor income estimation in Cocco, Gomes, and Maenhout (2005) We decompose the two separate components using data from the social security administration, which reports that social security paid out $994 billion USD while defined benefits paid $733 billion USD. So pension payments (social security) correspond to about 42% (58%) of total retirement income, thus giving us values of λ^{ss} and λ^{db} equal to 0.3926 and 0.2895, respectively.

To calibrate the portfolio allocation of the representative Defined Benefit Pension Fund (DBPF) we use data from the St. Louis Fed. We obtain an average risky share of 69% based on the following assumptions. First, we discard the category "other", since it is unclear how this should be classified.[28] Second, we assign to "bonds and bills" a risky weight of 0.1, to reflect that these are not completely riskless, particular in the case of corporate bonds.[29] Third, we assign a risky weight of 0.9 to CIS holdings to reflect the modest holdings of cash and government bonds included in these investment vehicles.

In our baseline model we consider two calibrations of the portfolio allocation rule (α^P). In the first case we set b^P equal to zero and therefore set a^P to match the average risky share in the data (69%). This has the advantage of eliminating one parameter from the model but implies that the portfolio allocation of the pension fund does not respond to the interest rate. Therefore, we also consider an alternative version where we set $b^P = -2$. This implies that the risky share of the Pension fund (α^P) increases (decreases) to 74.23% (63.77%) when the risk-free rate is 2 standard deviations below (above) its unconditional mean. We then re-calibrate a^P so that we again obtain an average risky share of 69%.

3 Baseline results

The results for our baseline model are shown in Table 2, where we also report the corresponding moments in the data. The asset pricing data is taken from CRSP, while stock market participation

[28]This category represents a modest allocation of 10.56%, so if we classified these as risky assets, our calibrated risky share would only increase to 72%, while if we classified them as riskless asset, it would decrease to 62%.

[29]Unfortunately, the data does not disaggregate between corporate bonds and government bonds.

is computed as the historical average from the Survey of Consumer Finances. Since ours is a real model, we take the mean and volatility of the real risk-free from Croce et al. (2012), who adjust the nominal rate for inflation expectations. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis, and we use the full annual sample from 1930 to 2018.

[INSERT TABLE 2 HERE]

As discussed in Section 2.4, we consider two different versions of the baseline model, regarding the margin of adjustment in response to fluctuations in the endowment of the pension fund. In one version, the adjustments are made fully by changing the contribution rate of employees (τ_t^{db}), as computed from equation ((21)). In the other version, we adjust the contributions of the employers, using equations ((23)) and ((24)).[30] The corresponding results are in columns 3 and 4 of Table 2, respectively. Going forward, we only consider the first version of the baseline model in our different experiments and comparative statics to avoid excessive repetition. Therefore, that is the version that we have calibrated to match the data. For the same reason, we report results for the alternative version using the same parameter values, instead of re-calibrating the model. This will allow us to isolate the different economic implications of these two modelling choices.

In addition, as discussed in the calibration section, we also consider two alternative parameterizations of the risky share equation for the Defined Benefit Pension Fund (equation (19)). In the first one we set α^P equal to a constant, while in the second we allow the asset allocation of the fund to respond to interest rates, capturing a search-for-yield behavior. The results in columns 3 and 4 consider the first formulation, while the results in column 5 consider the second one (for the case where the contribution rate of employees is the margin of adjustment, as in column 3).

3.1 Baseline model with τ^{db} adjustment

Figure 1 plots the life-cycle wealth accumulation of the two types of agents. With their high discount factor ($\beta^B = 0.969$), Type-B agents have a high savings rate from early on. On the other hand, type-A agents, with a much lower discount factor ($\beta^A = 0.839$), only accumulate significant

[30]In reality, we have mix of both, but we did not want to introduce an additional free parameter in the model.

wealth closer to retirement. The latter group captures households with low savings rates without imposing the extreme assumption of "hand-to-mouth" behavior, thus allowing them to re-optimize in response to different economic environments.

The asset pricing moments implied by this version of the model are presented in column 3 of Table 2. The low discount factor of the type-A households drives up the risk-free rate in our economy.[31] Nevertheless, we still obtain a low average value (1.14%) because the type-A agents accumulate limited wealth, and therefore bond prices are primarily determined by the discount factor of the type-B agents. The model also matches extremely well the level and volatility of the real riskless rate: 1.34% versus 1.35% in the data.

The baseline economy (with τ^{db} adjustment) matches exactly the Sharpe ratio in the data (0.34), although the equity premium is lower than its empirical counterpart (4.65% versus 7.55%) because in our economy stock return volatility is also lower (13.67% versus 19.81%). In other words, the model is able to match the market price of risk but, since we have lower quantity of risk than in the data, the equity premium is also lower. We could have increased the volatility of equity returns in the model by choosing a higher volatility of stochastic depreciation. However, such a calibration generates a counter-factually high consumption volatility.

The model also delivers an average stock market participation rate of 52.3%, which matches well the 51.1% historical average in the Survey of Consumer Finances (SCF).[32] Type-B agents, given their high discount factor, have a strong incentive to pay the stock market participation cost early in life,and therefore they quickly become stockholders. The average participation rate among these households is 86%. On the other hand, Type-A agents have a low discount factor and, for most of their lives, only accumulate limited savings for precautionary reasons. Therefore they have a limited incentive to pay the entry cost (see Gomes and Michaelides (2005)).[33] Only as they approach retirement do their savings become more significant and, as a result, some of the individuals in this group also become stockholders temporarily. They exit the stock market again

[31] The impact of β^A on the risk-free rate is conditional on the stock market participation rate for these households remaining relatively low. If we increase β^A such that the majority of these agents decide to pay the participation cost then, over a certain range of the discount factor, their overall demand for bonds might actually fall.

[32] The SCF sample is from 1998 to 2019.

[33] This is both because of their low wealth accumulation, and because the optimal unconstrained portfolio would be quite conservative due to the background risk effect (see, for example, Deaton (1991), Hubbard et al. (1995), Carroll (1997), Gourinchas and Parker (2002), and Cocco et al. (2005).

late in life, as their wealth falls towards zero. Consequently, the average stock market participation among the type-B households is 18.6%.

The total value of pension fund assets in the model is an endogenous variable. Importantly we find that this value matches closely with the one in data. The St. Louis Fed reports pension fund assets to GDP in the U.S. from 1996 to 2017. The average is 101%, but there is a clear positive trend. In 1996 this ratio was 67%, while in 2017, the value was 144.6%. Since we calibrate the asset pricing and macro moments using longer time series, we take a conservative approach and target 67% as the value of pension wealth to GDP in our model.[34] In our baseline economy, the corresponding value is 74%, thus very close to our empirical target.

3.2 Baseline model with adjustment in employer contributions

Column 4 of Table 2 reports results for the alternative version of the baseline model, where the pension fund's endowment is kept constant through adjustments in the contributions made by employers/firms. In such a setting, the volatility induced by the returns on the pension fund's endowment is reflected in higher volatility of firm profits, i.e. equity returns, instead of higher volatility of net wages. As a result, the volatility of returns in this version of the model is 17.7%, compared with 13.7% in the previous specification.

A crucial difference relative to the previous specification is that this additional risk only affects stockholders. This can be seen in the volatility of consumption of the two groups. For type-B households, mostly shareholders, this volatility is now 2.67% instead of 2.49% in the previous version of the model. By contrast, for type-A households, with a participation rate of only 21%, the standard deviation of consumption growth is significantly lower: 1.73% versus 3.42%.

Since stockholders accumulate much more wealth than non-stockholders, they are better able to smooth consumption. This explains why the increase in the volatility of consumption growth for type-B households is much smaller than the corresponding decrease for type-A households. This also explains why the standard deviation of aggregate consumption is significantly smaller: 2.14% versus 2.91% in the alternative set-up. Since the volatility of stockholders' consumption and the volatility of stock returns both increase, this version of the model delivers a higher risk

[34]Taking the average historical value would make the results in our calibrated economy even more different from those obtained when ignoring the pension fund endowment.

premium (4.95%). However, the increase in the average risk premium is smaller than the increase in its standard deviation, leading to a lower Sharpe ratio (0.28).

In reality, fluctuations in pension fund wealth lead to adjustments in contribution rates from both employees and firms, so a weighted-average of the two versions of the model that we are presenting here. Since we do not want to introduce another free parameter in the model, we consider these two limit cases, and, to avoid excessive repetition of results, in the remainder of the paper, we limit our attention to the economy with τ^{db} adjustments only.

3.3 Baseline model with search-for-yield behavior by DBPF

In our most general formulation, we express the portfolio rule of the pension as a function of the bond price (equation (19)). More precisely, its risky share increases when interest rates decrease (b is negative), consistent with a search-for-yield behavior. In the previous results (columns 3 and 4 of Table 2) we consider a simplified version where we set $b^P = 0$, while in column 5 (of Table 2) we consider the more general case.

We set $b^P = -2$, such that the risky share of the Pension fund (α^P) increases (decreases) to 74.23% (63.77%) when the risk-free rate is 2 standard deviations below (above) its unconditional mean. We then re-calibrate a^P such the the average allocation of the pension fund matches the average risky share in the data (69%), as in the previous cases. We consider the case were the pension fund's endowment is kept constant through adjustments in employer contributions (τ^{db}), so these results should be compared to those in column 3.

We find that the two formulations (columns 3 and 5) yield very similar conclusions. The volatility of consumption, stock market participation, equity return, risk-free rate and Sharpe ratio are all extremely similar in both cases. Since these two versions of the model deliver almost identical results, going forward we only consider the specification with $b^P = 0$, thus eliminating one parameter from the model.

4 Model without DBPF endowment and additional comparative statics

In this section, we present an alternative model where we do not consider the endowment of the defined benefit pension fund (DBPF). In this alternative model ("noDBPF"), the pension fund is a simple pass-through entity, just like social security. More precisely, we now have $\bar{W}^P$=0 at all times. The results are shown in Table 3, which also includes the results for the baseline model (column 3) and the values in the data (column 6). For direct comparison with the previous economy, we report values obtained both for the baseline calibration (column 4) and for a re-calibrated version of model (column 5).

4.1 Results for noDBPF model with the same calibration

To understand the economic differences between the two versions of the model, we first compare the two economies for the same calibration of the structural parameters.

[INSERT TABLE 3 HERE]

Since the DBPF does not have an accumulated stock of wealth, it does not benefit from the return on those assets as an additional source of income. This implies that the average (defined-benefit) contribution rate for households increases from 2.82% in the baseline economy to 6.26% in this alternative formulation. A higher contribution rate implies a lower volatility of consumption growth for two reasons. First, because disposable income is now a smaller fraction of total wage income. Second, because the contributions are no longer subject to fluctuations induced by changes in the wealth of the DBPF. As a result, the volatility of consumption growth is now 1.87%, versus 2.91% in the baseline economy. These effects are particularly important for households financing their consumption mostly out of their current disposable income, namely the type-A agents, for whom the standard deviation of consumption growth falls from 3.42% to 1.65%.

Another first-order implication of assuming that the DBPF does not have an accumulated stock of wealth is that there is less total financial wealth in the economy. It is important to remember that households do not have an incentive to accumulate more wealth since they have the same level of retirement income as before. This is not a model where we have closed down the defined benefit fund (we will consider that experiment later). It is simply a model where we have failed to account for its initial endowment.[35] Moreover, the lower volatility of income also decreases precautionary savings, so that households actually have an even lower incentive to accumulate wealth than in our baseline model. As a result, for the same parameter values, capital accumulation is 30% lower (3.24 versus 4.69), and it is even lower than the total capital held by households in the previous economy (3.67), reflecting the lower precautionary savings motive.

The lower capital stock leads to a higher equity return (9.86% versus 5.79%), and likewise the demand for bonds is also reduced, leading to a higher risk-free rate (4.90% versus 1.14%). Therefore, the model delivers a slightly higher equity premium and higher Sharpe ratio than the baseline economy (respectively 4.96% and 0.37), but at the expense of a counterfactually high risk-free rate. In the next section, we consider a re-calibrated version of the model, which delivers a low mean risk-free and find that, in that case, the Sharpe ratio is much lower.

4.2 Re-calibrated noDBPF model

We now re-calibrate the noDBPF model to match the risk-free rate and stock market participation. For simplicity, we will refer to this as the "r-noDBPF economy", and the results are shown in column 5 of Table 3. In the new calibration, we increase the EIS and discount factor of the type-B households to 0.6 and 0.983, respectively, and the discount factor of type-A agents to 0.876.

4.2.1 Results

With the new preference parameters, agents have a stronger preference for savings, and this is reflected in a higher capital stock (5.21). Likewise, the demand for bonds also increases, and the riskless rate in the re-calibrated model is much lower than with the previous calibration. It is now almost identical to the one in the baseline economy (1.18% compared with 1.14%). However, the increase in wealth accumulation, particularly in the demand for capital, drives down the return on equity to 4.40%, corresponding to an equity premium of 3.22%, which compares with 4.65%

[35] Models that do include that endowment are ignoring wealth that was accumulated in the past, when these funds were first created. In those economies, internal consistency implies that pension payments are fully financed by the contributions of current workers, which are therefore higher than in our baseline economy.

in the baseline economy. In terms of the market price of risk, the r-noDBPF economy delivers a Sharpe ratio of 0.24 versus 0.34 in the baseline economy and in the data. This sequence of results is merely a reflection of the riskless rate puzzle. The previous calibration of the noDBPF model delivers a high Sharpe ratio/equity premium only at the expense of a counter-factually high riskless rate. Once we re-calibrate the model to match the riskless rate, the equity premium is significantly lower. The lower equity premium in this equilirium also explains why, despite households accumulating much more wealth than under the previous calibration, the participation rate is essentially unchanged and, therefore, still in line with the data.

Finally, in the re-calibrated economy, the volatility of consumption growth is still substantially lower (2.23% compared with 2.91% in the baseline model). This occurs because, in the r-noDBPF economy, households do not face the risk of changes in their pension contributions due to the funding ratio of the DBPF, a feature that we discuss in more detail below. This lower volatility partially explains the lower equity premium, but the Sharpe ratio comparison makes it clear that this is only part of the story. With the caveat that have incomplete markets, hence the standard consumption CAPM formula for the risk premium does not apply, we can reach the same conclusion by observing that the ratio of the standard deviation of consumption growth in the two models is 30%, while the ratio of the equity premium is higher (44%).

4.2.2 Discussion

What explains the higher equity premium and higher Sharpe ratio in our baseline economy when compared with those in the r-noDBPF model? One important difference between the two economies is that, by ignoring the endowment of the DB pension fund, the r-noDBPF economy is ignoring an important source of risk for households. When DB pension funds are in a difficult financial situation, they must increase the required contribution rates from current employees and/or decrease the accrual of pension payments or, in extreme circumstances, decrease the pension benefits already in place. These potential changes in contribution rates and/or benefits not only represent an additional risk for households, this is a risk that is correlated with the stock market. To the extent that the endowment of the DBPF is partially invested in equities, realized stock returns will impact the evolution of its funding position over time. As a result, households decrease their demand for equities and, in equilibrium, this implies a higher market Sharpe ratio.[36]

In our model, for tractability, fluctuations in the financial situation of the pension fund are translated each year into changes in the contribution rate. In reality, DB pension funds adjust those rates infrequently, typically in response to significant changes in their funding ratio. Modelling this would require us to add the funding ratio of the DBPF as an additional state variable, and to make arbitrary assumptions with regards to the rules for adjusting the contribution rates. Regardless, this would not change the fact that households would ultimately be facing this risk. Instead of it being reflected in one-for-one yearly adjustments, it would lead to large discrete jumps in some years, but either way it will be present. It is quite possible that the alternative formulation would have even larger quantitative implications for asset prices.[37]

4.3 Additional Comparative Statics

The DB pension fund affects the equilibrium in our economy because of its portfolio allocation, and because it forces households to save for retirement. In this section, we discuss additional comparative statics which enhance our understanding of these two different mechanisms. The results are shown in Table 4. In the first comparative statics, in column 4, we consider a different portfolio allocation for the pension fund. More precisely, we increase its risky asset share (α^{db}) from our baseline value of 0.69 to 0.80. In column 5, we report results for a re-calibration of the model where we have increased the discount factors of the two types of agents are increased by 0.02 (hence we have β^A=0.859 and β^B = 0.989).

[INSERT TABLE 4 HERE]

When we increase the risky share of the DB pension fund (column 4 of Table 4), the riskless rate increases to 1.72%, as the overall demand for bonds in the economy falls. Interestingly, the return on equity remains unchanged, and therefore the equity premium falls to 4.07% and the

[36] It is important to remember that, at the moment, we are not comparing our baseline steady-state with a DC economy. This is a comparison with an otherwise identical economy where we do not consider the endowment of the DBPF, and instead assume higher pension contribution rates.

[37] As discussed, we could have adjusted the benefit payments instead, or in addition to the contribution rate. We choose to use just one margin of adjustment for simplicity and because, in practice, contribution rates are the typical margin of adjustment, with defaults on promised payments being rare events. But again, regardless of the specific formulation, households face the corresponding risk.

Sharpe ratio falls to 0.3. With DBPF investing more in equities, the return on equity would have been expected to fall. However, we observe that the total capital stock is actually slightly lower in the new economy. This is explained by a reduction in the demand for equities by households, in response to the higher riskless rate and the increase in background risk arising from the higher risky share of the pension fund. In the new equilibrium, the capital held by households falls from 3.67 to 3.60, and the average stock market participation rate falls from 52.3% to 45.6%.

Increasing the discount factor of households (column 5 of Table 4) naturally delivers higher aggregate savings. In equilibrium, the rates of return on both assets fall. The risk-free rate is now -2.57%, and the average return on equity becomes 3.09%. The increased demand for savings also leads to a significantly higher stock market participation rate (69.9%), total capital (6.59) and capital held by households (5.58). In equilibrium, the volatility of consumption growth increases, which, combined with the substantial drop in the riskfree rate, leads to a higher Sharpe ratio on equities: 0.42. Although this calibration can deliver a higher market price of risk, we reject it because it significantly under-performs when matching the average historical returns (extremely low risk-free rate and equity return), and delivers an excessive volatility of consumption.

5 Economy with defined contribution pension scheme only

Defined benefit pension schemes face increasing funding problems due to a wide range of factors, namely increases in longevity not accompanied by increases in retirement age. As a result, in several countries these plans are being progressively replaced with defined contribution schemes, in which individuals are saving into their own private retirement accounts.[38]

5.1 The set-up

In the final section of the paper, we explore the potential asset pricing implications of the eventual/potential phasing out of DB pension plans. More precisely, we consider an alternative econ-

[38] In some countries, the contribution amount is determined by the rules of the pension plan, but in the U.S., this is typically at the discretion of the employee, subject to a cap.

omy where the defined benefit pension fund does not exist, so

$$\lambda^{db} = 0 \text{ and } W^P = 0. \tag{38}$$

As a result, households must finance their retirement consumption using their own personal savings and social security income.

Note that the analysis in this section differs from the one considered in sections 4.1 and 4.2. Before we considered an alternative equilibrium where the DB pension fund exists (so λ^{db} remained equal to its baseline value of 0.2895), but we set its endowment (W^P) to zero. In such a model, household retirement income was unchanged relative to the baseline economy. The only difference was how this retirement income was being financed. By contrast, in the current exercise, retirement income is now limited to social security transfers, and therefore, if households wish to keep their retirement consumption unchanged, they must now save more during their working lives.

It would be interesting to also solve for the transition dynamics, but this would be computationally challenging. We would have to introduce at least one additional state variable to capture the wealth/size of the DB pension fund, and potentially a second one to capture the fraction of households enrolled in the DB pension plan, or equivalently the fraction of their retirement savings that are being allocated to the DB pension plan. In addition, we would have to make specific arbitrary assumptions about how the DB plans would be phased out over time, and about households' expectations regarding this process.

5.2 Taxes and illiquidity

In our baseline economy, we do not tax the returns of the DBPF, consistent with the current tax regulations. Likewise, wealth accumulation in DC pension accounts is also tax-free.[39] Furthermore, individuals are not supposed to withdraw funds from their DC accounts before retiring, and doing so, incurs a 10% penalty, except under special circumstances.[40]

In the model, we do not distinguish between DC wealth and non DC-wealth at the house-

[39] An additional potential tax benefit of DC accounts is that households pay income taxes at their marginal tax rate during retirement, which might be a lower number. However, this is also the case with the DB system. In our model, income taxes are linear, so that effect is not present anyway.

[40] Special circumstances include facing a hardship event or a job loss.

hold level, as this would require adding one additional state variable and two additional choice variables.[41] Therefore, we model the tax benefits and illiquidity features of the DC account in a reduced form.

We incorporate the tax benefits by decreasing the tax rate on total household wealth accumulation by the same ratio as the percentage increase in their wealth relative to the baseline economy. So, for example, if household wealth in the new economy is 10% higher, we decrease the tax rate by 10%. Implementing this requires an iterative loop to find a fixed point. Likewise, we apply the 10% penalty withdrawal penalty to that a fraction of household dis-savings only.

Finally, as with defined benefit schemes, employee contributions to their DC pension account are typically accompanied by additional contributions from the employer, which can be linked to the size of the employee's contribution (i.e. "matching contributions"), depending on the specific features of the pension plan. Given our set-up, the contribution rate captures both the direct salary deductions taken from employee wages, and the top-up contributions made by firms since both represent a payment that is proportional to total wage compensation.

5.3 Results

5.3.1 Household wealth accumulation

When the DBPF is shut down, households must increase their personal saving to finance their retirement. On the other hand, since they no longer face fluctuations in their DB contribution rates (which are now zero), the volatility of their disposable labor income is smaller, and therefore precautionary savings should decrease. Consequently, the net effect on household wealth accumulation is ambiguous.

However, it is important to note households do not have an incentive to fully compensate for the "missing" DB retirement wealth. If they wanted to save more for retirement before, when the DB fund existed, they could have done so already. To the extent that some households were being "forced" to save more for retirement than their optimal private decision would imply, total retirement wealth accumulation should be lower than before. Combined with a reduction in

[41] The additional state variable would be the balance in the DC account relative to total wealth, or relative to non-DC wealth. The additional choice variables would be the contribution to the DC account and the portfolio allocation in the DC account. Gomes et al. (2009) solve such a model in partial equilibrium.

precautionary savings, this implies that total wealth should be smaller in the "DC-only economy".

Figure 2a plots wealth accumulation over the life-cycle for the two groups of agents, both in the baseline economy and in the DC-only economy. From mid-life onward, wealth accumulation is substantially higher for both type-A and type-B agents. The absence of a defined benefit pension leads them to increase their private savings to finance retirement. The savings behavior in early life, however, reflects both the increased retirement savings motive and the decreased precautionary savings motive.

The trade-off is clearly visible in Figure 2b, which plot the ratio of wealth accumulation in the two economies for the pre-retirement period. Type-A agents, with their low discount factor, only save early in life for precautionary reasons. Therefore, in the DC-only economy, they decrease their savings significantly at this stage of the life-cycle. Only from age 39 onward is their average wealth higher in the DC-only economy than in the baseline economy. By contrast, the type-B agents save for retirement from early on, and therefore their average wealth accumulation exceeds the one in the baseline economy already from age 25.

5.3.2 Asset pricing moments

In Table 5, we compare the results from the baseline economy and the DC-only economy. To facilitate the comparison, we consider two alternative versions of the DC-only economy: with and without the adjustments for tax benefits and illiquidity previously discussed. These two sets of results are labelled as "Case II" and "Case I", respectively. So, in "Case I", the returns on household wealth are fully taxed, and household wealth is fully liquid, as in our baseline economy. In "Case II", household savings, in addition to those in baseline economy, are tax-free but illiquid, and this is the case considered in figures 2a and 2b above.

[INSERT TABLE 5 HERE]

As shown in figures 2a and 2b, private household wealth accumulation increases in the DC-only economy, as households must now save more for retirement. However, as discussed, they do not have an incentive to fully compensate for the "missing retirement wealth", and also have a lower demand for precautionary savings, so total wealth in the economy is now lower. Since the supply of bonds is constant, the lower wealth accumulation must be reflected in a lower capital stock.

In the “Case I” scenario, we find that the aggregate capital falls to 4.49 (compared with 4.69 in the baseline economy). The impact on asset prices is more complicated, because the relative demand for bonds and stocks has changed for two reasons. First, because households have different demand curves for these two assets than the DB pension fund had. Second, because the demand curves of households have shifted in the new equilibrium. Therefore, we require a calibrated quantitative model to understand the impact on equilibrium returns.

Less capital in the economy implies a higher average return on equity (5.98%), but the reduction in savings also leads to a substantial increase in the riskless rate (2.21%). The equilibrium riskless rate increases by more than the return on capital, for two reasons. First, because the supply of bonds is fixed, while the supply of capital is endogenous. Second, because households now have more wealth accumulation, and their risky share is a decreasing function of wealth, hence their optimal portfolios would imply a lower risky share for the same equilibrium returns. As a result, the equity premium falls to 3.78%, and the Sharpe ratio is also lower (0.28).

As households increase their private savings they have a stronger incentive to pay the participation costs and invest in stocks, but this is counteracted by the lower average equity premium. These two effects are directly visible when we look at the behavior of the two types of agents separately. Among type-A households, those that relied more on the pension fund to finance their retirement, the first effect dominates, and stock market participation increases from 18.6% 25.8%. On the other hand, type-B households already had high savings, and therefore pension wealth represented a small fraction of their total retirement resources. As a result, the lower equity premium channel dominates, and stock market participation actually falls, from 86.0% to 81.1%. Combining the two groups, we find that the percentage of stockholders in the economy increases mildly from 52.3% to 53.4%.

When considering “Case II” (column 5 of Table 5), we find that wealth accumulation is now closer to the baseline economy, but still smaller (4.58 versus 4.69). First, there is a reduction in precautionary savings, since household disposable income is no longer subject to volatile DB contribution rates. This is reflected in a lower volatility of consumption growth (2.23%). Second, under the DB system, households with a low savings motive (type-A) were being forced to save too much for retirement, relative to their optimal consumption path.

Despite the similar total capital accumulation, the asset pricing moments are still very different across the two economies. Without the DBPF, the demand for bonds is substantially decreased, leading to a higher risk-free rate (2.50%). Since the standard deviation of consumption growth is lower (2.23%), there is a significant reduction in the equity premium and the Sharpe ratio, to 3.24% and 0.24, respectively. Relative to the "Case-I" economy, we have the same effects from the disappearance of the DBPF, but in addition we now have a higher capital stock, which is why the equity premium and the Sharpe ratio and both even lower in this setting.

Finally, relative to the baseline economy, we again observe a decrease in stock market participation among type-B households (because of the lower equity premium) and an increase in stock market participation among the type-A households (because they must now save more for retirement).

6 Conclusion

We solve a general equilibrium asset pricing model with an explicit defined benefit pension plan, and show that the augmented model is better able to jointly match the riskless rate, equity Sharpe ratio and volatility of consumption growth. The results highlight the importance of taking into account for the asset demands of the defined benefit pension fund and for its endowment of wealth. The model also identifies a new risk channel, coming from fluctuations in pension contributions, which are in turn linked to the funding ratio of the pension fund, and are consequently correlated with stock returns. Therefore, our paper is part of the growing literature emphasizing the importance of financial intermediaries in determining equilibrium asset prices. We further use the calibrated model to solve for an equilibrium where the DB pension plan has been replaced by a DC pension plan, a trend that we are observing in several countries, due to the funding problems of most existing DB pension plans. The new economy is characterized by less precautionary savings and more retirement savings, which differentially affect wealth accumulation at different stages of the life cycle. In the new steady-state, the riskless rate is higher and the Sharpe ratio on equities is lower.

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Table 1: Calibrated Parameters

Aggregate Variables		Household Variables	
Technology		Preferences	
π_r	16/37	γ^A	6
α	34%	γ^B	6
Mean(δ)	10%	β^A	0.876
Vol(δ)	10%	β^B	0.983
		ψ^A	0.2
Debt and Taxes		ψ^B	0.6
B/Y	40%		
τ^B	20%	Participation Costs	
τ^K	40%	F^0	6%
τ^E	100%	F^1	2%
Pension Fund		Retirement Income	
α^P	69%	λ^{ss}	0.3926
		λ^{db}	0.2895

Table 1 reports the baseline calibration of the different parameters of the model. The household-level income processes are given by a combination of values from Cocco et al. (2005), Guvenen et al. (2014) and own estimations, as described in the calibration section.

Table 2: Baseline Results.

Variable	Moment	Baseline Model			Data
		(τ_{db} adjust.) ($b^P = 0$)	(τ_{kdb} adjust.) ($b^P = 0$)	(τ_{db} adjust.) ($b^P = -2$)	
r_f	Mean	1.14%	1.19%	1.15%	0.86%
r_f	St. Dev.	1.34%	1.68%	1.43%	1.35%
r^m	Mean	5.79%	6.14%	5.80%	8.17%
r^m	St. Dev.	13.67%	17.7%	13.67%	19.81%
$r^m - r_f$	Mean	4.65%	4.95%	4.65%	7.55%
$\frac{Mean(r^m - r_f)}{Std.Dev.(r^m - r_f)}$		0.34	0.28	0.34	0.34
Cons. growth (all)	St. Dev.	2.91%	2.14%	2.90%	2.90%
Cons. growth (A)	St. Dev.	3.42%	1.73%	3.43%	(-)
Cons. growth (B)	St. Dev.	2.49%	2.67%	2.47%	(-)
K	Mean	4.69	4.53	4.67	(-)
$Kprivate$	Mean	3.67	3.52	3.66	(-)
W^P/Y	Mean	0.74	0.75	0.74	0.67
K/Y	Mean	2.35	2.30	2.35	(-)
Participation (all)	Mean	52.3%	53.9%	52.4%	51.1%
Participation (A)	Mean	18.6%	21.3%	18.9%	(-)
Participation (B)	Mean	86.0%	86.7%	85.8%	(-)

Table 2 reports asset pricing moments, consumption volatility, stock market participation, capital/output ratio and pension wealth to GDP for the baseline model (columns 3, 4 and 5), and in the data (column 6). The results in Columns 3 and 5 refer to a version of the model where the pension fund's wealth is kept balanced by changes in the contribution rate of employees only, while those in Column 4 consider a formulation where the contributions of the employers are the margin of adjustment. In Columns 3 and 4 we consider a version of the model where the risky share of the pension fund is constant, while for the results in Column 5 the pension fund's asset demand responds to movements in interest rates. The asset pricing data is taken from CRSP. The mean and volatility of the real risk free is taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis, and we use the full annual sample from 1930 to 2018. Pension wealth data is also from the Federal Reserve Bank of St. Louis and we consider the 1996 value as described in the main text.

Table 3: Baseline and noDBPF Models: Comparison.

Variable	Moment	Models			Data
		Baseline	noDBPF	r-noDBPF	
r_f	Mean	1.14%	4.90%	1.18%	0.86%
r_f	St. Dev.	1.34%	1.46%	1.09%	1.35%
r^m	Mean	5.79%	9.86%	4.40%	8.17%
r^m	St. Dev.	13.67%	13.81%	13.61%	19.81%
$r^m - r_f$	Mean	4.65%	4.95%	3.22%	7.55%
$\frac{Mean(r^m - r_f)}{Std.Dev.(r^m - r_f)}$		0.34	0.37	0.24	0.34
Cons. growth (all)	St. Dev.	2.91%	1.86%	2.23%	2.90%
Cons. growth (A)	St. Dev.	3.42%	1.65%	1.43%	(-)
Cons. growth (B)	St. Dev.	2.49%	2.15%	3.14%	(-)
K	Mean	4.69	3.24	5.21	(-)
$Kprivate$	Mean	3.67	3.24	5.21	(-)
W^P/Y	Mean	0.74	0	0	0.67
K/Y	Mean	2.35	1.85	2.53	(-)
Participation (all)	Mean	52.3%	52.4%	52.2%	51.1%
Participation (A)	Mean	18.6%	22.6%	18.5%	(-)
Participation (B)	Mean	86.0%	82.2%	86.0%	(-)

Table 3 reports asset pricing moments, consumption volatility, stock market participation and the capital/output ratio for the baseline model (column 3), the noDBPF model for the same parameter values (column 4), the re-calibrated noDBPF ("r-noDBPF") model (column 5) and in the data (column 6). The asset pricing data is taken from CRSP. The mean and volatility of the real risk free is taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis, and we use the full annual sample from 1930 to 2018.

Table 4: Baseline Model: Comparative Statics.

Variable	Moment	Baseline Model: Calibrations			Data
		Baseline	α^P=0.8	Higher βs	
r_f	Mean	1.14%	1.72%	-2.57%	0.86%
r_f	St. Dev.	1.34%	1.36%	0.87%	1.35%
r^m	Mean	5.79%	5.78%	3.09%	8.17%
r^m	St. Dev.	13.67%	13.68%	13.59%	19.81%
$r^m - r_f$	Mean	4.65%	4.07%	5.66%	7.55%
$\frac{Mean(r^m - r_f)}{Std.Dev.(r^m - r_f)}$		0.34	0.30	0.42	0.30
Cons. growth (all)	St. Dev.	2.91%	2.94%	3.28%	2.90%
Cons. growth (A)	St. Dev.	3.42%	3.47%	4.04%	(-)
Cons. growth (B)	St. Dev.	2.49%	2.50%	2.56%	(-)
K	Mean	4.69	4.61	6.59	(-)
$Kprivate$	Mean	3.67	3.60	5.58	(-)
W^P/Y	Mean	0.74	0.75	0.66	0.67
K/Y	Mean	2.35	2.33	2.95	(-)
Participation (all)	Mean	52.3%	45.6%	69.9%	51.1%
Participation (A)	Mean	18.6%	9.0%	46.2%	(-)
Participation (B)	Mean	86.0%	82.2%	93.6%	(-)

Table 4 reports asset pricing moments, consumption volatility, stock market participation and the capital/output ratio for the baseline model for different parameter calibrations. Column 3 reports results for the baseline calibration, while the values in column 4 refer to a version of the model where the risky share of the pension fund (α^P) is set to 0.8. In column 5 the discount factors of the two types of agents are increased by 0.02 (hence we have β^A=0.859 and $\beta^B = 0.989$). The asset pricing data is taken from CRSP. The mean and volatility of the real risk free is taken from Croce et al. (2012). Stock market participation is computed from the Survey of Consumer Finances. The consumption data is taken from the NIPA tables provided by the Federal Reserve Bank of St. Louis, and we use the full annual sample from 1930 to 2018.

Table 5: Baseline Economy versus DC-only Economy

Variable	Moment	Baseline	DC-Only	
			Case I	Case II
r_f	Mean	1.14%	2.21%	2.50%
r_f	St. Dev.	1.34%	1.29%	1.72%
r^m	Mean	5.79%	5.98%	5.74%
r^m	St. Dev.	13.67%	13.69%	13.75%
$r^m - r_f$	Mean	4.65%	3.78%	3.24%
$\frac{Mean(r^m - r_f)}{Std.Dev.(r^m - r_f)}$		0.34	0.28	0.24
Cons. growth (all)	St. Dev.	2.91%	1.87%	2.23%
Cons. growth (A)	St. Dev.	3.42%	1.53%	1.75%
Cons. growth (B)	St. Dev.	2.49%	2.30%	2.78%
K	Mean	4.69	4.49	4.58
$Kprivate$	Mean	3.67	4.49	4.58
W^P/Y	Mean	0.74	0.0	0.0
K/Y	Mean	2.35	2.29	2.32
Participation (all)	Mean	52.3%	53.4%	55.2%
Participation (A)	Mean	18.6%	25.6%	27.8%
Participation (B)	Mean	86.0%	81.1%	82.6%

Table 5 reports asset pricing moments, consumption volatility, stock market participation and the capital/output ratio for the baseline economy (column 3), an alternative economy where the defined-benefit pension fund has been closed and a new equilibrium steady-state has been reached (column 4).

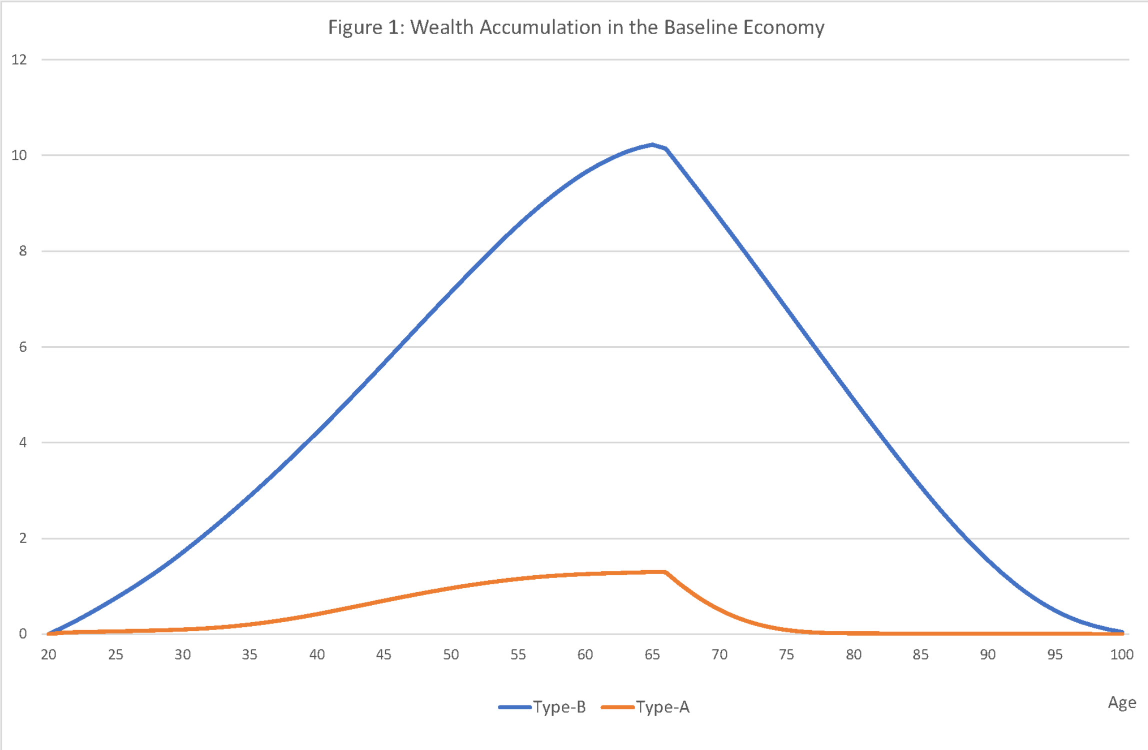

Figure 1 reports the average wealth accumulation over the life-cycle (ages 20 to 100) for both Type-A households and Type-B households, in the baseline economy.

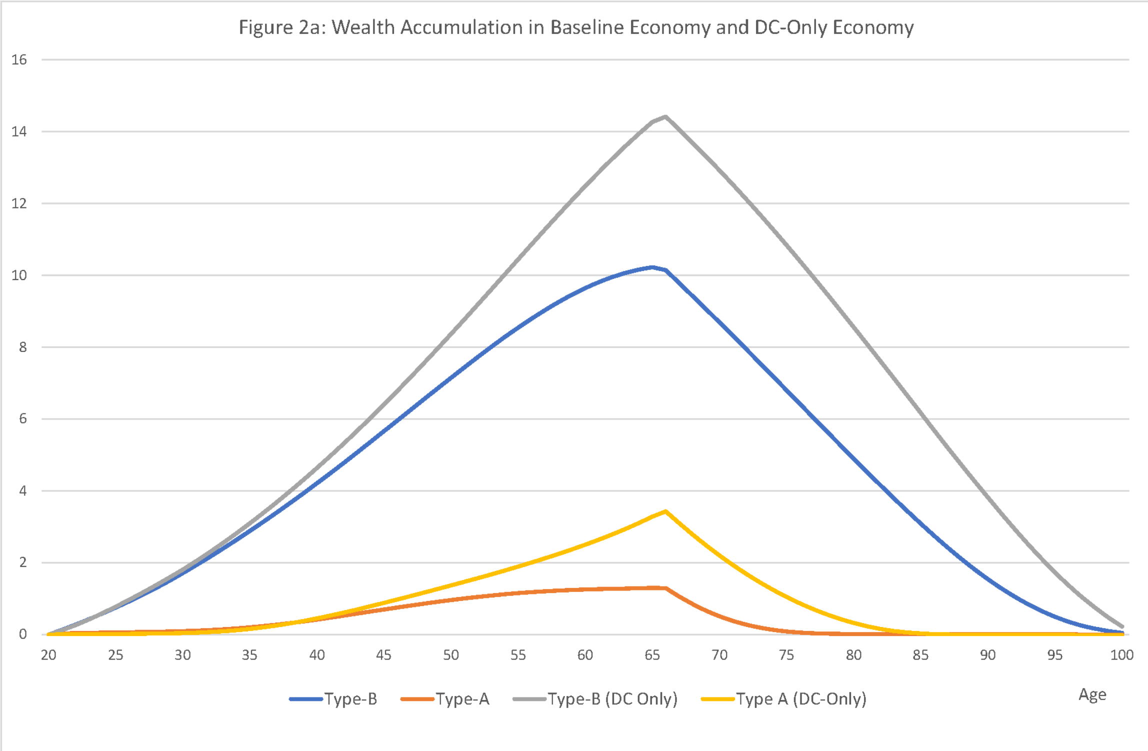

Figure 2a reports the average wealth accumulation over the life-cycle (ages 20 to 100) for both Type-A households and Type-B households, in the baseline economy and in the DC-only economy.

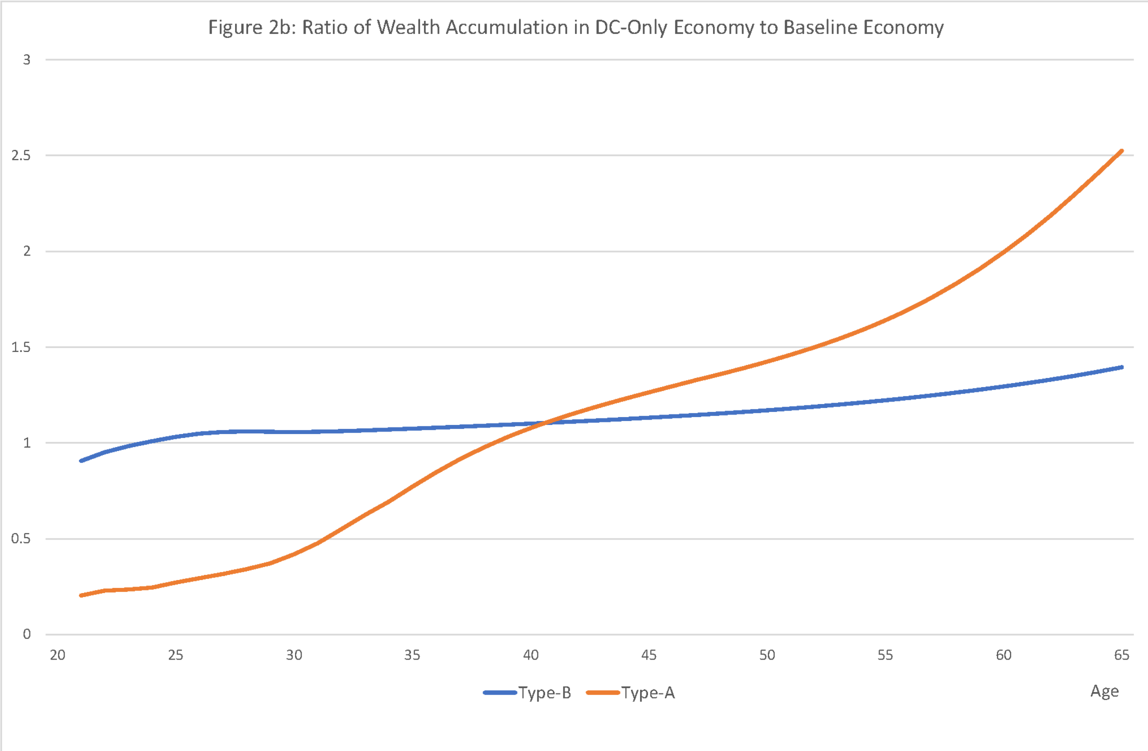

Figure 2b reports the ratio of average wealth accumulation over the life-cycle in the DC-only economy relative to the baseline economy before retirement (ages 20 to 65). Results are shown separately for the two types of agents, type-A and type-B.

Online Appendix

We follow a variant of the Krusell and Smith (1998) approach where households predict the next period capital stock using the first moment of the endogenously evolving wealth distribution. In our case we have two assets and we need to forecast the evolution of the bond price as well; the bond price is in turn clearing the government bond market every period.

There are seven state variables in this system; age (a), normalized cash on hand (x^i_{at}), the stock market participation status (E^i_a, a zero-one variable indicating whether the entry cost has been paid or not), and the four aggregate variables from the forecasting equations. The guess-and-verify equation for the log-capital stock is given by

$$log(k_{t+1}) = a_k + b_k \cdot log(k_t) \tag{39}$$

where each coefficient in the equation depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock. The guess-and-verify equation for the log-bond price is similarly given by

$$log(P^B_{t+1}) = a_P + b_P \cdot log(k_t) + c_P \cdot log(P^B_t) \tag{40}$$

where again each coefficient in the bond pricing function depends on the current realization of the aggregate productivity shock and the stochastic depreciation shock.

The full optimization problem is written as:

$$\begin{aligned} V_a(x^i_{at}, E^i_a; k_t, U_t, \eta_t, P^B_t) = & \underset{\{k^i_{a+1,t+1}, b^i_{a+1,t+1}\}^A_{a=1}}{Max} \{(1-\beta)(c^i_{at})^{1-1/\psi} \\ & +\beta(E_t[(\frac{P^i_{a+1,t+1}}{P^i_{at}}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V^{1-\rho}_{a+1}(x^i_{a+1,t+1}, E^i_{a+1}; k_{t+1}, U_{t+1}, \eta_{t+1}, P^B_{t+1})])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}}, \end{aligned} \tag{41}$$

subject to the constraints:

$$k^i_{a+1,t+1} \geq 0 \quad , \quad b^i_{a+1,t+1} \geq 0 \tag{42}$$

$$c^i_{at} + b^i_{a+1,t+1} + k^i_{a+1,t+1} = x^i_{at} \tag{43}$$

and

$$x^i_{a+1,t+1} = \begin{cases} \frac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)r^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{\left[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}\right]} & \\ +\varepsilon^i(1-\tau^s)w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} & a < a^R \\ \frac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)r^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau^K)r^B_{t+1})\right]}{\left[(P^i_{a+1,t+1}/P^i_{at})(1+g)^{\frac{1}{1-\alpha}}\right]} & \\ +(\lambda^{db}+\lambda^{ss})w_{t+1} - I^i_E F^0 w_{t+1} - I^i_S F^1 w_{t+1} & a > a^R \end{cases}, \quad (44)$$

the stochastic process for individual labor productivity, and the forecasting equations.

Cubic spline interpolations are used over the more dense individual wealth grids, while linear interpolation is sufficient for the less dense aggregate state interpolations (the next period capital stock and the bond price). Once the coefficients in the forecasting equations have converged in an outer loop, we check (i) that the relevant conditional R-2 results are very high (typically above 99.99 percent) and (ii) that all simulated individual and state variables are within the range assumed when setting the parameters of the model. Using a parallel grid (rather than a rectangular grid) for the bond pricing function as a function of the capital stock is also useful. The graph below shows the results from a simulation for the bond pricing function that illustrates the idea.